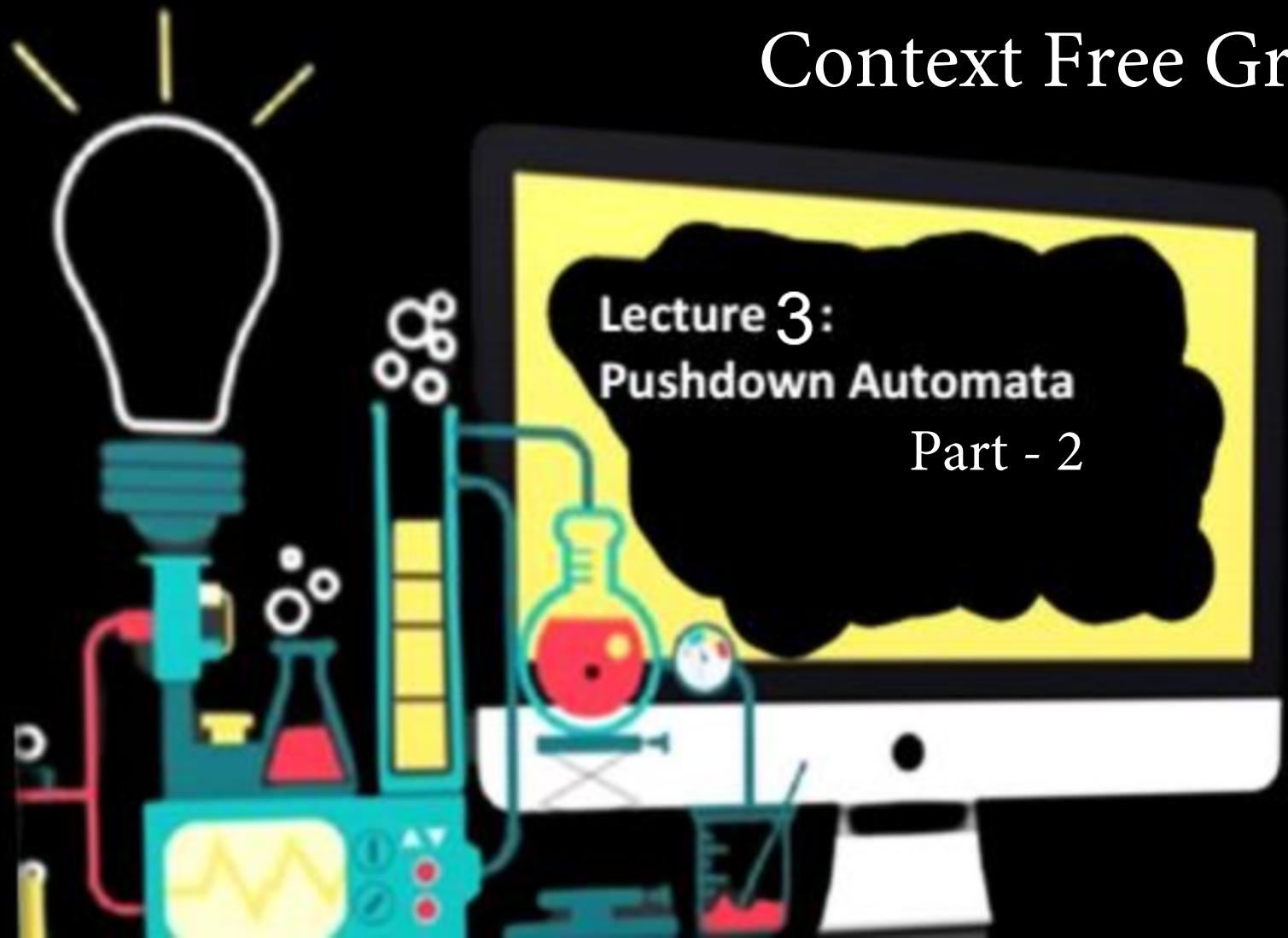


CS & IT Engineering

Context Free Grammar



Deva sir

Topics to be covered:

