

Quantum Principal Component Analysis

Tzu Hsuan Chang

December 3, 2020

1 Introduction

Density Matrix Exponentiation

$$\begin{aligned} Tr_P(e^{-i\hat{S}\Delta t} \rho \otimes \sigma e^{i\hat{S}\Delta t}) &= Tr_P[(\cos \Delta t + i\hat{S} \sin \Delta t) \rho \otimes \sigma (\cos \Delta t - i\hat{S} \sin \Delta t)] \\ &\approx Tr_P[\rho \otimes \sigma - i\Delta t[\rho, \sigma] + O(\Delta t^2)] \\ &= \sigma - i\Delta t[\rho, \sigma] + O(\Delta t^2) \\ &\approx e^{-i\rho\Delta t} \sigma e^{i\rho\Delta t} \end{aligned} \tag{1}$$

Where Tr_P is the partial trace ρ over the first variable and \hat{S} is the swap operator. Note that $\rho \otimes \sigma \hat{S} = \rho \otimes \sigma$. By repeating (1) n times,

$$\begin{aligned} &Tr_{P_1} Tr_{P_2} \dots Tr_{P_n} [e^{-i\hat{S}_n \Delta t} e^{-i\hat{S}_{n-1} \Delta t} \dots e^{-i\hat{S}_1 \Delta t} (\rho^{\otimes n} \otimes \sigma) e^{i\hat{S}_1 \Delta t} \dots e^{i\hat{S}_{n-1} \Delta t} e^{i\hat{S}_n \Delta t}] \\ &= Tr_{P_n} [e^{-i\hat{S}_n \Delta t} Tr_{P_{n-1}} [e^{-i\hat{S}_{n-1} \Delta t} (\dots Tr_{P_1} [e^{-i\hat{S}_1 \Delta t} (\rho^{\otimes n} \otimes \sigma) e^{i\hat{S}_1 \Delta t}] e^{i\hat{S}_{n-1} \Delta t}] e^{i\hat{S}_n \Delta t}] \\ &\approx e^{-i\rho n \Delta t} \sigma e^{i\rho n \Delta t} \end{aligned} \tag{2}$$

2 Algorithm