432 Class 18

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2024-03-21

## Today’s Topics

1. Project B Instructions
2. Asbestos Exposure in the US Navy: A new example predicting an ordered multi-categorical outcome
   * Proportional Odds Logistic Regression Models
     + with MASS::polr()
     + with rms::lrm()
   * Fitting Ordinal Logistic Regressions with rms::orm()

Chapter 27 of the Course Notes describes this material.

## Setup

knitr::opts\_chunk$set(comment=NA)  
options(width = 80)  
  
library(janitor); library(broom); library(gt)

Attaching package: 'janitor'

The following objects are masked from 'package:stats':  
  
 chisq.test, fisher.test

library(mosaic) ## but just for favstats

Registered S3 method overwritten by 'mosaic':  
 method from   
 fortify.SpatialPolygonsDataFrame ggplot2

The 'mosaic' package masks several functions from core packages in order to add   
additional features. The original behavior of these functions should not be affected by this.

Attaching package: 'mosaic'

The following objects are masked from 'package:dplyr':  
  
 count, do, tally

The following object is masked from 'package:Matrix':  
  
 mean

The following object is masked from 'package:ggplot2':  
  
 stat

The following objects are masked from 'package:stats':  
  
 binom.test, cor, cor.test, cov, fivenum, IQR, median, prop.test,  
 quantile, sd, t.test, var

The following objects are masked from 'package:base':  
  
 max, mean, min, prod, range, sample, sum

library(nnet)  
library(MASS)

Attaching package: 'MASS'

The following object is masked from 'package:dplyr':  
  
 select

library(conflicted)  
  
library(rms)

Loading required package: Hmisc

Attaching package: 'Hmisc'

The following objects are masked from 'package:dplyr':  
  
 src, summarize

The following object is masked from 'package:gt':  
  
 html

The following objects are masked from 'package:base':  
  
 format.pval, units

library(tidyverse)

── Attaching core tidyverse packages ──────────────────────── tidyverse 2.0.0 ──  
✔ forcats 1.0.0 ✔ stringr 1.5.1  
✔ lubridate 1.9.3 ✔ tibble 3.2.1  
✔ purrr 1.0.2 ✔ tidyr 1.3.0  
✔ readr 2.1.5

conflicts\_prefer(dplyr::filter, dplyr::select, dplyr::summarize, dplyr::count)

[conflicted] Will prefer dplyr::filter over any other package.

[conflicted] Will prefer dplyr::select over any other package.  
[conflicted] Will prefer dplyr::summarize over any other package.  
[conflicted] Will prefer dplyr::count over any other package.

theme\_set(theme\_bw())

# Project B instructions

## Project B deadlines

Everything is found at <https://thomaselove.github.io/432-2024/projB.html>.

In project B, you will create two research questions that can be addressed with data you can conveniently obtain that can be used to address each of your questions, with models that you learned in the 431-432 sequence. You will fit one regression model for each research question.

* Each of your models must include 2-8 predictors.
* Each model must include at least one predictor that is not included in the other model.
* Each model will need to describe observations from the same pool of “subjects”, so that a single tibble services the whole Project.

## Your Two Models

The model for your first outcome must be either:

* A model for a multi-categorical outcome
* A model for a count outcome, or
* A Cox model for a time-to-event outcome with censoring

The model for your second outcome must use a different approach than you used for Outcome 1. For this model, you can use any of the three options above, or you may use:

* A linear or binary logistic model fit using a Bayesian engine, or
* A weighted linear regression model.

## Deliverables

1. A Proposal Form to be submitted no later than noon on Wednesday 2024-04-10. (details on next three slides.)
2. An in-person or Zoom Presentation
   * April 30 and May 1 are CWRU Reading Days and May 2 is when CWRU final exams begin. Our last 432 class is Thursday 2024-04-25.
3. A Portfolio developed with Quarto (along with some data) submitted to Canvas by noon on Tuesday 2024-05-07.

## Project B Proposal Form

Now open at <https://bit.ly/432-2024-projectB-proposal-form>

The form allows you to:

* register a brief description of your project for our approval
* specify presentation times when you (and your partner if applicable) are available.

Please read the instructions at <https://thomaselove.github.io/432-2024/projB.html> before you submit the form.

## Information the form requires

* You’ll need to assert that you have already successfully ingested the data into R.
* You’ll need to tell us the two types of models you plan to fit.
* You’ll need to describe the variables you intend to use as outcomes in your models.
  + These two outcomes need to be clearly linked to your research questions, and you’ll need to tell us something about each of their distributions.
* You’ll need to specify how many cases are in your data with complete data on each of your study’s two outcomes.
* You’ll need to specify the title of your project.
* You’ll need to specify your research questions.
* You’ll need to specify who your partner is, if you have one.