↳ Construction of PDA

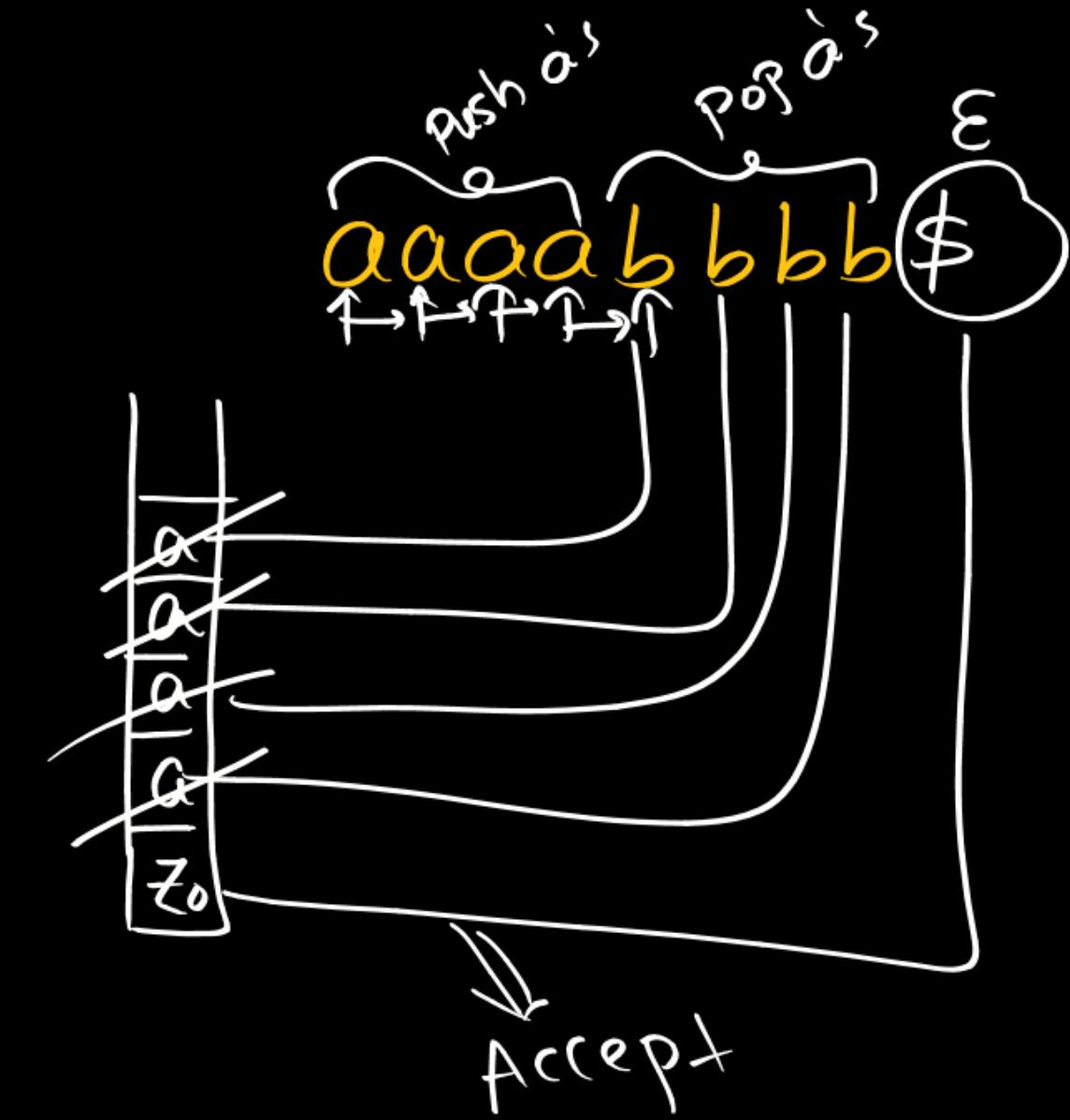
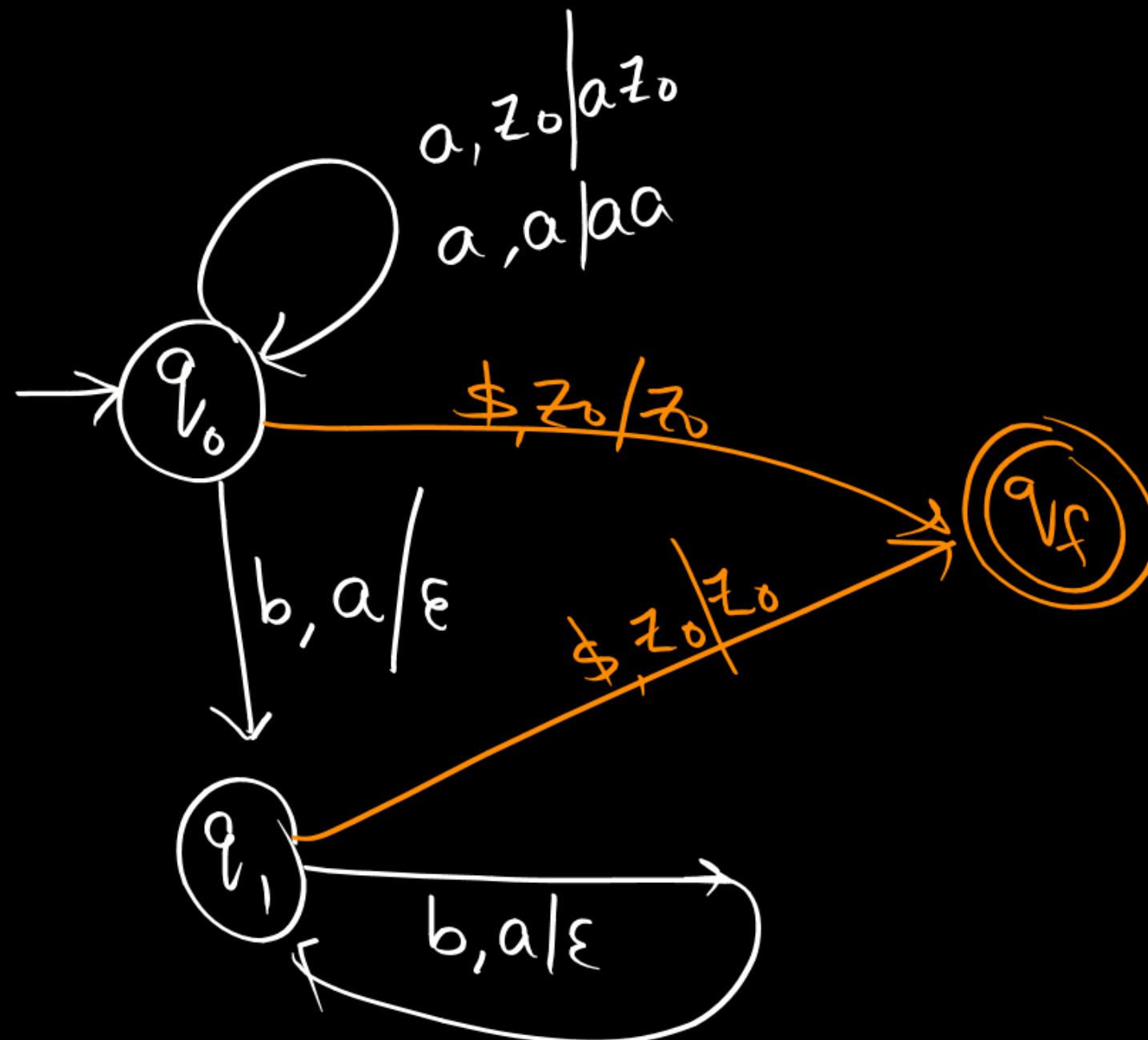
Topics Covered in Previous Session:

↳ PDA Basics

Pushdown Automata

P
W

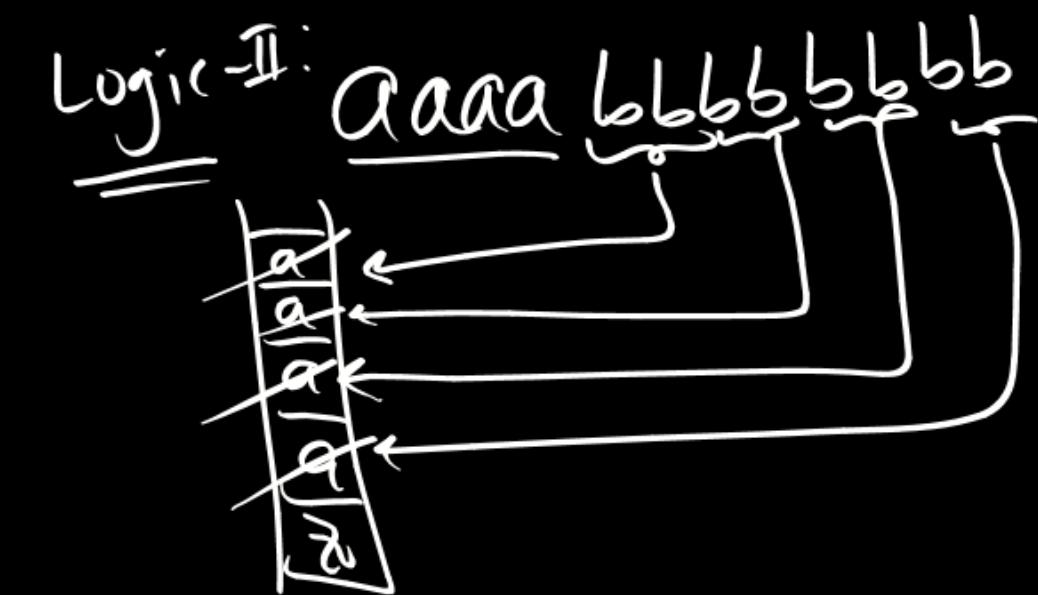
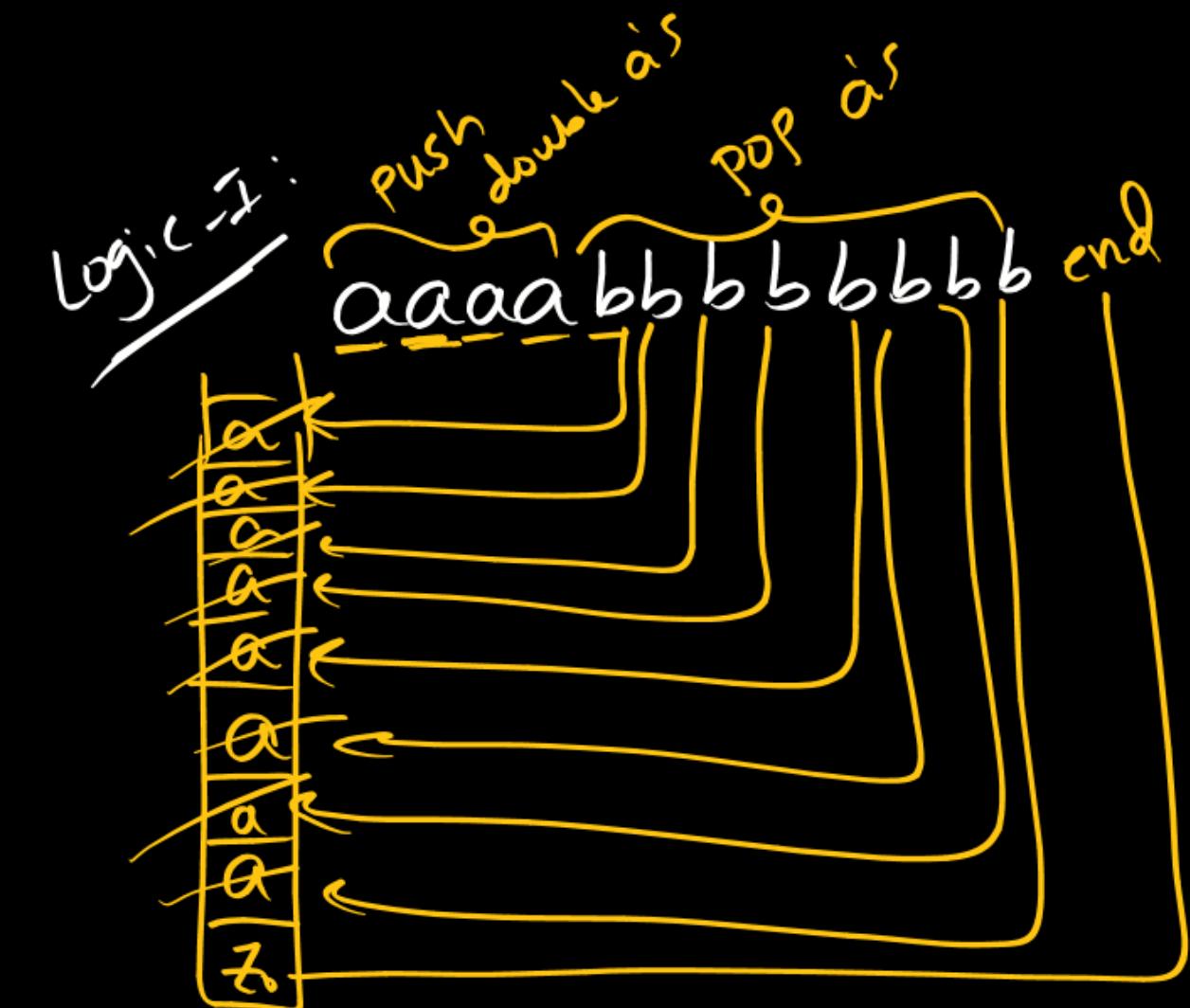
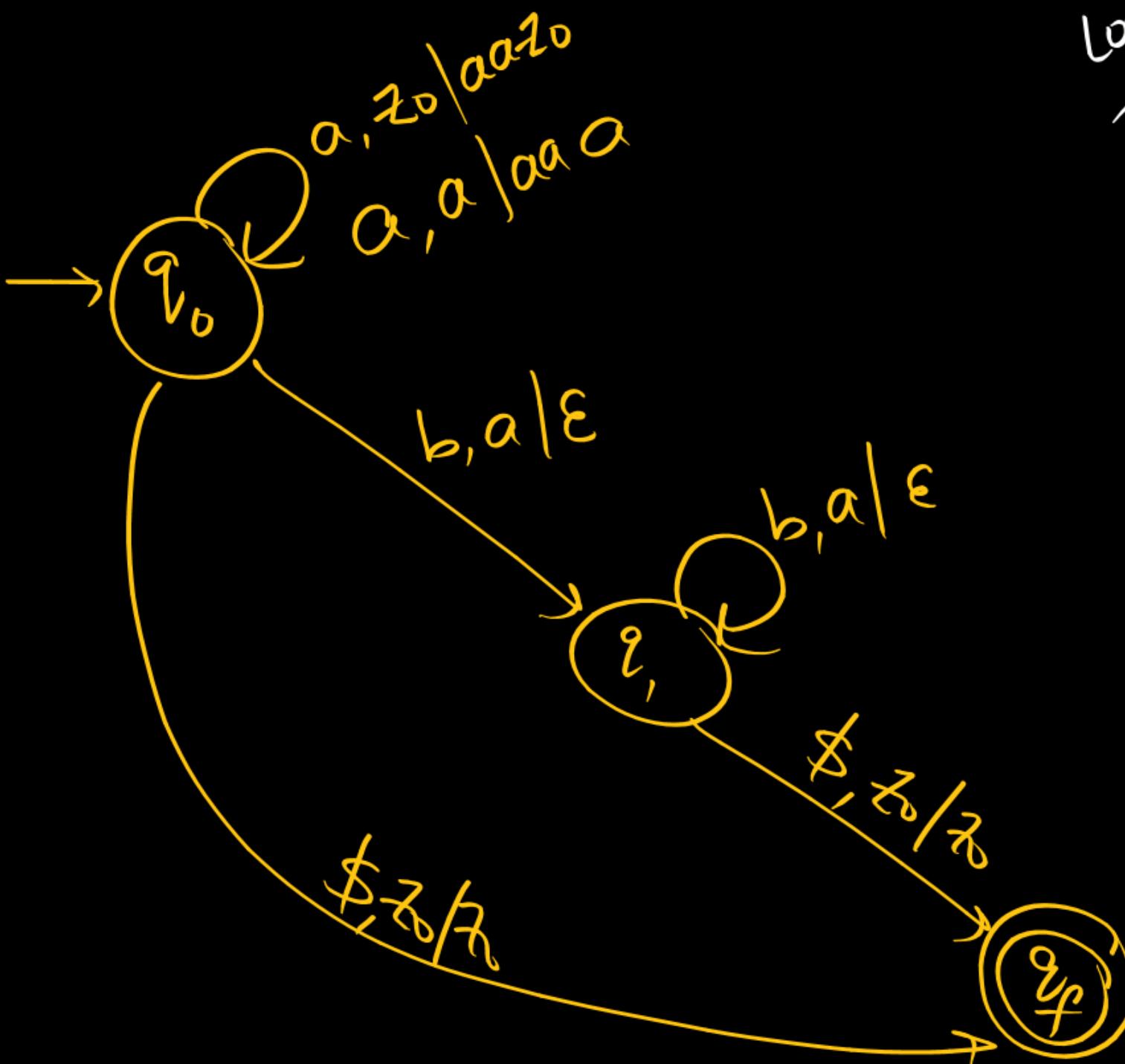
① $\{a^n b^n \mid n \geq 0\}$



