CS & IT Engineering

Finite Automata:

Closure Properties - 2

Lecture No:12

Deva sir

Previous Class Summary:

↳ Closure properties of finite, Infinite regular languages

Topics to be covered Today:

→ closure properties of regulars

Closure Properties for regular languages :

B ① Union

B ② Intersection

U ③ Complement

B ④ Difference

B ⑤ concatenation

U ⑥ Reversal

U ⑦ Kleene star

U ⑧ Kleene plus

U ⑨ Subset

B ⑩ Symmetric Difference

U ⑪ Substitution

U ⑫ Homomorphism

U ⑬ ϵ-free Homomorphism

U ⑭ Inverse Homomorphism

U ⑮ prefix(L)

U ⑯ Suffix

U ⑰ Substring

B ⑱ Quotient

U ⑲ $Half(L) = \frac{1}{2}(L)$

U ⑳ Second Half(L)

U ㉑ one-third(L)

U ㉒ Middle $\frac{1}{3}(L)$

U ㉓ Last $\frac{1}{3}(L)$

㉔ Finite Union

㉕ Finite Intersection

㉖ Finite Difference

㉗ Finite Concatenation

㉘ Finite Subset

㉙ Finite Substitution

㉚ Infinite Union

㉛ Infinite Intersection

㉜ Infinite Difference

㉝ Infinite Concatenation

㉞ Infinite Subset

㉟ Infinite Substitution

Remember "Not closed operations" for regulars

Subset, $\underbrace{\text{Inf}}_{6}$ ....

Closure Properties

*** (9) Subset

↳ not closed for regulars

↳ closed for finite languages

Subset (Regular) $\Rightarrow$ May or may not regular

$a^* b^*$ (regular)

ϕ . . . $a^n b^n$

reg nonreg

finite set

$\{a^n b^n \mid n < 10\}$

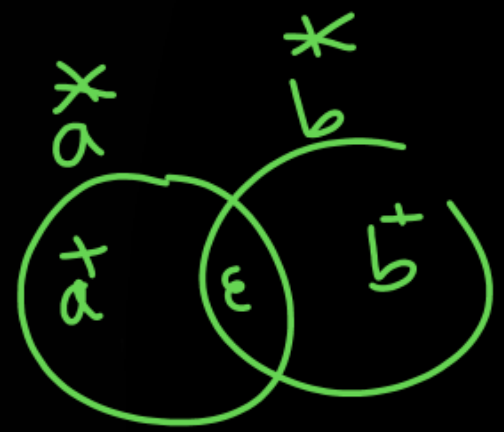
ϕ $\{\varepsilon\}$ . . . $\{a^n b^n \mid n < 10\}$

