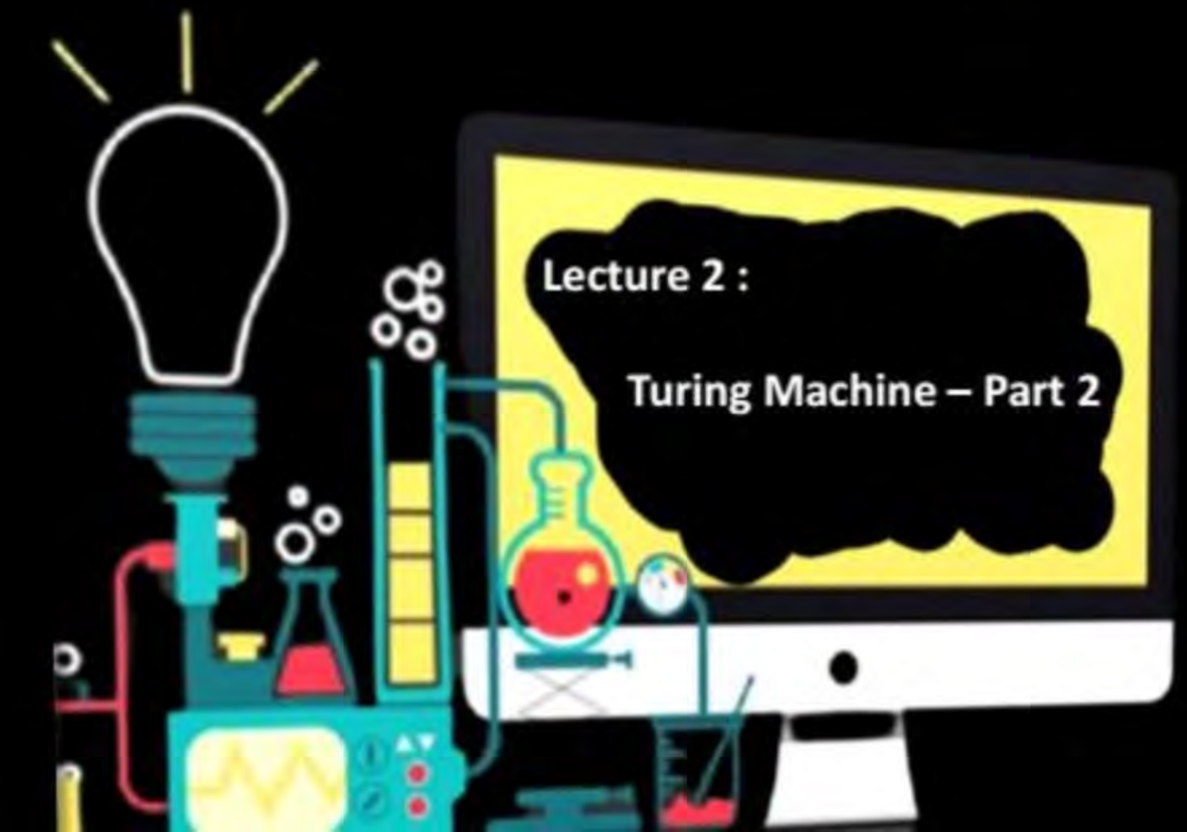
CS & IT Engineering



Deva sir

Topics to be covered:

- construction of TM
- closure properties of Recursives and REs

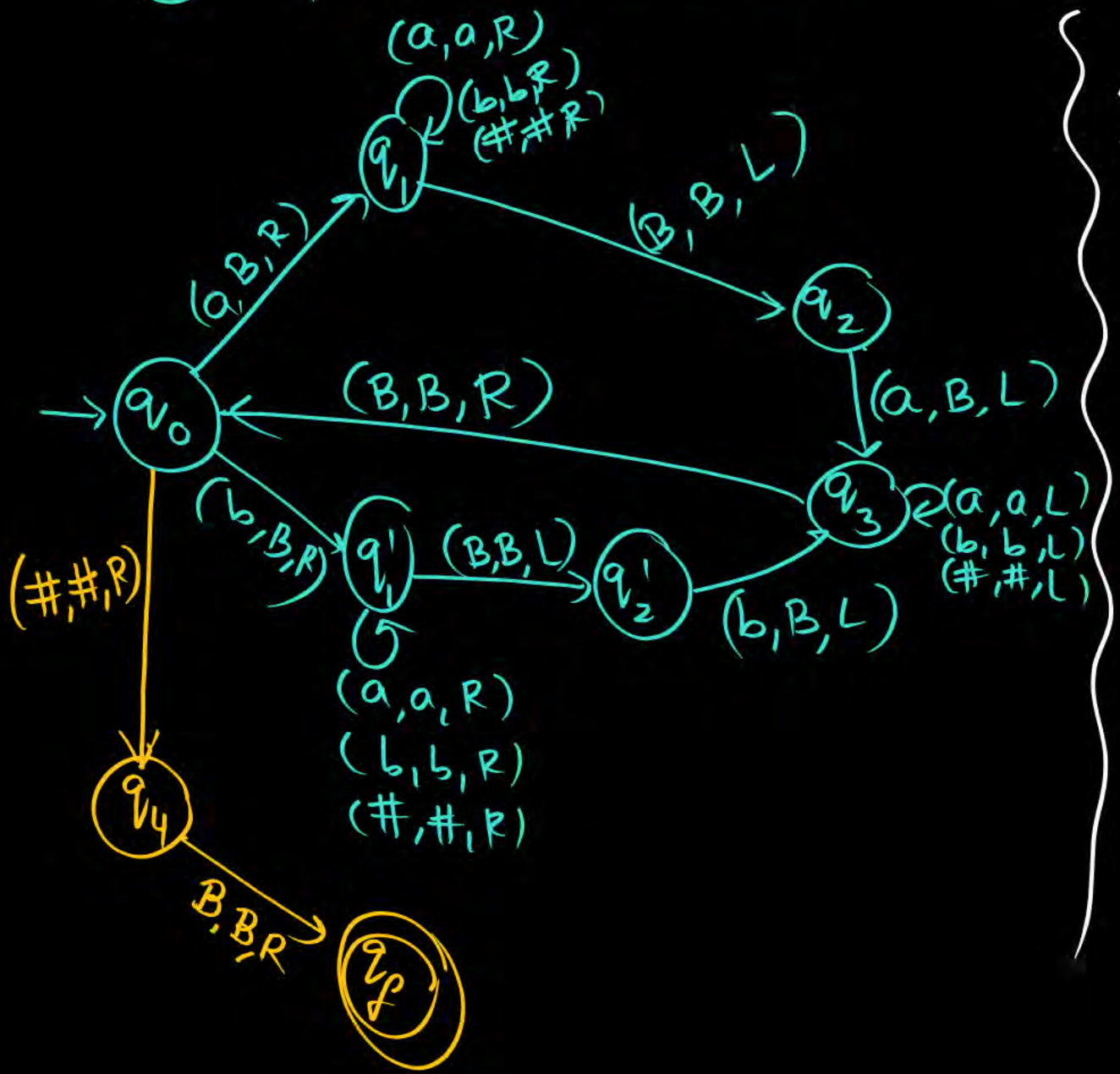
Topics Covered in Previous Session:

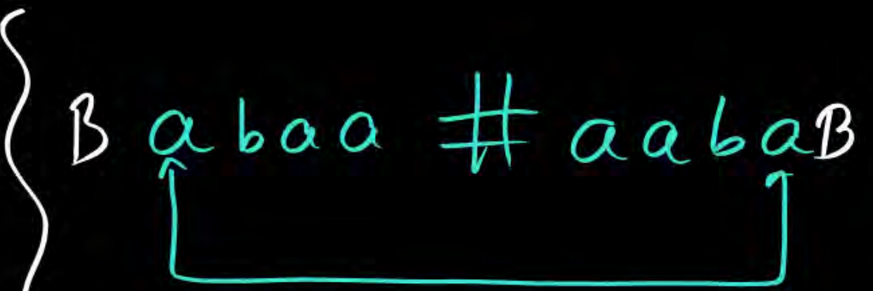
↳ TM construction

$a^n b^n$	$w\#w^R$
$a^n b^n c^n$	1's comp
ww^R	2's comp
$a^* b^*$	$f(x) = x+1$
	$f(x) = x-1$
	unary addition

TM

(5) $\{w\#w^R \mid w \in \{a,b\}^*\}$





q_0 : B (a) B (b)

q_1' & q_1 : Skip as, bs, # to reach B

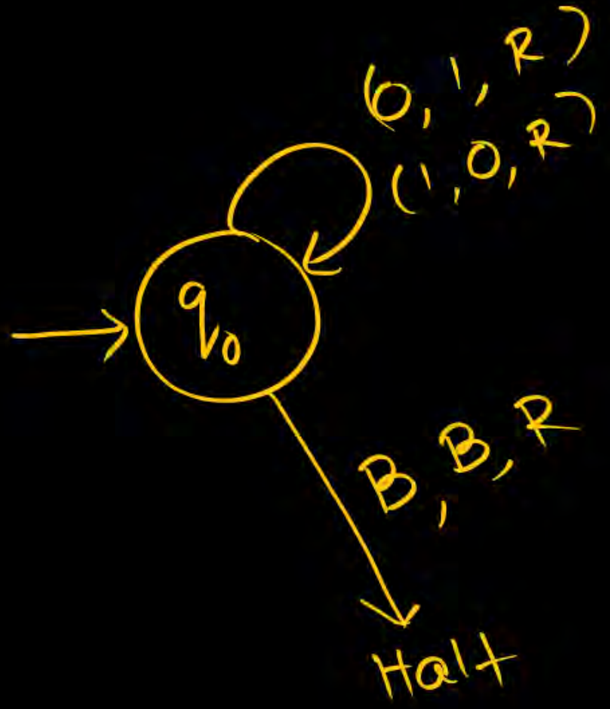
q_2 : B (a)

q_2' : B (b)

