Topic: Poisson distributions

Question: A carpenter is able to build 3 chairs per day, on average. Find the probability that he can build 5 chairs tomorrow.

Answer choices:

A $P(5) \approx 0.1008$

B $P(5) \approx 0.1404$

C $P(5) \approx 0.0136$

D $P(5) \approx 0.2729$

Solution: A

We know this is a Poisson experiment with the following given values:

$\lambda = 3$, the average number of chairs built in a day

$x = 5$, the number of chairs required to be built tomorrow

The Poisson probability is

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$P(5) = \frac{3^5 e^{-3}}{5!}$$

$$P(5) \approx 0.1008$$

So the probability the carpenter will build five chairs tomorrow is approximately 0.1008 or $10.08\,\%$.

Topic: Poisson distributions

Question: Let X be the number of typos on a page in a printed book, with a mean of 3 typos per page. What is the probability that a randomly selected page has at most one typo on it?

Answer choices:

A $P(X \leq 1) \approx 0.0498$

B $P(X \leq 1) \approx 0.1404$

C $P(X \leq 1) \approx 0.1494$

D $P(X \leq 1) \approx 0.1992$

Solution: D

The probability that there is at most one typo on the page is the probability that there are no typos on the page ($X = 0$), plus the probability that there is one typo on the page ($X = 1$).

$$P(X \leq 1) = P(X = 0) + P(X = 1)$$

$$P(X \leq 1) = \frac{3^0 e^{-3}}{0!} + \frac{3^1 e^{-3}}{1!}$$

$$P(X \leq 1) = e^{-3} + 3e^{-3}$$

$$P(X \leq 1) = 4e^{-3}$$

$$P(X \leq 1) \approx 0.1992$$

There is an approximately 0.1992 or 19.92% chance of finding at most one typo on a randomly selected page, when the average number of typos per page is 3.

Topic: Poisson distributions

Question: There are 40 students in a college math course, and each one of them has a 4.5% chance of forgetting their calculator on any given day. What is the probability that exactly 5 of them will forget their calculator today?

Answer choices:

A $P(5) \approx 95.26\%$

B $P(5) \approx 97.4\%$

C $P(5) \approx 2.6\%$

D $P(5) \approx 36.97\%$

Solution: C

This is a binomial experiment with $n=40$, $p=0.045$, and $x=5$. Because we have at least 20 "attempts," and because the probability of a "success" is less than $5\,\%$, we can use the Poisson formula to estimate this binomial probability.

$$P(x)=\frac{(np)^x e^{-np}}{x!}$$

$$P(5)=\frac{(40\cdot 0.045)^5 e^{-40\cdot 0.045}}{5!}$$

$$P(5)=\frac{1.8^5 e^{-1.8}}{120}$$

$$P(5)\approx 0.0260$$

So the chance that exactly 5 of the college math students forget their calculator today is approximately $2.6\,\%$.