CS & IT Engineering

Deva sir

Previous Class Summary:

- NFA with ε-moves
  - ε-NFA
  - ε-closure (state)
  - ε-NFA ⇒ NFA
  - ε-NFA ⇒ DFA

Topics to be covered Today:

↳ practice

Class : Understand

$\Downarrow$

practice : Apply

$\Downarrow$

practice/Revision : Quickly

Let L be the language represented by the regular expression $\Sigma^*0011\Sigma^*$ where $\Sigma=\{0,1\}$. What is the minimum number of states in a DFA that recognizes $\overline{L}$ (complement of L)?

(GATE – 15 – SET3)

(a) 4 ~~(b)~~ 5

(c) 6 (d) 8

Shortcut:

$L = \Sigma^* 0011 \Sigma^*$

$\boxed{\text{contains } 0011}$

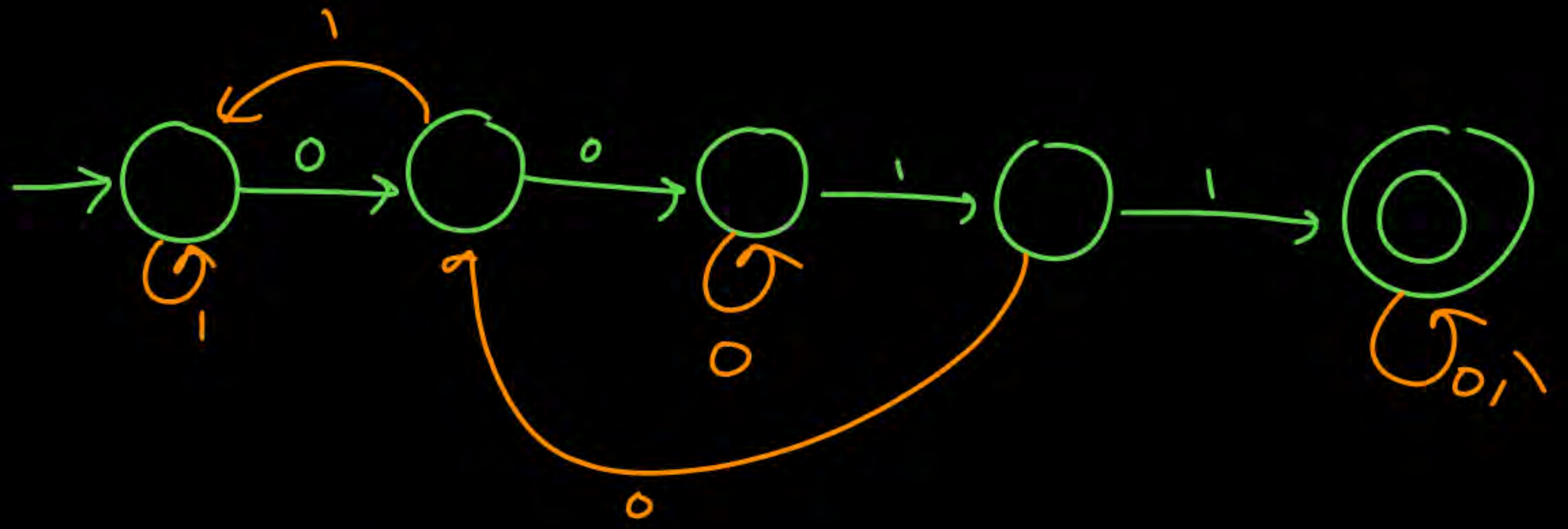
Min DFA ⇒ 5 states

$\overline{L}$ ⇒ How many states in DFA?

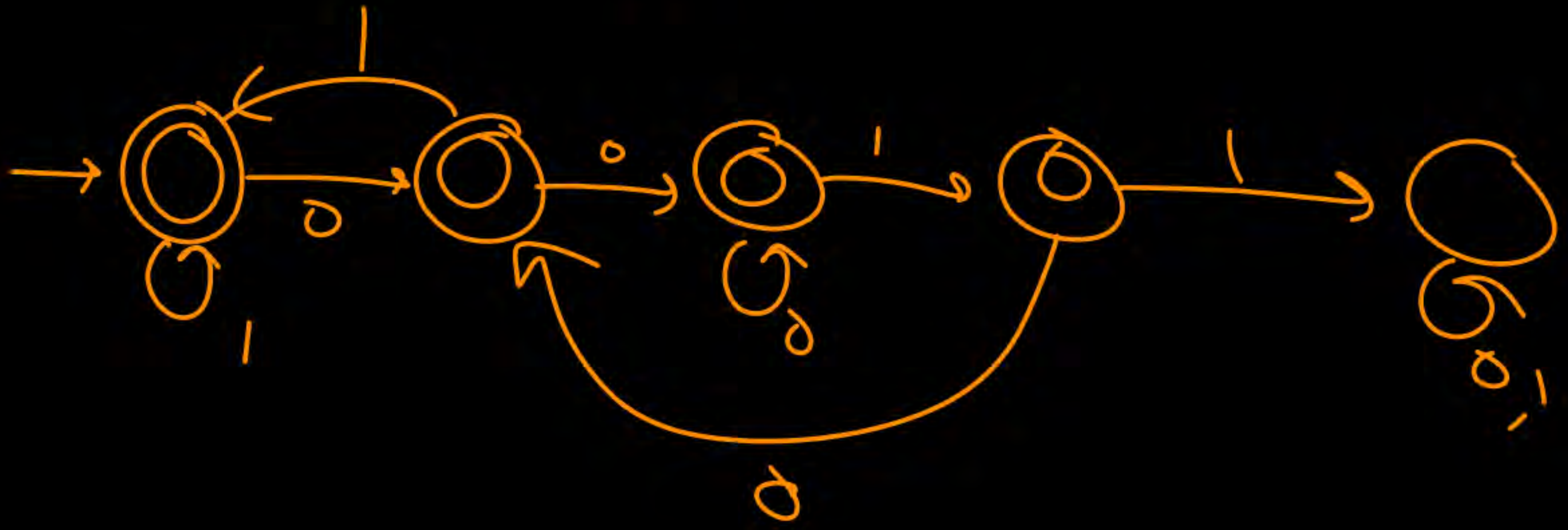
5 states

Lengthy:

L :



$\bar{L}$:



Which one of the following regular expressions represents the language: the set of all binary strings having two consecutive 0's and two consecutive 1's? **(GATE – 16 – SET1)**

(a) $(0+1)^* \, 0011(0+1)^* + (0+1)^* \, 1100(0+1)^*$

(b) $(0+1)^* \, (00(0+1)^* \, 11 + 11(0+1)^* \, 00)(0+1)^*$

(c) $(0+1)^* \, 00(0+1)^* + (0+1)^* \, 11(0+1)^*$

(d) $00(0+1)^* \, 11 + 11(0+1)^* \, 00$

$\times$ 001011

both $\underbrace{00 \text{ and } 11}$

min:
0011
1100

00
we don't need

10011

$X \, 00 \, X \, 11 \, X + X \, 11 \, X \, 00 \, X$

Let $r = 1(1+0)^*$, $s = 11^*0$ and $t = 1^*0$ be three regular expressions. Which one of the following is true? **(GATE - 91)**

(a) $L(s) \subseteq L(r)$ and $L(s) \subseteq L(t)$

(b) $L(r) \subseteq L(s)$ and $L(s) \subseteq L(t)$

(c) $L(s) \subseteq L(t)$ and $L(s) \subseteq L(r)$

(d) $L(t) \subseteq L(s)$ and $L(s) \subseteq L(r)$

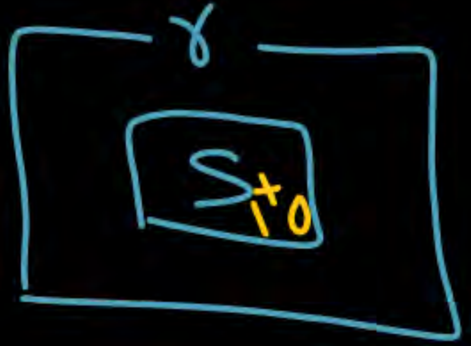
same

$r = 1X = \{1, \dots\}$

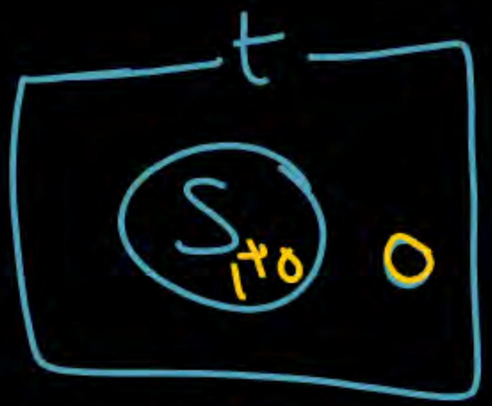
$s = 1^+0 = \{10, \dots\}$

$t = 1^*0 = \{0, \dots\}$

$0 \quad \underbrace{1^+0}_{s}$



$L(s) \subseteq L(r)$



$L(s) \subseteq L(t)$

Which of the following regular expression identities are true?