q_3 : Reverse scan to find B

TM

(6) One's complement of binary

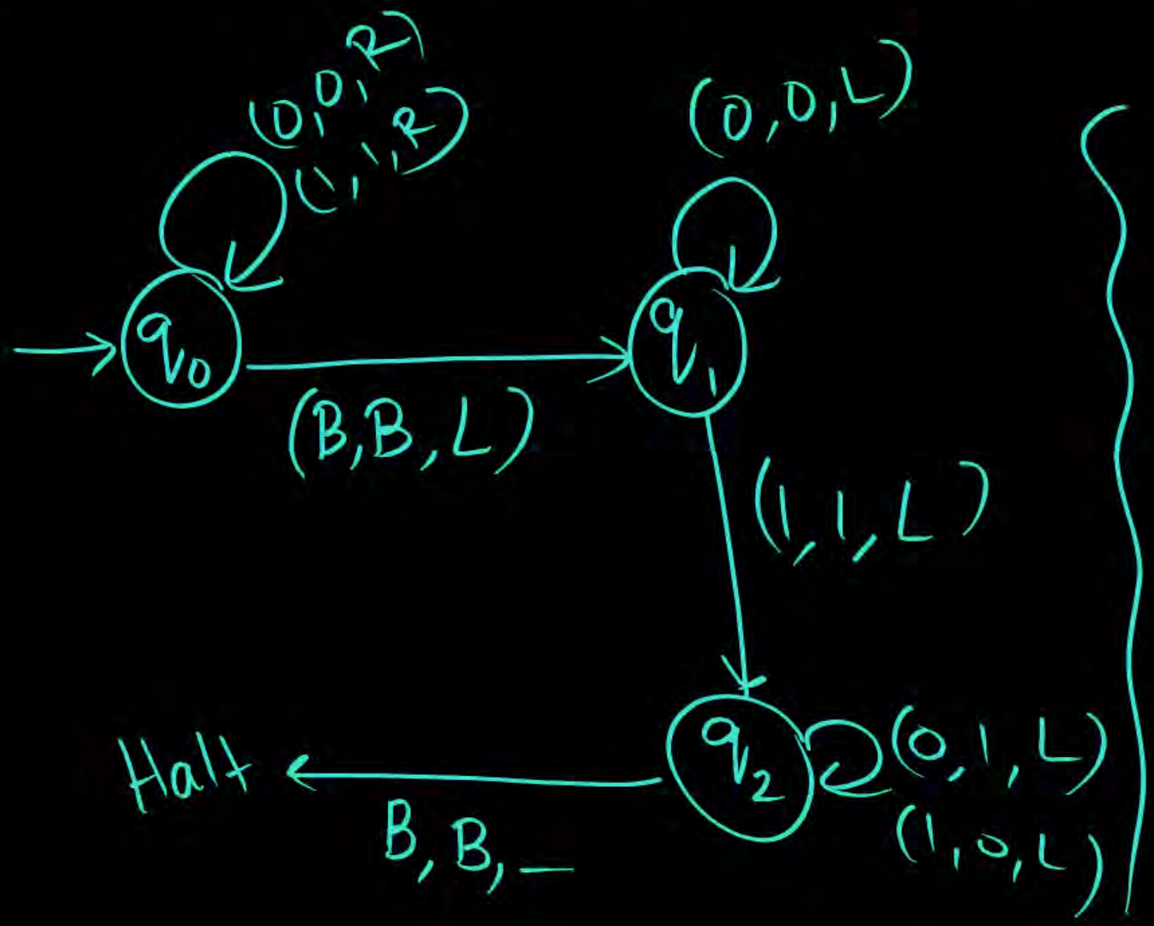


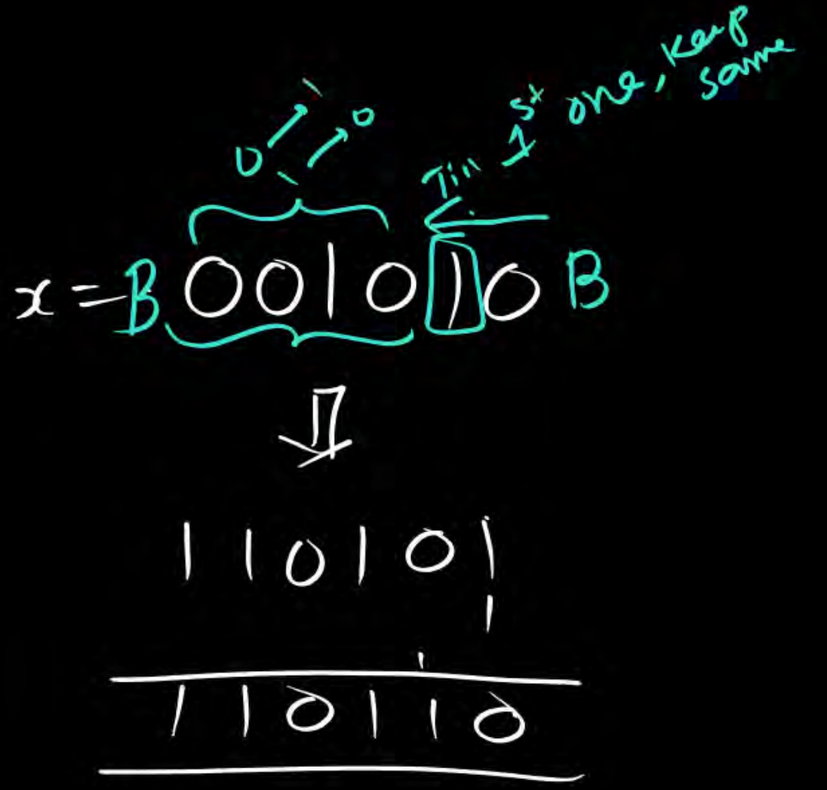
1 0 1 0 0

B 0 1 0 1 1 B

TM

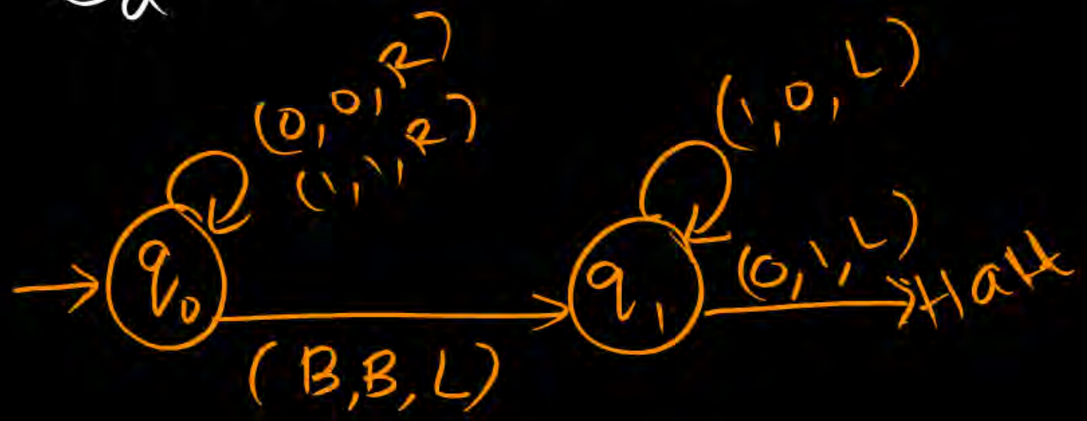
(7) 2's complement of binary input





TM

⑧a Increment $f(x) = x+1$, x is binary



Binary: 0101

⑧b $f(x) = x+1$; x is unary

Unary:

5+1=6

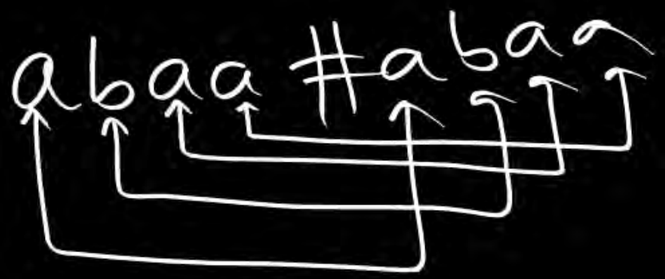
TM

(9)$_a$ $f(x) = x - 1$ when x is binary

(9)$_b$ $f(x) = x - 1$ when x is unary

(10) Unary Addition

(11) $\{w\#w \mid w \in \{a,b\}^*\}$



B aa # aaaaaB

(a, B)

