432 Class 07

https://thomaselove.github.io/432-2023/

2023-02-07

Today's Agenda

- A First Example: Space Shuttle O-Rings
- Predicting a Binary outcome using a single predictor
  - using a linear probability model
  - using logistic regression and `glm`
  - using logistic regression and `lrm`

See Chapters 19-21 in our Course Notes for more on these models.

Today's R Setup

```
knitr::opts_chunk$set(comment = NA)

library(faraway) # data source
library(broom)
library(knitr)
library(patchwork)
library(rms)
library(tidyverse)

theme_set(theme_bw())
```

Challenger Space Shuttle Data

The US space shuttle Challenger exploded on 1986-01-28. An investigation ensued into the reliability of the shuttle's propulsion system. The explosion was eventually traced to the failure of one of the three field joints on one of the two solid booster rockets. Each of these six field joints includes two O-rings which can fail.

The discussion among engineers and managers raised concern that the probability of failure of the O-rings depended on the temperature at launch, which was forecast to be 31 degrees F. There are strong engineering reasons based on the composition of O-rings to support the judgment that failure probability may rise monotonically as temperature drops.

We have data on 23 space shuttle flights that preceded *Challenger* on primary o-ring erosion and/or blowby and on the temperature in degrees Fahrenheit. No previous liftoff temperature was under 53 degrees F.

The "O-rings" data

```
orings1 <- faraway::orings |>
    tibble() |>
    mutate(burst = case_when( damage > 0 ~ 1,
                              TRUE ~ 0))

orings1 |> summary()
```

```
      temp          damage          burst
 Min.   :53.00   Min.   :0.0000   Min.   :0.0000
 1st Qu.:67.00   1st Qu.:0.0000   1st Qu.:0.0000
 Median :70.00   Median :0.0000   Median :0.0000
 Mean   :69.57   Mean   :0.4783   Mean   :0.3043
 3rd Qu.:75.00   3rd Qu.:1.0000   3rd Qu.:1.0000
 Max.   :81.00   Max.   :5.0000   Max.   :1.0000
```

- damage = number of damage incidents out of 6 possible
- we set burst = 1 if damage > 0

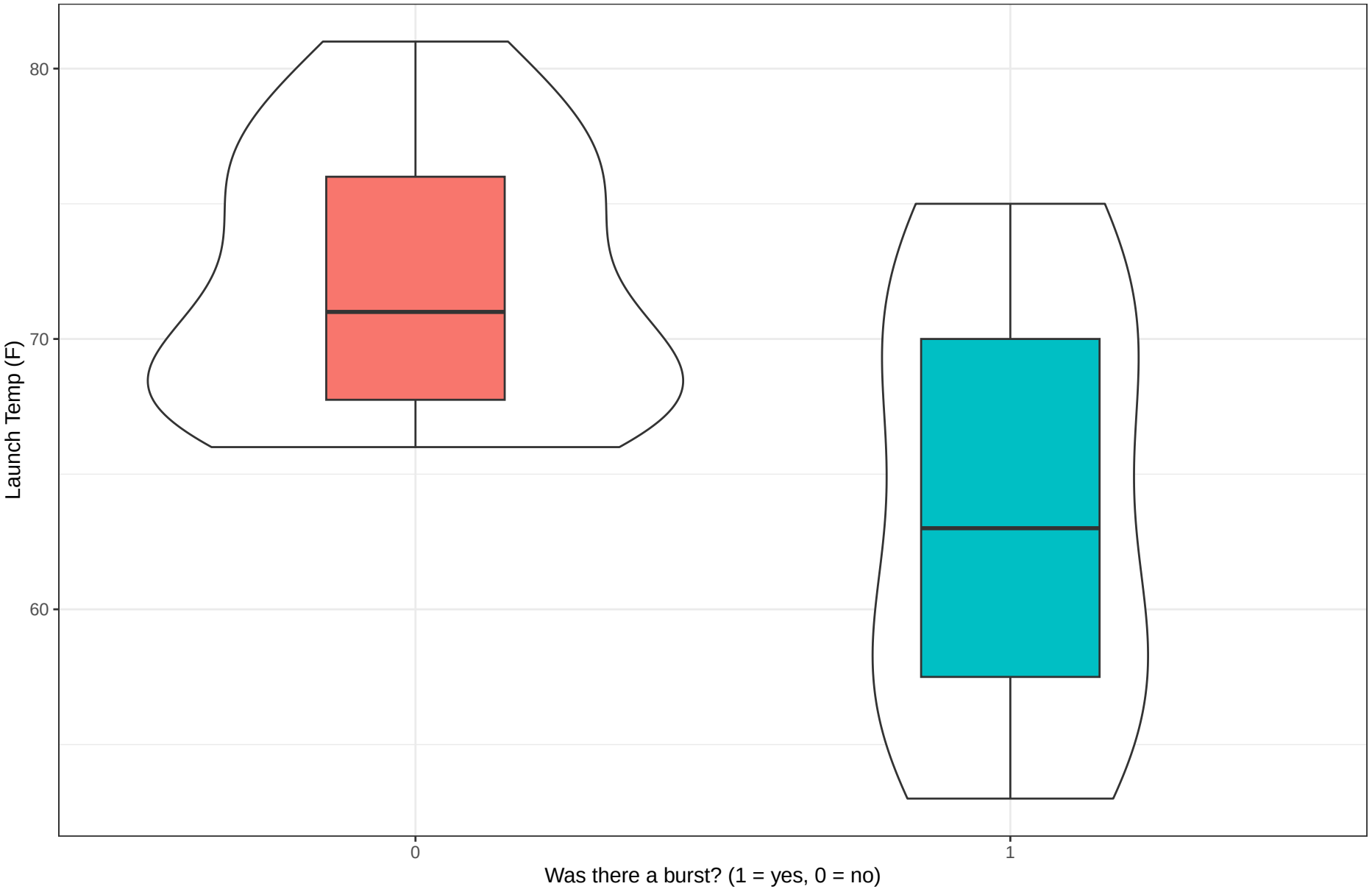
Code to plot burst and temp in our usual way...

```
ggplot(orings1, aes(x = factor(burst), y = temp)) +
    geom_violin() +
    geom_boxplot(aes(fill = factor(burst)), width = 0.3) +
    guides(fill = "none") +
    labs(title = "Are bursts more common at low temperatures?"
         subtitle = "23 prior space shuttle launches",
         x = "Was there a burst? (1 = yes, 0 = no)",
         y = "Launch Temp (F)")
```

Plotted Association of `burst` and `temp`

Are bursts more common at low temperatures?

23 prior space shuttle launches



What if we want to predict Prob(burst) using temp?

