

CS & IT Engineering



Finite Automata
DFA – Part 3
Lecture:03



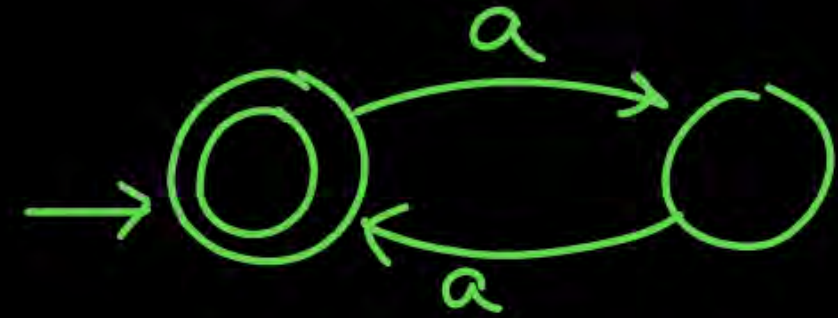
Deva sir

Topics: To Be Covered

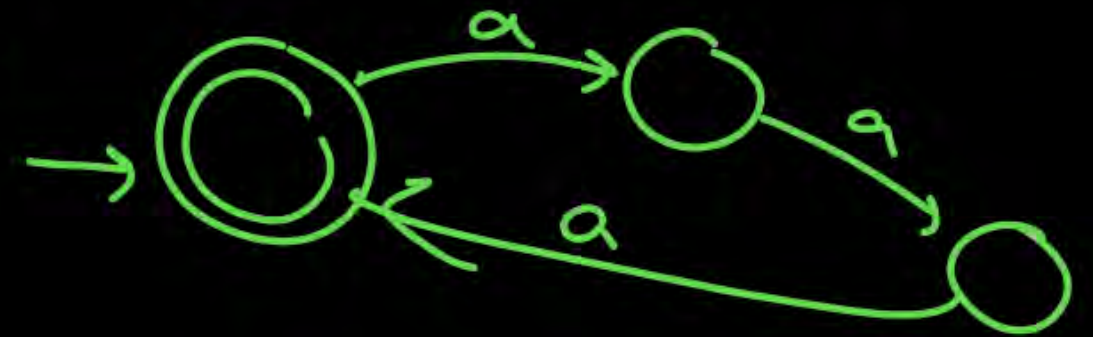
→ Model VI onwards

Model-II [languages over 1 symbol]

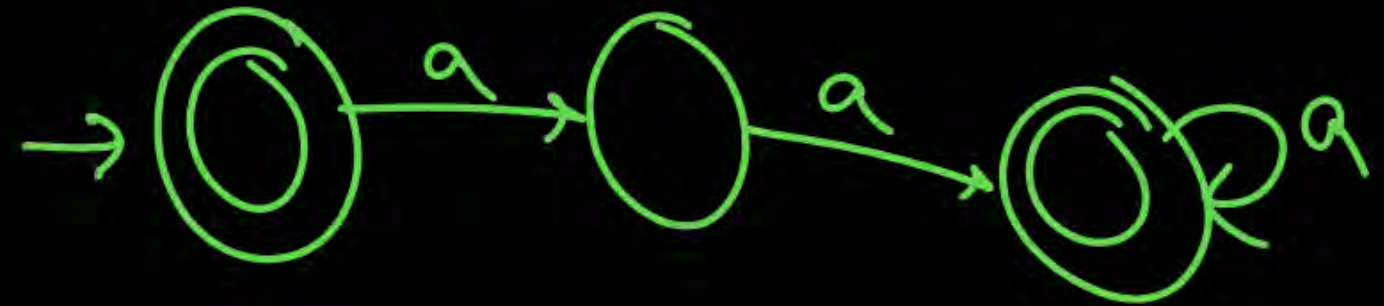
(21) $L = (aa)^* = \{\epsilon, a^2, a^4, a^6, \dots\}$



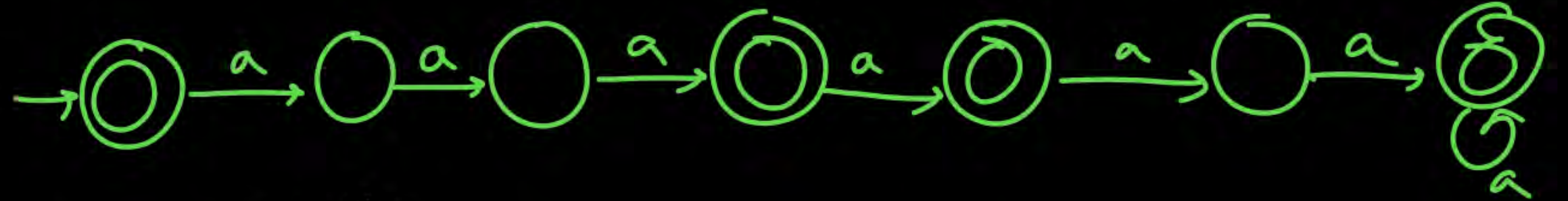
(22) $L = (aaa)^* = \{\epsilon, a^3, a^6, a^9, \dots\}$



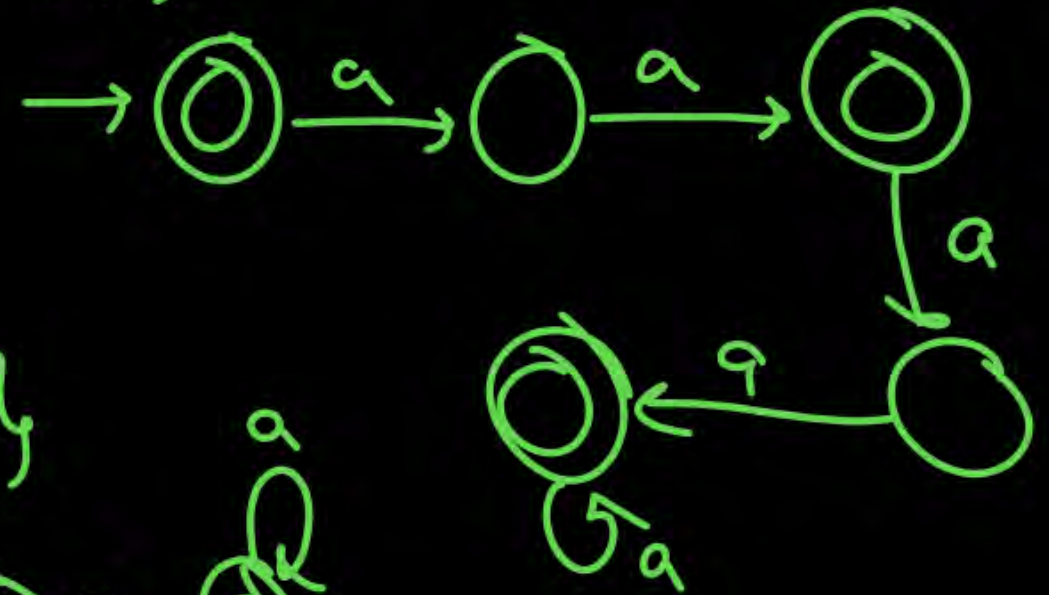
(23) $L = (aa+aaa)^*$
 $= \{\epsilon, a^2, a^3, a^4, a^5, \dots\}$
 $= a^* - \{a\}$



(24) $L = (aaa + aaaa)^* = (a^3 + a^4)^* = \{\epsilon, a^3, a^4, \boxed{a^4}, a^7, a^8, a^9, \dots\}$



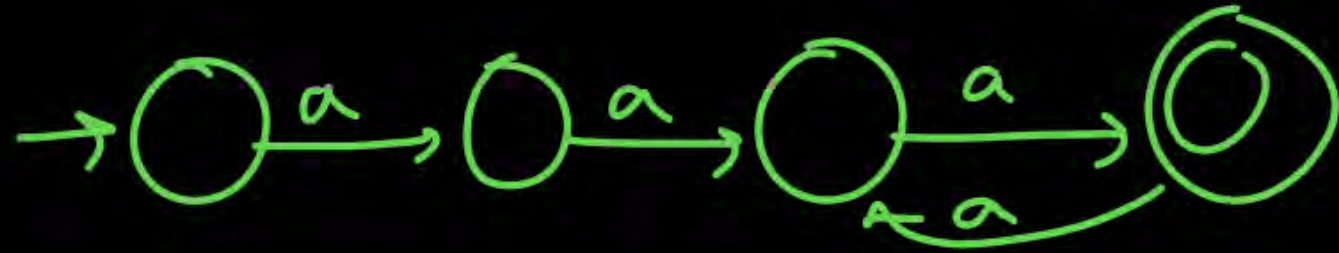
(25) $L = (aa + a^5)^*$
 $= \{\epsilon, a^2, \boxed{a^4}, a^5, a^6, a^7, a^8, a^9, \dots\}$



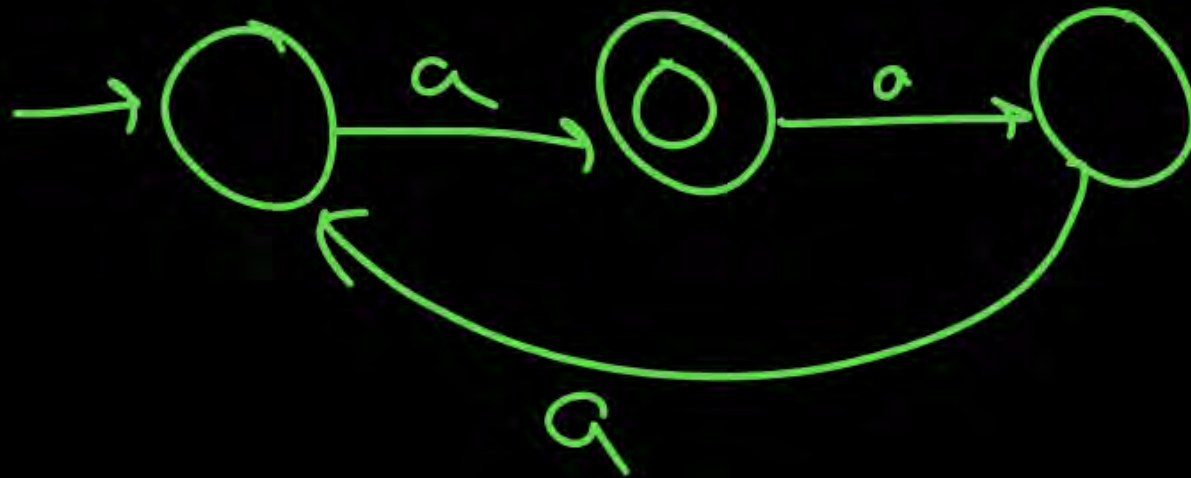
(26) $L = (a^3 + a^5)^*$
 $= \{\epsilon, a^3, a^5, a^6, \boxed{a^8}, a^9, a^{10}, \dots\}$



$$(27) \quad L = \{ a^{2n+3} \} = \{ \underline{a}^3, a^5, a^7, a^9, \dots \}$$



$$(28) \quad L = \{ a^{3n+1} \} = \{ \underline{a}^1, a^4, a^7, a^{10}, \dots \}$$



