Probability & Statistics Workbook Solutions

Hypothesis testing

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INFERENTIAL STATISTICS AND HYPOTHESES

■ 1. A current pain reliever has an 85% success rate of treating pain. A company develops a new pain reliever and wants to show that its success rate of treating pain is better than the current option. Decide if the hypothesis statement would require a population proportion or a population mean, then set up the statistical hypothesis statements for the situation.

Solution:

We're interested in finding out if the new pain reliever has a better success rate than the current one. Since we're given a percentage of success, we'll be using a population proportion p, instead of a population mean μ. And since we're looking at how much better the pain reliever will perform, we use the $>$ symbol in our alternative hypothesis, which means the null hypothesis has to have the $\leq$ symbol.

$$H_0 : p \leq 0.85$$

$$H_a : p > 0.85$$

■ 2. A research study on people who quit smoking wants to show that the average number of attempts to quit before a smoker is successful is less than 3.5 attempts. How should they set up their hypothesis statements?

Solution:

We're interested in finding out if the mean number of attempts is less than 3.5, so we'll be using a population mean μ. And since we're looking at whether the mean is less than 3.5, we use the $<$ symbol in our alternative hypothesis, which means the null hypothesis has to have the $\geq$ symbol.

$$H_0 : \mu \geq 3.5$$

$$H_a : \mu < 3.5$$

3. A factory creates a small metal cylindrical part that later becomes part of a car engine. Because of variations in the process of manufacturing, the diameters are not always identical. The machine was calibrated to create cylinders with an average diameter of $1/16$ of an inch. During a periodic inspection, it became clear that further investigation was needed to determine whether or not the machine responsible for making the part needed recalibration. Write statistical hypothesis statements.

Solution:

The factory wants the mean diameter of the parts it produces to match the diameter that they need, $1/16$ of an inch. That means this is an example of a statistical hypothesis statement that uses the population mean.

Both parts that are too small or too large could create problems, which means the alternative hypothesis needs to have a $\neq$ sign. Which means the null hypothesis will include an $=$ sign.

$$H_0 : \mu = \frac{1}{16}$$

$$H_a : \mu \neq \frac{1}{16}$$

4. A marketing study for a clothing company concluded that the mean percentage increase in sales could potentially be over 17% for creating a clothing line that focused on lime green and polka dots. Which hypothesis statements do they need to write in order to test their theory?

Solution:

The claim of the marketing study is that creating the clothing line that focuses on lime green and polka dots will increase sales by over 17%. Which means the alternative hypothesis would need to include the $>$ sign, and therefore that the null hypothesis has to include a $\leq$ sign.

$$H_0 : \mu \leq 0.17$$

$$H_a : \mu > 0.17$$

5. A food company wants to ensure that less than 0.0001% of its product is contaminated. Which hypothesis statements will it write if it wants to test for this?

Solution:

The food company wants the proportion of contaminated product to be less than 0.0001%, so they'll be using a population proportion p. And since they're looking at whether the proportion is less than 0.0001%, they'll use the $<$ symbol in the alternative hypothesis, which means the null hypothesis has to have the $\geq$ symbol.

$$H_0: \mu \geq 0.0001\%$$

$$H_a: \mu < 0.0001\%$$

6. A new medication is being developed to prevent heart worms in dogs, and the developer wants it to work better than the current medication. The current medication prevents heart worms at a rate of 75%. What hypothesis statements should they write if they want to test whether or not the new medication works better than the existing one?

Solution:

The developer wants the proportion of dogs in which heart worm is prevented by their medication to be greater than 0.75, so they'll be using a population proportion p. And since we're looking at whether the proportion is greater than 0.75, we use the $>$ symbol in our alternative hypothesis, which means the null hypothesis has to have the $\leq$ symbol.

$$H_0 : \mu \leq 0.75$$

$$H_a : \mu > 0.75$$

SIGNIFICANCE LEVEL AND TYPE I AND II ERRORS

1. You're running a statistical test on a new pharmaceutical drug. The stakes are high, because the side effects of the drug could potentially be serious, or even fatal. If you want to reduce the Type I and Type II error rates as low as possible to avoid rejecting the null when it's true or accepting the null when it's false, what should you do when you take the sample for your test?

