Estimating and Interpreting Effect Sizes: A Note for 432 Class 08

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A Sample Data Set

Consider the `small.csv` data available on our site, which is modeled on the public Framingham data set available from BIOLINCC[1]. From the BIOLINCC documentation:

[1] The Framingham data from this source are not appropriate for publication or project use because they have been anonymized by permuting the results of individual subjects.

> The Framingham Heart Study is a long term prospective study of the etiology of cardiovascular disease among a population of free living subjects in the community of Framingham, Massachusetts. The Framingham Heart Study was a landmark study in epidemiology in that it was the first prospective study of cardiovascular disease and identified the concept of risk factors and their joint effects.

Available Variables

```
smalldat <- read_csv(here("data/small.csv"))
```

The `smalldat` data contains 150 observations on the following variables[2]:

[2] The *educ* levels are: 1_Low, 2_Middle, 3_High and 4_VHigh, which stands for Very High

Variable	Description
subject	Subject identification code
smoker	1 = current smoker, 0 = not current smoker
totchol	total cholesterol, in mg/dl
age	age in years
sex	subject's sex (M or F)
educ	subject's educational attainment (4 levels)

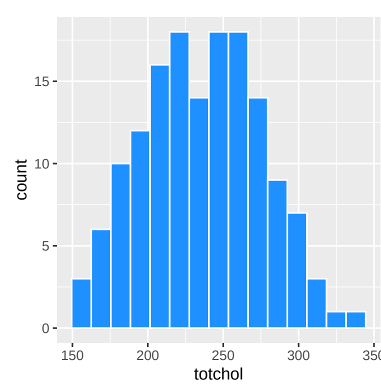


Figure 1: Histogram of totchol values

Effect Interpretation in A Linear Regression Model

```
m1 <- lm(totchol ~ age + sex + factor(educ),
         data = smalldat)
```

```
kable(tidy(m1, conf.int = TRUE), digits = 3)
```

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	171.197	20.201	8.475	0.000	131.268	211.126
age	1.202	0.367	3.270	0.001	0.475	1.928

term	estimate	std.error	statistic	p.value	conf.low	conf.high
sexM	3.612	6.441	0.561	0.576	-9.119	16.343
factor(educ)2_Middle	11.044	7.702	1.434	0.154	-4.180	26.268
factor(educ)3_High	-2.459	9.390	-0.262	0.794	-21.019	16.101
factor(educ)4_VHigh	10.927	9.780	1.117	0.266	-8.405	30.258

1. What is the effect of *age* on *totchol* in Model `m1`?

term	estimate	std.error	statistic	p.value	conf.low	conf.high
age	1.202	0.367	3.27	0.001	0.475	1.928

The coefficient of the *age* effect on *totchol* is 1.202. Suppose we have two subjects, Doris and Emily, who are the same sex and have the same level of education, but Doris is one year older than Emily. Our model predicts that Doris' total cholesterol will be 1.202 mg/dl higher than Emily's.

The 95% confidence interval for this estimated *age* coefficient is (0.475, 1.928), so holding everything else constant, it seems that older *age* is associated with higher *totchol* in this model.

2. What is the effect of *sex* on `totchol` in Model `m1`?

term	estimate	std.error	statistic	p.value	conf.low	conf.high
sexM	3.612	6.441	0.561	0.576	-9.119	16.343

The model is parametrized to incorporate the *sex* information with an indicator (and factor) variable called *sexM* which is interpreted as taking the value 1 when *sex* = M, and 0 otherwise. The coefficient of the *sexM* effect on *totchol* is 3.612. Suppose we have two subjects, David and Emily, who are the same age, have the same level of education, but David is male and Emily is female. Our model predicts that David's total cholesterol will be 3.612 mg/dl higher than Emily's.

The 95% confidence interval for this estimated *sexM* coefficient is (-9.119, 16.343), which suggests that the effect of *sex* on *totchol* could be quite small.

3. What is the effect of *educ* on *totchol* in Model `m1`?

The *educ* variable splits the subjects into four categories. In this model the "1_Low" category is used as the baseline, and we have estimates for "2_Middle" (as compared to "1_Low"), for "3_High" (as compared to "1_Low") and for "4_VHigh" (as compared to "1_Low".)

term	estimate	std.error	statistic	p.value	conf.low	conf.high
factor(educ)2_Middle	11.044	7.702	1.434	0.154	-4.180	26.268
factor(educ)3_High	-2.459	9.390	-0.262	0.794	-21.019	16.101
factor(educ)4_VHigh	10.927	9.780	1.117	0.266	-8.405	30.258

The coefficient of the *educ* effect comparing the "2_Middle" group to the baseline "1_Low" group on *totchol* is 11.044.