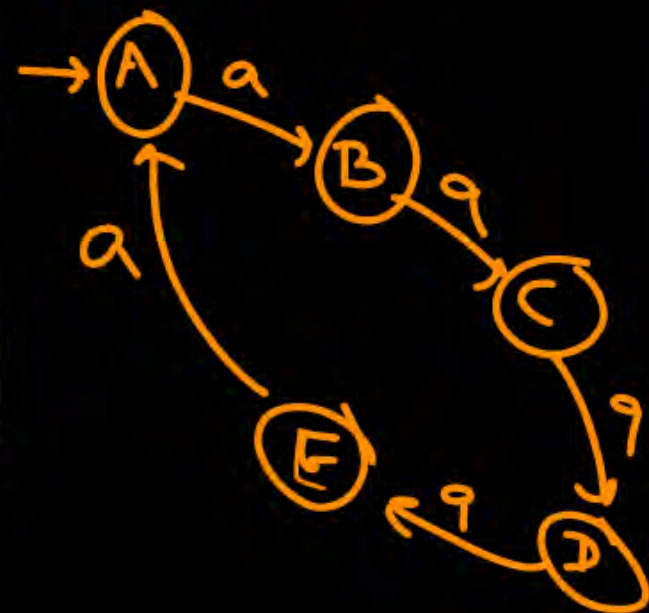
(29) $\{ a^{Kn+C} \}_{n \geq 0}$ \Rightarrow Case I ($K > C$) : K states
 Case II ($K \leq C$) : $C+1$ states
 K, C are constants

Case I: $K > C$

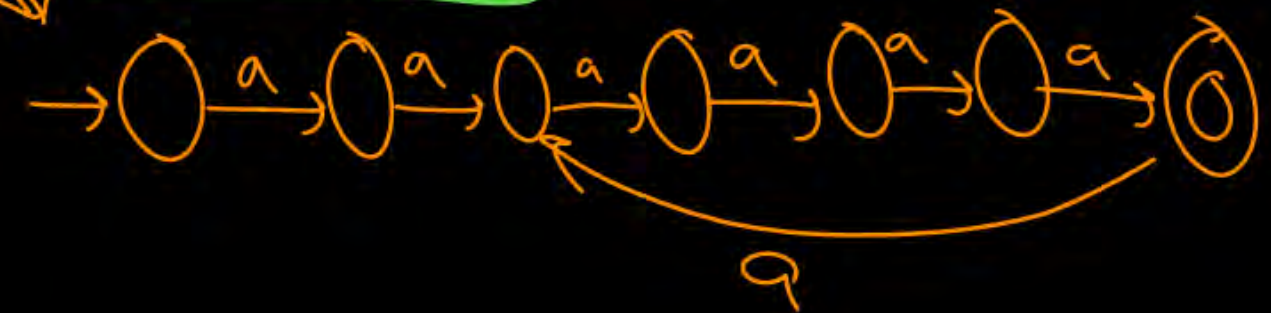
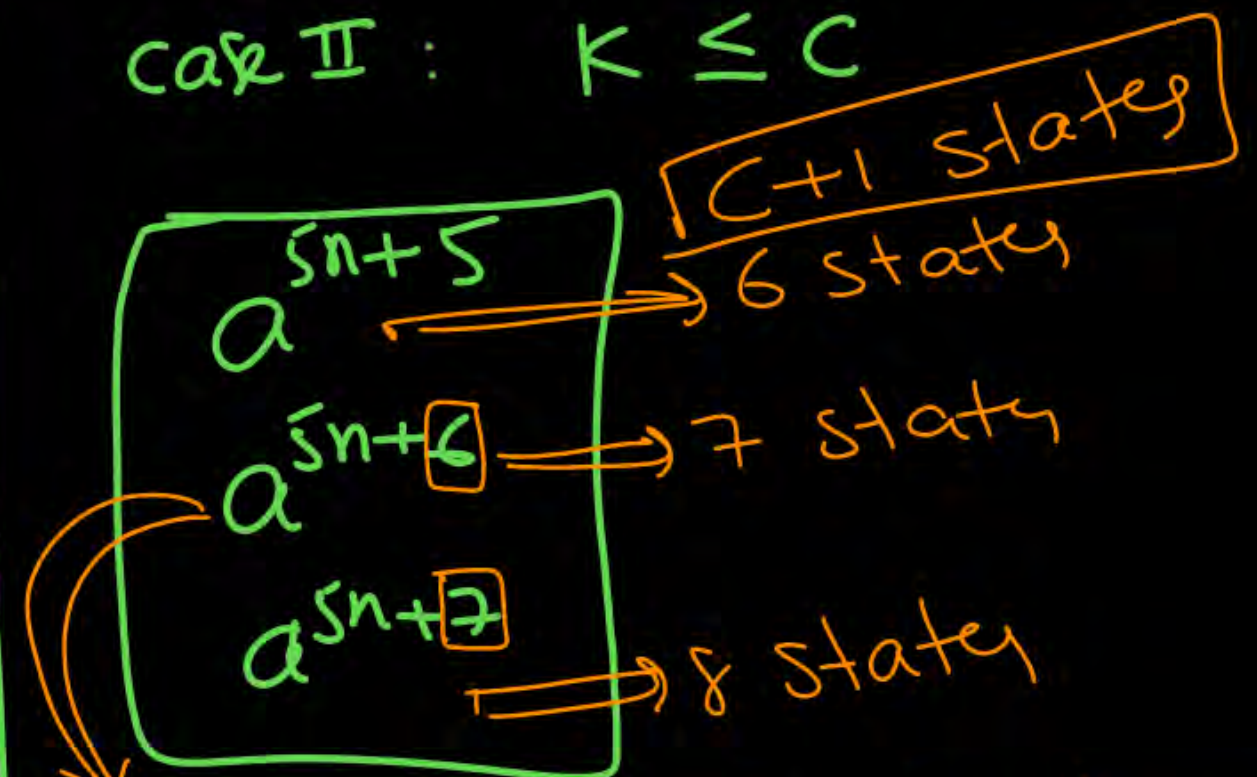
C represents remainder

$5n+0$ a A is final	$5n+3$ a D is final
$5n+1$ a B is final	$5n+4$ a E is final
$5n+2$ a C is final	

5 states
(K states)



Case II: $K \leq C$



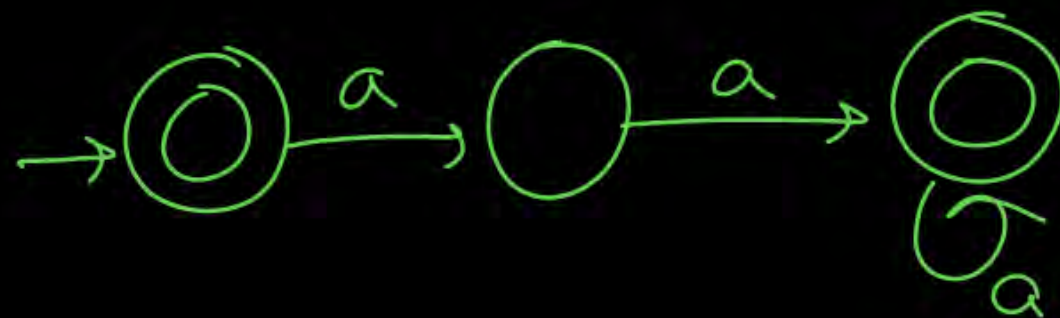
$$(30) \{a^{\text{prime}}\}^* \Rightarrow \{a^{\text{prime}}\}^0 \cup \{a^{\text{prime}}\}^1 \cup \{a^{\text{prime}}\}^2 \cup \dots$$

$$\{a^2, a^3, a^5, a^7, a^{11}, \dots\}^* = \{\epsilon, a^2, a^3, a^4, a^5, a^6, \dots\}$$

