



Multiple View Geometry: Exercise Sheet 6

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Part I: Theory

1. The non-zero essential matrix $E = \hat{T}R$ with $T \in \mathbb{R}^3$ and $R \in \text{SO}(3)$ has the singular value decomposition $E = U\Sigma V^T$. Let $R_Z(\pm\frac{\pi}{2})$ be the rotation by $\pm\frac{\pi}{2}$ around the z -axis.

According to the lecture (Chapter 5, Slide 9), there exist exactly two options for (\hat{T}, R) :

$$(\hat{T}_1, R_1) = \left(UR_Z\left(+\frac{\pi}{2}\right) \Sigma U^T, \quad UR_Z\left(+\frac{\pi}{2}\right)^T V^T \right) \quad (1)$$

$$(\hat{T}_2, R_2) = \left(UR_Z\left(-\frac{\pi}{2}\right) \Sigma U^T, \quad UR_Z\left(-\frac{\pi}{2}\right)^T V^T \right) \quad (2)$$

Show that by using (1) and (2), the following properties hold:

- (a) $\hat{T}_1, \hat{T}_2 \in \text{so}(3)$ (i.e. \hat{T}_1, \hat{T}_2 are skew-symmetric matrices)
 - (b) $R_1, R_2 \in \text{SO}(3)$ (i.e. R_1, R_2 are rotation matrices)
2. Consider the matrices $E = \hat{T}R$ and $H = R + Tu^T$ with $R \in \mathbb{R}^{3 \times 3}$ and $T, u \in \mathbb{R}^3$. Show that the following holds:
- (a) $E = \hat{T}H$
 - (b) $H^T E + E^T H = 0$

3. Let $F \in \mathbb{R}^{3 \times 3}$ be the fundamental matrix for the cameras C_1 and C_2 . Show that the following holds for the epipoles e_1 and e_2 :

$$F e_1 = 0 \quad \text{and} \quad e_2^T F = 0$$

Part II: Practical Exercises

1. Download the package `ex06.zip` from the website and extract the images `batinria0.tif` and `batinria1.tif`.
2. **Get the 2D coordinates of corresponding point pairs:** Show the first image and mark at least 8 points. You can retrieve the pixel coordinates of mouse clicks with the command `[x, y] = ginput(gcf)`. Then show the second image and click at the corresponding points in the same order. Again you can get the pixel coordinates with `ginput`. Now you should have the 2D coordinates of corresponding point pairs.
3. **Implement the 8-point algorithm** from the lecture and run it with these point pairs. To this end, you have to transform the coordinates. The intrinsic camera matrices are:

$$K1 = \begin{pmatrix} 844.310547 & 0 & 243.413315 \\ 0 & 1202.508301 & 281.529236 \\ 0 & 0 & 1 \end{pmatrix}$$

$$K2 = \begin{pmatrix} 852.721008 & 0 & 252.021805 \\ 0 & 1215.657349 & 288.587189 \\ 0 & 0 & 1 \end{pmatrix}$$

4. Reconstruct the depths of the points as described in Chapter 5 on Slides 17 and 18.
5. Visualize 3D points and cameras to check whether the results are plausible. You may find the function `plotCamera` useful.

Hints:

- The file `additional_information.txt` provides $K1$ and $K2$
- Use `kron` and `reshape`
- If U or V of the SVD of E have a negative determinant, multiply them by -1 , respectively, such that both have a positive determinant.