Pushdown Automata

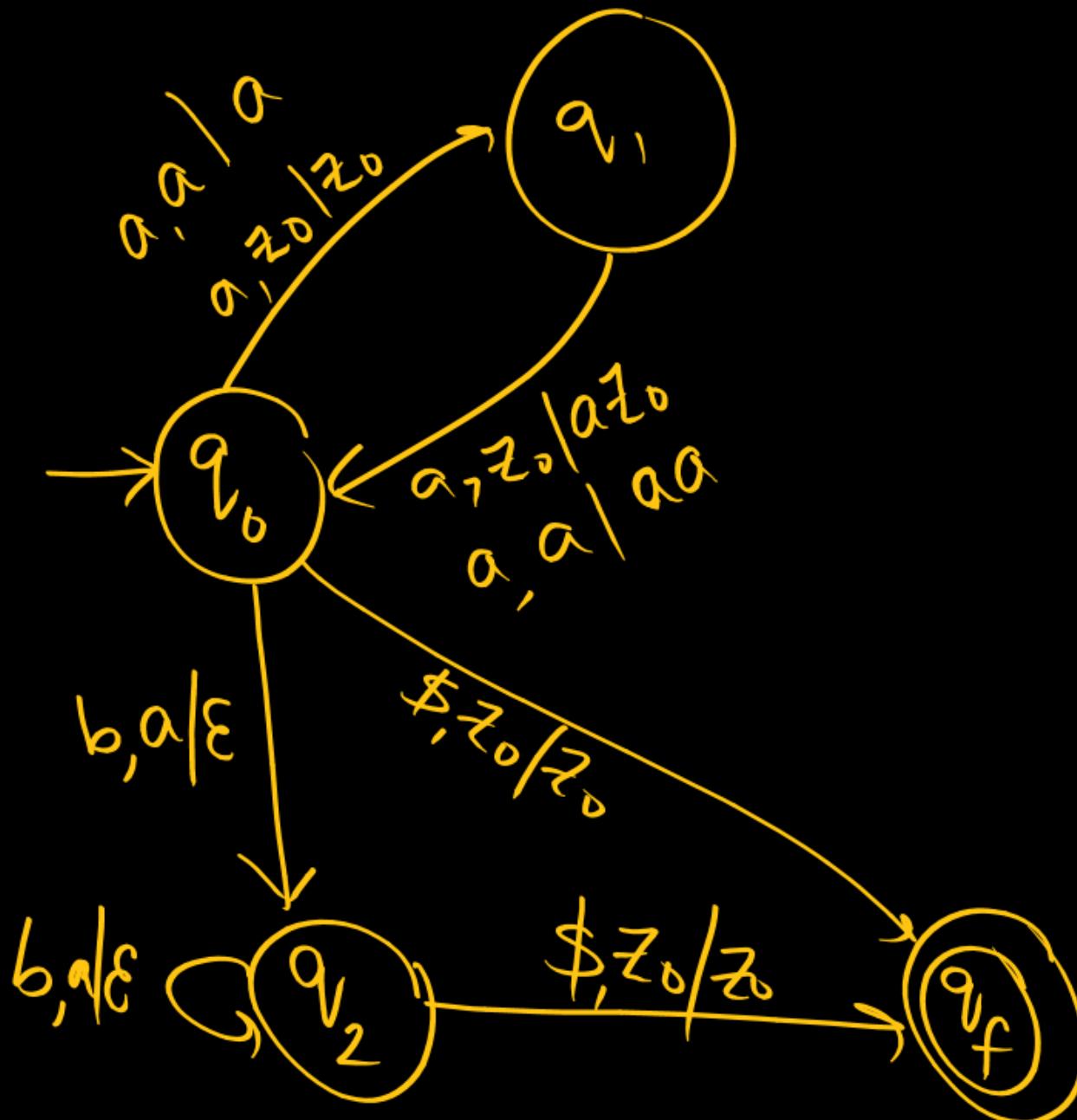
P
W

$$\textcircled{2} \quad \{a^n b^{2n} \mid n \geq 0\}$$



Pushdown Automata

③ $\{a^{2n} b^n \mid n \geq 0\}$



Pushdown Automata

P
W

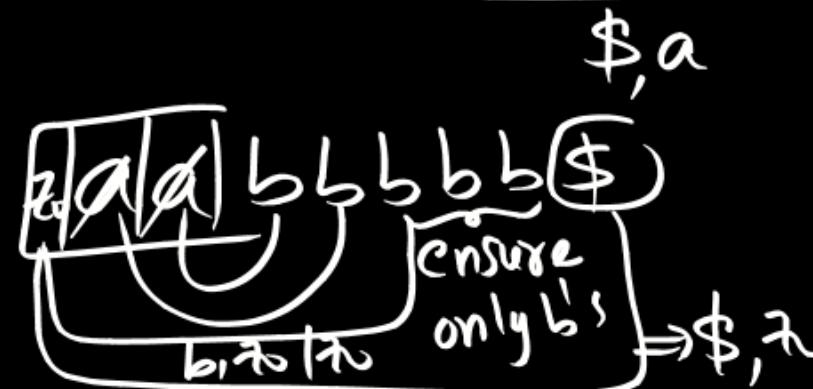
$$\textcircled{4} \quad \{a^m b^n \mid m > n\}$$

$\#a's > \#b's$



$$\textcircled{5} \quad \{a^m b^n \mid m < n\}$$

$\#a's < \#b's$



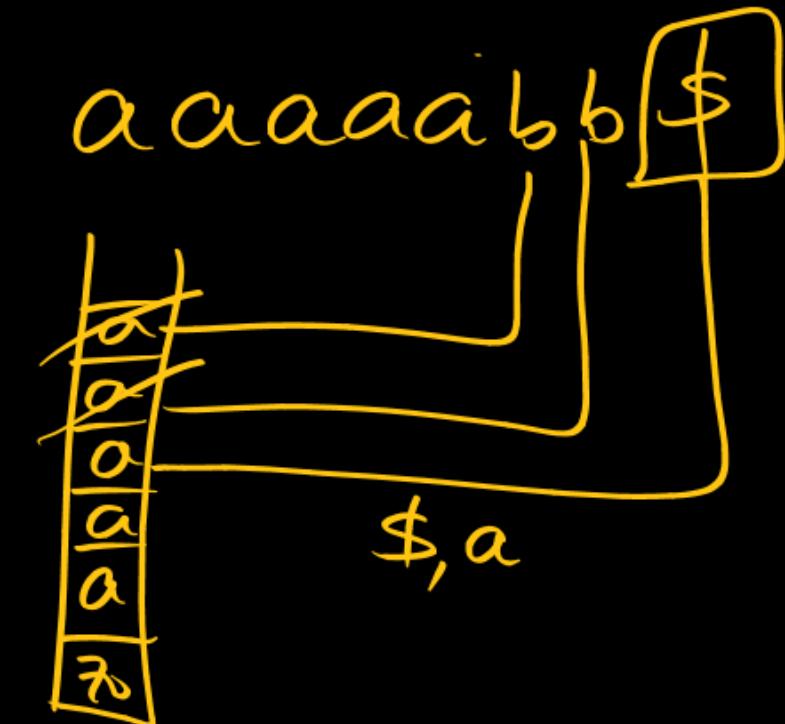
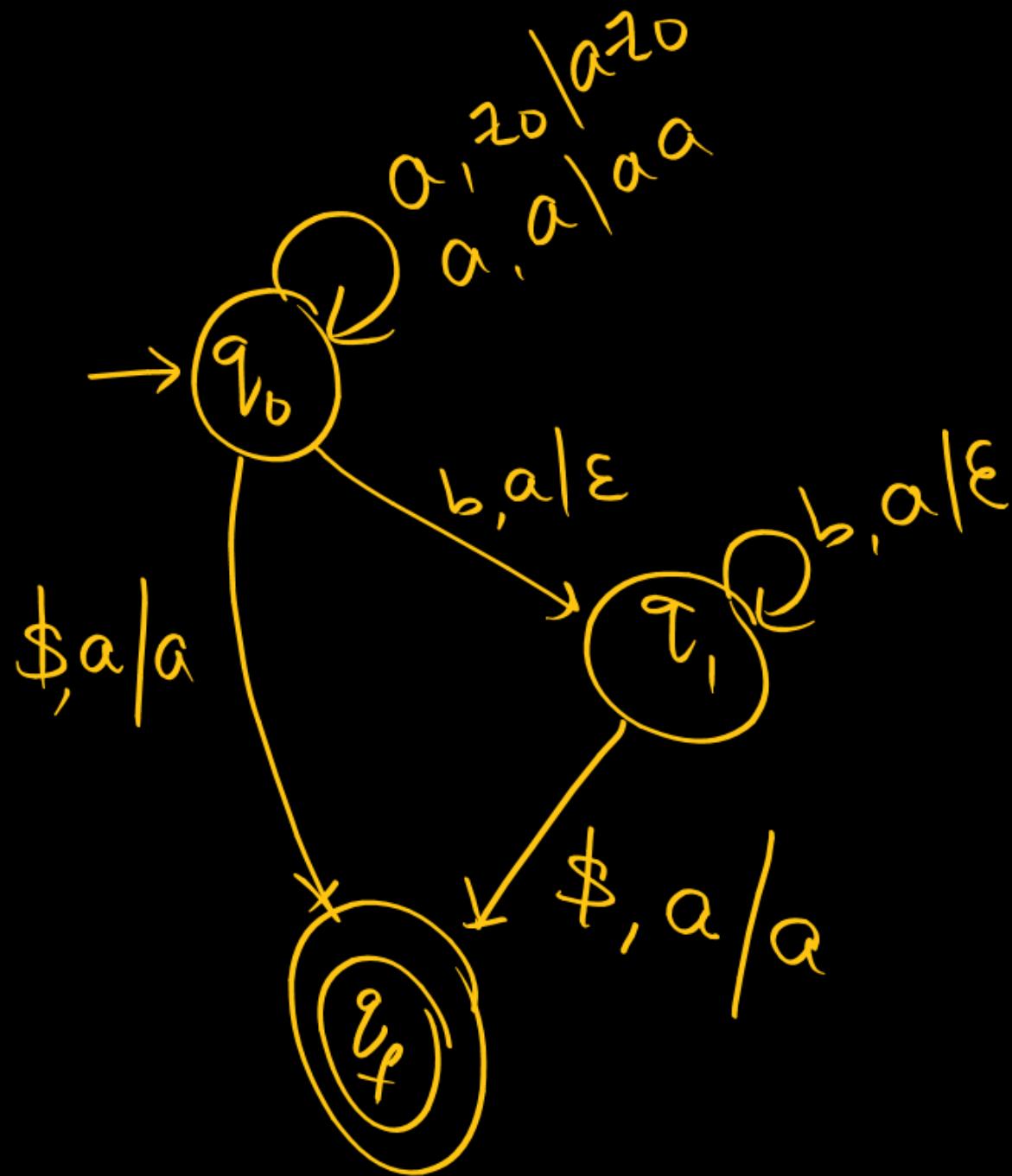
$$\textcircled{6} \quad \{a^m b^n \mid \underbrace{m \neq n}_{m > n \text{ or } m < n}\}$$

