Problem Sheet 0: Trigonometric Identities

Cezar Ionescu

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1. Fill in the table and prove the derived rules from the slides.

2. Simplify the following expressions:

   1) $\cos\left(\frac{\pi}{6} + x\right) - \cos\left(\frac{\pi}{6} - x\right)$

   2) $\cos(x - 330°) - \cos(120° - x) + \sin(270° - x)$

   3) $\sin\left(\frac{2\pi}{3} - x\right) + \cos\left(\frac{5\pi}{6} - x\right)$

   4) $\frac{1-\cos^2(2x)}{2\sin x}$

3. Prove the following identities:

   1) $\frac{\sin x + \cos x \tan y}{\cos x - \sin x \tan y} = \tan(x+y)$

   2) $\tan\left(\frac{\pi}{4} + x\right) = \frac{\cot x + 1}{\cot x - 1}$

   3) $\frac{\tan x}{\tan(2x)} = \frac{1}{2} - \frac{1}{2}\tan^2 x$

   4) $\tan^2\left(\frac{\pi}{4} + \frac{x}{2}\right) = \frac{1+\sin x}{1-\cos x}$

   5) $\cos x \cos y = \frac{1}{2}\left(\cos(x+y) + \cos(x-y)\right)$

   6) $\sin^2(45° + 2x) = \frac{1+\sin(2x)}{2}$

   7) $\tan^2 x = \frac{1-\cos(2x)}{1+\cos(2x)}$

   8) $\cot^2 x - \tan^2 x = \frac{4\cot(2x)}{\sin(2x)}$

   9) $\frac{\sin(2x)+\sin x}{\cos(2x)+\cos x} = \tan\left(\frac{3}{2}x\right)$

4. (Arens et al., 4.12 page 138) One of the following identities contains a typo. Correct it and prove both identities.

   a) $\sin(x+y)\sin^2\frac{x-y}{2} = \frac{1}{2}\sin(x+y) - \frac{1}{4}\sin(2x) - \frac{1}{4}\sin(2y)$

   b) $\cos(3(x+y)) = 4\cos^3(x+y) - 3\cos x\cos y - 3\sin x\sin y$