KPZ in 1D physics : driven-dissipative Bose gaz to relaxation of phonons for isolated Bose gas

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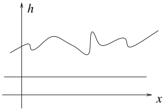
Group meeting, 18$^{\text{th}}$ of September 2023

- Observation of KPZ universal scaling in a one-dimensional polariton condensate, Q. Fontaine et al. 2022
- From GPE to KPZ : finite temperature dynamical structure factor of the 1D Bose gas, Kulkarni and Lamacraft (2012)

Outline

Growing of surface



PKZ equation

$$\partial_t h = \nu \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + \eta(x,t) \; , \; \langle \eta(x',t')\eta(x,t)\rangle = 2D\delta(x-x')\delta(t-t')$$

Without non-linear term :

$$\langle h_k^2 \rangle \rightarrow \frac{D}{\nu k^2}, \; \langle (h(x) - h(0))^2 \rangle \rightarrow \frac{D}{\nu} |x|$$

Gaussian distributions

Non-linear term : normal of the surface $\neq z$: $dh = dn \cos(\partial_x h)$

Results on KPZ

$$\partial_t h = \nu \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + \eta(x,t)$$

Mathematic results : “Exact scaling functions for one-dimensionalstationary KPZ growth”, Michael Prähofer and Herbert Spohn, (2004)

$$C(x,t) = \langle (h(x,t) - h(0,0) - \langle h \rangle_t)^2 \rangle$$

$$\begin{cases} C(x,0) = \frac{D}{\nu}|x| \\ C(x,t) \simeq \left(\frac{\lambda A^2}{2} t\right)^{2/3} g\left(\frac{x}{(2\lambda^2 A)^{1/3} t^{2/3}}\right) & , x,t \to \infty \text{ with } A = D/\nu \end{cases}$$

with

$$\begin{cases} g(y) \to c_0 > 0 \text{ for } y \to 0 \\ g(y) \simeq c_\infty |y|, \text{ for } |y| \to \infty \end{cases}$$

g exactly known (complicated)

Outline

1D Bose gas with losses and gain : Fokker Planck

Bose gas with losses

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[\hat{H}, \rho] + \Gamma \sum_r \left(\psi_r \rho \psi_r^+ - \frac{1}{2}\{\rho, \psi^+ \psi\} \right),$$

Wigner distribution :

$$W(\{\psi_r, \psi_r^*\}) = \int \prod_r d^2\lambda_r \frac{e^{-(\lambda_r \psi_r^* + \lambda_r^* \psi_r)}}{\pi^2} \chi(\{\lambda_r, \lambda_r^*\}),$$

with

$$\chi(\{\lambda_r, \lambda_r^*\}) = \mathrm{Tr}\left\{ \rho \exp\left[\sum_r (\lambda_r \hat{\psi}_r^\dagger - \lambda_r^* \hat{\psi}_r) \right] \right\}.$$

One can show

$$\left.\frac{\partial W}{\partial t}\right|_{\mathrm{Loss}} = \frac{\Gamma}{2} \sum_r \left\{ -\frac{\partial}{\partial \psi_r}\psi_r + \frac{\partial}{\partial \psi_r^*}\psi_r^* + \frac{\partial^2}{\partial \psi_r \partial \psi_r^*} \right\} W \qquad (1)$$

From Fokker-Planck to stocastic GP

Fokker-Planck equation $\Rightarrow$ stocastic equation

$$i\hbar d\psi = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}\psi + g|\psi|^2\psi - i\frac{\Gamma}{2}\psi \right) dt + d\xi,$$

where $\langle d\xi^*(z) d\xi(z') \rangle = \Gamma dt \delta(z - z')/2$

Idem for gain terms
Case of loss + gain + dissipative terms :
Altman, Diehl, Sieberer et al. PRX 2015, PRB 2015

$$\partial_t \psi = -\frac{\delta H_d}{\delta \psi^*} - i\frac{\delta H_c}{\delta \psi^*} + \eta$$

$$H_l = \int dx \left(r_l |\psi|^2 + K_l |\partial_x \psi|^2 + \frac{1}{2} u_l |\psi|^4 \right)$$

From stocastic GP to KPZ

Pase/density representation : $\psi = \sqrt{\rho} e^{i\theta}$

$$\text{SGP} \Rightarrow \begin{cases} \partial_t \theta = ... + \frac{1}{2} \frac{\partial_x^2 \sqrt{\rho}}{\sqrt{\rho}} \\ \frac{1}{2\rho} \partial_t \rho = -r_d \rho ...... \end{cases}$$

Key point : take $\partial_t \rho \simeq 0$, neglect quantum pressure

Then one obtain

KPZ for θ

$$\partial_t \theta = \nu \partial_x^2 \theta + \frac{\lambda}{2} (\partial_x \theta)^2 + \eta$$

