$$V^{hfs} = \hbar A_{hfs}\, \vec{I}\cdot\vec{J} + \hbar B_{hfs} \frac{6(\vec{I}\cdot\vec{J})^2 + 3\vec{I}\cdot\vec{J} - 2\vec{I}^2\vec{J}^2}{2I(2I-1)2J(2J-1)} \quad (1)$$

$$\langle nJFM | V^{hfs} | nJFM \rangle = \frac{1}{2}\hbar A_{hfs} G + \hbar B_{hfs} \frac{\frac{3}{2}G(G+1) - 2I(I+1)J(J+1)}{2I(2I-1)2J(2J-1)} \quad (2)$$

$$G = F(F+1) - I(I+1) - J(J+1)$$

$$\vec{E} = \frac{1}{2}\vec{\mathcal{E}} e^{-i\omega t} + cc = \frac{1}{2}\mathcal{E}\vec{u} e^{-i\omega t} + cc \quad (3)$$

$$V^E = -\vec{E}\cdot\vec{d} = -\frac{1}{2}\mathcal{E}\vec{u}\cdot\vec{d} e^{-i\omega t} - \frac{1}{2}\mathcal{E}^*\vec{u}^*\cdot\vec{d} e^{i\omega t}, \quad (4)$$

Appendix A

$E_i = \hbar\omega_i$

$$\dot{\rho}_{ba} = -i(\omega_b - \omega_a - i\gamma_{ba}/2)\rho_{ba} - \frac{i}{2\hbar}\left(\mathcal{E}\vec{u} e^{-i\omega t} + \mathcal{E}^*\vec{u}^* e^{i\omega t}\right)\cdot \underset{\langle b|d|a\rangle}{d_{ba}} (\rho_{bb} - \rho_{aa}) \quad (A1)$$

$\vec{d}_{ba} = \langle b|\vec{d}|a\rangle$ $\quad \vec{d} = d_{ba}|b\rangle\langle a| + cc$ $\quad \gamma_{ba}$ linewidth $|b\rangle \leftrightarrow |a\rangle$

↳ généralement $\gamma_{ba} = \gamma_a + \gamma_b$

↳ decay rates of the population level

$\underbrace{\omega - |\omega_b - \omega_a| \gg \gamma_{ba},\ \Omega = d_{ba}\mathcal{E}/\hbar}$

↳ $\rho_{bb} \simeq 0$, $\rho_{aa} \simeq 1$ $\quad$, $\rho_{ba} = \rho^+_{ba} e^{-i\omega t} + \rho^-_{ba} e^{i\omega t}$

↳ variation lente dans le temps

$$\rho^+_{ba} = \frac{\mathcal{E}\vec{u}\cdot\vec{d}_{ba}}{2\hbar}\cdot\frac{1}{\omega_{ba} - \omega - i\gamma_{ba}/2} \qquad \rho^-_{ba} = \frac{\mathcal{E}^*\vec{u}^*\cdot\vec{d}_{ba}}{2\hbar}\frac{1}{\omega_{ba} - \omega - i\gamma_{ba}/2} \quad (A2)$$

dipole $\vec{p} \equiv \langle\vec{d}\rangle = \vec{d}_{ba}\rho_{ab} + \vec{d}_{ab}\rho_{ba} = \cancel{\frac{\vec{\mathcal{P}} e^{-i\omega t} + cc}{2}}$ $\quad \vec{\mathcal{P}} = 2(\vec{d}_{ba}\rho^{+*}_{ba} + d_{ab}\rho^+_{ba})$

$$\vec{\mathcal{P}} = \vec{d}_{ab}\frac{\mathcal{E}\vec{u}\cdot\vec{d}_{ba}}{\hbar}\frac{1}{\omega_{ba} - \omega - i\gamma_{ba}/2} + \vec{d}_{ba}\frac{\mathcal{E}\vec{u}\cdot\vec{d}_{ab}}{\hbar}\frac{1}{\omega_{ba} + \omega + i\gamma_{ba}/2}. \quad (A3)$$