(GATE - 92)

(a) $r(^*) = r^*$ (b) $(r^*s^*)^* = (r+s)^*$

(c) $(r+s)^* = r^* + s^*$ (d) $r^*s^* = r^* + s^*$

wff :

$(r)^*$

r^*

r^+

$(r)^+$

$\boxed{\cancel{r^{(*)}}}$ not wff

$\cancel{r^{(+)}}$ not wff

$\cancel{a(+)b}$ not wff

wff

$a+b$

$(a)+b$

$a+(b)$

$(a)+(b)$

$(a+b)$

Consider the language L given by the regular expression $(a+b)^{*}b(a+b)$ over the alphabet {a, b}. The smallest number of states needed in a deterministic finite-state automaton (DFA) accepting L is________. **(GATE – 17 – SET1)**

$= 4$ //

2^{nd} symbol from end is 'b'
(n^{th})

⇓

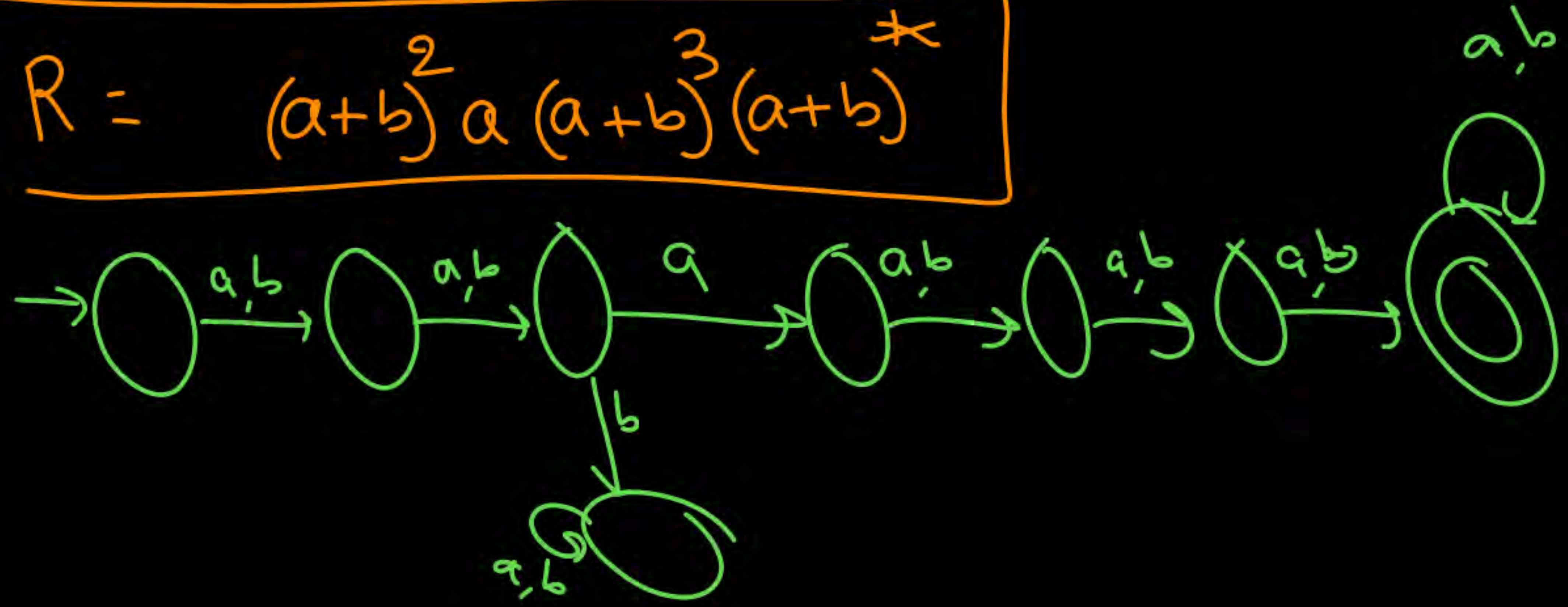
$2^{n} = 2^{2} = 4$

The minimum possible number of states of a deterministic finite automaton that accepts the regular language **(GATE – 17 – SET2)** $L = \{w_1 a w_2 \mid w_1, w_2 \in \{a, b\}^*, |w_1| = 2, |w_2| \geq 3\}$ is ________.

$= 8$

$\underbrace{w_1}_{=2} a \underbrace{w_2}_{\geq 3}$

$$\boxed{R = (a+b)^2 a (a+b)^3 (a+b)^*}$$



H.W.:

Model-XII :

85 $\{w_1 a w_2 \mid w_1, w_2 \in \{a,b\}^*, |w_1| = 2, |w_2| = 3\}$

86 $\{$ '' $\mid$ '' $, |w_1| = 2, |w_2| \geq 3\}$

87 $\{$ '' $\mid$ '' $, |w_1| = 2, |w_2| \leq 3\}$

88 $\{$ '' $\mid$ '' $, |w_1| \leq 2, |w_2| = 3\}$

89 $\{$ '' $\mid$ '' $, |w_1| \leq 2, |w_2| \leq 3\}$

90 $\{$ '' $\mid$ '' $, |w_1| \leq 2, |w_2| \geq 3\}$

91 $|w_1| \geq 2, |w_2| = 3$

92 $|w_1| \geq 2, |w_2| \leq 3$

93 $|w_1| \geq 2, |w_2| \geq 3$

Compiler

The lexical analysis for a modern computer language such as java needs the power of which one of the following machine models in a necessary and sufficient sense? **(GATE - 11)**

(a) Finite state automata

(b) Deterministic pushdown automata

(c) Non –deterministic pushdown automata

(d) Turing machine

The number of substrings (of all lengths inclusive) that can be formed from a character string of length n is **(GATE – 89 & 94)**

~~(a) n~~ ~~(b) n^2~~

~~(c) $\frac{n(n-1)}{2}$~~ ~~(d) $\frac{n(n+1)}{2}+1$~~

no option

min $n \Leftarrow$ Discrete maths : $\Rightarrow$ max $\frac{n(n+1)}{2}$

min $n+1 \Leftarrow$ TOC : $\Rightarrow$ $\frac{n(n+1)}{2} + 1$

zero length

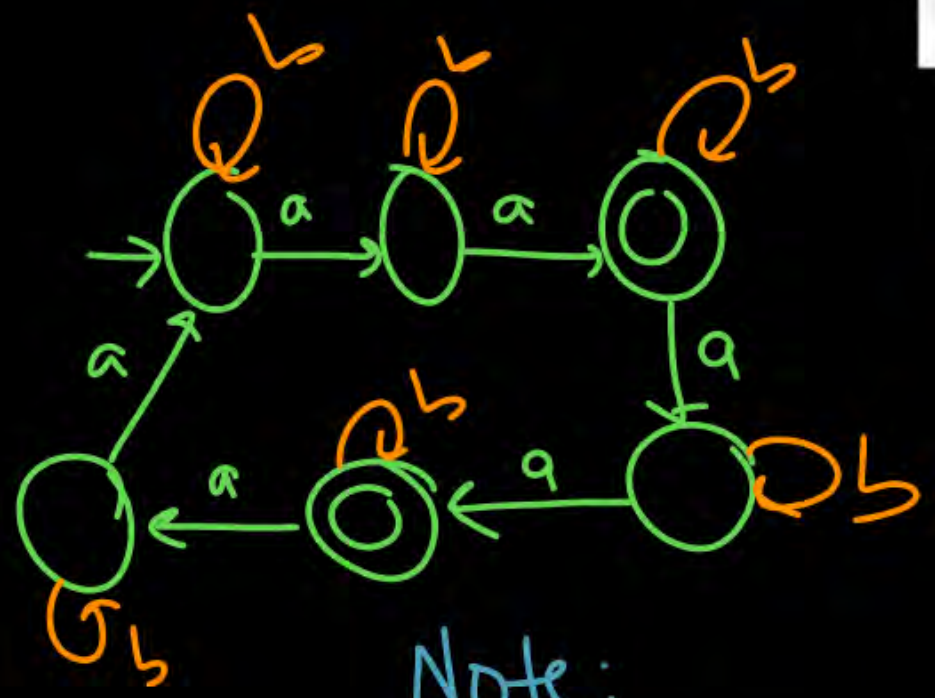
Only when all characters are distinct in the string

Consider the following language.

$L = \{x \in \{a, b\}^* \mid \text{number of } a\text{'s in } x \text{ divisible by 2 but not divisible by 3}\}$

The minimum number of states in DFA that accepts L is __________

GATE 2020



$L = \{x \in \{a,b\}^* \mid \text{no. of } a\text{'s}(x) \text{ is div by 2 but not divisible by 3}\}$

Note:

$L = \{x \in a^* \mid \underbrace{n_a(x) \text{ is div by 2}} \text{ but not } \underbrace{\text{div by 3}}\}$

$= \{a^2, a^4, a^8, a^{10}, a^{14}, a^{16} \ldots\}$

$\cancel{\varepsilon}, a^2, a^4, \cancel{a^6}, a^8, a^{10}, \cancel{a^{12}}, a^{14}, a^{16}, \cancel{a^{18}}$

$\varepsilon, a^3, a^6, a^9, a^{12}, \ldots$

No option is correct

Which one of the following regular expressions represents the set of all binary strings with an $\boxed{\text{odd number of 1's}}$?

A. $((0+1)^*1(0+1)^*1)^*10^*$ → even no. of 1's Possible

B. $(0^*10^*10^*)^*0^*\boxed{1}$ → not generating all strings having odd no. of 1's

C. $\boxed{1}0^*(0^*10^*10^*)^*$

D. $(0^*10^*10^*)^*10^*$ → 1, 01 ✗

10 odd no. of 1's

01

GATE 2020

Even no. of 1's

$0^*(0^*10^*10^*)^*0^*$

odd no. of 1's

$0^*(0^*10^*10^*)^*0^*10^*$

$0^*10^*(0^*10^*10^*)^*0^*$

Given a language L, define L^i as follows:

$$L^0 = \{\varepsilon\}$$

$$L^i = L^{i-1} \bullet L \text{ for all } i > 0$$

The order of a language L is defined as the smallest k such that $L^k = L^{k+1}$. Consider the language L_1 (over alphabet 0) accepted by the following automaton.