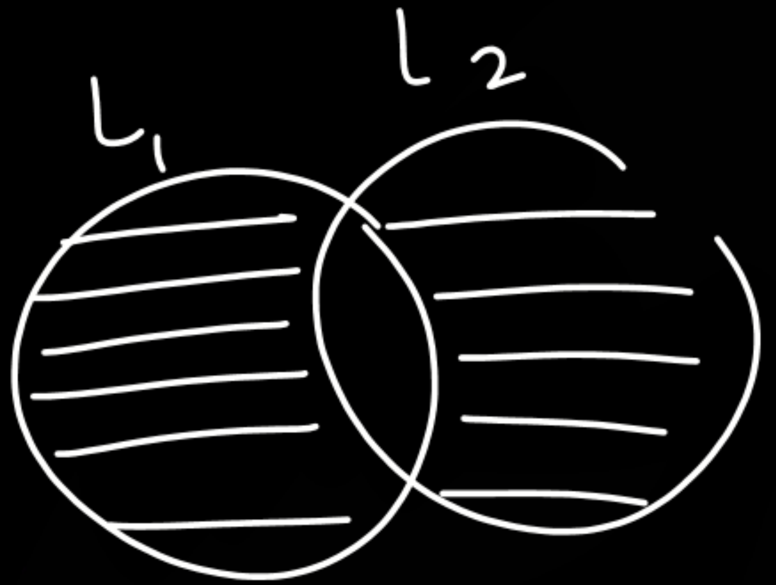
fin fin fin

10 Symmetric Difference

$$L_1 \triangle L_2 = (L_1 - L_2) \cup (L_2 - L_1)$$

$$= (L_1 \cup L_2) - (L_1 \cap L_2)$$





→ closed for regulars

$$\left.\begin{array}{l} L_1 = a^* \\ L_2 = b^* \end{array}\right\} \Rightarrow L_1 \triangle L_2 = a^+ + b^+$$

(11) $f(L)$ Substitution(L)

$\hookrightarrow$ closed for regulars

$\Sigma \rightarrow$ set of symbol

$\Sigma^* \rightarrow$ set of strings

$2^{\Sigma^*} \rightarrow$ set of languages

$L = \underbrace{a b^* a^+}_{\text{Regular}} \longrightarrow \boxed{f : \Sigma \rightarrow 2^{\Delta^*}} \longrightarrow \underbrace{f(L)}_{\text{regular}} = f(a) . [f(b)]^* [f(a)]^+$

$\text{regular} = 01^* [(101)^*]^* [01^*]^*$

$f(a) = \underbrace{01^*}_{\text{Reg}}$ $\qquad$ $f(b) = \underbrace{(101)^*}_{\text{Reg}}$

$\Sigma = \{a, b\}$

$\Delta = \{0, 1\}$

$f(L)$: $\boxed{\text{Every symbol}}$ of L over Σ is substituted with $\boxed{\text{Some regular language}}$ over Δ

$$f : \underbrace{\Sigma}_{\text{Set of symbols}} \longrightarrow \underbrace{P(\Delta^*)}_{\text{Set of languages}}$$

(12) Homomorphism $h(L)$:

$L = aba^* \longrightarrow \boxed{h : \Sigma \to \Delta^*} \longrightarrow h(L) = h(a).\,h(b)\,[h(a)]^*$

$= 011\,\varepsilon\,(011)^*$

$= (011)^+$

$h(a) = \underbrace{011}_{\text{String}}$ $\qquad h(b) = \underbrace{\varepsilon}_{\text{String}}$

(13) ε-free homomorphism

$$\varepsilon\text{-free } h : \Sigma \longrightarrow \Delta^{+}$$

$$L = ab^* \longrightarrow \boxed{\varepsilon \text{ free } h} \longrightarrow \varepsilon \text{ free } h(L) = (01).(011)^*$$

$$h(a) = \underbrace{01}_{\text{non empty}} \qquad h(b) = \underbrace{011}_{\text{non empty string}}$$

(14) $h^{-1}(L)$:

$L = \{0111, 1111\}$ $\longrightarrow$ $\boxed{h^{-1}}$ $\longrightarrow$ $h^{-1}(L) = \{abbb, acb, abc, bbbb, bcb, cc, cbb, bbc\}$

