masters theorem for dividing functions:

general form: $T(n) = aT(n/b) + f(n)$

assume:

$a \geq 1$ $\quad f(n) = \theta(n^k \log^p n)$
$b > 1$

- From this form, we find two values:
  1. $\log_b^a$
  2. K

↳ Based on those two values, there are three cases:

- Case 1: if $\log_b^a > k$, then $\theta(n^{\log_b^a})$
- case 2: if $\log_b^a = k$,

  if $P > -1$, $\theta(n^k \log^{P+1} n)$
  if $P = -1$, $\theta(n^k \log\log n)$
  if $P < -1$, $\theta(n^k)$

- case 3: if $\log_b^a < K$,

  if $P \geq 0$, $\theta(n^k \log^P n)$
  if $P < 0$, $\theta(n^k)$

Example of recurrence relation:

- case 1: $\quad$ * $T(n) = aT(n/b) + f(n)$ * $f(n) = \theta(n^k \log^p n)$

$T(n) = 2T(n/2) + 1$

$a = 2$
$b = 2$
$f(n) = \theta(1)$
$= \theta(n^0 \log^0 n)$
↳ $k = 0$ $\quad P = 0$

$\log_b^a = \log_2^2 = 1$, $k = 0$

↓

$\log_b^a > K$ * $1 > 0$ * case 1

↓

$\theta(n^{\log_b^a})$

↓

$\theta(n^1)$

- case 1:

$T(n) = 4T(n/2) + n$ $\quad \downarrow * n^1$ $\quad * \ T(n) = aT(n/b) + f(n) \quad * f(n) = \theta(n^k \log^p n)$

① lets find $\log^a_b$ and k

$* \ \theta(n^{\log^a_b})$

$\log^4_2 = 2$, $k = 1$, $P = 0$ $\rightarrow$ satisfies case #1 $\rightarrow \theta(n^2)$

- case 1:

$T(n) = 8T(n/2) + n \quad \longrightarrow \theta(n^3)$

$\log^8_2 = 3$, $k = 1$,

- case 1:

$T(n) = 9T(n/3) + 1 \quad \nearrow n^0$

$\log^9_3 = 2$, $k = 0 \rightarrow 2 > 0$

$\hookrightarrow \log^a_b > k \quad \longrightarrow \theta(n^2)$

- case 2:

$T(n) = 2T(n/2) + n$

$\log^2_2 = 1$, $k = 1$, $P = 0$

$\log^a_b = k$, $1 = 1$

$\longrightarrow$ if $\log^a_b = k$,

$\longrightarrow$
- if $P > -1$, $\theta(n^k \log^{p+1} n)$
- if $P = -1$, $\theta(n^k \log\log n)$
- if $P < -1$, $\theta(n^k)$

$\rightarrow \theta(n \log n)$

- case 2:

*can be written as $+ n^1 \times \log^{-1} n$

* P is in denom $= -1$

$T(n) = 2T(n/2) + \frac{n}{\log n}$

$\log^2_2 = 1$, $k = 1$, $P = -1$ $\longrightarrow$ if $\log^a_b = k$,
- if $P > -1$, $\theta(n^k \log^{p+1} n)$
- $\rightarrow$ if $P = -1$, $\theta(n^k \log\log n)$
- if $P < -1$, $\theta(n^k)$

$\rightarrow \theta(n \log\log n)$

- case 3:

$T(n) = T(n/2) + n^2$ $\quad$ if $\log^a_b < k$,

$\log^1_2 = 0$, $k = 2$, $0 < 2$ $\longrightarrow$

$\rightarrow$ if $P \geq 0$, $\theta(n^k \log^p n)$
if $P < 0$, $\theta(n^k)$

$\theta(n^2)$