Note that none of the *educ* levels show especially large differences from the baseline group, and each of their 95% confidence intervals contains zero.

- Suppose we have two subjects, Lola and Mina, who are the same age, and the same sex, but Lola is in the "1_Low" education group and Mina is in the "2_Middle" education group.
- Our model predicts that Mina's total cholesterol will be 11.044 mg/dl higher than Lola's.

The coefficient of the *educ* effect comparing the "3_High" group to the baseline "1_Low" group on *totchol* is -2.459.

- Suppose we have two subjects, Lola and Heidi, who are the same age, and the same sex, but Lola is in the "1_Low" education group and Heidi is in the "3_High" education group.
- Our model predicts that Heidi's total cholesterol will be 2.459 mg/dl **lower** than Lola's.

Finally, the coefficient of the *educ* effect comparing the "4_VHigh" group to the baseline "1_Low" group on *totchol* is 10.927.

- Suppose we have two subjects, Lola and Vera, who are the same age, and the same sex, but Lola is in the "1_Low" education group and Vera is in the "4_VHigh" education group.
- Our model predicts that Vera's total cholesterol will be 10.927 mg/dl higher than Lola's.

What if we include a Spline or an Interaction?

Suppose we fit a new model to predict *totchol* using a five-knot spline in *age* and the interaction of sex and educational attainment. How does that change our interpretation of the effect sizes?

None of these coefficients show particularly large effects, and zero is contained in each of the 95% confidence intervals provided in the table summarizing model m2.

```
d <- datadist(smalldat); options(datadist = "d")

m2 <- ols(totchol ~ rcs(age, 5) + sex*catg(educ),
          data = smalldat, x = TRUE, y = TRUE)

kable(summary(m2), digits = 2)
```

	Low	High	Diff.	Effect	S.E.	Lower 0.95	Upper 0.95	Type
age	42	57	15	9.99	9.40	-8.59	28.57	1
sex - M:F	1	2	NA	9.11	9.87	-10.41	28.64	1
educ - 2_Middle:1_Low	1	2	NA	11.36	9.81	-8.04	30.76	1
educ - 3_High:1_Low	1	3	NA	7.80	12.41	-16.73	32.34	1
educ - 4_VHigh:1_Low	1	4	NA	16.17	14.63	-12.75	45.10	1

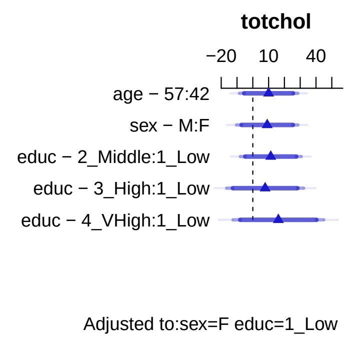


Figure 2: Summary plot of model m2

The `kable` approach I used in these notes hides the adjusted values specified at the bottom of the `summary` table for this `ols` model, but they are `Adjusted to: sex=F educ=1_Low`.

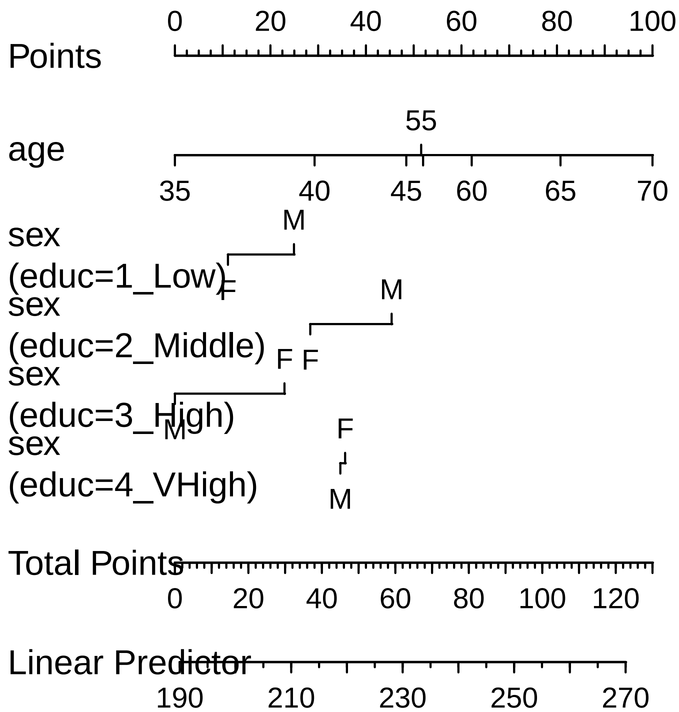
Now, how do we interpret these model `m2` results?

