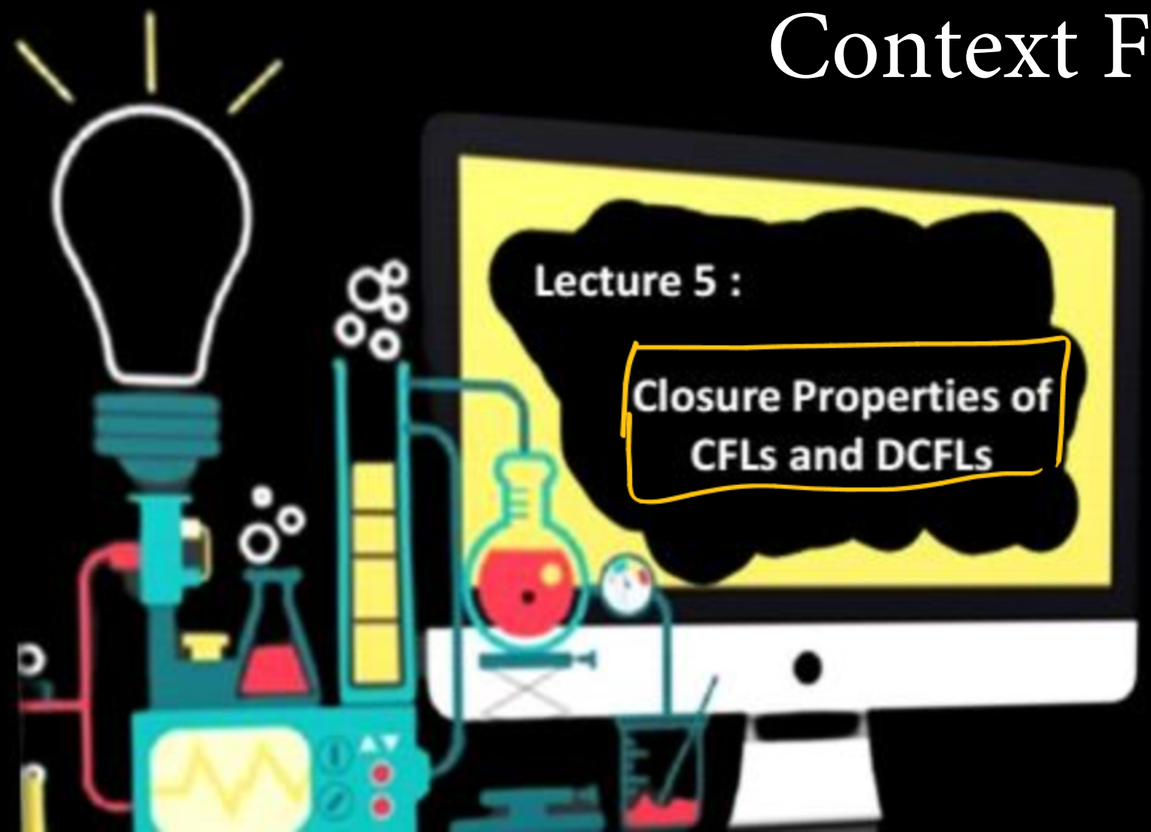
CS & IT Engineering

Context Free Grammar



Deva sir

Topics to be covered:

- Closure properties:
  - CFLs
  - DCFLs

Topics Covered in Previous Session:

- CFL ✓
- DCFL ✓

Closure Properties of DCFLs:

1. Union
2. Intersection
3. Complement
4. Difference
5. Concatenation
6. Kleene star
7. Kleene plus
8. Reversal
9. Subset
10. Prefix
11. Suffix
12. Substring
13. Substitution
14. Homomorphism
15. ε-free homomorphism
16. Inverse Homomorphism
17. Finite Union
18. Finite Intersection
19. Finite Difference
20. Finite Concatenation
21. Finite Subset
22. Finite Substitution
23. Infinite Union
24. Infinite Intersection
25. Infinite Difference
26. Infinite Concatenation
27. Infinite Subset
28. Infinite Substitution

Closure Properties of CFLs

1. Union
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(29) Regular closures:

i) $L \cup Reg$

ii) $L \cap Reg$

iii) $L - Reg$

iv) $Reg - L$

Closure Properties

SI6 ✗ → For Regular languages: We have remembered "not closed operations"

(SI$_6$)✗ Subset, Inf($\cup, \cap, -, \cdot, \subseteq, f$)

CPIF ✓ → For DCFLs: We will remember "closed operations"

(CPIF)✓

Complement, Prefix, Inverse Homomorphism, Finite Subset

ICDSF$_2$I$_6$ ✗ → For CFLs: We will remember "Not closed operations"

