COMPUTER SCIENCE

Database Management System

FD's & Normalization

Lecture_08

Vijay Agarwal sir

TOPICS TO BE COVERED

01 Minimal Cover

02 Finding Number of Super keys

- Membership set
- Equality b/w 2 FD Set
- <u>Minimal Cover</u>
  - Cross Check $F \equiv G$?
  - Procedure to Find Minimal Cover.

Canonical Cover

- Sets of functional dependencies may have redundant dependencies that can be inferred from the others
  - For example: $A \rightarrow C$ is redundant in: $\{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$

Q. $AB \rightarrow C$, $D \rightarrow E$, $E \rightarrow C$ is a minimal cover for the set of functional dependencies $AB \rightarrow C$, $D \rightarrow E$, $AB \rightarrow E$, $E \rightarrow C$.

$F \equiv G$?

F Cover G : True

G Cover F : True

$\Downarrow$

False [Not minimal cover]

Q.

Given the following two statements:

S1: Every table with two single-valued attributes is in 1NF, 2NF, 3NF and BCNF.

S2: $AB \rightarrow C$, $D \rightarrow E$, $E \rightarrow C$ is a minimal cover for the set of functional dependencies $AB \rightarrow C$, $D \rightarrow E$, $AB \rightarrow E$, $E \rightarrow C$.

Which one of the following is CORRECT?

[MCQ: 2014: 2M]

(A) S1 is TRUE and S2 is FALSE.

(B) Both S1 and S2 are TRUE.

(C) S1 is FALSE and S2 is TRUE.

(D) Both S1 and S2 are FALSE.

Procedure to find minimal set

Step (1)

Split the FD such that RHS contain single Attribute.

Ex. $A \rightarrow BC$, $\Rightarrow$ $A \rightarrow B$ and $A \rightarrow C$

Step (2)

Find the redundant attribute on L.H.S and delete them.

Ex. $AB \rightarrow C$,

A – Can be deleted $[B]^+=[A]$ B^+ Contains 'A'

OR

B can be delete if A^+ contain 'B' $[A]^+=[\ldots B]$

Step

(3)

Find the redundant FD and delete them from the set

Ex. $\{A \rightarrow B,\ B \rightarrow C,\ A \rightarrow C\}$ → Extra FD

$\{A \rightarrow B,\ B \rightarrow C\}$

$(A)^+ = (AC)$ $(B)^+ = (B)$ $(A)^+ = (ABC)$

Example1:

$$[AB \rightarrow CD,\ A \rightarrow E,\ E \rightarrow C]$$

Step1: $AB \rightarrow C$, $AB \rightarrow D$, $A \rightarrow E$, $E \rightarrow C$

Step2: $AB \rightarrow C$

$(A)^+ = [AEC]$; B is Not Extra

$(B)^+ = [B]$; A is Not Extra

$\boxed{① AB \rightarrow C \quad ② AB \rightarrow D \quad ③ A \rightarrow E \quad ④ E \rightarrow C}$

Step3

× ① $\cancel{AB \rightarrow C}$ ✓ ② $AB \rightarrow D$ ✓ ③ $A \rightarrow E$ ✓ ④ $E \rightarrow C$

$(AB)^+ = [ABDEC]$ $(AB)^+ = [ABEC]$ $(A)^+ = [A]$ $(E)^+ = [E]$

Korth

$AB \rightarrow C$

B extra

$(A)^+ = [AEC]$

$(B)^+ = [B]$

$\boxed{\begin{matrix} AB \rightarrow D \\ A \rightarrow E \\ E \rightarrow C \end{matrix}}$ Ans

Example1:

$$[AB \to CD,\ A \to E,\ E \to C]$$

Step1 : $AB \to C$, $AB \to D$, $A \to E$, $E \to C$

Step2 : $AB \to C$

$(A)^+ = [AEC]$; B is Not Extra

$(B)^+ = [B]$; A is Not Extra

$$\boxed{A \to C \quad AB \to D \quad A \to E \quad E \to C}$$

① $\cancel{A \to C}$ ✓② $AB \to D$ ✓③ $A \to E$ ✓④ $E \to C$

$(A)^+ = [AEC]$ $(AB)^+ = (ABEC)$ $(A)^+ = (A)$ $(E)^+ = [E]$

Korth

$(A)^+ = [AEC]$

$(B)^+ = (B)$

B extra

$\boxed{AB \to C}$

$AB \to D$

B is extra

$A \to C$

$$\boxed{\begin{matrix} AB \to D \\ A \to E \\ E \to C \end{matrix}}$$ Ans

Example 2:

