# B. Derivatives

## B1. Calculating derivatives

Calculate the following derivatives (using pen and paper):

i. $\frac{d}{dx}(x^2 - 4x^3)$  ii. $\frac{d}{dx}\sin(x^2)$  iii. $\frac{d}{dx}(\ln(x)\, e^x)$

iv. $\frac{d}{dx}\pi$  v. $\frac{d^2}{dx^2}x^{3/2}$  vi. $\frac{d}{dx}\left(1+\frac{1}{x}\right)^3$

vii. $\frac{d}{dx}\sin(\cos(x))$  viii. $\frac{d^2}{dx^2}\sin(x)$  ix. $\frac{d}{dy}\left(y-\frac{1}{y}\right)$

## B2. The limit definition of a derivative

Find the derivative of $f(x) = 3x^2 + 7x - 1$ using the limit definition of a derivative,

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

and confirm that it matches what you expect.

## B3. The derivative of $x^x$

For some functions, like $f(x) = x^x$, it is not straight-forward to find the derivative using standard rules. In such cases, we usually have to have a really good idea! Recall that for $z > 0$, $z = e^{\ln z}$, and as such, we can rewrite the function $f(x) = e^{\ln f(x)}$. With this really good idea, calculate the derivative of $x^x$ (for $x > 0$).

## B4. A mysterious derivative

Professor Mathson (not her again!) tells you that the function $f$ has the property:

$$f = \frac{d^4 f}{dx^4}$$

She then asks you to suggest no less than *three* candidates for what the function $f$ might be.

## B5. L'Hôpital's rule

Where applicable, use l'Hôpital's rule to evaluate the following limits. Otherwise, find a different way of evaluating the limit:

i. $\lim_{x\to 0}\frac{\tan x}{x}$  ii. $\lim_{x\to -\infty} xe^x$  iii. $\lim_{x\to\infty} xe^x$

## B6. Big-Oh

In the study of the run time of algorithms, it is a well-known fact that, as $n$ approaches infinity, $O(\log n)$ algorithms are faster than $O(n^k)$ algorithms for any $k > 0$. Use l'Hôpital's rule to show this. For simplicity, use $\ln n$ instead of $\log n$.