(ICDSF$_2$I$_6$)✗

Intersection, complement, Difference, Subset,
Finite Intersection, Finite Difference, Inf($\cup, \cap, -, \cdot, \subseteq f$)

① Union

- It is not closed for DCFLs
- It is closed for CFLs

$DCFL_1 \cup DCFL_2 \Rightarrow$ Need not be DCFL (Always CFL)

$CFL_1 \cup CFL_2 \Rightarrow$ Always CFL

Example:

$L_1 = \{a^n b^n\}$ DCFL

$L_2 = \{a^n b^{2n}\}$ DCFL

$L_1 \cup L_2 = \{a^i b^j \mid j = i \text{ OR } j = 2i\}$ not DCFL

$L_1 = CFL = a^n b^n \Rightarrow S_1 \to aS_1b/\varepsilon$

$L_2 = CFL = a^n b^{2n} \Rightarrow S_2 \to aS_2bb/\varepsilon$

$L_1 \cup L_2 \Rightarrow$ CFL

$S \to S_1 \mid S_2$

$S_1 \to aS_1b/\varepsilon$

$S_2 \to aS_2bb/\varepsilon$

① $L_1 = \{a^n b^n\}$, $L_2 = a^* b^*$ $\Rightarrow L_1 \cup L_2 = a^* b^*$

② L_1 is CFL, L_2 is DCFL $\Rightarrow$ $L_1 \cup L_2 \Rightarrow$ Always CFL
(may or may not be DCFL)
(may or may not be Reg)
(may or may not be Fin)

② Intersection

- not closed for CFLs
- not closed for DCFLs

$DCFL_1 \cap DCFL_2 \Longrightarrow$ Need not be DCFL (need not be CFL) (Always CSL)

$CFL_1 \cap CFL_2 \Longrightarrow$ Need not be CFL (Always CSL)

$L_1 = a^n b^n c^*$

$L_2 = a^* b^n c^n$

$\Rightarrow L_1 \cap L_2 = a^n b^n c^n$

not DCFL

not CFL

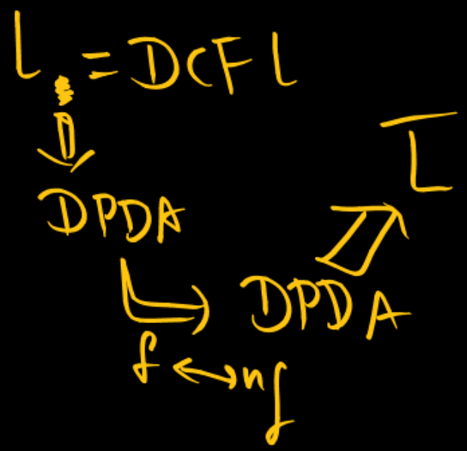
CFL $\cap$ DCFL $\Rightarrow$ Need not be DCFL
Need not be CFL
Always CSL

③ Complement

- closed for DCFLs
- Not closed for CFLs

$\overline{DCFL} \Rightarrow DCFL$

$\overline{CFL} \Rightarrow$ may or may not be CFL



Closure Properties

① $L = \underbrace{\{a^n b^n\}}_{\text{DCFL}} \Rightarrow \overline{L}$ is DCFL

② $L = \underbrace{\{w w^R \mid w \in \{a,b\}^*\}}_{\text{CFL but not DCFL}} \Rightarrow \overline{L}$ is CFL but not DCFL

③ $L = \underbrace{\{ww \mid w \in \{a,b\}^*\}}_{\text{not CFL}} \Rightarrow \overline{L}$ is $\overline{\{ww \mid w \in \{a,b\}^*\}}$ CFL but not DCFL

④ $L = \underbrace{\{w \# w^R \mid w \in \{a,b\}^*\}}_{\text{DCFL}} \Rightarrow \overline{L}$ is DCFL