- The **age** interpretation is that if we have two subjects, Al and Bob, who are the same sex and have the same education level, but Al is age 42 and Bob is age 57, then model `m2` projects that Bob's *totchol* will be 9.993 mg/dl higher than will Al's.
- Because of the interaction between *sex* and *educ* in our model `m2`, we must select an *educ* level in order to cleanly interpret the effect of *sex* on *totchol*. The **sex - M:F** interpretation compares M(ale) to F(emale) *sex* while requiring[3] that *educ* = `1_Low`. The result is that if we have two subjects, Carl and Diane, who are the same age and *each is in the low education group*, but Carl is Male and Diane is Female, then model `m2` predicts that Carl's *totchol* will be 9.115 mg/dl higher than will Diane's.
- Because of the interaction between *sex* and *educ* in our model `m2`, we must select a *sex* in order to cleanly interpret the effect of *educ* on *totchol*. The **educ - 2_Middle:1_Low** term, for instance, compares "2_Middle" education to "1_Low" education while requiring that *sex* is Female[4]. The result is that if we have two subjects, Lola and Mina, who are the same age and *each is Female*, but Lola is in the "1_Low" education group and Mina is in the "2_Middle" education group, then model `m2` predicts that Mina's *totchol* will be 11.363 mg/dl higher than will Lola's.

Here is a nomogram of model `m2`.

[3] We know this because of the `Adjusted to: sex = F, educ=1_Low` note at the bottom of the `summary` output for the `ols` model.

[4] Again, we know this because of the `Adjusted to: sex = F, educ=1_Low` note.


Points
0
20
40
60
80
100
age
55
35
40
45
60
65
70
sex
(educ=1_Low)
M
F
sex
(educ=2_Middle)
M
F F
sex
(educ=3_High)
M
sex
(educ=4_VHigh)
F
M
Total Points
0
20
40
60
80
100
120
Linear Predictor
190
210
230
250
270

Effect Estimates in A Logistic Regression fit with `glm`

In a binary **logistic** model, where we predict the log odds of smoking (*smoker* = 1), we will exponentiate so as to interpret the odds ratio estimates associated with each coefficient.

```
m3 <- glm(smoker ~ age + sex + factor(educ),
          data = smalldat, family = binomial)
```

```
kable(tidy(m3, exponentiate = TRUE, conf.int = TRUE),
      digits = 3)
```

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	19.054	1.152	2.557	0.011	2.082	195.209
age	0.943	0.021	-2.782	0.005	0.903	0.982
sexM	1.795	0.356	1.643	0.100	0.897	3.637
factor(educ)2_Middle	0.690	0.428	-0.866	0.386	0.295	1.589
factor(educ)3_High	0.725	0.519	-0.619	0.536	0.258	2.005
factor(educ)4_VHigh	0.339	0.571	-1.895	0.058	0.105	1.008

1. What is the effect of *age* on the odds of being a *smoker* in Model `m3`?

term	estimate	std.error	statistic	p.value	conf.low	conf.high
age	0.943	0.021	-2.782	0.005	0.903	0.982

The estimated odds ratio for the *age* effect on *smoker* is 0.943. Suppose we have two subjects, Doris and Emily, who are the same sex and have the same level of education, but Doris is one year older than Emily. Our model predicts that Doris' odds of smoking will be 0.943 times as high as Emily's.

The 95% confidence interval for this estimated odds ratio for the *age* effect on being a *smoker* is (0.903, 0.982). This confidence interval for the odds ratio does not include one, and again we see that holding everything else constant, older *age* is associated with lower odds of being a *smoker* in this model.

2. What is the effect of *sex* on the odds of being a *smoker* in Model `m3`?

term	estimate	std.error	statistic	p.value	conf.low	conf.high
sexM	1.795	0.356	1.643	0.1	0.897	3.637

The model is parametrized to incorporate the *sex* information with an indicator (and factor) variable called *sexM* which is interpreted as taking the value 1 when *sex* = M, and 0 otherwise. The estimated odds ratio describing the *sexM* effect on being a *smoker* is 1.795. Suppose we have two subjects, David and Emily, who are the same age, have the same level of education, but David is male and Emily is female. Our model predicts that David's odds of being a *smoker* are 1.795 times the odds that Emily is a *smoker*.

The 95% confidence interval for the odds ratio estimate of the effect of *sexM* on being a *smoker* is (0.897, 3.637). The effect of *sex* on the odds of being a *smoker* appears modest, and 1 is included in the confidence interval.