$$\textcircled{7} \quad \{a^m b^n \mid \underbrace{m \geq n}_{m > n \text{ or } m = n}\}$$

$$\textcircled{8} \quad \{a^m b^n \mid \underbrace{m \leq n}_{m < n \text{ or } m = n}\}$$

Pushdown Automata

④ $\{a^m b^n \mid m > n\}$
 $m, n \geq 0$



aaa

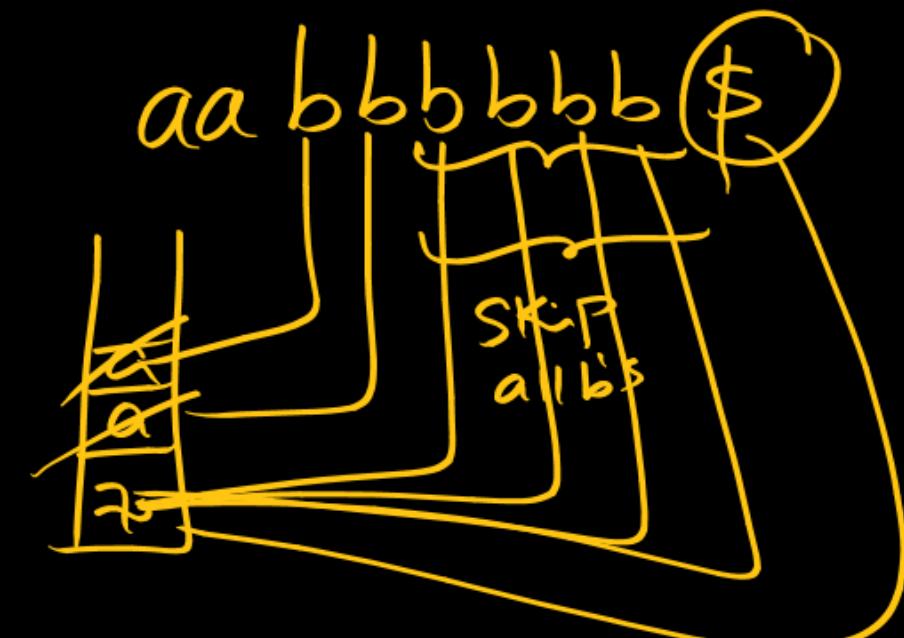
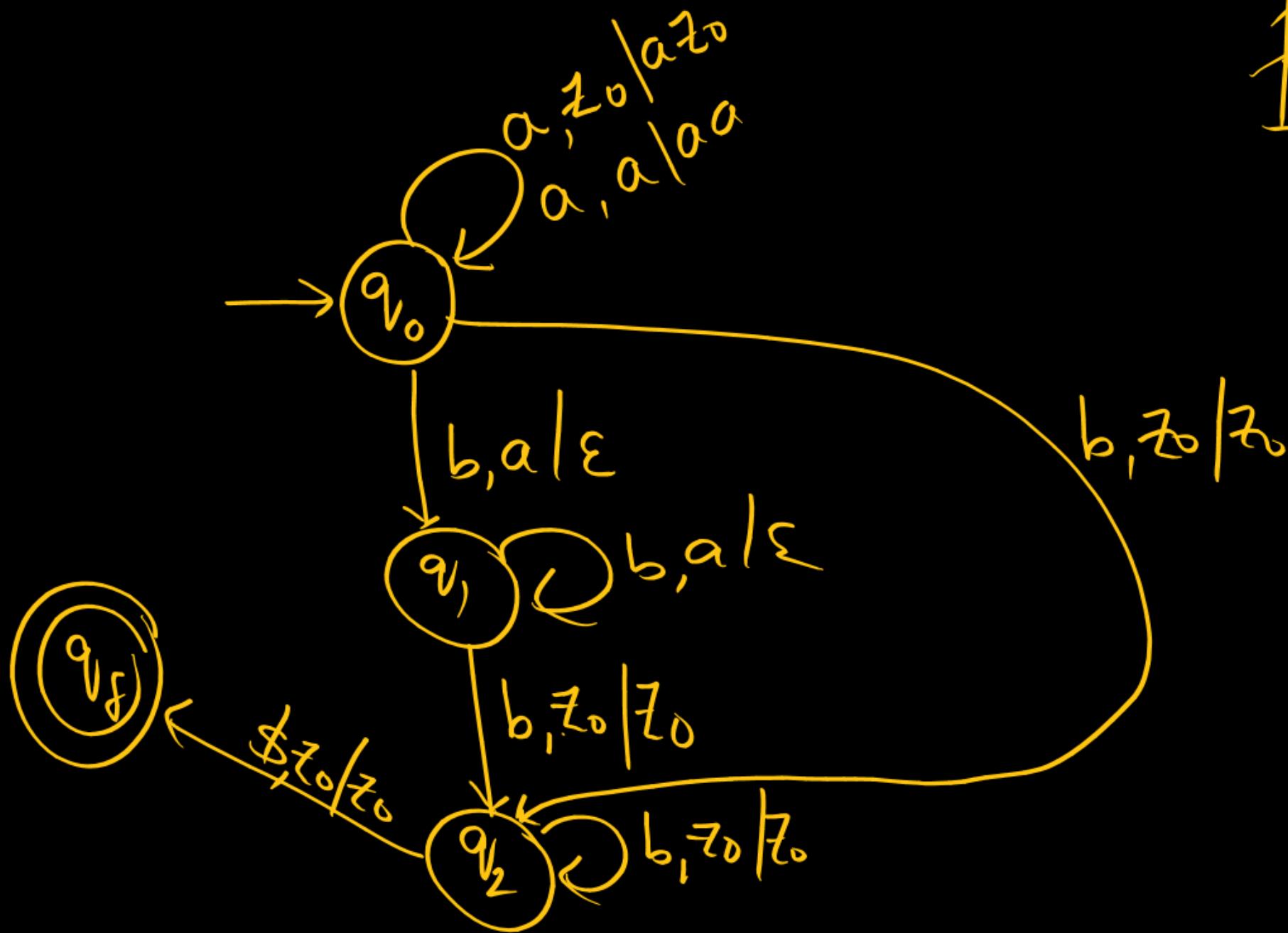
$a^3 b^2$