Solution:

The only way to reduce both the Type I error rate and Type II error rate simultaneously is to increase the sample size. Therefore, if it's important that you reduce error rate as low as possible, you should take the largest sample you can.

2. If the probability of making a Type II error in a statistical test is 5%, what is the power of the test?

Solution:

The power of a statistical test is the probability that we'll reject the null hypothesis when it's false (make that particular correct choice).

	H_0 is true	H_0 is false
Reject H_0	Type I error P(Type I error)=alpha	**CORRECT** **Power**
Accept H_0	CORRECT	Type II error P(Type II error)=beta

Power is always equivalent to $1-\beta$, and β is another name for Type II error rate. So

Power $= 1-\beta$

Power $= 1-$ Type II error rate

Power $= 1-0.05$

Power $= 0.95$

The power of the statistical test, given that the probability of making a Type II error is 5%, is Power $= 95\%$.

3. On average, professional golfers make 75% of putts within 5 feet. One golfer believes he does better than this, and wants to use a statistical test to see whether or not he's correct. Unbeknownst to him, in actuality this golfer makes 7 out of 10 of these kinds of putts. When he takes a sample of his putts, he finds $\hat{p}=0.92$. What kind of error might he be in danger of making?

Solution:

The golfer's null and alternative hypotheses are

$$H_0 : p \leq 0.75$$

$$H_a : p > 0.75$$

In reality, his null hypothesis is true, but based on the sample proportion $\hat{p} = 0.92$, he may be in danger of rejecting the null when he shouldn't.

	H_0 is true	H_0 is false
Reject H_0	**Type I error** **P(Type I error)=alpha**	CORRECT
Accept H_0	CORRECT	Type II error P(Type II error)=beta

Which means the golfer is in danger of making a Type I error.

4. The average age of a guest at an amusement park is 15 years old. One amusement park believes the average age of their guests is younger than this, and wants to use a statistical test to see whether or not they're correct. Unbeknownst to them, in actuality the average guest age at this particular amusement park is 12 years old. When they take a sample of his guests, they find $\bar{x} = 16$ years. What kind of error might they be in danger of making?

Solution:

The park's null and alternative hypotheses are

$$H_0 : \mu \geq 15$$

$$H_a : \mu < 15$$

In reality, they're null hypothesis is false, but based on the sample mean $\bar{x} = 16$, they may be in danger of accepting the null when they shouldn't.

	H_0 is true	**H_0 is false**
Reject H_0	Type I error P(Type I error)=alpha	CORRECT
Accept H_0	CORRECT	**Type II error P(Type II error)=beta**

Which means the amusement park is in danger of making a Type II error.

5. Of all political donations, 70% come from corporations and lobbies, not from individual citizens. One politician believes he receives less than 70% of his own donations from corporations and lobbies, and wants to use a statistical test to see whether or not he's correct. Unbeknownst to him, in actuality the proportion of his donations that come from corporations and lobbies is 65%. When he takes a sample of his donations that come from corporations and lobbies, he finds $\hat{p} = 0.72$. What kind of error might he be in danger of making?

Solution:

The politician's null and alternative hypotheses are

$$H_0 : p \geq 0.7$$

$$H_a : p < 0.7$$

In reality, his null hypothesis is false, but based on the sample proportion $\hat{p} = 0.72$, he may be in danger of accepting the null when he shouldn't.

	H_0 is true	H_0 is false
Reject H_0	Type I error P(Type I error)=alpha	CORRECT
Accept H_0	CORRECT	**Type II error P(Type II error)=beta**

Which means the politician is in danger of making a Type II error.

