

Deep Learning Labwork 2

2440048 - Thien An NGUYEN

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1 Introduction

In this lab work, we focus on Linear Regression, where I implemented the algorithm in Python from scratch.

Linear Regression aims to model the relationship between inputs and outputs by fitting a straight line to the observed data. This line is represented by a linear equation. I will, of course, use Gradient Descent to update the weights. For this labwork, assume we only have 2 weights to update for the equation $y = w_0 + w_1x$

Linear Regression typically uses a Mean Square Error Cost Function. ($MSE = \frac{1}{N} \sum_{i=1}^N (y_i - (w_0 + w_1x_i))^2$ or scaled version $MSE = \frac{1}{2} * \frac{1}{N} \sum_{i=1}^N (y_i - (w_0 + w_1x_i))^2$). The gradient will be calculated for each w_0 and w_1 , and then they will be updated.

2 Implementation

2.1 The code

The linear regression function:

```
def linear_regression(x, y, F, lr, iter, threshold):
    assert len(x) == len(y), "length must be the same"
    W = [0, 0]
    loss_list = []
    weights_states = []

    loss = F(x, y, W)
    print(f"Initial loss: {loss}")
    loss_list.append(loss)
    print(f"Initial weights: {W}")

    for i in range(iter):
        # check if loss is less than threshold
        if loss < threshold:
```

```

        break
    # calculate gradient
    J_w0 = F_w0(x, y, W)
    J_w1 = F_w1(x, y, W)

    # update the weights
    W[0] = W[0] - lr * J_w0
    W[1] = W[1] - lr * J_w1

    # calculate loss
    loss = F(x, y, W)
    loss_list.append(loss)
    weights_states.append(W[:])

    return W, loss_list, weights_states

```

The cost function calculation

```

def F(x, y, W):
    J = 0
    for i in range(len(x)):
        J += (W[0] + W[1]*x[i] - y[i])**2
    J = J/(2*len(x))
    return J

```

Derivative or Gradient for w_0 and w_1

```

def F_w0(x, y, W):
    J_w0 = 0
    N = len(x)
    for i in range(len(x)):
        J_w0 += (W[0] + W[1]*x[i] - y[i])
    J_w0 = J_w0/N
    return J_w0

def F_w1(x, y, W):
    J_w1 = 0
    N = len(x)
    for i in range(len(x)):
        J_w1 += (W[0] + W[1]*x[i] - y[i])*x[i]
    J_w1 = J_w1/N
    return J_w1

```

Numeric version of all of the above:

```

def linear_regression_2(x, y, F, lr, iter, threshold):
    assert len(x) == len(y), "length must be the same"
    W = [0, 0]

```

```

loss_list = []
weights_states = []

loss = F(x, y, W)
print(f"Initial loss: {loss}")
loss_list.append(loss)
print(f"Initial weights: {W}")

for i in range(iter):
    # Check if loss is less than the threshold
    if loss < threshold:
        break
    # calculate gradient
    W_0_h = [W[0] + 1e-5, W[1]]
    W_1_h = [W[0], W[1] + 1e-5]

    W_0_h_ = [W[0] - 1e-5, W[1]]
    W_0_h_ = [W[0], W[1] - 1e-5]

    J_w0 = (F(x, y, W_0_h) - F(x, y, W_0_h_)) / 2e-5
    J_w1 = (F(x, y, W_1_h) - F(x, y, W_0_h_)) / 2e-5

    # update the weights
    W[0] = W[0] - lr * J_w0
    W[1] = W[1] - lr * J_w1

    # calculate loss
    loss = F(x, y, W)
    loss_list.append(loss)
    weights_states.append(W[:])

return W, loss_list, weights_states

```

2.2 Explanation

My code takes the following inputs:

- x: The inputs for a batch (here, since the data is small, so it's the whole input)
- F: The cost function (loss function) (Here is MSE)
- lr: The learning rate (or the step we want to take follows the Gradient.
- iter: Max number of iterations
- threshold: The threshold at which, if the changes after each step are not too big anymore, we stop.

In my linear regression function, the weights are first initialized as $[0,0]$, but we can modify this later and use it as an argument for the function. The loss is calculated, and then we apply the Gradient Descent. Here, I used the scaled version of MSE. For w_0 the analytic Gradient are pre-computed as $\frac{\partial J}{\partial w_0} = \frac{1}{N} \sum_{i=1}^N (w_0 + w_1 x_i - y_i)$ and w_1 is $\frac{\partial J}{\partial w_1} = \frac{1}{N} \sum_{i=1}^N (w_0 + w_1 x_i - y_i) x_i$. They will be updating my LR step towards the minimum. The numeric version is the same, but I calculated it using the central derivative formula.

