Annexe A

Action de $\hat{P}$ et $\hat{H}$ sur $|\{\theta_a\}\rangle$

L'état $|\{\theta_a\}\rangle$ s'écrit :

$$|\{\theta_a\}\rangle \quad = \quad \frac{1}{\sqrt{N!}} \int dx_1 \cdots dx_N \varphi_{\{\theta_a\}}(\{x_a\}) \hat{\boldsymbol{\Psi}}^\dagger(x_1) \cdots \hat{\boldsymbol{\Psi}}^\dagger(x_N)\, |\emptyset\rangle\,. \tag{A.1}$$

A.1 Action de $\hat{\boldsymbol{P}}$ sur $|\{\theta_a\}\rangle$

A.1.1 Utilisation de la définition de $\hat{P}$ intégrée par parties

Cela peut être expliqué en utilisant l'opérateur du moment $\hat{\boldsymbol{P}}$ (1.30) comme exemple. Tout d'abord, nous intégrons (1.30) par parties pour représenter $\hat{\boldsymbol{P}}$ sous la forme (avec $\hbar = m = 1$) :

$$\hat{\boldsymbol{P}} \quad = \quad i \int \left[\hat{\boldsymbol{\partial}}_x \hat{\boldsymbol{\Psi}}^\dagger(x) \right] \hat{\boldsymbol{\Psi}}(x) dx \tag{A.2}$$

A.1.2 Application à l'état à N particules

On fais agir $\hat{\boldsymbol{P}}$ de (A.2) sur l'état $|\{\theta_a\}\rangle$ de (A.1) :

$$\hat{\boldsymbol{P}}\, |\{\theta_a\}\rangle \quad = \quad \frac{i}{\sqrt{N!}} \int dx \int d^N z\, \varphi_{\{\theta_a\}}(\{x_a\}) \left[\hat{\boldsymbol{\partial}}_x \hat{\boldsymbol{\Psi}}^\dagger(x) \right] \tag{A.3}$$

$$\times \sum_{k=1}^{N} \hat{\boldsymbol{\Psi}}^\dagger(z_1) \cdots [\hat{\boldsymbol{\Psi}}(x), \hat{\boldsymbol{\Psi}}^\dagger(z_k)] \cdots \hat{\boldsymbol{\Psi}}^\dagger(z_N) |\emptyset\rangle \tag{A.4}$$

En utilisant les règles de commutation (1.17) il vient que

$$\hat{\boldsymbol{P}}\, |\{\theta_a\}\rangle \quad = \quad \frac{i}{\sqrt{N!}} \int d^N z\, \varphi_{\{\theta_a\}}(\{x_a\}) \tag{A.5}$$

$$\times \sum_{k=1}^{N} \hat{\boldsymbol{\Psi}}^\dagger(z_1) \cdots \left[\hat{\boldsymbol{\partial}}_{z_k} \hat{\boldsymbol{\Psi}}^\dagger(z_k) \right] \cdots \hat{\boldsymbol{\Psi}}^\dagger(z_N) |\emptyset\rangle. \tag{A.6}$$

Nous intégrons maintenant par parties par rapport à z_k pour obtenir

$$\hat{\boldsymbol{P}}\, |\{\theta_a\}\rangle \quad = \quad \frac{1}{\sqrt{N!}} \int d^N z \left\{ -i \sum_{k=1}^{N} \hat{\boldsymbol{\partial}}_{z_k} \varphi_{\{\theta_a\}}(\{x_a\}) \right\} \times \hat{\boldsymbol{\Psi}}^\dagger(z_1) \cdots \hat{\boldsymbol{\Psi}}^\dagger(z_N) |\emptyset\rangle. \tag{A.7}$$