Le Stark shift δE_a of the energie level of $|a\rangle$ $\quad \delta E_a = -\frac{1}{2}\overline{\vec{p}(t)\cdot\vec{E}(t)} = -\frac{1}{4}\mathrm{Re}[\vec{\mathcal{P}}\cdot\mathcal{E}^*\vec{u}^*]. \quad (A4)$

$$\delta E_a = -\frac{|\mathcal{E}|^2}{4\hbar}\mathrm{Re}\left(\frac{|\vec{u}\cdot\vec{d}_{ba}|^2}{\omega_{ba} - \omega - i\gamma_{ba}/2} + \frac{|\vec{u}\cdot\vec{d}_{ab}|^2}{\omega_{ba} + \omega + i\gamma_{ba}/2}\right) \quad (A5)$$

$$\delta E_a = -\frac{|\mathcal{E}|^2}{4\hbar}\sum_b \mathrm{Re}\left(\frac{|\langle b|\vec{u}\cdot\vec{d}|a\rangle|^2}{\underbrace{\omega_b - \omega_a}_{\omega_{ba}} - \omega - i\gamma_{ba}/2} + \frac{|\langle a|\vec{u}\cdot\vec{d}|b\rangle|^2}{\omega_{ba} + \omega + i\gamma_{ba}/2}\right) \quad (5)$$

$$\delta E_a = \langle a|V^{EE}|a\rangle \qquad V^{EE} = \frac{|\mathcal{E}|^2}{4}\left[(\vec{u}^*\cdot\vec{d})R_+(\vec{u}\cdot\vec{d}) + (\vec{u}\cdot\vec{d})R_-(\vec{u}^*\cdot\vec{d})\right] \quad (6)$$

$$R_+ = -\frac{1}{\hbar}\sum_b \mathrm{Re}\left(\frac{1}{\omega_{ba} - \omega - i\gamma_{ba}/2}\right)|b\rangle\langle b|, \quad R_- = -\frac{1}{\hbar}\sum_b \mathrm{Re}\left(\frac{1}{\omega_{ba} + \omega + i\gamma_{ba}/2}\right)|b\rangle\langle a| \quad (7)$$

$$H_{int} = V^{hfs} + V^{EE} \quad (8), \quad |(nJ)FM\rangle \equiv |nJFM\rangle, \quad V^{EE} = \sum_{FMF'M'} V^{EE}_{FMF'M'} |(nJ)FM\rangle\langle(nJ)F'M'| \quad (9)$$

$$\langle(nJ)FM|V^{EE}|(nJ)F'M'\rangle$$

$$V^{EE}_{FMF'M'} = \frac{1}{4}|\mathcal{E}|^2 \sum_{\substack{K=0,1,2\\ q=-K,\dots,K}} \alpha^{(K)}_{nJ} \{\vec{u}^* \otimes \vec{u}\}_{Kq} \times (-1)^{J+I+K+q-M} \sqrt{(2F+1)(2F'+1)} \times \begin{pmatrix} F & K & F' \\ M & q & -M' \end{pmatrix} \begin{Bmatrix} F & K & F' \\ J & I & J \end{Bmatrix} \quad (10)$$

$$\alpha^{(K)}_{nJ} = (-1)^{K+J+1}\sqrt{2K+1} \times \sum_{n'J'} (-1)^{J'} \begin{Bmatrix} 1 & K & 1 \\ J & J' & J \end{Bmatrix} |\langle n'J'\|d\|nJ\rangle|^2 \times \frac{1}{\hbar} \mathrm{Re}\left(\frac{1}{\omega_{n'J'nJ} - \omega - i\gamma_{n'J'nJ}/2} + \frac{1}{\omega_{n'J'nJ} + \omega + i\gamma_{n'J'nJ}/2} \right) \quad (11)$$

$\omega_{n'J'} - \omega_{nJ}$ $\qquad$ $\gamma_{n'J'} + \gamma_{nJ}$

avec $K = 0, 1, 2$

reduced dynamical scalar, vector, tensor polarizabilities of the atom in the fine-structure level $|nJ\rangle$

$\begin{pmatrix} j_1 & j_2 & j \\ m_1 & m_2 & m \end{pmatrix}$ Wigner 3j $\qquad$ $\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$ Wigner 6-j

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Appendix B. Cartesian coordinate $\{x, y, z\}$