$h(a) = 0 \quad h(b) = 1 \quad h(c) = 11$

$$\begin{cases} h^{-1}(0) = a \\ h^{-1}(1) = b \\ h^{-1}(11) = c \end{cases}$$

0111
a b b b
a c b
a b c

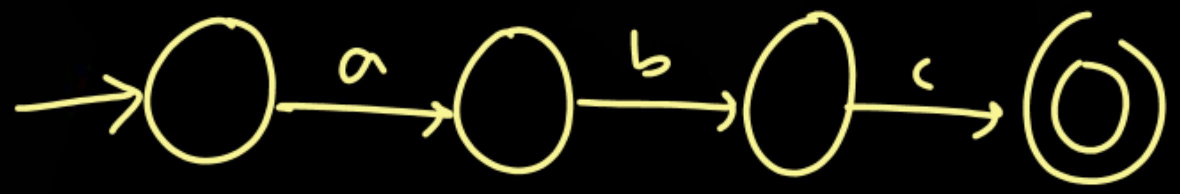
1111
b b b b
c b b
b c b
b b c
c c

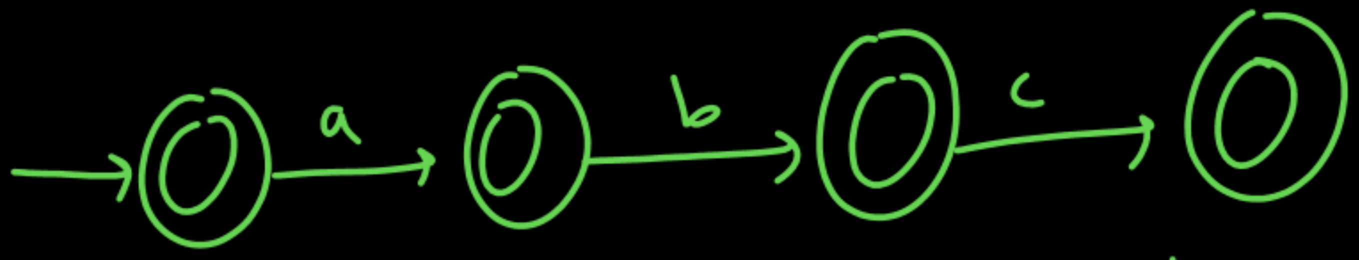
15. Prefix (Regular) ⇒ Regular

16. Suffix (Regular) ⇒ Regular

17. Substring (Regular) ⇒ Regular

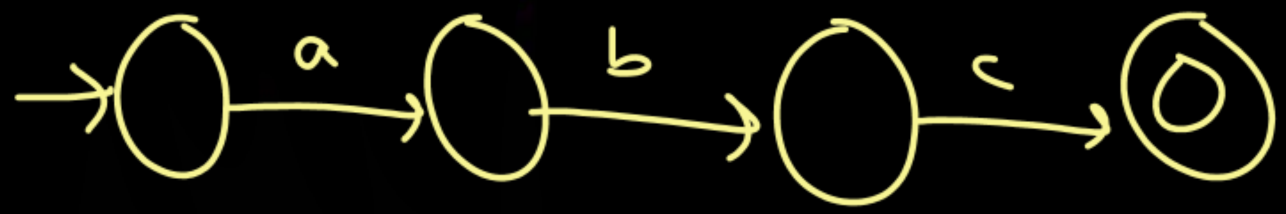
$L = \{abc\} \Longrightarrow \text{prefix}(L) = \{\varepsilon, a, ab, abc\}$