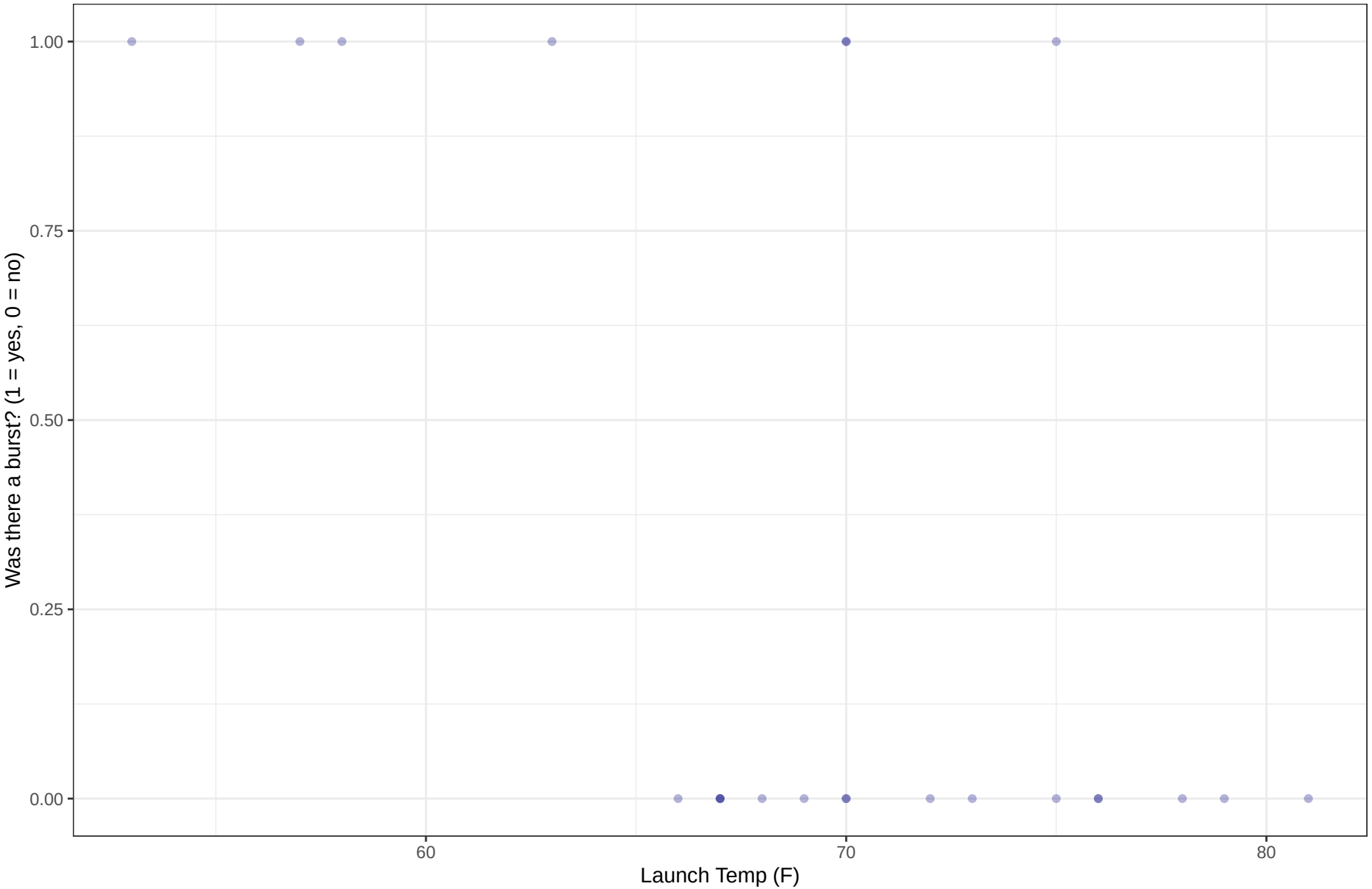
We want to treat the binary variable `burst` as the outcome, and `temp` as the predictor...

```
ggplot(orings1, aes(x = temp, y = burst)) +
    geom_point(col = "navy", alpha = 0.3) +
    labs(title = "Are bursts more common at low temperatures",
         subtitle = "23 prior space shuttle launches",
         y = "Was there a burst? (1 = yes, 0 = no)",
         x = "Launch Temp (F)")
```

Plot of Prob(burst) by temperature at launch

Are bursts more common at low temperatures

23 prior space shuttle launches



Section 1

A Linear Probability Model, fit with `lm()`

Fit a linear model to predict Prob(burst)?

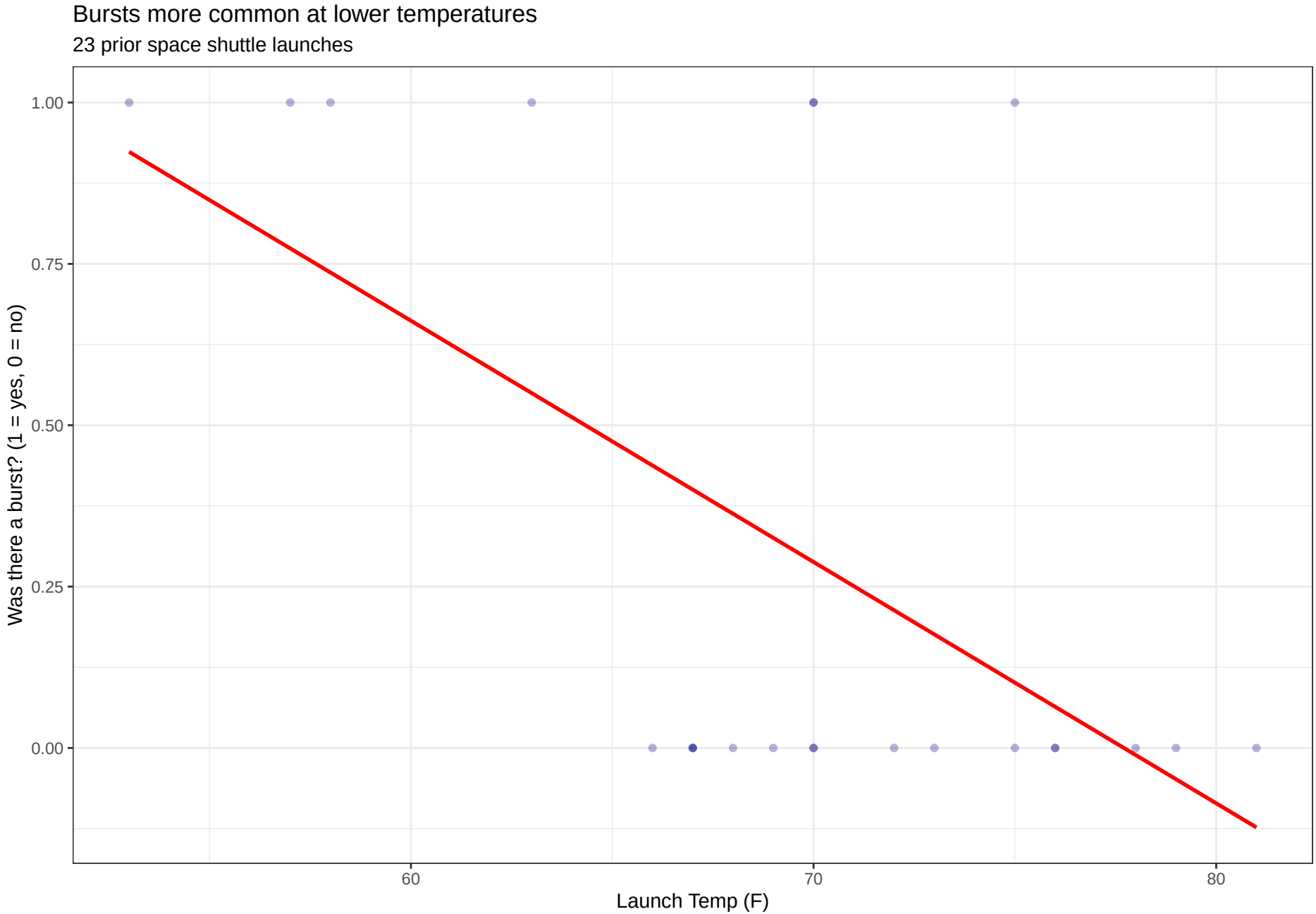
```
mod1 <- lm(burst ~ temp, data = orings1)

tidy(mod1, conf.int = T) |> kable(digits = 3)
```

term	estimate	std.error	statistic	p.value	conf.low	conf.high
(Intercept)	2.905	0.842	3.450	0.002	1.154	4.656
temp	-0.037	0.012	-3.103	0.005	-0.062	-0.012

- This is a **linear probability model**.

