Topic: Sampling distribution of the sample proportion

Question: A math class in a school wants to survey the students in the school to find the proportion of them who carry a blue backpack. There are $2,000$ students in the school. Which sample meets all conditions of a normal sampling distribution?

Answer choices:

A The class surveys 250 students by randomly asking students during lunch.

B The class surveys 100 students by going to freshmen classes.

C The class surveys 500 students by going to senior classes.

D The class surveys 100 students by randomly asking students during passing period throughout the school.

Solution: D

Surveying 100 students randomly during a passing period is a valid sampling distribution because it's random and keeps the number of subjects in the sample below 10%, which maintains independence.

The other choices either don't keep the sample below 10%, or aren't taken randomly because they're surveying only freshmen or seniors.

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Question: A restaurant wants to know the percentage of their customers who order desert. The restaurant has $1{,}500$ customers in one week and finds by randomly surveying 100 customers that 35 of them order desert. What is the standard deviation for the sample?

Answer choices:

A $\sigma_{\hat{p}} = 0.047697$

B $\sigma_{\hat{p}} = 0.015096$

C $\sigma_{\hat{p}} = 0.052303$

D $\sigma_{\hat{p}} = 0.084900$

Solution: A

To verify normality, our sample space should be random, no more than 10% of the population, and the expected number of successes and failures should each be at least 10.

Independence: $\frac{100}{1,500} = 0.067 = 6.7\% \leq 10\%$

Successes: $100(0.35) = 35 \geq 10$

Failures: $100(0.65) = 65 \geq 10$

The sample space was random, so we've met the conditions of normality. To find standard deviation of a sample, we use the formula for standard error of the proportion.

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\sigma_{\hat{p}} = \sqrt{\frac{0.35(1-0.35)}{100}}$$

$$\sigma_{\hat{p}} = \sqrt{\frac{0.35(0.65)}{100}}$$

$$\sigma_{\hat{p}} = \sqrt{\frac{0.2275}{100}}$$

$$\sigma_{\hat{p}} \approx 0.047697$$

Topic: Sampling distribution of the sample proportion

Question: A group of scientists is studying $10{,}000$ manatees and finds that $20\,\%$ are calves. You want to verify the claim, but can't conduct a study of all $10{,}000$, so you randomly sample 500. What's the probability that your results are within $5\,\%$ of the first study?

Answer choices:

A $13.5\,\%$

B $73.72\,\%$

C $99.48\,\%$

D $99.8\,\%$

Solution: C

To verify normality, our sample space should be random, no more than 10% of the population, and the expected number of successes and failures should each be at least 10.

Independence: $\frac{500}{10{,}000} = 0.05 = 5\% \leq 10\%$

Successes: $500(0.2) = 100 \geq 10$

Failures: $500(0.8) = 400 \geq 10$

The sample space was random, so we've met the conditions of normality. Now we'll find the mean and standard error for the sample.

$$\mu_{\hat{p}} = p = 0.2$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.2(0.8)}{500}} \approx 0.0179$$

We need to find the probability that our results are within 5% of the population proportion $p = 20\%$. This means, how likely is it that the mean of the sample proportion falls between 15% and 25%?

$$\frac{0.05}{0.0179} \approx 2.79$$

We want to know the probability of $P(-2.79 < z < 2.79)$. Using a z-table, -2.79 gives us 0.0026 and 2.79 gives us 0.9974.

$$P(-2.79 < z < 2.79) = 0.9974 - 0.0026$$

$$P(-2.79 < z < 2.79) = 0.9948$$

So there's a 99.48% chance that our sample proportion will fall within 5% of the first study's claim.