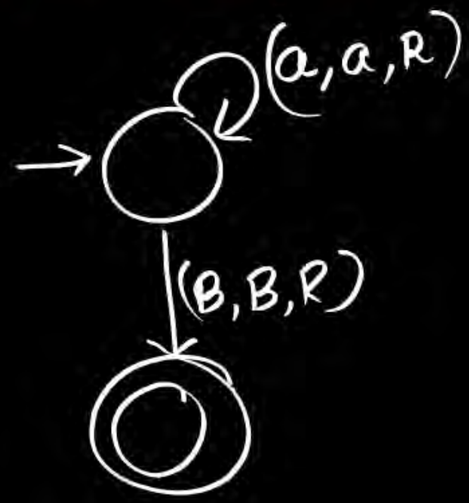
2 + 5 = 7

in unary

B aaaaaaa B

TM

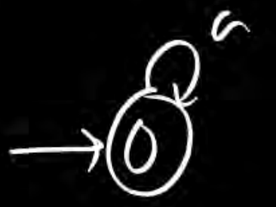
Try to understant following TM's

① 

or 

$L = a^*$

FA



$= a^*$

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② 

D a b a a B B B

Never halts for every string in $(a+b)^*$

TM

Closure properties for Recursive languages

1. Union
2. Intersection
3. Complement
4. Difference
5. concatenation
6. Reversal
7. Kleene star
8. Kleene plus

~~9~~ Subset

~~10~~ Substitution (f)

~~11~~ Homomorphism (h)

12. ε-free homomorphism
13. Inverse Homomorphism

Remember not closed operations:

$\subseteq, f, h, f_{\text{finite}}, \text{Inf}(\cup, \cap, -, ;, \subseteq, f)$

14. Finite union
15. Finite $\cap$
16. Finite Difference
17. Finite Concatenation
18. Finite Subset
19. Finite Substitution

~~20~~ to ~~25~~ : $\text{Inf}(\cup, \cap, -, ;, \subseteq, f)$

26. $L \cup Reg$
27. $L \cap Reg$
28. L / Reg
29. $L - Reg$
30. $Reg - L$

L is Recursive

TM

Closure properties for RELs

1. Union
2. Intersection
3. ~~Complement~~
4. ~~Difference~~
5. Concatenation
6. Reversal
7. Kleene star
8. Kleene plus
9. ~~Subset~~
10. Substitution
11. Homomorphism
12. ε-free homomorphism
13. Inverse Homomorphism
14. Finite union
15. Finite $\cap$
16. ~~Finite Difference~~
17. Finite Concatenation
18. Finite Subset
19. Finite Substitution

~~20~~ to ~~25~~ : Inf $(\cup, \cap, -, \cdot, \subseteq, f)$

Remember not closed operations:

$\bar{L}$, Difference, $\subseteq$, Finite Difference, Inf $(\cup, \cap, -, \cdot, \subseteq, f)$, Reg-L

