Quantum Principal Component Analysis

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1 Introduction

Density Matrix Exponentiation

$$
\begin{aligned}
Tr_P(e^{-i\hat{S}\Delta t}\rho\otimes\sigma e^{i\hat{S}\Delta t}) &= Tr_P[\,(\cos\Delta t + i\hat{S}\sin\Delta t)\ \rho\otimes\sigma\ (\cos\Delta t - i\hat{S}\sin\Delta t)] \\
&\approx Tr_P[\,\rho\otimes\sigma - i\Delta t[\rho,\sigma] + O(\Delta t^2)] \\
&= \sigma - i\Delta t[\rho,\sigma] + O(\Delta t^2) \\
&\approx e^{-i\rho\Delta t}\ \sigma\ e^{i\rho\Delta t}
\end{aligned} \tag{1}
$$

Where Tr_P is the partial trace ρ over the first variable and $\hat{S}$ is the swap operator. Note that $\rho\otimes\sigma\hat{S} = \rho\otimes\sigma$. By repeating (1) n times,

$$
\begin{aligned}
&Tr_{P_1}Tr_{P_2}...Tr_{P_n}[\,e^{-i\hat{S}_n\Delta t}e^{-i\hat{S}_{n-1}\Delta t}...e^{-i\hat{S}_1\Delta t}(\rho^{\otimes n}\otimes\sigma)e^{i\hat{S}_1\Delta t}...e^{i\hat{S}_{n-1}\Delta t}e^{i\hat{S}_n\Delta t}] \\
&= Tr_{P_n}[\ e^{-i\hat{S}_n\Delta t}\ Tr_{P_{n-1}}[\,e^{-i\hat{S}_{n-1}}(...Tr_{P_1}[\,e^{-i\hat{S}_1\Delta t}(\rho^{\otimes n}\otimes\sigma)e^{i\hat{S}_1\Delta t}]\,)e^{i\hat{S}_{n-1}}]\ \ e^{i\hat{S}_n\Delta t}\ ] \\
&\approx e^{-i\rho n\Delta t}\ \sigma\ e^{i\rho n\Delta t}
\end{aligned} \tag{2}
$$