CS & IT Engineering

Deva sir

Topics to be covered:

1. CFG
2. Types of CFG
3. Derivation of a string
4. CFL Vs CFG

Topics Covered in Previous Session:

↳ Doubts ⇒ Algorithms

Grammar $(G) = (V, T, P, S)$

1) Regular Grammars $\Rightarrow$ LLG & RLG

$V \to VT^* / T^*$ $\quad$ $V \to T^*V \mid T^*$

2) Linear Grammars $\Rightarrow$ $\boxed{V \to T^* V T^* \mid T^*}$

Every LLG is LG

Every RLG is LG

Eg :- $S \to abSa \mid \varepsilon$

3) Context Free Grammar

$\boxed{V \to (VUT)^*}$

$S \to aSBA \mid ab$

$A \to ABa \mid \varepsilon$

$B \to ab$

- It generates a CFL
- It is used to represent Syntax of programming language

1. How to derive a string? (LMD, RMD, Parse Tree)
2. Types of CFGs (Ambiguous CFG and Unambiguous CFG)
3. CFG Vs CFL

Derivation of a String:

① Left Most Derivation (LMD)

② Right most Derivation (RMD)

} Linear

③ Parse Tree (Non-linear)
(Derivation Tree)

LMD: In each sentential form, left most nonterminal is Substituted - to derive a string

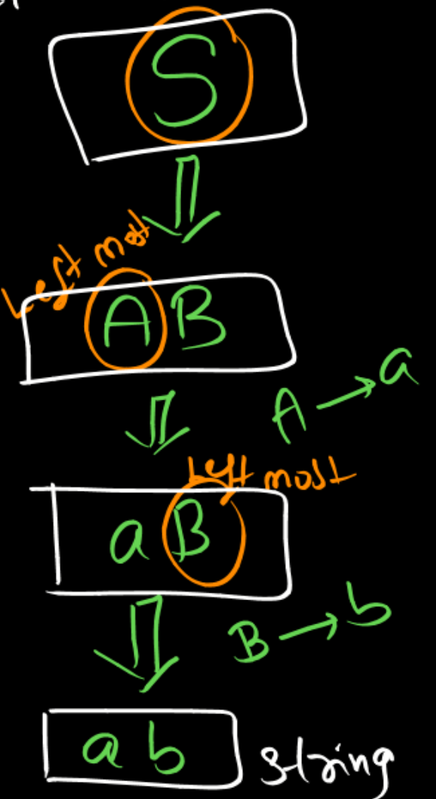
$S \rightarrow AB \mid a$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$

$w = ab$

LMD



Derivative step

Sentential Form

String

No. of derivative steps = No. of Substitutions

Sentential form:

↳ Sequence of terminals and non-terminals derived from start symbol

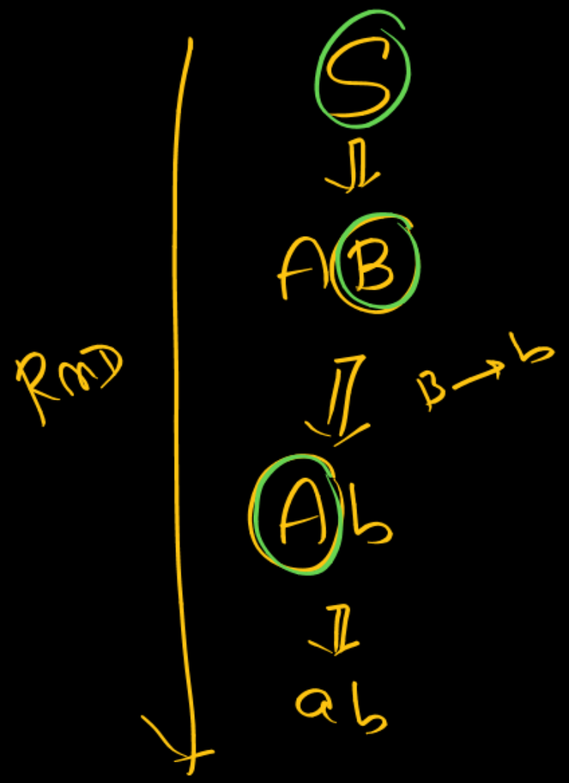
RMD: In each sentential form, right most non-terminal substituted - to derive a string.