$\vec{A} = (A_x, A_y, A_z)$

$A_q \equiv A_{1q} \quad q = -1, 0, 1 \qquad \vec{A} \equiv A_1$

$$\begin{cases} A_{-1} = \{A_x - iA_y\}/\sqrt{2}, \\ A_0 = A_z, \\ A_1 = -(A_x + iA_y)/\sqrt{2} \end{cases} \quad (B1)$$

irreducible tensor of rank 1

$\{\vec{A} \otimes \vec{B}\}_K$? $K = 0, 1, 2$? vary

$$\{\vec{A} \otimes \vec{B}\}_{K,q} = \sum_{q_1 q_2} C^{Kq}_{1q_1 1q_2} A_{q_1} B_{q_2} \quad (B2)$$

$$C^{jm}_{j_1 m_1 j_2 m_2} = (-1)^{j_1 - j_2 + m} \sqrt{2j+1} \begin{pmatrix} j_1 & j_2 & j \\ m_1 & m_2 & -m \end{pmatrix} \quad (B3)$$

Clebsch-Gordan coefficients.

In general an irreducible tensor product of 2 irreducible tensors U_{K_1} et V_{K_2} est définie par le tenseur irréductible $\{U_{K_1} \otimes V_{K_2}\}_K$ of rang K

$$\{U_{K_1} \otimes V_{K_2}\}_{Kq} = \sum_{q_1 q_2} C^{Kq}_{K_1 q_1 K_2 q_2} U_{K_1 q_1} V_{K_2 q_2} \quad (B4)$$

$K = |K_1 - K_2|, |K_1 - K_2| + 1, \dots, K_1 + K_2 - 1, K_1 + K_2$

$q = -K, -K+1, \dots, K-1, K$

$$(U_K \cdot V_K) = \sum_q (-1)^q U_{K,q} V_{K,-q}. \quad (B5)$$

$$(\vec{A}\cdot\vec{B})(\vec{A}'\cdot\vec{B}') = \sum_{K=0,1,2} (-1)^K \{\vec{A}\otimes\vec{A}'\}_K \cdot \{\vec{B}\otimes\vec{B}'\}_K, \quad (B6)$$

$$(6) \Longrightarrow \quad V^{EE} = \frac{|\mathcal{E}|^2}{4} \sum_{K=0,1,2} (-1)^K \{\vec{u}^*\otimes\vec{u}\}_K \cdot \left[\{\vec{d}\otimes \mathcal{R}_+ \vec{d}\}_K + (-1)^K \{\vec{d}\otimes \mathcal{R}_- \vec{d}\}_K\right]. \quad (B7)$$

$$\{\vec{u}\otimes\vec{u}^*\}_K = (-1)^K \{\vec{u}^*\otimes\vec{u}\}_K$$

$$(B5)+(B7) \Longrightarrow \quad V^{EE} = \frac{|\mathcal{E}|^2}{4} \sum_{K=0,1,2} (-1)^K \sum_q (-1)^q \{\vec{u}^*\otimes\vec{u}\}_{K,q}$$
$$\times \left[\{\vec{d}\otimes \mathcal{R}_+ \vec{d}\}_{K,-q} + (-1)^K \{\vec{d}\otimes \mathcal{R}_- \vec{d}\}_{K,-q}\right]. \quad (B8)$$

$$\{\vec{u}^*\otimes\vec{u}\}_{0,0} = -\frac{1}{\sqrt{3}}, \quad (B9)$$

$$\{\vec{u}^*\otimes\vec{u}\}_{1,0} = \frac{|u_1|^2 - |u_{-1}|^2}{\sqrt{2}},$$
$$\{\vec{u}^*\otimes\vec{u}\}_{1,1} = -\frac{u_0 u_{-1}^* + u_0^* u_1}{\sqrt{2}} \quad (B10)$$
$$\{\vec{u}^*\otimes\vec{u}\}_{1,-1} = \frac{u_0 u_1^* + u_0^* u_{-1}}{\sqrt{2}}$$

$$\{\vec{u}^*\otimes\vec{u}\}_{2,0} = \frac{3|u_0|^2 - 1}{\sqrt{6}}$$
$$\{\vec{u}^*\otimes\vec{u}\}_{2,1} = -\frac{u_0 u_{-1}^* - u_0^* u_1}{\sqrt{2}}$$
$$\{\vec{u}^*\otimes\vec{u}\}_{2,-1} = -\frac{u_0 u_1^* - u_0^* u_{-1}}{\sqrt{2}}$$
$$\{\vec{u}^*\otimes\vec{u}\}_{2,2} = -u_1 u_{-1}^* \quad (B11)$$
$$\{\vec{u}^*\otimes\vec{u}\}_{2,-2} = -u_{-1} u_1^*$$