: L_1

The order of L_1 is 2

GATE 2018

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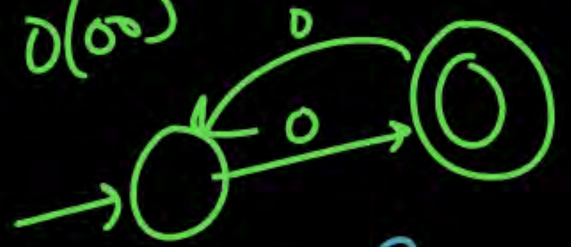
$(00)^*$

$0(00)^*$



$(\varepsilon + 0(00)^*)^2$

$$L^0 = \{\varepsilon\}$$

$$L^i = L^{i-1} . L$$

$$\boxed{L^k = L^{k+1} \text{ for smallest } k \qquad O(L) = k = 2}$$

$$L = \varepsilon + 0(00)^*$$

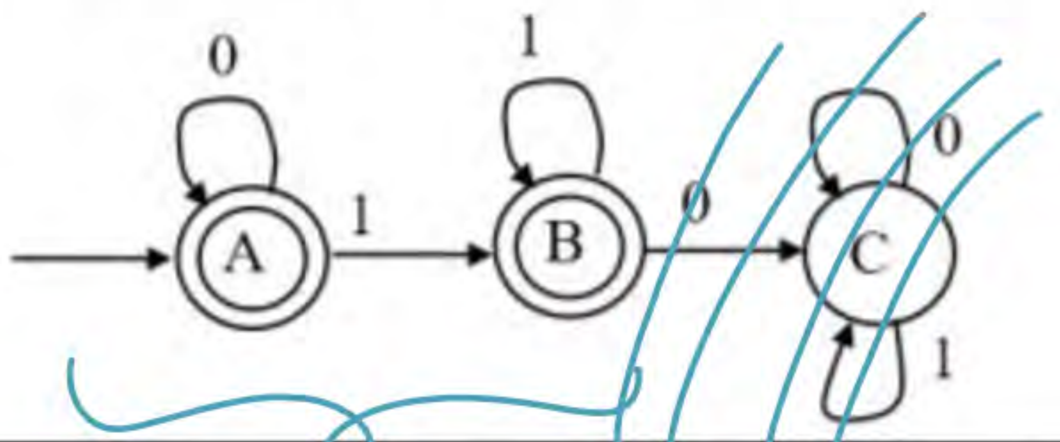
$$L^0 = \varepsilon$$

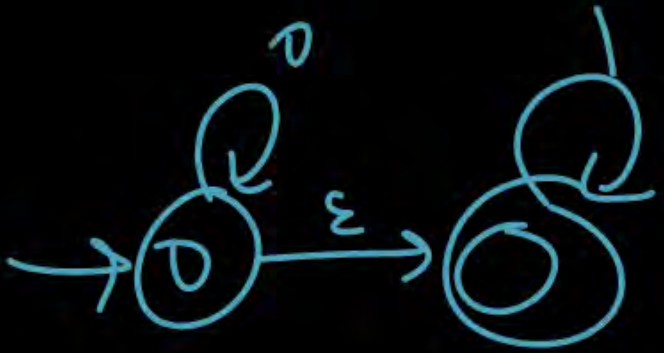
$$L^1 = L^0 . L = L = \varepsilon + 0(00)^* \rightarrow L^0 \neq L^1$$

$$L^2 = L^1 . L = L . L = (\varepsilon + 0(00)^*) . (\varepsilon + 0(00)^*) = 0^* \rightarrow L^1 \neq L^2$$

$$L^3 = L^2 . L = 0^* . L = 0^* \rightarrow L^2 = L^3$$

The regular expression for the language recognized by the finite state automation of the below figure is _____ **(GATE - 94)**





0^*1^*

$A = 0^*$

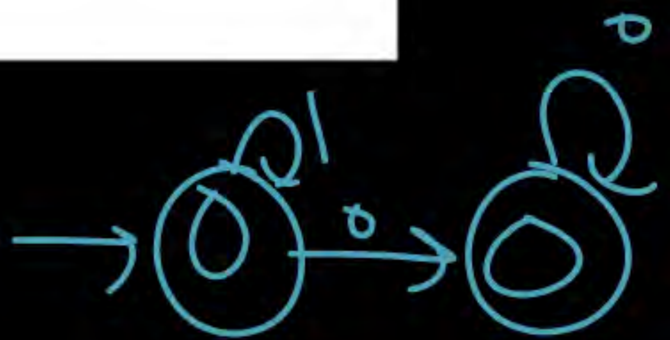
$B = 0^*1\,1^* = 0^*1^+$

Union

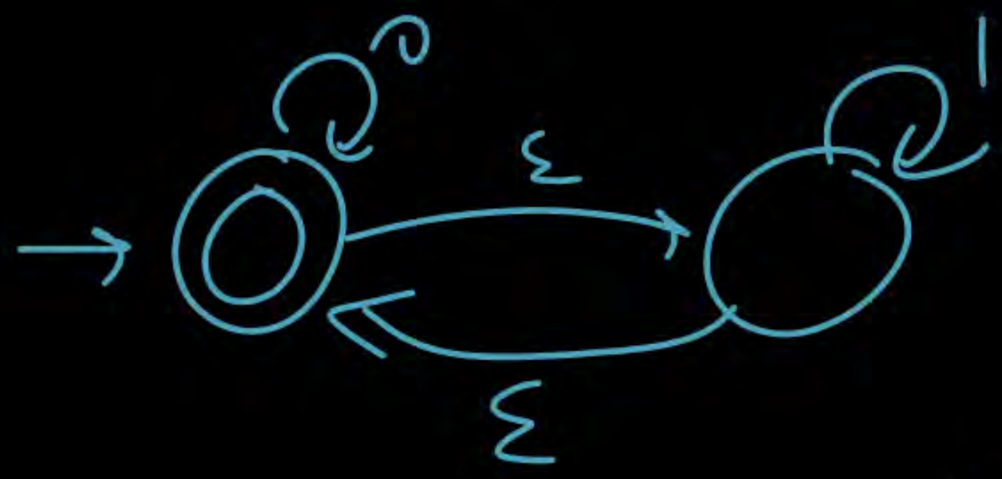
$A + B = 0^* + 0^*1^+$

$= 0^*(\varepsilon + 1^+) = 0^*1^*$

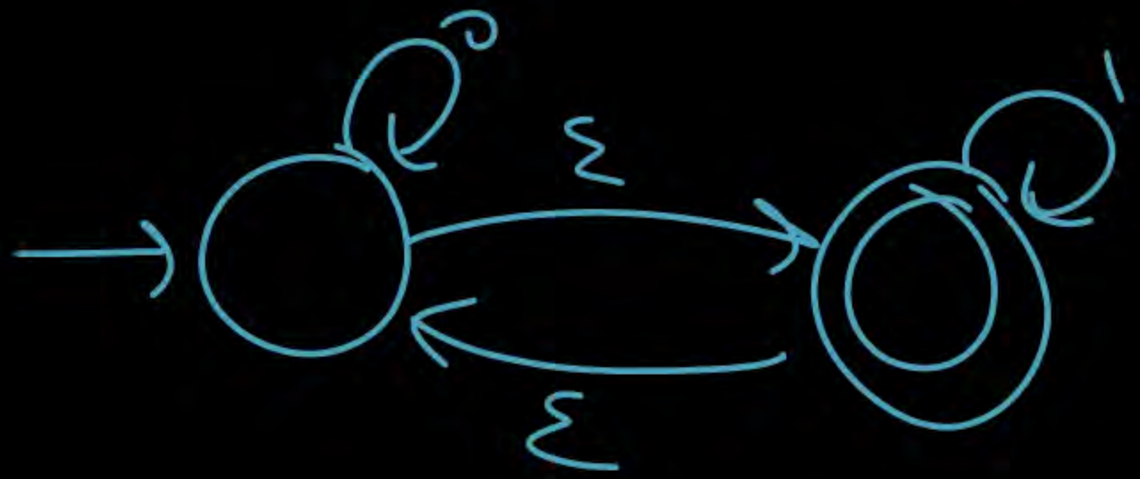
A. 0^*1^*
B. 0^+1^*
C. $(0+1)^*$
D. 0^*1^+
E. None of these



1^*0^*



$$= (0+1)^*$$



$$= (0+1)^*$$

Use Elimination technique:

Valid / Invalid

$0^*(10+1)^*$

Which one of the following regular expressions over {0, 1} denotes the set of all strings **not** containing 100 as a substring? **(GATE - 97)**

~~(a) $0^*(1^+0)^*$~~ → 1

~~(b) $0^*\boxed{101}0^*$~~ → 101 is min

~~(c) $0^*1^*01^*$~~ → min = 0 → generates only Some strings

(d) $0^*(10+1)^*$ ✓

$0^*(1^+0)^*$

~~DFA ⇓ Convert into Reg Exp~~

0100 ✗

1001 ✗

1000 ✗

1100 ✗

Valid	Invalid
min ε ✓	100 ✗
0 ✓	
1 ✓	
00 ✓	
01 ✓	
10 ✓	
11 ✓	
000 ✓	
001 ✓	
010 ✓	
011 ✓	
101 ✓	
110 ✓	

Let L be the set of all binary strings whose last two symbols are the same .The number of states in the minimum state deterministic finite-state automaton accepting L is

(GATE - 98)

