Domain Wall Dynamics for a 1D Bose gas

Léa Dubois[1], G. Themèze, J. Dubail and I. Bouchoule,

1 Laboratoire Charles Fabry, Institut d'Optique, Université Paris-Saclay
* lea.dubois@universite-paris-saclay.fr

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Abstract

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1 Introduction

Gaining insight on the out-of-equilibrium dynamics of many-body quantum systems is the tremendously difficult and it is the goal of an active research field. One particular class of systems where important progress have been done is the class of integrable one-dimensional systems. Owing to their infinite number of local conserved charges, to describe the local properties of equilibrium states, that arrise after relaxation, one needs a whole function, the rapidity distribution[]. The latter can be viewed as the distribution in velocity space of the infinite-lifetime quasi-particles in the system. Large scale dynamics is accounted for by a generalized hydrodynamic (GHD) effective theory[], which assume local equilibrium. A paradigmatic situation that can be handled by this theory is the dynamics induced by a partite quench [1,2], dubbed domain-wall protocol in this paper. In this protocol the Hamiltonian governing the dynamic is translation invariant but the initial state is the junction of two-semifinite homogeneous systems prepared each in a different equilibrium state of the Hamiltonian. The GHD theory predicts that, at time long enough such that diffusion effects become negligible [3] and Euler-scale hydrodynamics is valid, the time evolution is ballistic. An interesting feature of this protocol is that the local state, within the merging region, is expected to presents features caracteristic of zero-temperature systems. Thus this protocol could be used to reveal power-law singularities of correlation function characteristic of zero-temperature Luttinger liquid [4], providing a local probe is performed.

In this paper, we experimentally realize an instance of the domain-wall protocol using an ultra-cold atomic Bose gas, well described by the Lieb-Liniger model of one-dimensionnal Bosons with contact repulsive intercations [5,6], which is an integrable model. The domain wall consists in our experiment in the junction of a gas prepared in an equilibrium state on the one side, and the vaccuum on this other side. It is prepared starting from an homogeneous cloud, by the sudden removal of its left part. For different evolution times, we record the density profile of the border between the two zones, dubbed the border profile. We find that the border profile shows a ballistic behavior, as expected from GHD theory at Euler scale.

The border profile, for clouds prepared with deep evaporative cooling, is in fair agreement with GHD predictions assuming the semin-inifite gas is in its ground state, altgough deviations are present. We show that, from the border profile, it is in principle possible to reconstruct the rapidity distribution characterising the initial gas. This protocol can thus be used as a generalized thermometry. However, the reconstruction method suffers from a high sensitivity to experimental noise in the tail of the border profile which prevent us to reconstruct faithfully the initial rapidity distribution. Instead we use anstaz parametrized by a few parameters to extract the rapidity distributions of the initial gas from a fit to the border profile.

Finally, we use a newly developed techniques [7] to probe the local rapidity distribution within the border. The latter is expected to be highly asymetric for an initial state whose rapidity distribution is substantially broader and smoother than that of the ground state: while one of its border reflects the broad character of the initial rapidity distribution, the other border present the sharp feature expected for the ground state. Our expereimental data show such an asymetric behavior, although the above feature is softened by the finite spatial resolution of the local rapidity distribution measurement.

2 Experimental setup

[Premier jet : Léa]

2.1 Atom chip experiment

- Explain that the shape of the longitudinal potential locally at the center of the chip can be developed in polynomials. We can restrict ourselves to the first 4 predominant terms such as the potential is writen $V(x) = \sum_{i=1}^{4} a_i x^i$. Each coefficient a_i can be adjusted by changing the currents flowing through the four wires that generate the longitudinal potential.

