CS & IT ENGINEERING

DIGITAL LOGIC

Number System

Lecture No. 02

By- CHANDAN SIR

TOPICS TO BE COVERED

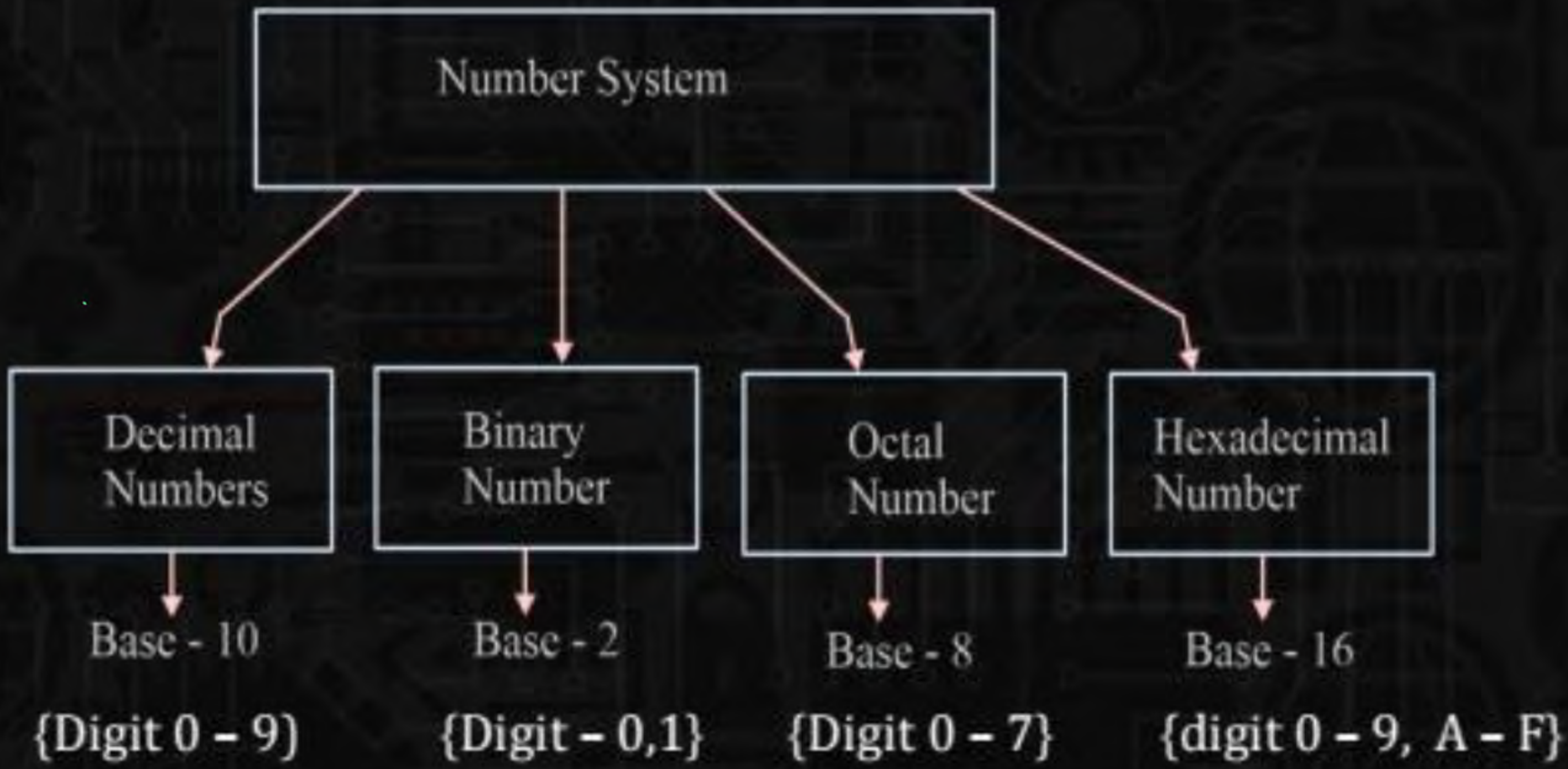
01 Types of Number System

02 Practice

03 Discussion

Base (Radix)

Total number of digit used in the system



Decimal Number System

...	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}...
...	a_4	a_3	a_2	a_1	a_0	a_{-1}	a_{-2}	a_{-3} ...