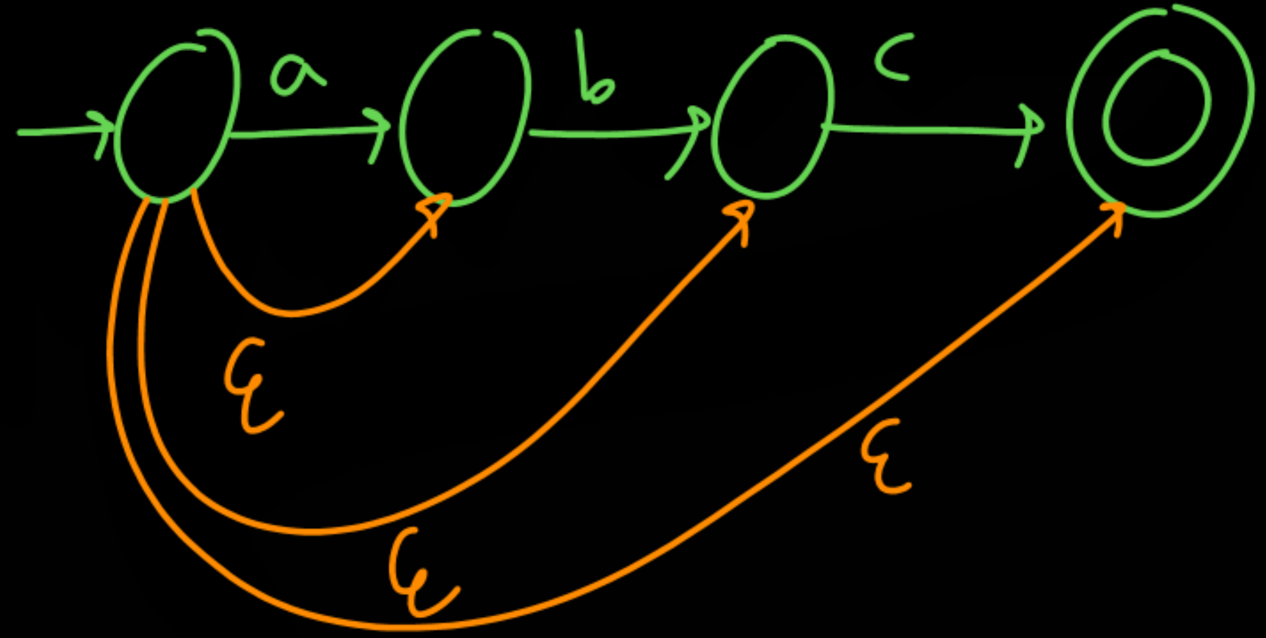




"Make every state as final from initial state to final state path"

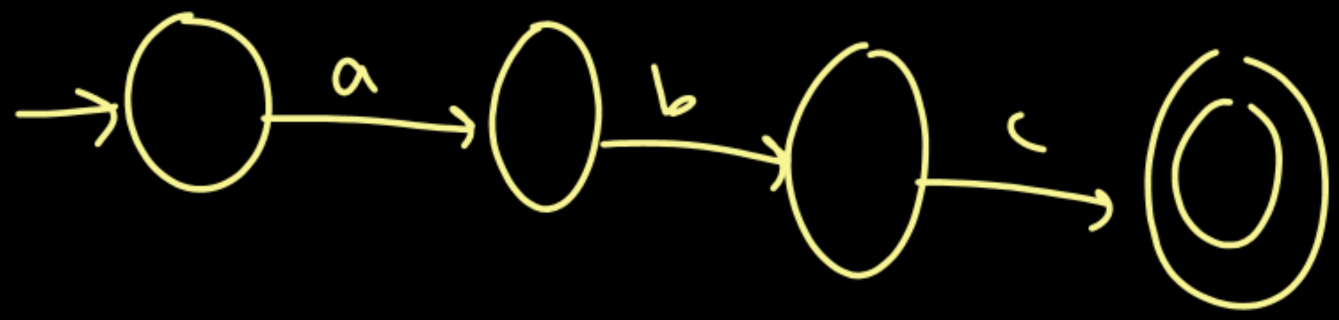
$L = \{abc\} \Longrightarrow \text{Suffix}(L) = \{\varepsilon, c, bc, abc\}$