## Signing Up for Presentation Times

Project B Presentations will be held on…

* Tuesday 2024-04-30 (CWRU Reading Day) between 8 AM and Noon and between 1 and 6 PM, either in person or via Zoom
* Wednesday 2024-05-01 (CWRU Reading Day) between Noon and 7 PM via Zoom
* Thursday 2024-05-02 (Final Exams Begin) between 1 and 6 PM via Zoom

You will specify at least five one-hour time slots within these windows (using at least two different days) as times when you (and your partner, if any) are available to give your project.

Some people will have special circumstances that make it difficult for them to meet during some of these times. There is an opportunity on the form for you to provide those details to Professor Love.

# POLR and Ordinal Regression Models

## Asbestos exposure in the U.S. Navy

These data describe 83 Navy workers[[1]](#footnote-31), engaged in jobs involving potential asbestos exposure.

* The workers were either removing asbestos tile or asbestos insulation, and we might reasonably expect that those exposures would be different.
* We’d expect more exposure with insulation removal.

## Asbestos exposure in the U.S. Navy

Data describe 83 Navy workers[[2]](#footnote-33) with potential asbestos exposure..

* The workers either worked with general ventilation (like a fan or naturally occurring wind) or negative pressure (where a pump with a High Efficiency Particulate Air filter is used to draw air (and fibers) from the work area.)
* We’d expect more exposure with general ventilation.

## Asbestos exposure in the U.S. Navy

83 Navy workers[[3]](#footnote-35) with potential asbestos exposure…

* The duration of a sampling period (in minutes) was recorded, and their asbestos exposure was classified as:
  + low exposure (< 0.05 fibers per cubic centimeter),
  + action level (between 0.05 and 0.1) and
  + above the legal limit (more than 0.1 fibers per cc).
* Sampling periods ranged from 30 to 300 minutes.

## Ingest and clean asbestos data

asbestos <- read\_csv("c18/data/asbestos.csv", show\_col\_types = FALSE) |>  
 clean\_names() |>  
 mutate(across(where(is\_character), as\_factor),  
 exposure = fct\_relevel(exposure, "1\_Low", "2\_Action", "3\_AboveLimit"),  
 exposure = factor(exposure, ordered = TRUE),  
 worker = as.character(worker))  
  
summary(asbestos |> select(-worker))

exposure task ventilation duration   
 1\_Low :45 Tile :37 NP :49 Min. : 30.0   
 2\_Action : 6 Insulation:46 General:34 1st Qu.: 85.0   
 3\_AboveLimit:32 Median :138.0   
 Mean :147.1   
 3rd Qu.:212.5   
 Max. :300.0

## Our Outcome and Modeling task

* exposure is determined by taking air samples in a circle of diameter 2.5 feet around the worker’s mouth and nose.

Our planned predictors for exposure are:

* task (Tile or Insulation),
* ventilation (Negative Pressure (NP) or General), and
* duration (in minutes).

## Effects of Task and Ventilation

We anticipated greater exposure with Insulation, rather than Tile, and with General ventilation vs. Negative Pressure.

asbestos |> tabyl(task, exposure) |>   
 gt() |> tab\_options(table.font.size = 20)

| task | 1\_Low | 2\_Action | 3\_AboveLimit |
| --- | --- | --- | --- |
| Tile | 32 | 2 | 3 |
| Insulation | 13 | 4 | 29 |

asbestos |> tabyl(ventilation, exposure) |>   
 gt() |> tab\_options(table.font.size = 20)

| ventilation | 1\_Low | 2\_Action | 3\_AboveLimit |
| --- | --- | --- | --- |
| NP | 39 | 2 | 8 |
| General | 6 | 4 | 24 |

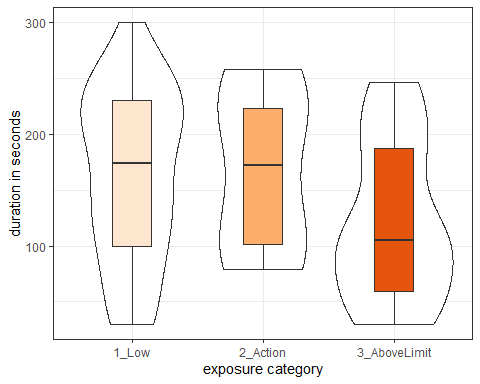
## Exposure and Duration

Is there a strong relationship of exposure and duration?

