**Topic**: Confidence interval for a population mean

**Question**: The height of students in your school is normally distributed with a standard deviation of $\sigma = 4$ inches. You take a sample of $50$ of your classmates and get a sample mean of $\bar{x} = 66$ inches. What is the confidence interval for a confidence level of $95\%$?

**Answer choices**:

A $(a, b) \approx (64.54,67.46)$

B $(a, b) \approx (64.89,67.11)$

C $(a, b) \approx (65.07,66.93)$

D $(a, b) \approx (65.74,66.26)$

**Solution**: B

A $95\%$ confidence level is associated with $z$-scores of $z = \pm 1.96$.

If we plug everything we know into the confidence interval formula for a known population standard deviation, we get

$$(a, b) = \bar{x} \pm z^* \cdot \frac{\sigma}{\sqrt{n}}$$

$$(a, b) = 66 \pm 1.96 \cdot \frac{4}{\sqrt{50}}$$

$$(a, b) = 66 \pm 1.1087$$

Therefore, we can say that the confidence interval is

$$(a, b) = (64.8913, 67.1087)$$

$$(a, b) \approx (64.89, 67.11)$$

We could also express this as the sample mean plus or minus the margin of error, or $66 \pm 1.1087$ inches. We're $95\%$ certain that the actual population mean of height of students in our school is between $64.89$ inches and $67.11$ inches.

**Topic**: Confidence interval for a population mean

**Question**: The weight of chickens on a farm is normally distributed with a standard deviation of $\sigma = 3.5$ ounces. What is the smallest sample you can take if you want a margin of error of $\pm 2.5$ ounces, and you want to be $99\%$ confident?

**Answer choices**:

A $n = 10$ chickens

B $n = 13$ chickens

C $n = 14$ chickens

D $n = 30$ chickens

**Solution**: C

The margin of error formula is

$$ME = z^* \cdot \frac{\sigma}{\sqrt{n}}$$

Since we want to find a sample size, solve this for $n$.

$$ME\sqrt{n} = z^*\sigma$$

$$\sqrt{n} = \frac{z^*\sigma}{ME}$$

$$n = \left(\frac{z^*\sigma}{ME}\right)^2$$

Now we can plug the values we were given into this equation, remembering that a confidence level of $99\%$ is associated with critical values of $z = \pm 2.58$.

$$n = \left(\frac{2.58 \cdot 3.5}{2.5}\right)^2$$

$$n = \left(\frac{2.58 \cdot 3.5}{2.5}\right)^2$$

$$n \approx 13.05$$

Because we can't sample $0.05$ of a chicken, we round up to $n = 14$ chickens. Then we can say that, to meet that threshold, and keep a margin of error of $\pm 2.5$ at $99\,\%$ confidence, we'd need to take a sample size of at least $n = 14$ chickens.

**Topic**: Confidence interval for a population mean

**Question**: You want to know the mean number of daylight hours in a day in your city (the time between sunrise and sunset) over the course of a year. You take a random sample of $30$ days throughout the year and get a sample mean of $\bar{x} = 13.15$ hours and standard error of the mean $s = 0.85$ hours. What is the confidence interval for a confidence level of $90\%$?

**Answer choices**:

A $(a, b) \approx (12.89,13.41)$

B $(a, b) \approx (12.85,13.45)$

C $(a, b) \approx (12.75,13.55)$

D $(a, b) \approx (12.28,14.02)$

**Solution**: A

A $90\%$ confidence level is associated with $z$-scores of $z = \pm 1.65$. We don't know population standard deviation, so we'll substitute sample standard deviation. Because our sample is at least $30$, we can still use the confidence interval formula.

$$(a, b) = \bar{x} \pm z^* \cdot \frac{s}{\sqrt{n}}$$

$$(a, b) = 13.15 \pm 1.65 \cdot \frac{0.85}{\sqrt{30}}$$

$$(a, b) = 13.15 \pm 0.2561$$

Therefore, we can say that the confidence interval is

$$(a, b) = (12.8939, 13.4061)$$

$$(a, b) \approx (12.89, 13.41)$$

We could also express this as the sample mean plus or minus the margin of error, or $13.15 \pm 0.2561$ hours. We're $90\%$ certain that the actual population mean of hours of daylight in a day is between $12.89$ hours and $13.41$ hours.