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Team INTERVAL Classroom Programme

CRASH COURSE

CLASS 11 PHYSICS

SHORT NOTES

INTERVAL
Individual Tuition Concept

CHAPTER 1

UNITS AND MEASUREMENTS

Physical quantities

The quantities which can be measured directly or indirectly are called physical quantities.

- A definite amount of physical quantity is taken as its standard unit.
- The standard unit should be easily reproducible, internationally accepted.

Fundamental units

Those physical quantities which are independent to each other are called fundamental quantities and their units are called fundamental units.

Derived units

Those physical quantities which are derived from fundamental quantities are called derived quantities and their units are called derived units.

System of units

A complete set of fundamental and derived units is called a system of unit.

- CGS System In this system, the unit of length is centimetre, the unit of mass is gram and the unit of time is second.
- FPS System In this system, the unit of length is foot, the unit of mass is pound and the unit of time is second.
- MKS System In this system, the unit of length is metre, the unit of mass is kilogram and the unit of time is second.

The international system of units is now accepted internationally and is called SI system. There are seven base units and two supplementary units in SI system. These units with their names and symbols are given below.

Quantities	Units	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	S
Temperature	kelvin	K
Electric current	ampere	A
Amount of substance	mole	mole
Luminous intensity	candela	Cd

Significant figures

In the measured value of a physical quantity, the number of digits about the correctness of which we are sure plus the next doubtful digit, are called the significant figures.

Rules to find significant figures

- All non-zero digits are significant
  Eg., 2.483 contains four significant figures.
- All zeroes appearing between two non-zero digits are significant
  Eg., 200.9 has four significant figures.
- The trailing zeroes in a number without a decimal point are insignificant
  Eg., in, 2304000. There are four significant figures only. The three zeroes appearing at the end are not significant.
- The trailing zeroes in a number having a decimal point are significant
  Eg., the number 308.600 has six significant figures.
- If a number is less than one, the zeroes on the left of the first non-zero digit are not significant

Eg., In the number 0.002783, there are four significant figures only. The three zeroes appearing in the beginning are not significant.

Rules for arithmetic operations with significant figures

- In multiplication or division, the computed result should not contain greater number of significant digits than in the observation which has the fewest significant digits.
- In addition or subtraction of given numbers, the same number of decimal places is retained in the result as are present in the number with minimum number of decimal places.

Rounding off the Uncertain Digits

- If the insignificant digit to be dropped is more than 5, the preceding digit is raised by 1. Let the insignificant digit in the number 3.78 be 8. Since 8 > 5, we raise the preceding digit 7 by 1. Hence, the number becomes 3.8.
- If the insignificant digit to be dropped is less than 5, the preceding digits is left unchanged. Let the insignificant digit in the number 3.74 be 4. Since 4 < 5, we keep the preceding digit 7 unchanged. Hence the number becomes 3.7.
- If the insignificant digit to be dropped is 5, the preceding digit is raised by 1 if it is odd, and is left unchanged if it is even. Let 5 be the insignificant digit in the numbers 3.745 and 3.775. In the first number, since the preceding digit 4 is even, it remains as such and the number becomes 3.74. In the second number, the preceding digit 7 is odd, hence it is raised by 1 and the number is written as 3.78.
- When a complex multi-step calculation is involved, all the numbers occurring in the intermediate steps should retain a digit more than the

significant digits present in them. The final answer at the end of the calculation, can then be rounded off to the appropriate significant figures.

- The exact numbers like π, 2, 3, 4 etc. that appear in formulae and are known to have infinite significant figures, can be rounded off to a limited number of significant figures as per the requirement.

Dimensions of physical quantities

Dimensions of any physical quantity are those powers which are raised on fundamental units to express its unit. The expression which shows how and which of the base quantities represent the dimensions of a physical quantity, is called the dimensional formula.

Dimensional formula of some physical quantities

Area $[L^2]$

Volume $[L^3]$

Velocity $[LT^{-1}]$

Acceleration $[LT^{-2}]$

Force $[MLT^{-2}]$

Applications of Dimensions

The concept of dimensions and dimensional formulae are put to the following uses:

- Checking the results obtained
- Conversion from one system of units to another
- Deriving relationships between physical quantities
- Scaling and studying of models.

Homogeneity Principle

If the dimensions of left hand side of an equation are equal to the dimensions of right hand side of the equation, then the equation is dimensionally correct. This is known as homogeneity principle.

Mathematically [LHS] = [RHS]

CHAPTER 2

MOTION IN A STRAIGHT LINE

- Kinematics is the branch of Physics which describes the motion of objects without going in to the causes of motion.
- Motion is change in position of an object with time.
- The motion along a straight line is called rectilinear motion.

Instantaneous velocity and speed

Instantaneous velocity gives the velocity of a particle at a particular instant of time. Thus the instantaneous velocity of a particle at a given instant of time is defined as the limit of average velocity as the time interval Δt, becomes infinitesimally small i.e.,

$$v = \lim_{\triangle t \to 0} \frac{\triangle x}{\triangle t} = \frac{dx}{dt}$$

Similarly instantaneous speed is the speed at a particular time.

Acceleration

Average acceleration of a particle is ratio of the change in velocity to the time interval.

Acceleration at any instant is called instantaneous acceleration.

$$\lim_{\triangle t \to 0} \frac{\triangle v}{\triangle t} = \frac{dv}{dt}$$

The acceleration at an instant is the slope of the tangent to the v–t curve at that instant.

Uniform Acceleration

A body is said to be in uniform acceleration if velocity changes equally in equal intervals of time.

The slope of velocity-time graph gives the acceleration of the object.

Kinematics equations for uniform accelerated motion

For objects in uniformly accelerated rectilinear motion, the five quantities, displacement s, time taken t, initial velocity u, final velocity v and acceleration a are related by a set of simple equations called kinematic equations of motion:

$$v = u + at$$

$$s = ut + \frac{1}{2} at^2$$

$$v^2 = u^2 + 2as$$

Velocity-time relation

Consider a body moving along a straight line with uniform acceleration 'a'. Let 'u' be initial velocity and 'v' be the final velocity at time t.

We know acceleration a = Change in velocity / Time interval

$$a = \frac{v - u}{t}$$

$$at = v - u$$

$$v = u + at$$

Position-Time Relation

Consider a body moving along a straight line with uniform acceleration a. Let 'u' be initial velocity and 'v' be the final velocity. 'S' is the displacement travelled by the body during the time interval 't'.

Displacement of the body during the time interval t,

S = average velocity × time

$$s = \left(\frac{v+u}{2}\right)t \ldots\ldots\ldots..(1)$$

$$\text{But}, v = u + at \ldots\ldots\ldots..(2)$$

Substitute eq (2) in eq (1), we get

$$s = \left(\frac{u+at+u}{2}\right)t$$

$$s = \left(\frac{2u+at}{2}\right)t$$

$$s = \frac{2ut+at^2}{2}$$

$$s = ut + \frac{1}{2}at^2$$

Position-Velocity Relation

$$s = \left(\frac{v+u}{2}\right)t \ldots\ldots\ldots..(1)$$

$$\text{But}, v = u + at$$

$$\frac{v-u}{a} = t \ldots\ldots\ldots..(2)$$

Substitute eq (2) in eq (1)

$$s = \left(\frac{v + u}{2}\right)\left(\frac{v - u}{a}\right)$$

$$s = \left(\frac{v^2 - u^2}{2a}\right)$$

$$2as = v^2 - u^2$$

$$2as = v^2 - u^2$$

$$v^2 = u^2 + 2as$$

CHAPTER 3

MOTION IN A PLANE

Scalars and Vectors

A quantity which has only magnitude and no direction is called a scalar quantity. A physical quantity which has both magnitude and direction is called a vector quantity.

Different Types of Vectors

- Two vectors of equal magnitude, in same direction are called equal vectors.
- Two vectors of equal magnitude but in opposite directions are called negative vectors.
- A vector whose magnitude is zero is known as a zero or null vector. Its direction is not defined. It is denoted by 0.
- A vector having unit magnitude is called a unit vector.

Multiplication of Vectors by Real Numbers

When a vector $\vec{A}$ is multiplied by a real number n, the quantity obtained is a vector $n\vec{A}$ whose magnitude is n times that of the original vector. $|n\vec{A}| = n|\vec{A}|$. Its direction might be the same or opposite to that of the original vector depending upon whether n is positive or negative.

- If n is a positive number, $n\vec{A}$ and $\vec{A}$ have the same direction.

- If n is a negative number, $n\vec{A}$ and $\vec{A}$ have opposite directions.
- If n is zero, the magnitude of $n\vec{A}$ is also zero. Such a vector, whose magnitude is zero is called a zero vector or a null vector and is denoted by $\vec{0}$. Since the magnitude of a null vector is zero, its direction cannot be specified.
- If n is a scalar quantity rather than just being a pure number, then the dimension of $n\vec{A}$ is the product of dimensions of n and A.

Addition of vectors

Triangle law of vectors

If two vectors acting at a point are represented in magnitude and direction by the two sides of a triangle taken in one order, then their resultant is represented by the third side of the triangle taken in the opposite order.

Parallelogram law of vectors

If two vectors acting at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram draw from a point, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram draw from the same point.

Properties of vector addition

- Vector addition is commutative

  A + B = B + A
- Vector addition is associative

  A + (B + C) = B + (C + A) = C + (A + B)

- Vector addition is distributive

  m (A + B) = m A + m B

Scalar or Dot product of two vectors

The scalar product of two vectors is equal to the product of their magnitudes and the cosine of the smaller angle between them.

$$A.B = AB \ Cos\theta$$

- Scalar product is commutative
- Scalar product is distributive
- Scalar product of two perpendicular vectors is zero

Vector or Cross product of two vectors

The vector product of two vectors is equal to the product of their magnitudes and the sine of the smaller angle between them.

$$AxB = AB \ Sin\theta$$

- Vector product is not commutative
- Vector product is distributive
- Vector product of two parallel vectors is zero

Resolution of vectors

If any vector A subtends an angle θ with x-axis,

then its horizontal component, $A_x = A \cos\theta$

Vertical component, $A_y = A \sin\theta$

Magnitude of vector $A = \sqrt{A_x^2 + A_y^2}$

$\tan\theta = A_y / A_x$

Motion in a plane

Position vector,

$$\vec{r} = x\hat{i} + y\hat{j}$$

Displacement vector,

$$\overrightarrow{\triangle r} = \triangle x\hat{i} + \triangle y\hat{j}$$

Velocity,

$$\bar{v} = \frac{\triangle r}{\triangle t} = \frac{\triangle x\hat{i} + \triangle y\hat{j}}{\triangle t}$$

$$\text{Or, } \bar{v} = v_x\hat{i} + v_y\hat{j}$$

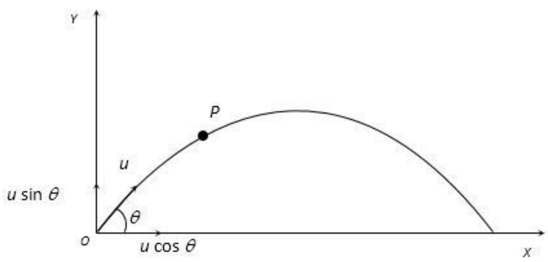
Acceleration,

$$\bar{a} = \frac{\triangle v}{\triangle t} = \frac{\triangle v\hat{i} + \triangle v\hat{j}}{\triangle t}$$

$$\text{Or, } \bar{a} = a_x\hat{i} + a_y\hat{j}$$

Projectile motion

When a particle is thrown obliquely near the surface of earth it moves along a curved path (known as parabolic path). Such a particle is called a projectile and its motion is called projectile motion.



