

Let $(a_n)_n$ be a positive sequence of real numbers which converges to zero. Let $B = \{b_1, b_2, b_3, \dots\}$ be a countable subset of $[0, 1]$. Consider the function:

$$f(x) = \begin{cases} a_n, & x = b_n \\ 0, & x \notin B. \end{cases}$$

Show that f is Riemann integrable on $[0, 1]$.