Topic: Bayes' theorem

Question: When should you use Bayes' Theorem?

Answer choices:

A When you have $P(A \cap B)$ but want to find $P(A)$.

B When you have $P(A \mid B)$ but want to find $P(B \mid A)$.

C When you have $P(A)$ but want to find $P(B)$.

D When you have $P(A \mid B)$ but want to find $P(A)$.

Solution: B

Bayes' Theorem is used when you have a conditional probability of two events, and you're interested in the reversed conditional probability. For example, when you have $P(A \mid B)$ but want to find $P(B \mid A)$.

Topic: Bayes' theorem

Question: Three factories A, B, and C produce car seats. A customer received a defective car seat and the manager at factory C would like to know if it came from her factory. She defines these events:

A represents a car seat from factory A.

B represents a car seat from factory B.

C represents a car seat from factory C.

D represents a defective car seat.

Which probability is the manager looking for?

Answer choices:

A $P(D \mid C)$

B $P(C \mid D)$

C $P(D \cap C)$

D $P(C \cap D)$

Solution: B

The manager is looking for the probability the car seat came from factory C given that it was defective. This means she's looking for $P(C|D)$.

You may be tempted to think that answer choices C and D represent correct answers, since they both give the probability that both events C and D occur. The only subtle difference is that, in answer choices C and D, we'd be calculating the probability that the car seat is defective *and* that the car seat comes from factory C. In other words, the probability that both of those events occur. Which means we're considering the probability that the car seat is defective.

However, we've already been told that the car seat is defective. So the probability that D occurs is no longer in question, it's already "given," so we shouldn't consider it. We're looking only for the probability of C, and we're assuming that D has already occurred, since the customer has already definitely received a defective car seat.

Topic: Bayes' theorem

Question: Which choice is equivalent to $P(C|D)$?

Answer choices:

A $\dfrac{P(D|C) \cdot P(C)}{P(D)}$

B $\dfrac{P(C \cap D)}{P(D)}$

C $\dfrac{P(C \cup D)}{P(D)}$

D Both A and B

Solution: D

Bayes' theorem is

$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}$$

This problem uses different variables. If you replace A with C and B with D, then Bayes' theorem is

$$P(C \mid D) = \frac{P(D \mid C) \cdot P(C)}{P(D)}$$

The multiplication rule says that $P(C \cap D) = P(C) \cdot P(D \mid C)$, which means we could also write Bayes' theorem as

$$P(C \mid D) = \frac{P(D \mid C) \cdot P(C)}{P(D)} = \frac{P(C \cap D)}{P(D)}$$