3 Experiments

Here is the data that we are dealing with:

Input: $[10.0, 20.0, 40.0, 60.0, 80.0]$
Output: $[55.0, 80.0, 100.0, 120.0, 150.0]$

Where each input has a corresponding output.

We will use the $f(x) = x^2$ function for the experiment and experiment with the learning rate of 0.0001, 0.0003, and 0.001. With max iteration is 20.

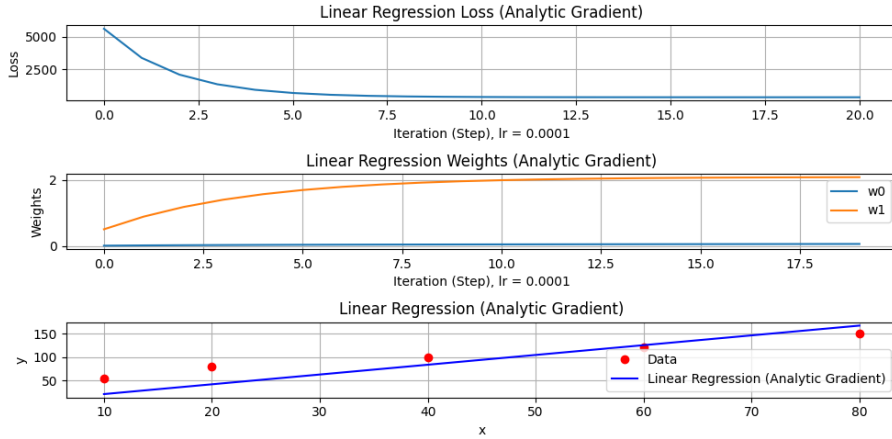


Figure 1: Linear Regression with Learning Rate 0.0001

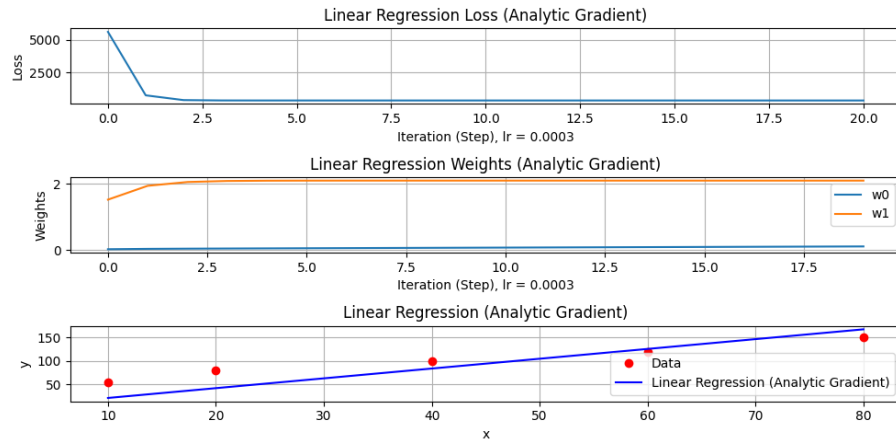


Figure 2: Linear Regression with Learning Rate 0.0003

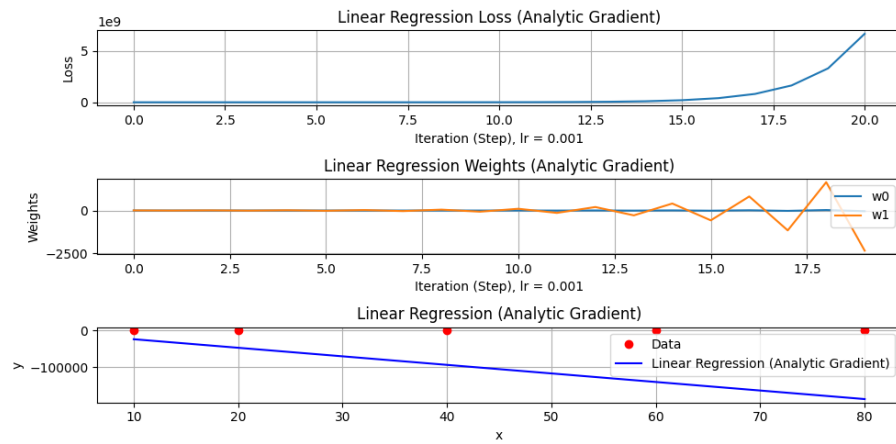


Figure 3: Linear Regression with Learning Rate 0.001

You can see that for the first two cases, the loss reduced and converged. The weights are updated accordingly. For the last case, it diverged since the learning rate was too high. These have a similar phenomenon to Gradient Descent. The line on the 2 cases fits quite well with the data, while the last case doesn't fit at all.