Multiple View Geometry: Exercise Sheet 7

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Part I: Theory

1. **Coimages of Points and Lines**

   Suppose $p_1, p_2 \in \mathbb{R}^3$ are two points on the line $L \subset \mathbb{R}^3$. Let $x_1, x_2 \in \mathbb{R}^3$ be the images of the points p_1, p_2 in homogeneous coordinates, respectively, and let $l \in \mathbb{R}^3$ be a vector that spans the coimage of the line L. All vectors are given in the image coordinate system.

   Furthermore suppose $L_1, L_2 \subset \mathbb{R}^3$ are two lines intersecting in the point $p \in \mathbb{R}^3$. Let $x \in \mathbb{R}^3$ be the image of the point p in homogeneous coordinates and let $l_1, l_2 \in \mathbb{R}^3$ be vectors that span the coimages of the lines L_1, L_2, respectively.

   Draw a picture and convince yourself of the following relationships:

   (a) Show that
   $$l \sim \hat{x_1} x_2, \qquad x \sim \hat{l_1} l_2,$$

   (b) Show that there exist $r, s, u, v \in \mathbb{R}^3$ such that,
   $$l_1 \sim \hat{x} u, \qquad l_2 \sim \hat{x} v, \qquad x_1 \sim \hat{l} r, \qquad x_2 \sim \hat{l} s$$

   where $\sim$ means equivalence in the sense of homogeneous coordinates.

2. **Rank Constraints**

   Let $x_1, x_2 \in \mathbb{R}^3$ be two image points in homogeneous coordinates with projection matrices $\Pi_1, \Pi_2 \in \mathbb{R}^{3\times 4}$. Show that the rank constraint
   $$\operatorname{rank}\begin{pmatrix} \hat{x_1}\Pi_1 \\ \hat{x_2}\Pi_2 \end{pmatrix} \leqq 3$$
   ensures that x_1 and x_2 are images (projections) of the same three-dimensional point X.

3. **Projection and Essential Matrix**

   Suppose two projection matrices $\Pi = [R, T]$ and $\Pi' = [R', T'] \in \mathbb{R}^{3\times 4}$ are related by a common transformation H of the form
   $$H = \begin{bmatrix} I & 0 \\ v^\top & v_4 \end{bmatrix} \in \mathbb{R}^{4\times 4} \quad \text{where } v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}.$$

   That is, $[R, T]H \sim [R', T']$ are equal up to scale.

   Show that Π and Π' give the same essential matrices ($E = \hat{T}R$ and $E' = \hat{T}'R'$) up to a scale factor.

Part II: Practical Exercises

Epipolar lines

1. Download the package `ex07.zip` from the website. Extract the images `batinria0.pgm` and `batinria1.pgm`. Their corresponding camera calibration matrices can be found in the file `calibration.txt`.

2. Show the two images with matlab and select a point in the first image. You can use the command `[x,y]=ginput(n)` to retrieve the image coordinates of a mouse click.

3. Think about where the corresponding epipolar line l_2 in the second image could be.

4. Now compute the epipolar line $l_2 = Fx_1$ in the second image corresponding to the point x_1 in the first image. To this end you will need to compute the fundamental matrix F between the two images. Use the calibration data from the file `calibration.txt`.

   Remark: Note that l_2 does not directly encode the epipolar line itself. Rather, l_1 is the coimage of the epipolar plane in the second coordinate system from which the epipolar line can be computed. This representation is chosen due to the easy formula shown above.

5. Test your program for different points x_1. What do you observe?

6. *Bonus: Determine the best matching point on the epipolar line via normalized cross correlation.*