Mapping the Magnetic Field of a Short, Thick Solenoid

William Thomas Campagna
University Physics II, Austin Peay State University, Clarksville, TN 37041
wcampagna@my.apsu.edu

Abstract

We built and calibrated a coil probe to map the axial magnetic field of a short, thick solenoid, then compared our measurements to three theoretical models. An ideal solenoid is infinitely long and thin and, due to the geometry of such an object, produces a constant magnetic field. Because of the importance of geometry of a solenoid, the field produced by a short, thick solenoid is dependent on the position of the solenoid. First, we derived the field at the center of Helmholtz coils from the Biot-Savart law and identified the region of uniform field as a function of coil spacing. Experimentally, we constructed a 23 turn coil probe optimized for signal strength and resolution, calibrating its voltage output against two pairs of Helmholtz coils. Using the coil probe, we collect data on the 3,400 turn solenoid's field as we move the probe through the center of the solenoid. Next, we verified the classic Helmholtz spacing by mapping the field with a commercial Gauss sensor. We compare the measurements to three models: the infinite solenoid, the circular-coil approximation, and the thin, finite length solenoid. We then plotted the measured field versus position and performed numerical fits.

Introduction

Magnetic field mapping is imperative to the design of electromagnets, precision actuators, and processes such as magnetic confinement fusion. A short, thick solenoid varies from the idealized infinite-length solenoids found in theory. Despite this, there is a law that can be used to relate current I to the generated magnetic field B at a distance z along the central axis. This law is referred to as Biot-Savart's law. From Biot-Savart law for a steady current I with N number of turns, through means of calculus and algebraic operations we derive the axial field of a single loop of radius a carrying current I at a distance x along its axis is:

$$B = \frac{\mu_0 NI}{2a} \Rightarrow B_{loop}(x) = \frac{\mu_0 I a^2}{2(a^2+x^2)^{3/2}},$$

Where $\mu_0 = 1.257 \times 10^{-6}$ Tm/A is the permittivity constant.

Helmholtz coils are two identical circular loops separated by their radius that produce an approximately uniform field region. Each coil has current flowing through in the same direction, producing a central-axial magnetic field that's maximum is in the center. They serve as a calibration standard via Faraday induction. The field vectors combine due to the principle of superposition when the coils are separated by a distance of R creating a uniform field in the region between the coils.

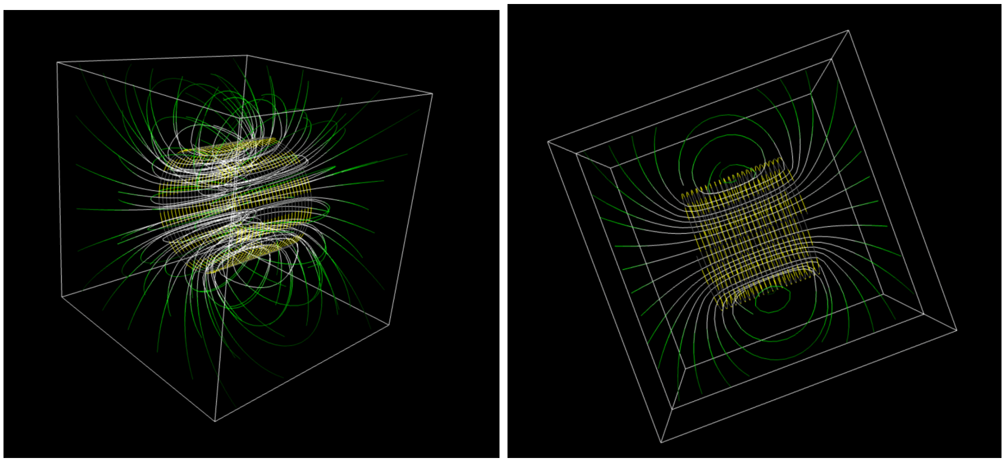

Figure 1: Simulated magnetic fields of a solenoid. On the left is a three dimensional view of the fields, on the right is a slice of the X Plane. The field lines shown on the right give a clear view of the magnetic field on the inside of a short solenoid.

The magnetic field is related to the electric field by Faraday's law of induction. Variations in the magnetic field can be mapped as a function for position within the field when an emf is produced inside the coils. Referring back to the ideal solenoid with a turn density of n and length l, the magnetic field is:

$$B_{ideal} = \mu_0 nl$$

Notice that in an ideal solenoid, there is no term for position within the field. This is because the field is uniform when the solenoid's length is infinite.

