Topic: Permutations and combinations

Question: The math club is meeting after school to study for their exams. Thirty members show up for the meeting and study groups of five to a group will be formed using random assignment. How many different groups of five could be formed?

Answer choices:

A 6 groups

B 150 groups

C $142{,}506$ groups

D $17{,}100{,}720$ groups

Solution: C

This is a combination question where $n = 30$ and $k = 5$. The order in which we choose our 5 members doesn't matter in this situation.

If Person A, B, C, D, and E end up in a group together, this is equivalent to Person E, D, C, B, and A ending up in a group together.

We'll use the combination formula.

$$_nC_k = \binom{n}{k} = \binom{30}{5} = \frac{n!}{k!(n-k)!} = \frac{30!}{5!25!} = 142{,}506 \text{ groups}$$

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Question: Four children are sledding in a toboggan. How many ways can the children arrange themselves on the toboggan?

Answer choices:

A 4 ways

B 16 ways

C 24 ways

D 256 ways

Solution: C

This is a permutation question. We have 4 people we're arranging and we'll arrange those 4 people as many different ways as we can. Set $n=4$ and $k=4$ and use the permutations formula.

$$_nP_k=\frac{n!}{(n-k)!}=\frac{4!}{0!}=\frac{4!}{1}=4!=(4)(3)(2)(1)=24 \text{ ways}$$

Topic: Permutations and combinations

Question: Sawyer is taking a 5-question biology test, and realizes after reading through them that he doesn't know any of the answers. He decides to guess on every question. How many different ways could he get exactly 3 of the 5 questions correct?

Answer choices:

A 10 ways

B 15 ways

C 60 ways

D 125 ways

Solution: A

To figure out how many different ways could Sawyer could answer exactly 3 of the 5 questions correctly, we need the formula for combinations.

We have 5 questions and want to know how many ways we can pick 3 of the 5 questions. The order won't matter, which is why we need the combination, and not the permutation. For example, getting questions $\#1$, $\#2$, and $\#3$ correct is the same as getting questions $\#2$, $\#1$, and $\#3$ correct.

Therefore, we find the combination $_5C_3$.

$$_nC_k = \binom{n}{k} = \binom{5}{3} = \frac{n!}{k!(n-k)!} = \frac{5!}{3!2!} = 10 \text{ ways}$$

There are 10 different ways that Sawyer could answer exactly 3 of the 5 questions correctly.