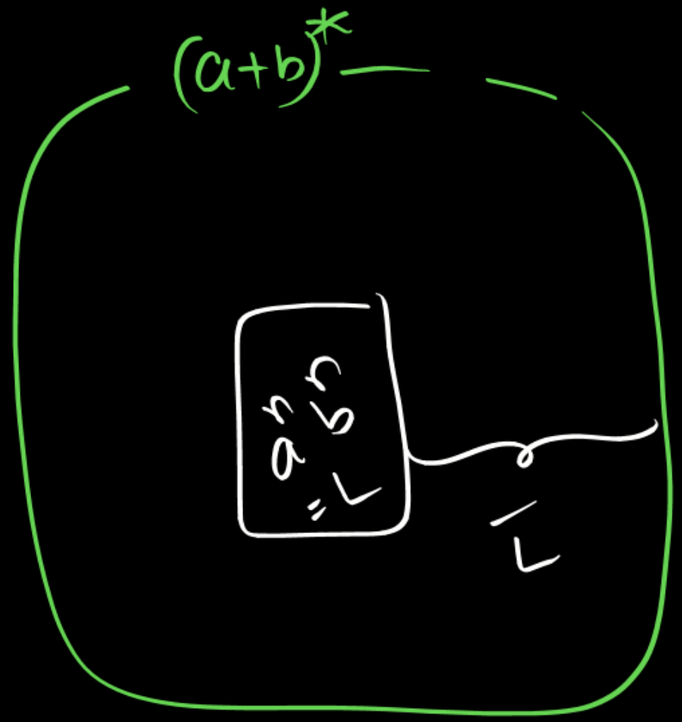
⑤ $L = \underbrace{\{a^n b^n c^n\}}_{\text{not CFL}} \Rightarrow \overline{L}$ is CFL but not DCFL