$a^m b^n \mid m > n$

Pushdown Automata

P
W

$$⑤ \{a^m b^n \mid m < n\}$$



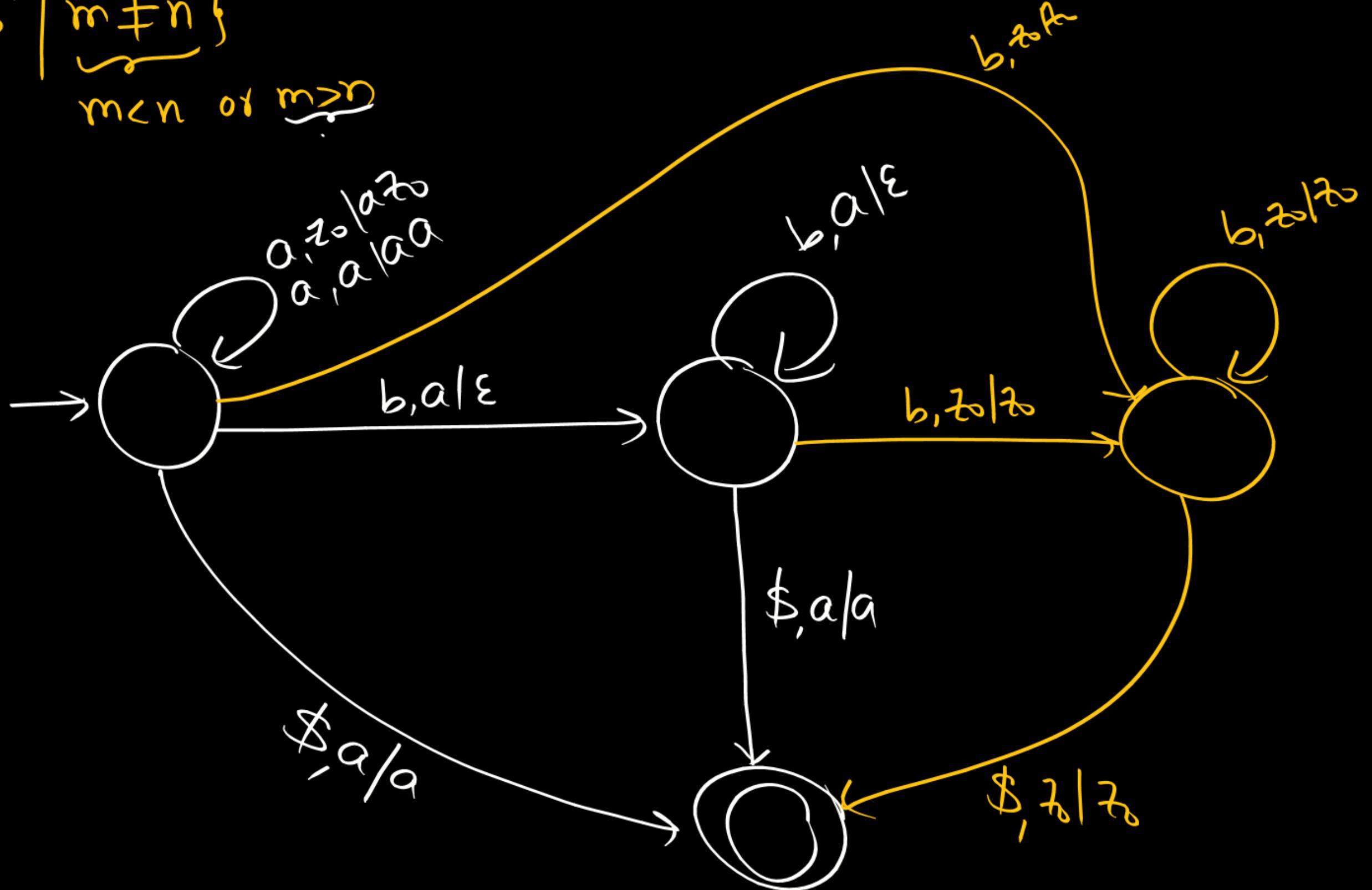
b
bb
bbb

Pushdown Automata

P
W

$$⑥ \{ a^m b^n \mid \underbrace{m \neq n} \}$$

$m < n$ or $m > n$



Pushdown Automata

XV. $\left\{ \begin{array}{l} \textcircled{7} \quad \{a^m b^n \mid m \geq n\} \\ \textcircled{8} \quad \{a^m b^n \mid m \leq n\} \end{array} \right.$

Pushdown Automata

P
W

$$\textcircled{9} \quad \left\{ w \mid w \in \{a,b\}^*, n_a(w) = n_b(w) \right\}$$

$$\textcircled{10} \quad \left\{ w \mid " , n_a(w) > n_b(w) \right\}$$

$$\textcircled{11} \quad \left\{ w \mid " , n_a(w) < n_b(w) \right\}$$

$$\textcircled{12} \quad \left\{ w \mid " , n_a(w) \neq n_b(w) \right\}$$

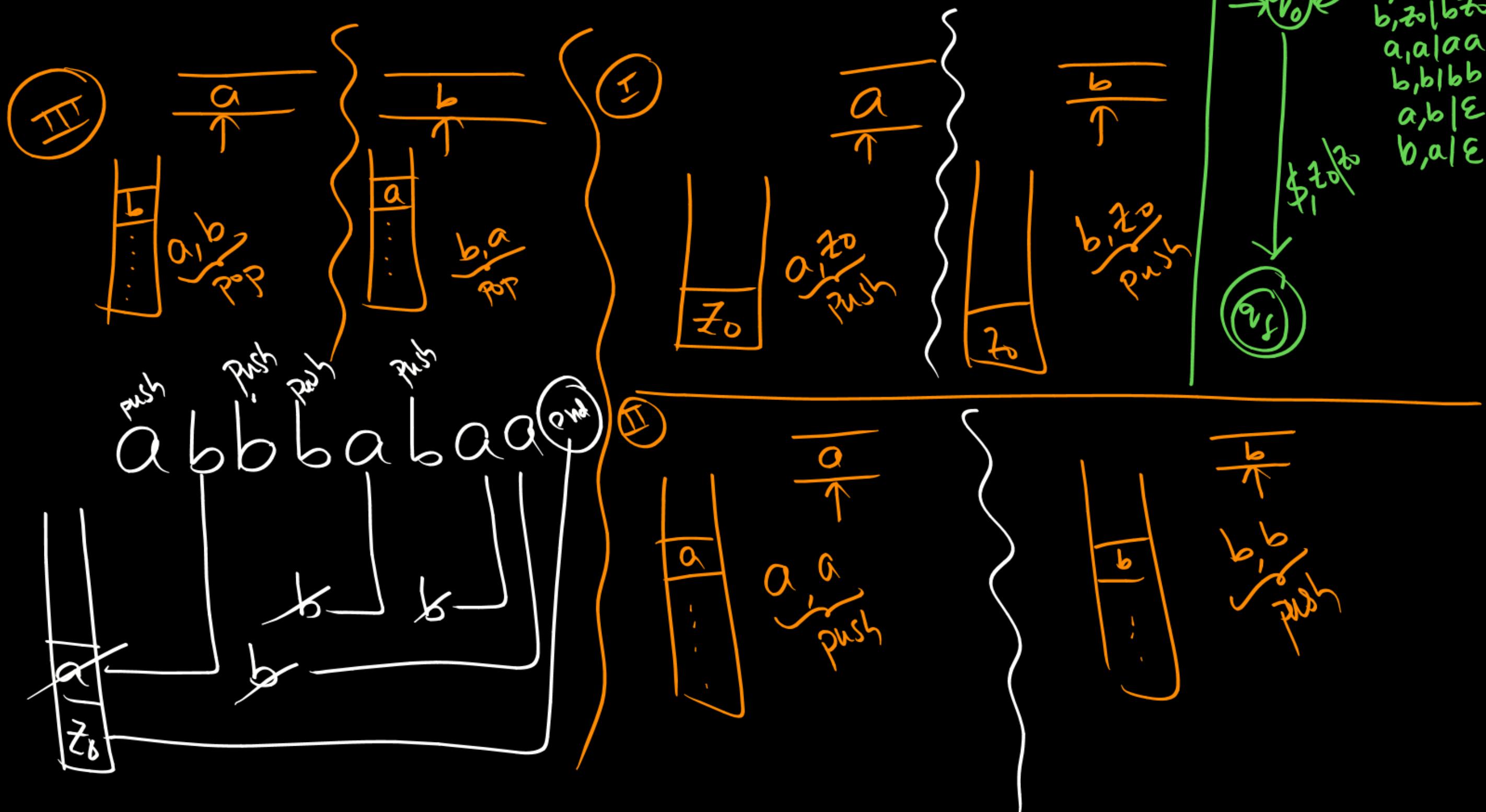
$$\textcircled{13} \quad \left\{ w \mid " , n_a(w) \geq n_b(w) \right\}$$