favstats(duration ~ exposure, data = asbestos) |>  
 gt() |> fmt\_number(columns = mean:sd, decimals = 2) |>  
 tab\_options(table.font.size = 20)

| exposure | min | Q1 | median | Q3 | max | mean | sd | n | missing |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1\_Low | 30 | 100.0 | 174 | 230.00 | 300 | 162.56 | 76.09 | 45 | 0 |
| 2\_Action | 79 | 101.5 | 172 | 223.00 | 258 | 166.50 | 75.77 | 6 | 0 |
| 3\_AboveLimit | 30 | 59.5 | 105 | 187.25 | 246 | 121.66 | 70.72 | 32 | 0 |

ggplot(asbestos, aes(x = exposure, y = duration)) +  
 geom\_violin() +  
 geom\_boxplot(aes(fill = exposure), width = 0.3) +  
 guides(fill = "none") +  
 scale\_fill\_brewer(type = "seq", palette = "Oranges") +  
 labs(y = "duration in seconds", x = "exposure category")



# Fitting polr models with the MASS::polr function

## Proportional-Odds Cumulative Logit

We’ll use the polr function in the **MASS** package.

* Clearly, exposure group (3) Above legal limit, is worst, followed by group (2) Action level, and then group (1) Low exposure.
* We’ll have two binary (1/0) predictors (one for task and one for ventilation) and one quantitative predictor (for duration).

## Equations to be Fit

* The model will have two logit equations: one comparing group (1) to group (2) and one comparing group (2) to group (3), and three slopes, for a total of five free parameters.

and

## Model Equations

Note that the intercept term is the only piece that varies across the two equations shown in the previous slide.

* A positive coefficient means that increasing the value of that predictor tends to *lower* the exposure category, and thus *reduce* the asbestos exposure.

### Fitting the Model

modelA <- polr(exposure ~ task + ventilation + duration,   
 data=asbestos, Hess = TRUE)

## Model Summary

summary(modelA)

Call:  
polr(formula = exposure ~ task + ventilation + duration, data = asbestos,   
 Hess = TRUE)  
  
Coefficients:  
 Value Std. Error t value  
taskInsulation 2.251342 0.644792 3.4916  
ventilationGeneral 2.156960 0.567540 3.8005  
duration -0.000708 0.003799 -0.1864  
  
Intercepts:  
 Value Std. Error t value  
1\_Low|2\_Action 2.3508 0.8682 2.7077  
2\_Action|3\_AboveLimit 2.8972 0.8916 3.2493  
  
Residual Deviance: 99.87952   
AIC: 109.8795

## Direction of Model Effects

Here are coefficient estimates for the three predictors.

tidy(modelA) |> filter(coef.type == "coefficient") |>  
 gt() |> fmt\_number(decimals = 3) |>  
 tab\_options(table.font.size = 20)

| term | estimate | std.error | statistic | coef.type |
| --- | --- | --- | --- | --- |
| taskInsulation | 2.251 | 0.645 | 3.492 | coefficient |
| ventilationGeneral | 2.157 | 0.568 | 3.801 | coefficient |
| duration | -0.001 | 0.004 | -0.186 | coefficient |

* The estimated slope for task = Insulation is 2.25.
  + Since the slope is positive, task = Insulation produces a *reduced* exposure level compared to task = Tile when ventilation and duration are held constant.

## Effect of Task via Odds Ratio + CI

tidy(modelA, exponentiate = TRUE, conf.int = TRUE) |>  
 filter(coef.type == "coefficient") |>  
 gt() |> fmt\_number(decimals = 3) |> tab\_options(table.font.size = 20)

| term | estimate | std.error | statistic | conf.low | conf.high | coef.type |
| --- | --- | --- | --- | --- | --- | --- |
| taskInsulation | 9.500 | 0.645 | 3.492 | 2.826 | 36.787 | coefficient |
| ventilationGeneral | 8.645 | 0.568 | 3.801 | 2.917 | 27.465 | coefficient |
| duration | 0.999 | 0.004 | -0.186 | 0.992 | 1.007 | coefficient |

* Assuming ventilation and duration remain constant, suppose Al has task = Insulation and Bob has task = Tile.
* modelA: Odds of reduced exposure are 9.5 (95% CI 2.8 to 36.8) times as large for Al as they are for Bob.