Equation of path of a projectile

$$y = x\tan\theta - \frac{g}{2(u\cos\theta)^2}x^2$$

Time of Flight (T)

The time taken by a projectile to return its initial elevation after projection is known as time of flight. It is denoted by (T) and given by

$$T = \frac{2u\sin\theta}{g}$$

Maximum Height Attained

The maximum vertical height traveled by the projectile during its journey is called the maximum height attained by the projectile. It is denoted by H_{max} and given by

$$H_{max} = \frac{u^2\sin^2\theta}{2g}$$

Horizontal Range

The maximum horizontal distance between the points of projection and the point of horizontal plane where the projectile hits is called horizontal range. It is denoted by R and give by

$$R = \frac{u^2\sin 2\theta}{g}$$

The range of projectile will be maximum if $\theta = 45^0$

Uniform circular motion

The motion of an object along the circumference of a circle is called circular motion. When an object follows a circular path at a constant speed, the motion is called uniform circular motion.

- The time taken by the object to complete one full revolution is called the period.
- The number of revolutions completed per second is called the frequency of the circular motion.

$$\nu = 1/T$$

- The angle in radians swept out by the radius vector in a given interval of time is called the angular displacement of the object.
- The rate of change of angular displacement is called the angular velocity.

$$\omega = \frac{d\theta}{dt}$$

CHAPTER 4

LAWS OF MOTION

Inertia

The inability of a body to change by itself it's state of rest or uniform motion along a straight line is called inertia.

Newton's first law of motion

A body continue to be in its state of rest or of uniform motion along a straight line, unless it is acted upon by some external force to change the state.

Momentum

It is defined as the quantity of motion contained in a body. It is measured as the product of mass of the body and its velocity and has the same direction as that of the velocity. It is a vector quantity. It is represented by p. The SI unit of momentum is kg m/s.

$$p = mv$$

Newton's second law of motion

The rate of change of linear momentum of a body is directly proportional to the external force applied on the body and this change takes place always in the direction of the applied force.

Mathematical Formulation of Second Law

Consider a body of mass m moving with some initial velocity v. If an unbalanced force F is applied, the velocity will change from v to v + Δv. The change in momentum will be $\Delta p = m\Delta v$ which changes from p (initial momentum) to p + Δp (final momentum).

According to the second law,

$$F \propto \frac{dP}{dt}, \qquad F = \frac{dP}{dt}$$

The momentum of a body is defined as,

$$P = mv$$

$$\frac{dP}{dt} = \frac{d(mv)}{dt}$$

$$F = m\frac{dv}{dt}$$

$$F = ma$$

Application of Newton's Second Law of Motion

- Cricket player lowers his hand while catching the ball: The player increases the time during which the high velocity of moving ball reduces to zero. If we increase t, F decreases, so force of impact on palm of the fielder reduces.
- A karate player can break a pile of tiles with a single blow of his hand: Because he strikes the pile of tiles with his hand very fast, during which the entire momentum of the fast moving hand is reduced to zero in very short interval of time. This exerts a very large force on the pile of tiles which is sufficient to break them, by a single blow of his hand.

- In a high jump athletic event, the athletes are allowed to fall either on a sand bed or cushioned bed: This is because to increase the time of athletes fall to stop after making the high jump, which decreases rate of change of momentum and decreases force of impact.

Impulse

The product of force and time, which is the change in momentum of the body remains a measurable quantity. This product is called impulse.

Impulse = Force × time duration

= Change in momentum

Large force acting for a short time is called impulsive force.

Newton's third law of motion

To every action, there is always an equal and opposite reaction.

Applications of Third Law

Recoiling of a gun: When a bullet is fired from a gun, it exerts a forward force on the bullet and the bullet exerts an equal and opposite force on the gun. Due to high mass of the gun, it moves a little distance backward and gives a backward jerk to the shoulder of the gunman.

To walk, we press the ground in backward direction with foot: When we walk on the ground, our foot pushes the ground backward and in return the ground pushes our foot forward.

Conservation of momentum

When there is no external force on a body (or system), the total momentum remains constant or the total momentum of an isolated system of interacting particles is conserved.

Consider two objects A and B of masses m_1 and m_2 moving along the same direction at different velocities u_1 and u_2 respectively.

$$m_1u_1 + m_2u_1 = m_1v_1 + m_2v_2$$

Total momentum before collision = Total momentum after collision

Equilibrium of a particle

Equilibrium of a particle in mechanics refers to the situation, when the net external force on the particle is zero.

Friction

Friction is the force that develops at the surfaces of contact of two bodies and opposes their relative motion.

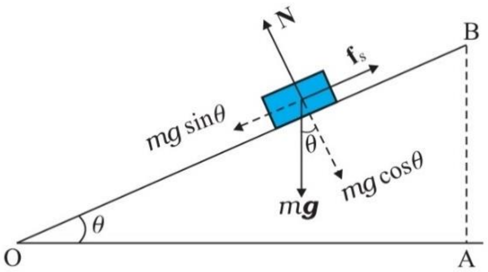
Types of friction

- Static friction: The opposing force that comes into play when one body tends to move over the surface of another (but the actual motion has yet not started)
  The magnitude of static frictional force is also proportional to normal force.
  $f_s = \mu_s N$

- Limiting friction: The maximum value of static friction is called limiting friction.
- Kinetic friction or dynamic friction: Kinetic friction or dynamic friction is the opposing force that comes into play when one body is actually moving over the surface of another body.
  The magnitude of the kinetic friction force f_k is found experimentally to be approximately proportional to the magnitude N of the normal force. In such cases, we represent the relationship by the equation
  $f_k = \mu_k N$
  where μ_k is a constant called the coefficient of kinetic friction.
- Sliding friction: The opposing force that comes into play when one body is actually sliding over the surface of the other body is called sliding friction.

Angle of Repose (θ)

It is the maximum angle of inclination (θ) of a rough inclined plane with horizontal such that the block kept on it remains at rest.



At angle of repose,

$$\text{Driving force} = \text{Limiting friction}$$

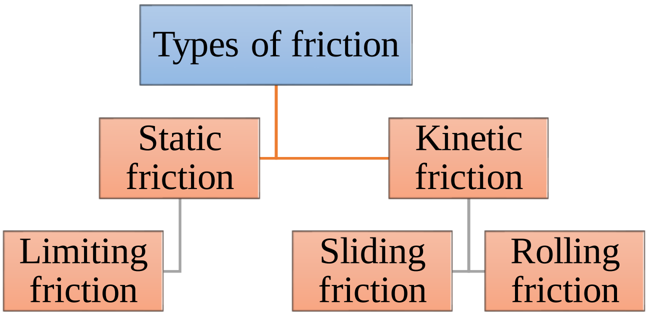
$$mg \sin\theta = \mu_s N$$

$$mg \sin\theta = \mu_s\, mg \cos\theta$$

$$\tan\theta = \mu_s s$$

Angle of friction = Angle of repose

- Rolling friction: The opposing force that comes into play when one body is actually rolling over the surface of the other body is called rolling friction.



Advantages of friction

- Walking is possible due to friction.
- Two body sticks together due to friction.
- Brake works on the basis of friction.
- Writing is not possible without friction.
- The transfer of motion from one part of a machine to other part through belts is possible by friction.

Disadvantages of friction

- Friction always opposes the relative motion between any two bodies in contact. Therefore extra energy has to be spent in overcoming friction. This reduces the efficiency of machine.
- Friction causes wear and tear of the parts of machinery in contact. Thus their lifetime reduces.

- Frictional force result in the production of heat, which causes damage to the machinery.

Circular motion

When a body moves along the circumstances of a circle, there is an acceleration towards it's centre. This acceleration is called centripetal acceleration. The force providing this acceleration is called centripetal force.

$$\text{Centripetal force, f} = \frac{mv^2}{R}$$

CHAPTER 6

WORK, ENERGY AND POWER

The scalar product

The scalar product or dot product of any two vectors A and B, denoted as A.B is defined as

$$A.B = A\,B\cos\theta$$

Where θ is the angle between the two vectors A and B.

Properties of scalar product

- The scalar product follows the commutative law

$$\vec{A}.\vec{B} = \vec{B}.\vec{A}$$

- Scalar product obeys the distributive law

$$\vec{A}.\left(\vec{B} + \vec{C}\right) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$$

- $\vec{A}.\left(\lambda\vec{B}\right) = \lambda\left(\vec{A}.\vec{B}\right)$
- $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$
- $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$
- $\vec{A}.\vec{A} = AA\cos 0 = A^2$

The work-energy theorem

The change in kinetic energy of a particle is equal to the work done on it by the net force.

Proof:

We know $v^2 = u^2 + 2as$

$v^2 - u^2 = 2as$

Multiplying both sides with m/2; we get

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas$$

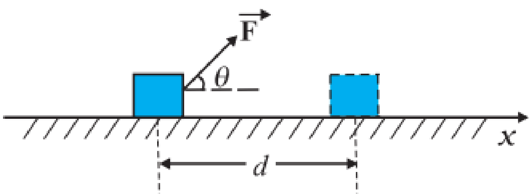
$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = Fs$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = W \qquad [\text{Since } W = Fs]$$

$$k_f - k_i = W$$

Work

The work done by the force is defined to be the product of component of the force in the direction of the displacement and the magnitude of this displacement.



Consider a constant force $\vec{F}$ acting on an object of mass m. The object undergoes a displacement d in the positive x-direction as shown in the figure. The projection of $\vec{F}$ on d is $F\cos\theta$.

Hence work done $w = F\cos\theta.\ d$

$W = Fd\cos\theta$

$$W = F.d$$

No work is done if:

- The displacement is zero
- The force is zero
- The force and displacement are mutually perpendicular ($\theta = \pi/2$)

There are three types of workdone

- Positive workdone:

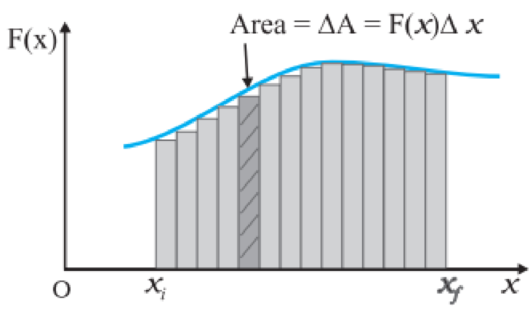
  Work will be positive, if the displacement has a component in the direction of the force. The angle between force and displacement is zero for positive workdone.

  When q = 0 w = Fd
- Negative workdone:

  Work will be negative, if the displacement has a component opposite to the force F. The angle between force and displacement lies between 90° and 180°.
- Zero work done:

  Work will be zero, if there is no component along the direction of force. The angle between applied force and displacement is 90°.