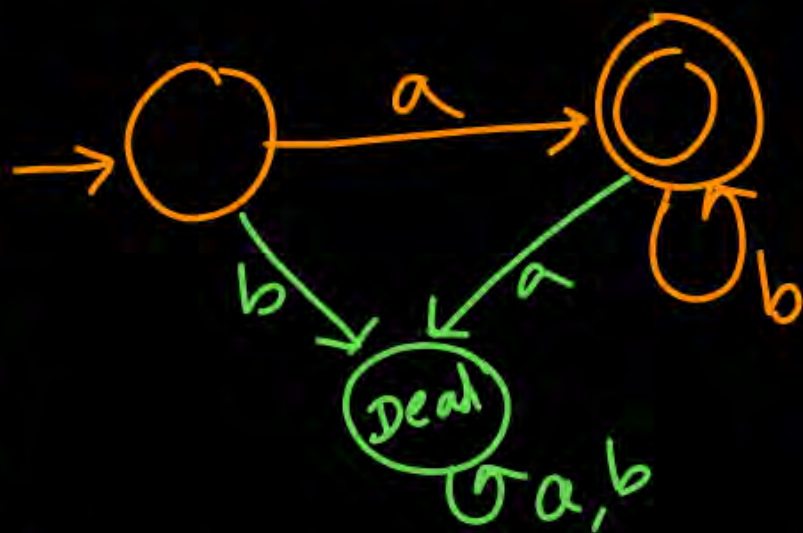
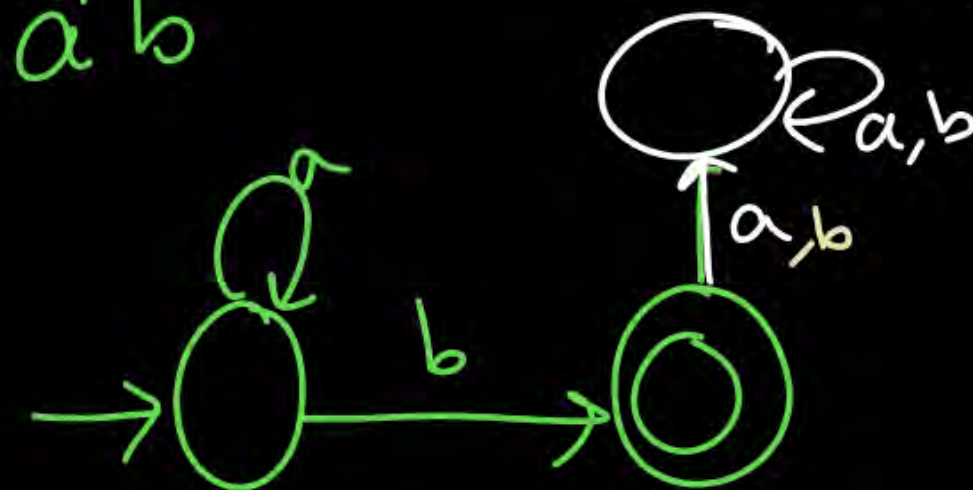
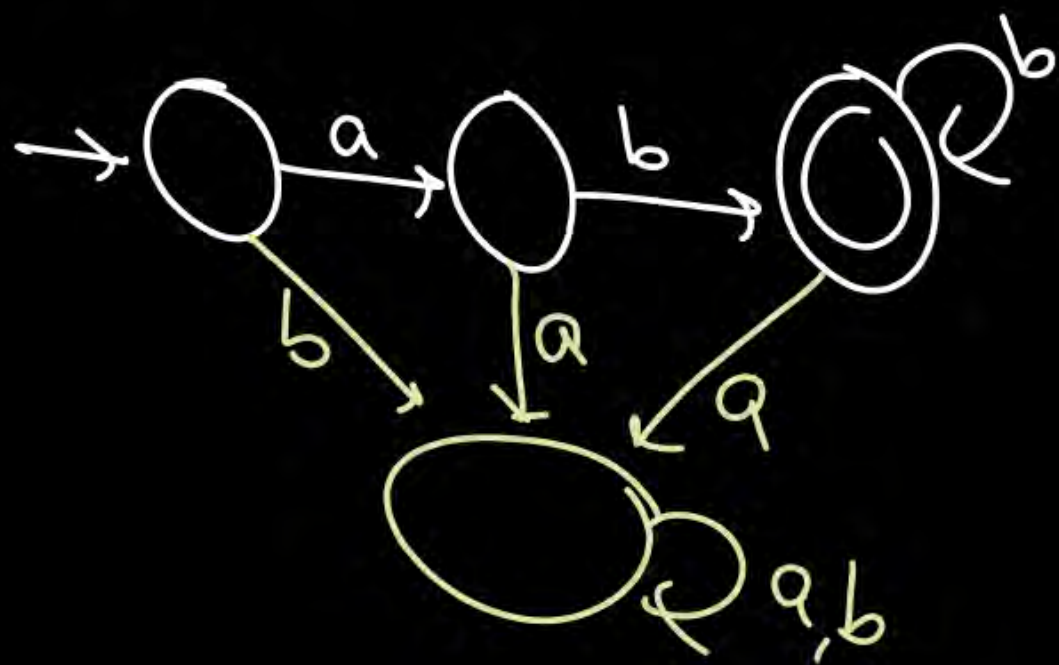
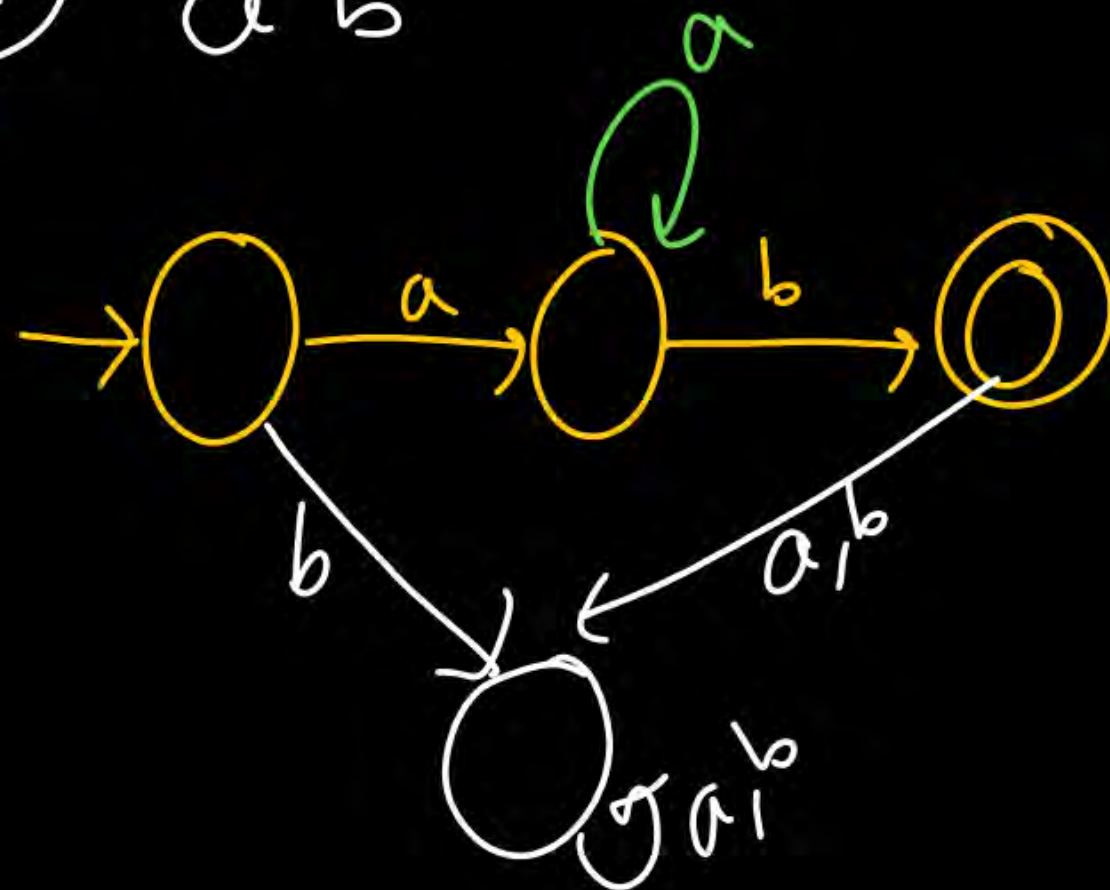
$$(a^2 + a^3 + a^5 + a^7 + a^{11} + \dots)^*$$

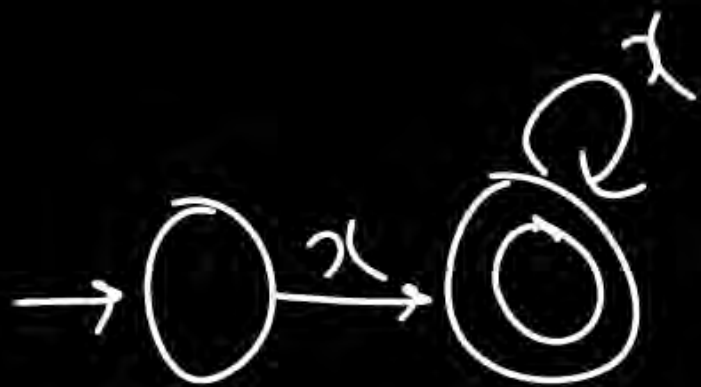


except "a", all strings possible



Model-VI [Sequence Based]