## Ventilation Effect

tidy(modelA, exponentiate = TRUE, conf.int = TRUE) |>  
 filter(coef.type == "coefficient") |>  
 gt() |> fmt\_number(decimals = 3) |> tab\_options(table.font.size = 20)

| term | estimate | std.error | statistic | conf.low | conf.high | coef.type |
| --- | --- | --- | --- | --- | --- | --- |
| taskInsulation | 9.500 | 0.645 | 3.492 | 2.826 | 36.787 | coefficient |
| ventilationGeneral | 8.645 | 0.568 | 3.801 | 2.917 | 27.465 | coefficient |
| duration | 0.999 | 0.004 | -0.186 | 0.992 | 1.007 | coefficient |

* Assuming task and duration remain constant, modelA suggests the odds of reduced exposure are 8.65 (95% CI 2.9, 27.5) times as large when using General ventilation.
* Impact of duration appears quite small: odds ratio is essentially 1, with 95% CI (0.99, 1.01).

## modelA: Equation 1

tidy(modelA) |>   
 gt() |> fmt\_number(decimals = 3) |> tab\_options(table.font.size = 20)

| term | estimate | std.error | statistic | coef.type |
| --- | --- | --- | --- | --- |
| taskInsulation | 2.251 | 0.645 | 3.492 | coefficient |
| ventilationGeneral | 2.157 | 0.568 | 3.801 | coefficient |
| duration | -0.001 | 0.004 | -0.186 | coefficient |
| 1\_Low|2\_Action | 2.351 | 0.868 | 2.708 | scale |
| 2\_Action|3\_AboveLimit | 2.897 | 0.892 | 3.249 | scale |

* 2.35 is the estimated log odds of falling into category (1) low exposure versus all other categories, when all other predictors (task, ventilation and duration) are zero.

## modelA: Equation 2

tidy(modelA) |>   
 gt() |> fmt\_number(decimals = 3) |> tab\_options(table.font.size = 20)

| term | estimate | std.error | statistic | coef.type |
| --- | --- | --- | --- | --- |
| taskInsulation | 2.251 | 0.645 | 3.492 | coefficient |
| ventilationGeneral | 2.157 | 0.568 | 3.801 | coefficient |
| duration | -0.001 | 0.004 | -0.186 | coefficient |
| 1\_Low|2\_Action | 2.351 | 0.868 | 2.708 | scale |
| 2\_Action|3\_AboveLimit | 2.897 | 0.892 | 3.249 | scale |

* 2.90 is the estimated log odds of falling into category (1) or (2) versus category (3), when all other predictors (task, ventilation and duration) are zero.

## modelA First Equation

tidy(modelA) |>   
 gt() |> fmt\_number(decimals = 3) |> tab\_options(table.font.size = 20)

| term | estimate | std.error | statistic | coef.type |
| --- | --- | --- | --- | --- |
| taskInsulation | 2.251 | 0.645 | 3.492 | coefficient |
| ventilationGeneral | 2.157 | 0.568 | 3.801 | coefficient |
| duration | -0.001 | 0.004 | -0.186 | coefficient |
| 1\_Low|2\_Action | 2.351 | 0.868 | 2.708 | scale |
| 2\_Action|3\_AboveLimit | 2.897 | 0.892 | 3.249 | scale |

## modelA Second Equation

tidy(modelA) |>   
 gt() |> fmt\_number(decimals = 3) |> tab\_options(table.font.size = 20)

| term | estimate | std.error | statistic | coef.type |
| --- | --- | --- | --- | --- |
| taskInsulation | 2.251 | 0.645 | 3.492 | coefficient |
| ventilationGeneral | 2.157 | 0.568 | 3.801 | coefficient |
| duration | -0.001 | 0.004 | -0.186 | coefficient |
| 1\_Low|2\_Action | 2.351 | 0.868 | 2.708 | scale |
| 2\_Action|3\_AboveLimit | 2.897 | 0.892 | 3.249 | scale |

# Comparing polr models

## modelA vs. “Intercept only” model

model.1 <- polr(exposure ~ 1, data=asbestos)  
anova(model.1, modelA) |>   
 gt() |> tab\_options(table.font.size = 20)

| Model | Resid. df | Resid. Dev | Test | Df | LR stat. | Pr(Chi) |
| --- | --- | --- | --- | --- | --- | --- |
| 1 | 81 | 147.61971 |  | NA | NA | NA |
| task + ventilation + duration | 78 | 99.87952 | 1 vs 2 | 3 | 47.74019 | 2.41857e-10 |