$$\hat{\text{burst}} = 2.905 - 0.037(\text{temp})$$

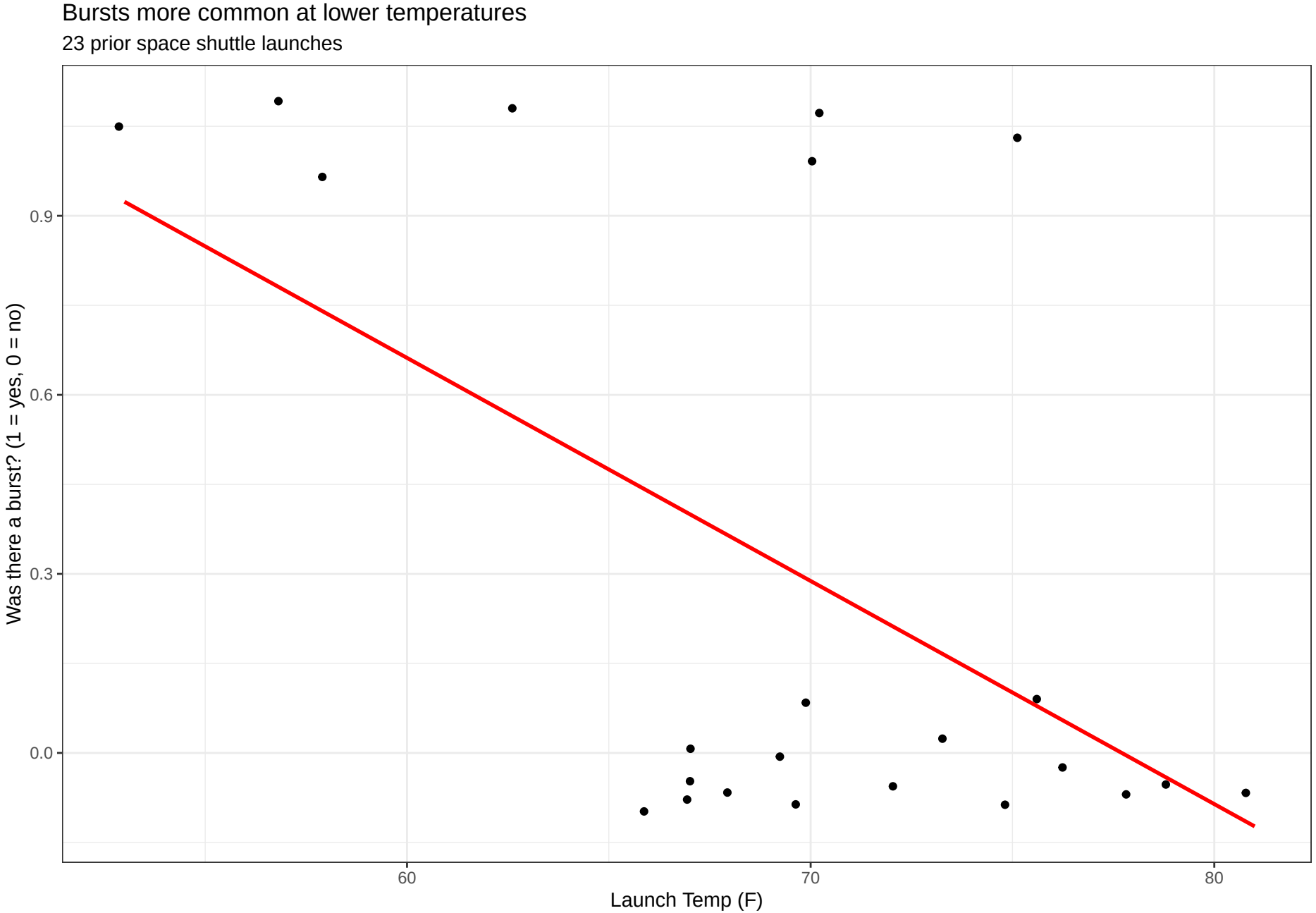
Add linear probability model to our plot?



Add vertical jitter and our mod1 model?

```
ggplot(orings1, aes(x = temp, y = burst)) +
    geom_jitter(height = 0.1) +
    geom_smooth(method = "lm", se = F, col = "red",
                formula = y ~ x) +
    labs(title = "Bursts more common at lower temperatures",
         subtitle = "23 prior space shuttle launches",
         y = "Was there a burst? (1 = yes, 0 = no)",
         x = "Launch Temp (F)")
```

Resulting plot with points jittered and linear model



Making Predictions with mod1

```
mod1$coefficients
```

```
(Intercept)        temp
 2.90476190 -0.03738095
```

- What does mod1 predict for the probability of a burst if the temperature at launch is 70 degrees F?

```
predict(mod1, newdata = tibble(temp = 70))
```

```
        1
0.2880952
```

- What if the temperature was actually 60 degrees F?

Making Several Predictions with mod1

Let's use our linear probability model mod1 to predict the probability of a burst at some other temperatures...

```
newtemps <- tibble(temp = c(80, 70, 60, 50, 31))

augment(mod1, newdata = newtemps)
```

```
# A tibble: 5 x 2
   temp .fitted
  <dbl>   <dbl>
1    80 -0.0857
2    70  0.288
3    60  0.662
4    50  1.04
5    31  1.75
```

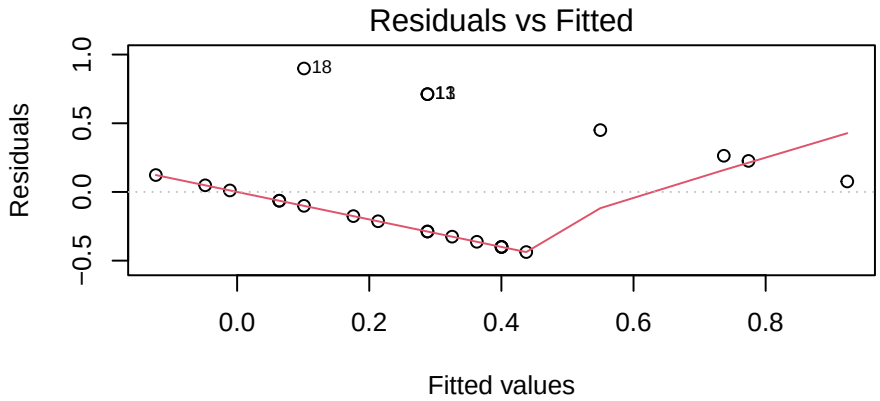
- Uh, oh.

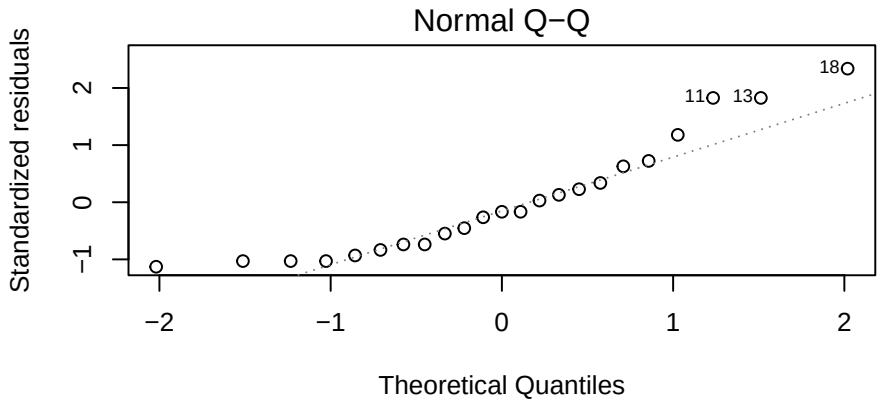
Can we build residual plots?

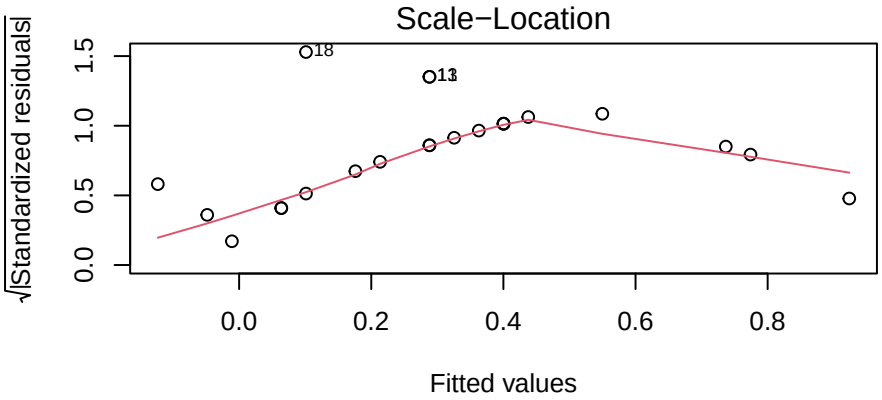
```
par(mfrow = c(2,2)); plot(mod1); par(mfrow = c(1,1))
```