(31) ab^* (32) a^*b (33) ab^+ $ab b^*$ min = ab (34) a^+b $a a^* b$ 



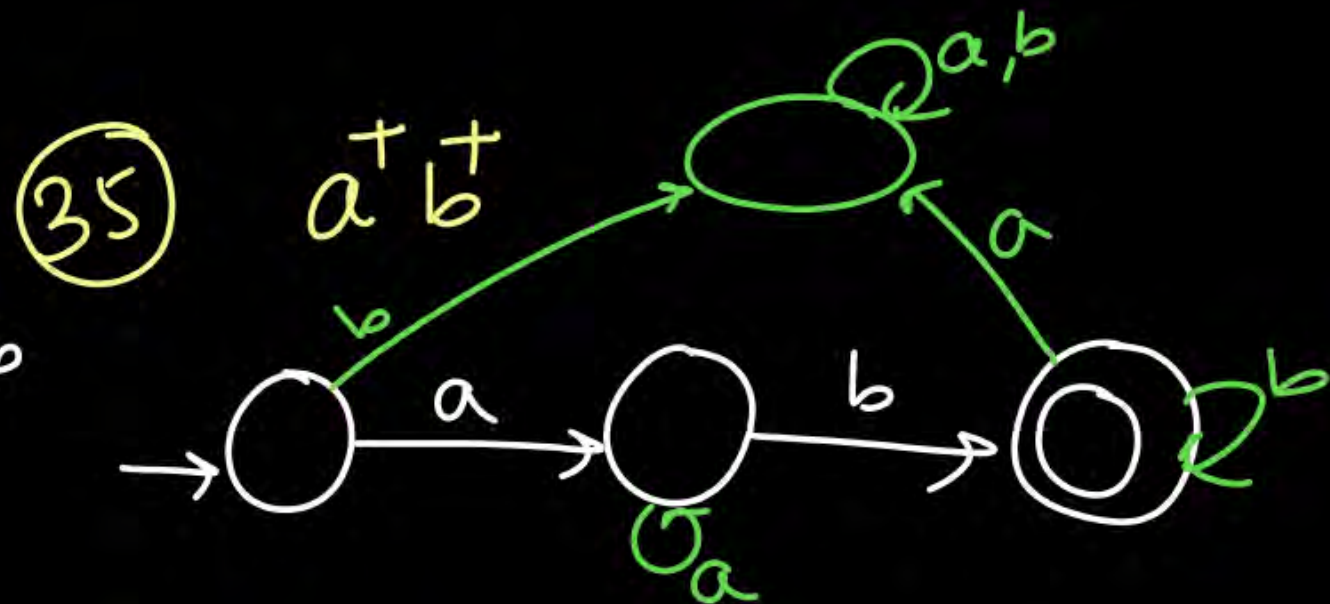
$$x^+ = x \boxed{x^*}$$



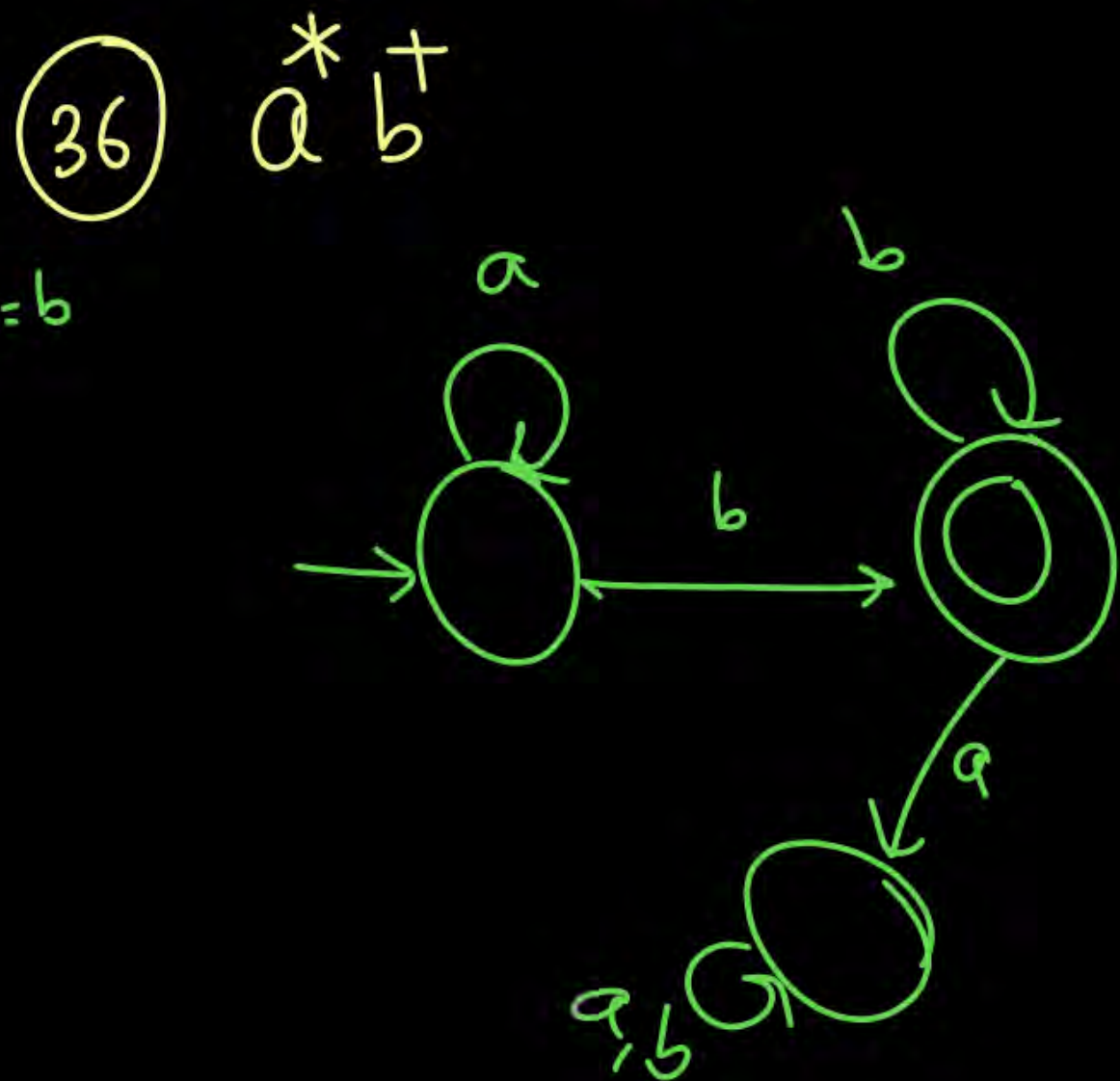
$$\boxed{x^*}$$



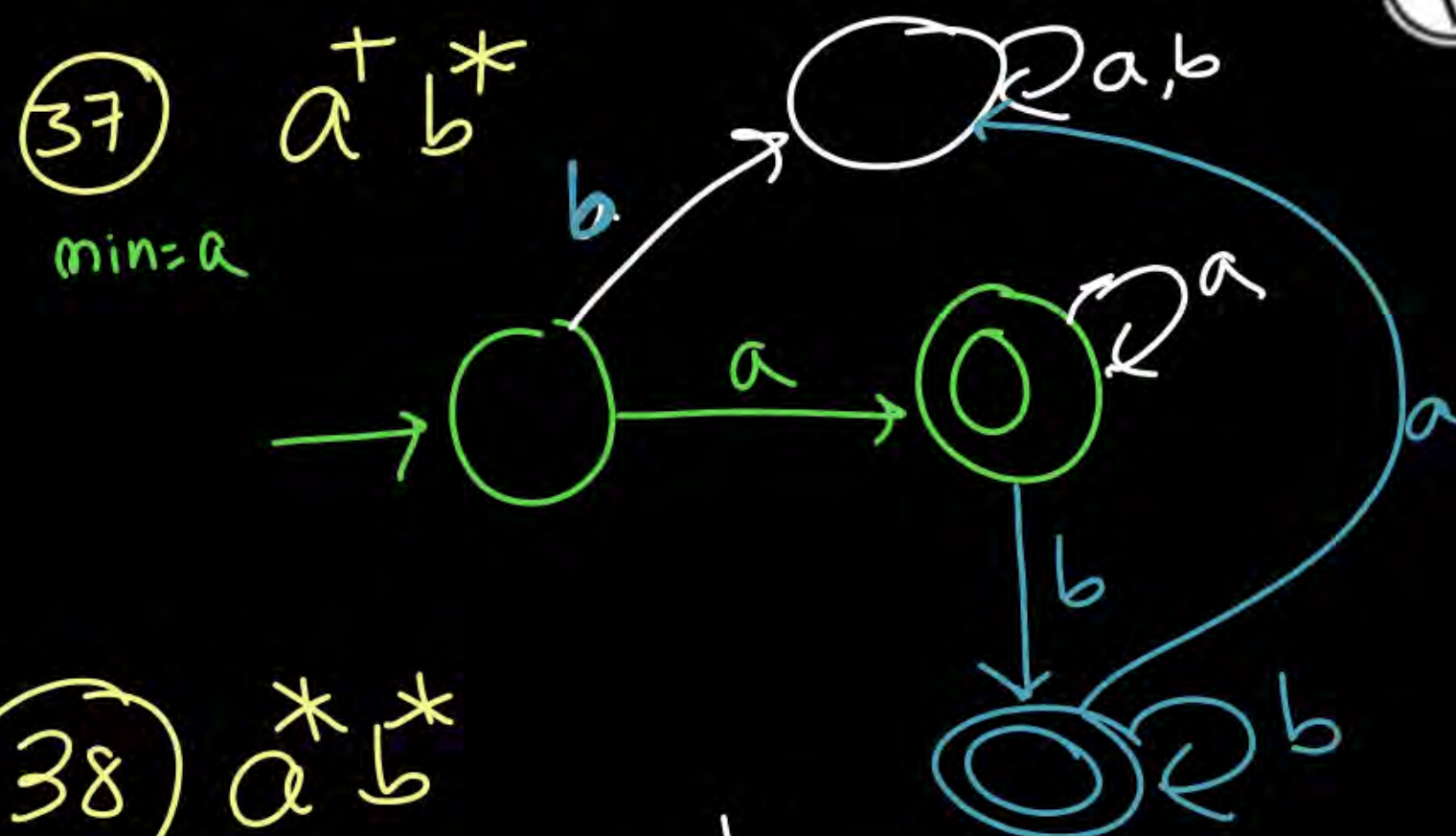
min=ab



min=b



min=a

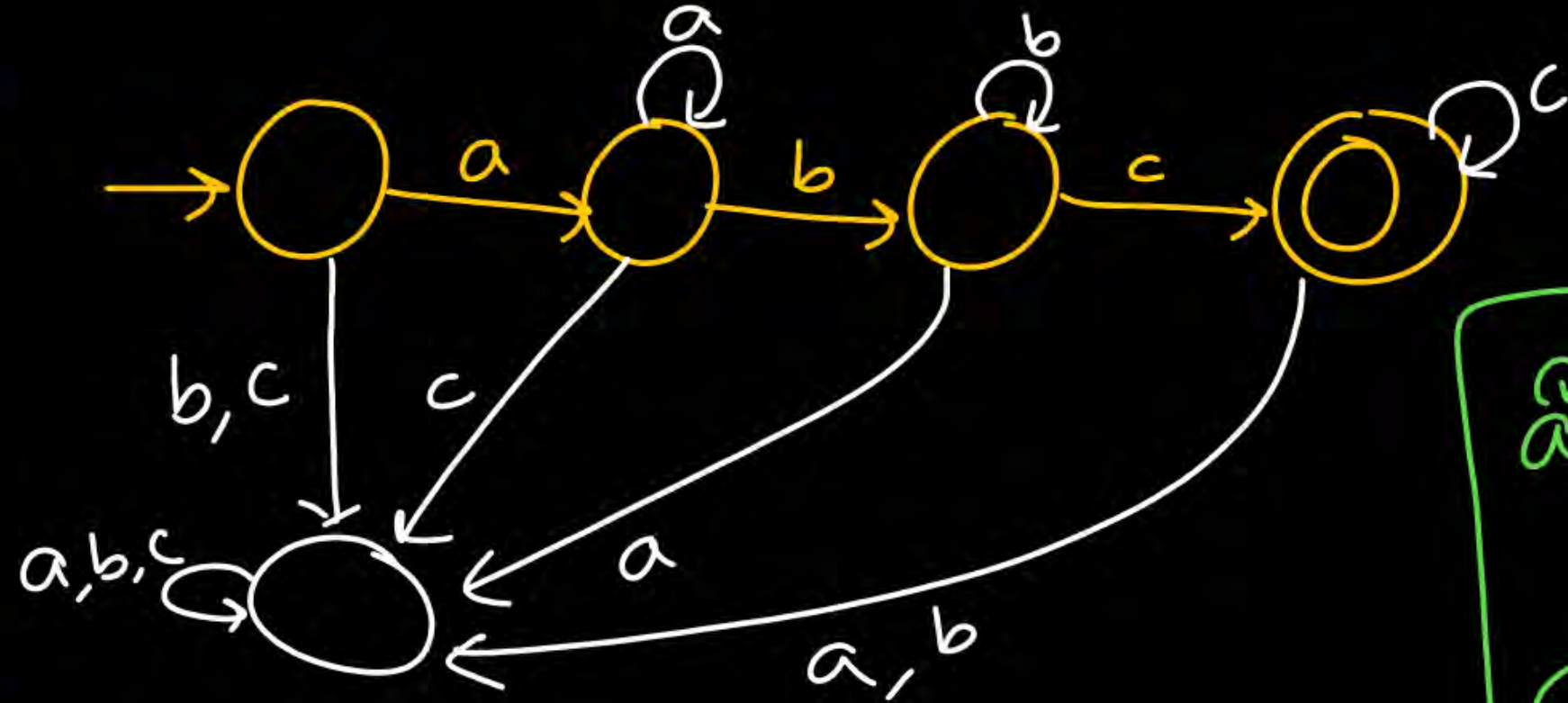
(38) a^*b^*

```

graph LR
    Start((( ))) -- b --> Accept((( )))
    Accept -- a --> Start
    Accept -- b --> qb((q_b))
    qb -- a --> qb
    qb -- b --> Accept
  
```

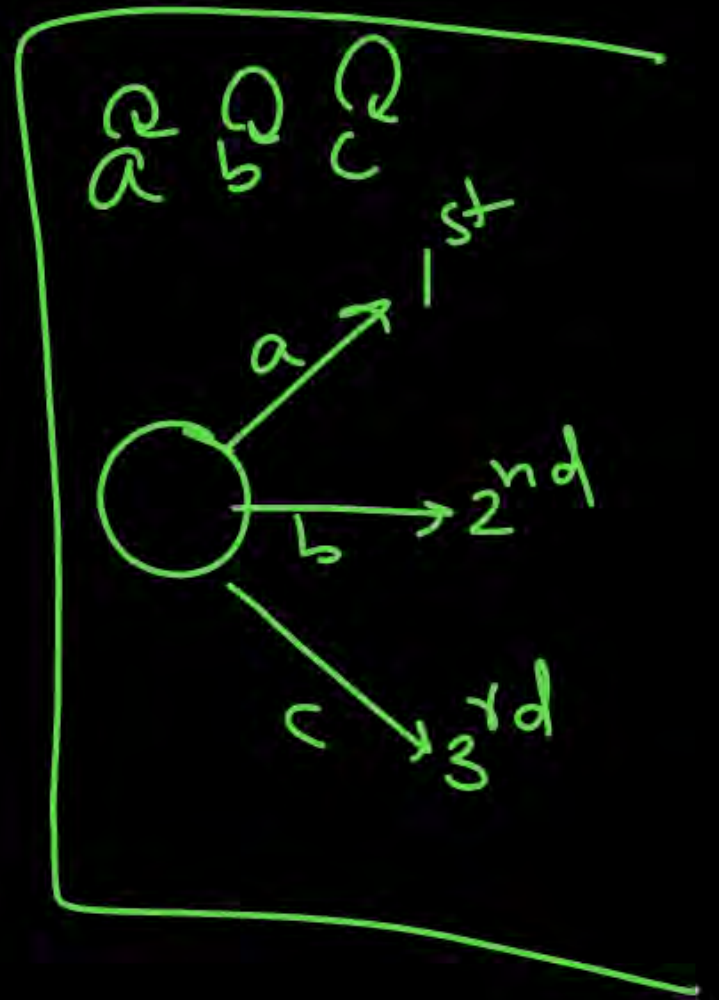
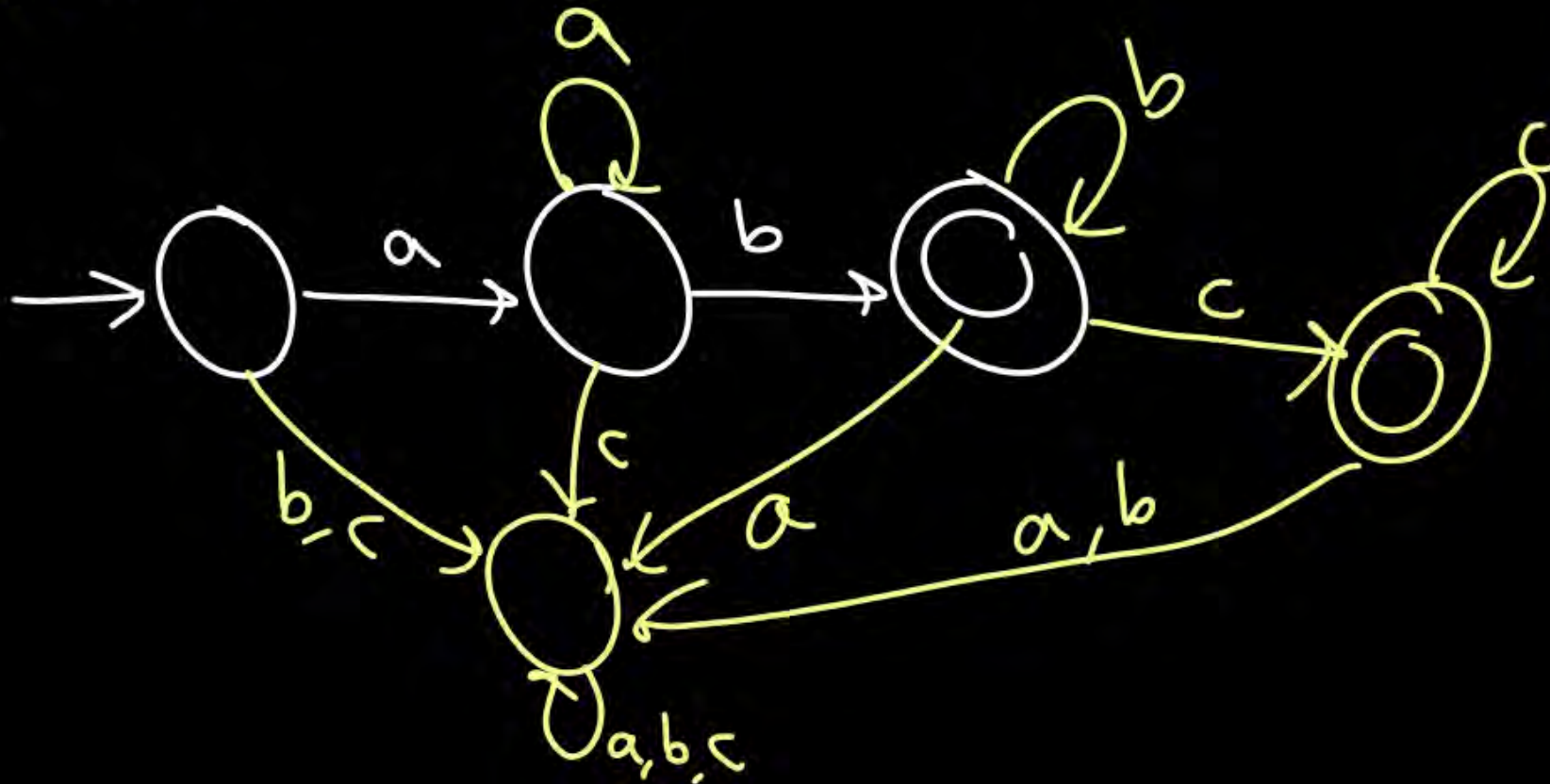