$$\textcircled{14} \quad \left\{ w \mid " , n_a(w) \leq n_b(w) \right\}$$

Pushdown Automata

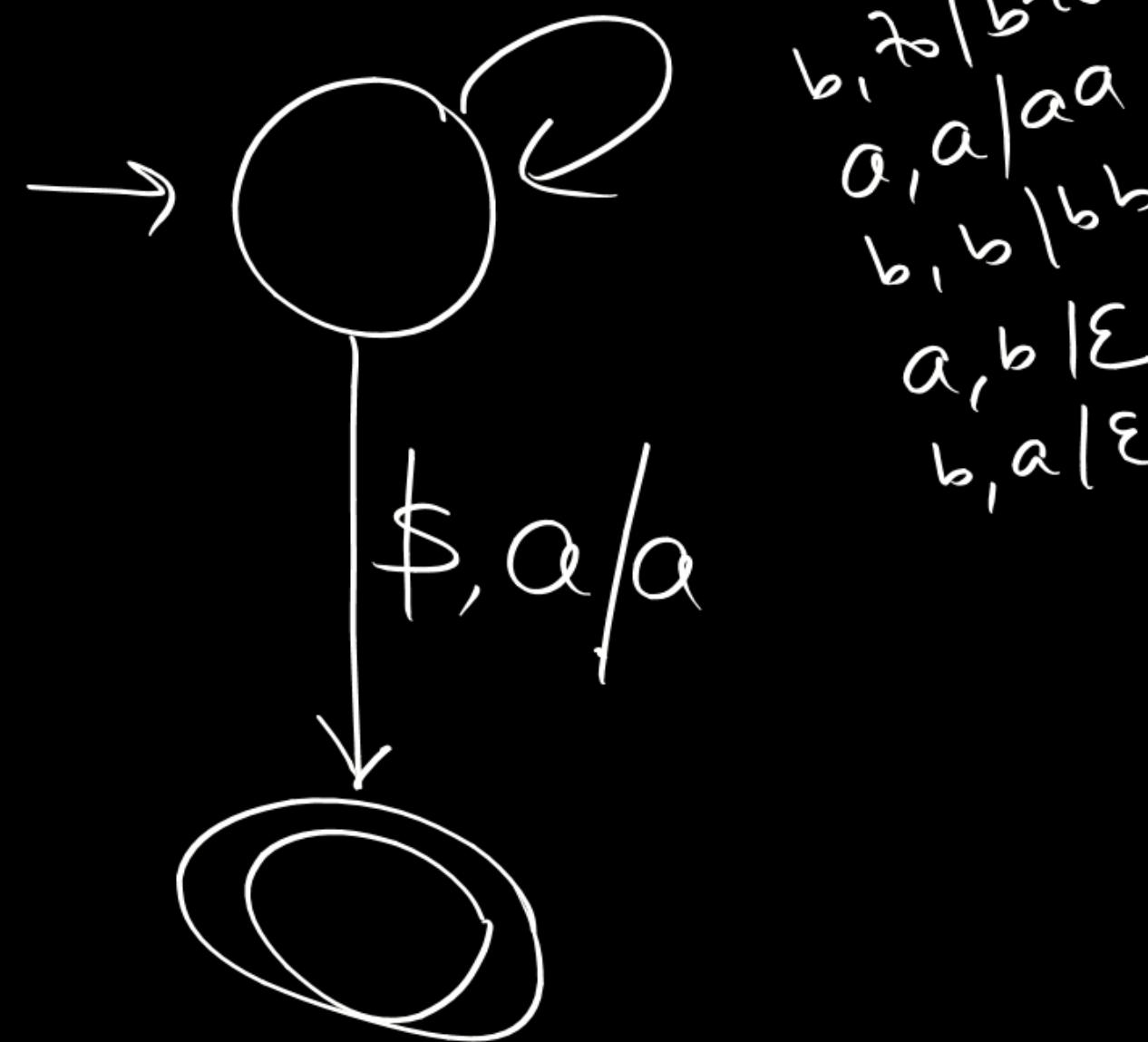
$$⑨ \quad \{ \omega \mid \omega \in \{a, b\}^*, \ n_a(\omega) = n_b(\omega) \}$$

P
W



10

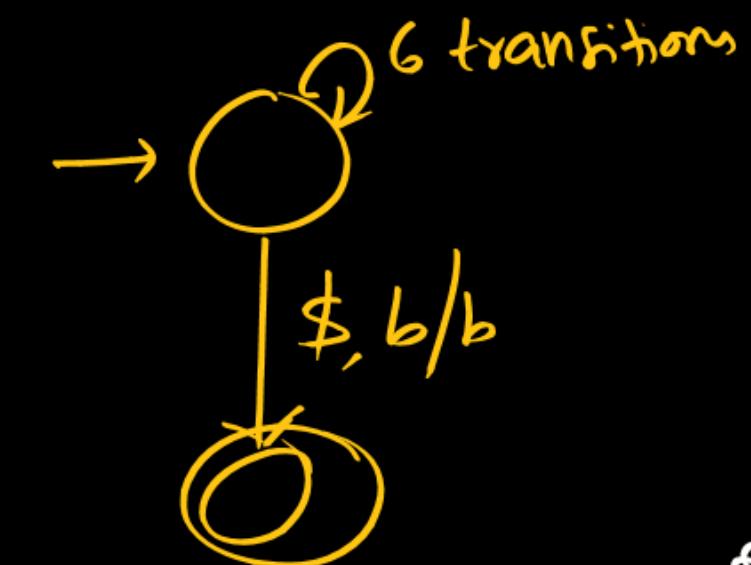
$$\{ w \mid w \in \{a,b\}^*, n_a(w) > n_b(w) \}$$