A. 2

~~B.~~ 5

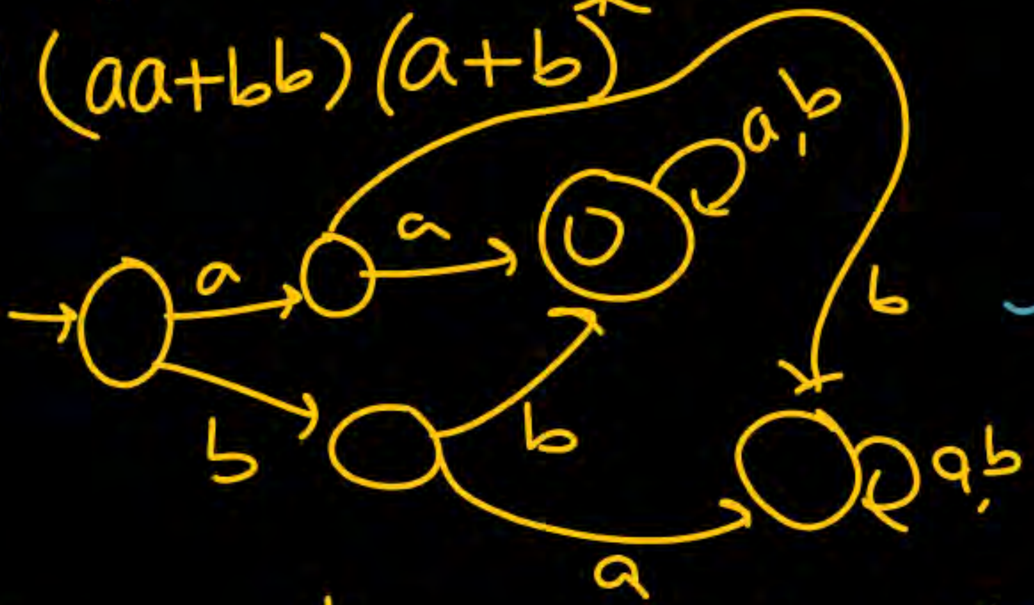
C. 8

D. 3

E. None of these

$\Rightarrow$ 5 states

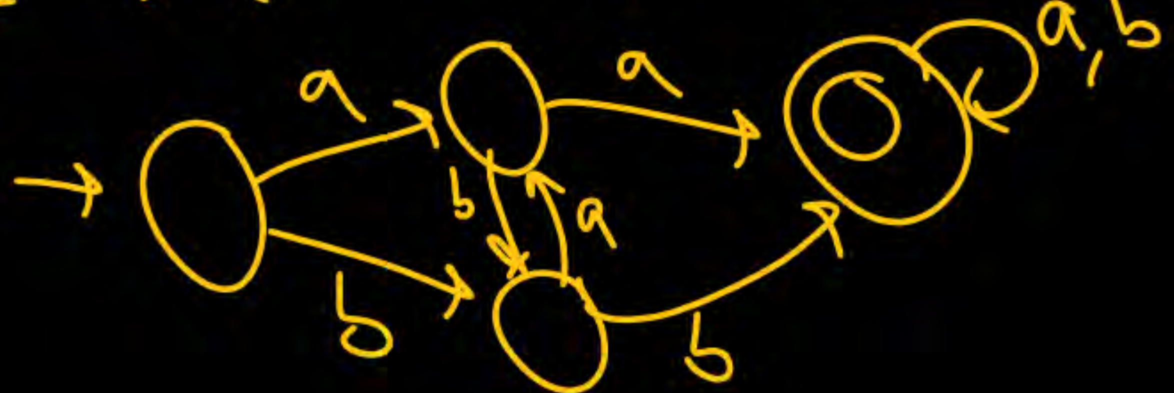
① $(aa+bb)(a+b)^*$



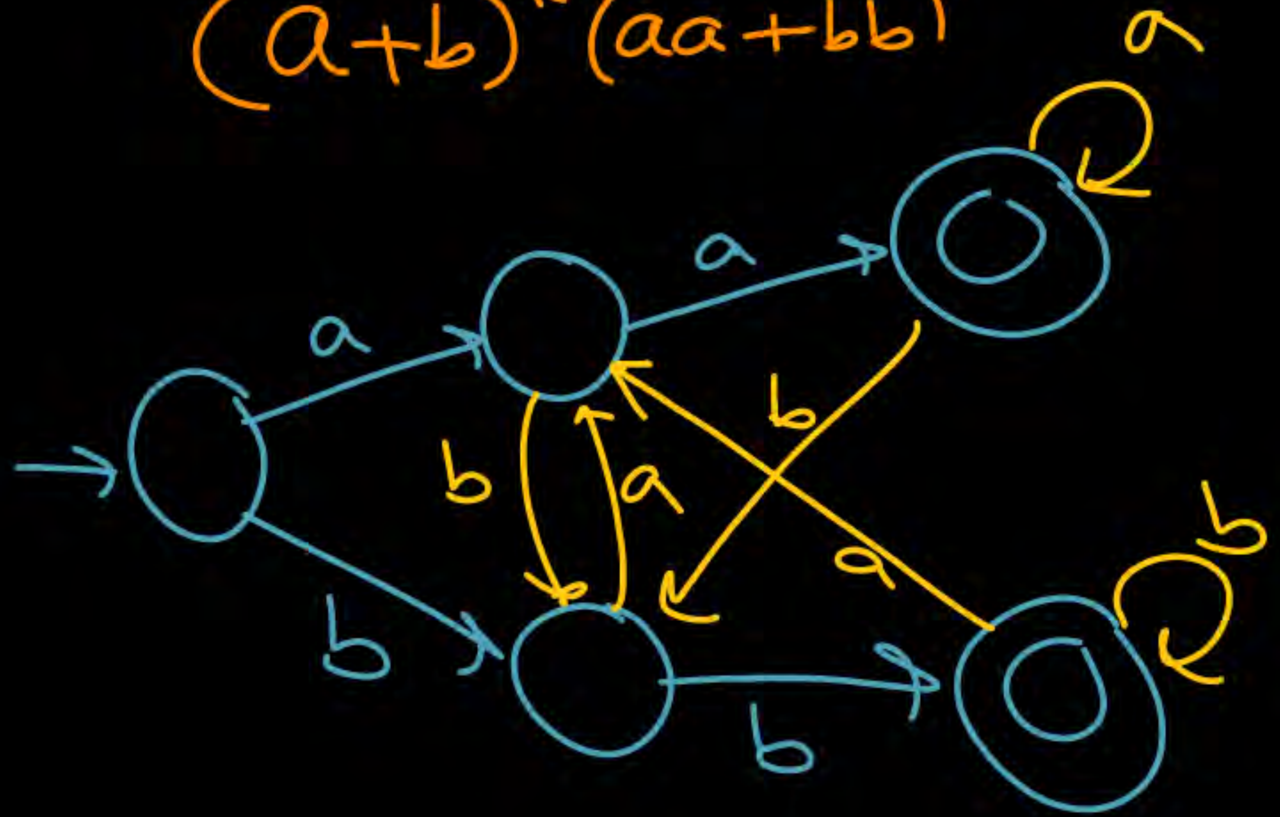
② $(a+b)^*(aa+bb)(a+b)^*$

$\Downarrow$

4 states



$(a+b)^*(aa+bb)$



no option

What can be said about a regular language L over {a} whose minimal finite state automaton has two states? **(GATE - 2000)**

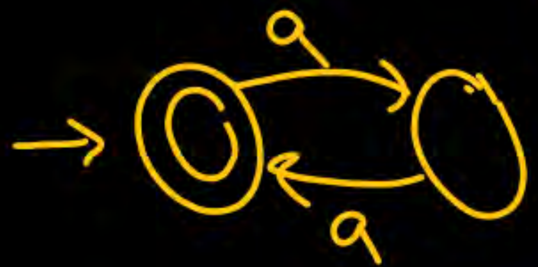
~~(a)~~ L must be $\{a^n \mid n \text{ is odd}\}$

~~(b)~~ L must be $\{a^n \mid n \text{ is even}\}$

~~(c)~~ L must be $\{a^n \mid n \geq 0\}$

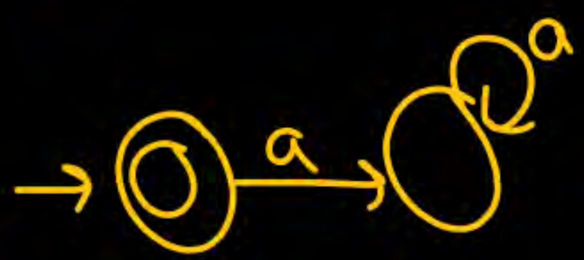
~~(d)~~ Either L must be $\{a^n \mid n \text{ is odd}\}$ or L must be $\{a^n \mid n \text{ is even}\}$

2 states FA $\Sigma = \{a\}$

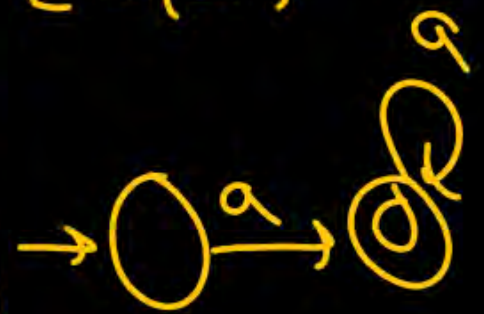


$= (aa)^*$

$= a(aa)^*$



$= \varepsilon$



$= a^+$

Consider a DFA over $\sum = \{a, b\}$ accepting all strings which have number of a's divisible by 6 and number of b's divisible by 8. What is the minimum number of states that the DFA will have?

OR

bw not

(GATE - 01)

(a) 8 (b) 14

(c) 15 (d) 48

$\underbrace{6 \times 8}$

$= 48$

Home work

Consider the following languages:

$L_1 = \{w\,w \mid w \in \{a, b\}^*\}$ → ?

$L_2 = \{ww^R \mid w \in \{a,b\}^*, w^R$ is the reverse of $w\}$ → ?

regular $L_3 = \{0^{2i} \mid i$ is an integer$\}$ $= (00)^*$

$L_4 = \{0^{i^2} \mid i$ is an integer$\}$ → ?

Which of the languages are regular?