(39) $a^+b^+c^+$

min = abc



(40) $a^+b^+c^*$

min = ab



(41) $a^+ b^* c^+$ (42) $a^* b^+ c^+$ (43) $a^* b^* c^+$ (44) $a^* b^+ c^*$ (45) $a^+ b^* c^*$ (46) $a^* b^* c^*$ (47) $b^+ a^+$ (48) $b^+ c^* a^+$ (49) $a^* c^* b^+$ (50) $b^* a^*$

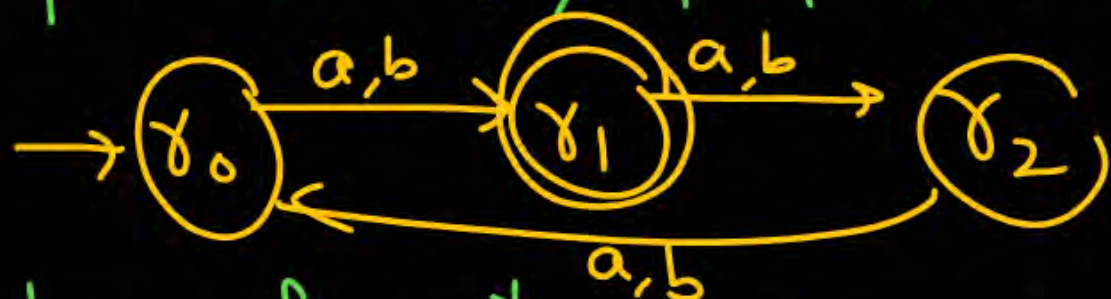
Model-VII [Divisible/modulo/Remainder]

(51) $\{w \mid w \in \{a,b\}^*, |w| \text{ is divisible by } 3\}$

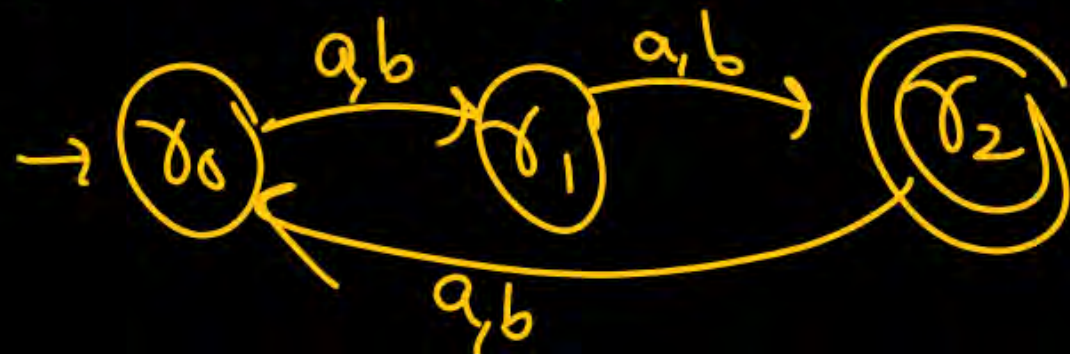
0 mod 3
1 mod 3
2 mod 3



(52) $\{w \mid w \in \{a,b\}^*, |w| \% 3 = 1\}$



(53) $\{w \mid w \in \{a,b\}^*, |w| \% 3 = 2\}$



Note:

$|w|$ is divisible by n

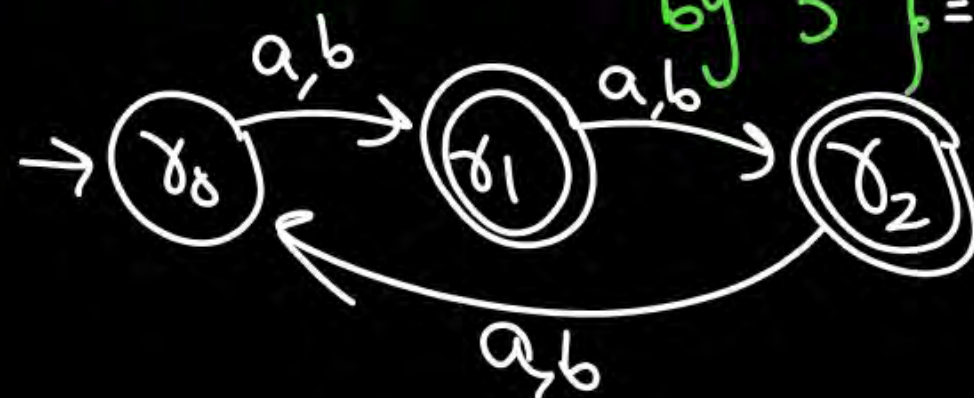


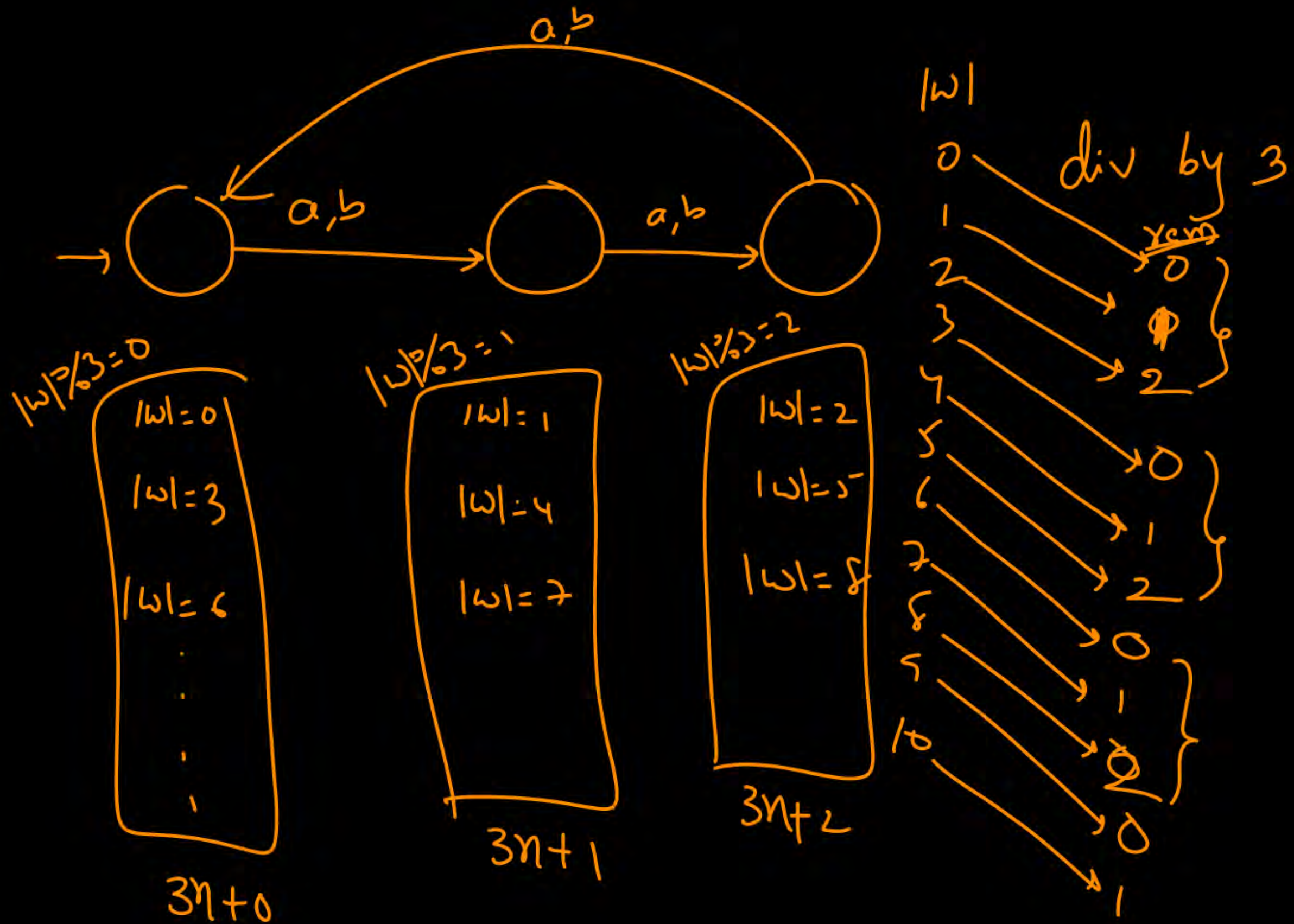
n states

rem =

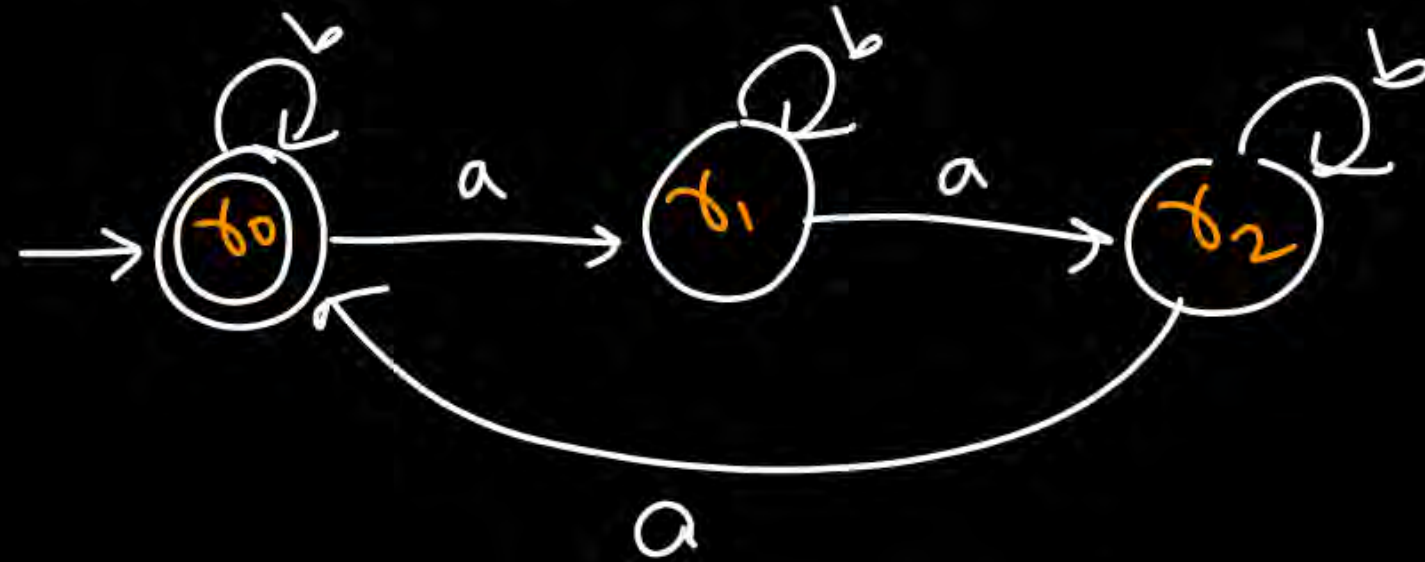


(54) $\{w \mid w \in \{a,b\}^*, |w| \text{ is not div by } 3\} = \overline{(51)}$





(55) $\{w \mid w \in \{a,b\}^*, n_a(w) \text{ is divisible by } 3\}$



q_1 is final

(56)

$\{w \mid w \in \{a,b\}^*, n_a(w) \% 3 = 1\}$

q_2 is final

(57)

$\{w \mid \text{ " }, n_a(w) \% 3 = 2\}$

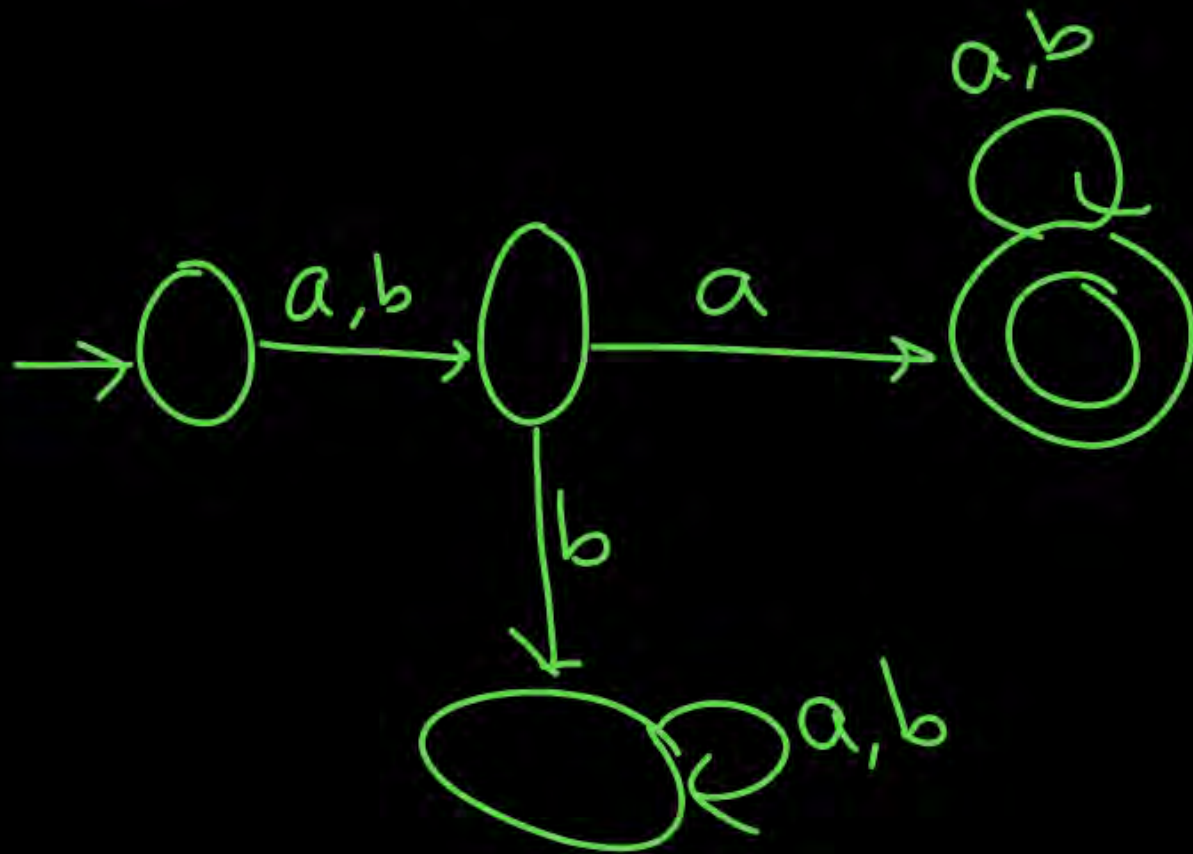
q_1 and q_2 are final

(58)

$\{w \mid \text{ " }, n_a(w) \% 3 \neq 0\}$

Model - VIII [Position Based]

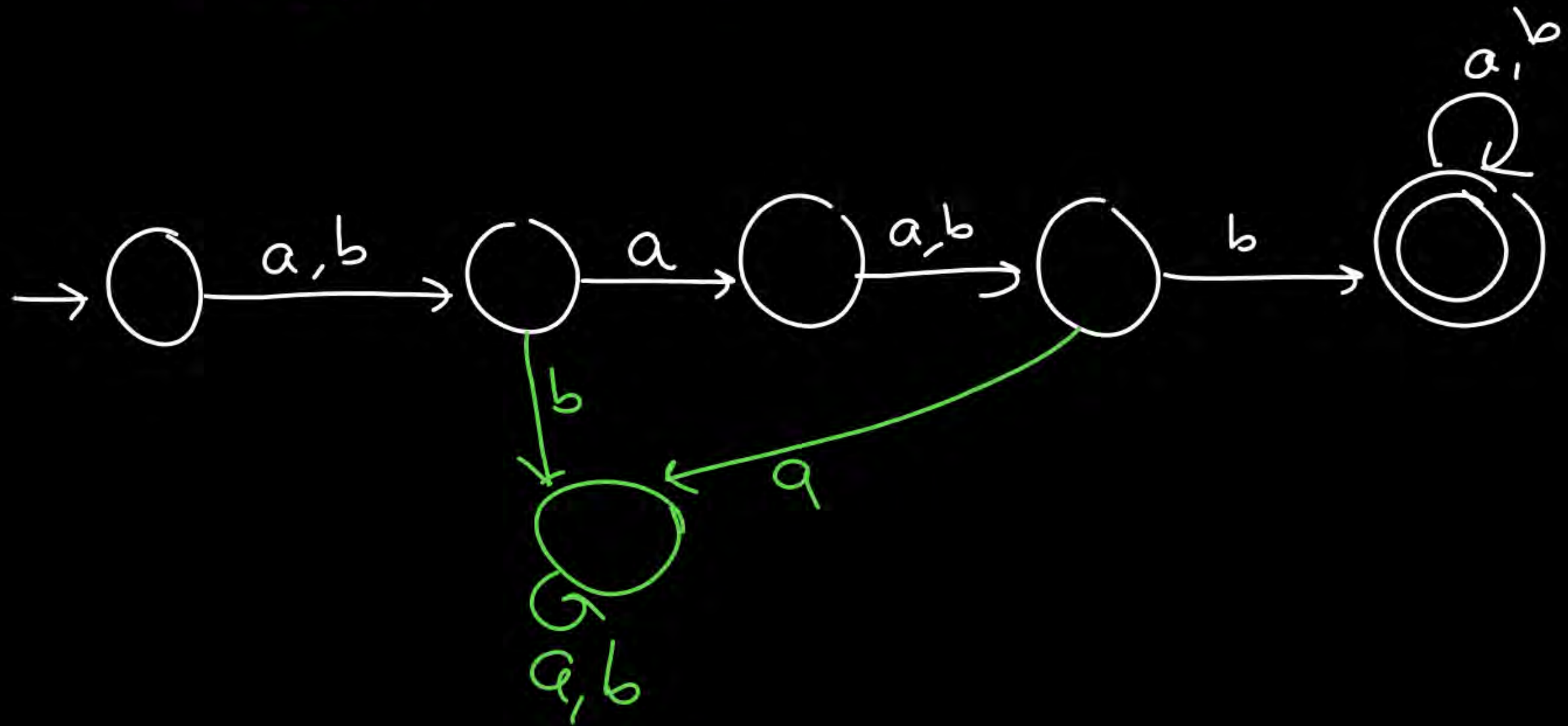
(59) $L = \{w \mid w \in \{a,b\}^*, \text{ 2nd symbol of } w \text{ is 'a'}\}$
 2nd symbol from LHS is 'a'
 begin