$$[A \rightarrow C,\ AC \rightarrow D,\ E \rightarrow AD,\ E \rightarrow H]$$

Example3:

$$[B \rightarrow A,\ D \rightarrow A,\ AB \rightarrow D]$$

$$\boxed{\begin{matrix} D \rightarrow A \\ B \rightarrow D \end{matrix}}$$ Ans

Navathe

$AB \to C$

A is extra $\quad (B)^+ = [\ldots\ldots \underline{A}]$; A is extra

B is extra $\quad [A]^+ = [\ldots\ldots B]$; B is extra.

Example 4:

$[A \rightarrow BC,\ CD \rightarrow E,\ E \rightarrow C,\ D \rightarrow AEH,\ ABH \rightarrow BD,\ DH \rightarrow BC]$

Step1: Split the FD. Such that R.H.S Contain Single Attribute.

$A \rightarrow B$, $A \rightarrow C$, $\cancel{C}D \rightarrow E$, $E \rightarrow C$, $D \rightarrow A$, $D \rightarrow E$, $D \rightarrow H$, $A\cancel{B}H \rightarrow B$, $A\cancel{B}H \rightarrow D$, $D\cancel{H} \rightarrow B$, $D\cancel{H} \rightarrow C$

Step2: Find the Redundant (Extra) Attribute on L.H.S & Delete them.

(i) $\cancel{C}D \rightarrow E$; $[C]^+ = [C]$, D is Not Extra $\quad [D]^+ = [DAC\ldots]$, C is extra Attribute

(ii) $D\cancel{H} \rightarrow B$, $DH \rightarrow C$; $[H]^+ = [H]$; D is Not extra $\quad [D]^+ = [DH\ldots]$ H is extra Attribute

(iii) $A\cancel{B}H \rightarrow B$, $A\cancel{B}H \rightarrow D$: $[AH]^+ = [AHB]$ B is extra

AB

BH

AH

$AH \rightarrow B$

$[A]^+ = [ABC]$

$[H]^+ = [H]$

$D \rightarrow E$, $D \rightarrow B$, $AH \rightarrow D$. $AH \rightarrow B$

$D \rightarrow C$

Example 4:

[A → BC, CD → E, E → C, D → AEH, ABH → BD, DH → BC]

A→B A→C D→E E→C D→A ~~D→E~~ D→H. AH→B

AH→D D→B D→C

① A→B $(A)^+ = [AC]$

② A→C $(A)^+ = (AB)$

③ D→E $(D)^+ = (DABCH)$

④ E→C $(E)^+ = (E)$

⑤ D→A $(D)^+ : (DEBCH)$

⑥ D→H $(D)^+ = (DEACB)$

⑦ ~~AH→B~~ $(AH)^+ = (AHDBC)$

⑧ AH→D $(AH)^+ = (AHBC)$

⑨ ~~D→B~~ $(D)^+ = (DCABEH)$

⑩ ~~D→C~~ $(D)^+ : (DABCE)$

$A \to B$, $A \to C$, $D \to E$, $D \to A$, $D \to H$, $E \to C$, $AH \to D$

Ans

OR

$A \to BC$, $D \to AEH$, $E \to C$, $AH \to D$

Ans

Note

Minimal Cover May or May Not be Unique. ie We Can have More than one Minimal Cover.

Q. The following functional dependencies hold true for the relational schema R{V, W, X, Y, Z}:

$$V \to W$$
$$VW \to X$$
$$Y \to VX$$
$$Y \to Z$$

Which of the following is irreducible equivalent for this set of functional dependencies? [MCQ:2017]

(A)
$V \to W$
$V \to X$
$Y \to V$
$Y \to Z$

(B)
$V \to W$
$W \to X$
$Y \to V$
$Y \to Z$

(C)
$V \to W$
$V \to X$
$Y \to V$
$Y \to X$
$Y \to Z$

(D)
$V \to W$
$W \to X$
$Y \to V$
$Y \to X$
$Y \to Z$

Q.

Consider the following FD Set:
$\{P \to QR, Q \to PR, R \to PQ\}$ which of the following is/are the minimal cover for the above FD set?

