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1 Categories

1.1 Basic Definitions

Definition 1.1.1. A *category* C consists of three ingredients:

- A class $\mathrm{obj}(C)$ of *objects*,
- a set of *morphisms* $\mathrm{Hom}(A,B)$ for every ordered pair (A,B) of objects, and
- *composition*: $\mathrm{Hom}(A,B) \times \mathrm{Hom}(B,C) \to \mathrm{Hom}(A,C)$ defined by $(f,g) \mapsto g \circ f$ for every ordered triple of objects.

These ingredients are subject to the following axioms:

(1) The Hom sets are pairwise disjoint, that is, each $f \in \mathrm{Hom}(A,B)$ has a unique domain and codomain.

(2) For each object A, there is an *identity morphism* $1_A \in \mathrm{Hom}(A,A)$ and $1_B \in \mathrm{Hom}(B,B)$ such that:

$$f \circ 1_A = f$$
$$1_B \circ f = f$$

for all $f : A \to B$.

(3) Composition is associative: given $A \xrightarrow{f} B \xrightarrow{g} C \xrightarrow{h} D$, then:

$$h \circ (g \circ f) = (h \circ g) \circ f.$$

Example 1.1.1.

(1)

Definition 1.1.2. A category $\mathcal{S}$ is a *subcategory* of a category C if:

(1) $\mathrm{obj}(\mathcal{S}) \subseteq \mathrm{obj}(C)$.

(2)