A.2 Action de $\hat{\boldsymbol{H}}$ sur $|\{\theta_a\}\rangle$

A.2.1 Réecriture de l'hamiltonien

L'hamiltonien (1.39) se réécrit avec $\hbar = 2m = 1$ et $c = g/2$:

$$\hat{\boldsymbol{H}} = \int dx \left[-\left[\hat{\boldsymbol{\partial}}_x^2 \hat{\boldsymbol{\Psi}}^\dagger(x)\right] \hat{\boldsymbol{\Psi}}(x) + c\hat{\boldsymbol{\Psi}}^\dagger(x)\hat{\boldsymbol{\Psi}}^\dagger(x)\hat{\boldsymbol{\Psi}}(x)\hat{\boldsymbol{\Psi}}(x)\right] . \tag{A.8}$$

A.2.2 Application à l'état à N particules

On fais agir $\hat{\boldsymbol{H}}$ définie en (A.8) sur l'état $|\{\theta_a\}\rangle$ de (A.1) :

$$\hat{\boldsymbol{H}}\,|\{\theta_a\}\rangle = \begin{cases} -\frac{1}{\sqrt{N!}} \int dx \int d^N z\, \varphi_{\{\theta_a\}}(\{x_a\}) \left[\hat{\boldsymbol{\partial}}_x^2 \hat{\boldsymbol{\Psi}}^\dagger(x)\right] \hat{\boldsymbol{\Psi}}(x)\, \hat{\boldsymbol{\Psi}}^\dagger(z_1)\cdots\hat{\boldsymbol{\Psi}}^\dagger(z_N)|\emptyset\rangle & \text{(A.9)} \\ +\frac{c}{\sqrt{N!}} \int dx \int d^N z\, \varphi_{\{\theta_a\}}(\{x_a\})\, \hat{\boldsymbol{\Psi}}^\dagger(x)\hat{\boldsymbol{\Psi}}^\dagger(x)\hat{\boldsymbol{\Psi}}(x)\hat{\boldsymbol{\Psi}}(x)\hat{\boldsymbol{\Psi}}^\dagger(z_1)\cdots\hat{\boldsymbol{\Psi}}^\dagger(z_N)|\emptyset\rangle & \text{(A.10)} \end{cases}$$

Les règles de commutations (1.17) impliquent que

$$\begin{aligned} [\hat{\boldsymbol{\Psi}}(x), \hat{\boldsymbol{\Psi}}^\dagger(z_1)\cdots\hat{\boldsymbol{\Psi}}^\dagger(z_N)] &= \textstyle\sum_{i=0}^N \hat{\boldsymbol{\Psi}}^\dagger(z_1)\cdots\hat{\boldsymbol{\delta}}(x-z_i)\cdots\hat{\boldsymbol{\Psi}}^\dagger(z_N) \\ \left[\hat{\boldsymbol{\partial}}_x\hat{\boldsymbol{\Psi}}^\dagger(x), \hat{\boldsymbol{\Psi}}^\dagger(z)\right] &= 0 \end{aligned} \tag{A.11}$$

En utilisant ces dernier règles de commutations et la définition d'état de Fock (1.16) , la premiers partie de l'application de l'hamiltonien sur l'état $|\{\theta_a\}\rangle$, (A.10) de simplifie en

$$-\frac{1}{\sqrt{N!}} \int d^N z\, \varphi_{\{\theta_a\}}(\{x_a\}) \sum_{i=1}^N \hat{\boldsymbol{\Psi}}^\dagger(z_1)\cdots\left[\hat{\boldsymbol{\partial}}_{z_i}^2 \hat{\boldsymbol{\Psi}}^\dagger(z_i)\right]\cdots\hat{\boldsymbol{\Psi}}^\dagger(z_N)|0\rangle \tag{A.12}$$

Et en faisant deux integration par partie selon la variable z_i , cette premier partie devient

$$-\frac{1}{\sqrt{N!}} \int d^N z \sum_{i=1}^N \hat{\boldsymbol{\partial}}_{z_i}^2\, \varphi_{\{\theta_a\}}(\{x_a\})\, \hat{\boldsymbol{\Psi}}^\dagger(z_1)\cdots\hat{\boldsymbol{\Psi}}^\dagger(z_N)|0\rangle \tag{A.13}$$

