Quantum Phase Estimation

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1 Introduction

Quantum phase estimation (QPE) is used to find the eigenvalues of a unitary matrix. Suppose we want to find the eigenvalues $e^{2\pi i\theta_i}$ corresponding to the eigenvector $|u_i\rangle$ of an unitary operator $\hat{U}$ such that $\hat{U}\,|u_i\rangle = e^{2\pi i\theta_i}\,|u_i\rangle$. The QPE have the following operation,

$$|0\rangle\,|u_i\rangle \longrightarrow \left|\tilde{\theta}_i\right\rangle |u_i\rangle\,, \tag{1}$$

where $\tilde{\theta}_i$ is an estimate for θ_i.

As shown on figure 1, the QPE circuit write the phase of $\hat{U}$ to n ancillary qubits $|0\rangle^{\otimes n}$ in the Fourier basis and using inverse QFT to transform them back to the computational basis. The following is the mathematical details.

Mathematical details

As shown in figure 1, assuming ψ is the eigenvector of the unitary operator $\hat{U}$ with eigenvalue $e^{2\pi i\theta}$. Initially, we have

$$\psi_0 = |0\rangle^{\otimes n}\,\psi. \tag{2}$$

After applying n-bit Hadamard gates on the ancillary qubits,

$$\psi_1 = \frac{1}{2^{n/2}}(|0\rangle + |1\rangle)^{\otimes n}\psi. \tag{3}$$

References

[1] A. Asfaw, L. Bello, Y. Ben-Haim, S. Bravyi, N. Bronn, L. Capelluto, A. C. Vazquez, J. Ceroni, R. Chen, A. Frisch, J. Gambetta, S. Garion, L. Gil, S. D. L. P. Gonzalez, F. Harkins, T. Imamichi, D. McKay, A. Mezzacapo, Z. Minev, R. Movassagh, G. Nannicni, P. Nation, A. Phan, M. Pistoia, A. Rattew, J. Schaefer, J. Shabani, J. Smolin, K. Temme, M. Tod, S. Wood, and J. Wootton. Learn quantum computation using qiskit, 2020.

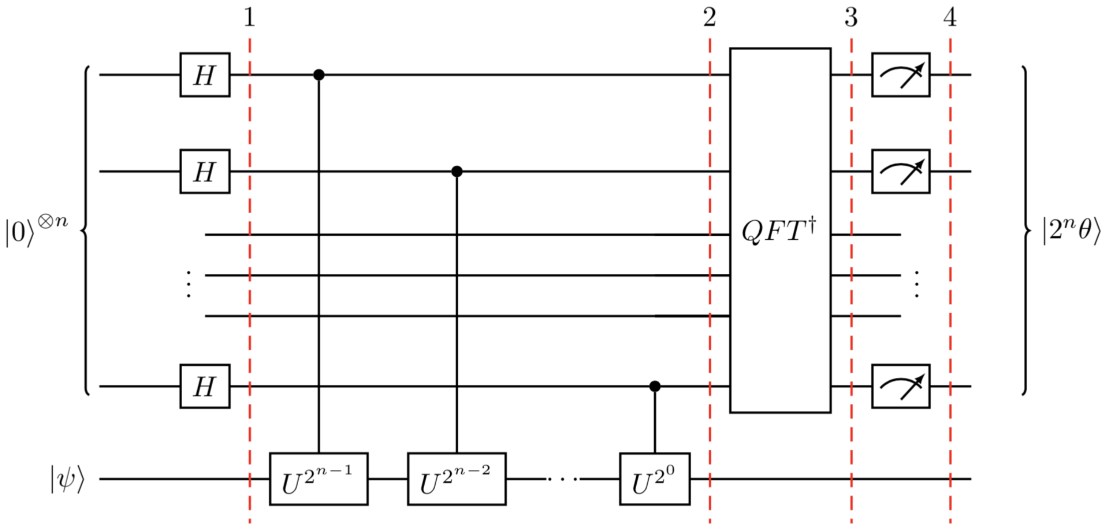

Figure 1: Quantum phase estimation circuit.[1]