Multiple View Geometry: Exercise Sheet 1

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Exercise: April 21st, 2021

Part I: Theory

1. Show for each of the following sets (1) whether they are linearly independent, (2) whether they span $\mathbb{R}^3$ and (3) whether they form a basis of $\mathbb{R}^3$:

   (a) $B_1 = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

   (b) $B_2 = \left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\}$

   (c) $B_3 = \left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}$

2. Which of the following sets forms a group (with matrix-multiplication)? Prove or disprove!

   (a) $G_1 := \left\{ A \in \mathbb{R}^{n \times n} | \det(A) \neq 0 \wedge A^\top = A \right\}$

   (b) $G_2 := \{ A \in \mathbb{R}^{n \times n} | \det(A) = -1 \}$

   (c) $G_3 := \{ A \in \mathbb{R}^{n \times n} | \det(A) > 0 \}$

3. Prove or disprove: There exist vectors $\mathbf{v}_1, ..., \mathbf{v}_5 \in \mathbb{R}^3 \setminus \{\mathbf{0}\}$, which are pairwise orthogonal, i.e.

$$\forall i, j = 1, ..., 5 : \quad i \neq j \implies \langle \mathbf{v}_i, \mathbf{v}_j \rangle = 0$$

Part II: Practical Exercises

1. Basic image processing

   (a) Download `lena.png`, provided in `ex01.zip`.

   (b) Load the image into the workspace.

   (c) Determine the size of the image and show the image.

   (d) Convert the image to gray scale and determine the maximum and the minimum value of the image.

   (e) Apply a gaussian smoothing filter and save the output image.

   (f) Show 1) the original image, 2) the gray scale image and 3) the filtered image in one figure and give the figures appropriate titles.

   (g) Compare the gray scale image and the filtered image for different values of the smoothing.

2. Basic operations

   (a) Let $A = \begin{pmatrix} 2 & 2 & 0 \\ 0 & 8 & 3 \end{pmatrix}$ and $b = \begin{pmatrix} 5 \\ 15 \end{pmatrix}$. Solve $Ax = b$ for x.

   (b) Define a matrix B equal to A.

   (c) Change the second element in the first row of A to 4.

   (d) Compute the following:
   $c = 0$;
   `for` $i \in \{-4, 0, 4\}$
   $\quad c = c + i * A^\top * b$
   `end`
   print c

   (e) Compare `A .* B` and `A' * B` and explain the difference.

3. Write a function `approxequal(`x, y, ϵ`)` checking if two vectors x and y are almost equal, i.e. if

   $$\forall i : \quad |x_i - y_i| \leq \epsilon \,.$$

   The output should be logical 1 or 0.

   If the input consists of two matrices, your function should compare the columns of the matrices if they are almost equal. In this case, the output should be a vector with logical values 1 or 0.

4. Write a function `addprimes(`s, e`)` returning the sum of all prime numbers between and including s and e.

   Use the Matlab-function `isprime`.