Workdone by a Variable Force



If the displacement Δx is small, we can take the force F (x) as approximately constant and the work done is then

$$\Delta W = F(x)\Delta x$$

$$W = \int_{x_1}^{x_2} F(x)\Delta x$$

In the limit Δx tends to zero

$$W = \int_{x_1}^{x_2} F(x)dx$$

Kinetic Energy

Kinetic energy is the energy possessed by the body because of it's motion. The kinetic energy of a body of mass m and velocity v,

$$K.E = \frac{1}{2}mv^2$$

Work Energy Theory for a Variable Force

Work energy theorem for a variable force can be derived from work-energy theorem of constant force. According work energy theorem for constant force, Change in kE = work done

dk = dw

dk = F dx

Integrating from the initial position (x_i) to (x_f) we get

$$\int_{k_i}^{k_f} dk = \int_{x_i}^{x_f} F dx$$

$$[K]_{k_i}^{k_f} = \int_{x_i}^{x_f} F dx$$

$$k_f - k_i = \int_{x_i}^{x_f} F dx$$

Potential Energy

Potential energy is energy stored in a system as a result of internal forces that depend on the position of interacting objects in the system.

Consider a mass ‘m’ on the surface of the earth. If this mass is raised to height ‘h’ against force of gravity,

Work done w = Force × displacement

w = mg × h

w = mgh

This work gets stored as gravitational potential energy.

ie; Gravitational energy V = mgh.

Relation between gravitational potential and gravitational force

If we take negative of the derivative of V(h) with respect to height (h), we get

$$-\frac{dV(h)}{dh} = -\frac{d}{dh}(mgh)$$

$$-\frac{dV(h)}{dh} = -mg$$

$$-\frac{dV}{dh} = F$$

The gravitational force is the negative derivative of the gravitational potential.

Relation between kinetic energy and gravitational potential energy

PE at a height h, V = mgh

When the object is released from a height it gains KE

$$K = \frac{1}{2}mv^2$$

$$v^2 = u^2 + 2as$$

$$u = o, a = g, v^2 = 2gh$$

$$K = \frac{1}{2}mx2gh$$

$$K = mgh$$

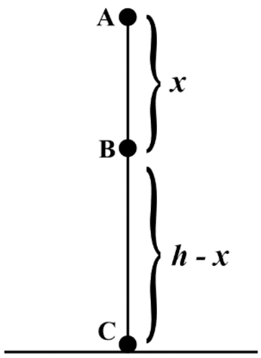
$$\text{Kinetic energy} = \text{ Potential energy}$$

Properties of conservative force

- A force is conservative, if it can be derived from a scalar quantity (ie F = – dV/dh)
- The workdone by the conservative force depends only on the endpoints.
- The workdone by conservative force in a closed path is zero.

The Conservation of Mechanical Energy

The principle of conservation of total mechanical energy can be stated as, the total mechanical energy of a system is conserved if the forces, doing work on it, are conservative.



Consider a body of mass 'm' at a height H from the ground.

- At point A

  Potential energy at A,

  PE = mgh

  Kinetic energy, KE = ½ mv^2 = 0

  (Since the body at rest, v = 0).

  ∴ Total mechanical energy = PE + KE = mgh + 0 = mgh ……….. (1)

- At point B

  The body travels a distance x when it reaches B. The velocity at B, can be found using the formula.

  $v^2 = u^2 + 2as$

  $v^2 = 0 + 2\ gx$

  ∴ KE at B, = ½ mv^2 = ½ m2gx

  = mgx

P.E. at B, = mg (h – x)

Total mechanical energy = PE + KE

= mg (h – x) + mgx = mgh ……….. (2)

- At point C

Velocity at C can be found using the formula

$v^2 = u^2 + 2as$

$v^2 = 0 + 2gh$

∴ KE at C, = ½ mv^2 = ½ m2gh = mgh

P.E. at C = 0

Total energy = PE + KE

= 0 + mgh = mgh ……….. (3)

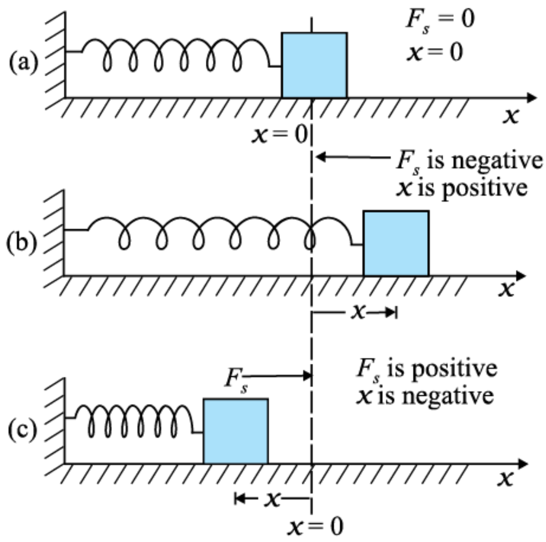
From eqns (1), (2) and (3), it is clear that the total mechanical energy is conserved during the free fall.

The potential energy of a spring

Hooks law: The restoring force developed in the spring is proportional to the displacement x and it is opposite to the displacement,

$$F = -kx$$

Where k is a constant called the spring constant.



Consider a block of mass m attached to a spring and resting on a smooth horizontal surface. The other end of the spring is attached to a rigid wall. Let the spring be pulled through a distance x.

Then the spring force F = − kx

The work done by the spring force is

$$W = \int_0^x F dx$$

$$W = -\int_0^x kx\, dx$$

$$W = -\frac{1}{2}kx^2$$

This work is stored as potential energy of spring

$$P.E = \frac{1}{2}kx^2$$

Energy of an oscillating spring at any point

If the block of mass 'm' (attached to massless spring) is extended to x_m and released, it will oscillate in between $+x_m$ and $-x_m$. The total mechanical energy at any point x, (lies between $-x_m$ and $+x_m$) is

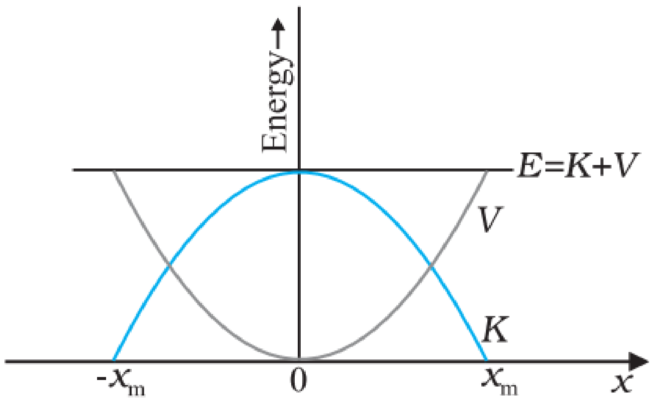
$$\frac{1}{2}kx_m{}^2 = \frac{1}{2}kx^2 + \frac{1}{2}kv^2$$

This block mass 'm' has maximum velocity at equilibrium position (x = 0). At this position, the potential energy stored in a spring is completely converted in to kinetic energy.

$$\frac{1}{2}kx_m^2 = \frac{1}{2}mv_m^2$$

$$V_m = \sqrt{\frac{k}{m}}x_m$$

Graphical variation of energy



Power

Power of a body is defined as the rate at which the body can do the work.

$$P = \frac{W}{t}$$

The instantaneous power

The instantaneous power is defined as the limiting value of the average power as time interval approaches zero.

$$P = \frac{dW}{t}$$

The work dW done by a force F for a displacement dr is

$$dW = F.dr$$

$$P = F.\frac{dr}{dt}$$

$$P = F.v$$

Where v is the instantaneous velocity when the force is F

Collisions

The total energy and total linear momentum are conserved during all type of collisions.

- Elastic collision

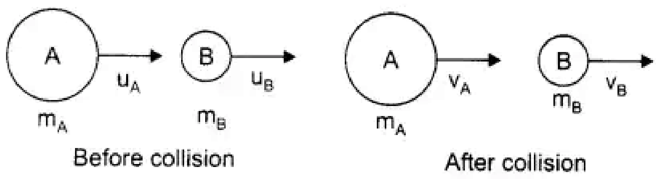
  Elastic collision is one in which both momentum and kinetic energy are conserved.

- Inelastic collision

  Inelastic collision is one in which the momentum is conserved, but KE is not conserved.

Collisions in One Dimension

If the initial velocities and final velocities of both the bodies are along the same straight line, then it is called a one-dimensional collision, or head-on collision.



Consider two masses m_1 and m_2 making elastic collision in one dimension. By the conservation of momentum.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \ldots\ldots\ldots\ldots (1)$$

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \ldots\ldots\ldots\ldots (2)$$

By the conservation of kinetic energy

$$\frac{1}{2}m_1{u_1}^2 + \frac{1}{2}m_2{u_2}^2 = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 \ldots\ldots\ldots\ldots (3)$$

$$\frac{1}{2}m_1{u_1}^2 - \frac{1}{2}m_1{v_1}^2 = \frac{1}{2}m_2{v_2}^2 - \frac{1}{2}m_2{u_2}^2$$

$$\frac{1}{2}m_1({u_1}^2 - {v_1}^2) = \frac{1}{2}m_2({v_2}^2 - {u_2}^2)$$

$$m_1({u_1}^2 - {v_1}^2) = m_2({v_2}^2 - {u_2}^2) \ldots\ldots\ldots\ldots (4)$$

$$\frac{\text{eqn (4)}}{\text{eqn (2)}}$$

$$\frac{m_1({u_1}^2 - {v_1}^2)}{m_1(u_1 - v_1)} = \frac{m_2({v_2}^2 - {u_2}^2)}{m_2(v_2 - u_2)}$$

$$\frac{({u_1}^2 - {v_1}^2)}{(u_1 - v_1)} = \frac{({v_2}^2 - {u_2}^2)}{(v_2 - u_2)}$$

$$u_1 + v_1 = v_2 + u_2 \ldots\ldots\ldots\ldots (5)$$

$$u_1 - u_2 = -(v_1 - v_2) \ldots\ldots\ldots\ldots (6)$$

ie, relative velocity before collision is numerically equal to relative velocity after collision.

From eqn (5)

$$v_2 = u_1 + v_1 - u_2$$

Substituting in eqn (1)

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2(u_1 + v_1 - u_2)$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2u_1 + m_2v_1 - m_2u_2$$

$$m_1u_1 + m_2u_2 - m_2u_1 + m_2u_2 = m_1v_1 + m_2v_1$$

$$(m_1 - m_2)u_1 + 2m_2u_2 = (m_1 + m_2)v_1$$

$$v_1 = \frac{(m_1 - m_2)u_1}{(m_1 + m_2)} + \frac{2m_2u_2}{(m_1 + m_2)} \ldots\ldots\ldots\ldots (7)$$

Similarly, $$v_2 = \frac{(m_2 - m_1)u_2}{(m_1 + m_2)} + \frac{2m_1u_1}{(m_1 + m_2)} \ldots\ldots\ldots\ldots (8)$$

Case 1: If two masses are equal, $m_1 = m_2 = m$

Substituting in eqns (7) and (8)

$$v_1 = \frac{2mu_2}{2m} = u_2$$

$$v_2 = \frac{2mu_1}{2m} = u_1$$

ie., the bodies will exchange their velocities

Case 2: If one mass dominates, $m_2 >> m_1$ and $u_2 = 0$

$m_1 + m_2 = m_2$ and $m_1 - m_2 = -m_2$

$$v_1 = \frac{(m_1 - m_2)u_1}{(m_1 + m_2)} = \frac{m_2u_1}{m_2} = -u_1$$

$$v_2 = \frac{2m_1u_1}{m_1 + m_2} = \frac{2 \text{ x } 0 \text{ x } u_1}{m_2} = 0$$

(Since m_1 is very small, it can be neglected)

The heavier mass comes to rest while the lighter mass reverses its velocity.