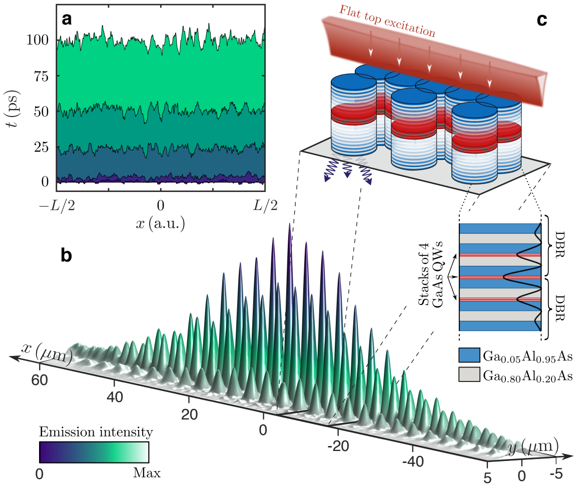
Ingredient : Adiabatic following for ρ, saturation of the gain (or effective 2 body losses) for the term in ν

Equation first derived by Altman, Diehl and co-authors in 2015

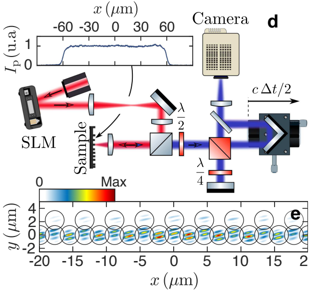
Outline

The polariton experiment at C2N

- A 1D lattice (3 sites per cell)
- A flat pump



Measure of $g^{(1)}$ function



Interferometer :

$$I(x,y) = |\psi(x)e^{iky} + \psi(-x)|^2 = |\psi(x)|^2 + |\psi(-x)|^2 + 2\mathcal{R}_e\left(\psi(x)^*\psi(-x)e^{-iky}\right)$$

Visibilité des franges : $V = 2\frac{\langle\psi^*(x)\psi(-x)\rangle}{|\psi(x)|^2+|\psi(-x)|^2}$

$$\boxed{\Rightarrow \text{measure of } g^{(1)}(2x,\Delta t) = \langle\psi^*(x,t)\psi(-x,t+\Delta t)\rangle / \sqrt{I(x)I(-x)}}$$

From $g^{(1)}$ to θ

Density fluctuation neglected :

$$g^{(1)}(x) = \langle e^{i(\theta(x)-\theta(0))^2} \rangle$$

Expansion small fluctuations :

$$\langle e^{i(\theta(x)-\theta(0))^2} \rangle \simeq 1 - \frac{1}{2} \langle (\theta(x) - \theta(0))^2 \rangle \simeq e^{-\frac{1}{2}\langle (\theta(x)-\theta(0))^2 \rangle}$$

NB : exact if gaussian fluctuations

Phase correlation measurement

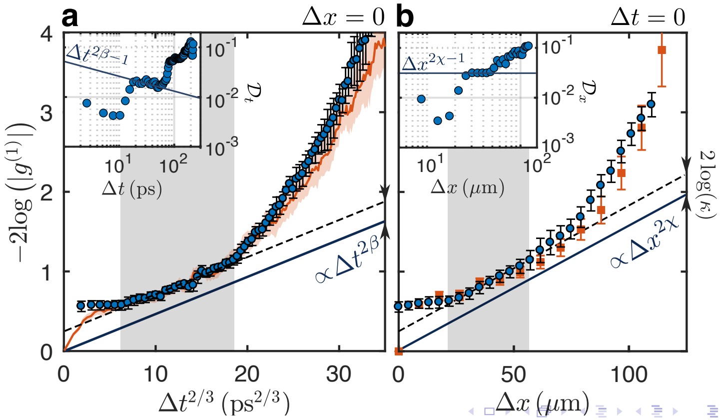
$$\langle (\theta(x,t) - \theta(0,0))^2 \rangle$$

The quantity considered in KPZ scaling (up to the drift !)

Observation of KPZ scaling

Recall :

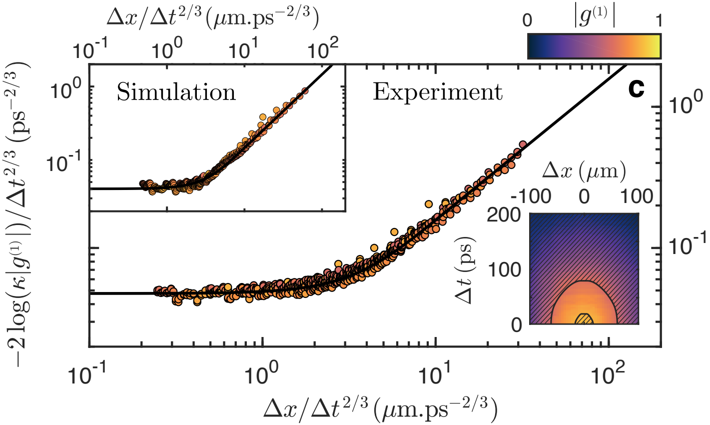
$$\langle (h(x,t) - h(0,0) - \langle h \rangle_t)^2 \rangle = \simeq \left(\frac{\lambda A^2}{2} t\right)^{2/3} g\left(\frac{x}{(2\lambda^2 A)^{1/3} t^{2/3}}\right) \quad , x, t \to \infty$$