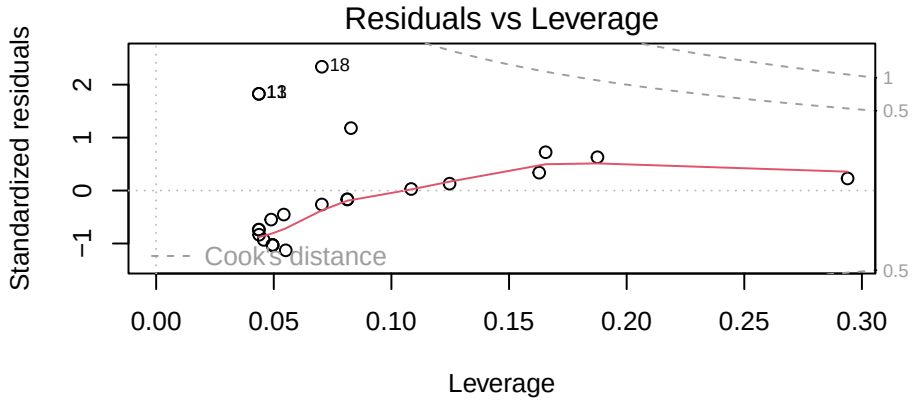
See next slide for results...

Residual Plots for `mod1`?









- Uh, oh.

Models to predict a Binary Outcome

Our outcome takes on two values (zero or one) and we then model the probability of a "one" response given a linear function of predictors.

Idea 1: Use a *linear probability model*

- Main problem: predicted probabilities that are less than 0 and/or greater than 1
- Also, how can we assume Normally distributed residuals when outcomes are 1 or 0?

Idea 2: Build a *non-linear* regression approach

- Most common approach: logistic regression, part of the class of *generalized* linear models

Section 2

A Logistic Regression Model, fit with `glm()`

The Logit Link and Logistic Function

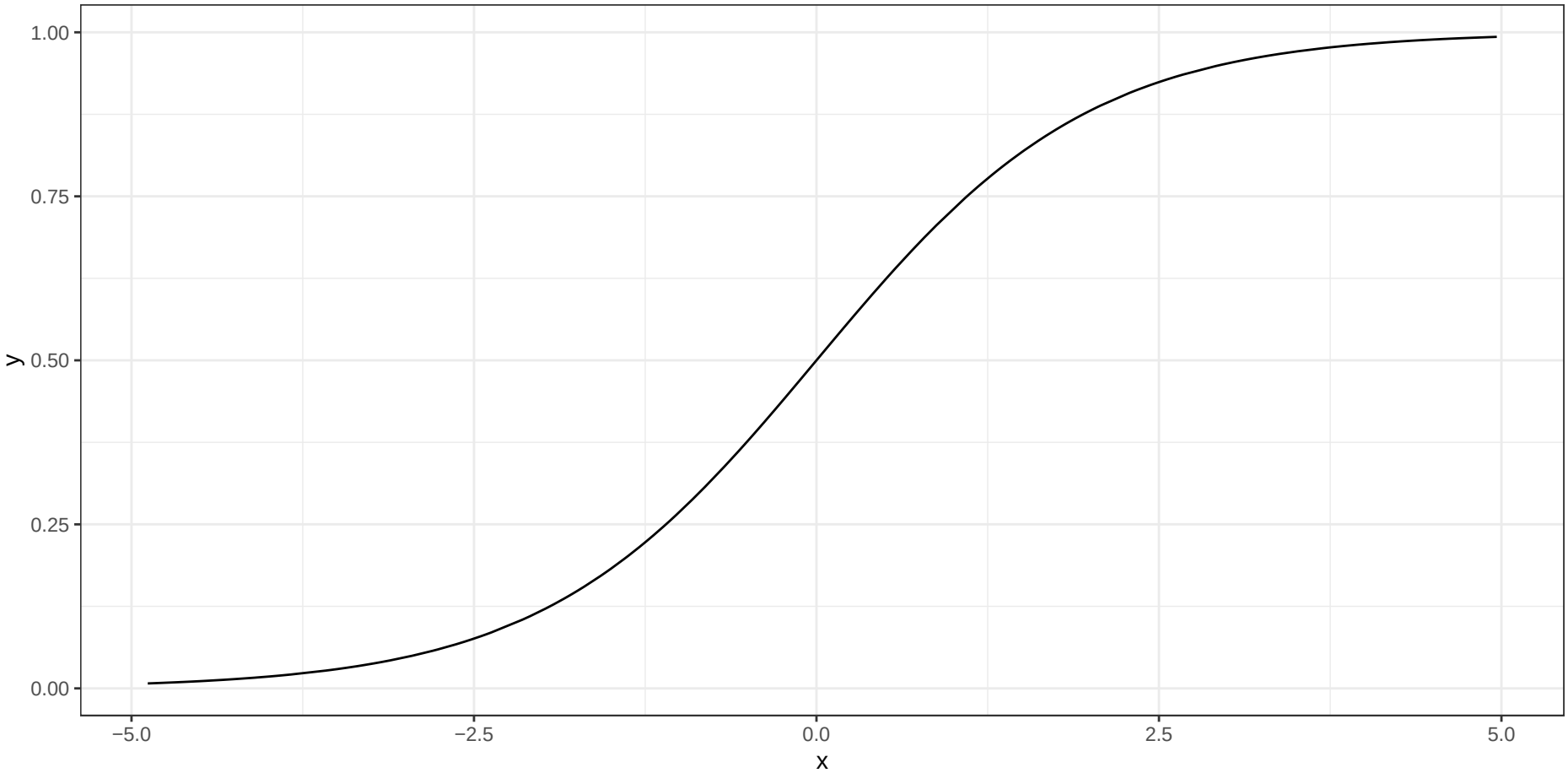
The function we use in logistic regression is called the **logit link**.

$$logit(\pi) = log\left(\frac{\pi}{1-\pi}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_k X_k$$

The inverse of the logit function is called the **logistic function**. If logit(π) $= \eta$, then $\pi = \frac{exp(\eta)}{1+exp(\eta)}$.

- The logistic function $\frac{e^x}{1+e^x}$ takes any value x in the real numbers and returns a value between 0 and 1.

The Logistic Function $y = \frac{e^x}{1+e^x}$



The logit or log odds

We usually focus on the **logit** in statistical work, which is the inverse of the logistic function.

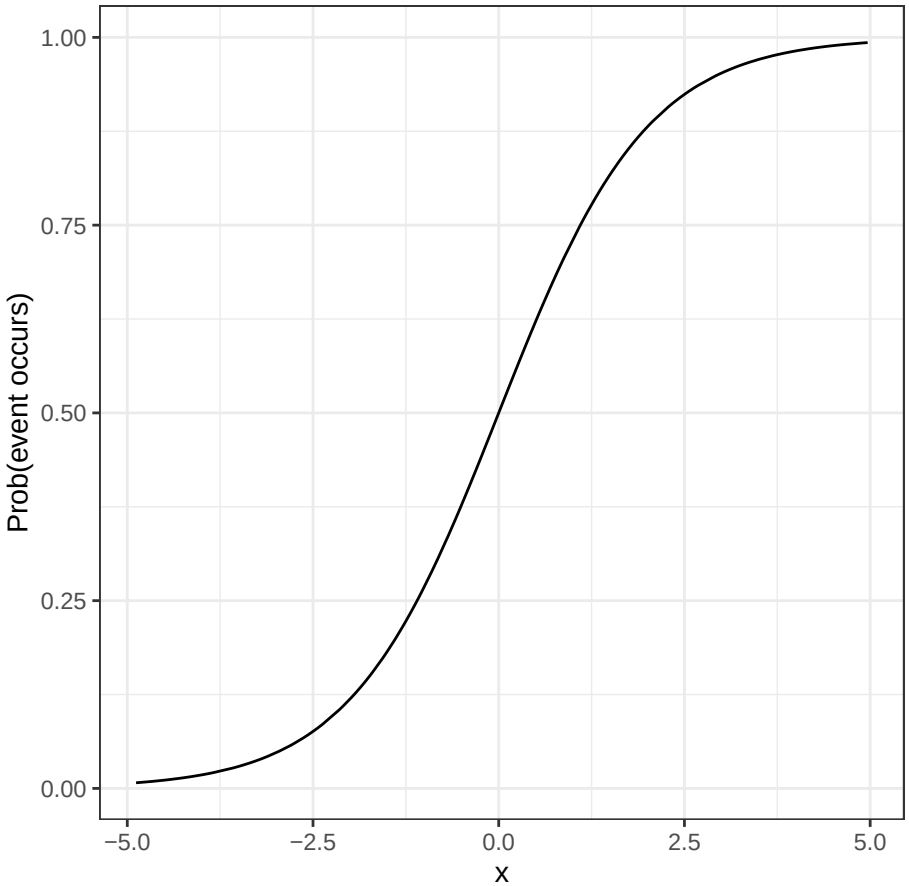
- If we have a probability $\pi < 0.5$, then $logit(\pi) < 0$.
- If our probability $\pi > 0.5$, then $logit(\pi) > 0$.
- Finally, if $\pi = 0.5$, then $logit(\pi) = 0$.