(GATE - 01)

(a) Only L_1 and L_2 (b) Only L_2, L_3 and L_4

(c) Only L_3 and L_4 (d) Only L_3

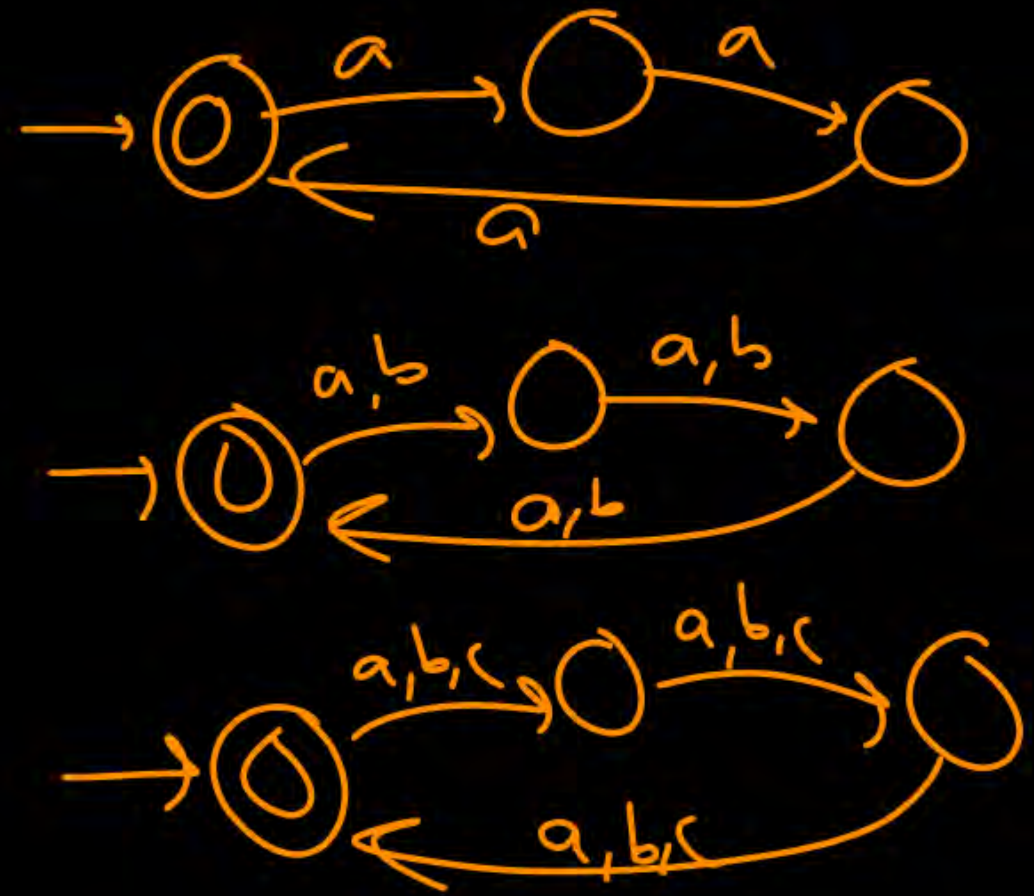
The smallest finite automation which accepts the language

L={x | length of x is divisible by 3} has

(GATE - 02)

(a) 2 states (b) 3 states

(c) 4 states (d) 5states



$x \in \Sigma^*$

$\Sigma = \{a\}$

$\Sigma = \{a, b\}$

$\Sigma = \{a, b, c\}$

⋮

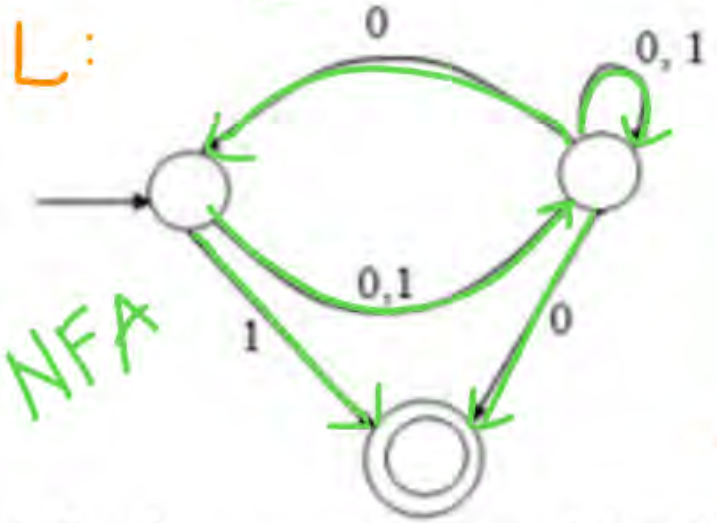
$L_1 \supseteq L$

***

Consider the NFA M shown below.

L:

M:



(a) $L_1 = \{0, 1\}^* - L$

(b) $L_1 = \{0, 1\}^*$

(c) $L_1 \subseteq L$

(d) $L_1 = L$

ε ✗

0 ✗

Let the language accepted by M be L. Let L_1 be the language accepted by the NFA, M_1 obtained by changing the accepting state of M to a non-accepting state and by changing the non-accepting state of M to accepting states. Which of the following statements is true?

only in DFA

⇓

Interchanging finals & non finals we will get complement of language

ε ✓

0 ✓

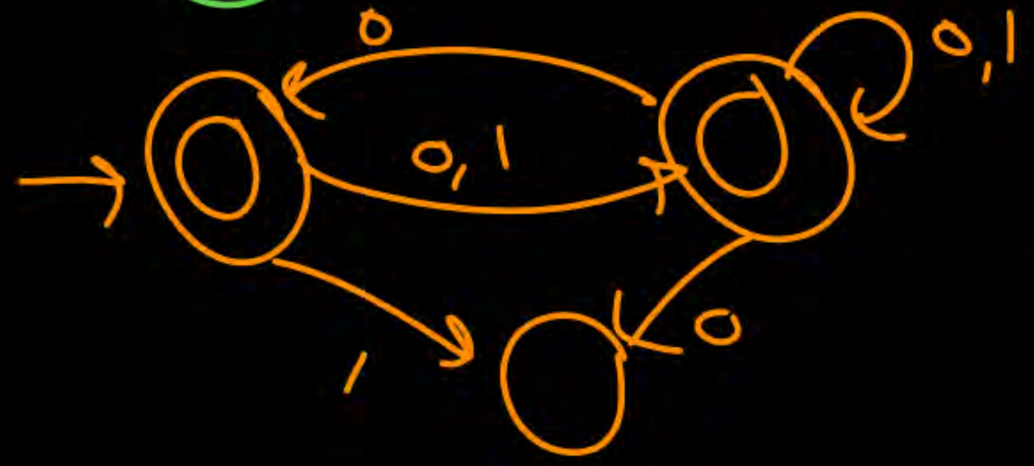
1 ✓

⋮

1

L_1

M_1 :



$L_1 = (0+1)^*$

The following finite state machine accepts all those binary strings in which the number of 1's and 0's are respectively

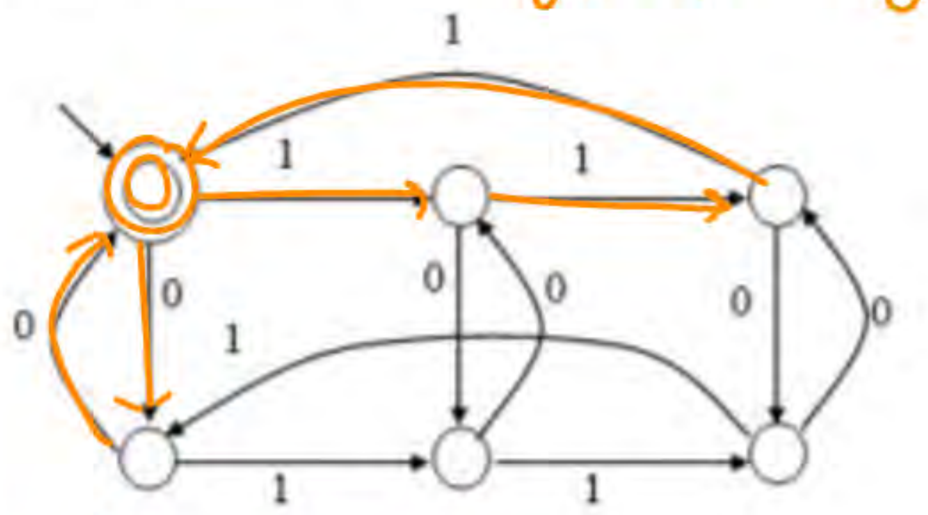
(GATE - 04)

div by 3 div by 2

ε

00

111



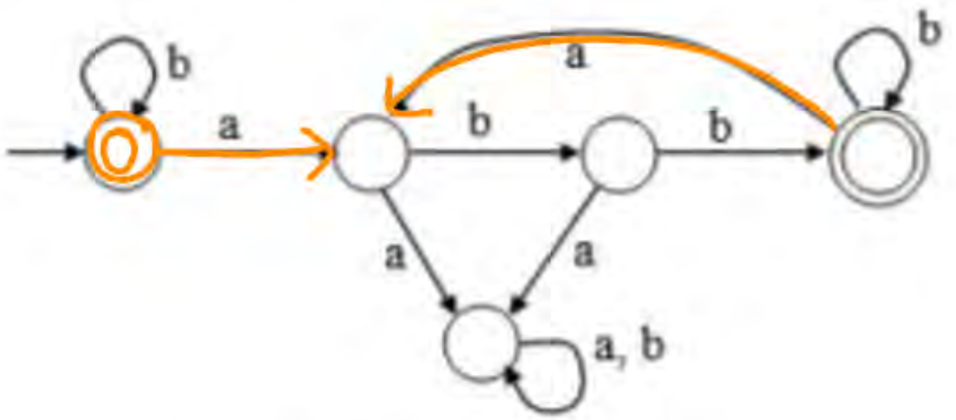
(a) Divisible by 3 and 2

(b) Odd and even

(c) Even and odd

(d) Divisible by 2 and 3

Consider the machine M:



abbb ✓

The language recognized by M is:

(GATE - 05)

abbb ← (a) {w ∈ {a,b}* | every a in w is followed by exactly ~~two~~ b's}

(b) {w ∈ {a,b}* | every a in w is followed by at least two b's} ⟹ ε ✓ b ✓ bb ✓ ⋮

abbaa ← (c) {w ∈ {a,b}* | w contains the substring 'abb'}

(d) {w ∈ {a,b}* | w does not contain 'aa' as a substring}

↳ ε should be accepted

Every student is clever

If object is student then object must be clever

$$\forall x \left(S(x) \Rightarrow C(x) \right)$$

*** Home work

If s is a string over $(0+1)^*$ then let $n_0(s)$ denote the number of 0's in s and $n_1(s)$ the number of 1's in s. Which one of the following languages is not regular? **(GATE - 06)**

(a) $L = \{s \in (0+1)^* \mid n_0(s) \text{ is a 3-digit prime}\}$

*** (b) $L = \{s \in (0+1)^* \mid \text{for every prefix } s' \text{ of } s, |n_0(s') - n_1(s')| \leq 2\}$

(c) $L = \{s \in (0+1)^* \mid |n_0(s) - n_1(s)| \leq 4\}$

(d) $L = \{s \in (0+1)^* \mid n_0(s) \bmod 7 = n_1(s) \bmod 5 = 0\}$

Done in class

Consider the regular language

$L = (111+11111)^*$. The minimum number of states in any DFA accepting this language is

(GATE - 06)

(a) 3 (b) 5 (c) 8 (d) 9

A minimum state deterministic finite automaton accepting the language

$L = \{w \mid w \in \{0, 1\}^*$, number of 0's and 1's in w are divisible by 3 and 5, respectively$\}$ has **(GATE - 07)**

(a) 15 states (b) 11 states

(c) 10 states (d) 9 states

H.W.