[MSQ]

(A) $P \to Q, Q \to R, R \to P$

(B) $P \to R, Q \to R, R \to PQ$

(C) $Q \to P, P \to R, R \to Q$

(D) $P \to QR, Q \to P, R \to P$

Ans (A, B, C & D)

[Home Work]

Finding Number of SUPER KEY :

Let R be the Relational Schema with n Attributes $A_1, A_2, A_3, \ldots, A_n$ How Many Super keys are there ?

(i) With Only Candidate key A_1 ?

(ii) With Only Candidate key A_1, A_2 ?

(iii) With Only Candidate key A_1A_2, A_3A_4 ?

(iv) With Only Candidate Key A_1A_2, A_2A_3 ?

(v) With Only Candidate key A_1, A_2, A_3 ?

Super key $\rightarrow$ Candidate key (C.K)

$\rightarrow$ C.K + Other Attributes

All possible Combination

(i) With Only Candidate key $\underline{A_1}$?

$A_1 A_2 A_3 A_4 A_5 \ldots \ldots A_n$

#Suber keys :

$\underline{A_1}$

$\underline{A_1}\ A_2$

$\underline{A_1}\ A_2\ A_3$

$\underline{A_1}\ A_2\ A_3\ A_4$

$\underline{A_1}\ A_2\ A_3\ A_4\ A_5$

$\vdots$

$\underline{A_1}\ \underbrace{A_2 A_3 A_4 A_5 \ldots \ldots A_n}_{n-1}$

$$\boxed{\#\text{Suberkeys} = 2^{n-1}}$$

Ans

eg R(ABCD) with C.K : A

① $\#\text{Suberkey} = 2^{n-1} \Rightarrow 2^{4-1} = 2^3$

$= 8$ Suberkeys

OR

② $R(\underline{A}\underbrace{BCD}_{3\text{ Attribute}}) = 2^3 = 8$ Suber keys

OR

A	B	C	D	
	0	0	0	→ A
	0	0	1	→ AD
	0	1	0	→ AC
	0	1	1	→ ACD
	1	0	0	→ AB
	1	0	1	→ ABD
	1	1	0	→ ABC
	1	1	1	→ ABCD

8 Suber key

A BCD

Single Attribute

A

AB

AC

AD

ABC

ACD

ABD

ABCD

8 Super keys

(ii) With only Candidate key A_1, A_2.