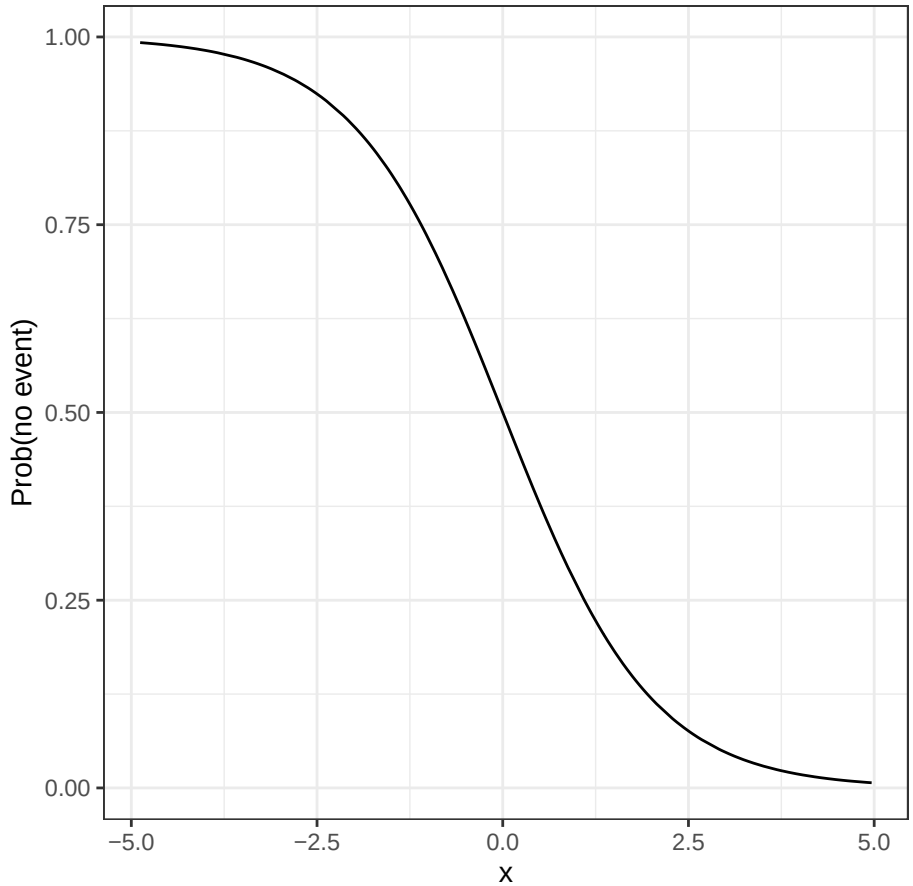
Why is this helpful?

- log(odds(Y = 1)) or logit(Y = 1) covers all real numbers.
- Prob(Y = 1) is restricted to [0, 1].

Predicting Pr(event) or Pr(no event)

- Can we flip the story?





Returning to the prediction of Prob(burst)

We'll use the `glm` function in R, specifying a logistic regression model.

- Instead of predicting $Pr(burst)$, we're predicting $log(odds(burst))$ or $logit(burst)$.

```
mod2 <- glm(burst ~ temp, data = orings1,
            family = binomial(link = "logit"))

tidy(mod2, conf.int = TRUE) |>
  select(term, estimate, std.error, conf.low, conf.high) |>
  kable(digits = c(0,4,3,3,3))
```

term	estimate	std.error	conf.low	conf.high
(Intercept)	15.0429	7.379	3.331	34.342
temp	-0.2322	0.108	-0.515	-0.061

Our model `mod2`

$$\log\left[\frac{P(\widehat{\text{burst}} = 1)}{1 - P(\widehat{\text{burst}} = 1)}\right] = 15.0429 - 0.2322(\text{temp})$$

- For a temperature of 70 F at launch, what is the prediction?

Let's look at the results

- For a temperature of 70 F at launch, what is the prediction?

log(odds(burst)) = 15.0429 - 0.2322 (70) = -1.211

- Exponentiate to get the odds, on our way to estimating the probability.

odds(burst) = exp(-1.211) = 0.2979

- so, we can estimate the probability by

$$Pr(burst) = \frac{0.2979}{(0.2979 + 1)} = 0.230.$$

Prediction from `mod2` for temp = 60

What is the predicted probability of a burst if the temperature is 60 degrees?

- log(odds(burst)) = 15.0429 - 0.2322 (60) = 1.1109
- odds(burst) = exp(1.1109) = 3.0371
- Pr(burst) = 3.0371 / (3.0371 + 1) = 0.752

Will augment do this, as well?

```
temps <- tibble(temp = c(60,70))

augment(mod2, newdata = temps, type.predict = "link")
```

```
# A tibble: 2 x 2
   temp .fitted
  <dbl>   <dbl>
1    60    1.11
2    70   -1.21
```

```
augment(mod2, newdata = temps, type.predict = "response")
```

```
# A tibble: 2 x 2
   temp .fitted
  <dbl>   <dbl>
1    60   0.753
2    70   0.230
```

