Econ 272

Homework 4

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Question 1. Consider the population regression testing the relationship whether college GPA is affected by hours of studying. $\text{GPA}_i = \beta_0 + \beta_1 \text{Study}_i + \delta \mathbf{X}_i + \epsilon_i$. $\mathbf{X}$ refers to other controls in the regression. Upon estimation in the data, the 95% confidence interval corresponding to β_1 is $(0.012, 0.024)$. Assume you have 200 degrees of freedom ($(n-k-1) = 200$).

(a) Can you reject the null hypothesis of $\beta_1 = 0$ at the 5% level of significance in a 2-sided test? What about the null hypothesis $\beta_1 = 0.02$ in a 2-sided test? Justify your reasoning.

Solution. Since $\beta_1 = 0$ is not within the interval $(0.012, 0.024)$, we reject the null-hypothesis. Since $\beta_2 = 0.02$ is within the interval $(0.012, 0.024)$, we fail to reject the null-hypothesis.

(b) Can you reject the null hypothesis of $\beta_1 = 0$ at the 1% level significance in a 2-sided test? What about the null hypothesis of $\beta_1 = 0$ at the 10% level of significance in a 1-sided test? Justify your reasoning and show any steps.

Solution.

Question 2. Consider the population regression testing the relationship whether college GPA is affected by hours of studying. $\text{GPA}_i = \beta_0 + \beta_1 \text{Study}_i + \delta \mathbf{X}_i + \epsilon_i$. $\mathbf{X}$ refers to other controls in the regression. Upon estimation in the data, $\widehat{\beta_1} = 0.025$ and the accompanying standard error is $\text{SE}(\widehat{\beta_1}) = 0.013$. Assume you have 200 degrees of freedom.

(a) Can you reject the null hypothesis of $\beta_1 = 0$ in a two-sided test at the 10 percent level of significance?

Solution. Our t-value is:

$$t_k = \frac{\widehat{\beta_1}}{\text{SE}(\widehat{\beta_1})} = \frac{0.025}{0.013} \approx 1.923$$

The critical value for a two-sided test with a 10% level of significance is $t_c \approx 1.652$. Since $t_k > t_c$, we reject the null-hypothesis.

(b) Given the estimated coefficient and standard error, what is the minimum level of Type-1 error with which you can reject the null of $\beta_1 = 0$ in a two-sided test? What would be your answer if you were considering a one-sided test? (The alternate hypothesis being $\beta_1 > 0$) Your answer in both instances should be a probability.

Solution. We can use t_k from part (a). For a two-sided test, we have that $2P(T > |t_k|) \approx 0.056$, so the minimum level of Type-1 error is 5.6%. For a one-sided test, we have $P(T > t_k) \approx 0.028$, so the minimum level of Type-1 error is 2.8%.

(c) Can you reject the null hypothesis of $\beta_1 = 0.04$ at the 10 percent level in a two-sided test? What about a one-sided test? Show your steps.

Solution. Our t-value is:

$$t_k = \frac{\widehat{\beta_1} - \beta_1}{\text{SE}(\widehat{\beta_1})} = \frac{0.025 - 0.04}{0.013} \approx -1.154$$

Using t_c from part (a), since $|t_k| \not> t_c$, we fail to reject the null-hypothesis. For a one-sided test, $t_c \approx 2.345$. Since $|t_k| > t_c$, we reject the null-hypothesis.

Question 3. Use *hw4data* for the following question. The data refers to manufacturing firms. A list of variable definitions are provided at the end of the problem set. Consider the population regression function:

$$\begin{aligned}\ln(\text{Output}_i) = \beta_0 + \beta_1 \ln(\text{Wages}_i) + \beta_2 \ln(\text{Capital}_i) + \beta_3 \ln(\text{Materials}_i) \\ + \beta_4 \text{Importer}_i + \beta_5 \text{Rural}_i + \beta_6 \text{Listed}_i + \beta_7 \text{Age}_i + \beta_8 \text{Age}_i^2 + \epsilon_i\end{aligned}$$

(a) How much additional output can a firm currently aged 5 years expect when it reaches the age of 10 years. How much additional output can a firm currently aged 50 years expect when it reaches the age of 55 years? (Note: you will first need to compute the natural log of output, wages paid, capital, and materials, and also generate the squared age variable).

Solution. From the regression, we found that $\widehat{\beta_7} = -0.0108897$ and $\widehat{\beta_8} = 0.0001206$. So we have:

$$\begin{aligned}\Delta \ln(\widehat{\text{Output}})_i &= (\widehat{\beta_7} \cdot 10 + \widehat{\beta_8} \cdot 10^2) - (\widehat{\beta_7} \cdot 5 + \widehat{\beta_8} \cdot 5^2) = -.04540269, \\ \Delta \ln(\widehat{\text{Output}})_i &= (\widehat{\beta_7} \cdot 55 + \widehat{\beta_8} \cdot 55^2) - (\widehat{\beta_7} \cdot 50 + \widehat{\beta_8} \cdot 50^2) = .00887165.\end{aligned}$$

(b) Based on your regression estimates, from what point can firm owners expect output to display an increasing relationship with age?

Solution. Consider:

$$\frac{\partial \ln(\widehat{\text{Output}_i})}{\partial \text{Age}_i} = \beta_7 + 2\beta_8 \text{Age}_i = 0$$

Solving for Age_i gives:

$$\text{Age}_i = -\frac{\beta_7}{2\beta_8}$$

This will be a local-minimum, whence after age 45 firm owners can expect output to display an increasing relationship with age.

(c) Based on your regression estimates, what will be the minimum Type-1 error you would need to tolerate to reject the null of $\beta_2 = 0.3$ using a two-sided test?

Solution. Our t-value is:

$$t_{\widehat{\beta_2}} = \frac{\widehat{\beta_2} - \beta_2}{\text{SE}(\widehat{\beta_2})} = \frac{0.207 - 0.3}{0.006} \approx -15.5$$

The probability that a Type-1 error occurs is basically 0.

(d) Based on your regression estimates, can you reject the null of $\beta_6 = -0.1$ at the 5% level using a two-sided test?

Solution. Our t-value is:

$$t_{\widehat{\beta_6}} = \frac{\widehat{\beta_6} - \beta_6}{\text{SE}(\widehat{\beta_6})} = \frac{-.0773319 + 0.1}{.0251632} \approx 0.9008$$

At the 5% level, the critical value is $t_c = 1.96$. Thus we fail to reject the null hypothesis.

(e) Based on your regression estimates, test the null hypothesis of $\beta_1 + \beta_2 + \beta_3 = 1$. Can you reject the null hypothesis at the 5% level of significance using a two-sided test? What about the 1% level of significance?

Solution. Using the *test* command in Stata gave Prob $> F = 0.0144$. So at the 5% level of significance, we reject the null hypothesis. For the 1% level of significance, we fail to reject the null hypothesis.

(f) Use an F-test to argue whether Age_i and Age_i^2 should be a part of the population regression function.

Solution. The value Stata reported is very small. This means Age_i and Age_i^2 are jointly statistically significant and should be included in the model.