2.2 Producing a initially semi infinite homogeneous 1D Bose gas

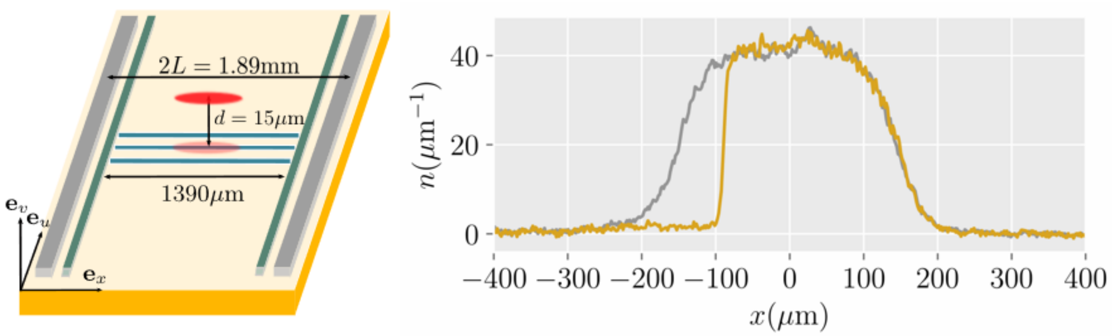

Figure 1: (a) — (b) The grey curve represents the linear density profile of gas confined within a quartic potential. The atomic cloud is then illuminated during 30μs by a near resonant light beam, shaped using a DMD. The resulting density profile is depicted in yellow.

3 GHD predictions

Upon time evolution, the initial sharp border of the cloud broadens and time derivative of local quantities decrease. After some time, upon coarse graining in position an time, one expects that the gas can locally be described by equilibirum states. Equilibrium states of the Lieb-Liniger model are entirely characterized by their rapidity distribution $\rho(\theta)$. Equivalently, equilibrium states can be parametrized by a function $\nu(x,t,\theta)$ dubbed the occupation factor distribution which takes values between 0 and 1 and which is related to ρ by $\nu(\theta) = \rho(\theta)/\rho_s(\theta)$, where $\rho_s(\theta) = 1/(2\pi)\left(1 + \int d\theta' \Delta(\theta-\theta')\rho(\theta')\right)$ and the function Δ is $\Delta(\Theta) = 2g/(g^2/\hbar + \hbar\Theta^2)$. The functions ν and ρ are in one-to-one correspondance and in the following we use either ρ or ν. Since local equilibrium is assumed, the system as a whole is described by a time and position dependent rapidity distribution $\rho(x,t,\theta)$ or equivalently by the occupation factor distribution $\nu(x,t,\theta)$. The former is particularly usefull to extract the linear density, which reads $n(x,t) = \int d\theta \rho(x,t,\theta)$.

The GHD theory provides a prediction for the time-evolution of ρ, or equivalently of ν. At large enough length scales, it reduces to its Euler-Scale approximation which, written in terms of ν, takes the convective form

$$\frac{\partial \nu}{\partial t} + v^{\mathrm{eff}}_{[\nu]} \frac{\partial \nu}{\partial x} = 0 \tag{1}$$

where the effective velocity $v^{\mathrm{eff}}_{[\nu]}$ is a functional of the local rapidity distribution which fulfills, for any rapidity θ, $v^{\mathrm{eff}}_{[\nu]}(\theta) = \theta - \int \frac{d\theta'}{2\pi} \Delta(\theta-\theta'')\rho(\theta')\left(v_{\mathrm{eff}}(\theta) - v_{\mathrm{eff}}(\theta')\right)$. For an initial domain-wall state whose discontinuity is located on $x = 0$, the solution of (**??**) is invariant along rays of constant velocity x/t and we indroduce the occupation factor distribution of the rays $\nu^*(v,\theta)$ such that

$$\nu(x,t,\theta)=\nu^*(x/t,\theta). \tag{2}$$

This equation implies that all local properties of the gas depend on x and t only through the quantity x/t. For the domain wall situation considered in this paper with, initially, a vaccum state for negative x and a state of occupation factor distribution ν_0 one the right, the function $\nu^*(v,\theta)$ is parametrised by an edge rapidity θ^* according to

$$\nu^*(v,\theta)=\left\{\begin{array}{l}\nu_0(\theta) \text{ if } \theta<\theta^* \\ 0 \text{ if } \theta>\theta^*\end{array}\right. \quad \text{where } v^{\text{eff}}_{[\nu^*]}(\theta^*)=v. \tag{3}$$

This equations can be solved numerically if the initial distribution $\nu_0(\theta)$ is known. Together with Eq.(2), it entirely describes the system after the Euler-scale has been reached. Note that to compute the linear density $n(x,t)$ in order to compare to experiments, one uses the relation $n(x,t)=\int d\theta\rho(x,t,\theta)$, such that the rapidity distribution needs to be computed from the knowledge of $\nu(x,t,\theta)$.