6. A coffee shop owner believes that he sells 500 cups of coffee each day, on average, and he wants to test this assumption. The truth is, he actually sells fewer than 500 cups each day. He takes a random sample of 10 days and records the number of cups he sells each of those days. What kind of error is the coffee shop owner in danger of making?

Day	1	2	3	4	5	6	7	8	9	10
Cups sold	488	502	496	506	492	489	510	511	506	500

Solution:

The owner's null and alternative hypotheses are

$$H_0 : \mu = 500$$

$$H_a : \mu \neq 500$$

If we look at the data, we can see that the sample mean is $\bar{x} = 500$. In reality, his null hypothesis is false, but based on the sample mean $\bar{x} = 500$, he may be in danger of accepting the null when he shouldn't.

	H_0 is true	**H_0 is false**
Reject H_0	Type I error P(Type I error)=alpha	CORRECT
Accept H_0	CORRECT	**Type II error P(Type II error)=beta**

Which means the coffee shop owner is in danger of making a Type II error.

TEST STATISTICS FOR ONE- AND TWO-TAILED TESTS

1. A local high school states that its students perform much better than average on a state exam. The average score for all high school students in the state is 106 points. A sample of 256 students at this particular school had an average test score of 129 points with a sample standard deviation of 26.8. Choose and calculate the appropriate test statistic.

Solution:

The sample is comparing average scores, which means the population parameter is a population mean (not a proportion) with an unknown standard deviation (since we have the sample standard deviation and not the population standard deviation).

The sample size is large enough at 256 high schoolers that we can assume the distribution is approximately normal. In this case, we use a t-test statistic.

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{129 - 106}{\frac{26.8}{\sqrt{256}}} \approx 13.73$$

2. A dietician is looking into the claim at a local restaurant that the number of calories in its portion sizes is lower than the national average. The national average is $1{,}500$ calories per meal. She samples 35 meals at

the restaurant and finds they contain an average of $1{,}250$ calories per meal with a sample standard deviation of 350.2. Choose and calculate the appropriate test statistic.

Solution:

The sample is comparing average number of calories, which means the population parameter is a population mean (not a proportion) with an unknown standard deviation (since we have the sample standard deviation and not the population standard deviation).

The sample size is large enough at 35 meals that we can assume the distribution is approximately normal. In this case, we use a t-test statistic.

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{1{,}250 - 1{,}500}{\frac{350.2}{\sqrt{35}}} \approx -4.22$$

■ 3. In a recent survey, 567 out of a 768 randomly selected dog owners said they used a kennel that was run by their veterinary office to board their dogs while they were away on vacation. The study would like to make a conclusion that the majority (more than $50\,\%$) of dog owners use a kennel run by their veterinary office when the owners go on vacation. Choose and calculate the appropriate test statistic.

Solution:

The sample size is large enough at 768 randomly selected individuals that we can state the distribution is approximately normal. We can show this by using the checks for the population proportion, $np \geq 10$ and $n(1-p) \geq 10$. The sample size is $n = 768$ and the population proportion is

$$\hat{p} = \frac{567}{768} \approx 0.738$$

Therefore,

$$n\hat{p} = (768)(0.738) = 567 \geq 10$$

$$n(1-\hat{p}) = (768)(1-0.738) = 201 \geq 10$$

So we can say that the test statistic will be the z-test statistic for a population proportion.

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.7383 - 0.5}{\sqrt{\frac{0.5(1-0.5)}{768}}} \approx 13.21$$

■ 4. You want to open a day care center, so you take a random sample of 500 households in your town with children under preschool age, and find that 243 of them were using a family member to care for those children. You want to determine if, at a statistically significant level, fewer than half of households in your town are using a family member to care for the kids.