### Can we compare AIC and BIC?

AIC(model.1, modelA)

df AIC  
model.1 2 151.6197  
modelA 5 109.8795

BIC(model.1, modelA)

df BIC  
model.1 2 156.4574  
modelA 5 121.9737

## modelA vs. “No duration” Model

Compare to a model with just Task and Ventilation

modelTV <- polr(exposure ~ task + ventilation, data=asbestos)  
anova(modelA, modelTV) |>   
 gt() |> tab\_options(table.font.size = 20)

| Model | Resid. df | Resid. Dev | Test | Df | LR stat. | Pr(Chi) |
| --- | --- | --- | --- | --- | --- | --- |
| task + ventilation | 79 | 99.91421 |  | NA | NA | NA |
| task + ventilation + duration | 78 | 99.87952 | 1 vs 2 | 1 | 0.03469476 | 0.8522367 |

AIC(modelA, modelTV)

df AIC  
modelA 5 109.8795  
modelTV 4 107.9142

BIC(modelA, modelTV)

df BIC  
modelA 5 121.9737  
modelTV 4 117.5896

## task\*ventilation interaction?

model.TxV <- polr(exposure ~ task \* ventilation, data=asbestos)  
anova(modelTV, model.TxV) |>   
 gt() |> tab\_options(table.font.size = 20)

| Model | Resid. df | Resid. Dev | Test | Df | LR stat. | Pr(Chi) |
| --- | --- | --- | --- | --- | --- | --- |
| task + ventilation | 79 | 99.91421 |  | NA | NA | NA |
| task \* ventilation | 78 | 99.64326 | 1 vs 2 | 1 | 0.2709469 | 0.6026973 |

AIC(modelTV, model.TxV)

df AIC  
modelTV 4 107.9142  
model.TxV 5 109.6433

BIC(modelTV, model.TxV)

df BIC  
modelTV 4 117.5896  
model.TxV 5 121.7375

## Fitting all of the models?

Well, not all of the models, but the interesting ones?

m1 <- polr(exposure ~ 1, data = asbestos)  
m2 <- polr(exposure ~ duration, data = asbestos)  
m3 <- polr(exposure ~ task, data = asbestos)  
m4 <- polr(exposure ~ ventilation, data = asbestos)  
m5 <- polr(exposure ~ task + ventilation, data = asbestos)  
m6 <- polr(exposure ~ task \* ventilation, data = asbestos)  
m7 <- polr(exposure ~ task + ventilation + duration, data = asbestos)  
  
anova(m2, m1)

Likelihood ratio tests of ordinal regression models  
  
Response: exposure  
 Model Resid. df Resid. Dev Test Df LR stat. Pr(Chi)  
1 1 81 147.6197   
2 duration 80 142.2944 1 vs 2 1 5.325273 0.02101831

## asbestos Likelihood Ratio Tests

| Model | Elements | DF | Deviance | Test | *p* |
| --- | --- | --- | --- | --- | --- |
| 1 | Intercept | 81 | 147.62 | – | – |
| 2 | Duration | 80 | 142.29 | vs 1 | 0.021 |
| 3 | Task | 80 | 115.36 | vs 1 | < 0.0001 |
| 4 | Ventilation | 80 | 115.45 | vs 1 | < 0.0001 |
| 5 | T+V | 79 | 99.91 | vs 3 | < 0.0001 |
| 6 | T\*V | 78 | 99.64 | vs 5 | 0.603 |
| 7 | T+V+D | 78 | 99.88 | vs 5 | 0.852 |

## Predictions with our T+V model

modelTV <- polr(exposure ~ task + ventilation, data=asbestos)  
asbestos <- asbestos |> mutate(TV\_preds = predict(modelTV))  
asbestos |> tabyl(TV\_preds, exposure) |> adorn\_title()

exposure   
 TV\_preds 1\_Low 2\_Action 3\_AboveLimit  
 1\_Low 42 3 10  
 2\_Action 0 0 0  
 3\_AboveLimit 3 3 22

* Predicting Low exposure led to 42 right and 13 wrong.
* We never predicted Action Level
* Predicting Above Legal Limit led to 22 right and 6 wrong.

Total: 64 right, 19 wrong. Accuracy = 64/83 = 77.1%

## Proportional odds assumption reasonable?

Alternative: fit a multinomial model?

mult\_TV <- multinom(exposure ~ task + ventilation,   
 data = asbestos, trace = FALSE)  
mult\_TV

Call:  
multinom(formula = exposure ~ task + ventilation, data = asbestos,   
 trace = FALSE)  
  
Coefficients:  
 (Intercept) taskInsulation ventilationGeneral  
2\_Action -3.423661 1.159959 2.316383  
3\_AboveLimit -3.117443 2.699791 2.495969  
  
