Math 548

Portfolio Assignment 1

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Problem 1. Let $T : \mathbf{R}^4 \to \mathbf{R}^4$ be an orthogonal projection to the 2-dimensional plane P spanned by the vectors $\vec{v} = (2, 0, 1, 0)$ and $\vec{w} = (-1, 0, 2, 0)$.

(1) Find (with proof) all the eigenvalues and eigenvectors, along with their geometric and algebraic multiplicities.

(2) Find the matrix T with respect to the standard basis. Is this matrix diagonalizable? Why or why not?

Proof. (1) Since T is a projection onto P, note that for any vector $v \in \mathbf{R}^4$, we have that $T(v) \in P$. Since T acts as the identity map on any vector in P, it must be the case that $T^2(v) = T(v)$.

Let w be an eigenvector of T. Since $T^2(v) = T(v)$ for all $v \in \mathbf{R}^4$, we get:

$$T(w) = \lambda w,$$
$$T(w) = T^2(w) = \lambda^2 w.$$

It must be the case that $\lambda^2 = \lambda$; i.e., $\lambda = 0$ or $\lambda = 1$.

$\square$

Problem 2.