Shuffling cards

A remarkable phenomenon of a deck of 52 cards is that if you shuffle it perfectly 8 times, the deck will return to its orignial sequence. The phenomenon holds true for decks with any number of cards, but may require a different number of shuffles.

The general solution for this problem can be written as;

$$2^s \bmod (n-1) + 1 = 2$$

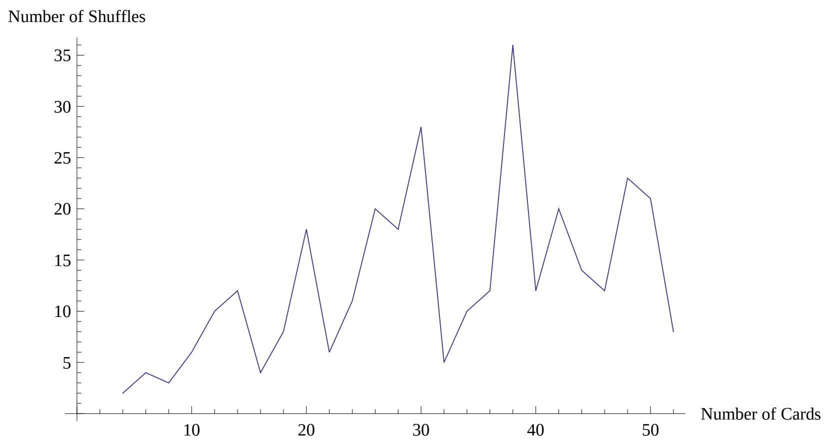
where n is the number of cards in the deck, and s is the number of shuffles required to return the deck to the orginal position. This intuitively makes since as the left-hand side of the equation describes the position of the original second card in the deck. When the LHS equals 2, the deck has returned to its orginal state. A modular function is required to deal with shuffles that place cards in positions higher than the number of cards in the deck by wrapping around the deck.

Solving for s yields the results;

$$s = \log_2((n-1)\,x + 1)$$

where x is the number of times the second card has wrapped around the deck.

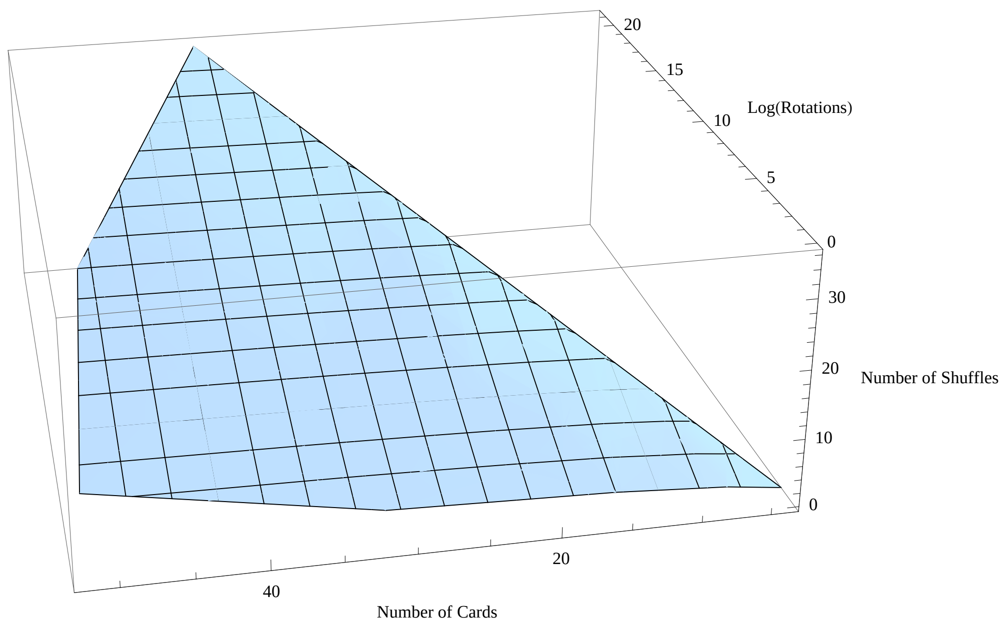
Finding integer solutions of n and s that are non-zero reveal the following pattern.



What an odd pattern. Though the number of shuffles needed to restore the deck seems to increase with the number of cards, the relationship is not very pretty.

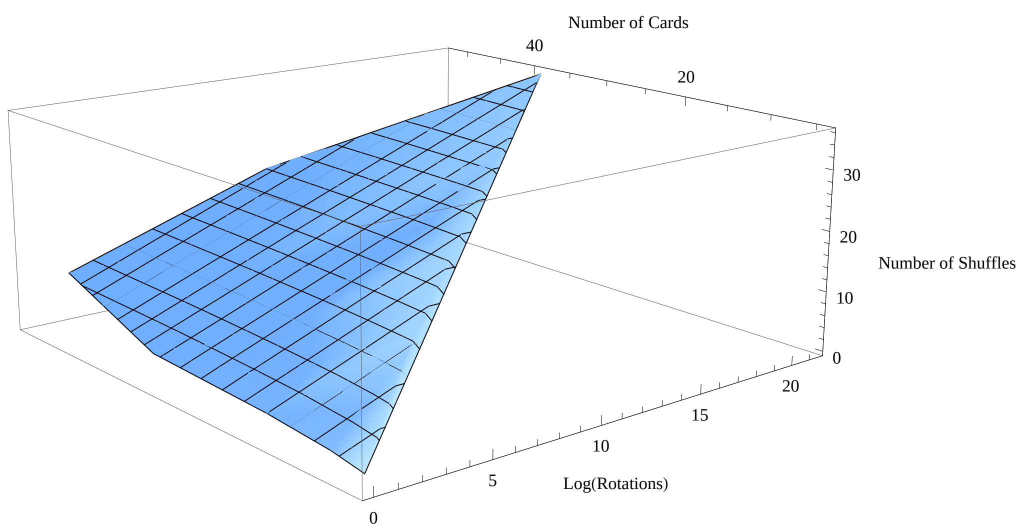
But lets not forget that the number of times a card wraps around the deck is an important component of this process. And observing the relationship between all three variables reveals remarkable symmetry.

Out[124]=



A view from another angle:

Out[125]=



And thus it is evident that as the number of cards in a deck increases, the number of shuffles required to restore the deck need not increase dramatically as long as the second card is lucky enough to find its orginal position before rotating around the whole deck too many times.

More thoughts may come of this...