1. Set up the hypothesis statements.

2. Check that the conditions for normality are met.

3. State the type of test: upper-tailed, lower-tailed, or two-tailed.

4. Calculate the test statistic using the appropriate formula.

Solution:

The hypothesis statements would be

$$H_0 : p \geq 0.5$$

$$H_a : p < 0.5$$

We need to see if we have an approximately normal distribution by using the checks for a population proportion. The sample size is from a simple random sample of $n = 500$ households. The proportion is the 243 out of the 500 households, so $\hat{p} = 243/500 = 0.486$.

$$n\hat{p} = (500)(0.486) = 423 \geq 10$$

$$n(1 - \hat{p}) = (500)(1 - 0.486) = 257 \geq 10$$

Because both values are greater than 10, the distribution is approximately normal. This is a lower-tailed test because the alternative hypothesis uses the $<$ sign.

This is a population proportion, so we'll calculate a z-test statistic for a population proportion.

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}} = \frac{0.486 - 0.50}{\sqrt{\frac{0.5(1 - 0.5)}{500}}} \approx -0.6261$$

5. The highest allowable amount of bromate in drinking water is 0.0100 mg/L^2. A survey of a city's water quality took 50 water samples in random locations around the city and found an average of 0.0102 mg/L^2 of bromate with a sample standard deviation of 0.0025 mg/L. The survey committee is interested in testing if the amount of bromate found in the water samples is higher than the allowable amount at a statistically significant level.

1. Set up the hypothesis statements.

2. Check that the conditions for normality are met.

3. State the type of test: upper-tailed, lower-tailed, or two-tailed.

4. Calculate the test statistic using the appropriate formula.

Solution:

The hypothesis statements would be

$$H_0 : \mu \leq 0.0100$$

$$H_a : \mu > 0.0100$$

The sample size is a simple random sample of 50 samples, so the distribution is approximately normal. This is an upper-tailed test because the alternative hypothesis uses the greater than sign.

This is a population mean with an unknown population standard deviation, so we'll calculate a t-test statistic with the population mean formula.

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{0.0102 - 0.0100}{\frac{0.0025}{\sqrt{50}}} \approx 0.5658$$

■ 6. A farmer reads a study that states: The average weight of a day-old chick upon hatching is $\mu_0 = 38.60$ grams with a population standard deviation of $\sigma = 5.7$ grams. The farmer wants to see if her day-old chicks have the same average. She takes a simple random sample of 60 of her day-old chicks and finds their average weight is $\bar{x} = 39.1$ grams.

1. Set up the hypothesis statements.

2. Check that the conditions for normality are met.

3. State the type of test: upper-tailed, lower-tailed, or two-tailed.

4. Calculate the test statistic using the appropriate formula.

Solution:

The hypothesis statements would be

$$H_0 : \mu = 38.60$$

$$H_a : \mu \neq 38.60$$

The sample size is a simple random sample of 60 of her day-old chicks so we can say the distribution is approximately normal. This is a two-tailed test because the alternative hypothesis uses the $\neq$ sign.

This is a population mean with a known population standard deviation, so we'll calculate a z-test statistic with the population mean formula.

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{39.10 - 38.60}{\frac{5.7}{\sqrt{60}}} \approx 0.6795$$

THE P-VALUE AND REJECTING THE NULL

1. A medical trial is conducted to test whether or not a new medicine reduces total cholesterol, when the national average is 230 mg/dL with a standard deviation of 16 mg/dL. The trial takes a simple random sample of 223 adults who take the new medicine, and finds $\bar{x} = 227$ mg/dL with standard error $\sigma_{\bar{x}} = 14$ mg/dL. What can the trial conclude at a significance level of $\alpha = 0.01$?

Solution:

The hypothesis statements will be

$$H_0 : \mu \geq 230$$

$$H_a : \mu < 230$$

The test statistic will be

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{227 - 230}{\frac{16}{\sqrt{223}}} = -\frac{3\sqrt{223}}{16} \approx -2.80$$

From the z-table we get 0.0026.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026

Because this is a lower-tail test, the p-value is just this value we found, $p = 0.0026$. Comparing this to $\alpha = 0.01$, we can see that $p \leq \alpha$, which means we'll reject the null hypothesis.

Therefore, the trial can conclude that the new medicine reduces cholesterol at the $p = 0.0026$ significance level.