$S \rightarrow AB \mid a$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$

$w = ab$



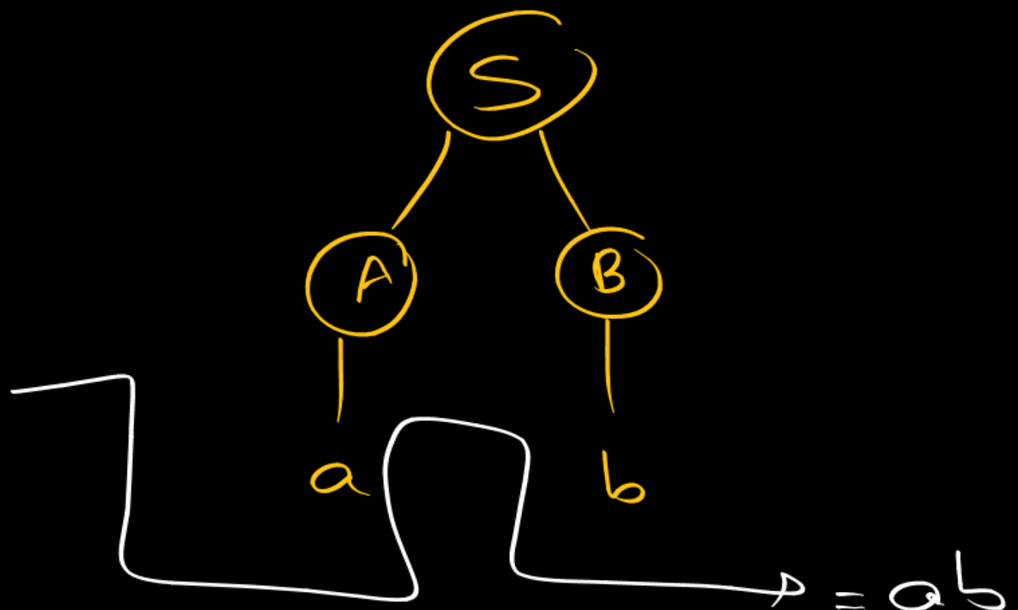
No. of steps = 3

Parse Tree :

$S \rightarrow AB \mid a$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$



$= ab$

$w = ab$

No. of steps = No. of non leaf nodes = 3

Root : start symbol

Non leaf node : Non-terminal

Leaf node : Terminal or ε

LMD Vs RMD Vs Parse tree:

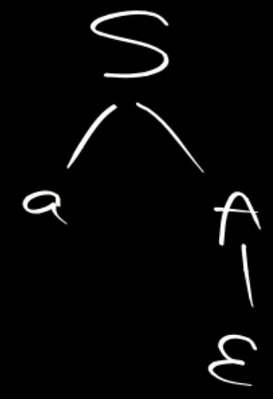
For particular string:

No. of LMDs = No. of RMDs = No. of Parse trees

$S \to aA \mid a$

$A \to \varepsilon \mid b$

$w = a$



2 Parse Trees
=
2 LMDs
=
2 RMDs

⟶ LMD and RMD need not be same

$S \rightarrow AB$

$A \rightarrow a$

$B \rightarrow b$

$w = ab$

LMD:

$S \Rightarrow AB$

$\Rightarrow aB$

$\Rightarrow ab$

RMD:

$S \Rightarrow AB$

$\Rightarrow Ab$

$\Rightarrow ab$

$S \rightarrow a$

LMD:

$S \Rightarrow a$

RMD

$S \Rightarrow a$

Types of CFGs :

① Ambiguous CFG

For Some string, more than one parse tree exist

Ambiguous CFG

$S \rightarrow AB \mid ab$

$A \rightarrow aB \mid a$

$B \rightarrow bB \mid b$

$w = ab$

↳ 2 parse trees

② Unambiguous CFG

For every string, exactly one parse tree

Unambiguous CFG

$S \rightarrow AB$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$

$ab \Rightarrow$ 1 PT

$aab \Rightarrow$ 1 PT

$abb \Rightarrow$ 1 PT

⋮

CFG Vs CFL :