Which of the following languages is regular?

(GATE - 07)

(a) $\{ww^R \mid w \in \{0, 1\}^+\}$

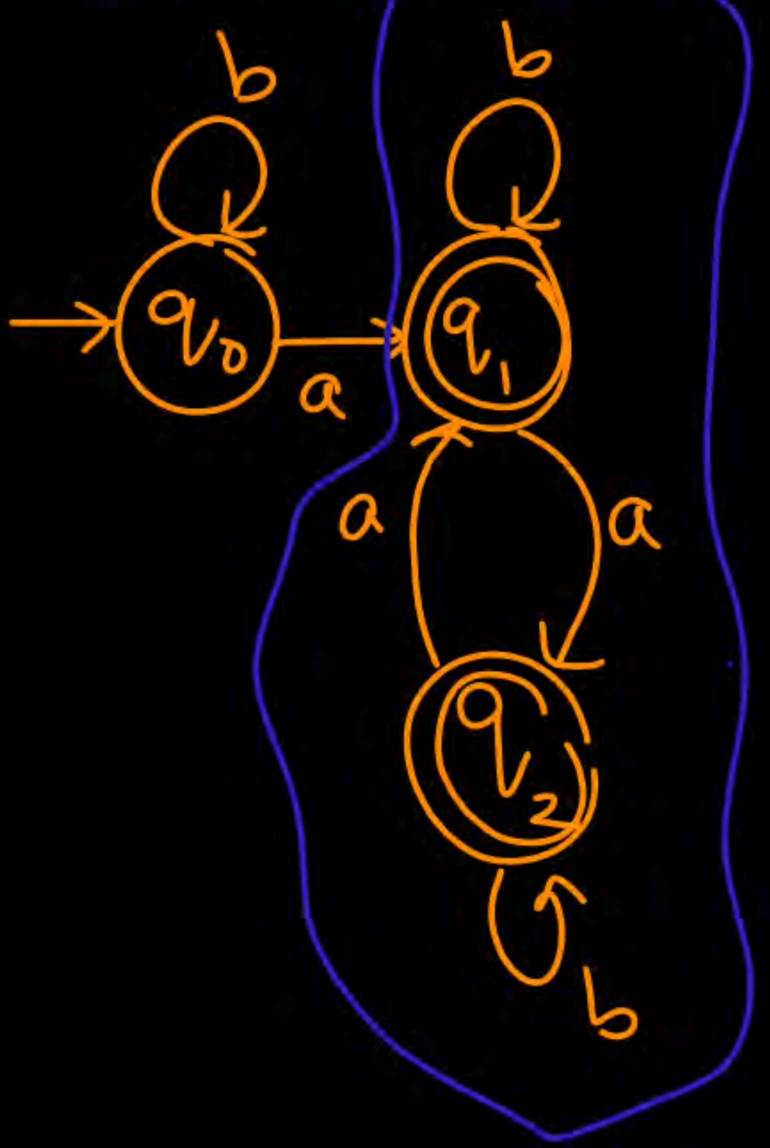
(b) $\{ww^R x \mid x, w \in \{0, 1\}^+\}$

(c) $\{wxw^R \mid x, w \in \{0, 1\}^+\}$

(d) $\{xww^R \mid x, w \in \{0, 1\}^+\}$

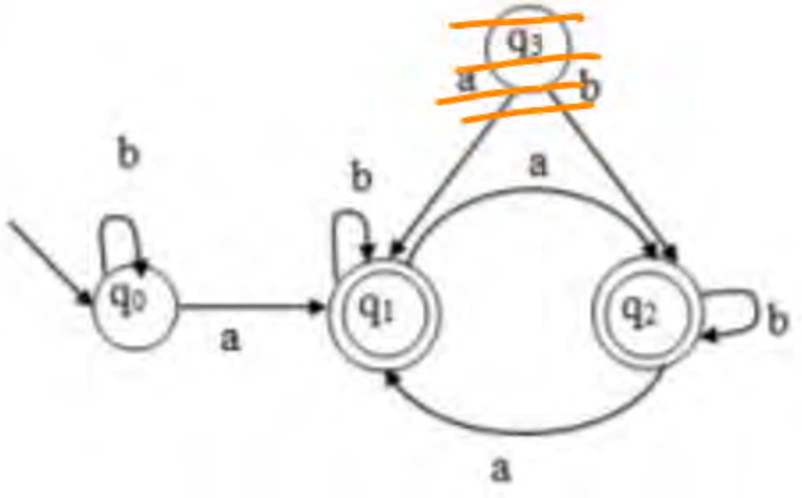
We will cover in Identification of regulars

$b^*a(a+b)^*$



Consider the following finite state automaton

(GATE - 07)



The language accepted by this automaton is given by the regular expression

(a) $b^*ab^*ab^*ab^*$ → exactly 3a's

~~(b)~~ $(a+b)^*$

(c) $b^*a(a+b)^*$ ✓

(d) $b^*ab^*ab^*$ → exactly 2a's

Contains 'a'

***

ε

0

1

00

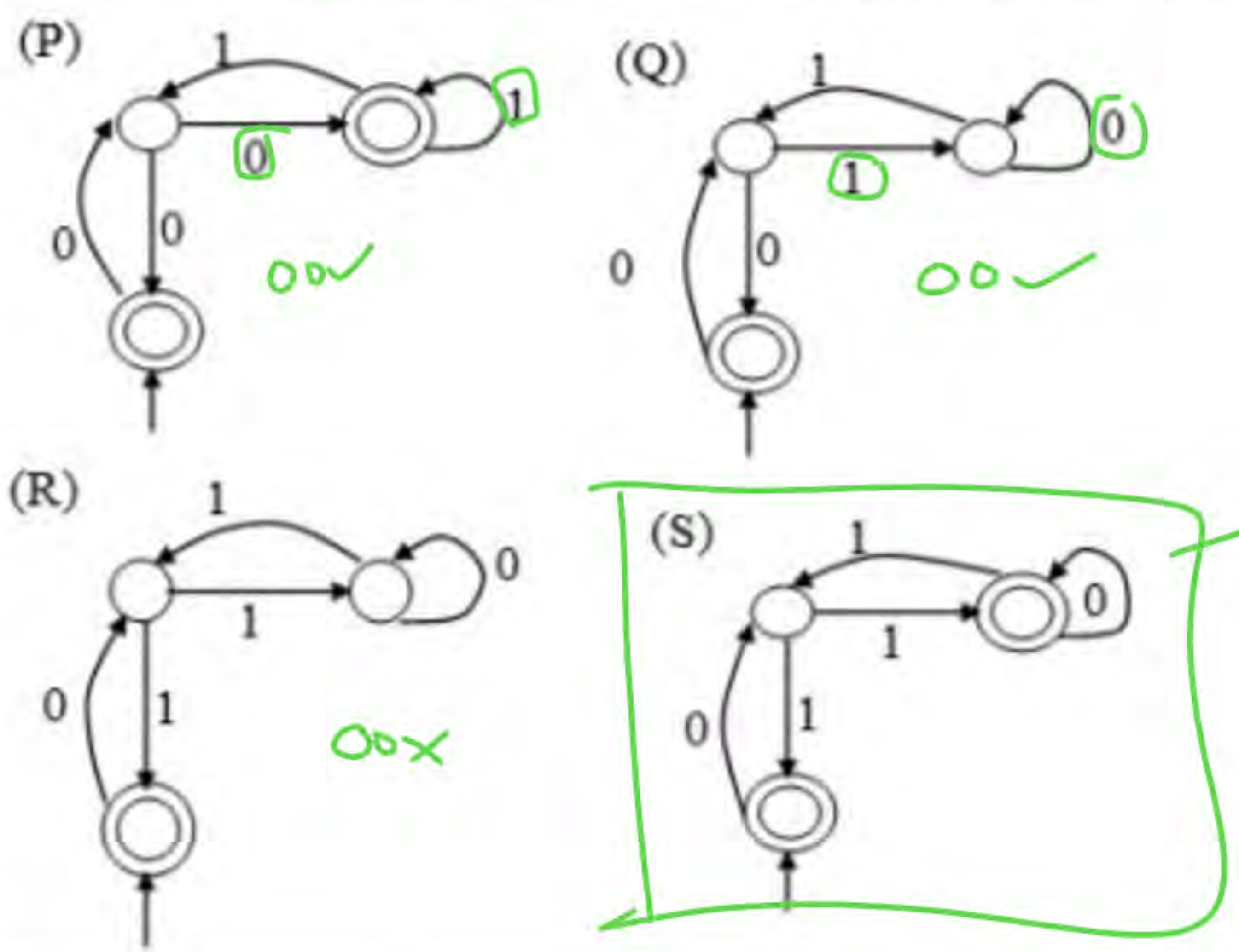
01

10

11

Match the following NFAs with the regular expressions they correspond to

(GATE - 08)



1. $\varepsilon+0(01^*1+00)^*01^*$
2. $\varepsilon+0(10^*1+00)^*0$
3. $\varepsilon+0(10^*1+10)^*1$
4. $\varepsilon+0(10^*1+10)^*10^*$

P

Q

R

S

(a) P-2, Q-1, R-3, S-4

(b) P-1, Q-3, R-2, S-4

(c) P-1, Q-2, R-3, S-4

(d) P-3, Q-2, R-1, S-4

H.W.