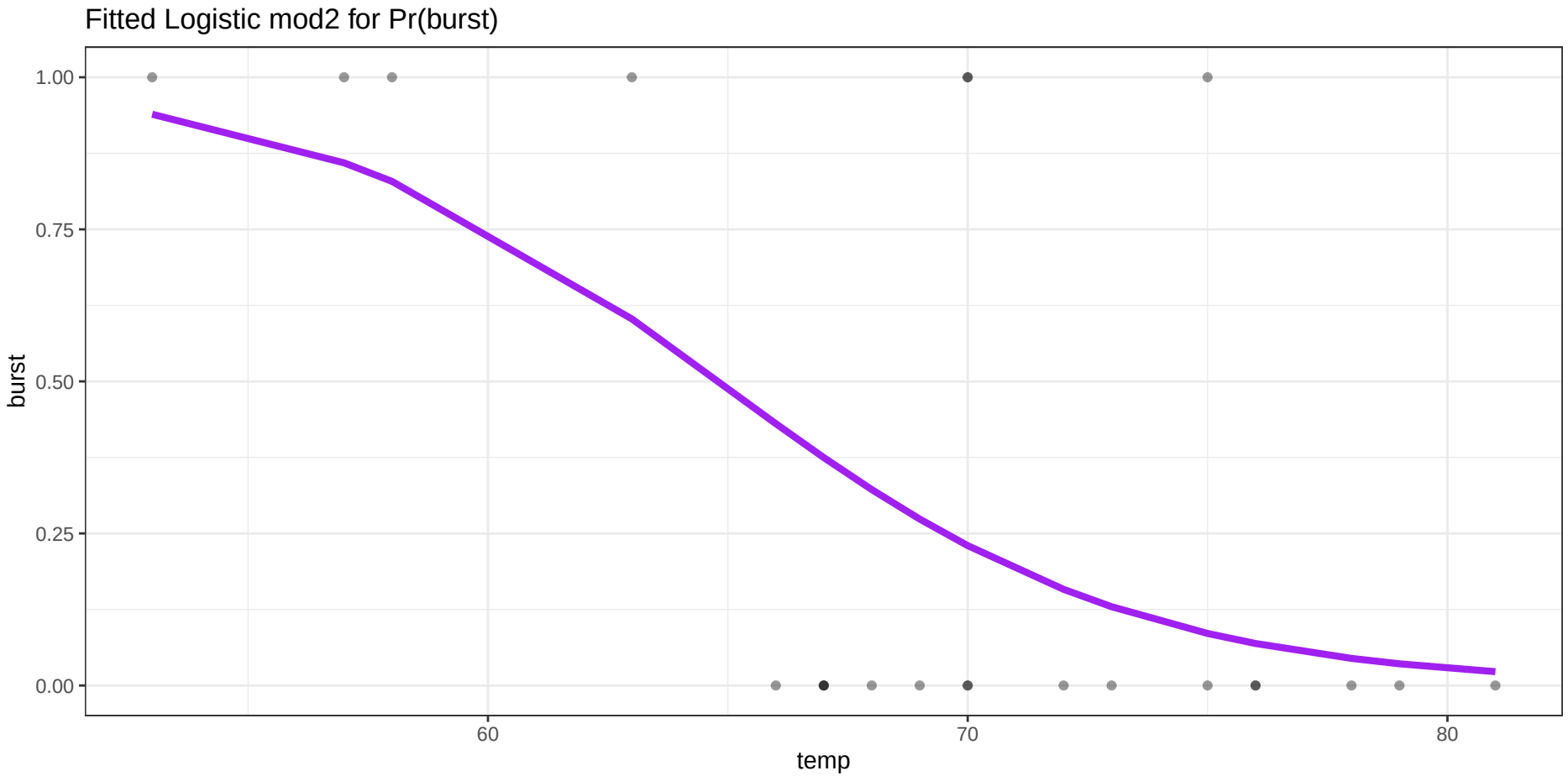
Plotting the Logistic Regression Model

Use the `augment` function to get the fitted probabilities into the original data, then plot.

```
mod2_aug <- augment(mod2, type.predict = "response")

ggplot(mod2_aug, aes(x = temp, y = burst)) +
  geom_point(alpha = 0.4) +
  geom_line(aes(x = temp, y = .fitted),
            col = "purple", size = 1.5) +
  labs(title = "Fitted Logistic mod2 for Pr(burst)")
```

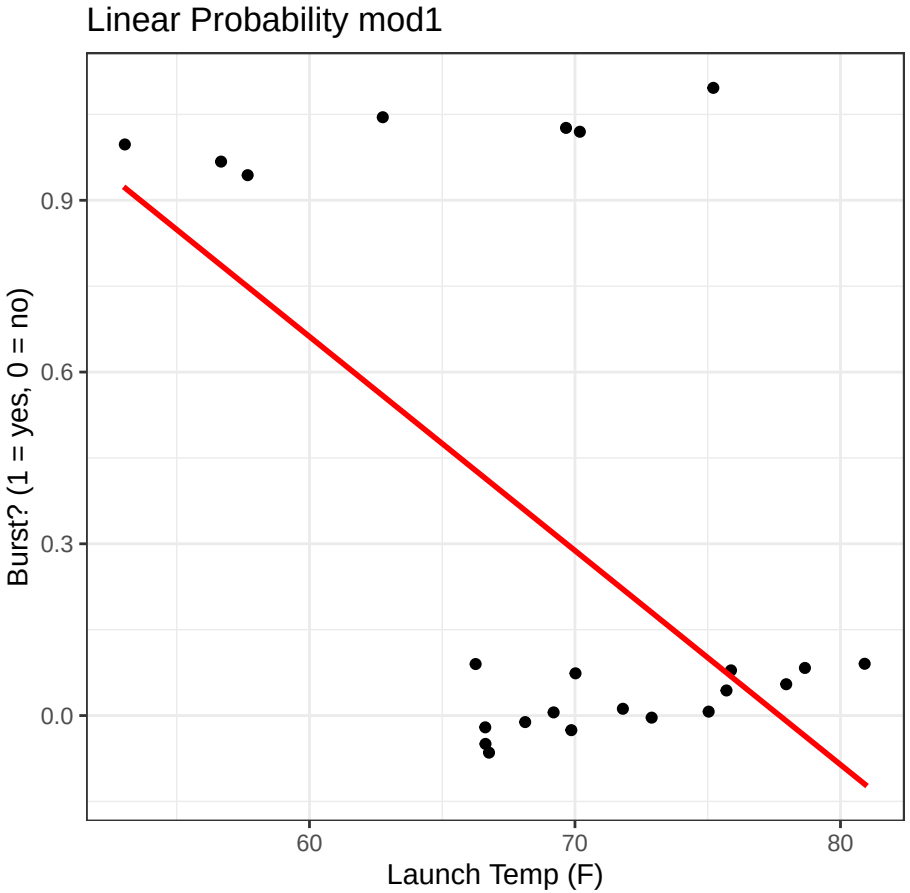
- Results on next slide

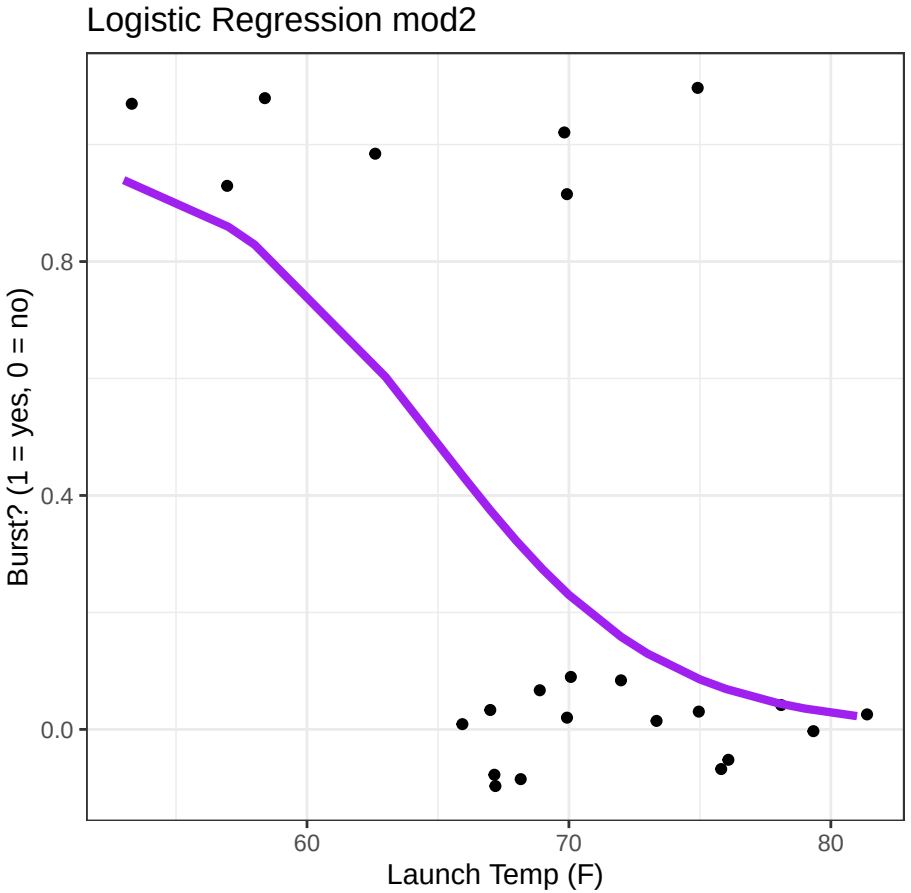
Plotting Model `m2`



Note that we're just connecting the predictions made for observed `temp` values with `geom_line`, so the appearance of the function isn't as smooth as the actual logistic regression model.

Comparing the fits of `mod1` and `mod2`...





Could we try exponentiating the mod2 coefficients?

How can we interpret the coefficients of the model?

$$logit(burst) = log(odds(burst)) = 15.043 - 0.232temp$$

Exponentiating the coefficients is helpful...

```
exp(-0.232)
```

```
[1] 0.7929461
```

Suppose Launch A's temperature was one degree higher than Launch B's.