① $S \rightarrow aS \mid b$

$L = a^* b$

② $S \rightarrow aS \mid a$

$L = aa^* = a^+$

③ $S \rightarrow aS \mid \varepsilon \Rightarrow L = a^*$

④ $S \rightarrow aS \mid bS \mid \varepsilon \Rightarrow L = (a+b)^*$

⑤ $S \rightarrow Sa \mid Sb \mid \varepsilon \Rightarrow L = (a+b)^*$

⑥ $S \rightarrow aSb \mid c$

c ✓

acb ✓

aacbb ✓

aaacbbb ✓

$L = \{ a^n c\, b^n \mid n \geq 0 \}$

S
||
aSb
||
aaSbb

⑦ $S \rightarrow aSb \mid \varepsilon$

$L = \{ a^n b^n \mid n \geq 0 \}$

(8) $S \rightarrow AB$

$A \rightarrow aAb \mid \varepsilon \Rightarrow a^n b^n$

$B \rightarrow cBd \mid \varepsilon \Rightarrow c^m d^m$

$$L = \{a^n b^n c^m d^m \mid n, m \geq 0\}$$

(9) $S \rightarrow aSb \mid A$

$A \rightarrow cAd \mid \varepsilon \Rightarrow c^n d^n$

$$L = \{a^k c^n d^n b^k\}$$

(10) $S \rightarrow aaSbb \mid \varepsilon$

$L = (aa)^n (bb)^n$

$= \{ a^{2n} b^{2n} \mid n \geq 0 \}$

(11) $S \rightarrow aSa \mid \varepsilon$

$L = a^n \varepsilon a^n = a^{2n} = (aa)^*$

(12) $S \rightarrow aaS \mid \varepsilon$

$L = (aa)^*$

(13) $S \rightarrow Saa \mid \varepsilon$

$L = (aa)^*$

(14) $S \rightarrow aSa \mid bSb \mid \varepsilon$

$L = \{ww^R \mid w \in \{a,b\}^*\}$

= Set of all even length palindromes

abaa ε aaba

(15) $S \rightarrow aSa \mid bSb \mid a \mid b$

$L = \{wxw^R \mid w \in \{a,b\}^*, x \in \{a,b\}\}$

= Set of all odd length palindromes

a ✓
b ✓
aaa
aba

(16) $S \rightarrow aSa \mid bSb \mid a \mid b \mid \varepsilon$

$L = \{w \mid w = w^R, w \in \{a,b\}^*\}$

= Set of all palindromes

(17) $S \rightarrow aSa \mid bSb \mid \#$

$L = \{ w \# w^R \mid w \in \{a,b\}^* \}$

(18) $S \rightarrow AB$

$A \rightarrow aA \mid \varepsilon \Rightarrow a^*$

$B \rightarrow bB \mid \varepsilon \Rightarrow b^*$

$L = a^* b^*$

(19) $S \to SS \mid \varepsilon$

$L = \{\varepsilon\}$

(20) $S \to SS \mid a$

$L = a^+$

a ✓

aa ✓

aaa ✓

a^4 ✓

a^5 ✓

(21) $S \to SA \mid \varepsilon$

$A \to aa$

$L = A^* = (aa)^*$

(22) $S \to SA \mid \varepsilon$

$A \to aAb \mid \varepsilon \Rightarrow \{a^n b^n\}$

$L = A^* = \{a^n b^n\}^*$

$= \{a^{n_1} b^{n_1} a^{n_2} b^{n_2} a^{n_3} b^{n_3} \ldots a^{n_k} b^{n_k}\}$

(23)

$S \rightarrow AB$

$A \rightarrow aAb \mid \varepsilon \rightarrow a^n b^n$

$B \rightarrow bB \mid \varepsilon \rightarrow b^*$

$L = \{a^n b^n b^*\}$

$= \{a^i b^j \mid i \leq j\}$

(24)

$S \rightarrow AB$

$A \rightarrow aA \mid \varepsilon$

$B \rightarrow aBb \mid \varepsilon$

$L = \{a^* a^n b^n\}$

$= \{a^i b^j \mid i \geq j\}$

(25) $S \rightarrow aSa \mid aaSaa \mid \varepsilon$

$L = (aa)^*$

ε ✓
aa ✓
$aaaa$ ✓
a^6 ✓
a^8 ✓

(26) $S \rightarrow aSb \mid aaSbb \mid \varepsilon$

$L = \{a^n b^n \mid n \geq 0\}$

Summary

① $S \rightarrow aS$

$L = \phi$

② $S \rightarrow AB$

$A \rightarrow a$ $\quad L = \phi$

Thank you