Which of the following are regular sets?

I. $\{a^n b^{2m} \mid n \geq 0, m \geq 0\}$

II. $\{a^n b^m \mid n = 2m\}$

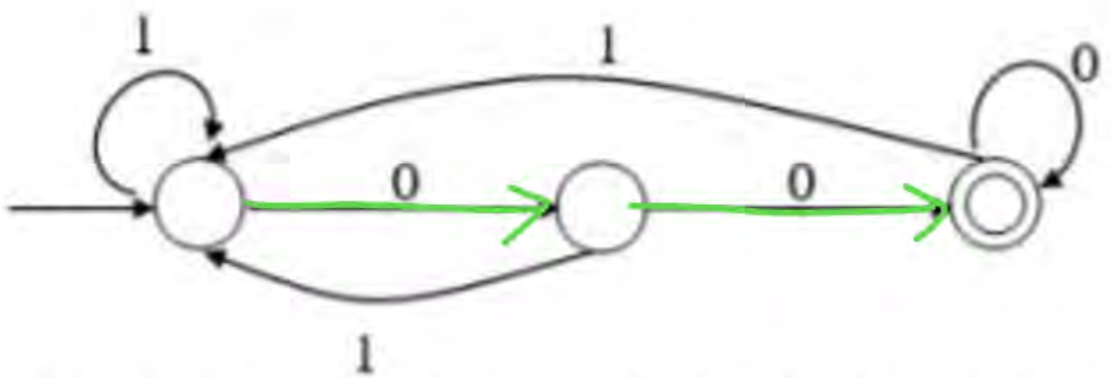
III. $\{a^n b^m \mid n \neq m\}$

IV. $\{xcy \mid x, y \in \{a, b\}^*\}$ **(GATE - 08)**

(a) I & IV only (b) I & III only

(c) I Only (d) IV only

Another topic



The above DFA accepts the set of all strings over {0, 1} that

(GATE - 09)

(a) Begins either with 0 or 1

(b) End with 0

(c) End with 00

(d) Contains the substring 00.

Let $L = \{w \in (0+1)^* \mid w$ has even number of 1's$\}$, i.e L is the set of all bit strings with $\boxed{\text{even number of 1's.}}$ Which one of the regular expressions below represents L?

(GATE - 10)

(a) $(0^*10^*1)^*$ → 0 is missing

(b) $0^*(10^*10^*)^*$

(c) $0^*(10^*1)^*0^*$ → 11011 ×

(d) $0^*1(10^*1)^*10^*$ → ε ×

ε ✓
0 ✓
00 ✓
11 ✓
...
11011 ✓

$0^* (\boxed{\cancel{0^*}} \mid 0^* \mid 0^*)^* \cancel{0^*}$

$\Rightarrow 0^*(10^*10^*)^*$

$(0^*10^*1)^*0^*$

Let w be any string of length n in $\{0, 1\}^*$. Let L be the set of all substrings of w. What is the minimum number of states in a non-deterministic finite automation that accepts L?

(GATE-10)

(a) $n - 1$
(b) n
(c) $n+1$
(d) 2^{n-1}

H.W.

***

Definition of the language L with alphabet {a} is given as following.

$L = \{a^{nk} | k > 0$, and n is a positive integer constant$\}$

What is the minimum number of states needed in a DFA to recognize L?

(GATE - 11)

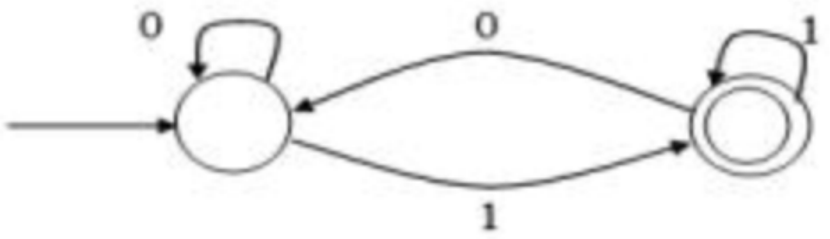
(a) $k + 1$ (b) $n + 1$

(c) 2^{n+1} (d) 2^{k+1}

HW.

Which of the regular expression given below represent the following DFA?

(GATE – 14-SET1)

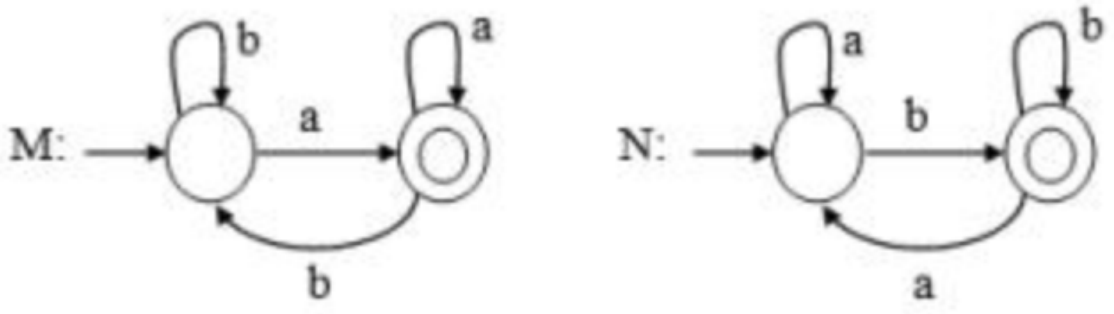


I. 0*1(1+00*1)*

II. 0*1*1+11*0*1

III. (0+1)*1

(a) I and II only
(b) I and III only
(c) II and III only
(d) I, II, and III



Consider the DFAs M and N given above. The number of states in a minimal DFA that accepts the languages $L(M) \cap L(N)$ is ________. **(GATE – 15 – SET1)**

The number of states in the minimal deterministic finite automaton corresponding to the regular expression $\underbrace{(0 + 1)^* (10)}$ is ___3___.

(GATE – 15- SET2)

Consider the alphabet $\Sigma = \{0, 1\}$, the null/empty string λ and the set of strings X_0, X_1, and X_2 generated by the corresponding non-terminals of a regular grammar X_0, X_1, and X_2 are related as follows.

$$X_0 = 1\,X_1$$
$$X_1 = 0\,X_1 + 1\,X_2$$
$$X_2 = 0\,X_1 + \{\lambda\}$$

Which one of the following choices precisely represents the strings in X_0?

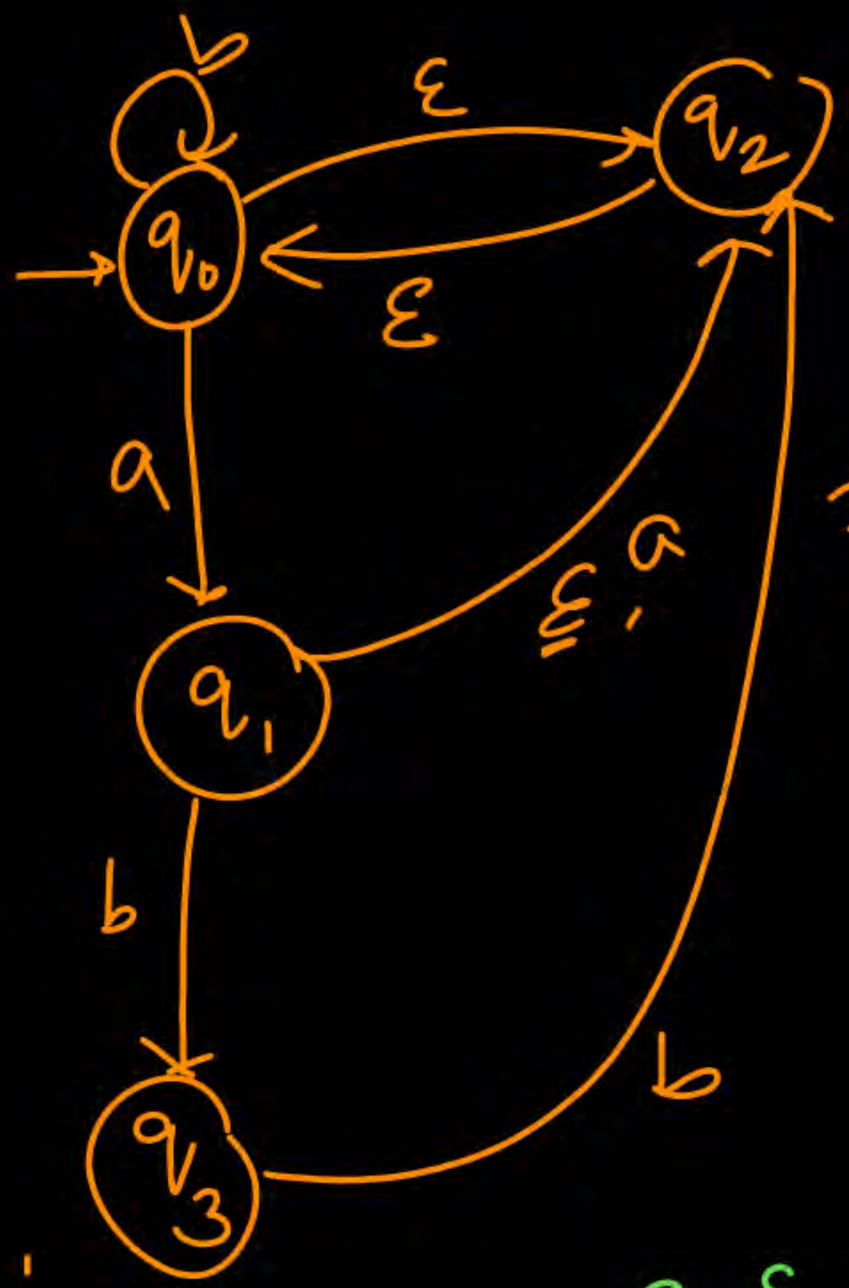
(GATE – 15- SET2)