Elastic Collisions in Two Dimensions

Consider the elastic collision of a moving mass m_1 with the stationary mass m_2.
Since momentum is a vector, it has 2 equations in x and y directions.

Equation for conservation of momentum in x direction

$$m_1u_1 = m_1v_1\cos\Theta_1 + m_2v_2\cos\Theta_2$$

Equation for conservation of momentum in y direction

$$= m_1v_1\sin\Theta_1 - m_2v_2\sin\Theta_2$$

Equation for conservation of kinetic energy, (KE is a scalar quantity)

$$\frac{1}{2}m_1{u_1}^2 = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2$$

CHAPTER 6

SYSTEM OF PARTICLES AND ROTATIONAL MOTION

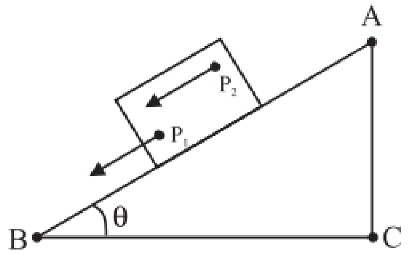
Basically a rigid body can have two types of motion.

- Translational
- Rotational motion

Translational motion

In pure translational motion at any instant of time, all particles of the body have the same velocity.

Eg: A block moving down an inclined plane.



Any point like P_1 or P_2 of the block moves with the same velocity at any instant of time.

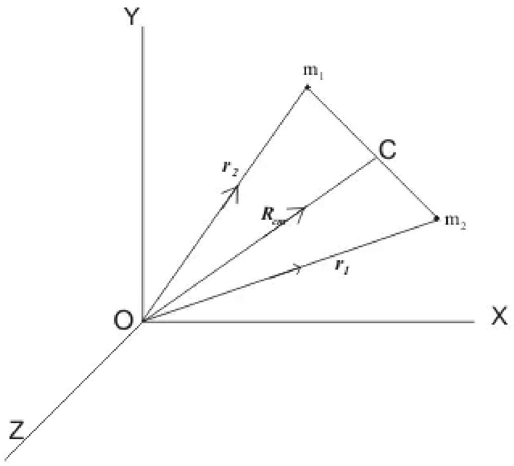
Rotational motion

In the rotation of a rigid body about a fixed axis every particle of the body moves in circles. The circular path of the particle will be perpendicular to the axis of rotation.

In pure rotational motion, angular velocity of all the points is same about the fixed axis.

Centre of mass

The centre of mass of a system of particles is the point where all the mass of the system may be assumed to be concentrated.



Consider two particles of masses m_1 and m_2 with position vectors $\vec{r}_1$ and $\vec{r}_2$ respectively with respect to the origin O. Now the position coordinate of the center of mass C is defined as.

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{M}$$

$$\text{Where } M = m_1 + m_2$$

X, Y and Z coordinate of centre of mass of two particle system

$$X = \frac{m_1x_1 + m_2x_2}{M}$$

$$X = \frac{m_1y_1 + m_2y_2}{M}$$

$$X = \frac{m_1 z_1 + m_2 z_2}{M}$$

If we have n particles of masses $m_1, m_2, \ldots\ldots m_n$

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_2\vec{r}_2 + \cdots + m_n\vec{r}_n}{M}$$

$$\vec{R} = \frac{\sum m_i\vec{r}_i}{M}$$

$$\text{Where } M = m_1 + m_2 + \cdots m_n$$

If the origin is chosen to be the centre of mass then $\vec{R} = 0$

$$0 = \frac{\sum m_i\vec{r}_i}{M}$$

$$\sum m_i\vec{r}_i = 0$$

Motion of Centre of Mass

Position vector of centre of mass

$$\vec{R} = \frac{\sum m_i\vec{r}_i}{M}$$

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_2\vec{r}_2 + \cdots + m_n\vec{r}_n}{M} \ldots\ldots\ldots\ldots(1)$$

Velocity of centre of mass

Differentiating the position vector of C.M., we get velocity of CM. ie;

$$\vec{V} = \frac{d\vec{R}}{dt}$$

$$\vec{V} = \frac{d}{dt}\left(\frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots + m_n\vec{r}_n}{M}\right)$$

$$\vec{V} = \frac{m_1\frac{d}{dt}\vec{r}_1 + m_2\frac{d}{dt}\vec{r}_2 + m_3\frac{d}{dt}\vec{r}_3 + \cdots + m_n\frac{d}{dt}\vec{r}_n}{M}$$

$$\vec{V} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v}_n}{M}$$

Acceleration of centre of mass

$$\vec{a} = \frac{d}{dt}\vec{v}$$

$$\vec{a} = \frac{d}{dt}\left(\frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v}_n}{M}\right)$$

$$\vec{a} = \frac{m_1\frac{d}{dt}\vec{v}_1 + m_2\frac{d}{dt}\vec{v}_2 + m_3\frac{d}{dt}\vec{v}_3 + \cdots + m_n\frac{d}{dt}\vec{v}_n}{M}$$

$$\vec{a} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots + m_n\vec{a}_n}{M}$$

Force on centre of mass

Force acting on the centre of mass = Total mass at the centre of mass × acceleration of the centre of mass.

$$\vec{F} = M\vec{a}$$

$$\text{But } F = F_{internal} + F_{extenal}$$

By Newton's third law, sum of the internal forces is zero.

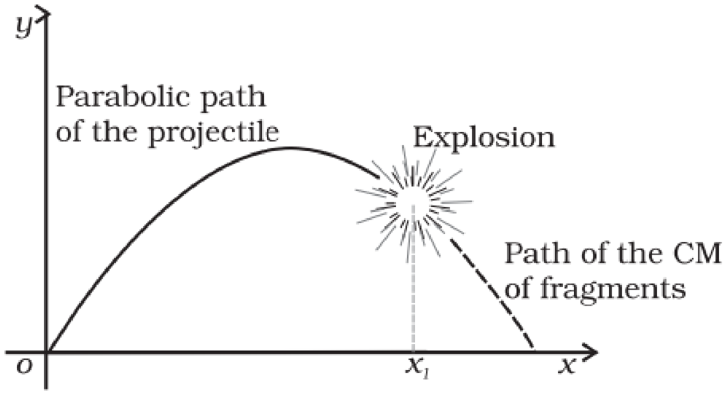
$$\therefore F = F_{extenal}$$

$$\vec{a} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots + m_n\vec{a}_n}{M}$$

$$M\vec{a} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots + m_n\vec{a}_n$$

$$\vec{F}_{ext} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots + \vec{F}_n$$

The centre of mass of a system of particles moves as if all the mass of the system was concentrated at the centre of mass and all the external forces were applied at that point.



The centre of mass of the fragments of the projectile continues along the same parabolic path which it would have followed if there were no explosion.

Linear Momentum of centre of mass

Velocity of centre of mass,

$$\vec{V} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v}_n}{M}$$

$$M\vec{V} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v}_n$$

$$\vec{P} = \vec{P}_1 + \vec{P}_2 + \vec{P}_3 + \cdots + \vec{P}_n$$

Law of Conservation of Momentum for a System of Particles

If Newton's second law is extended to a system of particles

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

When the sum of external forces acting on a system of particles is zero

$$\vec{F}_{ext} = 0$$

$$\frac{d\vec{P}}{dt} = 0$$

$$\vec{P} = \text{Constant}$$

Thus, when the total external force acting on a system of particles is zero, the total linear momentum of the system is constant. This is the law of conservation of the total linear momentum of a system of particles.

Vector product of two vectors

The vector product or cross product of any two vectors $\vec{A}$ and $\vec{B}$, denoted as $\vec{A}x\vec{B}$ is defined as

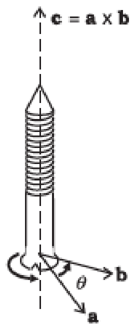
$$\vec{A}x\vec{B} = A\,B\sin\theta\,\hat{n}$$

Where θ is the angle between the two vectors $\vec{A}$ and $\vec{B}$. $\hat{n}$ is the unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

The direction of $\vec{A}x\vec{B}$ is given by right hand screw rule or right hand rule.

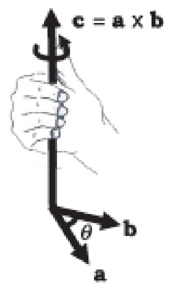
Right hand screw rule

If we turn the head of screw in the direction from $\vec{A}$ to $\vec{B}$, then the tip of the screw advances in the direction of $\vec{A}x\vec{B}$.



Right hand rule

If the fingers of right hand are curled up in the direction from $\vec{A}$ to $\vec{B}$, then the stretched thumb points in the direction of $\vec{A}x\vec{B}$.



Properties of cross product

- The vector product is not commutative

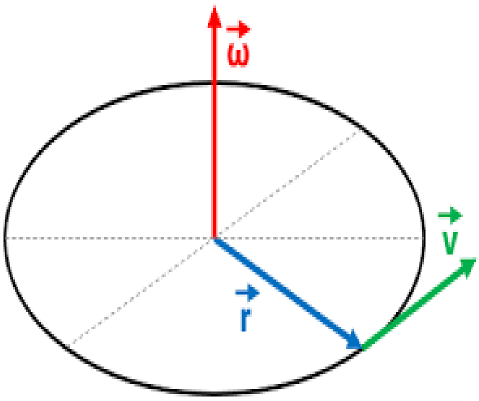
$$\vec{A}x\vec{B} \neq \vec{B}x\vec{A}$$

- Vector product obeys distributive law

$$\vec{A}x\left(\vec{B}+\vec{C}\right) \neq \vec{A}x\vec{B}+\vec{A}x\vec{C}$$

- $\vec{A}x\vec{A}=0$
- $\hat{\imath}x\hat{\imath}=0,\ \hat{\jmath}x\hat{\jmath}=0,\ \hat{k}x\hat{k}=0$
- $\hat{\imath}x\hat{\jmath}=\hat{k},\ \hat{\jmath}x\hat{k}=\hat{\imath},\ \hat{k}x\hat{\imath}=\hat{\jmath}$
- $\hat{\jmath}x\hat{\imath}=-\hat{k},\ \hat{k}x\hat{\jmath}=-\hat{\imath},\ \hat{\imath}x\hat{k}=-\hat{\jmath}$

Angular velocity and its relation with linear velocity

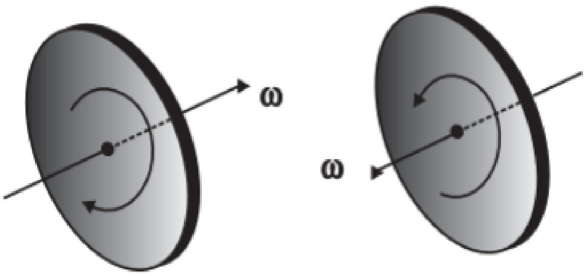


The angular velocity is a vector quantity. $\boldsymbol{\omega}$ is directed along the fixed axis as shown.

The linear velocity of the particle is

$$\vec{v} = \vec{\omega} \times \vec{r}$$

It is perpendicular to both $\vec{\omega}$ and $\vec{r}$ and is directed along the tangent to the circle described by the particle.



Angular acceleration

Angular acceleration is defined as the time rate of change of angular velocity.

