Topic: Binomial random variables

Question: Which random variable X follows a binomial distribution?

Answer choices:

A X is the number of attempts it takes a basketball player to make a free throw

B X is the amount of time it takes a runner to complete a marathon

C X is the number of red cards you're dealt in a 5-card hand of poker

D X is the number of times out of 10 tries that you roll a 4 on a six-sided die

Solution: D

In order for X to be a binomial random variable,

1. each trial must be independent,

2. each trial can be called a "success" or a "failure,"

3. there are a fixed number of trials, and

4. the probability of success on each trial is constant.

Answer choice A isn't a binomial random variable because we don't have a fixed number of trials.

Answer choice B isn't a binomial random variable because there's no success or failure, but rather a continuous numeric random variable.

Answer choice C isn't a binomial random variable because the trials aren't independent and the probability on each trial isn't constant. As you draw cards out of a deck, the probability of drawing a red changes because the number of cards you're drawing from decreases and your probability of getting a red card on any draw depends on what you were already dealt.

Answer choice D is a binomial random variable. A trial consists of rolling a die. Trials are independent when we roll a die because what we roll on each trial has no influence on what we'll roll next. Rolling a 4 will be considered a success. There are a fixed number of trials, $n = 10$. And the probability of success on each trial remains constant at $p = 1/6$.

Topic: Binomial random variables

Question: Let X be a binomial random variable with $n = 15$ and $p = 0.45$. Find $P(X = 9)$.

Answer choices:

A 0.000006

B 0.6

C 0.1048

D 6.75

Solution: C

The goal is to find the probability of exactly $k = 9$ successes.

$$P(k \text{ successes in } n \text{ trials}) = \binom{n}{k} p^k (1-p)^{n-k}$$

Start by finding the number of ways to have exactly 9 successes in 15 trials using the combination formula.

$$\binom{15}{9} = {}_{15}C_9 = \frac{15!}{9!(15-9)!} = 5{,}005$$

Now plug this value into the probability formula.

$$P(X = 9) = \binom{15}{9}(0.45)^9(1-0.45)^6$$

$$P(X = 9) = (5{,}005)(0.45)^9(1-0.45)^6$$

$$P(X = 9) = 0.1048$$

Topic: Binomial random variables

Question: Suppose 35% of our nation's high school seniors will be taking at least one AP Exam this year. We select 80 students at random from our nation. What is the probability that exactly 30 will be taking at least one exam?

Answer choices:

A 0.0824

B 0.375

C 0.1406

D 0.35

Solution: A

Let X be the number of seniors that take at least one AP exam out of 80 trials.

X follows a binomial distribution with a trial representing choosing a random student from our nation and recording whether or not they're taking an AP Exam. These trials will be independent and the probability of success remains constant at $p = 0.35$. And there is a fixed number of trials, $n = 80$.

$$X \sim B(80,0.35)$$

The goal is to find the probability of exactly $k = 30$ successes.

$$P(k \text{ successes in } n \text{ trials}) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

Start by finding the number of ways to have exactly 30 successes in 80 trials using the combination formula.

$$\binom{80}{30} = {}_{80}C_{30} = \frac{80!}{30!(80-30)!}$$

Now we can find the probability.

$$P(x = 30) = \binom{80}{30}(0.35)^{30}(1-0.35)^{50} = 0.0824$$