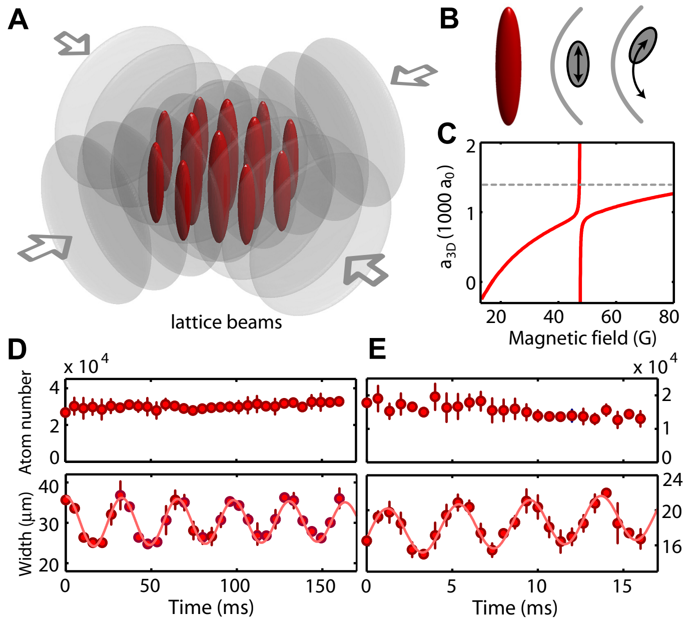

FIG. 1: **A**, Experimental setup. The lattice potential is created by two retro-reflected laser beams confining the atoms to an array of one-dimensional tubes with equipotential surfaces shown in red. **B**, Along each tube (left) we excite the lowest compressional mode (center) and compare its frequency to the dipole mode (right). **C**, The strength of the interatomic interaction is adjusted by tuning the s-wave scattering length $a_{3\mathrm{D}}$. The background scattering length rises gently from 0 to 1240 a_0 when the magnetic field B is tuned from 17 to 76 G. Further tuning is possible near a Feshbach resonance at $47.78(1)$ G to absolute values beyond 4000 a_0. The dashed line indicates $a_\perp/C$ for a transversal trap frequency of $\omega_\perp = 2\pi \times 13.1$ kHz. **D** and **E** present typical data sets for the compressional mode in the TG and sTG regime at $a_{3\mathrm{D}} = 875(1)$ a_0 and $a_{3\mathrm{D}} = 2300(200)$ a_0, respectively. The upper panels show the atom number, the lower panels show the $1/e$-cloud-width after time-of-flight. The solid lines in the lower panels are sinusoidal fits (see online material), yielding the oscillation frequencies $\omega_C = 2\pi \times 30.6(3)$ Hz and $\omega_C = 2\pi \times 241(1)$ Hz, respectively.

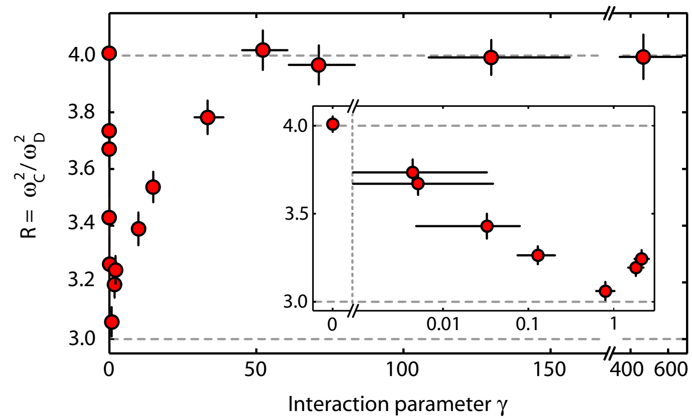

FIG. 2: Transition from the non-interacting regime via the mean-field TF regime into the TG regime. The squared frequency ratio $R = \omega_C^2/\omega_D^2$ of the lowest compressional mode with frequency ω_C and the dipole mode with frequency ω_D serves as an indicator for the different regimes of interaction. For increasing interactions from $\gamma = 0$ to $\gamma \approx 500$ the system passes from the ideal gas regime ($R = 4$) to the 1D TF regime ($R \approx 3$) and then deeply into the TG regime ($R = 4$). The inset shows the transition from the non-interacting regime to the mean-field regime in more detail. The vertical error bars refer to standard error and the horizontal error bars reflect the uncertainty in determining $a_{1\mathrm{D}}$ and $n_{1\mathrm{D}}$ (see online material). The horizontal error bar on the data point at $\gamma = 0$ (not shown in the inset) is ± 0.03 a_0.

phase, the sTG gas, should be accessible [13]. Is this excited phase stable, i.e. does it exist at all? The expectation is that the large kinetic energy inherited from the TG gas, in a Fermi-pressure like manner, prevents the gas from collapsing [20]. This stability can most simply be inferred from a Bethe-ansatz solution to the Lieb-Liniger model with attractive interactions [20, 21]. This ansatz yields for the energy per particle $E/N \approx \hbar^2\pi^2 n_{1\mathrm{D}}^2/[6m(1-n_{1\mathrm{D}}a_{1\mathrm{D}})^2]$, corresponding to the energy of a gas of hard rods [1], for which $a_{1\mathrm{D}}$ represents the excluded volume. This results in a positive inverse compressibility and also in an increased stiffness of the systems as long as $n_{1\mathrm{D}}a_{1\mathrm{D}}$ is sufficiently small. Interestingly, in this phase the density correlations are even stronger than in the TG gas, as they show a power-law decay that is slower than for a TG gas [13], indicating an effective long-range interaction.

We realize the crossover all the way from a non-interacting gas via the 1D mean-field Thomas-Fermi (TF) regime to a TG gas and then to a sTG gas. We exploit the fact that our 1D systems possess weak harmonic confinement along the axial direction characterized by the confinement length $a_\parallel$. Whereas the frequency ω_D of the lowest dipole mode depends only on the confinement, the frequency ω_C of the lowest axial compressional mode is sensitive to the various regimes of interaction [16]. For the non-interacting system one expects $R \equiv \omega_C^2/\omega_D^2 = 4$. This value then changes to $R = 3$ for weakly repulsive interactions in a 1D TF regime [7]. For increasing positive interaction strength, R is expected to change smoothly to 4 when entering the TG regime as the system becomes fermionized and hence effectively non-interacting. A rise beyond the value of 4, after crossing the CIR, would then constitute clear evidence for the sTG regime [13]. As $a_{1\mathrm{D}}$ is further increased, the system will finally become unstable and R is expected to turn over and drop towards zero. For a harmonically confined system, the point of instability is reached when the overall length of the system of hard rods, $Na_{1\mathrm{D}}$, becomes of the order of the size $\sqrt{N}a_\parallel$ for the wave function of N non-interacting fermions, i.e. $A \equiv Na_{1\mathrm{D}}/(\sqrt{N}a_\parallel) \approx 1$. We use A^2 as an alternative parameter to γ to characterize the strength of the interaction as it accounts for the harmonic confinement.

We start from a 3D Bose-Einstein condensate (BEC) with up to 2×10^5 Cs atoms with no detectable thermal fraction in a crossed-beam dipole trap with magnetic levitation [22]. Depending on the interaction regime to be studied, we then set the number of atoms in the BEC to values in the range of $(1-4) \times 10^4$ by means of forced radio-frequency evapora-