In (7) sans énergie perturbée $\omega_{nJFM} = \omega_{nJ}$ $\gamma_{nJFM} = \gamma_{nJ}$

$V^{EE}_{FMF'M'} \equiv \langle (nJ)FM | V^{EE} | (nJ)F'M' \rangle$ élément de matrice de Stark interaction opérateur V^{EE} dans la base hyperfine $\{|(nJ)FM\rangle\}$ pour nJ fixé

$$(B8) \Longrightarrow \quad V^{EE}_{FMF'M'} = \frac{|\mathcal{E}|^2}{4} \sum_{K=0,1,2} (-1)^K \sum_q (-1)^q \{\vec{u}^*\otimes\vec{u}\}_{Kq} \times \mathcal{O}^{Kq}_{FMF'M'}, \quad (B12)$$

$$\mathcal{O}^{Kq}_{FMF'M'} = \sum_{q_1 q_2} C^{K,-q}_{1q_1 1q_2} \sum_{n''J''F''M''} \langle nJFM | d_{q_1} | n''J''F''M'' \rangle \langle n''J''F''M'' | d_{q_2} | nJF'M' \rangle$$
$$\times \mathcal{R}^{(K)}_{n''J''nJ}$$

(B13)

avec

$$\mathcal{R}^{(K)}_{n''J''nJ} = -\frac{1}{\hbar}\,\mathrm{Re}\left(\frac{1}{\omega_{n''J''}-\omega_{nJ}-\omega-i\gamma_{n''J''nJ}/2}+\frac{1}{\omega_{n''J''}-\omega_{nJ}+\omega+i\gamma_{n''J''nJ}/2}\right) \quad (B14)$$

According to the <u>Wigner-Eckart theorem</u> $\langle nJFM|T_{Kq}|n'J'F'M'\rangle$ or T_{Kq} on quantum number M, M', q

↳ Wigner 3-j symbol

$$\langle nJFM|T_{Kq}|n'J'F'M'\rangle = (-1)^{F-M}\begin{pmatrix} F & K & F' \\ -M & q & M'\end{pmatrix}\langle nJF||T_K||n'J'F'\rangle. \quad (B15)$$

$$\langle nJF||T_K||n'J'F'\rangle = \sum_{MM'q}(-1)^{F-M}\begin{pmatrix} F & K & F' \\ -M & q & M'\end{pmatrix}\langle nJFM|T_{Kq}|n'J'F'M'\rangle \quad (B16)$$

avec la convention de normalisation $|\langle nJF||T_K||n'J'F'\rangle|^2 = \sum_{MM'q}|\langle nJFM|T_{Kq}|n'J'F'M'\rangle|^2$ (B17)

et le complex conjugate relation $\langle nJF||T_K||n'J'F'\rangle = (-1)^{F-F'}\langle n'J'F'||T_K||nJF\rangle$ (B18)

↳ Prenons (Since) le dipole électrique $\vec{d}$ (tenseur de rang 1, W-E application in spherical-tensor component operators d_q

$$\langle nJFM|d_q|n'J'F'M'\rangle = (-1)^{F-M}\begin{pmatrix} F & 1 & F' \\ -M & q & M'\end{pmatrix}\underbrace{\langle nJF||\vec{d}||n'J'F'\rangle} \quad (B19)$$

invariant factor is the reduced matrix element for the electric dipole operator $\vec{d}$

$$\mathcal{O}^{Kq}_{FMF'M'} = \sum_{n''J''F''}\langle nJF||\vec{d}||n''J''F''\rangle\langle n''J''F''||\vec{d}||nJF'\rangle\,\mathcal{R}^{(K)}_{n''J''nJ}\,\mathcal{N}^{KqF''}_{FMF'M'}, \quad (B20)$$

avec $$\mathcal{N}^{KqF''}_{FMF'M'} = \sqrt{2K+1}\sum_{q_1q_2M''}(-1)^{F+F''-M-M''-q}\begin{pmatrix} 1 & 1 & K \\ q_1 & q_2 & q\end{pmatrix}\begin{pmatrix} F & 1 & F'' \\ -M & q_1 & M''\end{pmatrix}\begin{pmatrix} F'' & 1 & F' \\ -M'' & q_2 & M'\end{pmatrix}. \quad (B21)$$

