*Introduction

Physical systems of many identical particles behave very differently depending on the distance and time scales at which they are probed. In a very dilute gas, on time scales not larger than the typical time between collisions, the particles are essentially non-interacting. Then two clouds of fluid can collide and simply pass through each other; one example of such phenomenon, familiar from astrophysics, is the one of clouds of stars in colliding galaxies. In contrast, on time scales much longer than the collision time, particles typically undergo a very large number of collisions, so that the fluid has time to locally relax to an equilibrium state. This local relaxation gives rise to hydrodynamic behavior, which is typically much more complex, non-linear, than simple free propagation. For example, one can think of two droplets of water that collide: those will not simply pass through each other. More likely their motion will be more complex, for instance they will coalesce brazier1972interaction.

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Fluid dynamics at short times is captured by an evolution equation for the phase-space density of particles $\rho(x,p,t)$ which takes the form of a free transport equation, or collisionless Boltzmann equation. Typically,

equation eq:liouville $\partial\partial t\rho(x,p,t) + v(p)\partial\partial x\rho(x,p,t) - \partial V(x)\partial x\partial\partial p\rho(x,p,t) \;=\; 0.$