$a_i \rightarrow$ Coefficient of decimal number system

$10^i \rightarrow$ Weight of decimal number system

Example : $(501.23)_{10}$

10^2	10^1	10^0	10^{-1}	10^{-2}
5	0	1	2	3

Base	Digit
2	0, 1
3	0, 1, 2
4	0, 1, 2, 3
5	0, 1, 2, 3, 4
6	0, 1, 2, 3,4,5
7	0, 1, 2, 3, 4, 5, 6
8	0, 1, 2, 3, 4, 5, 6, 7
9	0, 1, 2, 3, 4, 5, 6, 7, 8
10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
11	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A
12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B
13	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C
14	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D
15	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E
16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

$(12C)$

13, 14, 15, 16

minimum base

Binary Number System (Base (Radix) = 2)

...	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3} ...
...	a_4	a_3	a_2	a_1	a_0	a_{-1}	a_{-2}	a_{-3} ...

$2^i \rightarrow$ Weight of Binary number system

$a_i \rightarrow$ Coefficient of Binary number system $\{0, 1\}$

Example:- $(101.11)_2$

2^2	2^1	2^0	2^{-1}	2^{-2}
1	0	1	1	1

Octal Number System (Base (Radix) = 8)

...	8^3	8^2	8^1	8^0	8^{-1}	8^{-2}	8^{-3}...
...	a_3	a_2	a_1	a_0	a_{-1}	a_{-2}	a_{-3}...

$8^i \rightarrow$ Weight of Octal number system

$a_i \rightarrow$ Coefficient of Octal number system $\{0 - 7\}$

Example:- $(728.64)_8$

8^2	8^1	8^0	8^{-1}	8^{-2}
7	2	8	6	4

Hexadecimal Number System (Base (Radix) = 16)

...	16^3	16^2	16^1	16^0	16^{-1}	16^{-2}	16^{-3}...
...	a_3	a_2	a_1	a_0	a_{-1}	a_{-2}	a_{-3}...

$16^i \rightarrow$ Weight of Hexadecimal number system

$a_i \rightarrow$ Coefficient of Hexadecimal number system $\{0 - 9, A\text{–}F\}$

Example: $(A2C.F)_{16}$

16^2	16^1	16^0	16^{-1}
A	2	C	F

1. Base conversion
2. Magnitude Representation

In base conversion 2 key points are there:

(A) Any base to Decimal conversion

(B) Decimal to any other base conversion

$$(\quad)_{r_1} \longrightarrow (\quad)_{10} \longrightarrow (\quad)_{r_2}$$

(A) Any base to Decimal conversion :

$$\left(\overset{r^{3}}{a_3}\ \overset{r^{2}}{a_2}\ \overset{r^{1}}{a_1}\ \overset{r^{0}}{a_0}\ .\ \overset{r^{-1}}{a_{-1}}\ \overset{r^{-2}}{a_{-2}}\right)_r = (\quad)_{10}$$

$$\left(a_3 \times r^3 + a_2 \times r^2 + a_1 \times r^1 + a_0 \times r^0 + a_{-1} \times r^{-1} + a_{-2} \times r^{-2}\right)_{10}$$

Case (1) : Binary to Decimal conversion

$2^3 2^2 2^1 2^0 2^{-1} 2^{-2}$

Ex. $(1011.11)_2 = (\quad)_{10}$

$$\Rightarrow \left[(1\times2^3)+(0\times2^2)+(1\times2^1)+(1\times2^0)+(1\times2^{-1})+(1\times2^{-2})\right]_{10}$$

$$\Rightarrow [8+0+2+1+0.5+0.25]_{10}$$

$$\Rightarrow (11.75)_{10}$$

Case (2) : Octal to Decimal conversion

$8^2\,8^1\,8^0\,8^{-1}$

Ex. $(721.4)_8 = (\quad)_{10}$

$\Rightarrow \left[(7\times8^2)+(2\times8^1)+(1\times8^0)+(4\times8^{-1})\right]_{10}$

