Topic: Hypothesis testing for the population proportion

Question: We want to test the hypothesis that 10% of people are left-handed, so we collect a random sample of 500 people and find that 43 of them are left-handed. What can you conclude at a significance level of $\alpha = 0.10$?

Answer choices:

A   You'll reject the null hypothesis; your result is significant at the $p = 0.1314$ level

B   You'll reject the null hypothesis; your result is significant at the $p = 0.8686$ level

C   You'll reject the null hypothesis; your result is significant at the $p = 0.9000$ level

D   You'll fail to reject the null hypothesis

Solution: D

First build the hypothesis statements.

H_0: 10% of people are left-handed, $p = 0.1$

H_a: The proportion of left-handed people is different than 10%, $p \neq 0.1$

The sample proportion is

$$\hat{p} = \frac{x}{n} = \frac{43}{500} = 0.086$$

Then find the standard error of the proportion.

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.086(1-0.086)}{500}} \approx 0.0125$$

Now we have enough to find the z-value of the test-statistic.

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{0.086 - 0.10}{0.0125} \approx -1.12$$

The critical z-values for 90% confidence with a two-tail test are $z = \pm 1.65$. Since the test statistic we found is negative ($z = -1.12$), we'll compare it to $z = -1.65$.

Our z-value of $z = -1.12$ is not less than $z = -1.65$, and therefore falls in the region of acceptance, which means we'll fail to reject the null hypothesis and fail to conclude that the proportion of left-handed people is different than 10%.

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Question: We want to test the hypothesis that fewer than 80% of Americans eat breakfast, so we collect a random sample of 650 Americans and find that 496 of them eat breakfast. What can you conclude at a significance level of $\alpha = 0.05$?

Answer choices:

A   You'll reject the null hypothesis; your result is significant at the $p = 0.0136$ level

B   You'll reject the null hypothesis; your result is significant at the $p = 0.9500$ level

C   You'll reject the null hypothesis; your result is significant at the $p = 0.9864$ level

D   You'll fail to reject the null hypothesis

Solution: A

First build the hypothesis statements.

H_0: At least 80% of Americans eat breakfast, $p \geq 0.8$

H_a: Fewer than 80% of Americans eat breakfast, $p < 0.8$

The sample proportion is

$$\hat{p} = \frac{x}{n} = \frac{496}{650} \approx 0.7631$$

Then find the standard error of the proportion.

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.7631(1-0.7631)}{650}} \approx 0.0167$$

Now we have enough to find the z-value of the test-statistic.

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{0.7631 - 0.8}{0.0167} \approx -2.2096$$

The critical z-value for 95% confidence with a one-tail lower tail test is $z = -1.65$.

Our z-value of $z \approx -2.2096$ falls to the left of $z = -1.65$, and therefore falls in the region of rejection, which means we'll reject the null hypothesis and conclude that the proportion of Americans who eat breakfast is less than 80%.

We know our findings are significant at $\alpha = 0.05$, but we can find the p-value to state a higher level of significance that corresponds to $z \approx -2.2096$

and not just $z = -1.65$. The test statistic $z \approx -2.2096$ gives a value of 0.0136 in the z-table.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	**.0136**	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143

Which means the conclusion isn't only significant at $\alpha = 0.05$, but it's actually significant at 0.0136. As long as $\alpha \geq 0.0136$, we'll be able to reject H_0.

Topic: Hypothesis testing for the population proportion

Question: We want to test the hypothesis that more than 25% of NBA players (professional basketball players) started playing basketball before age 5, so we collect a random sample of 117 NBA players and find that 34 of them started playing before they turned 5. What can you conclude at a significance level of $\alpha = 0.01$?

Answer choices:

A   You'll reject the null hypothesis; your result is significant at the
         $p = 0.1660$ level

B   You'll reject the null hypothesis; your result is significant at the
         $p = 0.8340$ level

C   You'll reject the null hypothesis; your result is significant at the
         $p = 0.9900$ level

D   You'll fail to reject the null hypothesis

Solution: D

First build the hypothesis statements.

H_0: At most 25% of NBA players started playing before 5, $p \leq 0.25$

H_a: More than 25% of NBA players started playing before 5, $p > 0.25$

The sample proportion is

$$\hat{p} = \frac{x}{n} = \frac{34}{117} \approx 0.2906$$

Then find the standard error of the proportion.

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.2906(1-0.2906)}{117}} \approx 0.0420$$

Now we have enough to find the z-value of the test-statistic.

$$z = \frac{\hat{p}-p}{\sigma_{\hat{p}}} = \frac{0.2906 - 0.25}{0.0420} \approx 0.9667$$

The critical z-value for 99% confidence with a one-tail upper tail test is $z = 2.33$.

Our z-value of $z \approx 0.9667$ falls to the left of $z = 2.33$, and therefore falls in the region of acceptance, which means we'll fail to reject the null hypothesis and fail to conclude that the proportion of NBA players who started playing basketball before age 5 is more than 25%.