2. The national average length of pregnancy is 283.6 days with a population standard deviation of 10.5 days. A hospital wants to know if the average length of a pregnancy at their hospital deviates from the national average. They use a sample of $9{,}411$ births at the hospital to calculate a test statistic of $z = -1.60$. Set up the hypothesis statements and find the p-value.

Solution:

The hospital wants to know if mean length of pregnancy at their hospital is different than the national average in a significant way.

$$H_0 : \mu = 283.6$$

$$H_a : \mu \neq 283.6$$

Because the alternative hypothesis uses a $\neq$ sign, this is a two-tailed test. We were told in the problem that the test statistic is $z = -1.60$, so we'll look that up in the z-table.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	**.0548**	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559

For the lower tail, $z = -1.60$ gives an area of 0.0548. Now to calculate our p-value, we multiply this by 2.

$$p = 2(0.0548)$$

$$p = 0.1096$$

3. The highest allowable amount of bromate in drinking water is 0.0100 (mg/L)^2. A survey of a city's water quality took 31 water samples in random locations around the city and used the data to calculate a test statistic of $t = 2.04$. The city wants to know if the amount of bromate in their drinking water is too high. Set up the hypothesis statements and determine the type of test, then find the p-value.

Solution:

The city wants to know if the amount of bromate in their drinking water is higher than the allowable amount in a significant way.

$$H_0 : \mu \leq 0.0100$$

$$H_a : \mu > 0.0100$$

Because the alternative hypothesis uses a $>$ sign, this is an upper-tailed test. We were told in the problem that the test statistic is $t = 2.04$, so we'll look that up in the t-table, but we'll also need to know the degrees of freedom. We know the study included 31 samples, so degrees of freedom is $n - 1 = 31 - 1 = 30$.

	Upper-tail probability p									
df	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	**2.042**	2.457	2.750	3.385	3.646
	50%	**60%**	**70%**	**80%**	**90%**	**95%**	**98%**	**99%**	**99.8%**	**99.9%**
	Confidence level C									

Because the test statistic and degrees of freedom gives a p-value just under $p = 0.025$, we'll round the p-value to $p = 0.025$.

4. A paint company produces glow in the dark paint with an advertised glow time of 15 min. A painter is interested in finding out if the product behaves worse than advertised. She sets up her hypothesis statements as $H_0 : \mu \geq 15$ and $H_a : \mu < 15$, then calculates a test statistic of $z = -2.30$. What would be the conclusions of her hypothesis test at significance levels of $\alpha = 0.05$, $\alpha = 0.01$, and $\alpha = 0.001$?

Solution:

We need to look up $z = -2.30$ in the z-table.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110

For a lower-tail test, the p-value is given by this value we found in the z-table, so $p=0.0107$.

We know that

If $p \le \alpha$, reject the null hypothesis

If $p > \alpha$, do not reject the null hypothesis

Therefore,

- For $p=0.0107$ and $\alpha=0.05$, $p \le \alpha$, so she'd reject the null
- For $p=0.0107$ and $\alpha=0.01$, $p > \alpha$, so she'd fail to reject the null
- For $p=0.0107$ and $\alpha=0.001$, $p > \alpha$, so she'd fail to reject the null

5. An article reports that the average wasted time by an employee is 125 minutes every day. A manager takes a small random sample of 16 employees and monitors their wasted time, calculating that average wasted time for her employees is 118 minutes with a standard deviation of 28.7 minutes. She wants to know if 118 minutes is below average at a significance level of $\alpha=0.05$. She assumes the population is normally distributed.

1. State the population parameter and whether you should use a t-test or z-test statistic.

2. Check that the conditions for performing the statistical test are met.

3. Set up the hypothesis statements.

4. State the type of test: upper-tailed, lower-tailed, or two-tailed.

5. Calculate the test statistic using the appropriate formula.

6. Calculate the p-value.

7. Compare the p-value to the significance level and draw a conclusion.

Solution:

This is a population mean with an unknown population standard deviation because the manager is going to do her analysis based on the sample standard deviation. She also has a small sample size of 16 employees. This means we should use the t-test statistic because we have a small sample size and also an unknown population standard deviation.