Residual Deviance: 98.08263   
AIC: 110.0826

## Multinomial T+V model predicts…

asbestos <- asbestos |>   
 mutate(TVmult\_preds = predict(mult\_TV))  
asbestos |> tabyl(TVmult\_preds, exposure) |> adorn\_title()

exposure   
 TVmult\_preds 1\_Low 2\_Action 3\_AboveLimit  
 1\_Low 42 3 10  
 2\_Action 0 0 0  
 3\_AboveLimit 3 3 22

* Exactly the same predictions as our polr model.

asbestos |> count(TVmult\_preds, TV\_preds)

# A tibble: 2 × 3  
 TVmult\_preds TV\_preds n  
 <fct> <fct> <int>  
1 1\_Low 1\_Low 55  
2 3\_AboveLimit 3\_AboveLimit 28

## Compare Models with Likelihood Ratio Test?

(LL\_multTV <- logLik(mult\_TV)) # multinomial model: 6 df

'log Lik.' -49.04131 (df=6)

(LL\_polrTV <- logLik(modelTV)) # polr model: 4 df

'log Lik.' -49.9571 (df=4)

(G = -2 \* (LL\_polrTV[1] - LL\_multTV[1]))

[1] 1.831584

pchisq(G, 2, lower.tail = FALSE)

[1] 0.4001995

*p* = 0.4 testing the difference in goodness of fit between the proportional odds model and the more complex multinomial logistic regression model.

## AIC and BIC for multinomial vs. polr models

AIC(mult\_TV, modelTV)

df AIC  
mult\_TV 6 110.0826  
modelTV 4 107.9142

BIC(mult\_TV, modelTV)

df BIC  
mult\_TV 6 124.5957  
modelTV 4 117.5896

* mult\_TV is the multinomial model
* modelTV is the polr model

# Using rms to fit the POLR model via lrm()

## Proportional Odds Ordinal Logistic Regression with lrm()

d <- datadist(asbestos)  
options(datadist = "d")  
  
# note that exposure must be an ordered factor  
  
model\_TV\_LRM <- lrm(exposure ~ task + ventilation,  
 data = asbestos, x = TRUE, y = TRUE)

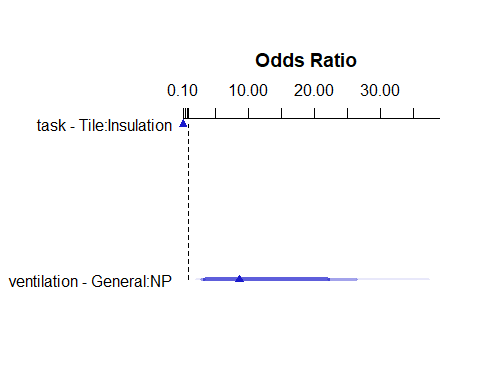
## The lrm() fit

model\_TV\_LRM

Logistic Regression Model  
  
lrm(formula = exposure ~ task + ventilation, data = asbestos,   
 x = TRUE, y = TRUE)  
  
 Model Likelihood Discrimination Rank Discrim.   
 Ratio Test Indexes Indexes   
Obs 83 LR chi2 47.71 R2 0.526 C 0.854   
 1\_Low 45 d.f. 2 R2(2,83) 0.423 Dxy 0.708   
 2\_Action 6 Pr(> chi2) <0.0001 R2(2,65) 0.505 gamma 0.839   
 3\_AboveLimit 32 Brier 0.127 tau-a 0.396   
max |deriv| 3e-10   
  
 Coef S.E. Wald Z Pr(>|Z|)  
y>=2\_Action -2.4751 0.5613 -4.41 <0.0001   
y>=3\_AboveLimit -3.0208 0.6005 -5.03 <0.0001   
task=Insulation 2.2868 0.6173 3.70 0.0002   
ventilation=General 2.1596 0.5675 3.81 0.0001

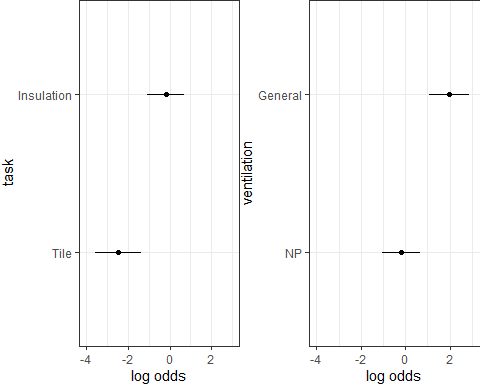
## Effects Plot after lrm()

plot(summary(model\_TV\_LRM))