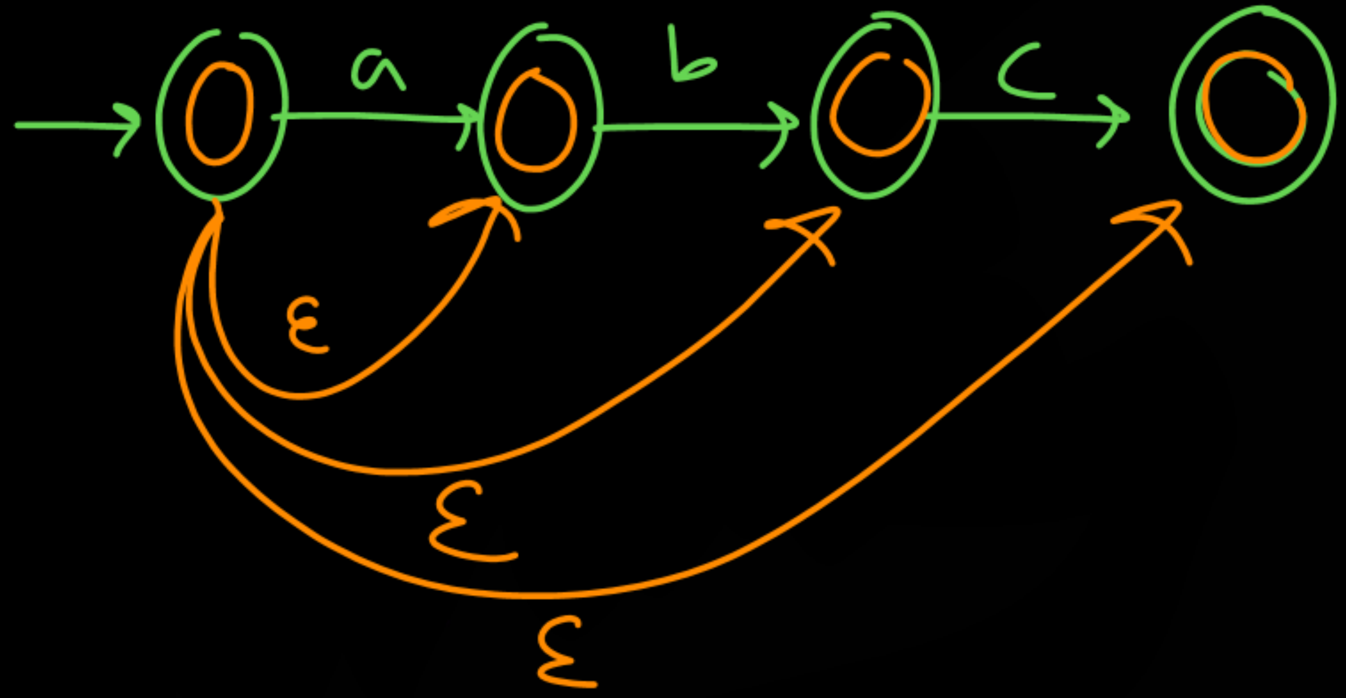




$L = \{abc\} \Longrightarrow$ Substring$(L) = \{\varepsilon, a, b, c, ab, bc, abc\}$

Subword(L)





18 Quotient(L_1, L_2)

$$L_1/L_2 = \{u \mid uv \in L_1,\ v \in L_2\}$$

$$\underbrace{11}\cancel{x}/\cancel{x} = 11$$

$$\underbrace{11}2\,3/3 = 112$$

$uv/v = u$

$\underbrace{ab}\boxed{c}/\boxed{c} = ab$

$abc/bc = a$

$abc/abc = \varepsilon$

$abc/a = \phi$

$abc/b = \phi$

$abc/ab = \phi$

① $L/\varepsilon = L$

$a/\varepsilon = a.\varepsilon/\varepsilon = a$

② $\varepsilon/L = $ either $\underbrace{\varepsilon}_{\text{If } L \text{ has } \varepsilon}$ or $\underbrace{\phi}_{\text{If } L \text{ has no } \varepsilon}$

③ $a^*/a = \{\varepsilon, a, aa, aaa, \ldots\}/a$

$= \{\underbrace{\varepsilon/a}_{\phi}, \underbrace{a/a}_{\varepsilon}, \underbrace{aa/a}_{a}, \underbrace{aaa/a}_{aa}, \ldots\}$

$= \{\varepsilon, a, aa, aaa, \ldots\}$

$= a^*$

$$\boxed{\begin{aligned} aa^* &= a^+ \\ a^*a &= a^+ \\ a^*/a &= a^* \end{aligned}}$$