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

If the axis of rotation is fixed, the direction of ω and hence α is fixed. In this case the vector equation reduces to scalar equation.

$$\alpha = \frac{d\omega}{dt}$$

Torque or Moment of Force

The tendency of a force to rotate the body to which it is applied is called torque. The torque, specified with regard to the axis of rotation, is equal to the magnitude of the component of the force vector lying in the plane perpendicular to the axis, multiplied by the shortest distance between the axis and the direction of the force component.

$$\tau = \vec{r} \times \vec{F}$$

$$\tau = rF\sin\theta$$

The unit of torque is Nm.

Angular momentum of a particle

Angular momentum is the rotational analogue of linear momentum. Angular momentum is a vector quantity. It could also be referred to as moment of (linear) momentum.

$$\vec{l} = \vec{r} \times \vec{p}$$

$$\vec{l} = rp\sin\theta$$

Relation between angular momentum and torque

$$\vec{l} = \vec{r} \times \vec{p}$$

Differentiating,

$$\frac{d\vec{l}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p})$$

$$\frac{d\vec{l}}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \, x \frac{d\vec{p}}{dt}$$

$$\vec{p} = m\vec{v}, \qquad \frac{d\vec{r}}{dt} = \vec{v}, \qquad \frac{d\vec{p}}{dt} = \vec{F}$$

$$\frac{d\vec{l}}{dt} = \vec{v} \times m\vec{v} + \vec{r} \, x\vec{F}$$

$$\frac{d\vec{l}}{dt} = 0 + \vec{r} \, x\vec{F}$$

$$\frac{d\vec{l}}{dt} = 0 + \vec{\tau}$$

$$\frac{d\vec{l}}{dt} = \vec{\tau}$$

Thus, the time rate of change of the angular momentum of a particle is equal to the torque acting on it.

Conservation of angular momentum

For a system of particles

$$\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$$

$$\text{If external torque}, \vec{\tau}_{ext} = 0$$

$$\frac{d\vec{L}}{dt} = 0$$

$$\vec{L} = \text{constant}$$

If the total external torque on a system of particles is zero, then the total angular momentum of the system is conserved.

Equilibrium of a rigid body

A rigid body is said to be in mechanical equilibrium, if both its linear momentum and angular momentum are not changing with time.

Condition for the translational equilibrium: If the total force on the body is zero, then the total linear momentum of the body does not change with time.

$$\mathbf{F}_1 + \mathbf{F}_2 + \ldots + \mathbf{F}_n = \sum_{i=1}^{n} \mathbf{F}_i = \mathbf{0}$$

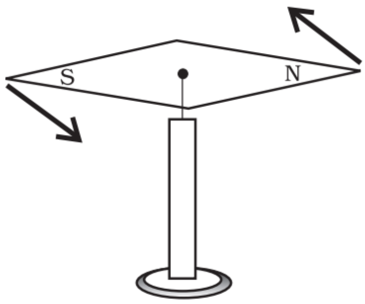
Condition for the rotational equilibrium: If the total torque on the rigid body is zero, the total angular momentum of the body does not change with time.

$$\boldsymbol{\tau}_1 + \boldsymbol{\tau}_2 + \ldots + \boldsymbol{\tau}_n = \sum_{i=1}^{n} \boldsymbol{\tau}_i = \mathbf{0}$$

Couple

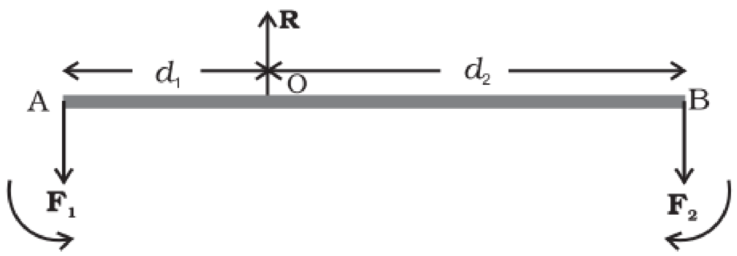
A pair of equal and opposite forces with different lines of action is known as a couple. A couple produces rotation without translation.

Our fingers apply a couple to turn the lid.



The Earth's magnetic field exerts equal and opposite forces on the poles of a compass needle. These two forces form a couple.

Principles of moments



Consider a lever pivoted at some origin (say fulcrum) in mechanical equilibrium. Let F_1 and F_2 he the forces acting at A and B as shown in figure. Let R be the reaction of the support at the fulcrum.

For translational equilibrium

$$R - F_1 - F_2 = 0$$

$$R = F_1 + F_2$$

For considering rotational equilibrium we take the moments about the fulcrum; the sum of moments must be zero,

$$d_1F_1 - d_2F_2 = 0$$

$$d_1F_1 = d_2F_2$$

In the case of lever, F_1 is used to lift some weight. Hence F_1 is called load and its distance from the fulcrum d_1 is called load arm. Force F_2 is the effort applied to lift the load, distance d_2 is called effort arm.

Hence $d_1F_1 = d_2F_2$ can be written as

load arm × load = effort arm × effort

The above equation expresses the principle of moments for a lever.

$$\text{Mechanical advantage (MA)} = \frac{F_1}{F_2} = \frac{d_2}{d_1}$$

Centre of gravity

The Centre of gravity of a body is the point where the total gravitational torque on the body is zero.

- The centre of gravity of the body coincides with the centre of mass. For a body is small, g does not vary from one point of the body to the other. Then the centre of gravity of the body coincides with the centre of mass.
- If the body is so extended that g varies from part to part of the body, then the centre of gravity and centre of mass will not coincide.

Moment of inertia

A body at rest cannot rotate by itself. A body in uniform rotation cannot stop by itself. This inability of a material body is called rotational inertia. It depends on the mass of the body and axis of rotation. In other words it depends on a quantity called moment of inertia.

Moment of inertia of a particle of mass ‘m’ rotating about an axis at a distance ‘r’ from it is given by

$$I = mr^2$$

In the case of a rigid body which consist of large no. of particles

$$I = \sum m_i {r_i}^2$$

Moment of inertia in rotational motion plays the same role as mass does in linear motion.

Rotational Kinetic energy

Consider a particle of mass m rotating about an axis of radius r with angular velocity ω

The kinetic energy of motion of this particle is

$$KE = \frac{1}{2}mv^2$$

$$v = r\omega$$

$$KE = \frac{1}{2}mr^2\omega^2$$

$$I = mr^2$$

$$\text{Rotational KE} = \frac{1}{2}I\omega^2$$

Radius of gyration

Radius of gyration of a body is the square root of ratio of moment of inertia and total mass of the body

$$k = \sqrt{\frac{I}{M}}$$

Kinematics of Rotational Motion about a Fixed Axis

The kinematical equations of linear motion with uniform (i.e. constant) acceleration

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v^2 = {v_0}^2 + 2ax$$

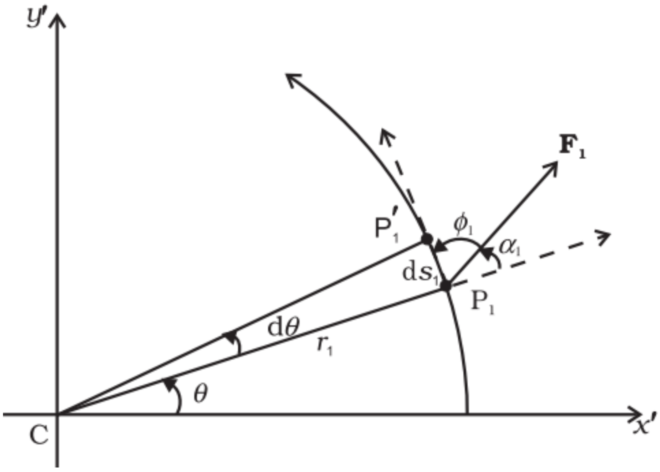
The corresponding kinematic equations for rotational motion with uniform angular acceleration are:

$$\omega = \omega_0 + \alpha t$$

$$\Theta = \Theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = {\omega_0}^2 + 2\alpha\,(\Theta - \Theta_0)$$

Work done by a torque



Work done by a force F acting on a particle of a body rotating about a fixed axis

Consider a particle at P_1. Let r_1 be the position vector at time t = 0. This position vector makes an angle Θ with x – axis. Let particle be acted by a force F_1. Due to this force the particle subtends an angle $d\Theta$ and reaches at ${p_1}^1$.

ds_1 is the linear displacement due to the force $F_{1.}$ This force makes angle α_1 with the position vector r_1. Φ_1 is the angle made by F_1 with linear displacement.

Form the triangle the workdone for small displacement ds_1,

$$dW = F_1 . ds_1$$

$$dW = F_1 ds_1 \cos\Phi_1$$

$$\text{but} = \Phi_1 + \alpha_1 = 90, \qquad \Phi_1 = 90 - \alpha_1$$

$$\cos(90 - \alpha_1) = \sin \alpha_1$$

$$dW = F_1 (rd\Theta) \sin \alpha_1$$

$$dW = r_1 F_1 \sin \alpha_1 d\Theta$$

$$dW = \tau d\Theta$$

$$W = \tau\Theta$$

Where τ is the total torque acting on the body.

Relation Connecting Angular Momentum and Moment of Inertia

Angular momentum of a particle

$$\vec{l} = \vec{r} \times \vec{p}$$

For a system of particles, $L = \Sigma\vec{l}$

$$\vec{L} = \Sigma\vec{r} \times \vec{p}$$

$$\vec{L} = \Sigma rp \sin 90\, \hat{k}$$

$$\vec{L} = \Sigma rp\, \hat{k}$$

$$\vec{L} = \Sigma mrv\, \hat{k} \qquad (p = mv)$$

$$\vec{L} = \Sigma rm(r\omega)\hat{k} \qquad (v = r\omega)$$

$$\vec{L} = \Sigma mr^2\omega\, \hat{k}$$

$$\vec{L} = I\omega\, \hat{k}$$

$$\vec{L} = I\,\vec{\omega}$$

Relation Connecting Torque and Angular Acceleration

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

$$\text{But} = \vec{L} = I\,\vec{\omega}$$

$$\vec{\tau} = \frac{dI\,\vec{\omega}}{dt}$$

$$\vec{\tau} = I\frac{d\,\vec{\omega}}{dt}$$

$$\vec{\tau} = I\,\vec{\alpha}$$

Conservation of angular momentum

If the external torque is zero, angular momentum is constant.

$$L = \text{constant}$$

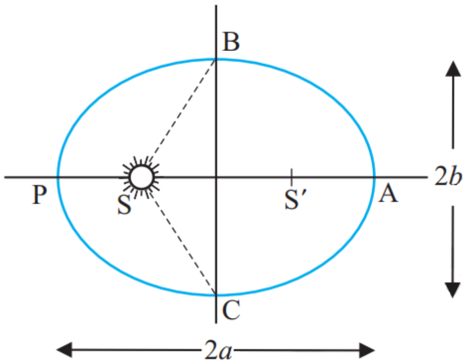
$$I\omega = \text{constant}$$

CHAPTER 7

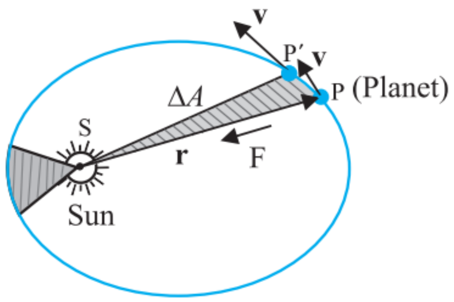
GRAVITATION

Kepler's Laws

❖ Kepler's first law (law of orbits): Every planet revolves around the sun in an elliptical orbit with the sun is situated at one of the foci of the ellipse.



