Topic: Mean, variance, and standard deviation

Question: Find the mean and population standard deviation for the data set.

$$6, 3, 3, 2, 2$$

Answer choices:

A $\mu = 3.2, \sigma \approx 1.4697$

B $\mu = 3.2, \sigma \approx 1.6431$

C $\mu = 3.0, \sigma \approx 1.4697$

D $\mu = 3.0, \sigma \approx 1.6431$

Solution: A

The formula for the population mean is

$$\mu = \frac{\sum_{i=1}^{N} x_i}{N}$$

This means we add up all of our numbers and divide by how many there are. Our numbers are 6, 3, 3, 2, 2, and we have 5 of them, so $N = 5$.

$$\mu = \frac{6 + 3 + 3 + 2 + 2}{5}$$

$$\mu = \frac{16}{5}$$

$$\mu = 3.2$$

To find the population standard deviation, we first need to find the population variance and then take the square root. The variance is

$$\sigma^2 = \frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}$$

This means we take each value and subtract the mean, then square it and add the sum. Then we divide the sum by the number of items in the data set. We know that $\mu = 3.2$ and $N = 5$, so we get

$$\sigma^2 = \frac{(6 - 3.2)^2 + (3 - 3.2)^2 + (3 - 3.2)^2 + (2 - 3.2)^2 + (2 - 3.2)^2}{5}$$

$$\sigma^2 = \frac{(2.8)^2 + (-0.2)^2 + (-0.2)^2 + (-1.2)^2 + (-1.2)^2}{5}$$

$$\sigma^2 = \frac{7.84 + 0.04 + 0.04 + 1.44 + 1.44}{5}$$

$$\sigma^2 = \frac{10.8}{5}$$

$$\sigma^2 = 2.16$$

Now take the square root of the population variance to find the population standard deviation.

$$\sqrt{\sigma^2} = \sqrt{2.16}$$

$$\sigma = \sqrt{2.16}$$

$$\sigma \approx 1.4697$$

Topic: Mean, variance, and standard deviation

Question: Find the mean and sample standard deviation for the data set.

$$2, 4, 7, 9, 10$$

Answer choices:

A $\bar{x} = 6.4, S \approx 3.0067$

B $\bar{x} = 6.4, S \approx 3.3615$

C $\bar{x} = 8.0, S \approx 3.0067$

D $\bar{x} = 8.0, S \approx 3.3615$

Solution: B

The formula for the sample mean is

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

This means we add up all of our numbers and divide by how many there are. Our numbers are $2, 4, 7, 9, 10$, and we have 5 of them, so $n = 5$.

$$\bar{x} = \frac{2 + 4 + 7 + 9 + 10}{5}$$

$$\bar{x} = \frac{32}{5}$$

$$\bar{x} = 6.4$$

To find the sample standard deviation, we first need to find the sample variance and then take the square root. The formula for the sample variance is

$$S^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}$$

This means we take each value and subtract the mean, then square it and add the sum. Then we divide the sum by the number of items in the data set, minus 1. We know that $\bar{x} = 6.4$ and $n - 1 = 5 - 1 = 4$, so we get

$$S^2 = \frac{(2 - 6.4)^2 + (4 - 6.4)^2 + (7 - 6.4)^2 + (9 - 6.4)^2 + (10 - 6.4)^2}{4}$$

$$S^2 = \frac{(-4.4)^2 + (-2.4)^2 + (0.6)^2 + (2.6)^2 + (3.6)^2}{4}$$

$$S^2 = \frac{19.36 + 5.76 + 0.36 + 6.76 + 12.96}{4}$$

$$S^2 = \frac{45.2}{4}$$

$$S^2 = 11.3$$

Now take the square root of the sample variance to get sample standard deviation.

$$\sqrt{S^2} = \sqrt{11.3}$$

$$S \approx 3.3615$$

Topic: Mean, variance, and standard deviation

Question: Consider the small population: 1, 2, 1. If each number is increased by 4, how will the population standard deviation change?

Answer choices:

A The population standard deviation will increase by 4 also.

B The population standard deviation will increase by 16.

C The population standard deviation will be multiplied by 4.

D The population standard deviation will be the same for both data sets.

Solution: D

The population standard deviation is meant to measure how far apart numbers are from one another. It's a measure of how much the data is spread out.

Adding 4 to each number doesn't change how far apart they are. You can calculate both population standard deviations to be sure.

For the original data set:

$$\mu = \frac{1+2+1}{3} = \frac{4}{3}$$

$$\sigma^2 = \frac{\left(1-\frac{4}{3}\right)^2 + \left(2-\frac{4}{3}\right)^2 + \left(1-\frac{4}{3}\right)^2}{3}$$

$$\sigma^2 = \frac{\left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2}{3}$$

$$\sigma^2 = \frac{\frac{1}{9}+\frac{4}{9}+\frac{1}{9}}{3}$$

$$\sigma^2 = \frac{\frac{6}{9}}{3}$$

$$\sigma^2 = \frac{6}{27}$$

$$\sigma^2 = \frac{2}{9}$$

$$\sigma = \frac{\sqrt{2}}{3}$$

For the new data set where we add 4 to each data point:

$$\mu = \frac{(1+4)+(2+4)+(1+4)}{3} = \frac{5+6+5}{3} = \frac{16}{3}$$

$$\sigma^2 = \frac{\left(5-\frac{16}{3}\right)^2+\left(6-\frac{16}{3}\right)^2+\left(5-\frac{16}{3}\right)^2}{3}$$

$$\sigma^2 = \frac{\left(\frac{15}{3}-\frac{16}{3}\right)^2+\left(\frac{18}{3}-\frac{16}{3}\right)^2+\left(\frac{15}{3}-\frac{16}{3}\right)^2}{3}$$

$$\sigma^2 = \frac{\left(-\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)^2+\left(-\frac{1}{3}\right)^2}{3}$$

$$\sigma^2 = \frac{\frac{1}{9}+\frac{4}{9}+\frac{1}{9}}{3}$$

$$\sigma^2 = \frac{\frac{6}{9}}{3}$$

$$\sigma^2 = \frac{6}{27}$$

$$\sigma^2 = \frac{2}{9}$$

$$\sigma = \frac{\sqrt{2}}{3}$$

Both population standard deviations are the same.