use the symmetries of the 3-j symbol et the sum rule

$$\sum_{m_4m_5m_6}(-1)^{j_4+j_5+j_6-m_4-m_5-m_6}\begin{pmatrix} j_5 & j_1 & j_6 \\ m_5 & -m_1 & -m_6\end{pmatrix}\begin{pmatrix} j_6 & j_2 & j_4 \\ m_6 & -m_2 & -m_4\end{pmatrix}\begin{pmatrix} j_4 & j_3 & j_5 \\ m_4 & -m_3 & -m_5\end{pmatrix}$$
$$= \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3\end{pmatrix}\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6\end{Bmatrix} \quad (B22)$$

$$\mathcal{N}^{KqF''}_{FMF'M'} = (-1)^{K+F'+M'}\sqrt{2K+1}\begin{pmatrix} F & K & F' \\ -M & -q & M'\end{pmatrix}\begin{Bmatrix} 1 & K & 1 \\ F & F'' & F'\end{Bmatrix} \quad (B23)$$

(B23) ⊕ (B26) ⊕ (B12)

$$V^{EE}_{FMF'M'} = \frac{|\mathcal{E}|^2}{4} \sum_{K=0,1,2} (-1)^K \sum_q (-1)^q \{\vec{u}^* \otimes \vec{u}\}_{Kq} (-1)^{F-M} \begin{pmatrix} F & K & F' \\ -M & -q & M' \end{pmatrix} \alpha^{(K)}_{nJFF'} \qquad (B24)$$

avec

$$\alpha^{(K)}_{nJFF'} = (-1)^{K+F+F'} \sqrt{2K+1} \sum_{n''J''F''} \begin{Bmatrix} 1 & K & 1 \\ F & F'' & F' \end{Bmatrix} \langle nJF \| \vec{d} \| n''J''F'' \rangle \langle n''J''F'' \| \vec{d} \| nJF' \rangle \mathcal{R}^{(K)}_{n''J''nJ}$$

are the reduced scalar ($K=0$), vector ($K=1$), tensor ($K=2$) polarizability coefficient for the hfs levels within a fine-structure manifold nJ. (B25)

For the tensor T_{Kq} that do not act on the nuclear spin degrees of freedom

$$\langle nJIF \| T_K \| n'J'I'F' \rangle = \delta_{II'} (-1)^{J+I+F'+K} \times \sqrt{(2F+1)(2F'+1)} \begin{Bmatrix} F & K & F' \\ J' & I & J \end{Bmatrix} \langle nJ \| T_K \| n'J' \rangle \qquad (B26)$$

Puisque le dipôle électrique $\vec{d}$ de l'atome ne se couple pas aux degrés de liberté nucléaires et est un tenseur de rang 1, l'utilisation de (B26) pour $\vec{d}$ donne

$$\langle nJF \| \vec{d} \| n'J'F' \rangle \equiv \langle nJIF \| \vec{d} \| n'J'IF' \rangle = (-1)^{J+I+F'+1} \sqrt{(2F+1)(2F'+1)} \begin{Bmatrix} F & 1 & F' \\ J' & I & J \end{Bmatrix} \langle nJ \| \vec{d} \| n'J' \rangle \qquad (B27)$$

(B27) et (B25)

$$\alpha^{(K)}_{nJFF'} = (-1)^{K+J+2I+F+2F'} \sqrt{(2F+1)(2F'+1)} \times \sum_{n''J''} (-1)^{J''} \langle nJ \| \vec{d} \| n''J'' \rangle \langle n''J'' \| \vec{d} \| nJ \rangle \mathcal{R}^{(K)}_{n''J''nJ}$$

$$\times \sqrt{2K+1} \sum_{F''} (-1)^{F''} (2F''+1) \begin{Bmatrix} 1 & F' & F'' \\ F & 1 & K \end{Bmatrix} \begin{Bmatrix} F & 1 & F'' \\ J'' & I & J \end{Bmatrix} \begin{Bmatrix} J'' & I & F'' \\ F' & 1 & J \end{Bmatrix} \qquad (B28)$$