26. $L \cup Reg$
27. $L \cap Reg$
28. L / Reg
29. $L - Reg$
30. ~~Reg - L~~

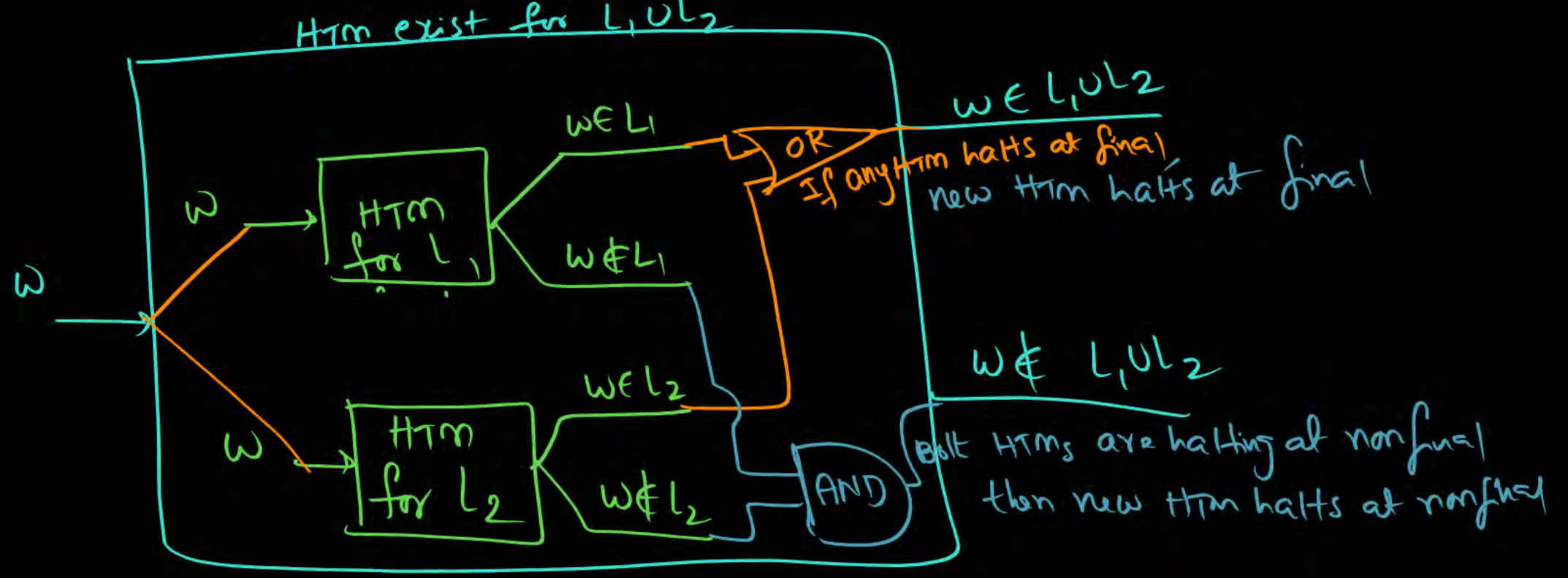
L is REL

TM

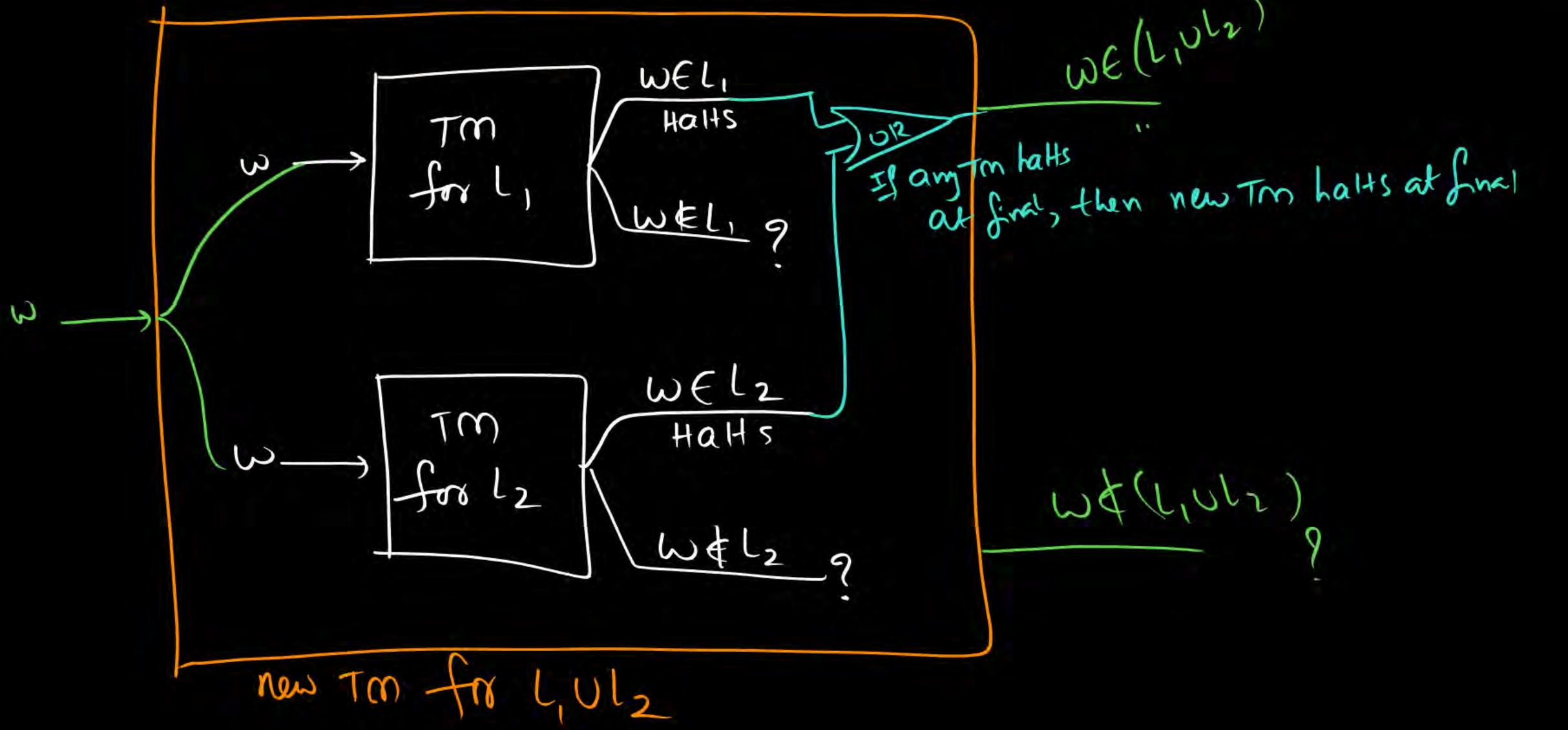
