Topic: "At least" and "at most," and mean, variance, and standard deviation

Question: Let X be a binomial random variable with $n = 15$ and $p = 0.45$. Find $P(X \leq 10)$.

Answer choices:

A 0.0515

B 0.9745

C 0.9231

D 0.0255

Solution: B

X follows a binomial distribution, but instead of finding the probability of exactly k successes in n trials, we're asked to find the probability of k or fewer successes in n trials. Specifically, find the chance of 10 or fewer successes in 15 trials, where the probability of success on any one trial is $p = 0.45$.

Find the probability of 0 success, 1 success, 2 successes, etc., up to 10 successes and then find the sum of those probabilities.

$$P(X \leq 10) = P(X = 0) + P(X = 1) + P(X = 2) + \dots + P(X = 10)$$

To find the probability for each value of k, we use the binomial probability formula.

$$P(k \text{ successes in } n \text{ trials}) = \binom{n}{k} p^k (1 - p)^{n-k}$$

The probability is

$$P(X \leq 10) = \binom{15}{0}(0.45)^0(1 - 0.45)^{15} + \binom{15}{1}(0.45)^1(1 - 0.45)^{14}$$

$$+ \dots + \binom{15}{10}(0.45)^{10}(1 - 0.45)^5$$

$$P(X \leq 10) = 0.9745$$

Topic: "At least" and "at most," and mean, variance, and standard deviation

Question: 32% of all internet users have Instagram accounts. Suppose 20 random internet users are selected. What's the probability that at least half of them have Instagram accounts?

Answer choices:

A 0.0440

B 0.9721

C 0.0719

D 0.0279

Solution: C

Let X be the number of Instagram users out of 20 internet users.

X follows a binomial distribution, but instead of finding the probability of exactly k successes in n trials, we're finding the probability of k or more successes in n trials. Specifically, we're finding the chance of 10 or more successes in 20 trials, where the probability of success on any one trial is $p = 0.32$.

We could find the probability of 10 success, 11 success, 12 successes, etc., up to 20 successes, and then find the sum of those probabilities. But it would be easier to find the complement, $P(X \le 9)$. We'll use the formula

$$P(k \text{ successes in } n \text{ trials}) = \binom{n}{k} p^k (1-p)^{n-k}$$

and then calculate the probability of 0 successes, 1 success, 2 successes, etc., all the way up to 9 successes.

$$P(X \le 9) = \binom{20}{0}(0.32)^0(1-0.32)^{20} + \binom{20}{1}(0.32)^1(1-0.32)^{19}$$

$$+ \dots + \binom{20}{9}(0.32)^9(1-0.32)^{11} = 0.9281$$

$$P(X \le 9) = 0.9281$$

And now find the probability of at least 10 successes out of 20 by subtracting $P(X \le 9)$ from 1.

$$P(X \ge 10) = 1 - P(X \le 9)$$

$$P(X \geq 10) = 1 - 0.9281$$

$$P(X \geq 10) = 0.0719$$

Topic: "At least" and "at most," and mean, variance, and standard deviation

Question: In January, 2018, 17% of teens said Instagram is the most important social media site. If we select an SRS of 150 teens, how many are expected to say Instagram is the most important social media site? With what standard deviation?

Answer choices:

A $\mu = 17$ and $\sigma = 2.55$

B $\mu = 25.5$ and $\sigma = 21.165$

C $\mu = 25.5$ and $\sigma = 8.824$

D $\mu = 25.5$ and $\sigma = 4.601$

Solution: D

Let X be the number of teens who say Instagram is the most important social media site.

X follows a binomial distribution, with a fixed number of trials $n = 150$ and a probability of success $p = 0.17$. We can find the expected value for the distribution.

$$\mu_X = E(X) = np$$

$$\mu_X = (150)(0.17)$$

$$\mu_X = 25.5 \text{ teens}$$

Find the variance for the distribution.

$$\sigma_X^2 = Var(X) = np(1 - p)$$

$$\sigma_X^2 = (150)(0.17)(1 - 0.17)$$

$$\sigma_X^2 = 21.165$$

Then we can find standard deviation.

$$\sigma_X = \sqrt{21.165}$$

$$\sigma_X = 4.601 \text{ teens}$$