(4) $a/a^* = a/\{\varepsilon, a, aa, aaa, \ldots\}$

$= \{a/\varepsilon, a/a, a/aa, \ldots\}$

ϕ

$= \{a, \varepsilon\}$

$= a+\varepsilon$

(5) $ab^*/a^* = ab^*/\{\varepsilon, a, aa, aaa, \ldots\}$

$= \{ab^*/\varepsilon,\ \underbrace{ab^*/a}_{\varepsilon},\ \underbrace{ab^*/\boxed{aa}}_{\phi},\ \underbrace{ab^*/\boxed{aaa}}_{\phi}, \ldots\}$

$= \{ab^*, \varepsilon\}$

$= \varepsilon + ab^*$

$\underbrace{ab^*}/a$

$\underbrace{a/a}_{\varepsilon},\ \underbrace{ab/a,\ abb/a, \ldots}_{\phi}$

$uv/v = u$

$ab/\varepsilon = ab$

$L = \{\varepsilon, a, ab, aab, abaa, abaaba, abbaab\}$

$\frac{1}{3}(L) = \{\varepsilon, a, ab\}$

$\text{middle}\frac{1}{3}(L) = \{\varepsilon, a, ba\}$

$\text{Last}\frac{1}{3}(L) = \{\varepsilon, b, ab\}$

19) $\rightarrow \text{Half}(L) = \{u \mid w \in L,\ w = uv,\ |u| = |v|\}$

$= \{\varepsilon, a, ab, abb\}$

20) $\rightarrow \text{Second Half}(L) = \{v \mid w \in L,\ w = uv,\ |u| = |v|\}$

$= \{\varepsilon, b, aa, aab\}$

21) $\rightarrow \text{one-third}(L) = \{x \mid w \in L,\ w = xyz,\ |x| = |y| = |z|\}$

22) $\rightarrow \text{middle}\frac{1}{3}(L) = \{y \mid \quad \}$

23) $\rightarrow \text{Last}\frac{1}{3}(L) = \{z \mid \quad \}$

aab

$|x| = |y|$

$|a| \neq |ab|$

$k \geq 2$ K-ary (24) Finite Union

$L_i \rightarrow$ regular $\quad L_1 \cup L_2 \cup L_3 \cup \ldots \cup L_K \Rightarrow$ Always Regular

K-ary (25) Finite Intersection

$L_1 \cap L_2 \cap L_3 \cap \ldots \cap L_K \Rightarrow$ Always Regular

K-ary (26) Finite Difference

$L_1 - L_2 - L_3 - L_4 \ldots - L_K \Rightarrow$ Always Regular

K-ary (27) Finite Concatenation

$L_1 \cdot L_2 \cdot L_3 \cdot L_4 \ldots L_K \Rightarrow$ Always Regular

Unary operation

28 Finite Subset

↳ closed for regular

$L \rightarrow$ Finite Subset $\Rightarrow$ Always finite

Finite Subset of a regular language is always finite (so, regular)