The conditions for performing a t-test with a population mean are an approximately normal distribution and a simple random sample, and we've been told in the problem that both of those conditions are met.

The manager wants to know if 118 minutes is below average. We're comparing 118 minutes to the stated average of 125 minutes. Since she wants to know if her measurement is below average, we should use the less than symbol in our alternative hypothesis.

$$H_0: \mu \geq 125$$

$$H_a: \mu < 125$$

The test statistic will be

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{118 - 125}{\frac{28.7}{\sqrt{16}}} \approx -0.9756$$

The next step is to find the p-value by looking up the test statistic in the t-table. To look up a t-value, we'll also need to know the degrees of freedom from the problem. We know the study included 16 samples, so the degrees of freedom are $16 - 1 = 15$.

We calculated the test statistic as $t \approx -0.9756$. We're looking for the area in the lower tail, but the table will give us the area in the upper tail when $t = 0.9756$. Remember these values are equal because the t-curve is symmetric. Now we look up where our test statistic and degrees of freedom intersect. The value we read from the t-table is somewhere between $p = 0.20$ and $p = 0.15$.

	Upper-tail probability p									
df	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.691	**0.866**	**1.074**	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
	50%	**60%**	**70%**	**80%**	**90%**	**95%**	**98%**	**99%**	**99.8%**	**99.9%**
	Confidence level C									

Regardless of the exact value of p between $p = 0.20$ and $p = 0.15$, at a significance level of $\alpha = 0.05$, we can say $p > \alpha$, so the manager will fail to reject the null hypothesis, and conclude that there's not enough evidence to conclude that her employees waste less time than the average rate of 125 minutes per day at the significance level of $\alpha = 0.05$.

6. We want to test if college students take fewer than than 5 years to graduate, on average, so we take a simple random sample of 36 students and record their years to graduate. For the sample, $\bar{x} = 4.9$ and $s = 0.5$. What can we conclude at 90% confidence?

Solution:

The hypothesis statements will be

$$H_0 : \mu \geq 5$$

$$H_a : \mu < 5$$

Find the test statistic.

$$z = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{4.9 - 5}{\frac{0.5}{\sqrt{36}}} = \frac{-0.1}{\frac{0.5}{6}} = \frac{-0.6}{0.5} = -1.2$$

If we look up this z-score in a z-table, we get 0.1151.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	**.1151**	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170

Because this is a lower-tail test, $p = 0.1151$. Compared to $\alpha = 0.10$, $p > \alpha$, which means we'll fail to reject the null hypothesis. Which means we can't support the hypothesis that college students, on average, take fewer than 5 years to graduate.

HYPOTHESIS TESTING FOR THE POPULATION PROPORTION

1. A large electric company claims that at least 80% of the company's $1{,}000{,}000$ customers are very satisfied. Using a simple random sample, 100 customers were surveyed and 73% of the participants were very satisfied. Based on these results, should we use a one- or two- tailed test, and should we accept or reject the company's hypothesis? Assume a significance level of 0.05.

Solution:

The first step is to state the null and alternative hypotheses for the survey.

$$H_0: \ p \geq 0.80$$

$$H_a: \ p < 0.80$$

These hypotheses require a one-tailed test, specifically a lower-tail test. The null hypothesis will be rejected only if the sample proportion is significantly less than 80%.

We calculate standard error based on the sample,

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.73(1-0.73)}{100}} = 0.0444$$

and then compute the z-score test statistic.

$$z = \frac{\hat{p} - p}{\sigma_p} = \frac{0.73 - 0.80}{0.0444} = -1.58$$

The z-table gives 0.0401 for a z-score of $z = -1.58$.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	**.0571**	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681

Since we have a one-tailed test, the p-value is $p = 0.0571$, and we were told in the problem that $\alpha = 0.05$. Because the p-value is greater than the α-level, $p \geq \alpha$, you'll fail to reject the null hypothesis.

