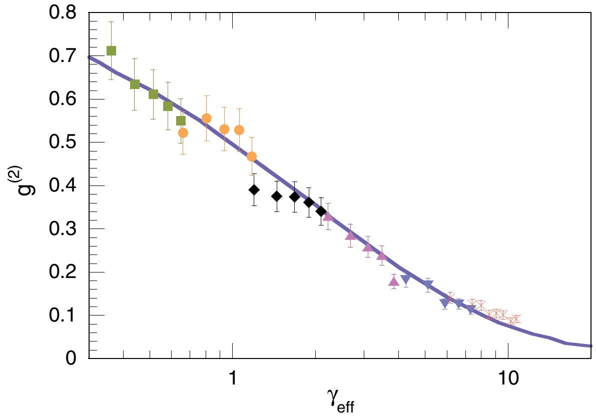

FIG. 4 (color online). The local pair correlation function vs the coupling strength. The solid blue line is the 1D Bose gas theory [10]. The points and associated error bars are generated from the same data used in Fig. 3. Here, $g^{(2)}$ is calculated for each point and the points are arranged according to the coupling parameter γ_{eff}. The data labels correspond to those in Fig. 3. A scale factor proportional to K_{3D} has been determined by a weighted least squares fit of the data to the theory. This value of K_{3D} accords with our direct measurement of K_{3D} using 3D BECs.

The solid curve in Fig. 4 is the result of the zero temperature 1D Bose gas theory for $g^{(2)}(\gamma)$ [10]. We use Eqs. (4) and (5) to plot the data from Fig. 3 (with corresponding labels) in Fig. 4. K_{3D} is left as a free parameter, so that it acts as a scaling factor for the data. A weighted least squares fit to the theory determines $K_{3D} = 4.3 \times 10^{-10}$ cm^3/s. Over our measured range between $\gamma_{\text{eff}} = 0.37$ and 11, $g^{(2)}(\gamma_{\text{eff}})$ varies by an order of magnitude. The agreement between theory and experiment is excellent over the whole range of γ_{eff}. In the weak coupling limit, $g^{(2)}$ approaches one, like a 3D BEC. Strong coupling makes $g^{(2)}$ approach zero, showing that strongly interacting bosons act like fermions.

Our experiment provides a more direct way to measure K_{3D}, using the Γ_L results for the 3D BECs, and Eq. (5) with $g^{(2)}$ set equal to 1. Averaging these results, we determine $K_{3D} = 4.7 \times 10^{-10}$ cm^3/s, with a statistical standard deviation ($\sigma_{K\text{st}}$) of 0.3×10^{-10} cm^3/s. The systematic uncertainty in this measurement of K_{3D} is 0.6×10^{-10} cm^3/s, larger than $\sigma_{K\text{st}}$, primarily due to our ± 5 μm uncertainty in the crossed dipole beam waist, which affects $\langle n_{3D} \rangle$ in Eq. (4).

We can compare the direct measurement of K_{3D} to the value determined from Fig. 4 in order to test the 1D Bose theory without any free parameters. Because the systematic uncertainty in the direct measurement of K_{3D} is highly correlated with the systematic uncertainty in $\langle n_{3D} \rangle$ for the 1D Bose gas, the systematic uncertainty in the scale for $g^{(2)}(\gamma_{\text{eff}})$ turns out to be less than 1% ($0.1 \times \sigma_{K\text{st}}/K_{3D}$). The no free parameter test is thus quite robust against systematic errors. The two separate determinations of K_{3D} agree to within 9%, or $1.3 \times \sigma_{K\text{st}}$. We have thus tested the 1D Bose theory to within this uncertainty.

In conclusion, we have created a 1D Bose gas, which is a rare example of an exactly theoretically solvable many-body system. The central result of the solutions, that bosonic wave functions overlap progressively less as the strength of their interactions is increased, is quantitatively confirmed in the experiment. The specific technique used here could also be used to study pair correlations at nonzero temperatures [30], in 2D Bose gases [31], and in a wide variety of lattice gases. The success of the experiment presented here suggests that similar experiments might be used to find the solutions of previously unsolvable many-body models [2–6].

We acknowledge discussions with Ken O'Hara and Kurt Gibble, and financial support from the National Science Foundation, Grant No. PHY-0457206.

[1] D. Jaksch and P. Zoller, Ann. Phys. (N.Y.) **315**, 52 (2005).
[2] A. Sørensen and K. Mølmer, Phys. Rev. Lett. **83**, 2274 (1999).
[3] J. Garcia-Ripoll, M. Martin-Delgado, and J. Cirac, Phys. Rev. Lett. **93**, 250405 (2004).
[4] L.-M. Duan, E. Demler, and M. Lukin, Phys. Rev. Lett. **91**, 090402 (2003).
[5] W. Hofstetter *et al.*, Phys. Rev. Lett. **89**, 220407 (2002).
[6] E. Demler and F. Zhou, Phys. Rev. Lett. **88**, 163001 (2002).
[7] E. H. Lieb and W. Liniger, Phys. Rev. **130**, 1605 (1963).
[8] C. N. Yang and C. P. Yang, J. Math. Phys. (N.Y.) **10**, 1115 (1969).
[9] M. Girardeau, J. Math. Phys. (N.Y.) **1**, 516 (1960).
[10] D. Gangardt and G. Shlyapnikov, Phys. Rev. Lett. **90**, 010401 (2003).
[11] M. Greiner *et al.*, Nature (London) **415**, 39 (2002).
[12] B. Paredes *et al.*, Nature (London) **429**, 277 (2004).
[13] T. Kinoshita, T. Wenger, and D. S. Weiss, Science **305**, 1125 (2004).
[14] H. Moritz *et al.*, Phys. Rev. Lett. **91**, 250402 (2003).
[15] T. Stöferle *et al.*, Phys. Rev. Lett. **92**, 130403 (2004).
[16] B. Laburthe-Tolra *et al.*, Phys. Rev. Lett. **92**, 190401 (2004).
[17] V. Dunjko, V. Lorent, and M. Olshanii, Phys. Rev. Lett. **86**, 5413 (2001).
[18] T. Kinoshita, T. Wenger, and D. S. Weiss, Phys. Rev. A **71**, 011602(R) (2005).
[19] M. Olshanii, Phys. Rev. Lett. **81**, 938 (1998).
[20] M. Yasuda and F. Shimizu, Phys. Rev. Lett. **77**, 3090 (1996).
[21] W. Ketterle and H.-J. Miesner, Phys. Rev. A **56**, 3291 (1997).
[22] C. McKenzie *et al.*, Phys. Rev. Lett. **88**, 120403 (2002).
[23] G. B. Partridge *et al.*, cond-mat/0505353.
[24] M. Greiner *et al.*, Phys. Rev. Lett. **94**, 110401 (2005).
[25] S. Fölling *et al.*, Nature (London) **434**, 481 (2005).
[26] M. Theis *et al.*, Phys. Rev. Lett. **93**, 123001 (2004).
[27] J. L. Bohn and P. S. Julienne, Phys. Rev. A **56**, 1486 (1997).
[28] A. Fioretti *et al.*, Eur. Phys. J. D **15**, 189 (2001).
[29] M. T. DePue *et al.*, Opt. Commun. **180**, 73 (2000).
[30] K. V. Kheruntsyan *et al.*, Phys. Rev. Lett. **91**, 040403 (2003).
[31] D. Rychtarik *et al.*, Phys. Rev. Lett. **92**, 173003 (2004).