❖ Kepler's second law (law of areas): The radius vector drawn from the sun to a planet sweeps out equal areas in equal intervals of time, i.e., the areal velocity of the planet around the sun is constant.



The law of areas can be understood as a consequence of conservation of angular momentum which is valid for any central force. A central force is such that the force on the planet is along the vector joining the Sun and the planet. Let the Sun be at the origin and let the position and momentum of the planet be denoted by r and p respectively. Then the area swept out by the planet of mass m in time interval Δt is ΔA given by

$$\Delta A = \frac{1}{2}(rxv\Delta t)$$

Hence,

$$\frac{\Delta A}{\Delta t} = \frac{1}{2}(rxp)/m \qquad (\text{since } v = \frac{p}{m})$$

$$\frac{\Delta A}{\Delta t} = \frac{L}{2m} \qquad (L = rxp)$$

But we know angular momentum of a planet is constant because the gravitational force is central force. Hence we get $\frac{\Delta A}{\Delta t}$ = a constant.

- Kepler's third law (law of periods): The square of the time period of revolution of a planet around the sun is directly proportional to the cube of semi-major axis of the elliptical orbit of the planet around the sun.

Universal law of Gravitation

According to Newton's law of gravitation, everybody in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

$$F = \frac{GMm}{r^2}$$

Where G is a constant called universal gravitational constant. The value of G is 6.67×10^{-11} Nm^2/Kg^2

Acceleration due to gravity of Earth

Acceleration due to gravity is the acceleration experienced by a body falling freely towards the earth.

Relation between acceleration due to gravity and gravitational constant

$$g = \frac{GM}{R^2}$$

The value of g is 9.8 m/s^2. There is a slight variation for g from place to place depending on the height or depth of the place.

Variation of 'g' with altitude (Height)

$$g_h = g\left(1 - \frac{2h}{R}\right)$$

The above equation shows that the value of acceleration due to gravity decreases with height.

Variation of 'g' with depth

$$g_d = g\left(1 - \frac{d}{R}\right)$$

The above equation shows that the value of g decreases with depth.

Gravitational Potential Energy

Gravitational potential energy is defined as the work done in bringing a mass from infinity to a point in the gravitational field of another body.

$$U = -\frac{GMm}{r}$$

Gravitational Potential

Gravitational potential at a point in a gravitational field is defined as the work done in bringing a body of unit mass from infinity to that point.

$$V = -\frac{GM}{r}$$

Escape Velocity

The minimum speed with which a body is projected so that it never returns to the earth is called escape velocity.

$$V_e = \sqrt{2gR}$$

This escape velocity estimated to be 11.2 km/s on the earth. Escape velocity is independent of mass of the escaping body.

Earth satellites

A body revolving around a planet in a fixed orbit is called a satellite. The natural satellite of earth is moon. Examples for artificial (manmade) satellites are Sputnik, Aryabhatta, INSAT etc.

Orbital Velocity

Orbital velocity of a satellite is the velocity with which it revolves round a planet in its fixed orbit.

$$V_o = \sqrt{\frac{gR^2}{R+h}}$$

If the satellite is very close to earth, then $R + h \approx R$

$$V_0 = \sqrt{gR_E}$$

The nearest orbit of a satellite is called minimum orbit and the corresponding velocity is called first cosmic velocity.

CHAPTER 8

MECHANICAL PROPERTIES OF SOLIDS

Elasticity

The property of a body by virtue of which, it tends to regain its original size and shape when the applied force is removed is known as elasticity.

Plasticity

The body which has no tendency to regain its original shape and get permanently deformed is called plastic body. This property is known as plasticity.

Stress

A force which changes the length, shape or volume of a body is called a deforming force. When an elastic body is subjected to a deforming force, a restoring force is developed in the body. This restoring force is equal in magnitude but opposite in direction to the applied force. The restoring force per unit area is known as stress.

Magnitude of the stress = F/A

Strain

The effect of stress on a body is called strain. Strain is measured as the ratio of the change in dimension produced to the original dimension.

$$Strain = \frac{change\ in\ dimension}{original\ dimension}$$

Different types of stress and strain

There are three ways in which a solid may change its dimensions when an external force acts on it. Hence there are three types of stress and corresponding strain.

- Linear Stress (longitudinal or tensile stress): It is the stress developed, when the applied force produces a change in the length of the body.

  Longitudinal strain = Change in length / Original length

- Volume stress (or Bulk stress): It is the stress developed in the body, when the applied force produces a change in the volume of the body.

  Volume Strain = Change in Volume / Original Volume

- Shearing Stress (or tangential stress): It is the stress developed in the body, when the applied force produces, a change in shape of the body.

  Shearing strain = $\triangle x$ / l = tan θ

Hooke's Law

According to Hooke's law, with in the elastic limit stress is directly proportional to stain. ie, stress $\propto$ strain.

Stress = k × strain, Where k is the proportionality constant and is known as modulus of elasticity.

Young's modulus

The ratio of longitudinal stress to the longitudinal strain is defined as the Young's modulus and is denoted by Y.

$$Y = \frac{FL}{A \triangle L}$$

Shear modulus

The ratio of shearing stress to the shearing strain is called the shear modulus of the material and is denoted by 'G'. It is also called rigidity modulus.

$$G = \frac{F}{A\theta}$$

Bulk modulus

The ratio of the volume stress to the corresponding volume strain is defined as bulk modulus. It is denoted by 'B'.

$$B = \frac{-PV}{\triangle V}$$

The reciprocal of bulk modulus is called compressibility and is denoted by K.

Poisson's ratio

Within the elastic limit, lateral strain is directly proportional to the longitudinal strain. The ratio of the lateral strain to the longitudinal strain is called Poisson's ratio.

$$Poisson's\ ratio, \sigma = \frac{lateral\ strain}{longitudinal\ strain}$$

Applications of elastic behaviour of materials

- Metallic ropes are used in the cranes for lifting the load because of its high elasticity.
- While designing bridges and buildings, beams and columns are widely used. The bending of beams and columns under a load depends upon Young's modulus of the material used. To reduce the bending of a beam for a given load, a material with a larger Young's modulus is to be used.

- The maximum height of the mountain is limited by the elastic properties of the rock which holds the mountain.

CHAPTER 9

MECHANICAL PROPERTIES OF FLUIDS

Pressure

The force acting per unit area normal to the surface is called pressure.

$$P=F/A$$

Pascal's Law

Pascal's law states that the pressure applied at any point on a continuous fluid in equilibrium is equally transmitted to all other points.

- A number of devices such as hydraulic lift, hydraulic jack and hydraulic brakes are based on Pascal's law. In these devices fluids are used for transmitting pressure.

Atmospheric pressure

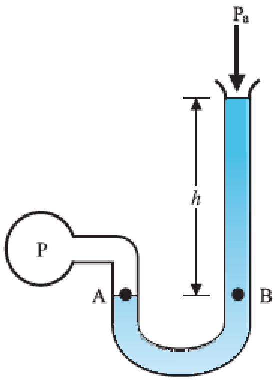
The pressure exerted by the atmosphere at any point is due to the weight of air above that point. At sea level the atmospheric pressure is maximum and is taken as 1 atmosphere.

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

Instruments used to measure pressure are called Manometers. Barometer is a device used to measure atmospheric pressure.

Open Tube Manometer

An open tube manometer is an instrument to measure pressure differences.



This device consists of a U shaped tube containing liquid of density r. One end is open to the atmosphere and other end is connected to the system whose pressure is to be measured.

The pressure 'P' at A is equal to the pressure at B.

$$\text{ie: } P = Pa + h\rho g$$

$$\text{(or) } P - Pa = h\rho g$$

P is called absolute pressure.

Streamline flow

- The study of fluids in motion is called fluid dynamics.
- The flow of the fluid is said to be steady, if at any point the velocity of each passing fluid particle remains constant in time.
- But the velocities at different points may differ.
- The path taken by a fluid particle under a steady flow is called a streamline. It is defined as a curve whose tangent at any point is in the direction of the fluid velocity at that point.
- A bundle of stream line is called tube of flow.

Turbulent Flow: When the speed of flow increases beyond a limiting value, called critical speed, the flow loses its steadiness and becomes turbulent.

Equation of continuity

Flow rate of a incompressible fluid is constant.

$$\text{ie } Av = \text{cont.}$$

Where A is the area of cross section of flow and v is the velocity of flow. 'Av' is also called volume flux.

Bernoulli's theorem

In a stream line flow of an ideal fluid, the sum of pressure energy per unit volume, potential energy per unit volume and kinetic energy per unit volume is always constant at all cross section of the liquid.

$$p + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

Applications of Bernoulli's Principle

- Speed of Efflux: Torricelli's Law: Torricellis law may be stated as the velocity of afflux through a hole at a depth 'h' will be equal to the velocity gained by a freely falling body when it travels a distance 'h'. ie; $v = \sqrt{2gh}$
- Dynamic lift: Dynamic lift is the force that acts on a body by virtue of its motion through a fluid (air).

Viscosity

Viscosity is liquid friction. When liquid layer moves over another liquid layer, there is a force of friction between the liquid layers, opposing the motion of layers.

The SI unit of viscosity is poiseiulle (PI). Its another units are NSm^{-2} or PaS.

Stokes' Law

When a spherical body of density 'ρ' moves through a fluid of density 'σ', the viscous force acting on it is given by,

$$F = 6\pi a\eta v$$

This is known as Stokes' law.

Where η is the coefficient of viscocity of the medium and v is the velocity of the body.

Terminal velocity

The constant velocity attained by a body as it falls down through a fluid medium is called the terminal velocity.

Reynolds Number

The turbulence in a fluid is determined by a dimension parameter called Reynolds number.

Reynolds number, $R_e = \frac{\rho v d}{\eta}$

Surface tension

Liquids acquire a free surface when poured in a container. These surfaces possess some additional. energy. This phenomenon is known as surface tension.

Surface energy is defined as the work done to increase the surface area of a liquid meniscus by unity.

Angle of contact

Angle of contact is the angle between the solid surface and the tangent drawn to the liquid surface at the point of contact inside the liquid.

Drops and bubbles

Due to surface tension, the liquid surface always tends to have the minimum surface area. For a given volume, a sphere has a minimum surface area. Hence, small drops and bubbles of a liquid assume spherical shape.

Capillary rise

A tube of very fine core is called a capillary tube. When a capillary tube is dipped in a liquid, the liquid immediately rises in the tube. This phenomenon is called capillarity or capillary rise.

CHAPTER 10

THERMAL PROPERTIES OF MATTER

Heat and Temperature

- Heat is the form of energy, which is transferred from one body to another because of their temperature difference.
- Temperature is the degree of hotness of a body.

Ideal gas equation

$$PV = \mu RT$$

μ is the number of moles in given gas and R is universal gas constant.

Thermal expansion

Thermal expansion defines the tendency of an object to change its dimension either in length, area, or volume due to heat. When the substance is heated it increases its kinetic energy. Thermal expansion is of three types:

- When temperature increases, length of a solid (rod like structure) increases. This is called linear expansion.
- When temperature increases area of a solid substance increases. This is called area expansion.
- When temperature increases, the volume of a substance (solid, liquid or gas) increases. This called volume expansion.

