Homework 1

Wilson Chen

August 27, 2025

Problem 1.1.8:

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 2 & -2 \end{bmatrix} \xrightarrow{R_3 \to \frac{1}{2} R_3} \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\therefore x_3 = -1$$

$$x_2 + 7x_3 = 0 \therefore x_2 = 7$$

$$x_1 + x_2 + 2x_3 = 0 \therefore x_1 = -5$$

$$\boxed{(x_1, x_2, x_3) = (-5, 7, -1)}$$

Problem 1.1.12:

$$\begin{bmatrix} 1 & -3 & 4 & -4 \\ 3 & -7 & 7 & -8 \\ -4 & 6 & 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 4 & -4 \\ 3 & -7 & 7 & -8 \\ -4 & 6 & 2 & 4 \end{bmatrix} \xrightarrow{R_3 \to R_1 + R_2} \begin{bmatrix} 1 & -3 & 4 & -4 \\ 3 & -7 & 7 & -8 \\ 0 & -4 & 13 & -8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 4 & -4 \\ 3 & -7 & 7 & -8 \\ 0 & -4 & 13 & -8 \end{bmatrix} \xrightarrow{R_2 \to R_2 + 3R_1} \begin{bmatrix} 1 & -3 & 4 & -4 \\ 0 & 2 & -5 & 4 \\ 0 & -4 & 13 & -8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 4 & -4 \\ 0 & 2 & -5 & 4 \\ 0 & -4 & 13 & -8 \end{bmatrix} \xrightarrow{R_3 \to R_3 + 2R_2} \begin{bmatrix} 1 & -3 & 4 & -4 \\ 0 & 2 & -5 & 4 \\ 0 & 0 & 3 & 0 \end{bmatrix}$$

$$\therefore x_3 = 0$$

$$2x_2 - 5x_3 = 4 \therefore x_2 = 2$$

$$x_1 - 3x_2 + 4x_3 = -4 \therefore x_1 = 2$$

$$\boxed{(x_1, x_2, x_3) = (2, 2, 0)}$$

Problem 1.1.20

$$\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ -2 & 3 & 2 & 1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ -2 & 3 & 2 & 1 & 5 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_4} \begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ -2 & 3 & 2 & 1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ -2 & 3 & 2 & 1 & 5 \end{bmatrix} \xrightarrow{R_4 \to R_4 + 2R_1 + R_3} \begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 3 & 3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 3 & 3 & 0 & 0 \end{bmatrix} \xrightarrow{R_4 \to R_4 - \frac{3}{2} R_2} \begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

This linear system is $\boxed{\text{consistent}}$ because the augmented matrix has no row of the form $[0 \ \cdots \ 0, \ b]$ where b is a non-zero number.

Problem 1.1.24

$$\begin{bmatrix} 1 & h & -3 \\ -2 & 4 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & h & -3 \\ -2 & 4 & 6 \end{bmatrix} \xrightarrow{R_1 \to R_1 + \frac{1}{2}R_2} \begin{bmatrix} 0 & 2h & 0 \\ -2 & 4 & 6 \end{bmatrix}$$

This linear system is consistent for $\boxed{h \in \mathbb{R}}$ because the augmented matrix will have no row of the form $[0 \quad \cdots \quad 0,\ b]$ where b is a non-zero number.

Problem 1.1.28
Elementary row operations on an augmented matrix never change the solution set of the associated linear system. (T/F)

This statement is $\boxed{\text{true}}$ as it is stated on the second paragraph of page 7.

Problem 1.1.30
Two matrices are row equivalent if they have the same number of rows (T/F)

This statement is $\boxed{\text{false}}$, since two matrices are row equivalent only if there is a sequence of elementary row operations that can transform one matrix into the other. (pg 7 par 1)

Problem 1.2.8

$$\begin{bmatrix} 1 & 4 & 0 & 7 \\ 2 & 7 & 0 & 11 \end{bmatrix} \xrightarrow{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 4 & 0 & 7 \\ 0 & -1 & 0 & -3 \end{bmatrix}$$

Pivot Columns: $1, 2$

Free Variables: x_3

$-x_2 + 0x_3 = -3 \therefore x_2 = 3$

$x_1 + 4x_2 + 0x_3 = 7 \therefore x_1 = -5$

$$\boxed{\begin{cases} x_1 = -5 \\ x_2 = 3 \\ x_3 \text{ is free} \end{cases}}$$

Problem 1.2.12

$$\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{bmatrix} \xrightarrow{R_3 \to R_1 + R_3} \begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Pivot Columns: $1, 3$

Free Variables: x_2, x_4

$x_1 - 7x_2 + 0x_3 + 6x_4 = 5 \therefore x_1 = 5 + 7x_2 - 6x_4$

$x_3 - 2x_4 + = -3 \therefore x_3 = -3 + 2x_4$

$$\boxed{\begin{cases} x_1 = 5 + 7x_2 - 6x_4 \\ x_2 \text{ is free} \\ x_3 = -3 + 2x_4 \\ x_4 \text{ is free} \end{cases}}$$

Problem 1.2.20a

a) is $\boxed{\text{consistent}}$ because the augmented matrix has no row of the form $[0 \quad \cdots \quad 0, \ b]$ where b is a non-zero number. However, as not every column has a pivot, this system is $\boxed{\text{not unique}}$.

Problem 1.2.20b

b) is $\boxed{\text{consistent}}$ because the augmented matrix has no row of the form $[0 \quad \cdots \quad 0, \ b]$ where b is a non-zero number. However, as not every column has a pivot, this system is $\boxed{\text{not unique}}$.

Problem 1.2.22

$$\begin{bmatrix} 1 & -3 & -2 \\ 5 & h & -7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & -2 \\ 5 & h & -7 \end{bmatrix} \xrightarrow{R_2 \to R_2 - 5R_1} \begin{bmatrix} 1 & -3 & -2 \\ 0 & h+15 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & -2 \\ 0 & h+15 & 3 \end{bmatrix} \xrightarrow{R_2 \to \frac{1}{h+15} R_2} \begin{bmatrix} 1 & -3 & -2 \\ 0 & 1 & \frac{3}{h+15} \end{bmatrix}$$

The matrix is the augmented matrix of a consistent linear system if $\boxed{h \neq -15}$, as $h = -15$ would result in a row of the form $[0 \ \ \cdots \ \ 0, \ b]$ where b is a non-zero number.

Problem 1.2.24a

$\boxed{(h,k) = (9,7)}$ will produce no solution, since the second row reduces to $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$, which is of the form $[0 \ \ \cdots \ \ 0, \ b]$ where b is a non-zero number.

Problem 1.2.24b

$\boxed{(h,k) = (5,6)}$ will produce one solution, since every column that is not augmented has a pivot.

Problem 1.2.24b

$\boxed{(h,k) = (9,6)}$ will produce infinite solutions, since not every column that is not augmented has a pivot.

Problem 1.2.26

The echelon form of a matrix is unique (T/F)

This statement is $\boxed{\text{false}}$. Only the row reduced echelon form of a matrix is unique, as defined by Theorem 1 on page 14.

Problem 1.2.28

The pivot positions in a matrix depend on whether row interchanges are used in the row reduction process (T/F)

This statement is $\boxed{\text{false}}$. As the book states, "the leading entries are always in the same positions in any echelon form obtained from a given matrix."