Finite Subset of any language is always finite

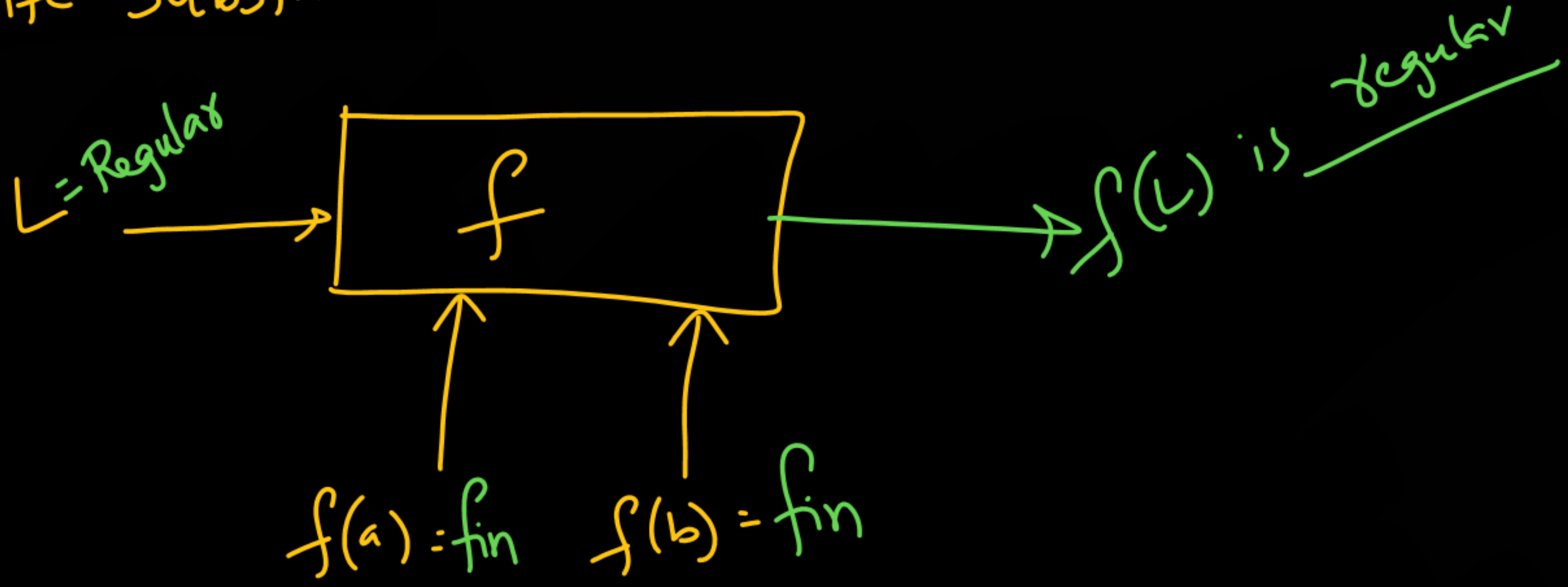
$a^* b^*$

$\Downarrow$ finite Subset

ϕ, $\{a, \varepsilon\}$, $\{a^n b^n \mid n < 100\}$ . . .

fin fin fin

unary 29 Finite Substitution



(30) Infinite Union $\Rightarrow$ Not closed for regulars

$L_i \rightarrow$ regular

$L_1 \cup L_2 \cup L_3 \cup \ldots \Rightarrow$ Need not be Regular

$\underset{\text{reg}}{\{\epsilon\}} \cup \underset{\text{reg}}{\{ab\}} \cup \underset{\text{reg}}{\{a^2b^2\}} \cup \ldots \Rightarrow \underbrace{a^n b^n}_{\text{non reg}}$

$\Sigma^* \cup L_2 \cup L_3 \cup \ldots \Rightarrow \underbrace{\Sigma^*}_{\text{reg}}$

(31) Infinite Intersection

$L_1 \cap L_2 \cap L_3 \cap L_4 \ldots \Rightarrow$ need not be regular

$\overline{\{\epsilon\}} \cap \overline{\{ab\}} \cap \overline{\{a^2b^2\}} \cap \ldots \Rightarrow \overline{\{a^n b^n\}}$ not reg

$\phi \cap L_2 \cap L_3 \cap \ldots \Rightarrow \underset{\text{reg}}{\phi}$

32. Infinite Difference
33. Infinite concatenation
34. Infinite Subset
35. Infinite substitution

Not closed for regulars

Subset

Inf $\cup$

Inf $\cap$

Inf $-$

Inf $\cdot$

$\subseteq_{Inf}$

$\int_{Inf}$

Not closed for regulars

H.W.:

F → Fin

I → Inf

R → Reg

NR → NonReg

1. $L_1 \cup L_2$
2. $L_1 \cap L_2$
3. $L_1 - L_2$
4. $\overline{L}$
5. $L_1 \cdot L_2$
6. L^*
7. L^{Rev}

Summary

→ closure properties ✓

Thank you