Specific heat capacity

Specific heat capacity of a substance is defined as amount of heat required to increase temperature of unit mass of substance by one unit.

$$s = \frac{\triangle Q}{m \triangle T}$$

Molar specific heat capacity of a substance is the amount of heat required to increase the temperature of 1 mole of substance by one unit.

$$s = \frac{1}{\mu} \frac{\triangle Q}{\triangle T}$$

Molar specific capacity are of two types

- molar specific capacity at constant volume (C_V)
- molar specific heat capacity at constant pressure (C_P).

Calorimetry

Calorimetry is the measurement of heat. Calorimeter is a device used to measure heat.

Change of State

A transition from one state (solid, liquid or gas) to another state is called change of state.

Change of state	Name of transition
Solid → Liquid	Melting
Liquid → gas	Vaporization
Liquid → solid	Fusion

Solid $\rightarrow$ gas (without forming liquid)	Sublimation

Melting point

The temperature at which solid and liquid coexist in thermal equilibrium with each other is called melting point. The melting point decreases with pressure

Boiling point

The temperature at which liquid and vapour state of substance coexist in thermal equilibrium with each other is called boiling point. The boiling point increases with increase in pressure.

Regelation

When pressure is applied, ice melts at low temperature. If pressure is removed, water refreezes. This refreezing is called regelation.

Latent Heat

The amount of heat per unit mass transferred during change of state of substance is called latent heat of substance for the process.

- Latent heat of fusion : It is the amount of heat required to convert one kilogram solid substance completely into liquid at its melting point, without any change in temperature.
- Latent heat of Vaporisation: It is the amount of heat required to convert one kilogram liquid substance completely into gas at its boiling point, without any change in the temperature.

Heat Transfer

Heat transfer occurs due to temperature difference. There are three distinct modes of heat transfer.

- Conduction: Conduction is the transfer of heat between two adjacent parts of a body because of their temperature difference.

The rate of flow of heat (H) is directly proportional to the temperature difference ($\triangle$T) and the area of cross section A and is inversely proportional to the length L.

$$H = \frac{KA \triangle T}{L}$$

K is called thermal conductivity.

- Convection: Convection is the mode of heat transfer by actual motion of matter. Convection is possible only in fluids.
- Radiation: Radiation is the mode of heat transfer without the need of any material medium.

The electromagnetic radiation entitled by a body by virtue of its temperature is called thermal radiation.

Black body radiation

A black body is one which absorbs radiations of all wavelengths incident on it. When a black body is heated it will emit radiations of all possible wavelengths. The wavelengths emitted by a perfect black body are called black body radiations.

Newtons laws of cooling

According to Newton's law of cooling the rate of loss of heat is directly proportional to the difference of temperature between the body and its surroundings.

$$\frac{dQ}{dt} = K(T_2 - T_1)$$

CHAPTER 11

THERMODYNAMICS

Thermal equilibrium

Two systems are said to be in thermal equilibrium, if their temperatures are the same. Then there will not be any heat flow from one system to another.

Zeroth Law of thermodynamics

The law states that, two systems which are in thermal equilibrium with a third system separately are in thermal equilibrium with each other.

Internal Energy

If we consider a bulk system consisting of a large number of molecules, then internal energy of the system is the sum of the kinetic energies and potential energies of these molecules.

Internal energy is a thermodynamic variable and hence it depends on state of the thermodynamic system.

First law of thermodynamics

The law states that, If an amount of heat is given to a system, a part of the heat is used to increase the internal energy and other part is used to do the external work.

$$\Delta Q = \Delta U + \Delta W$$

- If the entire heat supplied to the system is used to do work, then $\Delta Q = \Delta W$
- The work done against constant pressure, $\Delta W = P\Delta V$, $\Delta Q = \Delta U + P\Delta V$.

Mayor's Relation

$$Cp - Cv = R$$

If molar specific heat capacity of constant pressure is Cp and that at constant volume is Cv then Cp – Cv = R, for an ideal gas.

Thermodynamic state variables

The physical quantities which characterise a system are known as state variables. Eg: - Pressure, volume, temperature, mass, density, internal energy, heat capacity, specific heat capacity etc.

Thermodynamic state variables are of two types:

- Intensive state variables: They are the state variables which do not depend on the size of the system. Eg: - Pressure, temperature, density, specific heat capacity etc.
- Extensive sate variables: They are the state variables which depend on the size of the system. Eg: -volume, mass, heat capacity, internal energy etc.

Heat and work are not state variables.

Thermodynamic processes

Quasi-static process

The thermodynamic process in which thermodynamic variables (P, V, T… etc) changes so slowly that the system remains in thermal and mechanical equilibrium is called quasi-static (nearly static) process. In quasi-static process change in temperature or pressure will be infinitesimally small.

Isothermal process

It is a process taking place at constant temperature. Equation for isothermal process is

$$PV = \text{Constant} = \mu RT$$

Conditions for isothermal process

- The process must be slow
- There should be a perfect conducting wall (diathermic wall) between the system and surroundings.

Work done during an isothermal process

$$W = \mu RT \ln\left(\frac{V_2}{V_1}\right)$$

Adiabatic process

During an adiabatic process, no heat enters or leaves the system. In an adiabatic process all of the quantities P, V and T changes.

Equations for Adiabatic Process

$$PV^{\gamma} = \text{constant}$$

$$\gamma = \frac{c_p}{c_v}$$

Work done during an adiabatic process

$$W = \frac{1}{\gamma - 1}\mu R(T_1 - T_2)$$

- In isochoric process no work is done on or by gas because volume is constant.
- In an isobaric process, pressure is constant throughout the process.

- Cyclic process: In cyclic process, the system returns to its initial state such that change in internal energy is zero.

Second law of Thermodynamics

Kelvin – Planck Statement: No process is possible whose sole result is the absorption of heat from a reservoir and the complete conversion of heat into work.

Clausius Statement: No process is possible whose sole result is the transfer of heat from a cold reservoir to a hot reservoir.

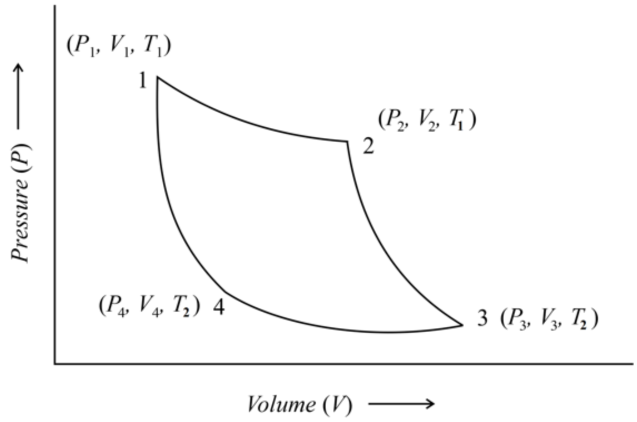
Reversible and Irreversible processes

- A thermodynamic process is said to be reversible if the process can bring both system and surrounding back to the original state without any change.
- Most thermodynamic processes are irreversible because the process involves dissipative effects like friction, viscous force etc. and during such process, system passes through non-equilibrium states.

Carnot engine

A reversible heat engine operating between two temperatures is called a Carnot engine.

The Carnot cycle consists of two isothermal processes and two adiabatic processes.



The work done by gas in one Carnot cycle

- Step 1: Isothermal expansion: The gas absorbs heat Q, from hot reservoir and undergoes Isothermal expansion.

  [$(P_1, V_1, T_1) \rightarrow (P_2, V_2, T_2)$]

$$W_1 = \mu R T_1 \ln\left(\frac{V_2}{V_1}\right) = Q_1$$

- Step 2: Adiabatic expansion

  [$(P_2, V_2, T_1) \rightarrow (P_3, V_3, T_2)$]

$$W_2 = \frac{\mu R}{\gamma - 1}(T_1 - T_2)$$

- Step 3: Isothermal compression: The gas releases heat Q_2 to cold reservoir at T_2.

  [$(P_3, V_3, T_2) \rightarrow (P_4, V_4, T_2)$]

$$W_3 = \mu R T_2 \ln\left(\frac{V_3}{V_4}\right) = Q_2$$

- Step 4: Adiabatic compression

  [$(P_4, V_4, T_2) \rightarrow (P_1, V_1, T_1)$]

$$W_2 = \frac{\mu R}{\gamma - 1}(T_1 - T_2)$$

The total work done by the gas in one complete cycle,

$W = W_1 + W_2 - W_3 - W_4$

$$W_1 = \mu R T_1 \ln\left(\frac{V_2}{V_1}\right) - \mu R T_2 \ln\left(\frac{V_3}{V_4}\right)$$

Efficiency of Carnot's engine

$$\eta = 1 - \frac{T_2}{T_1}$$

CHAPTER 12

KINETIC THEORY

Behaviour of gases

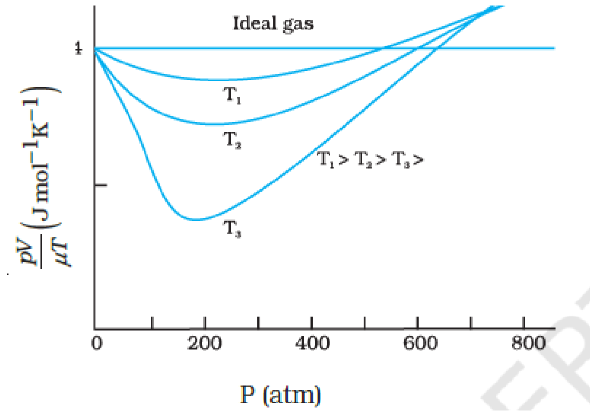
Ideal gas equation,

$$PV = \mu RT$$

A gas that satisfies the equation $PV = \mu RT$ at all temperatures and pressures is defined to be an ideal gas.

An ideal gas is a simple theoretical model of a gas. No real gas is truly ideal.

The figure below shows deviation from ideal gas behaviour for a real gas at three different temperatures.



It shows that the real gases approach ideal gas behaviour at low pressures and high temperatures.

Kinetic theory of an ideal gas

The theory is based on the following postulates.

- The gas is a collection of large number of molecules. The molecules are perfectly elastic hard spheres.

- The size of a molecule is negligible compared with the distance between the molecules.
- The molecules are always in random motion
- During their motion, the molecules collide with each other and with the walls of the containing vessel.
- The collisions are elastic and hence the total K.E energy and the total momentum of the colliding molecules before and after collisions are the same.
- The kinetic energy of a molecule is proportional to the absolute temperature of the gas.
- There is no force of attraction or repulsion between molecules.

Pressure of an ideal gas

$$P = \frac{1}{3} nm\bar{v}^2$$

Kinetic Interpretation of Temperature

$$K.E = \frac{3}{2} K_B T$$

That is, the average kinetic energy of a gas molecule is proportional to the absolute temperature. But it is independent of pressure, volume or nature of the gas.

Law of Equipartition of Energy

The law of equipartition of energy states that if a system is in equilibrium at absolute temperature T, the total energy is distributed equally in different degrees of freedom, the energy in each degree of freedom being equal to $\frac{1}{2} K_B T$

Degrees of freedom

Degrees of freedom is number of independent ways by which a molecule can possess kinetic energy of translation, rotation and vibration.

- A monoatomic atom has 3 degrees of freedom.
- If a molecule is restricted to move in plane. It has 2 degrees of freedom.
- If a molecule is restricted to move in a line, it has only 1 degrees of freedom.

A diatomic molecule has 3 rotational degrees of freedom. But we consider only 2 degrees of freedom. We neglect rotation along the line joining the atoms. Because it has very small moment of inertia.