$\underline{A_1}$

$\underline{A_1}\ A_2$

$\underline{A_1}\ A_2\ A_3$

$\vdots$

$\underline{A_1}\ \underbrace{A_2 A_3 \ldots\ldots A_n}_{(n-1)}$

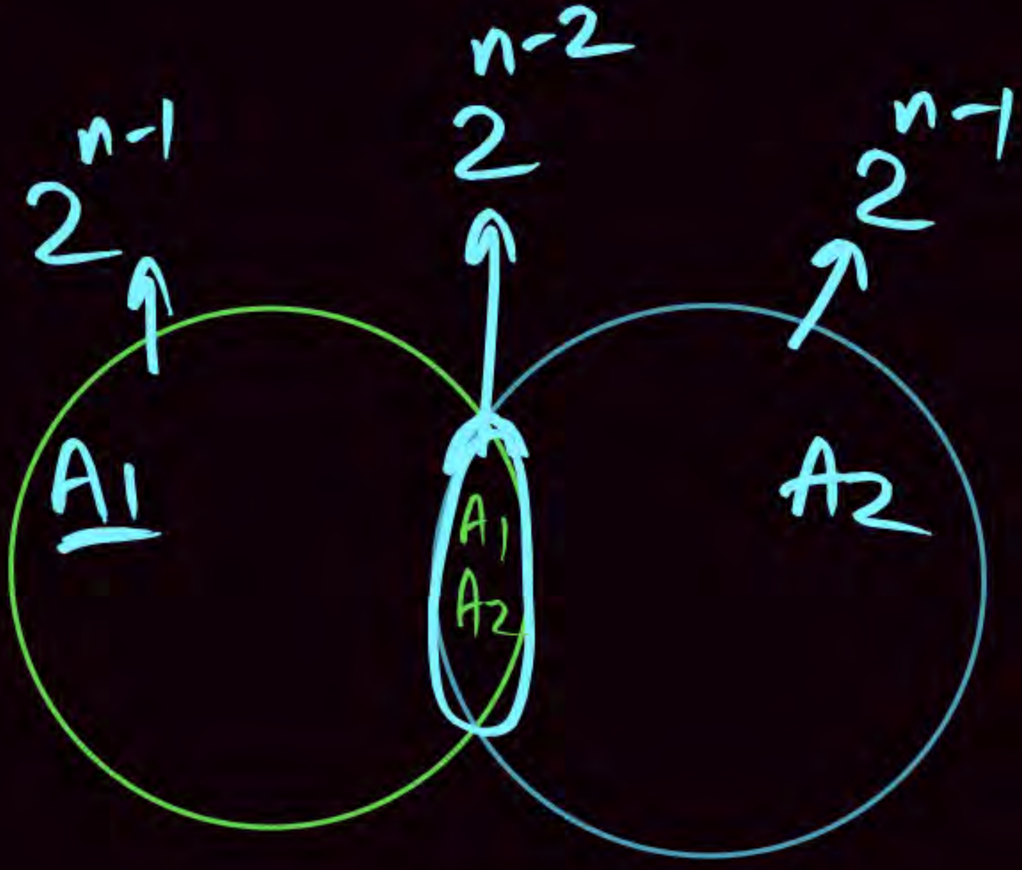
$\underline{A_2}$

$\underline{A_2}\ A_1$

$\underline{A_2}\ A_1\ A_3$

$\vdots$

$\underline{A_2}\ \underbrace{A_1 A_3 \ldots\ldots A_n}_{(n-1)}$



$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

$$\boxed{\#\text{Super keys} = 2^{n-1} + 2^{n-1} - 2^{n-2}}$$

eg R(ABCDE) with C.K : A, D

Super keys

$$\#\text{Super keys} = 2^{n-1} + 2^{n-1} - 2^{n-2}$$

(n is # of Attribute)

$$\Rightarrow 2^{5-1} + 2^{5-1} - 2^{5-2}$$

$$\Rightarrow 2^4 + 2^4 - 2^3 \Rightarrow 16 + 16 - 8$$

$$= 24 \text{ Super key}$$

(iii) with only candidate key A_1A_2, A_3A_4 ?

$$\boxed{\#\text{Super keys} = 2^{n-2} + 2^{n-2} - 2^{n-4}}$$

9 R(ABCDE) C.k [AB, DE]

Super key

$\Rightarrow 2^{5-2} + 2^{5-2} - 2^{5-4}$

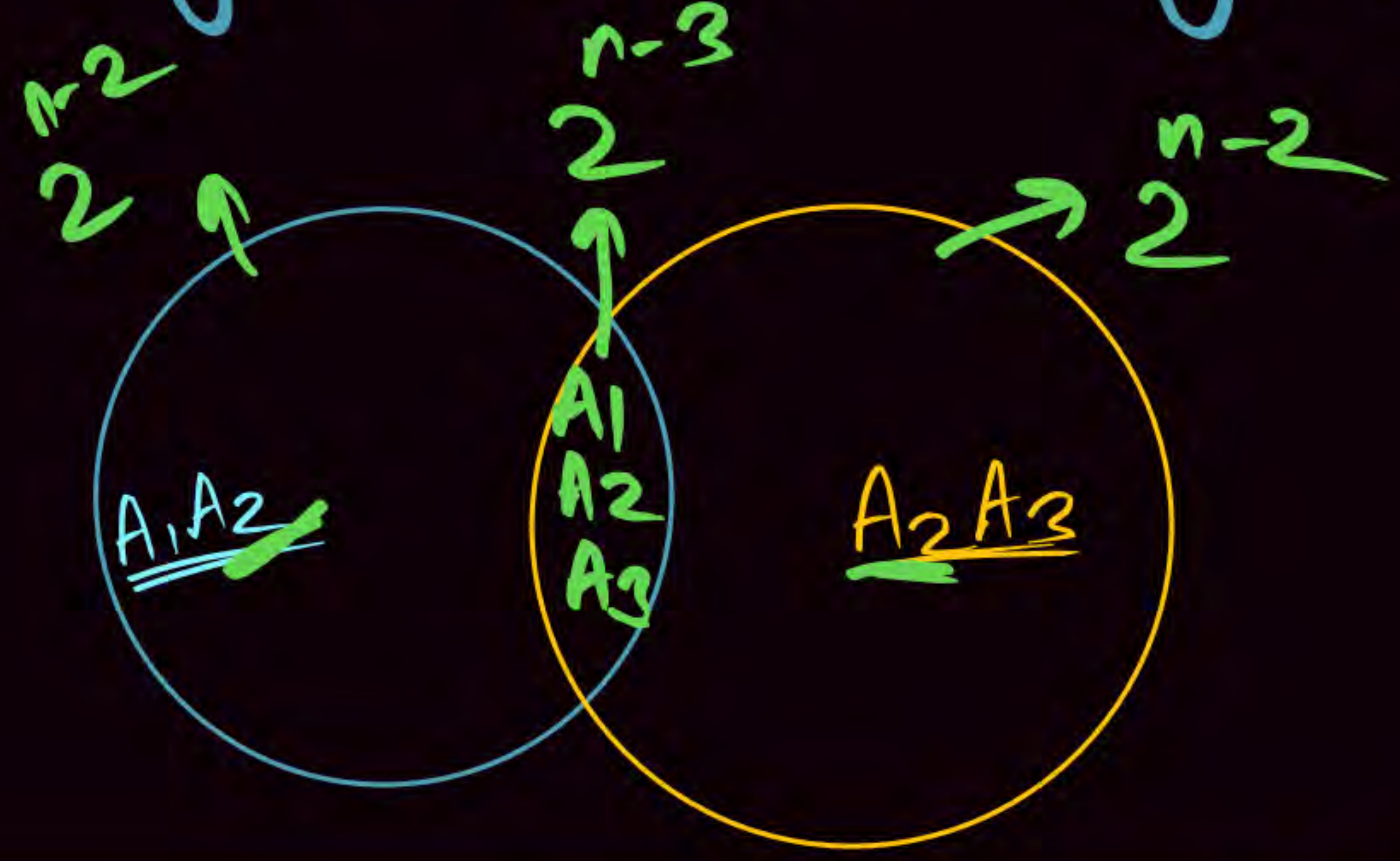
AB	DE
ABC	DEA
ABD	DEB
ABE	⋮
ABCD	⋮
ABDE	⋮
ABCE	⋮
ABCDE	