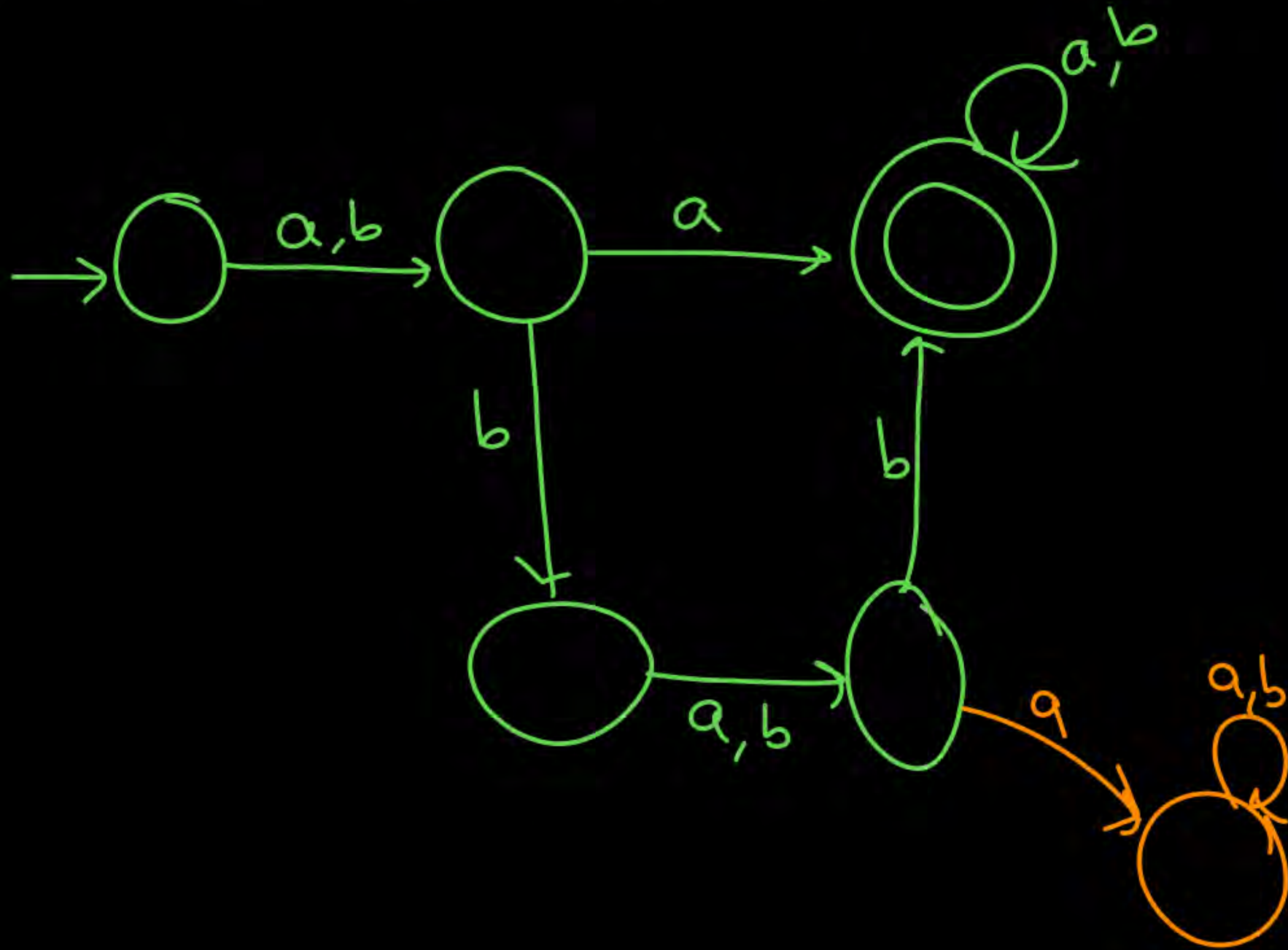
$$\boxed{\frac{(a+b)}{1^{st}} \quad \frac{a}{2^{nd}} \quad (a+b)^*}$$

$$(aa+ba)(a+b)^*$$

(60) $\{w/w \in \{a,b\}^*, 2^{\text{nd}} \text{ symbol is 'a' and } 4^{\text{th}} \text{ symbol is 'b'}\}$



61) $\{w \mid w \in \{a, b\}^*, \text{ 2nd symbol is 'a' } \boxed{\text{or}} \text{ 4th symbol is 'b'}\}$



If 2nd symbol is 'a'
 \Downarrow
 Don't look for 4th

If 2nd symbol is not 'a'
 \Downarrow
 look for 4th symbol

Note : I) If n^{th} symbol from begin is 'a' then
 $\Rightarrow (n+2)$ states in min DFA

$\Sigma = \{a, b\}$

*** II) If n^{th} symbol from end is 'a' then

$\Rightarrow 2^n$ states in min DFA

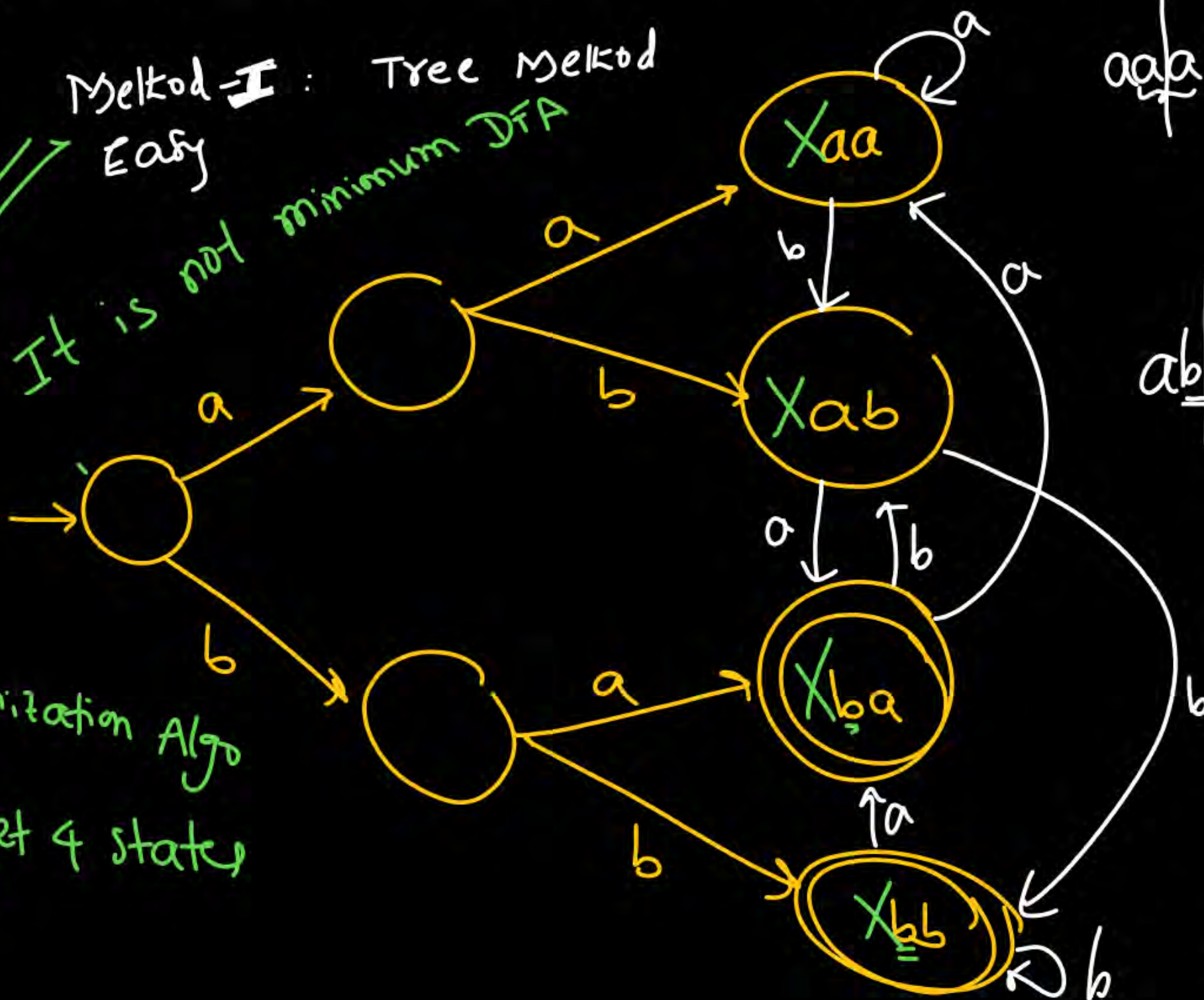
62) $\{w \mid w \in \{a,b\}^*, \text{ 2nd symbol of } w \text{ from end is 'b'}\}$

2 = 4 states

Method-I: Tree Method
Easy

It is not minimum DFA

Apply minimization Algo
↓
We will get 4 states



aba

abb

$(a+b)^*$

b
2nd last

(a+b)
last

aba

abb

baba

baab

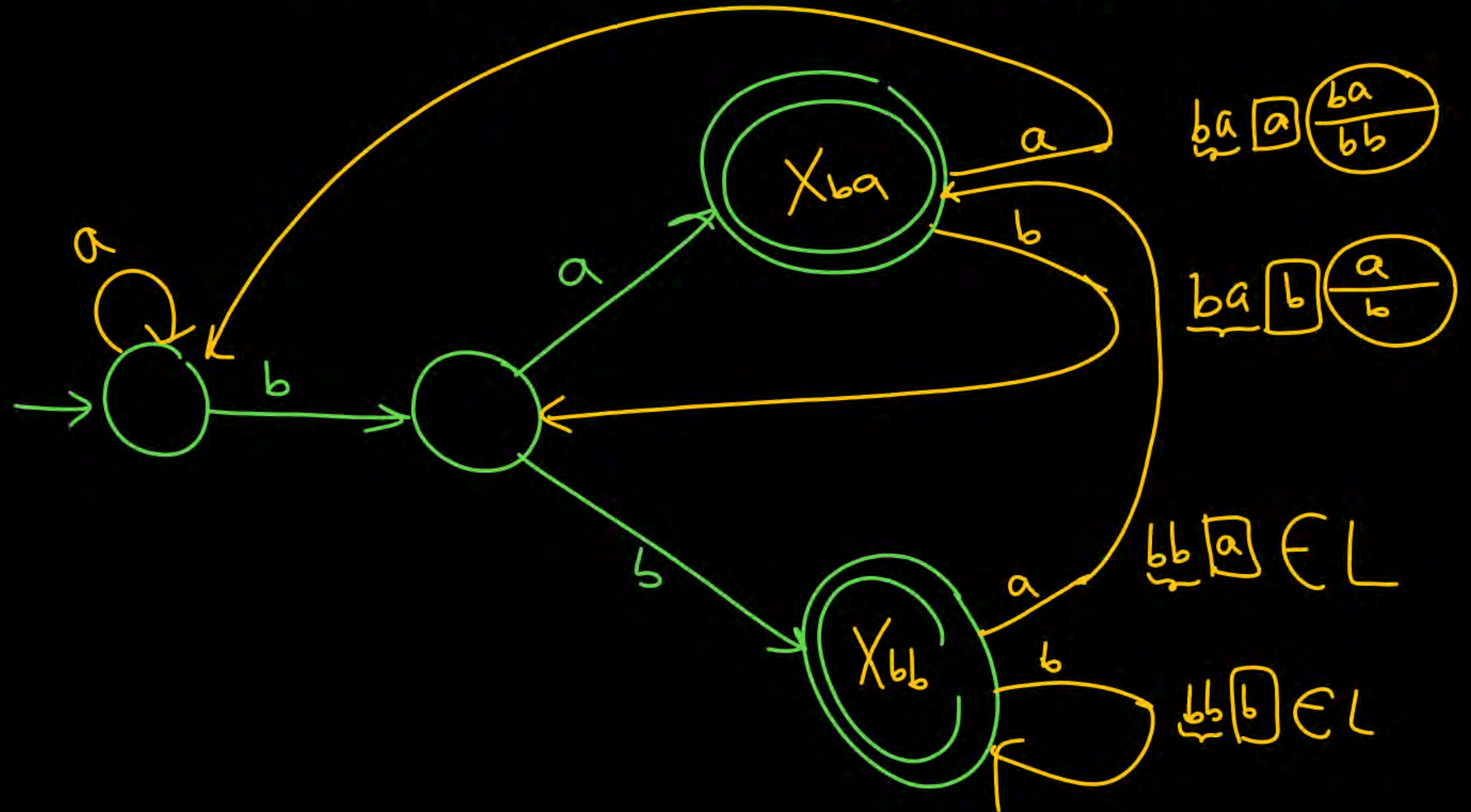
last 2 symbols

bbba

bbbb

Method 2: $(a+b)^* b (a+b)$
 $(a+b)^* (ba+bb)$

min = 'ba' or 'bb'