$$L = a^n b^n = \{a^m b^n \mid m = n\}$$

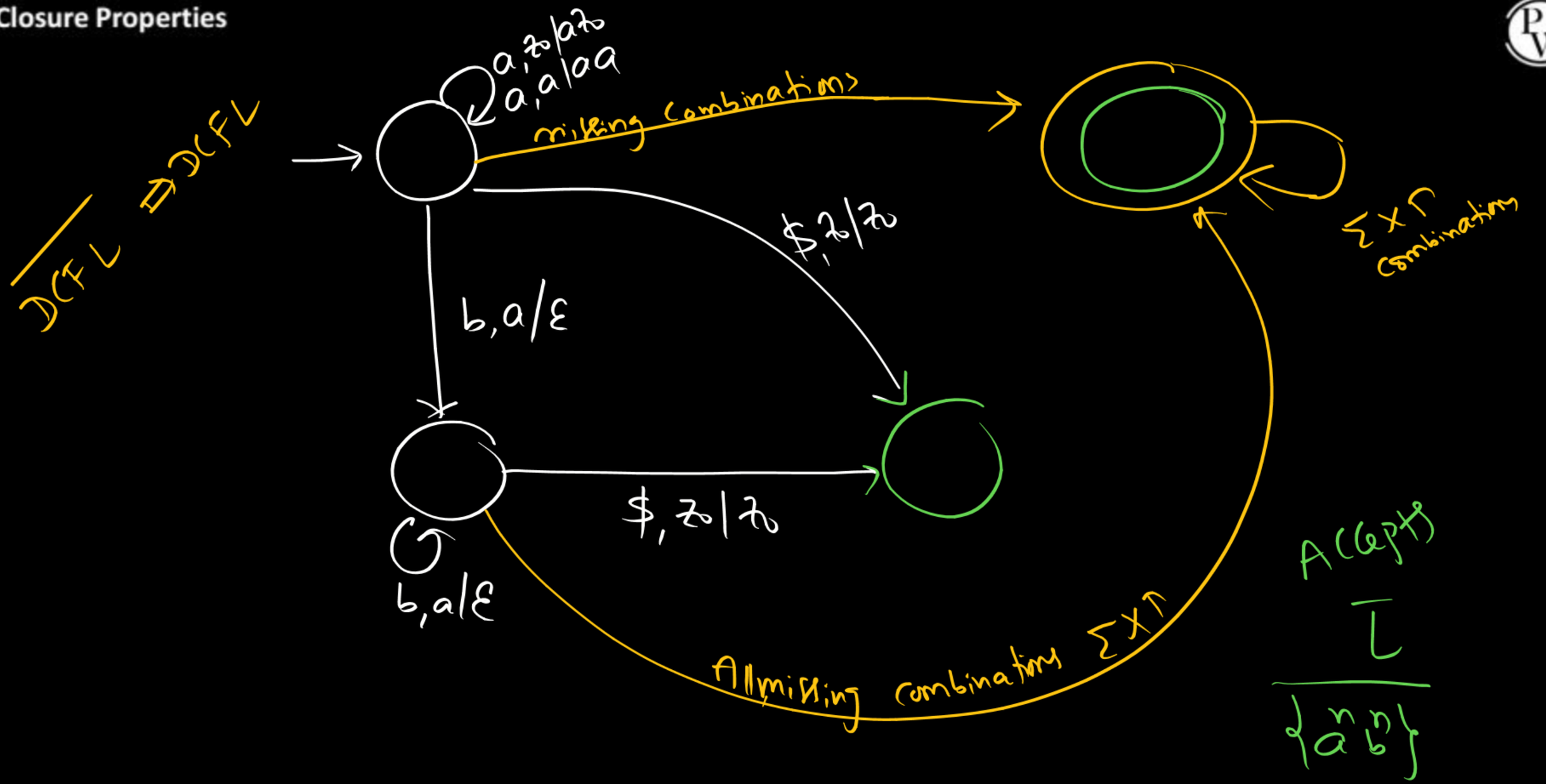
$$\bar{L} = (a+b)^* - L$$

$$= (a+b)^* - \{a^n b^n\}$$

$$\overline{\{a^n b^n\}} = (a+b)^* ba (a+b)^* \cup \{a^m b^n \mid m \neq n\}$$



Closure Properties



(4) Difference

$$L_1 - L_2 = L_1 \cap \overline{L_2}$$

→ Not closed for DCFLs

→ Not closed for CFLs

(5) Concatenation

- Not closed for DCFLs
- Closed for CFLs

$L_1 = \{a^n b^n\}_{n \geq 0}$

$L_2 = \{a^n b^{2n}\}_{n \geq 0}$

$L_1 \cdot L_2 \Rightarrow \{a^{n_1} b^{n_1} a^{n_2} b^{2n_2}\}$ It is CFL

$n_1 = 0$

aaaa b

push

not DCFL

$DCFL_1 \cdot DCFL_2 \Rightarrow$ Need not be DCFL (Always CFL)

$CFL_1 \cdot CFL_2 \Rightarrow$ Always CFL

S_1 S_2

$S \rightarrow S_1 \cdot S_2$

$S_1 \rightarrow$

$S_2 \rightarrow$

Closure Properties

(6) Kleene star

(7) Kleene plus

→ Not closed for DCFLs

→ Closed for CFLs

$(DCFL)^*$ ⟹ Need not be DCFL (Always CFL)

$(CFL)^*$ ⟹ CFL

	for DCFLs	for CFLs
$\cup$	✗	✓
$\cap$	✗	✗
$\overline{L}$	✓	✗
$L_1 - L_2$	✗	✗
$L_1 \cdot L_2$	✗	✓

$L = \{c\, a^n b^n\} \cup \{a^n b^{2n}\}$ is DCFL

$L^* = \left(\{c\, a^n b^n\}_{n \geq 0} \cup \{a^n b^{2n}\}_{n \geq 0} \right)^*$ is CFL but not DCFL

$n = 0$

$c\, a$

$\left(\quad\right)^2 = \left(c\quad\right)\left(a\quad\right)$

$L = \{a^n b^n\}$ DCFL

$L^* = \{a^n b^n\}^*$ is DCFL

L is CFL

$\Downarrow$

L^* is also CFL

$L = a^n b^n$ is CFL

$S \to aSb \mid \varepsilon$

$\Downarrow$

$X \to XS \mid \varepsilon$

$S \to aSb \mid \varepsilon$

$\Rightarrow L^* = \{a^n b^n\}^*$

⑧ Reversal

- → Not closed for DCFLs
- → closed for CFLs

$L = \{c a^n b^n\} \cup \{a^n b^{2n}\}$ is DCFL

$L^{Rev} = \{b^n a^n c\} \cup \{b^{2n} a^n\}$ is not DCFL

$(DCFL)^{Rev} \Rightarrow$ Need not be DCFL (Always CFL)

$(CFL)^{Rev} \Rightarrow$ always CFL

$(CFL)^{Rev} \Longrightarrow CFL$

Proof:

$L = a^n b^n$ is CFL

$\Downarrow$

$S \to aSb \mid \varepsilon$

$\Downarrow$ Rev every production

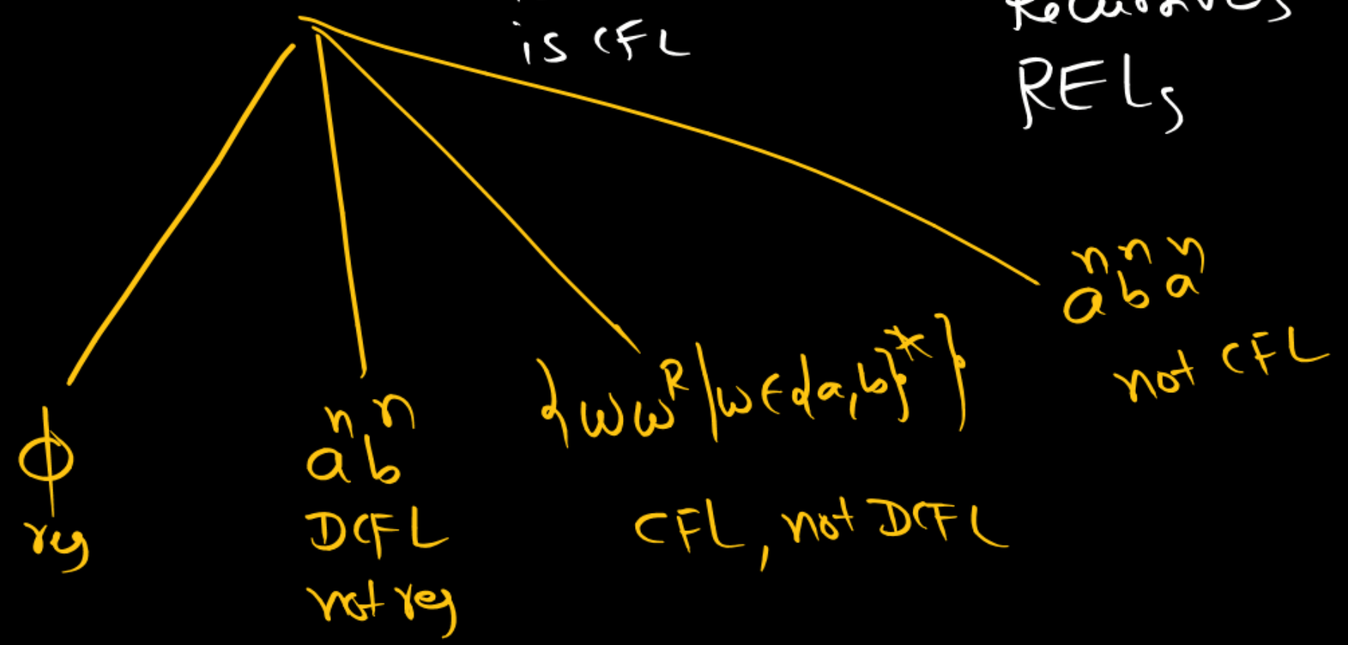
$\boxed{S \to bSa \mid \varepsilon} \Longrightarrow L^{Rev} = b^n a^n$

⑨ Subset

↳ not closed for regs

DCFLs
CFLs
CSLs
Recursives
RELs

$(a+b)^*$ is reg
is DCFL
is CFL



Note:

$\subseteq X$

h^{-1}

finite subset

Subset of regular is need not be regular

Subset of DCFL is need not be DCFL

Subset of CFL is need not be CFL

Closure Properties

* (10) Prefix(L) → closed for DCFLs
  → closed for CFLs

(11) Suffix(L)

(12) Substring(L)

→ Not closed for DCFLs

→ closed for CFLs

① $L = \{a^n b^n\}$ DCFL

$Prefix(L) = \{a^m b^n \mid m \geq n\}$ DCFL

Algo for CFL:

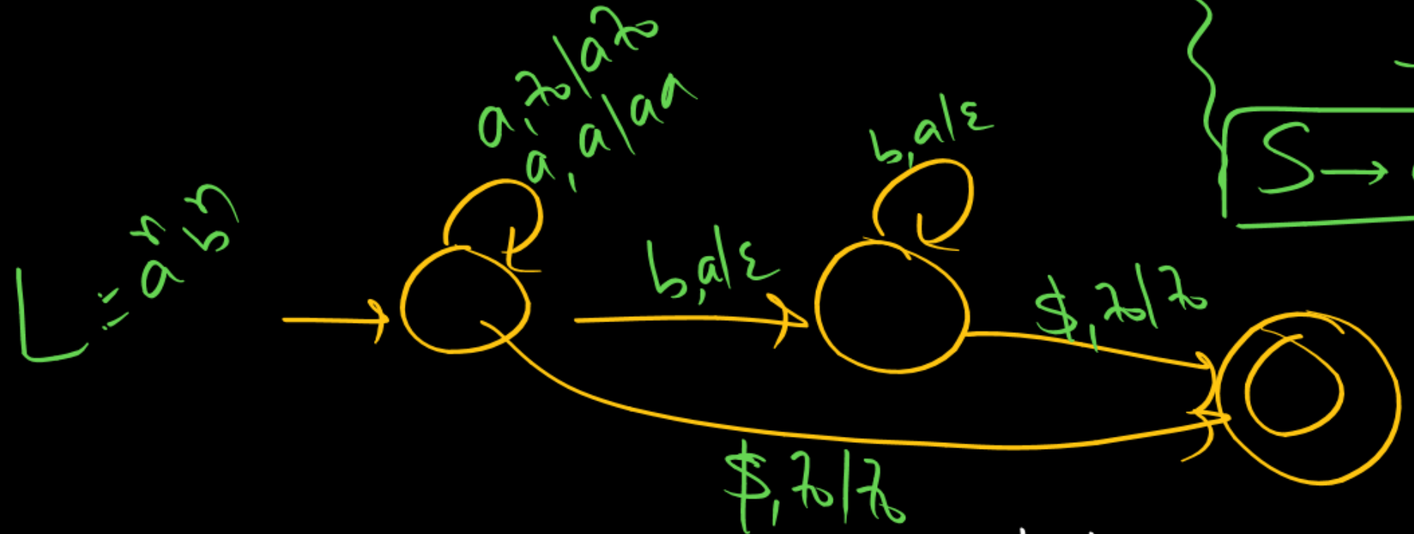
$L = a^n b^n$ CFL

$S \rightarrow aSb \mid \varepsilon$

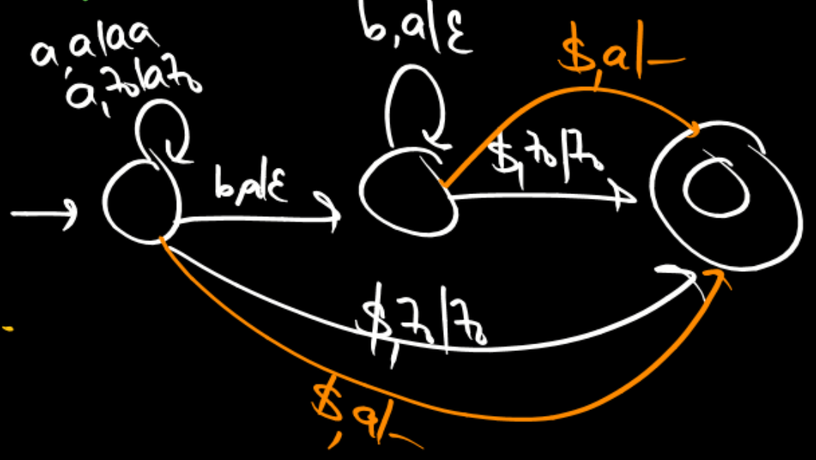
prefixes for every RHS

$S \rightarrow aSb \mid \varepsilon \mid a \mid aS$

$Prefix(L)$



$Prefix(L) = \{a^m b^n \mid m \geq n\}$



② $L = \{a^n b^n\}$

$\Downarrow$

$Suffix(L) = \{a^m b^n \mid m <= n\}$

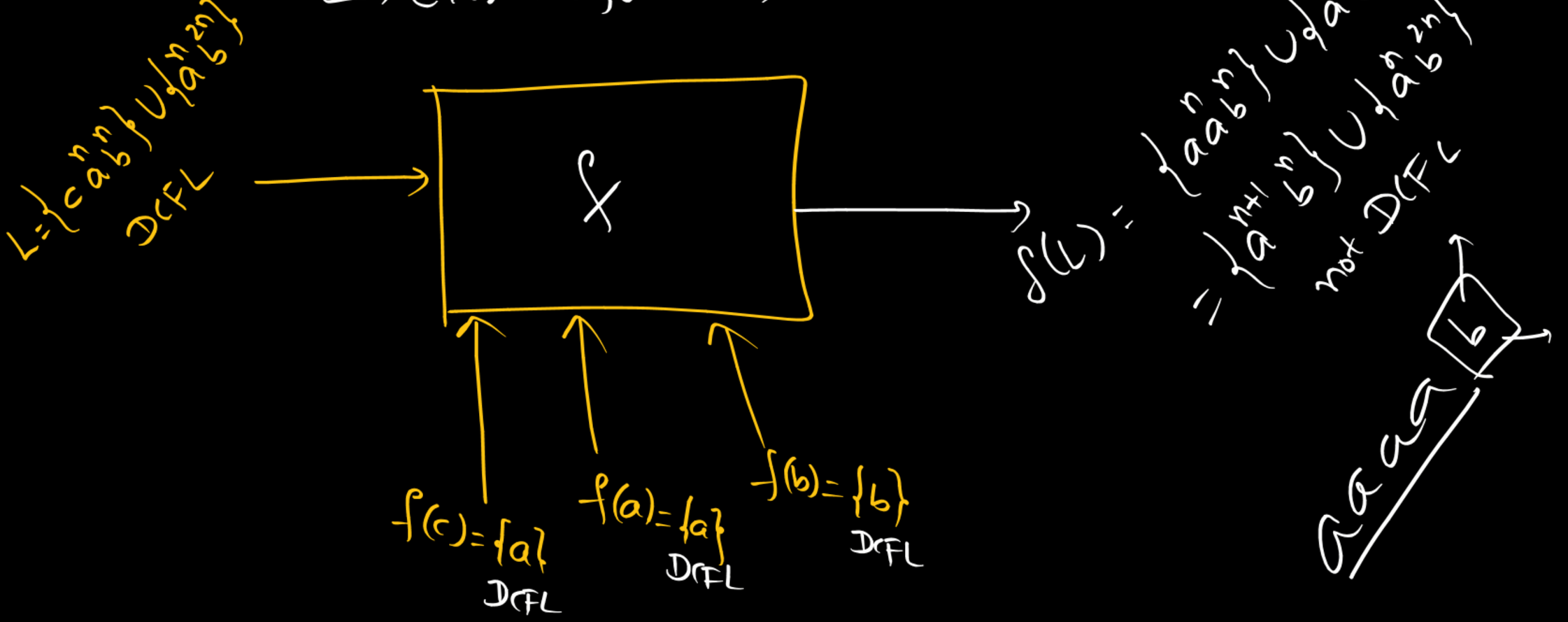
③ $L = \{a^n b^n\}$

$\Downarrow$

$Substring(L) = \{a^* b^*\}$

(13) Substitution

- Not closed for DCFLs
- Closed for CFLs



(14) Homomorphism

(15) ε-free Homomorphism

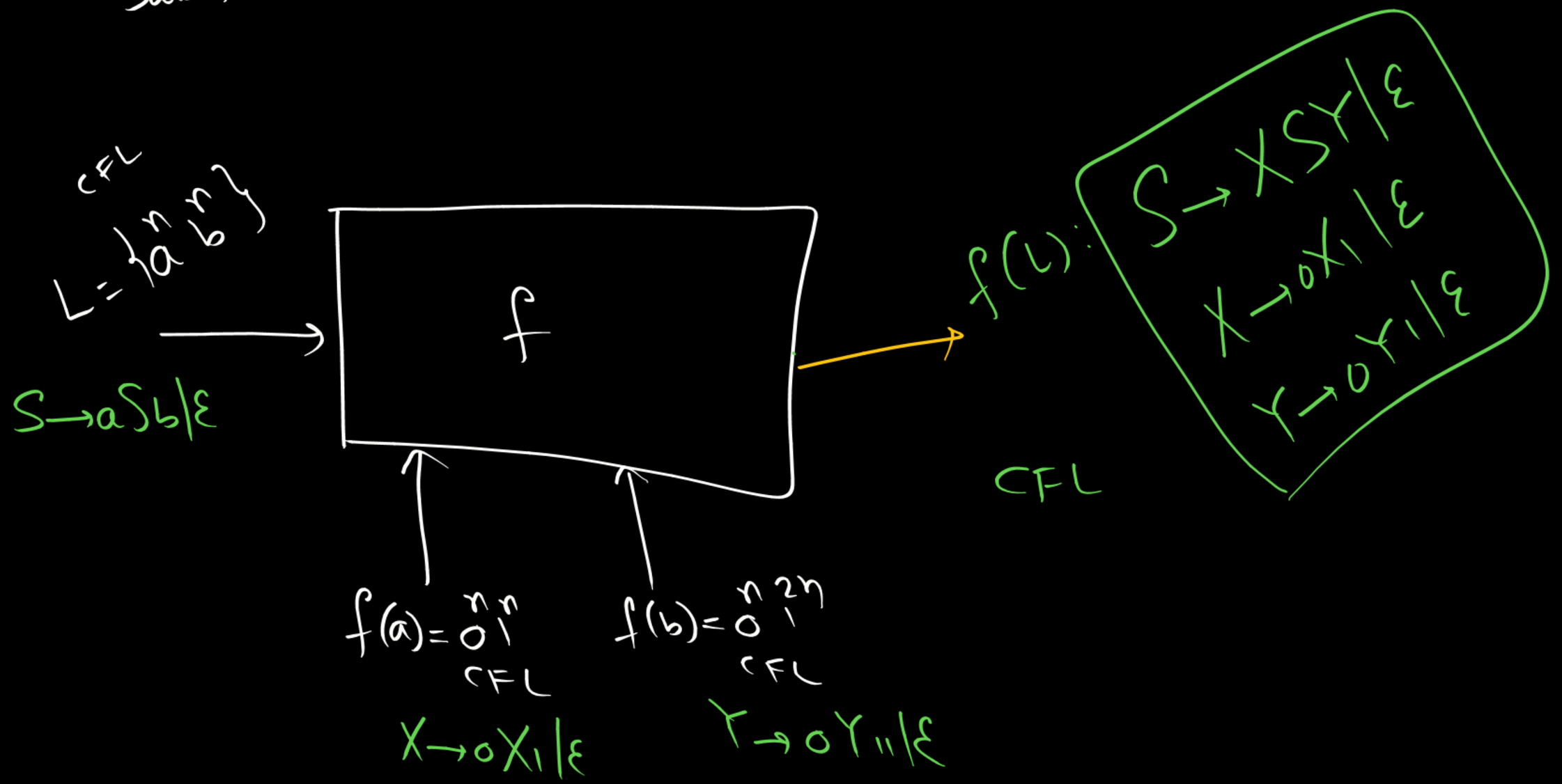
→ Not closed for DCFLs

→ closed for CFLs

*** (16) h^{-1}

→ closed for regs
Closed for DCFLs
closed for CFLs

Substitution (CFL) $\Longrightarrow$ CFL