Specific heat capacity

Internal Energy of one mole of monoatomic gas,

$$\mathrm{U} = \frac{3}{2}\mathrm{K_B T \ x\ N_A} = \frac{3}{2}\mathrm{RT}$$

$$C_V = \frac{dU}{dt} = \frac{3}{2}R$$

$$C_{P=}C_V + R = \frac{5}{2}R$$

$$\gamma = \frac{C_P}{C_V} = \frac{5}{3}$$

Diatomic Molecule

- Diatomic Rigid Rotator: Rigid rotator means the molecule does not vibrate.

A molecule of a diatomic gas has three translational degrees of freedom. But in addition it can also rotate about its centre of mass. The molecule thus has two rotational degrees of freedom.

$$\mathrm{U} = \frac{5}{2}\mathrm{K_B T \; x \; N_A} = \frac{5}{2}\mathrm{RT}$$

$$C_V = \frac{dU}{dt} = \frac{5}{2}R$$

$$C_{P=}C_V + R = \frac{7}{2}R$$

$$\gamma = \frac{C_P}{C_V} = \frac{7}{5}$$

- Diatomic Non-rigid Rotator

A non rigid rotator has 7 degrees of freedom.

$$\mathrm{U} = \frac{7}{2}\mathrm{K_B T \; x \; N_A} = \frac{7}{2}\mathrm{RT}$$

$$C_V = \frac{dU}{dt} = \frac{7}{2}R$$

$$C_{P=}C_V + R = \frac{9}{2}R$$

$$\gamma = \frac{C_P}{C_V} = \frac{9}{7}$$

Polyatomic Molecule

A polyatomic molecule has 6+2f degrees of freedom.

$$\mathrm{U} = (3 + \mathrm{f})\,\mathrm{K_B T \; x \; N_A} = (3 + \mathrm{f})\,\mathrm{RT}$$

$$C_V = \frac{dU}{dt} = (3 + f)\,R$$

$$C_{P=}C_V + R = (4 + f)\,R$$

$$\gamma = \frac{C_P}{C_V} = \frac{(4 + f)}{(3 + f)}$$

Mean free path

The mean free path is the average distance covered by a molecule between two successive collisions.

$$\mathrm{l} = \frac{1}{\sqrt{2}\pi \mathrm{n} \mathrm{d}^2}$$

CHAPTER 13

OSCILLATIONS

Periodic Motion

A motion which repeats at regular intervals of time is called a periodic motion.

Oscillation or Harmonic motion.

To and fro motion of a body about a mean position is called oscillation or harmonic motion.

- Oscillations with high frequency are usually called vibrations.
- Every oscillatory motion is necessarily periodic. But every periodic motion need not be oscillatory. For example, The motion of earth around the sun is periodic but it is not oscillatory.

- Time taken to complete one oscillation is called period.
- The number of oscillations per second is called frequency.

Displacement Variable

The physical quantity which changes with time in a Periodic motion is called displacement Variable or displacement.

Simple Harmonic Motion (SHM)

An oscillating particle is said to execute SHM if the restoring force on the particle at any instant of time is directly proportional to its displacement from the mean position and is always directed towards the mean position.

$$X(t) = A\cos(wt + \Phi)$$

Where

- x(t) = displacement x as a function of time t

- A = amplitude
- ω = angular frequency
- (ω t+ Φ) = phase (time-dependent)
- Φ = phase constant or initial phase

Simple Harmonic Motion and Uniform Circular Motion

Simple harmonic motion can be defined as the projection of uniform circular motion on a diameter of the circle.

Velocity and Acceleration in SHM

Velocity,

$$v(t) = \omega\sqrt{a^2 - y^2}$$

Acceleration

$$a = -\omega^2 y$$

Simple harmonic motion is the motion executed by a particle subject to a force, which is proportional to the displacement of the particle and is directed towards the mean position.

$$F = -m\omega^2 x(t) = -kx(t)$$

$$k = m\omega^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

- Time Period in SHM

$$T = 2\pi\sqrt{\frac{m}{k}}$$

- Frequency in SHM

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Energy in SHM

A particle executing SHM has kinetic and potential energies, both varying between the limits, zero and maximum.

Kinetic energy,

$$K = \frac{1}{2}kA^2 sin^2(\omega t + \Phi)$$

Potential energy,

$$U = \frac{1}{2}kA^2 cos^2(\omega t + \Phi)$$

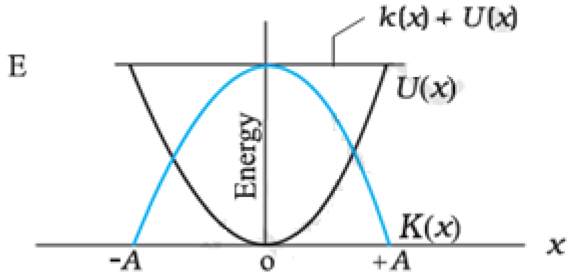
Total energy,

E=K+U

$$E = \frac{1}{2}kA^2$$

Total energy of a harmonic oscillation is independent of time, for any conservative force.

Graphical variation of PE, KE and TE of SHM



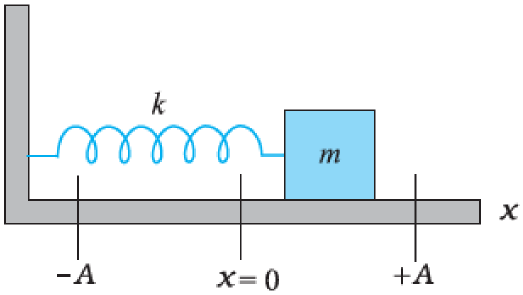
Some systems executing SHM

❖ Oscillation due to a spring

The force acting simple harmonic motion is proportional to the displacement and is always directed towards the centre of motion.

F α – x (or) F= kx

where k is called spring constant.

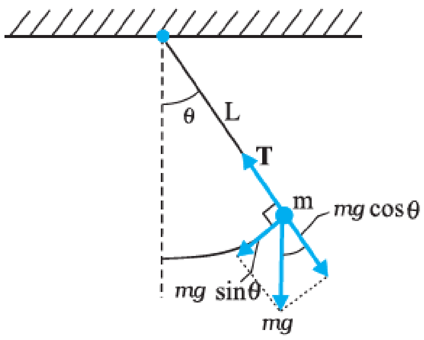


The time period,

$$T = 2\pi\sqrt{\frac{m}{k}}$$

- ❖ The Simple Pendulum

The forces acting on the both are force ‘T’, tension in the string and the gravitational force ‘mg’ as shown in figure.



$F_g\cos\theta$ cancels with the tension T in the string.
$F_g\sin\theta$ acts as the restoring force.

The time period,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

The time period of a simple pendulum depends on

- Length of the pendulum (L)
- Acceleration due to gravity (g).

CHAPTER 14

WAVES

Waves

Waves are the method of transfer of energy from one part of a medium to another without the actual flow of matter as a whole. There are three types of waves:

- Mechanical waves
- Electromagnetic waves
- Matter waves

Mechanical waves

They are the waves which necessarily need a medium for their propagation.

Eg :- water waves, sound waves etc. Mechanical waves cannot propagate through vacuum. Mechanical waves can be divided into two:

- Transverse waves
- Longitudinal waves

Electromagnetic waves

They are non-mechanical waves which do not require a medium for their propagation. They can travel through vacuum.

Matter waves

They are the waves associated with material particles.

Eg: Wave of moving electron, proton, etc.

Transverse and Longitudinal Waves

Transverse wave	Longitudinal wave
• The direction of vibrations of particles of medium is perpendicular to direction of propogation of wave. • They travel in the form of crust and troughs • Can be polarised Eg: Vibrations in stretched string, light etc.	• The direction of vibration of particles of medium is in the direction of propogation of wave. • They travel in the form of condensations and rare fractions. • Cannot be polarised Vibrations of tuning fork, sound wave, etc.

Displacement Relation of a progressive wave

To represent a travelling wave, we need a function of position ‘x’ and time‘t’.

- A transverse wave travelling in the +X direction can be represented as

$$y(x,t) = a \sin (kx - \omega t + \varphi)$$

- A transverse wave travelling in the - X direction can be represented as

$$y(x,t) = a \sin (kx + \omega t + \varphi)$$

Amplitude

Amplitude is the maximum displacement of the particles of the medium from their equilibrium position.

Displacement (y) may be positive or negative but ‘a’ is always positive.

Phase

The quantity kx-ωt+φ appearing as the argument of the sine function is called the phase of the wave.

Wavelength (λ)

Wavelength is the distance between two consecutive crests or troughs in a wave.

- The Speed of a Travelling Wave

$$v = \upsilon\lambda$$

- Speed of a transverse wave on a stretched string

$$v = \sqrt{\frac{T}{\mu}}$$

- T is the tension in the string
- μ is the linear mass density

Speed of a longitudinal wave (speed of sound)

$$v = \frac{\beta}{\rho}$$

- B → Bulk modulus
- ρ→ density of the medium.

The principle of superposition of waves

It states that when two or more waves pass through a media the net displacement of particle at any time is the algebraic sum of displacements due to each wave. (or) The overlapping waves algebraically add to produce a resultant wave.

Standing waves

When two waves of same amplitude and frequency travelling in opposite direction superimpose the resulting wave pattern does not move to either sides. This pattern is called standing wave.

$$y\ (x, t) = 2a \sin kx \cos \omega t$$

Nodes and Antinodes

The position of maximum amplitude in a standing wave is termed as anti node.

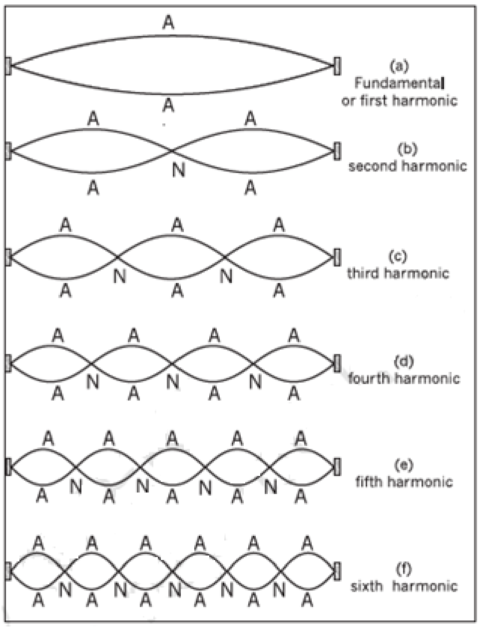
$$x = \frac{(2n+1)\lambda}{4}$$

And position of minimum amplitude (zero) is termed as node.

$$x = \frac{n\lambda}{2}$$

Standing waves on a stretched string

Consider a string stretched between two rigid supports. When string is excited stationary waves are produced. But string can vibrate only in some definite patterns. There patterns are called normal modes.



Frequency of vibration,

$$\nu_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

for n = 1, 2, 3,

This frequency is called fundamental frequency or first harmonic.

Second harmonic,

$$\nu_2 = \frac{v}{L}$$

Third harmonic,

$$\nu_3 = \frac{3v}{2L}$$

Thus collection of all possible mode is called harmonic series and n is called harmonic number.

Beats

When two sound waves of nearly same frequency and amplitude travelling in same direction super imposed and periodic variation of sound intensity (wavering of sound or waxing and waning of sound) is produced. This is called beats.