Solution for a system initially in the ground state. As an example, let us derive some implications of the above equations in the case the initial state is the ground state. The initial occupation factor distribution is then a Fermi sea. More precisely $\nu_0(\theta)=1$ if $|\theta|<\theta_m$ and zero otherwise, where the fermi radius θ_m depends on the initial linear density n_0. Some general features of the state of the system after the Euler-scale has been reached can be identified. The border has well defined fronts both in the empty region of $x<0$ and in the region of density n_0 for $x>0$. In the region $x<0$, the front is at $x/t=\theta_m$, since the effective velocity of a vanishingly narrow fermi sea is equal to its mean rapidity. The density vanishes for $x/t<-\theta_m$. In the region $x>0$, the front is at $x/t=c$, where $c=v^{\text{eff}}_{[\nu_0]}(\theta_m)$ is the speed of sound for the density n_0. For $x/t>c$, the system is not yet affected by the border deformation. Finally, for any x/t, the local state is a fermi sea, displaced by some quantity $v(x/t)$ in rapidity space, which corresponds to a local galilean boost of velocity $v(x/t)$. Exact solution can be derived in the two asymptotic regimes large and small densities.

In the hard core limit $\gamma=g/n_0\gg 1$, Eq.(3) is easily solved using the fact that, in this regime, $v^{\text{eff}}(\theta)=\theta$ regardless of the occupation factor distribution. We then use Eq.(2) and the fact that a fermi sea of radius θ_m corresponds to a linear density $n=m\theta_m/(\pi\hbar)$ in this regime to derive

$$n(x,t)=\frac{n_0}{2}\left(1+\frac{x}{t}\frac{m}{\pi\hbar n_0}\right). \tag{4}$$

We recover the results expected for a gas of free fermions, as expected from the mapping of the hard-core bosons to fermions, which presereves the density [8].

In the quasi-BEC regime $\gamma=g/n_0\ll 1$, we sole Eq. (3) using the fact in this regime that the effective velocity at the border of a Fermi sea of radius θ_m is $\theta_m/2$, in the frame where the Fermi sea is at rest. Then, using Eq.(2) and the fact that, in this regime, a fermi sea of radius θ_m corresponds to a linear density $n=m\theta_m^2/(4g)$, we obtain

$$n(x,t)=\frac{n_0}{9}\left(\frac{x}{t}\frac{\hbar}{\sqrt{mgn_0}}+2\right)^2. \tag{5}$$

We recover here the hydrodynamic predictions derived from the Gross-Pitaevski equation [9, 10], as exepcted since this classical field approach be a good description of the system in this regime.

4 Experimental data

[Premier jet : Léa] After preparing the initial state, the longitudinal confinement is removed while the transverse confinement is maintained to study the one-dimensional dynamics. The edge density profiles for different evolution times are shown in Fig.2 and represented in function of x/τ. The profiles overlap remarkably well, the ballisitic evolution expected at the Euler scale is experimentally observed from a deformation time of $\tau = 10$ms.