① Union

- closed for Recursive languages
- closed for REIs

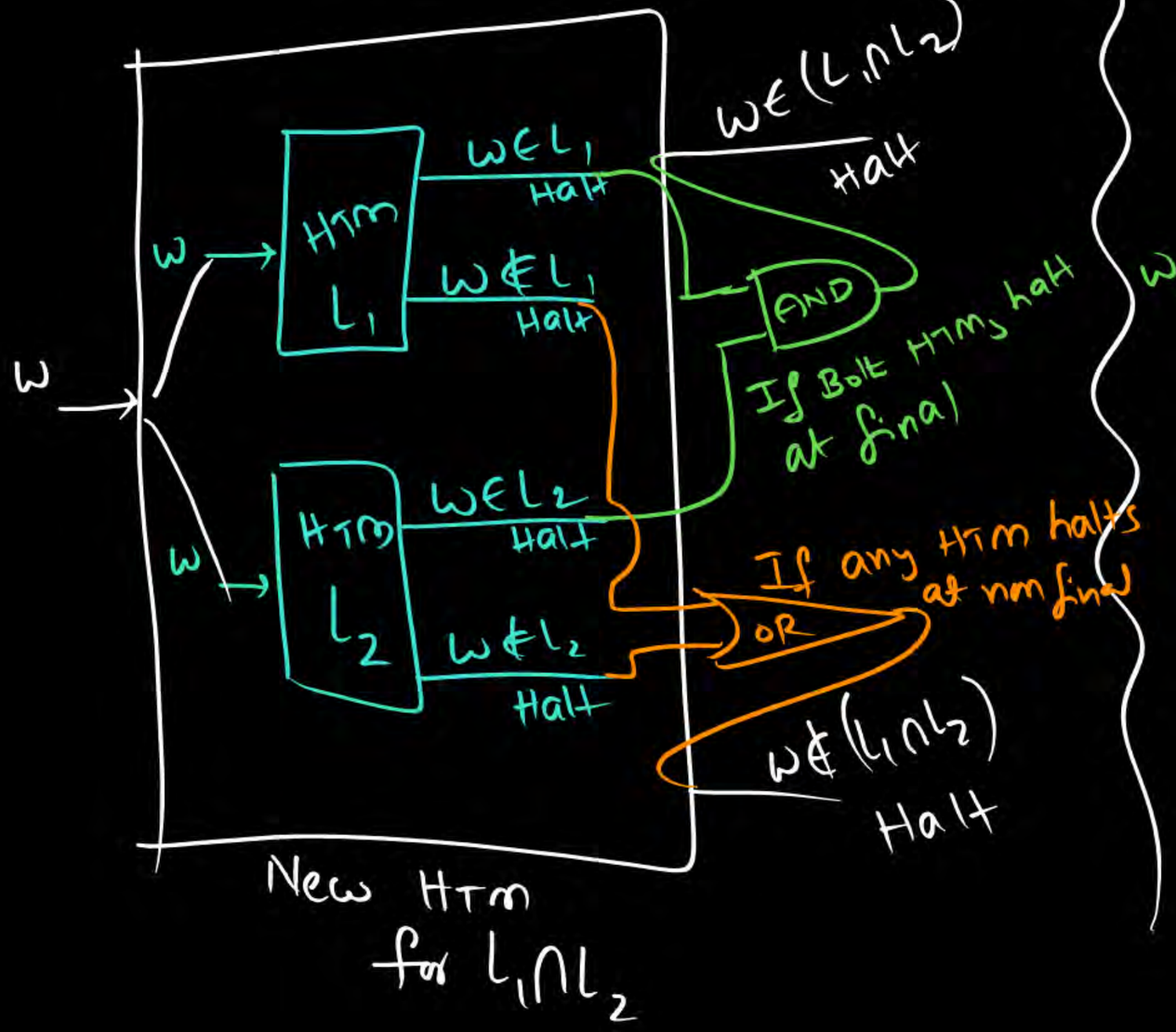
Recursive$_1$ ∪ Recursive$_2$ ⇒ Always Recursive