Closure Properties

		for DCFLs	for CFLs
(17)	finite union	✗	✓
(18)	finite Intersection	✗	✗
(19)	finite Difference	✗	✗
(20)	finite Concatenation	✗	✓
*** (21)	finite Subset	✓	✓
(22)	finite Substitution	✗	✓
*** (23) to (28) : Inf $(\cup, \cap, -, \cdot, ^*, \varepsilon, f)$		✗	✗

*** (29) Regular closures

$L_1 - L_2 = L_1 \cap \overline{L_2}$

For DCFLs

1) DCFL ∪ Regular $\Rightarrow$ DCFL
2) DCFL ∩ Regular $\Rightarrow$ DCFL
3) DCFL − Regular $\Rightarrow$ DCFL $\cap \overline{Reg}$ $\Rightarrow$ DCFL ∩ Reg $\Rightarrow$ DCFL
4) Regular − DCFL $\Rightarrow$ Reg ∩ $\overline{DCFL}$ $\Rightarrow$ Reg ∩ DCFL $\Rightarrow$ DCFL

For CFLs

5) CFL ∪ Regular $\Rightarrow$ CFL
6) CFL ∩ Regular $\Rightarrow$ CFL
7) CFL − Reg $\Rightarrow$ CFL ∩ $\overline{Reg}$ $\Rightarrow$ CFL ∩ Reg $\Rightarrow$ CFL
*** 8) Reg − CFL $\Rightarrow$ Reg ∩ $\overline{CFL}$ $\Rightarrow$ Reg ∩ CSL $\Rightarrow$ CSL (need not be CFL)

Closure Properties

$a^n b^n \cap a^* b^* \Rightarrow a^n b^n$

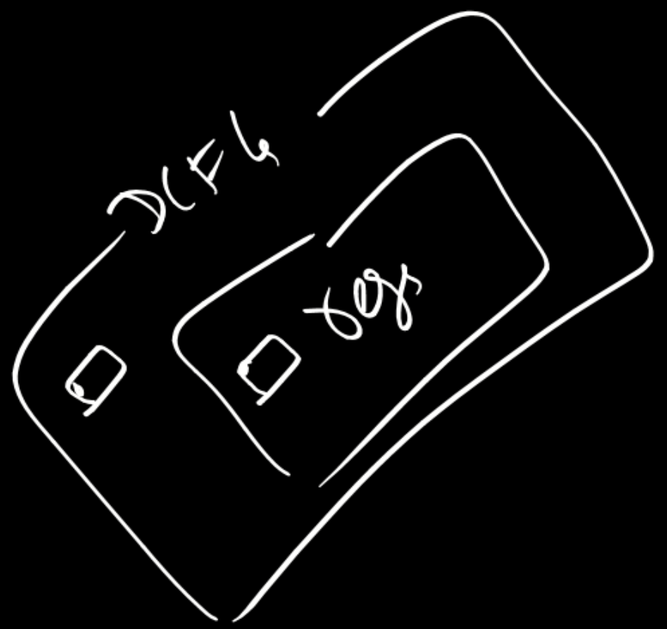
DCFL $\quad$ regular

DPDA X FA ⇒ DPDA

- U ✓
- ∩ ✓

PDA X FA ⇒ PDA

- U ✓
- ∩ ✗



Closure Properties

For DCFLs : ⇒

- Complement
- Prefix
- Inverse Hom
- Finite Subset

} Closed

For CFLs : ⇒

- Intersection
- Difference
- Subset
- Finite Intersection
- Finite Diff
- Inf $(\cup, \cap, -, \cdot, ^*, \subseteq, f)$

} Not closed

Summary

	$\cup$	$\cap$	$\overline{L}$	$\subseteq$
Finite languages	✓	✓	✗	✓
Infinite languages	✓	✗	✗	✗
Regulars	✓	✓	✓	✗
DCFLs	✗	✗	✓	✗
CFLs	✓	✗	✗	✗

Thank you