Topic: Geometric random variables

Question: Which of the following random variables follow a geometric distribution?

Answer choices:

A X is the number of attempts it takes a baseball player to get a hit

B X is the amount of time it takes a runner to complete a marathon

C X is the number of red cards you're dealt in a 5-card hand of poker

D X is the number of times out of 10 tries that you roll a 4 on a six-sided die

Solution: A

In order for X to be a geometric random variable,

- each trial must be independent,
- each trial can be called a “success” or a “failure,” and
- the probability of success on each trial is constant.

Answer choice A is a geometric random variable if we assume each attempt at a hit is a trial, and that these trials are independent. Consider a hit to be a success and anything else to be a failure. And assume the probability of a hit is constant.

Answer choice B is not a geometric random variable because there’s no success or failure, but rather a continuous numeric random variable.

Answer choice C is not a geometric random variable because the trials are not independent and the probability of success on each trial is not constant. As you draw cards out of a deck, the probability of drawing a red card changes as the number of cards you’re drawing from decreases and your probability of getting a red card on any draw depends on what you were already dealt.

Answer choice D is a binomial random variable. A trial consists of rolling a die. Trials are independent when we roll a die because what we roll on each trial has no influence on what we’ll roll next. Rolling a 4 will be considered a success. There are a fixed number of trials, $n = 10$. And the probability of success on each trial remains constant at $p = 1/6$.

Topic: Geometric random variables

Question: Let X be a geometric random variable with $p = 0.30$. Find $P(X = 4)$.

Answer choices:

A 0.0081

B 0.1029

C 1.2

D 0.0720

Solution: B

Let X be the trial where we get the first success. X follows a geometric distribution, and we want to find the probability of getting our first success on exactly the 4th trial, knowing the chance of success on any trial is $p = 0.30$.

To find the probability that a success S occurs on the nth trial, when a success has a probability of p, and therefore failure has a probability of $1 - p$, we'll use

$$P(S = n) = p(1 - p)^{n-1}$$

In this case, we'll set $n = 4$ and get

$$P(S = 4) = 0.30(1 - 0.30)^{4-1}$$

$$P(S = 4) = 0.1029$$

Topic: Geometric random variables

Question: Suppose $35\,\%$ of our nation's high school seniors will be taking at least one AP Exam this year. Suppose we select students at random and ask them if they'll be taking an AP Exam. What's the probability that we'll need to ask exactly 3 people to find someone who is taking the exam?

Answer choices:

A 0.0429

B 0.2389

C 0.1479

D 1.05

Solution: C

Let X be the trial when we find our first person taking at least one AP Exam. X follows a geometric distribution with a trial representing choosing a random student and recording whether he's taking an AP Exam or not. These trials will be independent and the probability of success remains constant at $p = 0.35$. There are not a fixed number of trials.

To find the probability that a success S occurs on the nth trial when a success has a probability of p, and therefore failure has a probability of $1 - p$, we'll use

$$P(S = n) = p(1 - p)^{n-1}$$

In this case, we'll set $n = 3$ and get

$$P(S = 3) = 0.35(1 - 0.35)^{3-1}$$

$$P(S = 3) = 0.1479$$