2. A university is conducting a statistical test to determine whether the percentage of its students who live on its campus is above the national average of 64%. They've calculated the test statistic to be $z = 1.40$. Set up hypothesis statements and find the p-value.

Solution:

The university wants to know if the proportion of students who live on campus is above the national average in a statistically significant way.

$$H_0 : p \leq 64\%$$

$$H_a : p > 64\%$$

Because the alternative hypothesis uses a $>$ sign, this is a one-tail, upper-tailed test. We were told that the test statistic is $z = 1.40$, so we'll look that up in the z-table.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	**.9192**	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441

This is an upper-tail test, which means the p-value is the area outside of 0.9192, or

$$p = 1 - 0.9192$$

$$p = 0.0808$$

3. A report claims that 60% of American families take fewer than 6 months to purchase a home, from the time they start looking to the time they make their first offer. A realtor wants to know if her clients purchase at the same rate, so she takes a simple random sample of 50 of her clients and finds a sample proportion $\hat{p} = 0.64$ and standard error $\sigma_{\hat{p}} = 0.05$. What can she conclude with 90% confidence?

Solution:

The first step is to state the null and alternative hypotheses for the survey.

$$H_0: \ p = 0.64$$

$$H_a: \ p \neq 0.64$$

These hypotheses require a two-tailed test. The null hypothesis will be rejected only if the sample proportion is significantly different than $64\,\%$.

The z-score test statistic will be

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{0.64 - 0.60}{0.05} = \frac{0.04}{0.05} = 0.8$$

The z-table gives 0.7881 for a z-score of $z = 0.80$.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389

This is an upper-tail test, which means the p-value is the area outside of 0.7881, or

$$p = 1 - 0.7881$$

$$p = 0.2119$$

Since we have a two-tailed test, the p-value is double this, or

$$p = 2(0.2119)$$

$$p = 0.4238$$

We were told in the problem that $\alpha = 0.10$. Because the p-value is greater than the α-level, $p \geq \alpha$, which means the realtor will fail to reject the null hypothesis. So she can't say that her clients purchase at a different rate than the report claims.

4. A gambler wins 48% of the hands he plays, but he feels like he's on a losing streak recently, winning fewer hands than normal. He takes a random sample of 40 of his recent hands, and finds the proportion of winning hands in the sample to be $\hat{p}=0.45$ with $\sigma_{\hat{p}}=0.02$. What can he conclude with 90% confidence?

Solution:

The first step is to state the null and alternative hypotheses for the survey.

$$H_0: \ p \geq 0.48$$

$$H_a: \ p < 0.48$$

These hypotheses require a one-tailed test, specifically a lower-tail test. The null hypothesis will be rejected only if the sample proportion is significantly lower than 48%.

The z-score test statistic will be

$$z=\frac{\hat{p}-p}{\sigma_{\hat{p}}}=\frac{0.45-0.48}{0.02}=\frac{-0.03}{0.02}=-1.5$$

The z-table gives 0.0668 for a z-score of $z=-1.50$.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681

This is a lower-tail test, which means this is also the p-value, $p = 0.0668$.

We were told in the problem that $\alpha = 0.10$. Because the p-value is less than the α-level, $p < \alpha$, the gambler will reject the null hypothesis, and conclude that he has in fact been on a losing streak at a statistically significant level.

5. A study claims that the proportion of new homeowners who purchase an internet subscription plan is 0.92. You take a random sample of 140 new homeowners to test this claim, and find $\hat{p} = 0.9$ with $\sigma_{\hat{p}} = 0.015$. What can you conclude at a significance level of $\alpha = 0.05$?

Solution:

The first step is to state the null and alternative hypotheses for the survey.

$$H_0: \ p = 0.92$$

$$H_a: \ p \neq 0.92$$

These hypotheses require a two-tailed test. The null hypothesis will be rejected only if the sample proportion is significantly different than 92%.