$\Rightarrow [448+16+1+0.5]_{10}$

$\Rightarrow (465.5)_{10}$

Case (3) : Hexadecimal to Decimal conversion

$16^2\ 16^1\ 16^0\ \ 16^{-1}$

Ex. $(A2B.C)_{16} = (\quad)_{16}$

$$\Rightarrow \left[(A\times16^2)+(2\times16^1)+(B\times16^0)+(C\times16^{-1})\right]_{10}$$

$$\Rightarrow \left[(10\times256)+(2\times16)+(11\times1)+(12\times16^{-1})\right]_{10}$$

$$\Rightarrow \left[2560+32+11+0.75\right]_{10}$$

$$\Rightarrow (2603.75)_{10}$$

Case (4) : Base 5 to Decimal conversion

$5^2\ 5^1\ 5^0\ 5^{-1}\ 5^{-2}$

Ex. $(432.22)_5 = (\quad)_{10}$

$\Rightarrow \left[(4\times5^2)+(3\times5^1)+(2\times5^0)+(2\times5^{-1})+(2\times5^{-2})\right]_{10}$

$\Rightarrow [100+15+2+0.4+0.08]_{10}$

$\Rightarrow (117.48)_{10}$

$$(\quad)_{10} \longrightarrow (\quad)_{8}$$

(B) Decimal to any other Base conversion

γ	a_3	a_2	a_1	a_0	*Remainder*
					b_0
					b_1
					b_2
					b_3

$$(\underbrace{a_3\ a_2\ a_1\ a_0}_{\text{Before Decimal}} \cdot \underbrace{a_{-1}\ a_{-2}\ a_{-3}}_{\text{After Decimal}})_{10} = (\quad)_r$$

$$0 \cdot a_{-1} a_{-2} a_{-3} \times r = x_0 \cdot x_{-1} x_{-2} \qquad x_0$$

$$0 \cdot x_{-1} x_{-2} \times r = x_1 \cdot x_{-3} x_{-4} \qquad x_1$$

$$0 \cdot x_{-3} x_{-4} \times r = x_2 \cdot x_{-5} x_{-6} \qquad x_2$$

$$(a_3 a_2 a_1 a_0 \cdot a_{-1} a_{-2} a_{-3})_{10} = (b_3 b_2 b_1 b_0 \cdot x_0 x_1 x_2)_r$$

Case (1) : Decimal to Binary Base conversion.

Ex. $(19.75)_{10} = (?)_2$

19 $\Leftarrow$ Before Decimal   R   After Decimal $\Rightarrow$ 0.75

		R
2	19	1
2	9	1
2	4	0
2	2	0
	1	1

$0.75 \times 2 = 1.5$ 1

$0.5 \times 2 = 1.0$ 1

$(10011.11)_2$ Ans

$(19.75)_{10} = (10011.11)_2$ Ans

Case (2) : Decimal to Octal Base conversion.

Ex.

$$(\underbrace{210}_{\text{Before Decimal}}.\underbrace{23}_{\text{After Decimal}})_{10} = (\quad)_8$$

		Remainder
8	210	2
8	26	2
	3	3

$0.23 \times 8 = 1.84 \quad 1$

$0.84 \times 8 = 6.72 \quad 6$

$0.72 \times 8 = 5.76 \quad 5$

$$(210.23)_{10} = (322.165)_8$$

Case (3): Decimal to Hexadecimal Base conversion.