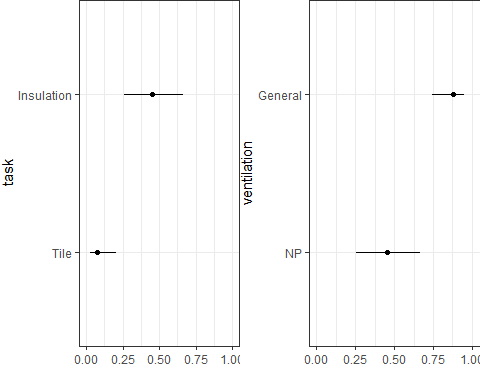
## lrm() fit, plotted on log odds scale

ggplot(Predict(model\_TV\_LRM), layout = c(1,2))



## lrm() fit, on probability scale

ggplot(Predict(model\_TV\_LRM, fun = plogis), layout = c(1,2))



## rms::validate results from lrm()

set.seed(432001)  
validate(model\_TV\_LRM)

index.orig training test optimism index.corrected n  
Dxy 0.7077 0.7194 0.7081 0.0112 0.6964 40  
R2 0.5260 0.5449 0.5210 0.0239 0.5021 40  
Intercept 0.0000 0.0000 0.0093 -0.0093 0.0093 40  
Slope 1.0000 1.0000 0.9414 0.0586 0.9414 40  
Emax 0.0000 0.0000 0.0149 0.0149 0.0149 40  
D 0.5627 0.5946 0.5554 0.0392 0.5235 40  
U -0.0241 -0.0241 -0.3980 0.3739 -0.3980 40  
Q 0.5868 0.6187 0.9534 -0.3347 0.9216 40  
B 0.1270 0.1205 0.1315 -0.0109 0.1379 40  
g 2.0639 2.1935 2.0426 0.1508 1.9131 40  
gp 0.3709 0.3755 0.3708 0.0047 0.3662 40

Validated = 0.5 + (0.6964/2) = 0.8482

## All possible combinations of T and V

newdat <- data.frame(  
 worker = c("New1", "New2", "New3", "New4"),  
 task = c("Tile", "Tile", "Insulation", "Insulation"),  
 ventilation = c("NP", "General", "NP", "General")  
) |>  
 mutate(task = factor(task),   
 ventilation = factor(ventilation))  
  
newdat ## note this is NOT a tibble

worker task ventilation  
1 New1 Tile NP  
2 New2 Tile General  
3 New3 Insulation NP  
4 New4 Insulation General

## Add individual predictions

We use predict() with type = "fitted.ind" here.

newdat\_aug <- cbind(newdat,   
 predict(model\_TV\_LRM, newdata = newdat, type = "fitted.ind"))  
  
newdat\_aug |> gt() |> fmt\_number(decimals = 3) |>  
 tab\_options(table.font.size = 20)

| worker | task | ventilation | exposure=1\_Low | exposure=2\_Action | exposure=3\_AboveLimit |
| --- | --- | --- | --- | --- | --- |
| New1 | Tile | NP | 0.922 | 0.031 | 0.046 |
| New2 | Tile | General | 0.578 | 0.125 | 0.297 |
| New3 | Insulation | NP | 0.547 | 0.129 | 0.324 |
| New4 | Insulation | General | 0.122 | 0.072 | 0.806 |

## Instead add fitted predictions?

Using type = "fitted" produces greater than or equal to predictions instead.

newdat\_aug2 <- cbind(newdat,   
 predict(model\_TV\_LRM, newdata = newdat, type = "fitted"))  
  
newdat\_aug2 |> gt() |> fmt\_number(decimals = 3) |>  
 tab\_options(table.font.size = 20)

| worker | task | ventilation | y>=2\_Action | y>=3\_AboveLimit |
| --- | --- | --- | --- | --- |
| New1 | Tile | NP | 0.078 | 0.046 |
| New2 | Tile | General | 0.422 | 0.297 |
| New3 | Insulation | NP | 0.453 | 0.324 |
| New4 | Insulation | General | 0.878 | 0.806 |

# Ordinal Logistic Regression with orm() from rms

## Ordinal Logistic Regression for T+V with orm

d <- datadist(asbestos)  
options(datadist = "d")  
  
model\_TV\_ORM <- orm(exposure ~ task + ventilation,  
 data = asbestos, x = TRUE, y = TRUE)  
  