The z-score test statistic will be

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{0.90 - 0.92}{0.015} = \frac{0.02}{0.015} \approx 1.33$$

The z-table gives 0.9082 for a z-score of $z \approx 1.33$.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319

The z-table gives an upper-tail probability, which means the p-value is

$$p = 1 - 0.9082$$

$$p = 0.0918$$

But since this is a two-tailed test, we double this to find the p-value, and we get

$$p = 2(0.0918)$$

$$p = 0.1836$$

We were told in the problem that $\alpha = 0.05$. Because the p-value is greater than the α-level, $p \geq \alpha$, you'll fail to reject the null hypothesis, which means that you can't conclude that the number of homeowners who purchase an internet subscription plan is different than $92\,\%$.

6. A recent study reported that the $15.3\,\%$ of patients who are admitted to the hospital with a heart attack die within 30 days of admission. The same study reported that $16.7\,\%$ of the $3{,}153$ patients who went to the

hospital with a heart attack died within 30 days of admission when the lead cardiologist was away.

Is there enough evidence to conclude that the percentage of patients who die when the lead cardiologist is away is any different than when they're present? Make conclusions at significance levels of $\alpha = 0.05$ and $\alpha = 0.01$.

1. State the population parameter and whether you should use a t-test or z-test statistic.

2. Check that the conditions for performing the statistical test are met.

3. Set up the hypothesis statements.

4. State the type of test: upper-tailed, lower-tailed, or two-tailed.

5. Calculate the test statistic using the appropriate formula.

6. Calculate the p-value.

7. Compare the p-value to the significance level and draw a conclusion.

Solution:

This is a population proportion because the data is looking at the proportion of heart attack patients admitted to the hospital who die within 30 days of admittance.

The sample size is large at $3{,}153$ with a population proportion of 16.7%, but to continue with the test we need to assume that the sample was a simple random sample (since it's not stated in the problem).

This sample size is large enough to meet the conditions:

$$np = (3{,}153)(0.167) = 527 \geq 10$$

$$n(1-p) = (3{,}153)(1-0.167) = 2{,}626 \geq 10$$

When these two conditions are met, then the distribution is approximately normal. Then we can continue with the hypothesis test.

According to the problem, we want to know if the percentage of patients who went to the hospital with a heart attack and died within 30 days of admission when the leading cardiologist was away differs from when they were not away. This means we need to use the $\neq$ symbol in our hypothesis statement.

$$H_0 : p = 0.153$$

$$H_a : p \neq 0.153$$

Since we're dealing with a population proportion, the z-test statistic will be

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.167 - 0.153}{\sqrt{\frac{0.153(1-0.153)}{3{,}153}}} \approx 2.1837$$

The next step is to find the p-value by looking up the test statistic in the z-table. Since this is a two-tailed test, we'll need to double the area we find in either the upper or lower tail. From the z-table, we find a value of 0.9854.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890

But this is the area below the upper tail. Before we can do anything else we need to find the area in the upper tail. The total area under the curve is 1, so we'll subtract this value from 1.

$$1 - 0.9854 = 0.0146$$

Now to calculate the p-value, we multiply the upper tail by 2.

$$p = 2(0.0146)$$

$$p = 0.0292$$

We know that

If $p \leq \alpha$, reject the null hypothesis

If $p > \alpha$, do not reject the null hypothesis

Therefore,

- For $p = 0.0292$ and $\alpha = 0.05$, $p \leq \alpha$, so we'd reject the null

- For $p = 0.0292$ and $\alpha = 0.01$, $p > \alpha$, so we'd fail to reject the null

Which means there's enough evidence to conclude that the percentage of patients who went to the hospital with a heart attack and died within 30 days of admission when the leading cardiologist was away is different than when the leading cardiologist is present, at a statistically significant level of $\alpha = 0.05$, but not at $\alpha = 0.01$.

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