$\Rightarrow 2^3 + 2^3 - 2^1$

$\Rightarrow 8 + 8 - 2$

$= 14$ Super keys.

(iv) With only Candidate key A_1A_2, A_2A_3 ?



$$\boxed{\#\text{Super keys} = 2^{n-2} + 2^{n-2} - 2^{n-3}}$$

eg R(ABCDE) C.K {AB, BC}

$$\# \text{Super keys} = 2^{n-2} + 2^{n-2} - 2^{n-3}$$

$$\Rightarrow 2^{5-2} + 2^{5-2} - 2^{5-3}$$

$$\Rightarrow 2^3 + 2^3 - 2^2$$

$$= 8 + 8 - 4$$

$= 12$ super key Ans

(v) with only C.K A_1, A_2, A_3 ?

$$\boxed{\#\text{Super keys} = 2^{n-1} + 2^{n-1} + 2^{n-1} - 2^{n-2} - 2^{n-2} - 2^{n-2} + 2^{n-3}}$$

$$\boxed{\begin{aligned} n(A \cup B \cup C) = {} & n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) \\ & + n(A \cap B \cap C) \end{aligned}}$$

Floor & Ceiling Concept

Floor

$\lfloor 3.0 \rfloor = 3$

$\lfloor 2.0 \rfloor = 2$

$\lfloor 2.9 \rfloor = 2$

$\lfloor 2.8 \rfloor = 2$

$\lfloor 2.7 \rfloor = 2$

$\lfloor 2.6 \rfloor = 2$

$\vdots$

$\lfloor 2.1 \rfloor = 2$

Ceiling

$\lceil 2.1 \rceil = 3$

$\lceil 2.2 \rceil = 3$

$\lceil 3.0 \rceil = 3$

$\lceil 2.3 \rceil = 3$

$\lceil 2.4 \rceil = 3$

$\lceil 2.5 \rceil = 3$

$\vdots$

$\lceil 2.9 \rceil = 3$

Maximum Number of Candidate keys = $^{n}C_{\lfloor \frac{n}{2} \rfloor}$

Where n is Number of Attribute.

6 Attribute

eg

R(ABCDEF) then What is Maximum # C.K ?