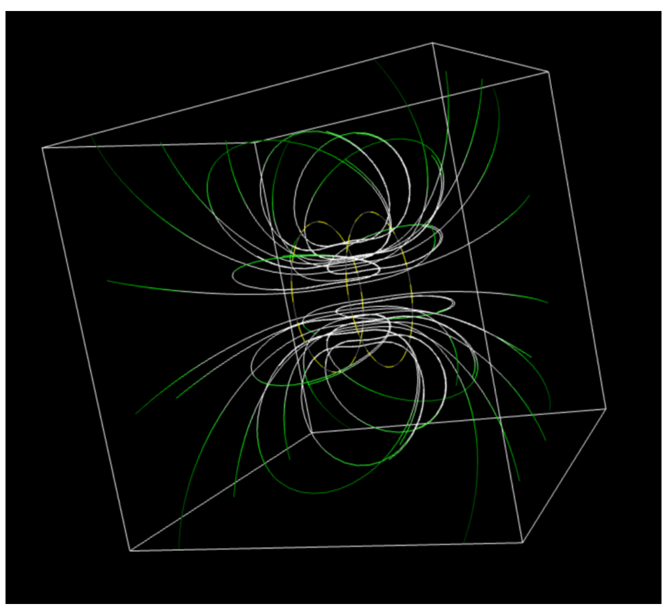

Figure 2: Simulated magnetic field between Helmholtz coils separated by their approximate radius. This shows the uniform nature of Helmholtz coils in this configuration.

In contrast, a real solenoid will have finite length L, so a different equation will be needed to solve for the magnetic field. At the ends of a finite solenoid there are field fringes that can only contribute a field equal to half the infinite solenoids field. We can produce a model of this field at all points along the axis by taking the difference between the angles, β and α, produced between any point and each end of the solenoid:

$$B_{real} = \frac{\mu_0 NI(cos\beta - cos\alpha)}{2L}$$

By comparing these models to experimental data from the short, thick solenoid we can analyze the effects of a non-ideal solenoid on the field it produces.

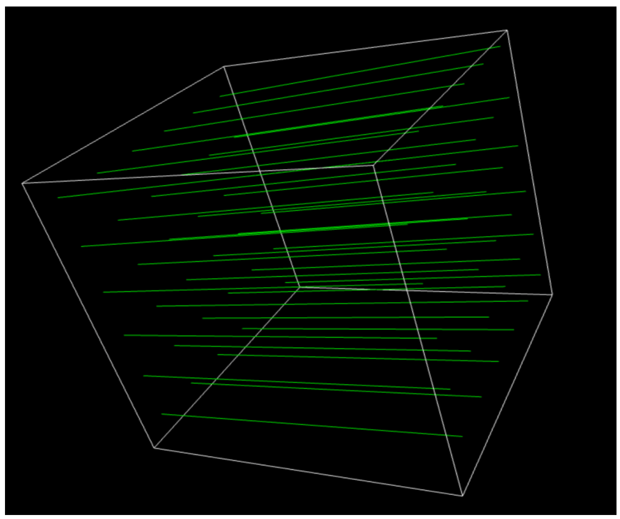

Figure 3: Simulation of ideal uniform field lines. Each line is parallel to all other lines and the density of the lines is constant.

Experimental Methods

We measured the various physical properties of our coils to determine each one's number of turns, the radius of the coils, and the length from one end to the other. This involved the small Helmholtz coil (400 turns, $a = 8.10$ cm) and the big Helmholtz coil (130 turns, $a = 15.0$ cm), as well as the solenoid which has 3400 turns, an inner/outer radii of 5.25 and 7.25 cm respectively, a length of 9 cm, and is driven at 61 V AC (2000 W). We use our 23 turn pickup coil to measure the magnetic flux through the production of emf. To set up our experiment we attach a meter stick to the table along with some clamps to hold it in place. Attached to one of those clamps is a rod which holds a smaller clamp horizontally such that we can attach our probe keeping it inline with the solenoid's center axis. This ensures a fixed position, allowing for accurate and consistent measurements throughout the experiment. We connect our pickup coil to a precise voltmeter for accurate data collection. After we have moved the pickup coil along the solenoids axis, we record the peak induced voltage and we convert it to B_{real}.