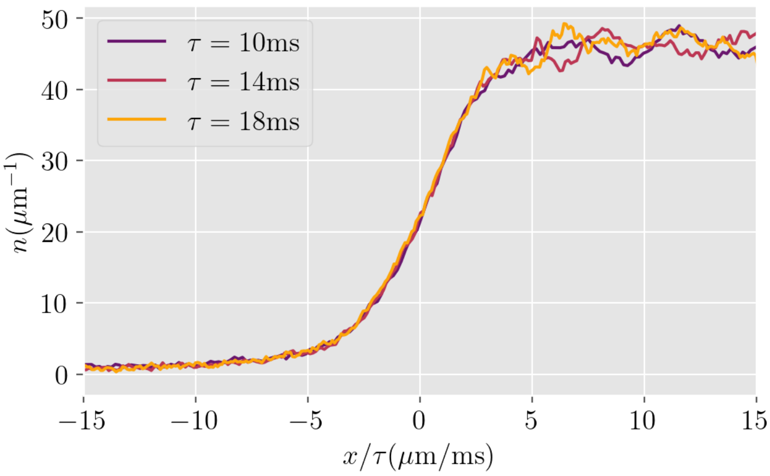

Figure 2: Caption

- Comparison with the ground state and $\gamma = 6 \times 10^{-3}$: FIGURE.
- Comparison with the ground state and $\gamma \to 0$: FIGURE and conclusion. The edge profile obtained at $T = 0$ with $\gamma = 6.10^{-3}$ is nearly identical to the parabola expected at $\gamma \to 0$. The deviations from the parabola observed experimentally are mainly due to non-zero entropy effects.

Transition : we would like to extract the rapidiy distribution from the edge profile

5 Extraction of the initial rapidity distribution

5.1 Directly from the edge profile

[Premier jet : Jérôme]

- Introducing the Jerôme's method

Transition : This method is highly sensitive to the linear density away from the edge. Since the signal-to-noise ratio in our experimental data is important in this region, The results obtained with this technique are not trustworthy.

5.2 Using GHD equations

- Thermal ansatz
- Non thermal ansatz

We propose to extract the rapidity distribution $\rho(\theta)$ of the initial homogeneous gas by fitting the experimental results with the GHD simulations. Extracting $\rho(\theta)$ remains a hard task since we would have an infinite number of fitting parameters — the whole rapidity distribution — with a limited calculation time. Thus, we have to limit the number of fitting parameters and choose an ansatz for the form of the rapidity distribution.

The first ansatz that we choose is the rapidity distribution for a Gibbs ensemble, the fitting parameters being the temperature T and the chemical potential μ.

6 Probing locally the rapidity distribution

[Premier jet : Guillaume]

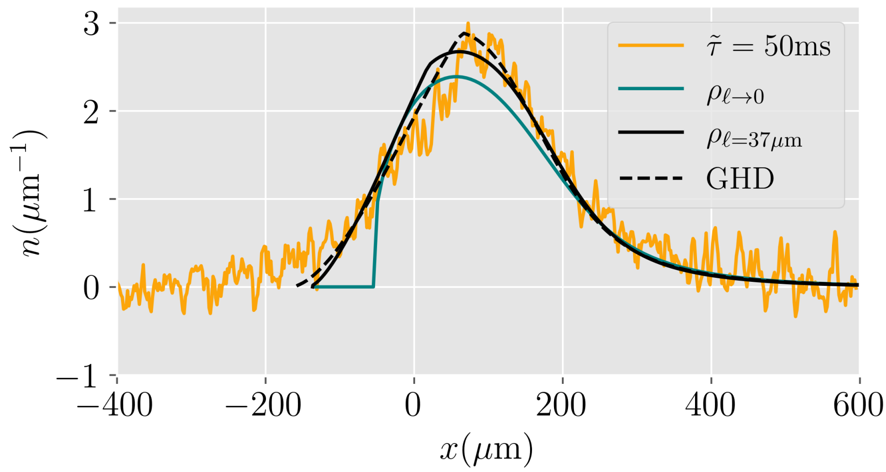

Figure 3: A second graph for this last part, change the format (.pdf). Add the profile n(-x) to add. Change the captions with using Π, the extensive rapidity distribution.

- GHD simulations predict locally a non thermal and asymetric rapidity distribution at the edge deformation. Maybe we can explain technical things about the simulations
- Describe the experimental protocol, referring to the previous article.
- Experimental data, profile asymmetry highlighted
- First comparison with the assumed homogeneous rapidity distribution in the slice
- Second comparison with the inhomogeneous rapidity distribution in the slice
- Third comparison with the GHD simulations to take into account the fact that the asymptotic regime is not completely reached.

7 Conclusion

- a protocol that could enable the study of zero-entropy physics

Acknowledgements

Funding information

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