(63) $\{w \mid w \in \{a, b\}^*, 3^{\text{rd}} \text{ symbol from end is 'a'}\}$

$\hookrightarrow 2^3 = 8$ states in DFA

(64) 10^{th} symbol from end is 'a' over $\Sigma = \{a, b\}$

$\hookrightarrow 2^{10} = 1024$ states in DFA

Model-IX [compound / composition based]



$2 \times 3 \Rightarrow 6$ states

$\{w \mid w \in \{a,b\}^*, n_a(w) \text{ is divisible by } 2,$

$n_b(w) \text{ is divisible by } 3\}$

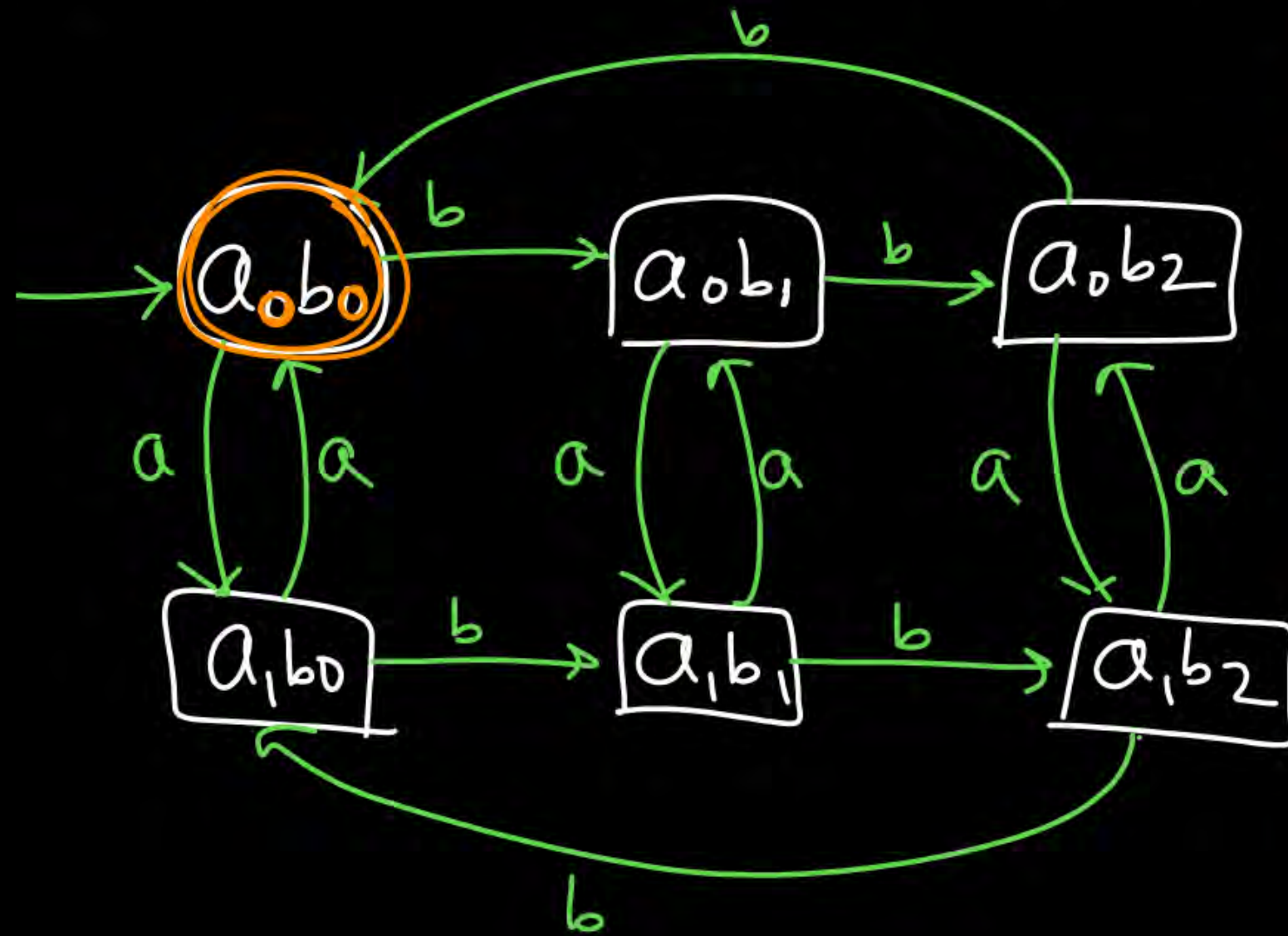


$2 \times 2 \Rightarrow 4$ states

$\{w \mid w \in \{a,b\}^*, n_a(w) \text{ is divisible by } 2,$

$n_b(w) \text{ is divisible by } 2\}$

65 $\#a's/2$ and $\#b's/3$



67 $\#a's/2$ but not $\#b's/3$

$\hookrightarrow \left. \begin{matrix} a_0b_1 \\ a_0b_2 \end{matrix} \right\} \text{finals}$

68 $\#b's/3$ but not $\#a's/2$

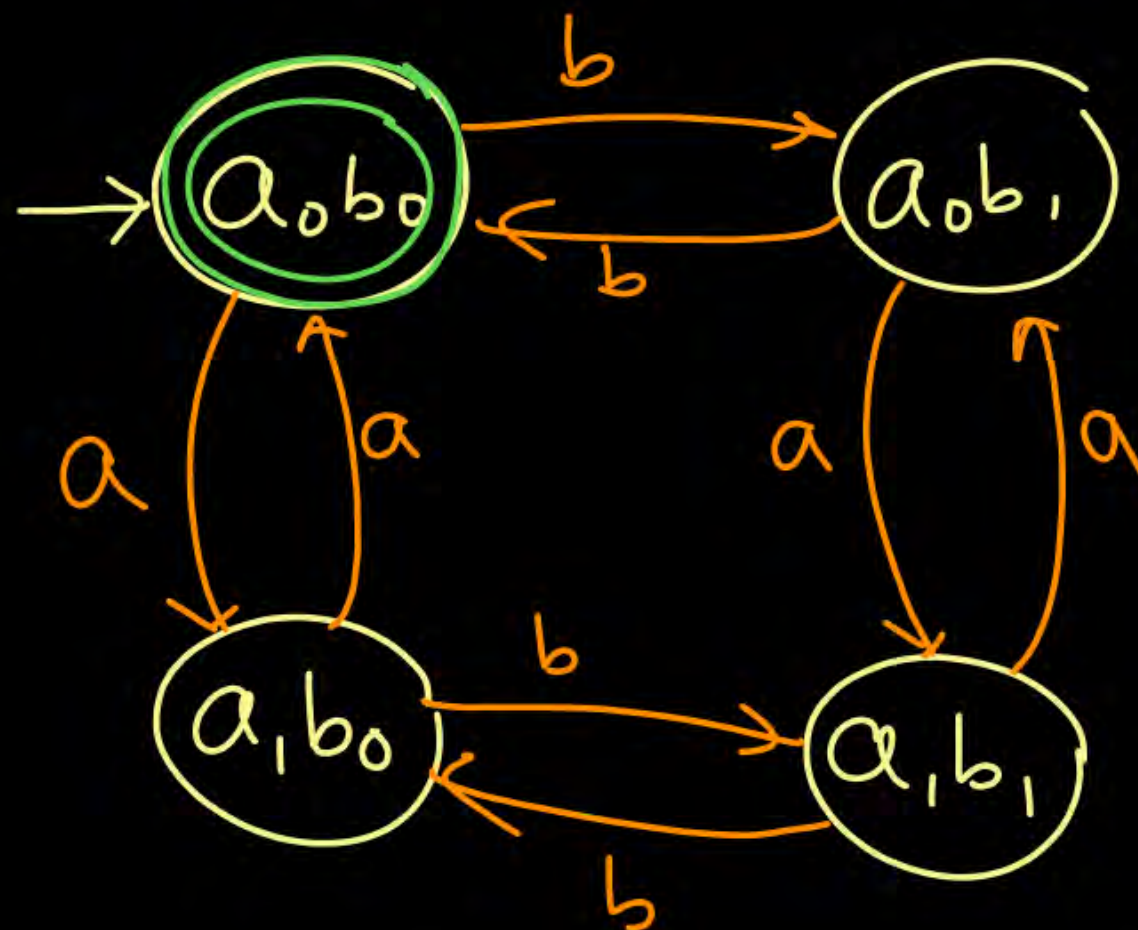
$\hookrightarrow a_1b_0$ is final

69 $\left. \begin{matrix} \#a's \% 2 = 1 \\ \#b's \% 3 = 2 \end{matrix} \right\}$

a_1b_2
final

(66)

 $\#a's/2$
 $\#a's = \text{even}$

 and $\#b's/2$
 $\#b's = \text{even}$
 a_0
 a_1
 b_0
 b_1
 $a_0 = a_2 = a_4$
 $a_1 = a_3 = a_5$


(70)

 $\#a's = \text{odd}, \#b's = \text{odd}$
 $\hookrightarrow a_1b_1$ is final

(71)

 $\#a's = \text{even}, \#b's = \text{odd}$
 $\hookrightarrow a_0b_1$ is final

(72)

 $\#a's = \text{odd}, \#b's = \text{even}$
 $\hookrightarrow a_1b_0$ is final

Model - Σ [Many things but simple thing]

(73) $\{w \mid w \in \{a,b\}^*, \boxed{|w| \text{ is div by 2}} \text{ OR } \boxed{|w| \text{ is div by 4}}\}$

(74) $\{w \mid \text{"}, \boxed{|w| \text{ is div by 2}} \text{ AND } \boxed{|w| \text{ is div by 4}}\}$

(75) $\{w \mid \text{"}, w \text{ starts with 'a' OR starts with 'b'}\}$

(76) $\{w \mid \text{"}, w \text{ ends with 'a' OR ends with 'b'}\}$

(77) $\{w \mid \text{"}, \boxed{w \text{ starts with 'a'}} \text{ AND } \boxed{w \text{ starts with 'b'}}\}$

(78) $\{w \mid \text{"}, w \text{ ends with 'a' AND } w \text{ ends with 'b'}\}$

1 state
 $\hookrightarrow L = \emptyset$

Model-XI [multiple conditions \rightarrow sequence]H.W.

(79) $\{w \mid w \in \{a, b\}^*, w \text{ starts with 'a', } w \text{ contains 'b'}\}$

(80) $\{w \mid \text{"}, w \text{ starts with 'a', } w \text{ ends with 'b'}\}$

(81) $\{w \mid \text{"}, w \text{ ends with 'a', } w \text{ contains 'b'}\}$

(82) $\{w \mid \text{"}, w \text{ starts and ends with different symbols}\}$

(83) $\{w \mid \text{"}, w \text{ starts and ends with same i/p symbol}\}$

(84) $(a+b)^*(aa+bb)$

(85) $(aa+bb)(a+b)^*$

(86) $(a+b)^*(aa+bb)(a+b)^*$

Summary

Sequence based

position based

Compound based

Multiple conditions 

```
graph LR; A[Multiple conditions] --- B[Simple]; A --- C[Sequence];
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Thank you