KPZ function

Recall :

$$\langle (h(x,t) - h(0,0) - \langle h \rangle_t)^2 \rangle =\simeq \left(\frac{\lambda A^2}{2} t\right)^{2/3} g\left(\frac{x}{(2\lambda^2 A)^{1/3} t^{2/3}}\right) \;, x, t \to \infty$$



Outline

From KPZ to noisy hydrodynamic equation

$$\partial_t \theta = \nu \partial_x^2 \theta + \frac{\lambda}{2} (\partial_x \theta)^2 + \eta$$

$v = \partial_x \theta$

$$\partial_t v = \nu \partial_x^2 v + \frac{\lambda}{2} \partial_x v^2 + \eta$$

$$\partial_t v + = \nu \partial_x^2 v + \frac{\lambda}{2} \partial_x v^2 + \eta$$

Case $\lambda = -1$:

Noisy burger equation

$$\partial_t v + v \partial_x v = \nu \partial_x^2 v + \eta$$

Outline

Hydro equation for conservative 1D gas

1D Bose gas withut dissipation. Classical field

$$i\partial_t\psi = -\frac{1}{2}\partial_x^2\psi + g|\psi|^2\psi$$

Density/phase representation (madelung transformation to look smart) :

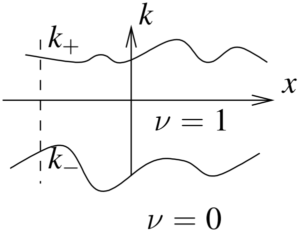
$$\begin{cases} \partial_t\rho + \partial_x(v\rho) = 0 \\ \partial_t v + v\partial_x v + g\partial_x\rho = \frac{1}{2}\partial_x\left(\frac{\partial_x^2\sqrt{\rho}}{\sqrt{\rho}}\right) \simeq 0 \text{ for long wavelengths} \end{cases}$$

Rieman invariant

$v_\pm = v \pm c\sqrt{\rho/\rho_0}$

$$\partial_t(v \pm 2c\sqrt{\rho/\rho_0}) + v_\pm\partial_x(v \pm 2c\sqrt{\rho/\rho_0}) = 0$$

Rieman equations recovered from GHD



GHD equations

$$\partial_t \nu + v_{\text{eff}} \partial_x \nu = 0$$

$$\Rightarrow \partial_t k_+ + v_{\text{eff}}(k_+) \partial_x k_+ = 0$$

$$(k_+ + k_-)/2 = v$$
$$k_+ - k_- = f(\rho)$$

Limite qBEC :

$$\begin{cases} f(\rho) = 4\sqrt{g\rho} \\ v_{\text{eff}}(k_+) = v + \sqrt{g\rho} \end{cases}$$

$\Rightarrow$ Rieman equation recovered

From Rieman to noisy burger

Another form of Rieman equations

$$\partial_t v_+ + v_+ \partial_x v_+ = \frac{1}{3} \left(\partial_t + v_+ \partial_x \right) v_-$$

Idea :

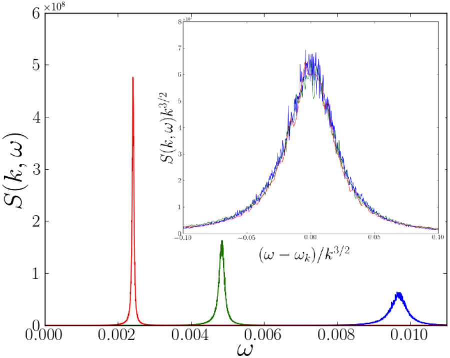
- r.h.s produces noise
- second derivative term arrises upon coarse graining

$\Rightarrow$ Noisy hydrodynamic equation $\Rightarrow$ KPZ physics ! ! !

Dynamical structure factor and KPZ

Result from Kulkarni and Lamacraft (2012)

$$S(k,\omega) = \langle \rho(k,\omega)\rho(0,0)\rangle = \frac{1}{\Gamma_k} f_{KPZ}\left(\frac{\omega \pm c|k|}{\Gamma_k}\right), \Gamma_k \propto \sqrt{T|k|^3}$$



Relaxation of phonons in 1D : link to KPZ

$$\Gamma_k = \alpha\sqrt{T}|k|^{3/2}$$

Result previously obtained by different means :

- Andreev , 1980
- Self-consistent Born Approximation (Buchhold, Diehk (2015))
- ...

A surprising result : Micheli and Scott (2022)

$$\Gamma_k = T\sqrt{\frac{g}{\rho}} f(k/\sqrt{g\rho})$$

Verified by numerics.

Effect of highly out-of-equilibrium ?