(i) If every Single Attribute is a C.K $= {}^6C_1 = 6$

(ii) If every 2 Attribute forms a CK $= {}^6C_2 = 15$

(iii) If every 3 Attribute forms a C.K $= {}^6C_3 = 20$

(iv) If every 4 Attributes forms a CK $= {}^6C_4 = 15$

(v) If every 5 Attribute forms a CK $= {}^6C_5 = 6$

(vi) If every 6 Attribute forms a C.K $= {}^6C_6 = 1$

# of Attribute

n: (6)

${}^nC_{\lfloor n/2 \rfloor}$

${}^6C_{\lfloor 6/2 \rfloor}$

$= {}^6C_3$

$= 20$

R(ABCDE) Finding # Super keys

(i) with only Candidate key = A

(ii) " " " " = A, D

(iii) " " " " = A, BC

(iv) " " " " = AB, CD

(v) " " " " = AB, AD

(vi) " " " " = ABC, DE

(vii) " " " " = ABC, CDE

(viii) " " " " = A, D, E

Closure of FD Set $[F]^+$

Set of all possible FD's which can be derived from given FD set is called closure of FD set. $[F]^+$

$[F]^+$ Closure of FD

R(AB)

ϕ	$A \rightarrow \phi$	$B \rightarrow \phi$	$AB \rightarrow \phi$
A	$A \rightarrow A$	$B \rightarrow A$	$AB \rightarrow A$
B	$A \rightarrow B$	$B \rightarrow B$	$AB \rightarrow B$
AB	$A \rightarrow AB$	$B \rightarrow AB$	$AB \rightarrow AB$

R(ABC)

ϕ	$A \rightarrow \phi$	$B \rightarrow \phi$
A	$A \rightarrow A$	$B \rightarrow A$
B	$A \rightarrow B$	$B \rightarrow B$
C	$A \rightarrow C$	$B \rightarrow C$
AB	$A \rightarrow AB$	$B \rightarrow AB$
BC	$A \rightarrow BC$	$B \rightarrow AC$
AC	$A \rightarrow AC$	$B \rightarrow BC$
ABC	$A \rightarrow ABC$	$B \rightarrow ABC$

$R(ABC)$ $[A \rightarrow B, B \rightarrow C]$ $[F]^{+} = 43$ Ans.

ϕ	0 attribute =	$\phi \rightarrow \phi$
A	1Attribute =	$[A]^{+} = [ABC] = 2^3$
B		$[B]^{+} = [BC] = 2^2$
C		$[C]^{+} = [C] = 2^1$
AB	2Attribute =	$[AB]^{+} = [ABC] = 2^3$
BC		$[BC]^{+} = [BC] = 2^2$
AC		$[AC]^{+} = [ABC] = 2^3$
ABC	3 Attribute=	$[ABC]^{+} = [ABC] = 2^3$

$$[A]^+ = \begin{bmatrix} A \to \phi, A \to A, A \to B, A \to C \\ A \to AB, A \to BC, A \to AC, A \to ABC \end{bmatrix}$$

$$[B]^+ = [B \to \phi,\ B \to B,\ B \to C,\ B \to BC]$$

$$[C]^+ = [C \to \phi,\ C \to C]$$

$$[AB]^+ = \begin{bmatrix} AB \to \phi, AB \to A, AB \to B, AB \to C \\ AB \to AB, AB \to BC, AB \to AC, AB \to ABC \end{bmatrix}$$

$$[BC]^+ = [BC \to \phi,\ BC \to B,\ BC \to C,\ BC \to BC]$$

$$[AC]^+ = \begin{bmatrix} AC \to \phi, AC \to A, AC \to B, AC \to C \\ AC \to AB, AC \to BC, AC \to AC, AC \to ABC \end{bmatrix}$$

$$[ABC]^+ = \begin{bmatrix} ABC \to \phi, ABC \to A, ABC \to B, ABC \to C \\ ABC \to AB, ABC \to BC, ABC \to AC, ABC \to ABC \end{bmatrix}$$

$R(AB)$ $[A \to B]$

ϕ 0 attribute = 1

A 1 Attribute = $[A]^+$ $[AB] = 2^2$

B $[B]^+ = [B] = 2^1$

AB 2 Attribute = $[AB]^+ = [AB] = 2^2$

$$\boxed{\begin{array}{c} 1 \\ \\ 4 \\ \\ 2 \\ \\ 4 \end{array}} \begin{array}{l} \\ \\ (A\to\phi, A\to A, A\to B, A\to AB) \\ \\ (B\to\phi, B\to B) \\ \\ \begin{pmatrix} AB\to\phi, AB\to A \\ AB\to B, AB\to AB \end{pmatrix} \end{array}$$

11 Ans.

Any Doubt ?

THANK YOU!