# note that exposure must be an ordered factor

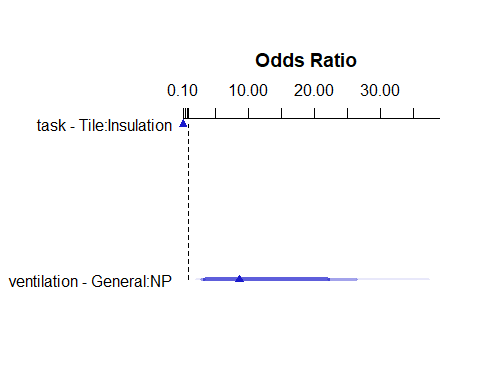
## model\_TV\_ORM fit with orm

model\_TV\_ORM

Logistic (Proportional Odds) Ordinal Regression Model  
  
orm(formula = exposure ~ task + ventilation, data = asbestos,   
 x = TRUE, y = TRUE)  
  
 Model Likelihood Discrimination Rank Discrim.   
 Ratio Test Indexes Indexes   
Obs 83 LR chi2 47.71 R2 0.526 rho 0.697   
 1\_Low 45 d.f. 2 R2(2,83) 0.423   
 2\_Action 6 Pr(> chi2) <0.0001 R2(2,65) 0.505   
 3\_AboveLimit 32 Score chi2 42.42 |Pr(Y>=median)-0.5| 0.301   
Distinct Y 3 Pr(> chi2) <0.0001   
Median Y 1   
max |deriv| 6e-05   
  
 Coef S.E. Wald Z Pr(>|Z|)  
y>=2\_Action -2.4751 0.5613 -4.41 <0.0001   
y>=3\_AboveLimit -3.0208 0.6005 -5.03 <0.0001   
task=Insulation 2.2868 0.6173 3.70 0.0002   
ventilation=General 2.1596 0.5675 3.81 0.0001

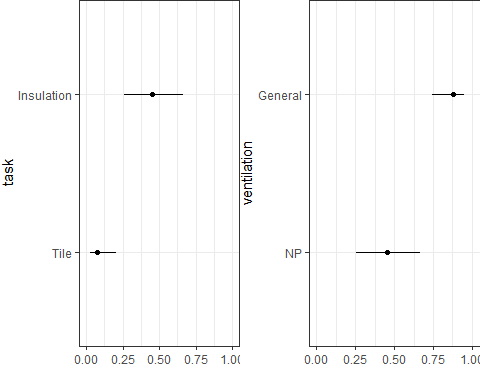
## Effects Plot from orm

plot(summary(model\_TV\_ORM))



## POLR model fit with orm, plotted

ggplot(Predict(model\_TV\_ORM, fun = plogis), layout = c(1,2))



## rms::validate results from orm

set.seed(432002)  
validate(model\_TV\_ORM)

index.orig training test optimism index.corrected n  
rho 0.6970 0.6916 0.6976 -0.0060 0.7031 40  
R2 0.5260 0.5288 0.5156 0.0132 0.5128 40  
Slope 1.0000 1.0000 0.9612 0.0388 0.9612 40  
g 2.0639 2.2268 2.0136 0.2132 1.8508 40  
pdm 0.3010 0.3109 0.3051 0.0058 0.2952 40

* rho = Spearman’s rank correlation between linear predictor and outcome
* R2 = Nagelkerke R-square

## Predicting with orm()

We can from the information below, estimate the model probability of obtaining each of the three possible results.

newdat\_aug3 <- cbind(newdat,   
 predict(model\_TV\_ORM, newdata = newdat, type = "fitted"))  
  
newdat\_aug2 |> gt() |> fmt\_number(decimals = 3) |>  
 tab\_options(table.font.size = 20)

| worker | task | ventilation | y>=2\_Action | y>=3\_AboveLimit |
| --- | --- | --- | --- | --- |
| New1 | Tile | NP | 0.078 | 0.046 |
| New2 | Tile | General | 0.422 | 0.297 |
| New3 | Insulation | NP | 0.453 | 0.324 |
| New4 | Insulation | General | 0.878 | 0.806 |

## Conclusions?

We can fit both POLR models and ordinal regression models with rms approaches, and we can also fit POLR with MASS::polr().

* All are designed for *ordinal* multi-categorical outcomes.
* Can compare results to what we would get with multinomial models, designed for *nominal* multi-categorical outcomes.

We’ll focus on regression for *nominal* multi-categorical outcomes next time.

1. Simonoff JS (2003) *Analyzing Categorical Data*. Chapter 10. [↑](#footnote-ref-31)
2. Simonoff JS (2003) *Analyzing Categorical Data*. Chapter 10. [↑](#footnote-ref-33)
3. Simonoff JS (2003) *Analyzing Categorical Data*. Chapter 10. [↑](#footnote-ref-35)