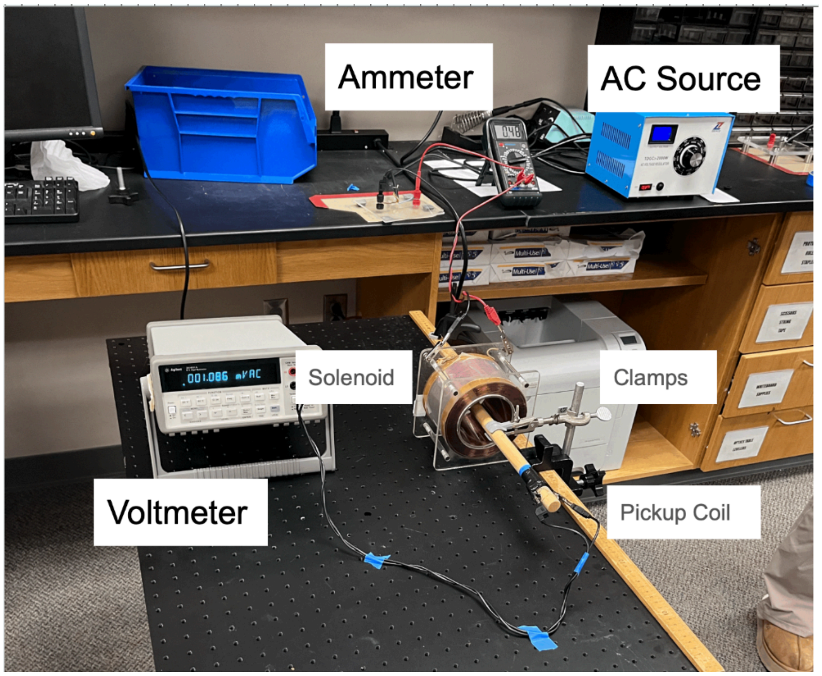

Figure 4: Experimental setup for solenoid field mapping.

Results and Analysis

Calibrations were made using the data from the small Helmholtz coils and the large Helmholtz coils. Both graphs produced a linear fit between the induced electric field and the current. The data for the small Helmholtz coil has a closer fit and reaches a greater induced electric field.

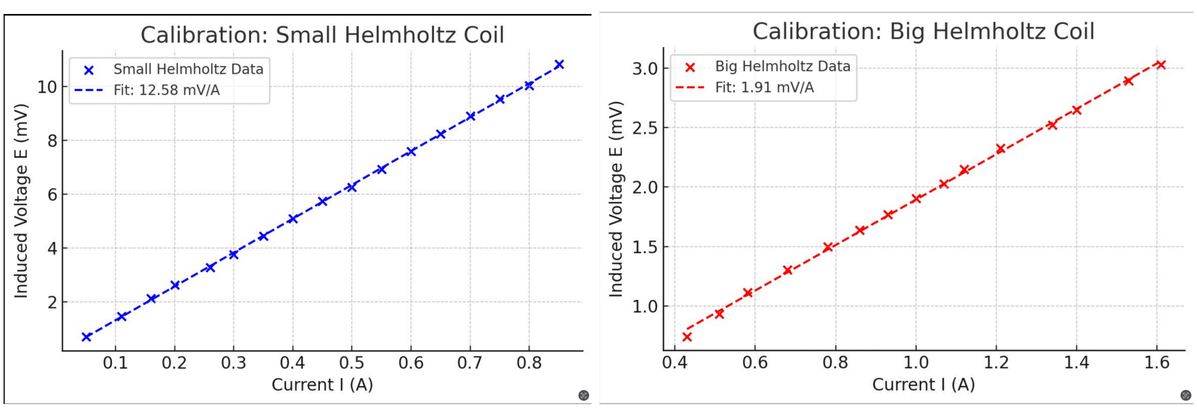

Figure 5: Calibration graphs relating induced voltage to current in a pair of Helmholtz coils. The graph on the left is from the Eisco model Helmholtz coils, and the curve fits very well. The graph on the right is from the Leybold model Helmholtz coil. The curve does not fit as well as the Leybold model is an older design that produces significantly smaller voltages.

Since the small Helmholtz coil produced a better, more neat graph, it was chosen to determine the calibration equation to convert induced voltage to magnetic field.

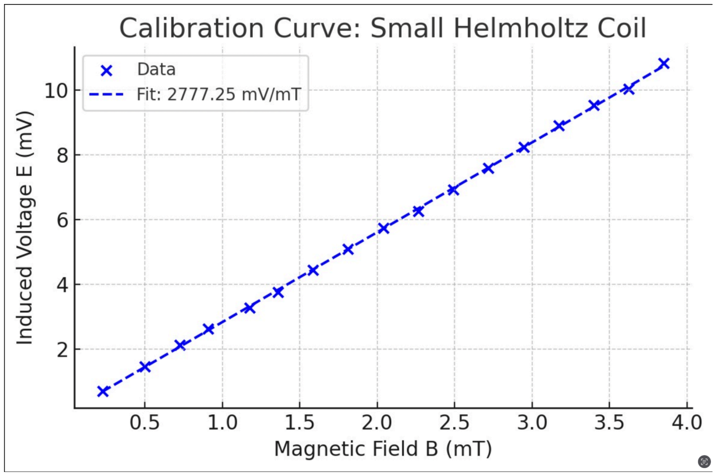

Figure 6: Eisco model Helmholtz coil final calibrated curve relating induced voltage to the magnetic field. This is the chosen calibration used for the data from the pickup coil.