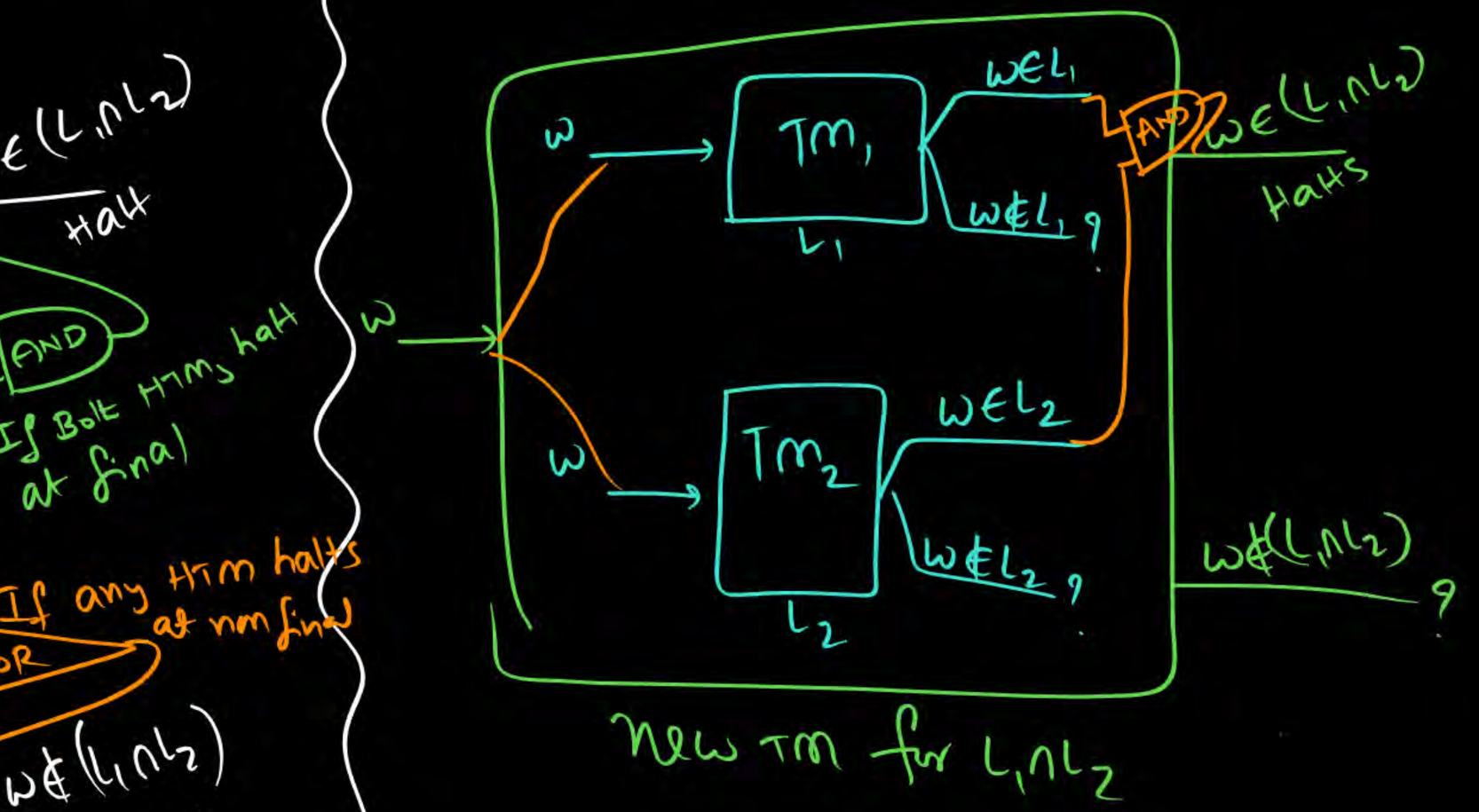
$REL_1 \cup REL_2 \Rightarrow REL$



Recursive$_1$ ∩ Recursive$_2$ ⇒ Recursive

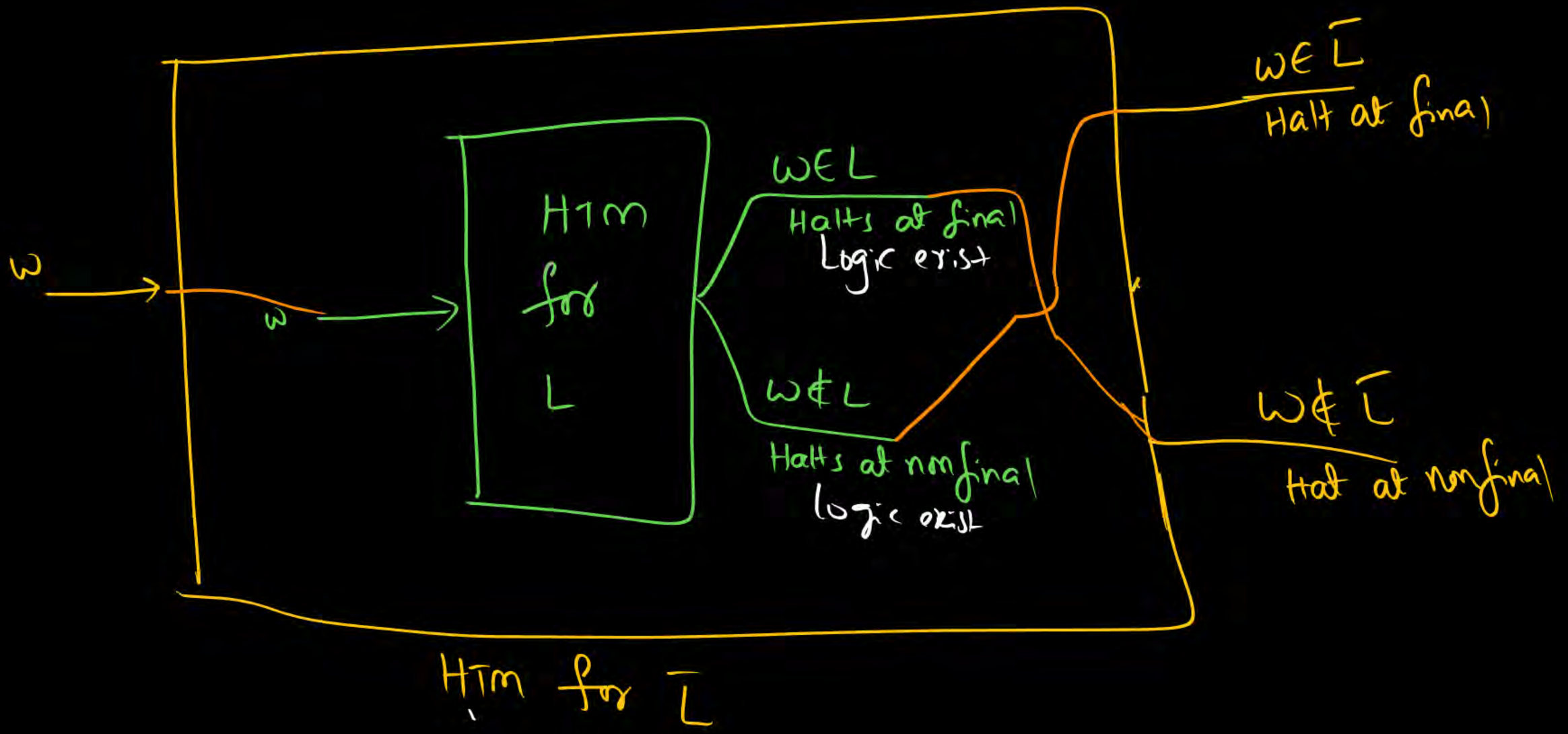


RE$_1$ ∩ RE$_2$ ⇒ RE



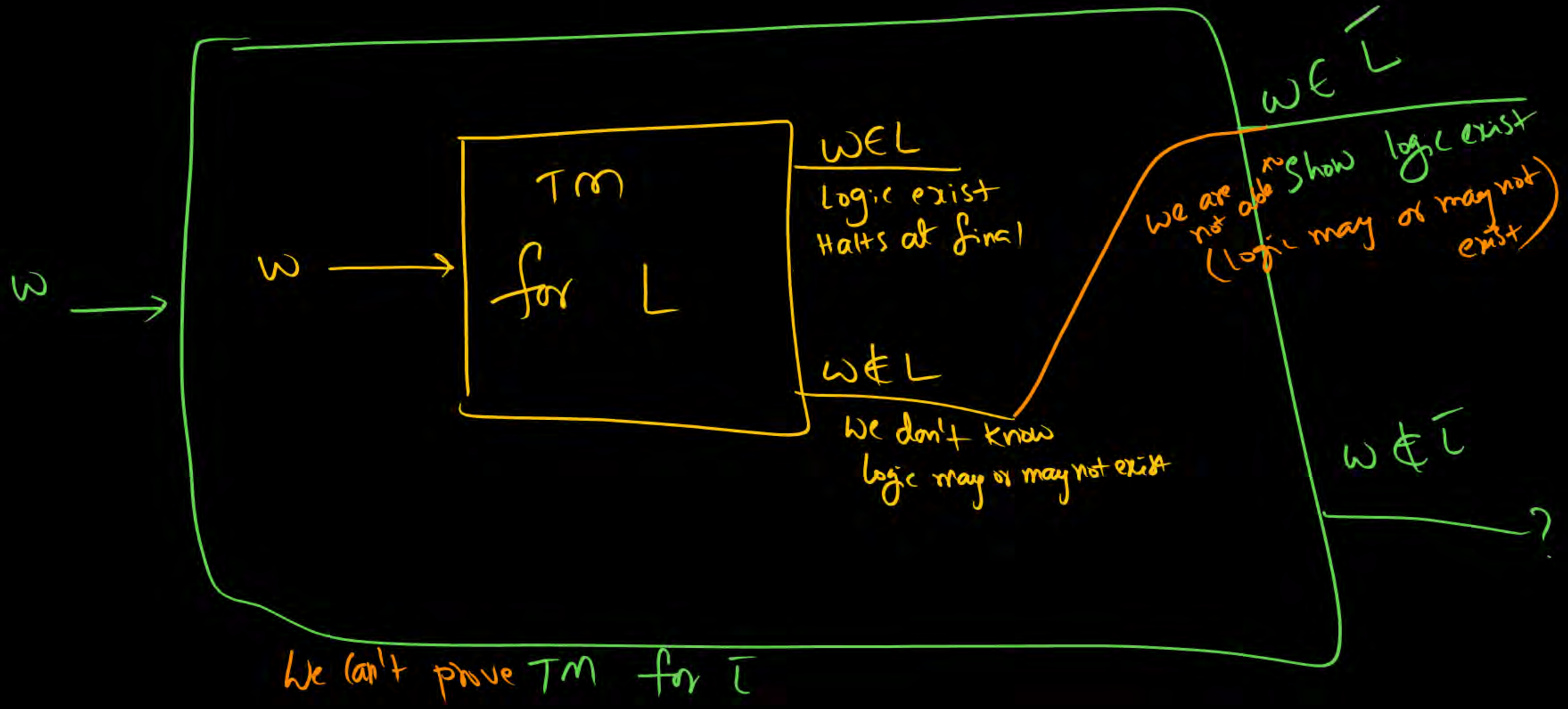
TM

Recursive $\Rightarrow$ Recursive [Decidable $\Rightarrow$ Decidable]



$\overline{REL} \Rightarrow$ Need not be REL

(Either Recursive or not REL)



TM

$\overline{REL}$ ⇨ Need not be REL

⇨ Either Recursive or not REL

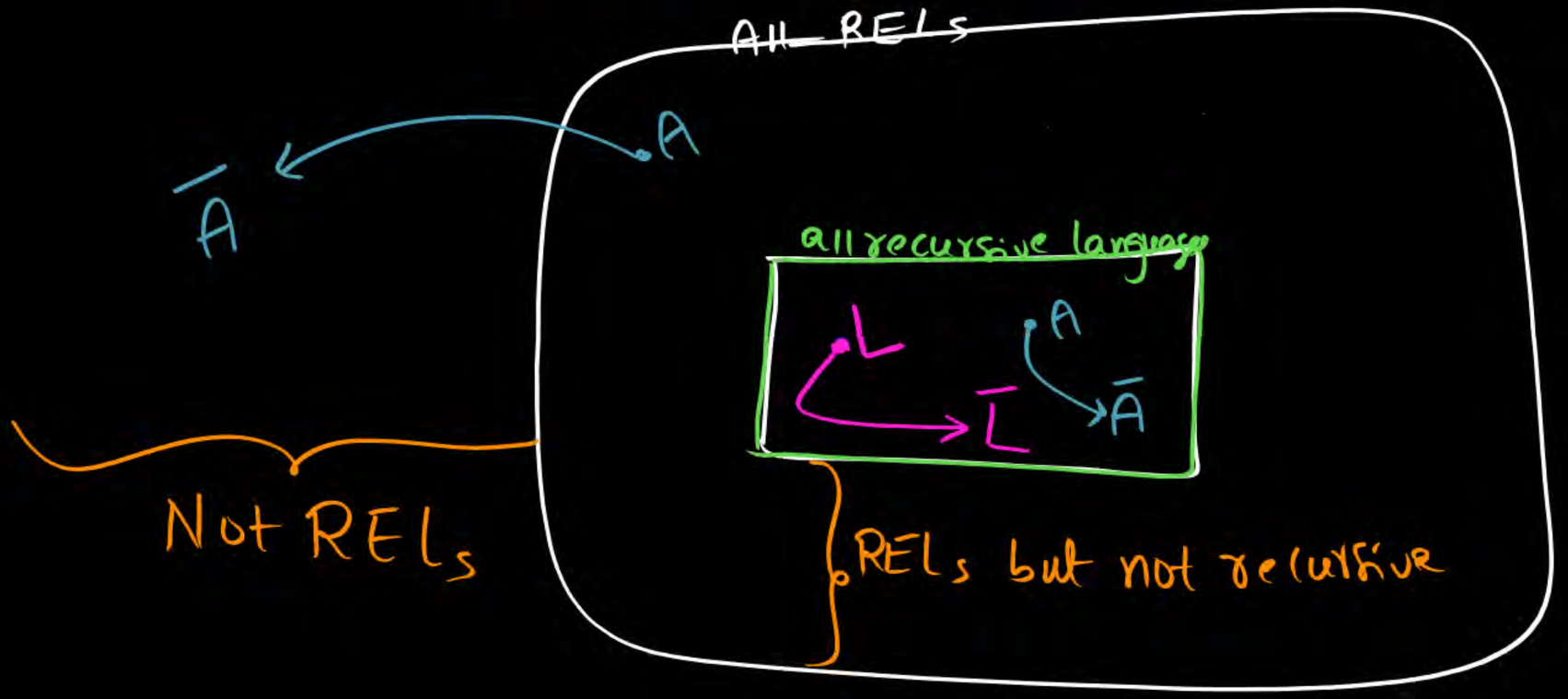
⇨ Never be "REL but not recursive"

A is REL

↳ may be inside Recursive

or

outside recursive



TM

① $Rec_1 \cup Rec_2 \Rightarrow$ Recursive

② $RE_1 \cup RE_2 \Rightarrow RE$

*** ③ $Rec \cup RE \Rightarrow RE$

④ $Rec_1 \cap Rec_2 \Rightarrow Rec$

⑤ $RE_1 \cap RE_2 \Rightarrow RE$

*** ⑥ $Rec \cap RE \Rightarrow RE$

⑦ $\overline{Rec} \Rightarrow Rec$

⑧ $\overline{RE} \Rightarrow$ "need not be RE" (either Rec or Not RE)

(Never be "RE but not Rec)

Rec (Recursive)

RE (Recursively Enumerable)

TM

$\overline{REL} \Rightarrow ?$

$\overline{Rel} \Rightarrow ?$

Summary

TM construction

Closure properties

Thank you