- The **odds** of Launch A having a burst are 0.793 times as large as they are for Launch B.
- Odds Ratio estimate comparing two launches whose `temp` differs by 1 degree is 0.793

Exponentiated and tidied slope of temp (mod2)

```
tidy(mod2, exponentiate = TRUE, conf.int = TRUE) |>
    filter(term == "temp") |>
    select(term, estimate, std.error, conf.low, conf.high) |>
    kable(digits = 3)
```

term	estimate	std.error	conf.low	conf.high
temp	0.793	0.108	0.597	0.941

- What would it mean if the Odds Ratio for temp was 1?
- How about an odds ratio that was greater than 1?

Section 3

A logistic regression model, fit with `lrm()` from **rms**

Fitting the model again

```
d <- datadist(orings1)
options(datadist = "d")

mod3 <- lrm(burst ~ temp, data = orings1, x = TRUE, y = TRUE)
```

as compared to

```
mod2 <- glm(burst ~ temp, data = orings1,
            family =binomial(link ="logit"))
```

will fit the same model.

mod3 Results

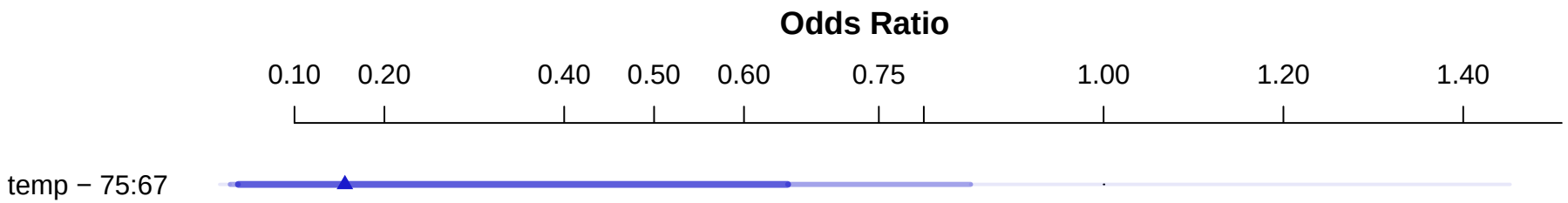
```
> mod3
Logistic Regression Model

 lrm(formula = burst ~ temp, data = orings1, x = TRUE, y = TRUE)

                       Model Likelihood     Discrimination    Rank Discrim.
                             Ratio Test            Indexes          Indexes
 Obs              23    LR chi2      7.95     R2       0.413    C       0.781
  0               16    d.f.            1     R2(1,23) 0.261    Dxy     0.562
  1                7    Pr(> chi2) 0.0048    R2(1,14.6)0.379    gamma   0.589
 max |deriv| 0.0002                         Brier    0.139    tau-a   0.249

          Coef    S.E.   Wald Z Pr(>|Z|)
 Intercept 15.0429 7.3786  2.04  0.0415
 temp      -0.2322 0.1082 -2.14  0.0320
```

summary(mod3) Results

```
> summary(mod3)
             Effects                  Response : burst

 Factor       Low High Diff. Effect   S.E.     Lower 0.95 Upper 0.95
 temp         67   75   8     -1.85730 0.86589 -3.554400  -0.16018
  Odds Ratio 67   75   8      0.15609       NA  0.028598   0.85199
```

Effects Plot

```
plot(summary(mod3))
```



Predictions from mod3

```
newdat <- tibble(temp = c(50, 60, 70, 80))

## predictions on the log odds scale
predict(mod3, newdata = newdat)
```

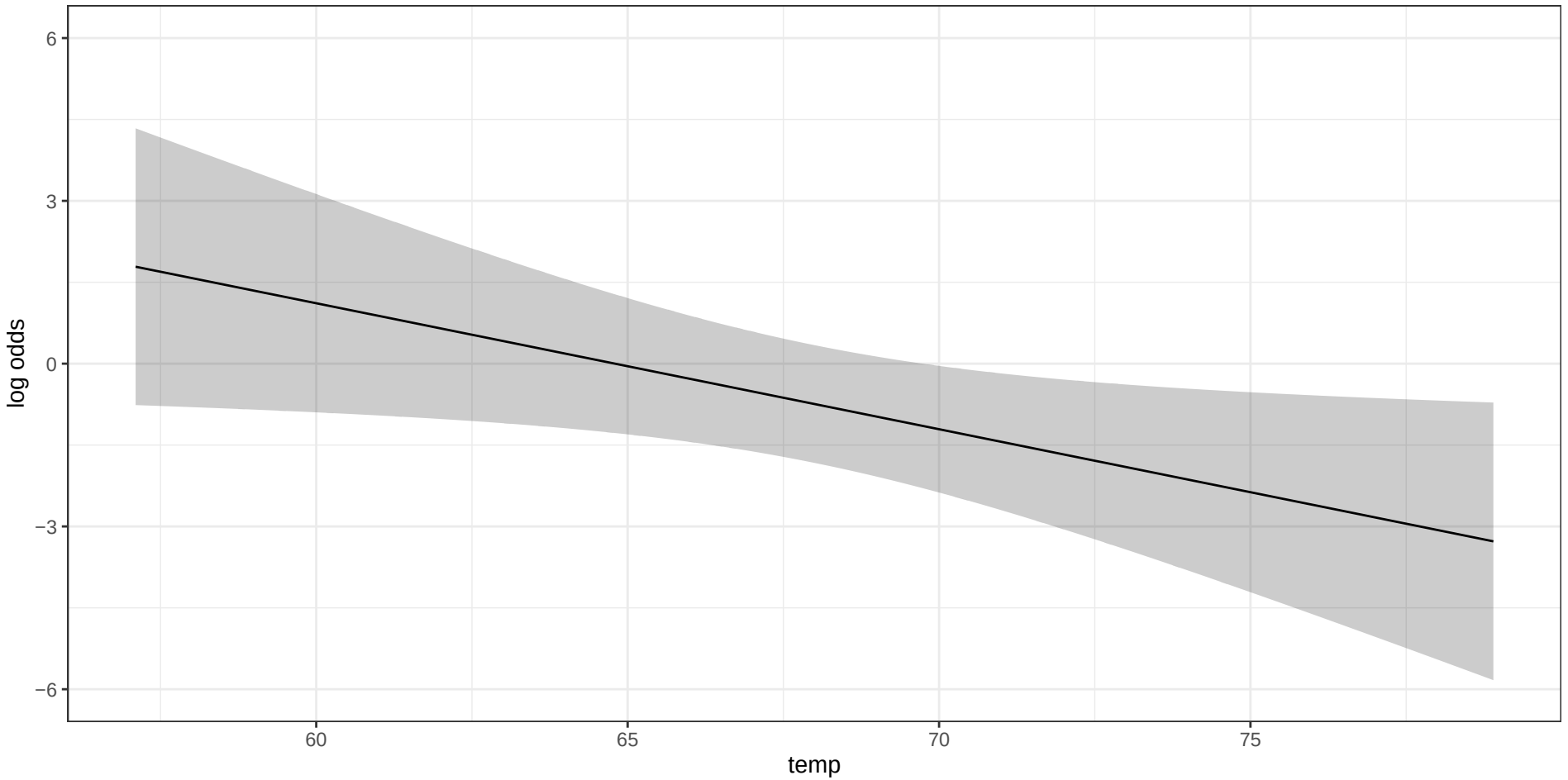
```
        1         2         3         4
 3.434751  1.113132 -1.208488 -3.530108
```

```
## predictions on the probability scale
predict(mod3, newdata = newdat, type = c("fitted"))
```

```
        1         2         3         4
0.9687731 0.7527125 0.2299686 0.0284676
```