3. What is the effect of *educ* on the odds of being a *smoker* in Model `m3`?

Again, the *educ* variable splits the subjects into four categories. In this model the "1_Low" category is used as the baseline, and we have estimates for "2_Middle" (as compared to "1_Low"), for "3_High" (as compared to "1_Low") and for "4_VHigh" (as compared to "1_Low".)

term	estimate	std.error	statistic	p.value	conf.low	conf.high
factor(educ)2_Middle	0.690	0.428	-0.866	0.386	0.295	1.589
factor(educ)3_High	0.725	0.519	-0.619	0.536	0.258	2.005
factor(educ)4_VHigh	0.339	0.571	-1.895	0.058	0.105	1.008

The estimated odds ratio describing the effect of *educ* being "2_Middle" instead of the baseline "1_Low" on the odds of being a *smoker* is 0.69.

No *educ* levels show meaningful differences from the baseline group, and their 95% confidence intervals all include 1, although the comparison of 4_VHigh to 1_Low only barely includes 1.

- Suppose we have two subjects, Lola and Mina, who are the same age, and the same sex, but Lola is in the "1_Low" education group and Mina is in the "2_Middle" education group.
- Our model predicts that Mina's odds of being a smoker will be 0.69 times the odds of Lola being a smoker.

The estimated odds ratio comparing the *educ* = "3_High" group to the baseline *educ* = "1_Low" group on *smoker* is 0.725.

- Suppose we have two subjects, Lola and Heidi, who are the same age, and the same sex, but Lola is in the "1_Low" education group and Heidi is in the "3_High" education group.

- Our model predicts that Heidi's odds of being a *smoker* will be 0.725 times the odds of Lola being a smoker.

Finally, The estimated odds ratio comparing the *educ* = "4_VHigh" group to the baseline *educ* = "1_Low" group on *smoker* is 0.339.

- Suppose we have two subjects, Lola and Vera, who are the same age, and the same sex, but Lola is in the "1_Low" education group and Vera is in the "4_VHigh" education group.
- Our model predicts that Vera's odds of being a *smoker* will be 0.339 times the odds of Lola being a smoker.

Estimates in The Same Logistic Regression fit with `lrm`

When we fit the same model as `m3` using `lrm`, we get identical results as we get from the `glm` fit for the categorical predictors, but there's a change in how the odds ratio for the quantitative predictor (*age*) is presented.

```
d <- datadist(smalldat); options(datadist = "d")

m3.lrm <- lrm(smoker ~ age + sex + educ,
         data = smalldat, x = TRUE, y = TRUE)

kable(summary(m3.lrm), digits = 3)
```

	Low	High	Diff.	Effect	S.E.	Lower 0.95	Upper 0.95	Type
age	42	57	15	-0.885	0.318	-1.508	-0.261	1
Odds Ratio	42	57	15	0.413	NA	0.221	0.770	2
sex - M:F	1	2	NA	0.585	0.356	-0.113	1.283	1
Odds Ratio	1	2	NA	1.795	NA	0.893	3.607	2
educ - 2_Middle:1_Low	1	2	NA	-0.370	0.428	-1.209	0.468	1
Odds Ratio	1	2	NA	0.690	NA	0.299	1.596	2
educ - 3_High:1_Low	1	3	NA	-0.321	0.519	-1.338	0.696	1
Odds Ratio	1	3	NA	0.725	NA	0.262	2.005	2
educ - 4_VHigh:1_Low	1	4	NA	-1.082	0.571	-2.201	0.037	1
Odds Ratio	1	4	NA	0.339	NA	0.111	1.038	2

Note that the odds ratio effect sizes and confidence intervals are identical to what we saw in the `glm` fit for the *sex* and *educ* variables here, but the *age* result is presented differently.

- The **age** interpretation is that if we have two subjects, Al and Bob, who are the same sex and have the same education level, but Al is age 42 and Bob is age 57, then model `m3` projects that Bob's odds of being a *smoker* will be 0.413 times higher than will Al's odds of being a smoker.
- After adjustment for *sex* and *educ*, increasing age appears to be associated with decreasing odds of smoking. Note, too, that the effect of *age* on the odds of being a *smoker* has a confidence interval for the odds ratio entirely below 1.