Ex.

$$(\underbrace{1228}_{\text{Before Decimal}}.\underbrace{55}_{\text{After Decimal}})_{10} = (\quad)_{16}$$

Before Decimal

After Decimal

$$\begin{array}{r|l l}
16 & 1228 & 12\ (C) \\
\hline
16 & 76 & 12\ (C) \\
 & 4 & 4
\end{array}$$

$0.55 \times 16 = 8.8 \quad 8$

$0.8 \times 16 = 12.8 \quad 12(C)$

$(1228.56)_{10} = (4CC.8C)_{16}$

$$(\quad)_{r_1} = (\quad)_{r_2}$$

$$(\quad)_{10}$$

$$(\quad)_{r_1} = (\quad)_{r_2}$$

$$r_1 = 2^m \qquad\qquad r_2 = 2^n$$

$$(\quad)_2$$

Some Special Case

Case (1): Binary to Octal base conversion

Ex. $(10110111)_2 = (\ ?\ )_8$

Octal → means base 8

$8 = 2^3$

Every three digits of binary represent one digit of octal

010	110	111
2	6	7

Hence $(10110111)_2 = (267)_8$ Ans

Q $(7635)_8 = (\ ?\ )_2$

2^3 Every digit should be written by 3 binary number.

$(7635)_8 = (111\ 110\ 011\ 101)_2$

Ans

Some Special Case

Case (2) : Binary to Hexadecimal base conversion

Ex. $(1011011)_2 = (\quad)_{16}$

Hexadecimal → means base 16

$16 = 2^4$

Every four digits of binary represent one digit of Hexadecimal.

0101 1011

5 11(B)

Hence $(1011011)_2 = (5B)_{16}$ Ans

Q $(AB2C)_{16} = (?)_2$

$2^{4} \rightarrow$ 4 bit

$A = 10$

$B = 11$

$C = 12$

$$(AB2C)_{16} = (1010\ 1011\ 0010\ 1100)_2$$

Ans

Q

$(2313)_4 = (?)_2$

$4 = 2^{2} \rightarrow 2\text{bit}$

$(10110111)_2$

Ans

Q. $(73467)_8 = (\ ?\ )_{16}$

Method ① $(73467)_8 = (\quad)_{10} = (\quad)_{16}$

Method ② $(73467)_8 = (\quad)_2 = (\quad)_{16}$

$8 = 2^{3}$

$(0111\ 0111\ 0011\ 0111)_2 =$

$(7737)_{16}$ Ans

BCD (Binary Coded Decimal)

In this each digit of the decimal number is represented by its four-bit binary equivalent. It is also called natural BCD or 8421 code. It is weighted code.

Excess – 3 Code: This is an non weighted binary code used for decimal digits. Its code assignment is obtained from the corresponding value of BCD after the addition of 3.

BCO (Binary Coded Octal): In this each digit of the Octal number is represented by its three-bit binary equivalent.

BCH (Binary Coded Hexadecimal): In this each digit of the hexadecimal number is represented by its four bit binary equivalent.

Decimal Digits	BCD 8421	Excess – 3	Octal digits	BCO	Hexadecimal Digits	BCH
0	0000	0011	0	000	0	0000
1	0001	0100	1	001	1	0001
2	0010	0101	2	010	2	0010
3	0011	0110	3	011	3	0011
4	0100	0111	4	100	4	0100
5	0101	1000	5	101	5	0101
6	0110	1001	6	110	6	0110
7	0111	1010	7	111	7	0111
8	1000	1011			8	1000
9	1001	1100			9	1001
					A	1010
					B	1011
					C	1100
					D	1101
					E	1110
					F	1111

Discussion

Decimal	Binary	Octal	
0	0	0	
1	1	1	
2	10	2	
3		3	
4		4	
5		5	
6		6	
7		7	
8		10	

9 11

De' octa

9 → 11

10 → 12

11 → 13

12 14

13 → 15

14 16

15 → 17

16 20

21

22

$$\begin{array}{r} (0.41)_8 \\ (0.42)_8 \\ \hline + \quad 1.03 \\ \hline \end{array}$$

