Topic: Sampling distribution of the sample mean

Question: A school finds that time spent studying for a test isn't normally distributed for their $2,800$ students. They would like to use the central limit theorem to normalize the data. Which sample allows them to use the central limit theorem?

Answer choices:

A The school samples 300 students randomly

B The school samples 200 students randomly

C The school samples 200 students from honors classes

D The school samples 25 students randomly

Solution: B

The random sample of 200 students will allow the school to use the central limit theorem. The sample is random, less than 10% of the population, and greater than 30, so it's large enough to normalize the data.

The other answer choices wouldn't allow the school to use the central limit theorem. Answer choice A samples more than 10% of the population which doesn't maintain independence, answer choice C isn't random because selecting students in honors classes will skew the data, and answer choice D doesn't have a big enough sample because a sample size smaller than 30 won't normalize the data.

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Question: A hospital finds that the average birth weight of a newborn is 7.5 lbs with a standard deviation of 0.4 lbs. The hospital randomly selects 45 newborns to test this claim. What is the standard deviation of the sampling distribution?

Answer choices:

A $\sigma_{\bar{x}} = 0.0596$

B $\sigma_{\bar{x}} = 1.118$

C $\sigma_{\bar{x}} = 0.0533$

D $\sigma_{\bar{x}} = 0.0089$

Solution: A

To find the standard deviation of the sampling distribution, we'll plug population standard deviation $\sigma = 0.4$ and the sample size $n = 45$ into the formula for sample standard deviation.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$\sigma_{\bar{x}} = \frac{0.4}{\sqrt{45}}$$

$$\sigma_{\bar{x}} = 0.0596$$

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Question: A company produces tires in a factory. Individual tires are filled to an approximate pressure of 36 PSI (pounds per square inch), with a standard deviation of 0.8 PSI. The pressure in the tires is normally distributed. The company randomly selects 125 tires to check their pressure. What is the probability that the mean amount of pressure in the tires is within 0.1 PSI of the population mean?

Answer choices:

A $8.38\,\%$

B $91.62\,\%$

C $71.55\,\%$

D $83.84\,\%$

Solution: D

To verify normality, our sample must be random, no more than 10% of the population, and (if the population is not normal) the sample size must be greater than 30.

The sample was collected randomly. It's safe to assume that 125 tires is less than 10% of the total tires produced in the factory. The population is normal, so the sample size doesn't have to be greater than 30, but 125 is greater than 30 anyway. The sample space meets the conditions of normality.

Find the standard deviation of the sampling distribution.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$\sigma_{\bar{x}} = \frac{0.8}{\sqrt{125}}$$

$$\sigma_{\bar{x}} = 0.07155$$

We want to know the probability that the sample mean $\bar{x}$ is within 0.1 PSI of the population mean. We need to express 0.1 in terms of standard deviations.

$$\frac{0.1}{0.07155} = 1.39754$$

This means we want to know the probability $P(-1.40 < z < 1.40)$. Using a z-table, a z-value of -1.40 gives 0.0808 and a z-value of 1.38 gives 0.9192. The probability under the normal curve between these z-scores is

$$P(-1.40 < z < 1.40) = 0.9192 - 0.0808$$

$$P(-1.40 < z < 1.40) = 0.8384$$

$$P(-1.40 < z < 1.40) = 83.84\%$$

There's an 83.84% chance that our sample mean will fall within 0.1 PSI of the population mean of 36 PSI.