



Multiple View Geometry: Solution Sheet 9

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Part I: Theory

1. Robust Least Squares

- (a) What situations can you think of where a robust loss function might be needed?
Missing (i.e. black or white) pixels in the image, dynamic changes of the scene, non-Lambertian surfaces like shiny or transparent objects, (local) changes in lighting conditions, ...

- (b) Write down the weight function for the Huber loss.

$$w_{\delta}(t) = \begin{cases} 1 & |t| \leq \delta \\ \frac{\delta}{|t|} & \text{else} \end{cases}$$

2. Optimization Techniques

Write down the update step $\Delta\xi$ for each of the following minimization methods:

- (a) Gradient descent, normal least squares,

$$\Delta\xi = -\lambda J^{\top} \mathbf{r}$$

- (b) Gradient descent, weighted least squares,

$$\Delta\xi = -\lambda J^{\top} W \mathbf{r}$$

- (c) Gauss-Newton, normal least squares,

$$\Delta\xi = -(J^{\top} J)^{-1} J^{\top} \mathbf{r}$$

- (d) Gauss-Newton, weighted least squares,

$$\Delta\xi = -(J^{\top} W J)^{-1} J^{\top} W \mathbf{r}$$

- (e) Levenberg-Marquardt, normal least squares,

$$\Delta\xi = -(J^{\top} J + \lambda \text{diag}(J^{\top} J))^{-1} J^{\top} \mathbf{r}$$

- (f) Levenberg-Marquardt, weighted least squares.

$$\Delta\xi = -(J^{\top} W J + \lambda \text{diag}(J^{\top} W J))^{-1} J^{\top} W \mathbf{r}$$

Note : for normal least square, updates are explained in chapter 7.

For weighted least square, we can use a change of variable to write the optimization problem exactly as a normal least square :

$$\sum_i w(r_i) r_i(\xi)^2 = \sum_i (\sqrt{w(r_i)} r_i(\xi))^2 = \sum_i \tilde{r}_i(\xi)^2 = \|\tilde{r}(\xi)\|^2$$

Then to get the new update steps, we just substitute r with $\tilde{r} = \sqrt{W}r$ and J with $\tilde{J} = \sqrt{W}J$ in the previous one.

(\tilde{J} is the jacobian of \tilde{r} , and \sqrt{W} is the matrix whose components are the square root of the components of W).