Pour la seconde partie , en remarquant que les règles de commutations (1.17) impliquent que

$$[\hat{\boldsymbol{\Psi}}(x)\hat{\boldsymbol{\Psi}}(x), \hat{\boldsymbol{\Psi}}^\dagger(z)] = 2\hat{\boldsymbol{\Psi}}(x)\hat{\boldsymbol{\delta}}(x-z) \tag{A.14}$$

et en remplaçant $\hat{\boldsymbol{\Psi}}(x)$ par $\hat{\boldsymbol{\Psi}}(x)\hat{\boldsymbol{\Psi}}(x)$ dans (A.11) il vient que

$$[\hat{\boldsymbol{\Psi}}(x)\hat{\boldsymbol{\Psi}}(x), \hat{\boldsymbol{\Psi}}^\dagger(z_1)\cdots\hat{\boldsymbol{\Psi}}^\dagger(z_N)] = 2\sum_{i=0}^N \hat{\boldsymbol{\Psi}}^\dagger(z_1)\cdots\hat{\boldsymbol{\Psi}}(x)\hat{\boldsymbol{\delta}}(x-z_i)\cdots\hat{\boldsymbol{\Psi}}^\dagger(z_N) \tag{A.15}$$

et en injectant (A.11), (A.15) devient

$$[\hat{\boldsymbol{\Psi}}(x)\hat{\boldsymbol{\Psi}}(x), \hat{\boldsymbol{\Psi}}^\dagger(z_1)\cdots\hat{\boldsymbol{\Psi}}^\dagger(z_N)] = \begin{cases} 2\sum_{i=0}^N \hat{\boldsymbol{\Psi}}^\dagger(z_1)\cdots\hat{\boldsymbol{\delta}}(x-z_i)\cdots\hat{\boldsymbol{\Psi}}^\dagger(z_N)\hat{\boldsymbol{\Psi}}(x) \\ + \\ 2\sum_{i=0}^N \sum_{j=i+1}^N \hat{\boldsymbol{\Psi}}^\dagger(z_1)\cdots\hat{\boldsymbol{\delta}}(x-z_i)\hat{\boldsymbol{\Psi}}^\dagger(z_{i+1})\cdots\hat{\boldsymbol{\delta}}(x-z_j)\cdots\hat{\boldsymbol{\Psi}}^\dagger(z_N) \end{cases} \tag{A.16}$$

En utilisant la régle de comutation (A.16) et la définition de l'état de Fock (1.16) , la seconde partie de (A.10) devient

$$\frac{1}{\sqrt{N!}}\int d^N z\ 2c \sum_{1\le i<j\le N} \hat{\boldsymbol{\delta}}(z_i - z_j)\, \varphi_{\{\theta_a\}}(\{x_a\})\, \hat{\boldsymbol{\Psi}}^\dagger(z_1)\cdot\cdots\cdot \hat{\boldsymbol{\Psi}}^\dagger(z_N)|\emptyset\rangle \tag{A.17}$$

en utilisant (A.13) et (A.17) , (A.13) devient

$$\hat{\boldsymbol{H}}\,|\{\theta_a\}\rangle \quad = \quad \frac{1}{\sqrt{N!}}\int d^N z \left[\hat{\boldsymbol{\mathcal{H}}}_N\, \varphi_{\{\theta_a\}}(\{x_a\})\right]\ \hat{\boldsymbol{\Psi}}^\dagger(z_1)\cdot\cdots\cdot \hat{\boldsymbol{\Psi}}^\dagger(z_N)|\emptyset\rangle \tag{A.18}$$

avec

$$\hat{\boldsymbol{\mathcal{H}}}_N \quad = \quad -\sum_{i=1}^{N} \hat{\boldsymbol{\partial}}^2_{z_i} + 2c \sum_{1\le i<j\le N} \hat{\boldsymbol{\delta}}(z_i - z_j) \tag{A.19}$$