(a) 10(0*+(10)*)1

(b) 10(0*+(10*))*1

(c) 1(0+10)*1

(d) 10(0+10)*1+110(0+10)*1



Let δ denote the transition function and $\hat{\delta}$ denote the extended transition function of the ε–NFA whose transition table is give below:

Design ε-NFA

δ	ε	a	b
$\rightarrow q_0$	$\{q_2\}$	$\{q_1\}$	$\{q_0\}$
q_1	$\{q_2\}$	$\{q_2\}$	$\{q_3\}$
q_2	$\{q_0\}$	ϕ	ϕ
q_3	ϕ	ϕ	$\{q_2\}$

$\{q_1, q_2, q_0\}$

Then $\hat{\delta}(q_2, aba)$ is (GATE – 17 – SET2)

(a) ϕ

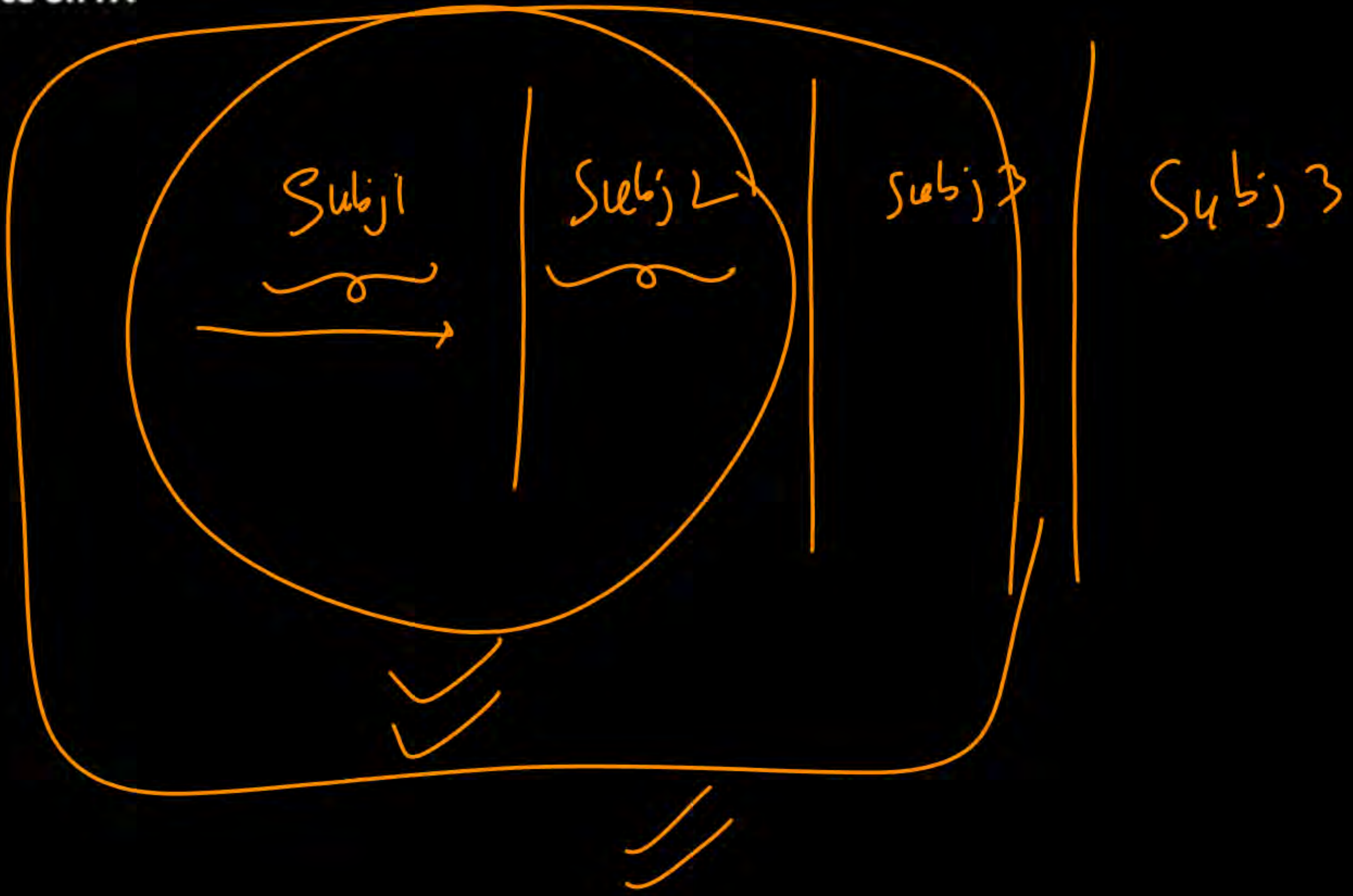
(b) $\{q_0, q_1, q_3\}$

(c) $\{q_0, q_1, q_2\}$

(d) $\{q_0, q_2, q_3\}$

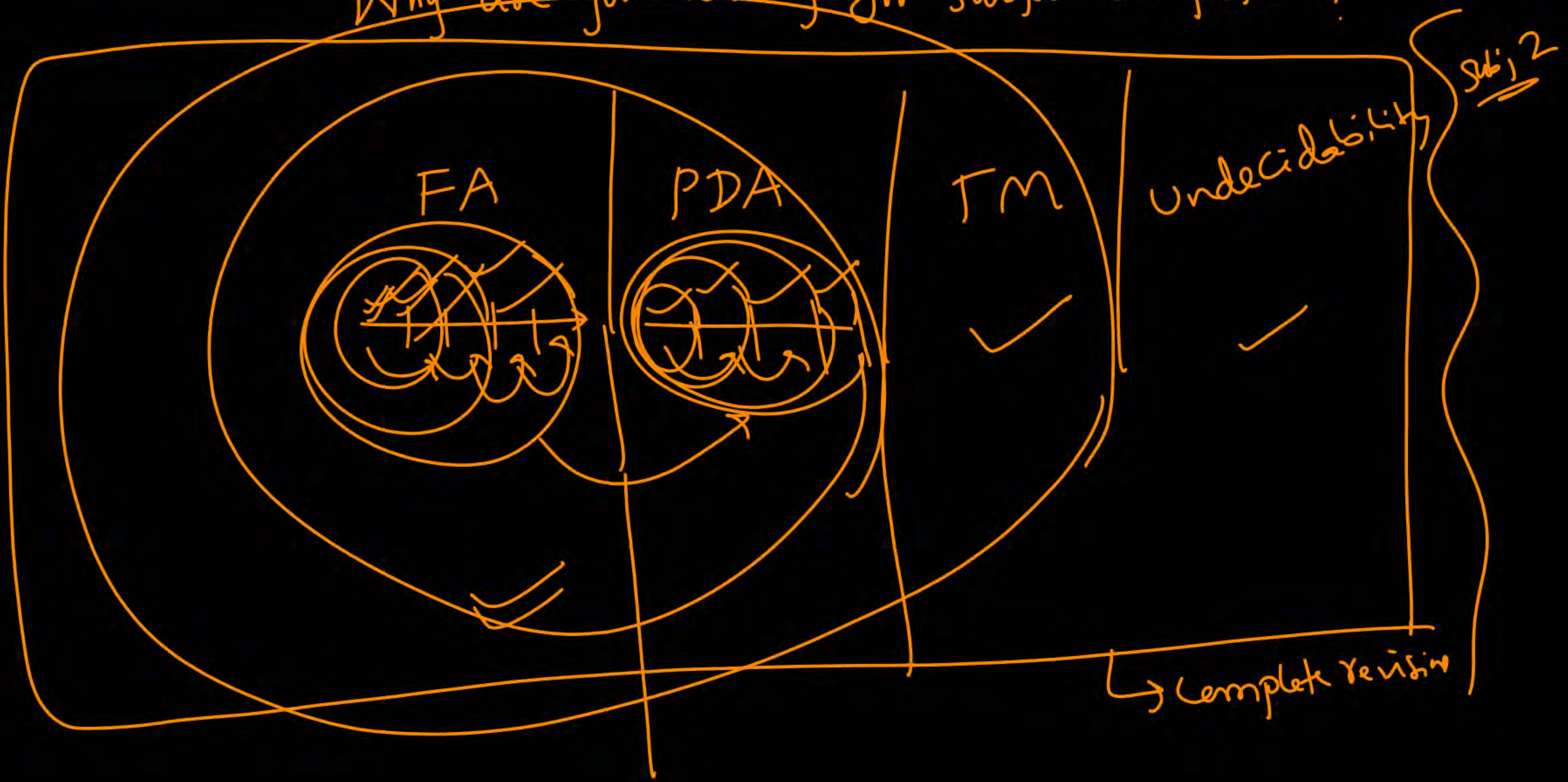
$q_2 \xrightarrow{\varepsilon} q_0 \xrightarrow{a} q_1 \xrightarrow{\varepsilon} q_2 \xrightarrow{\varepsilon} q_0 \xrightarrow{b} q_0 \xrightarrow{a} \boxed{q_1} \xrightarrow{\varepsilon} \boxed{q_2} \xrightarrow{\varepsilon} \boxed{q_0}$

Practice on FA



Practice on FA

Why are you waiting for Subject Completion ?



Summary

Practice on FA/RegExp

GATE

Thank you