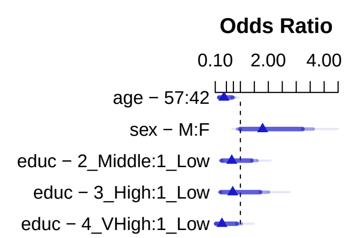


Figure 3: Summary plot of model m3.lrm

Estimates in A New Logistic Regression fit with `lrm`

Now, suppose we fit a new model to predict the log odds of being a *smoker* using a five-knot spline in *age* and the interaction of sex and educational attainment. How does that change our interpretation of the effect sizes?

```
d <- datadist(smalldat); options(datadist = "d")

m4 <- lrm(smoker ~ rcs(age,5) + sex * catg(educ),
          data = smalldat, x = TRUE, y = TRUE)

kable(summary(m4), digits = 3)
```

	Low	High	Diff.	Effect	S.E.	Lower 0.95	Upper 0.95	Type
age	42	57	15	-0.817	0.538	-1.872	0.237	1
Odds Ratio	42	57	15	0.442	NA	0.154	1.268	2
sex - M:F	1	2	NA	1.487	0.583	0.345	2.629	1
Odds Ratio	1	2	NA	4.422	NA	1.412	13.853	2
educ - 2_Middle:1_Low	1	2	NA	0.668	0.546	-0.402	1.739	1
Odds Ratio	1	2	NA	1.951	NA	0.669	5.690	2
educ - 3_High:1_Low	1	3	NA	0.019	0.699	-1.351	1.389	1
Odds Ratio	1	3	NA	1.019	NA	0.259	4.011	2
educ - 4_VHigh:1_Low	1	4	NA	-1.541	1.159	-3.813	0.731	1
Odds Ratio	1	4	NA	0.214	NA	0.022	2.078	2

Again, the `kable` approach I used in these notes hides the adjusted values specified at the bottom of the `summary` table for this `lrm` model (model `m4`), but they are `Adjusted to: sex=F educ=1_Low`.

Now, how do we interpret these model `m4` results?

- The **age** interpretation is that if we have two subjects, Al and Bob, who are the same sex and have the same education level, but Al is age 42 and Bob is age 57, then model `m4` projects that Bob's odds of being a smoker will be 0.442 times higher than Al's odds of being a smoker.
- Because of the interaction between *sex* and *educ* in our model `m4`, we must select an *educ* level in order to cleanly interpret the effect of *sex* on *smoker*. The **sex - M:F** interpretation compares M(ale) to F(emale) *sex* while requiring[5] that *educ* = `1_Low`. The result is that if we have two subjects, Carl and Diane, who are the same age and *each is in the low education group*, but Carl is Male and Diane is Female, then model `m4` predicts that Carl's odds of being a *smoker* will be 4.422 times higher than will Diane's.

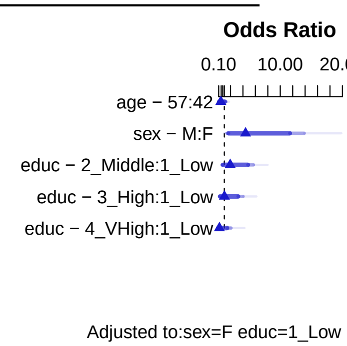


Figure 4: Summary plot of model m4

[5] We know this because of the `Adjusted to: sex = F, educ=1_Low` note at the bottom of the `summary` output.

- Because of the interaction between *sex* and *educ* in our model `m4`, we must select a *sex* in order to cleanly interpret the effect of *educ* on *totchol*. The **educ - 2_Middle:1_Low** term, for instance, compares "2_Middle" education to "1_Low" education while requiring that *sex* is Female[6]. The result is that if we have two subjects, Lola and Mina, who are the same age and *each is Female*, but Lola is in the "1_Low" education group and Mina is in the "2_Middle" education group, then model `m4` predicts that Mina's odds of being a *smoker* will be 1.951 times higher than will Lola's.

[6] `Adjusted to: sex = F, educ=1_Low` tells us this.

It should be easy to see that one is contained in each of the 95% confidence intervals summarizing model m4 except for the one for the main effect of sex, but we need to consider the impact of the interaction term, with `anova`. Here is the `anova` result for model `m4`.

```
anova(m4)
```

```
                Wald Statistics          Response: smoker

 Factor                                          Chi-Square d.f. P
 age                                              8.34       4   0.0799
  Nonlinear                                       1.78       3   0.6196
 sex  (Factor+Higher Order Factors)              12.64       4   0.0132
  All Interactions                               10.48       3   0.0149
 educ  (Factor+Higher Order Factors)             12.43       6   0.0531
  All Interactions                               10.48       3   0.0149
 sex * educ  (Factor+Higher Order Factors)       10.48       3   0.0149
 TOTAL NONLINEAR + INTERACTION                   12.60       6   0.0498
 TOTAL                                           21.69      11   0.0269
```

Finally, here is a nomogram of model `m4`.