La sommation sur F'' dans (B28) peut être commutée en utilisant

$$\sum_k (-1)^k (2k+1) \begin{Bmatrix} j_1 & j_2 & k \\ j_3 & j_4 & j_5 \end{Bmatrix} \begin{Bmatrix} j_3 & j_4 & k \\ j_6 & j_7 & j_8 \end{Bmatrix} \begin{Bmatrix} j_6 & j_7 & k \\ j_2 & j_1 & j_9 \end{Bmatrix} = (-1)^{-j_1-j_2-j_3-j_4-j_5-j_6-j_7-j_8-j_9} \times \begin{Bmatrix} j_5 & j_8 & j_9 \\ j_6 & j_1 & j_4 \end{Bmatrix} \begin{Bmatrix} j_5 & j_8 & j_9 \\ j_7 & j_2 & j_3 \end{Bmatrix} \qquad (B29)$$

Soit

$$\alpha^{(K)}_{nJFF'} = (-1)^{I+F'-J} \sqrt{(2F+1)(2F'+1)} \sum_{n''J''} \langle nJ \| \vec{d} \| n''J'' \rangle \langle n''J'' \| \vec{d} \| nJ \rangle \mathcal{R}^{(K)}_{n''J''nJ}$$

$$\times \sqrt{2K+1} \begin{Bmatrix} 1 & K & 1 \\ J & J'' & J \end{Bmatrix} \begin{Bmatrix} F & K & F' \\ J & I & J \end{Bmatrix} \qquad (B30)$$

⊕ (B14)

$$\alpha^{(K)}_{nJFF'} = (-1)^{J+I+F'+K} \sqrt{(2F+1)(2F'+1)} \times \begin{Bmatrix} F & K & F' \\ J & I & J \end{Bmatrix} \alpha^{(K)}_{nJ} \qquad (B31)$$

$$\alpha^{(K)}_{nJ} = (-1)^{2J+K+1} \sqrt{(2F+1)} \sum_{n'J'} \begin{Bmatrix} 1 & K & 1 \\ J & J' & J \end{Bmatrix} \langle nJ \| \vec{d} \| n'J' \rangle \langle n'J' \| \vec{d} \| nJ \rangle \frac{1}{\hbar} \mathrm{Re} \left(\frac{1}{\omega_{n'J'} - \omega_{nJ} - \omega - i\gamma_{n'J'nJ}/2} + \frac{(-1)^K}{\omega_{n'J'} - \omega_{nJ} + \omega + i\gamma_{n'J'nJ}/2} \right)$$

⊕

$$\langle nJ \| \vec{d} \| n'J' \rangle = (-1)^{J-J'} \langle n'J' \| \vec{d} \| nJ \rangle \Rightarrow (11) \qquad (B32)$$

R_+

$$\alpha^{(0)}_{nJ} = \frac{2}{\hbar\sqrt{3(2J+1)}} \sum_{n'J'} |\langle n'J' \| \vec{d} \| nJ \rangle|^2 \times \frac{\omega_{n'J'nJ}(\omega^2_{n'J'nJ} - \omega^2 + \gamma^2_{n'J'nJ}/4)}{(\omega^2_{n'J'nJ} - \omega^2 + \gamma^2_{n'J'nJ}/4)^2 + \gamma^2_{n'J'nJ}\omega^2} \; ; \; \alpha^{(2)}_{nJ} = -\frac{2\sqrt{5}}{\hbar} \sum_{n'J'} (-1)^{J+J'} \begin{Bmatrix} 1 & 2 & 1 \\ J & J' & J \end{Bmatrix} |\langle n'J' \| \vec{d} \| nJ \rangle|^2 R_+$$

(B33)

$$\alpha^{(1)}_{nJ} = \frac{2\sqrt{3}}{\hbar} \sum_{n'J'} (-1)^{J+J'} \begin{Bmatrix} 1 & 1 & 1 \\ J & J' & J \end{Bmatrix} |\langle n'J' \| \vec{d} \| nJ \rangle|^2 \times \frac{\omega(\omega^2_{n'J'nJ} - \omega^2 - \gamma^2_{n'J'nJ}/4)}{(\omega^2_{n'J'nJ} - \omega^2 + \gamma^2_{n'J'nJ}/4)^2 + \gamma^2_{n'J'nJ}\omega^2}$$

On neglige la linewidths $\Gamma_{n'J'nJ}$

On utilise the l'approximation $\omega_{nJFM} = \omega_{nJ}$ on neglige les effet de hfs splitting on the polarisation