We plot the experimental position vs. the magnetic field and apply the three models using the known dimensions of our solenoid and the same position values. We used the radius from the center of the solenoid's width and fit the solenoid model to the data using Python. The infinite-solenoid model predicts a uniform field of $B_{inf} = \mu_0 nI = 8.7$ mT across all positions. This overestimates the real peak (~5 mT) and fails to capture any edge fall-off. The circular loop approximation yields a peaked profile similar to what we expect but is too sharp and underestimates the field at the center (~4.5 mT). The finite-length solenoid model closely follows the calibrated data, predicting a maximum of approximately 4.8 mT at the midpoint and follows closely with the fall off.

The infinite model is only accurate deep inside a very long solenoid ($L \gg R$) and as such, is not applicable here. The single loop approximation works for rings but fails when predicting extended windings. The finite-length expression is the most appropriate analytic model for a short, thick solenoid. Measurement scatter (~ 0.02 mT) is fairly small when compared to the peak field.

The grey vertical lines approximate the length of the solenoid on the graph as a visualization aid for analyzing the difference in magnetic field as the probe moves through the solenoid. Moving left to right we see the magnetic field increasing gradually until we are in the center of the solenoid (0 cm) and then follows a symmetrical, decreasing curve as the probe leaves the magnetic field.

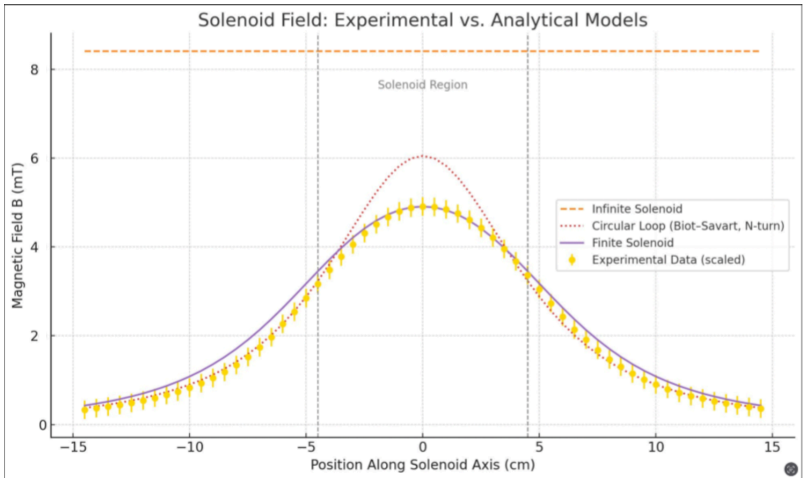

Figure 7: The magnetic field against position data compared against three models that are based on various solenoid shapes. Grey, vertical lines represent the length of the solenoid. The best fitting model is the finite solenoid model, showing a gradual drop-off of the field which better matches the experimental data. Graph was made with current 0.48 amps, not 0.20 amps.

Initially we measured our current to be 0.48 amps, but upon retesting it became apparent that our actual current was likely 0.20 amps. This adds to our uncertainty in collected data and analysis, but if we were to assume our actual current was 0.20 amps, our best fit would still be the finite solenoid's magnetic field. This is because the geometry of a finite solenoid, with N number of turns, provides a distinct field shape. Our close agreement between experimental data and the finite solenoid model suggests a highly accurate prediction of the axial magnetic field behavior.

Conclusion

We successfully mapped the magnetic field along a short, thick solenoid's central axis within close fitting to a known model for finite solenoids. The calibration of the pickup coil using Helmholtz coils was an effective method, though there is uncertainty due to the limitations in precision caused by background magnetic fields. Additionally, the pickup coil could have had more turns for a stronger signal while moving outside the solenoid.

This experiment can be expanded upon by varying the gauge of the copper wire as a smaller wire diameter would indicate a higher current-density, which would lead to a stronger magnetic field. This relationship is described by Ampere's Law, which relates current density to the curl of the magnetic field. This would make it easier for a pickup coil to gain a stronger signal and it may reveal important details about our finite solenoid model.

References

F. X. Hart, “The Magnetic Field Along the Axis of a Short, Thick Solenoid”, Phys. Teach. 56, 104 (2018);

Donev (2025). University Physics II, Lab Manual, Solenoid Project, APSU.

Falstad, P. (2004). *3-D magnetostatic fields applet*. Paul Falstad. https://www.falstad.com/vector3dm/vector3dm.html?f=SolenoidField&d=streamlines&sl=none&st=20&ld=6&a1=40&a2=30&a3=100&rx=5&ry=-30&rz=3&zm=1.2

Halliday & Resnick (2022). *Fundamentals of Physics* (12th ed.) (pp. 886-914). Cengage.