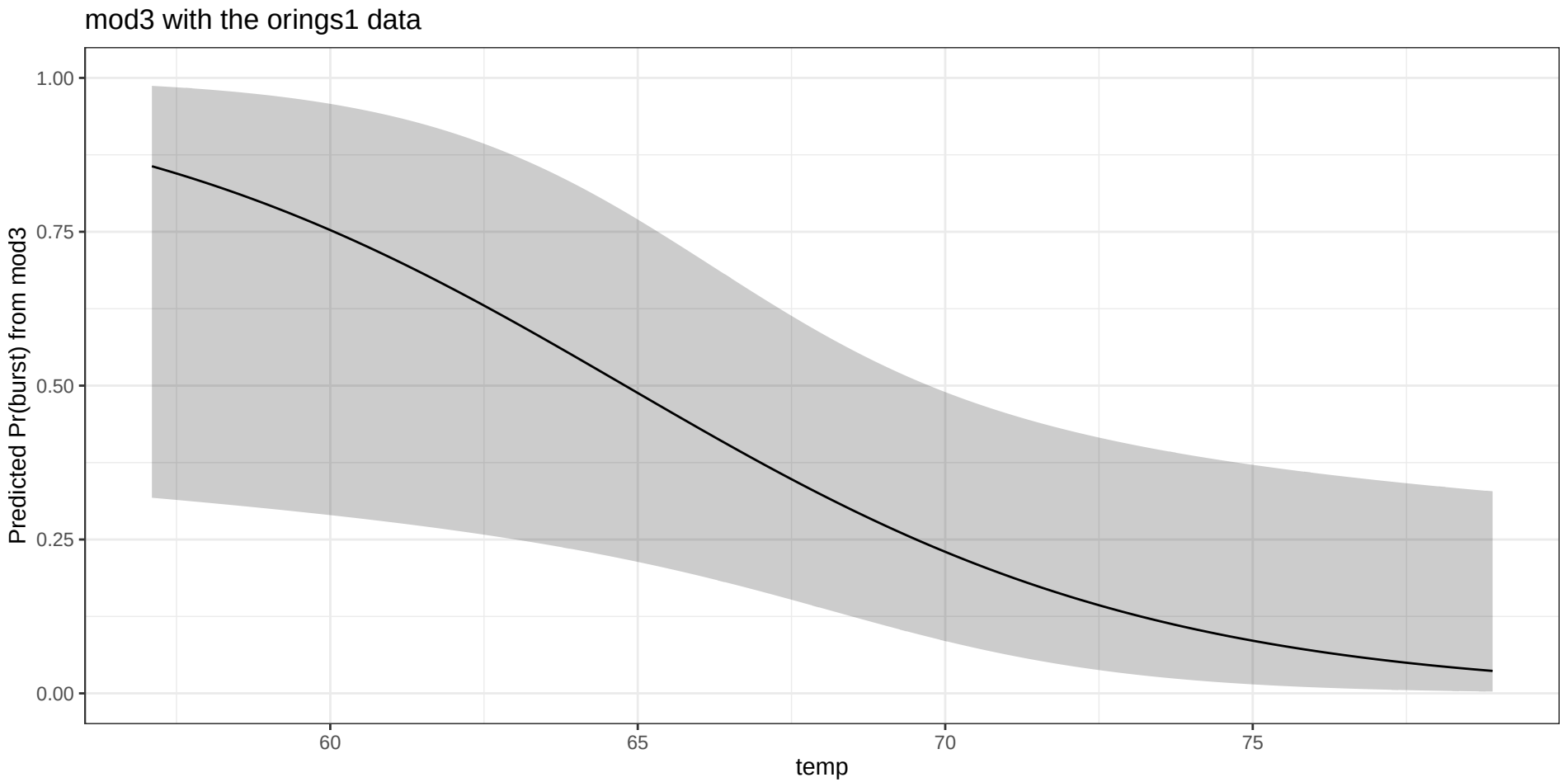
Plot in-sample predictions on log-odds scale

```
ggplot(Predict(mod3))
```



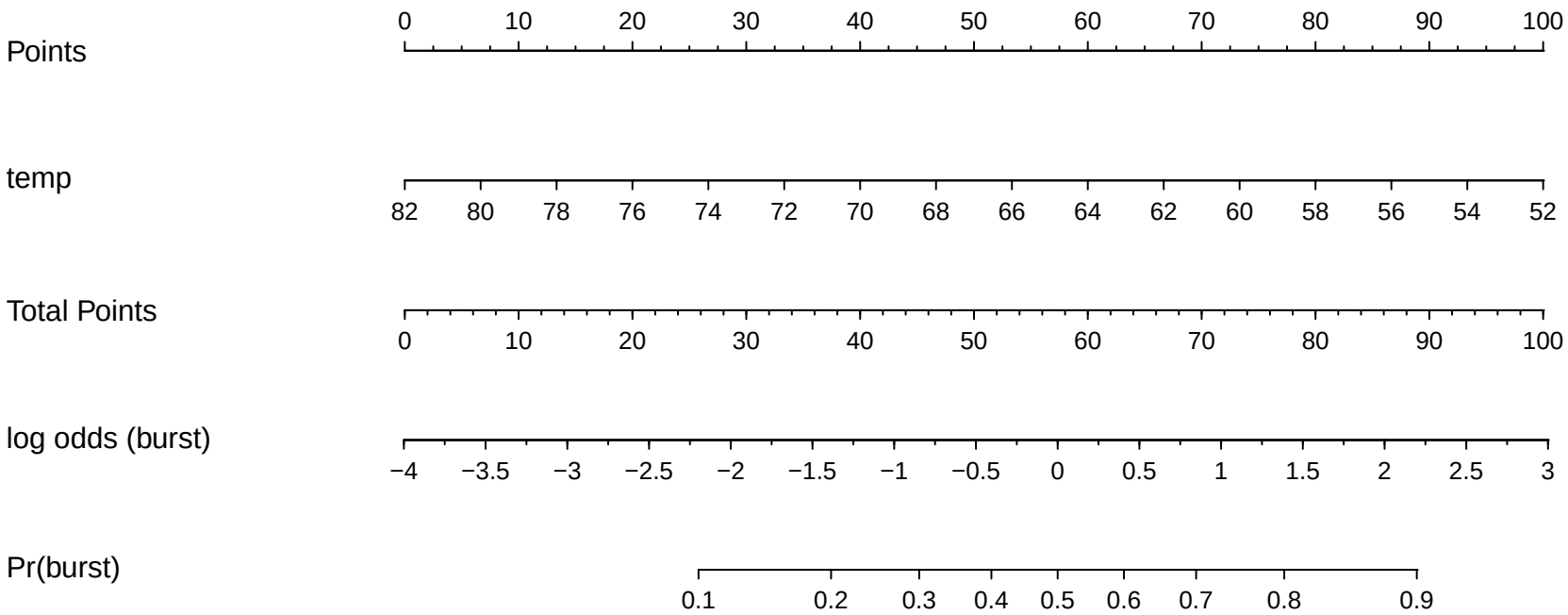
Plot in-sample predictions on probability scale

```
ggplot(Predict(mod3, fun = plogis)) +
  labs(y = "Predicted Pr(burst) from mod3",
       title = "mod3 with the orings1 data")
```



Nomogram for mod3

```
plot(nomogram(mod3, fun = plogis, funlabel = "Pr(burst)"),
     lplabel="log odds (burst)")
```



Regression on a Binary Outcome

Linear Probability Model (a linear model)

```
lm(event ~ predictor1 + predictor2 + ..., data = tibblename)
```

- Pr(event) is linear in the predictors

Logistic Regression Model (generalized linear model)

```
glm(event ~ pred1 + pred2 + ..., data = tibblename,
            family = binomial(link = "logit"))
```

or

```
dd <- datadist(tibblename); options(datadist = "dd")
lrm(event ~ pred1 + pred2 + ..., data = tibblename,
             x = TRUE, y = TRUE)
```

- Logistic Regression forces a prediction in (0, 1)
- log(odds(event)) is linear in the predictors

The logistic regression model

$$logit(event) = log\left(\frac{Pr(event)}{1 - Pr(event)}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_k X_k$$

$$odds(event) = \frac{Pr(event)}{1 - Pr(event)}$$

$$Pr(event) = \frac{odds(event)}{odds(event) + 1}$$

$$Pr(event) = \frac{exp(logit(event))}{1 + exp(logit(event))}$$

Next Time

- Binary regression models with multiple predictors
- Assessing the quality of fit for a logistic model