Dans la theorie des perturbation hfs splitting and Stark shift petite perturbation au meme ordre

si hfs splitting >> Stark shift Stark shift perturbation

$$V^{EE} = -\frac{1}{4}|\mathcal{E}|^2 \left\{ \alpha_F^{(s)} - i\alpha_F^{v} \frac{[\vec{u}^* \times \vec{u}] \cdot \vec{F}}{2F} + \alpha_F^{T} \frac{3[(\vec{u}^* \cdot \vec{F})(\vec{u} \cdot \vec{F}) + (\vec{u} \cdot \vec{F})(\vec{u}^* \cdot \vec{F})] - 2\vec{F}^2}{2F(2F-1)} \right\} \quad (B34)$$

element de matrix

$$V^{EE}_{MM'} = \frac{|\mathcal{E}|^2}{4} \sum_{K=0,1,2} (-1)^K \sum_q (-1)^q \{\vec{u}^* \otimes \vec{u}\}_{Kq} (-1)^{F-M} \begin{pmatrix} F & K & F \\ -M & -q & M' \end{pmatrix} \alpha_F^{(K)} \quad (B35)$$

où

$$\alpha_F^{s} = \frac{1}{\sqrt{3(2F+1)}} \alpha_F^{(0)}; \quad \alpha_F^{v} = -\sqrt{\frac{2F}{(F+1)(2F+1)}} \alpha_F^{(1)}, \quad \alpha_F^{T} = -\sqrt{\frac{2F(2F-1)}{3(F+1)(2F+1)(2F+3)}} \alpha_F^{(2)} \quad (B36)$$

and $$\alpha_F^{(K)} = (-1)^{K+F+1}(2F+1)\sqrt{2K+1} \sum_{n'J'} |\langle n'J' \| d \| nJ \rangle|^2 \sum_{F'} (-1)^{F'}(2F'+1) \begin{Bmatrix} 1 & K & 1 \\ F & F' & F \end{Bmatrix} \begin{Bmatrix} F & 1 & F' \\ J' & J & J \end{Bmatrix}^2 R^{(K)}_{n'J'nJ}$$

~~the~~

The compound tensor components $\{\vec{u}^* \otimes \vec{u}\}_{Kq}$ dans (10)

$$\{\vec{u}^* \otimes \vec{u}\}_{Kq} = \sum_{\mu,\mu' = 0,\pm 1} (-1)^{q+\mu'} u_\mu u^*_{-\mu'} \sqrt{2K+1} \begin{pmatrix} 1 & K & 1 \\ \mu & -q & \mu' \end{pmatrix} \quad (11)$$

Here $u_{-1} = (u_x - iu_y)/\sqrt{2}$, $u_0 = u_z$, $u_1 = -(u_x + iu_y)/\sqrt{2}$

$\langle n'J' \| d \| nJ \rangle$ of the electrique dipole can be obtained from the oscillator strengths

$$- f_{nJn'J'} = \frac{2m_e\omega_{n'J'nJ}}{3\hbar e^2} \frac{1}{2J+1} |\langle n'J' \| d \| nJ \rangle|^2 \quad (13)$$

or from the transition probability coefficients

$$- A_{n'J'nJ} = \frac{\omega^3_{n'J'nJ}}{3\pi\epsilon_0 \hbar c^3} \frac{1}{2J'+1} |\langle n'J' \| d \| nJ \rangle|^2 \quad (14)$$

$$V^{EE} = -\frac{1}{4}|\mathcal{E}|^2 \left\{ \alpha^s_{nJ} - i\alpha^v_{nJ} \frac{[\vec{u}^* \times u] \cdot J}{2J} + \alpha^{(T)}_{nJ} \frac{3[(\vec{u}^* \cdot \vec{J})(\vec{u} \cdot \vec{J}) + (\vec{u} \cdot \vec{J})(\vec{u}^* \cdot \vec{J})] - 2\vec{J}^2}{2J(2J-1)} \right.$$

$$\alpha^s_{nJ} = \frac{1}{\sqrt{3(2J+1)}} \alpha^{(0)}_{nJ}$$

$$\alpha^v_{nJ} = -\sqrt{\frac{2J}{(J+1)(2J+1)}} \alpha^{(1)}_{nJ}$$

$$\alpha^{(T)}_{nJ} = -\sqrt{\frac{2J(2J-1)}{3(J+1)(2J+1)(2J+3)}} \alpha^{(2)}_{nJ}$$