Erratum to: Section Chapter 14.3.2, pp307–308 Benchmarks and Examples for Thermo-Hydro-Mechanical/Chemical Processes in Porous Media
The analytical solutions of thermal stress analysis of hollow cylinder

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Analytical solution

For the hollow cylinder with the inner radius $R1$ and the outer radius $R2$ the following analytical solution for radial displacement u_r, stress σ_r and temperature in dependency on the radius was used.

$$u_r = -\frac{q\,R_1\,\beta}{2\,\psi\,\kappa} r \left(\ln\, r - \frac{1}{2}\right) + \frac{A_0}{2} r + \frac{A_1}{r} \quad \text{sign } - \text{is missing in the book} \tag{14.18}$$

$$\begin{aligned}\sigma_r = {} & \psi \left[-\frac{q\,R_1\,\beta}{2\,\psi\,\kappa} \cancel{r} \left(\ln\, r + \frac{1}{2}\right) + \frac{A_0}{2} - \frac{A_1}{r^2}\right] \\ & + \lambda \left[-\frac{q\,R_1\,\beta}{2\,\psi\,\kappa} \cancel{r} \left(\ln\, r - \frac{1}{2}\right) + \frac{A_0}{2} + \frac{A_1}{r^2}\right] \quad \text{this } r \text{ has to be dropped} \\ & - \beta \left[\frac{R_1\,q}{\kappa} \ln \left(\frac{R_2}{r}\right) + T_0 \right] \end{aligned} \tag{14.19}$$

$$T(r) = \frac{R_1\,q}{\kappa} \ln\left(\frac{R_2}{r}\right) + T_0 \tag{14.20}$$

where

$$\psi = \lambda + 2\,G \qquad \text{and} \qquad \beta = \alpha\,(3\,\lambda + 2\,G)$$

with

λ – Lamé elastic constant
G – shear modulus
α – thermal expansion coefficient
κ – thermal conductivity
$A_0,\ A_1$ – integration constants

At the outer surface of the hollow cylinder (where $r = R_2$) there is no deformation, that means the displacement u_{R2} is zero. Therefore equation (14.18) is set equal to zero for this boundary and adapted to A_0.

$$A_0 = -\frac{2\,A_1}{R_2^2} - 2B\left(\ln\, R_2 - \frac{1}{2}\right) \tag{14.21}$$

where

$$B = \frac{q\,R_1\,\beta}{2\,\psi\,\kappa}$$

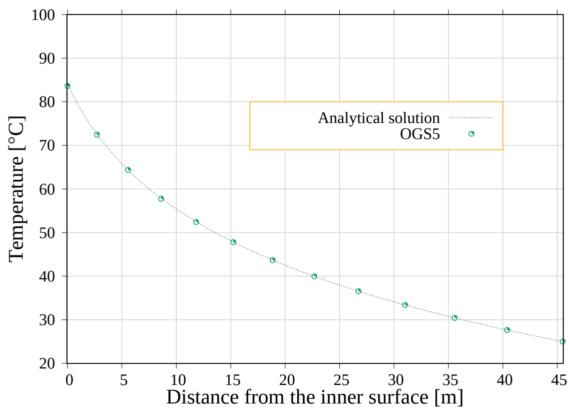
At the inner surface of the hollow cylinder (where $r = R_1$) no stress is effected by the expansion because this boundary is phreatic. Therefore equation (14.19) is set equal to zero and A_1 is calculated by using equation ().

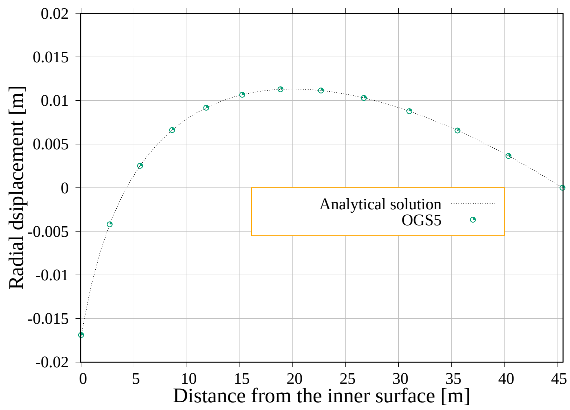
$$A_1 = \frac{\beta\left(\frac{R_1\,q}{\kappa}\ln\left(\frac{R_2}{r}\right)+T_0\right)+\lambda B\left(\ln R_1-\frac{1}{2}\right)+\psi B\left(\ln R_1+\frac{1}{2}\right)-\left(\frac{\lambda+\psi}{2}\right)2B\left(\ln R_2-\frac{1}{2}\right)}{\frac{\lambda-\psi}{R_1^2}-\frac{\lambda+\psi}{2}\cdot\frac{2}{R_1^2}}. \tag{14.22}$$

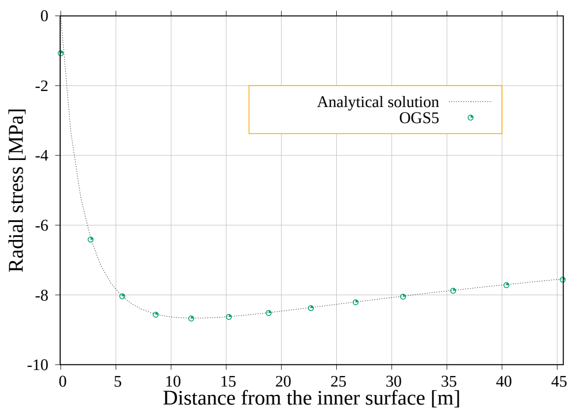
r should be R_1

After having solved this equation, A_1 is used to calculate A_0.

The solution figures in the book display the correct solutions as the following figures.







For verification of these analytical solutions, a C++ source code for computing the analytical solutions is provided via link:
https://github.com/wenqing/ExampleCollections/blob/master/ogs5/AnalyticalSolutionAxiTM/ana