Q. What is the possible base of the given number system?

$$\sqrt{41} = 5$$

A. 4

B. 5

C. 10

D. 6

$$\sqrt{41} = 5$$

$$\sqrt{(41)_r} = (5)_r$$

converting into Decimal

$$\sqrt{4\times r + 1\times r^0} = 5$$

$$4r + 1 = 25$$

$$4r = 24$$

$$\boxed{r = 6}$$

Q. What is the base of the given number system?

$(292)_{10} = (1204)_{?}$

A. 5 X

B. 6 ✓

C. 8

D. None

$(1204)_r = (292)_{10}$

$r^3 + 2r^2 + 4 = 292$

$r^3 + 2r^2 - 288 = 0$

(i) $5^3 + 2 \times 5^2 - 288$

$125 - 50 - 288 \neq 0$

(ii) $6^3 + 2 \times 6^2 - 288$

$= 0$

Q. 21DA having minimum base conversion to decimal is-

A. 6079

B. 10173

C. 2067

D. None

$(21DA)$

14, 15, 6

$(21DA)_{14} = (?)_{10}$

$2 \times 14^3 + 1 \times 14^2 + 13 \times 14^1 + 10 \times 14^0$

$(5876)_{10}$

Q. Two 2's complement number having sign bits x and y are added and the sign bit of the result is z.

Then, the occurrence of overflow is indicated by the Boolean function.

A. $x\,y\,z$

B. $\bar{x}\,\bar{y}\,\bar{z}$

C. $\bar{x}\,\bar{y}\,z + x\,y\,\bar{z}$

D. $x\,\bar{y}\,\bar{z}$

Q.

If $(12x)_3 = (123)_x$ then the value of x is

A. 3

B. 3 or 4

C. 2

D. None of these

$$(12x)_3 = (123)_x$$

$$(1\times 3^2 + 2\times 3 + x)_{10} = (x^2 + 2x + 3)_{10}$$

$$9 + 6 + x = x^2 + 2x + 3$$

$$x^2 + x - 12 = 0$$

$$(x-3)(x+4) = 0$$

$$x = 3 \quad x = -4$$

Q.

HW

Convert $(2724)_8$ to $(?)_5$

Q.

HW

The base of the number system for the addition operation $24 + 14 = 41$ to be true is

CHOICE (4)		Response
a.	8	
b.	6	
c.	5	
d.	7	

HW

Q. $(1217)_8$ is equivalent to

A. $(1217)_{16}$

B. $(028F)_{16}$

C. $(2297)_{10}$

D. $(0B17)_{16}$

Q.

H10

73_x (in base - x number system) is equal to 54_y (in base - y number system), the possible values of x and y are

A. 8, 16

B. 10,12

C. 9, 13

D. 8, 11

Q.

HW

Decimal 43 in Hexadecimal and BCD number system is respectively

A. B2, 0100 0011

B. 2B, 0100 0011

C. 2B, 0011 0100

D. B2, 0100 0100

Magnitude

	unsigned	Signed	Complement	
			1's	2's
+5	101	0101	0101	0101
-5	X	1101	1010	1011

2's

$+13 \longrightarrow 01101 \leftarrow$

$-13 \longrightarrow 10011$

Q If number given below are written in 1's complement then convert it into equivalont decimal.?

16 8 4 2 1

(1) 0 1 1 0 0 1 → +25 ✓

(2) 0 0 1 1 0 0 1 → +25 ✓

(3) 0 0 0 0 0 0 1 1 0 0 1 → +25 ✓

(4) 1 0 1 0 1 → −10

0 1 0 1 0 → +10

(5) 1 1 0 1 0 1 → −10

0 0 1 0 1 0

(6) 1 1 1 0 1 0 1 → −10

0 0 0 1 0 1 0

(7) 1 1 1 1 1 1 1 0 1 0 1 → −10

0 0 0 0 0 0 0 1 0 1 0

(8) 0 0 0 0 → +0

(9) 1 1 1 1 → −0

0 0 0 0

Q If number given below are written in 2's complement then convert it into equivalent decimal?

1. 011001 → +25
2. 0011001 → +25
3. 0000011001 → +25
4. 10101 → −11

   01011
5. 110101 → −11

   001011
6. 1110101 → −11
7. 1111110101 → −11
8. 0000 → +0
9. 1111 → −1

   0001

2's

Q 11001110 → -ve   Decimal = ?

→ -26 Ans

00110010

→ +26


THANK YOU GW SOLDIERS !