Topic: Normal distributions and z-scores

Question: Given an approximately normal distribution with a mean of 150 and a standard deviation of 28, approximately what percentage of the values are within the interval $(94,206)$?

Answer choices:

A $68\,\%$

B $95\,\%$

C $99.7\,\%$

D There is not enough information.

Solution: B

The empirical rule tells us approximately how much data is within one, two or three standard deviations from the mean.

We know the mean of the data is 150. Since the standard deviation is 28, the interval around one standard deviation is

$$(150 - 28, 150 + 28)$$

$$(122, 178)$$

The interval around two standard deviations is

$$(150 - 28 - 28, 150 + 28 + 28)$$

$$(94, 206)$$

The interval around three standard deviations is

$$(150 - 28 - 28 - 28, 150 + 28 + 28 + 28)$$

$$(66, 234)$$

We are asked about the interval $(94, 206)$, which is the interval around two standard deviations from the mean. According to the empirical rule, that interval contains 95% of the data.

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Question: A third grade class has a mean height of $50''$ with a standard deviation of $3''$. What is the approximate percentile of a third grader who is $53''$ tall?

Answer choices:

A 16%

B 32%

C 68%

D 84%

Solution: D

We're given a mean of 50 and a standard deviation of 3, and we want to know the percentile for someone who is $53''$ tall. We could do this calculation either with the empirical rule or with a z-score.

If you use the empirical rule, you'll need to know that 53 is one standard deviation above the mean. We know there is 68% of the data within one standard deviation, which means there is exactly half of that within one standard deviation, but above the mean only.

$$\frac{68\%}{2} = 34\%$$

Adding this to the 50% of the data below the mean, we can say that someone who is $53''$ tall is in the $50\% + 34\% = 84\%$ percentile.

If we do this with a z-score, then we use the formula to find the z-score and look up the percentage in the table.

$$z = \frac{x - \mu}{\sigma}$$

We know the mean is $\mu = 50$ and that the standard deviation is $\sigma = 3$. If we plug all of this, plus $53''$, into the z-score formula, we get

$$z = \frac{53 - 50}{3} = \frac{3}{3} = 1$$

We can look up 1.00 in a z-table, we find the value $.8413$, which tells us that we're in the 84.13 percentile.

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Question: One of the values in a standard normal distribution is 12, and its z-score is -0.80. If the mean of the distribution is 14, what is the standard deviation of the distribution?

Answer choices:

A -0.2119

B 0.2119

C 0.25

D 2.5

Solution: D

The formula for a z-score is

$$z = \frac{x - \mu}{\sigma}$$

We know the mean is $\mu = 14$, and we're looking for the standard deviation. The value of interest is 12, and we know the z-score is -0.80. Set up the formula and solve for the standard deviation.

$$-0.80 = \frac{12 - 14}{\sigma}$$

$$-0.80\sigma = 12 - 14$$

$$-0.80\sigma = -2$$

$$\sigma = \frac{-2}{-0.80}$$

$$\sigma = 2.5$$