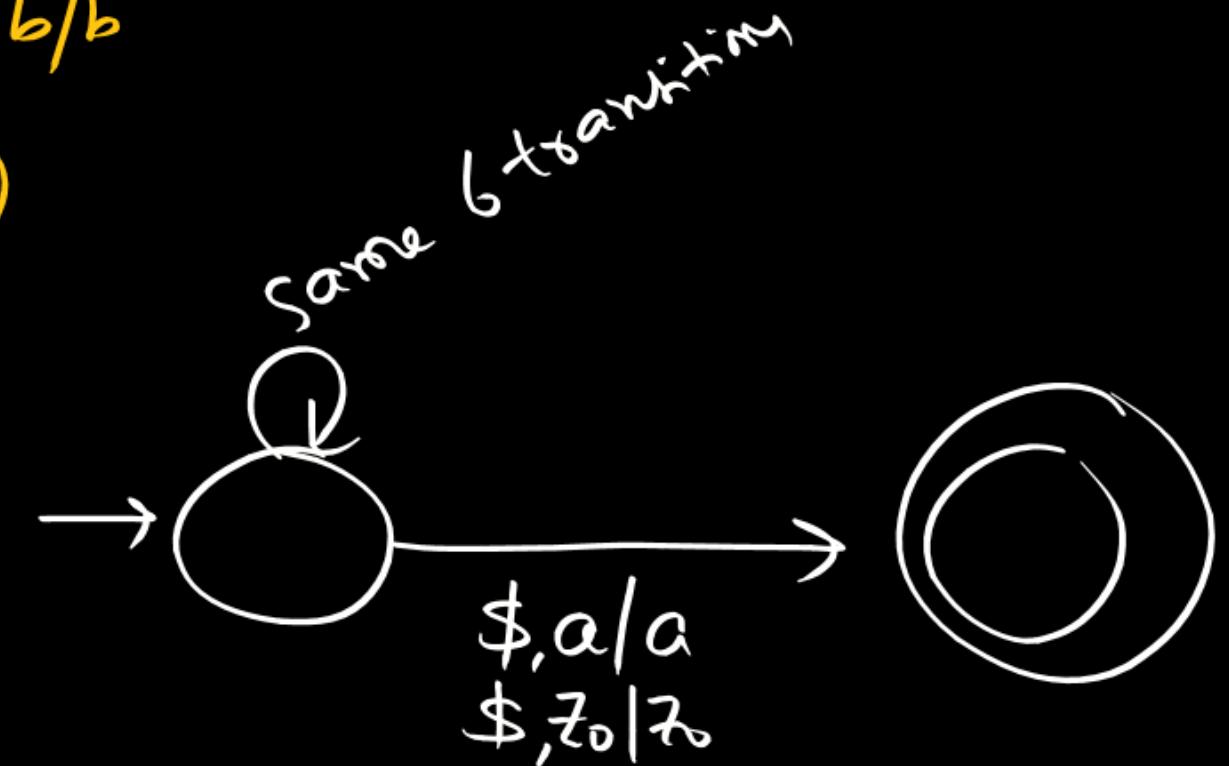
Pushdown Automata

P
W

⑪ $\{\omega \mid \omega \in \{a,b\}^*, n_a(\omega) < n_b(\omega)\}$



⑫ $\{\omega \mid \omega \in \{a,b\}^*, n_a(\omega) \geq n_b(\omega)\}$

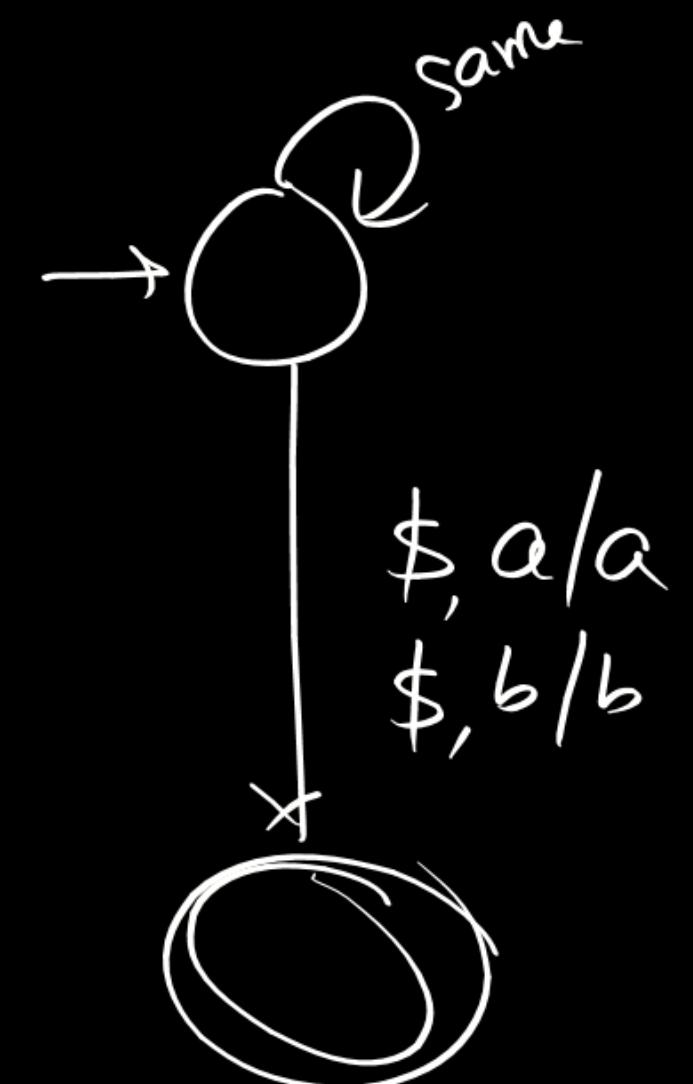


⑬ $\{\omega \mid \omega \in \{a,b\}^*, n_a(\omega) \leq n_b(\omega)\}$



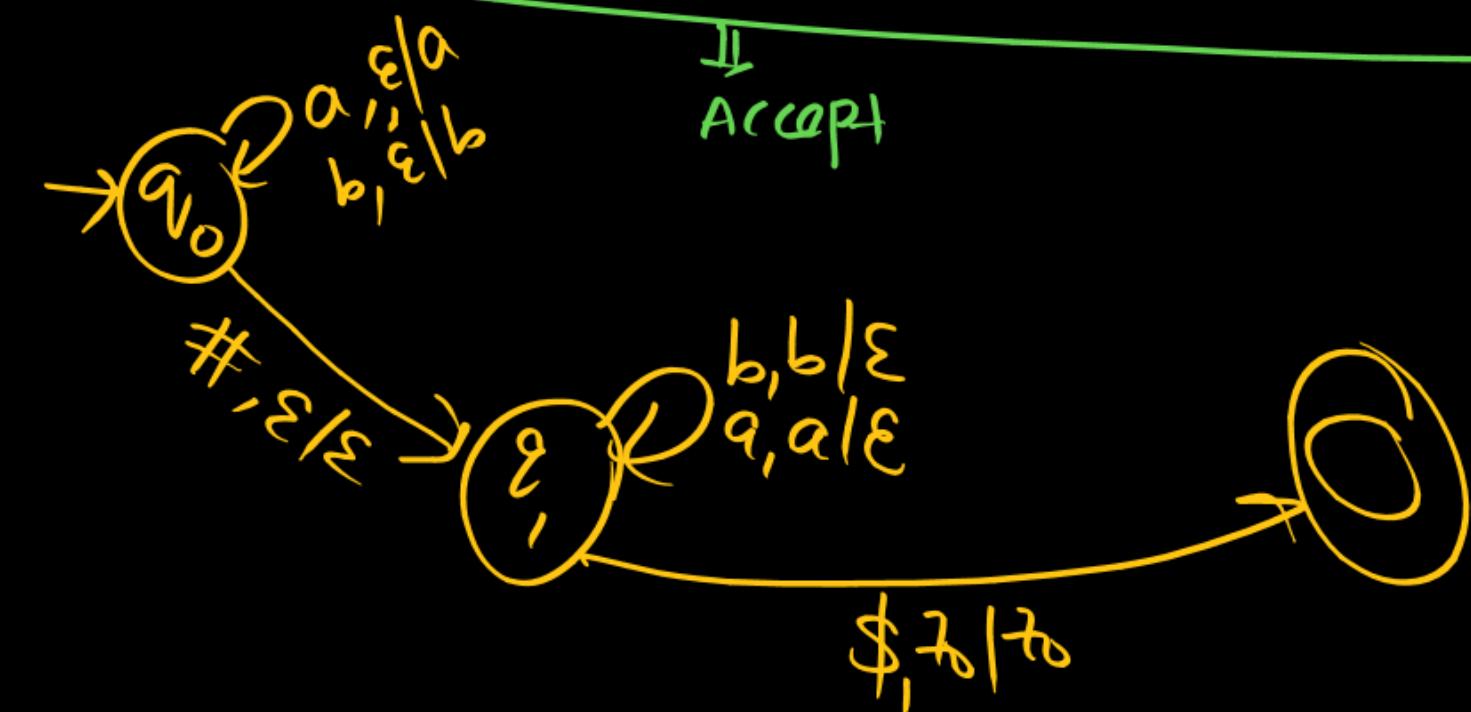
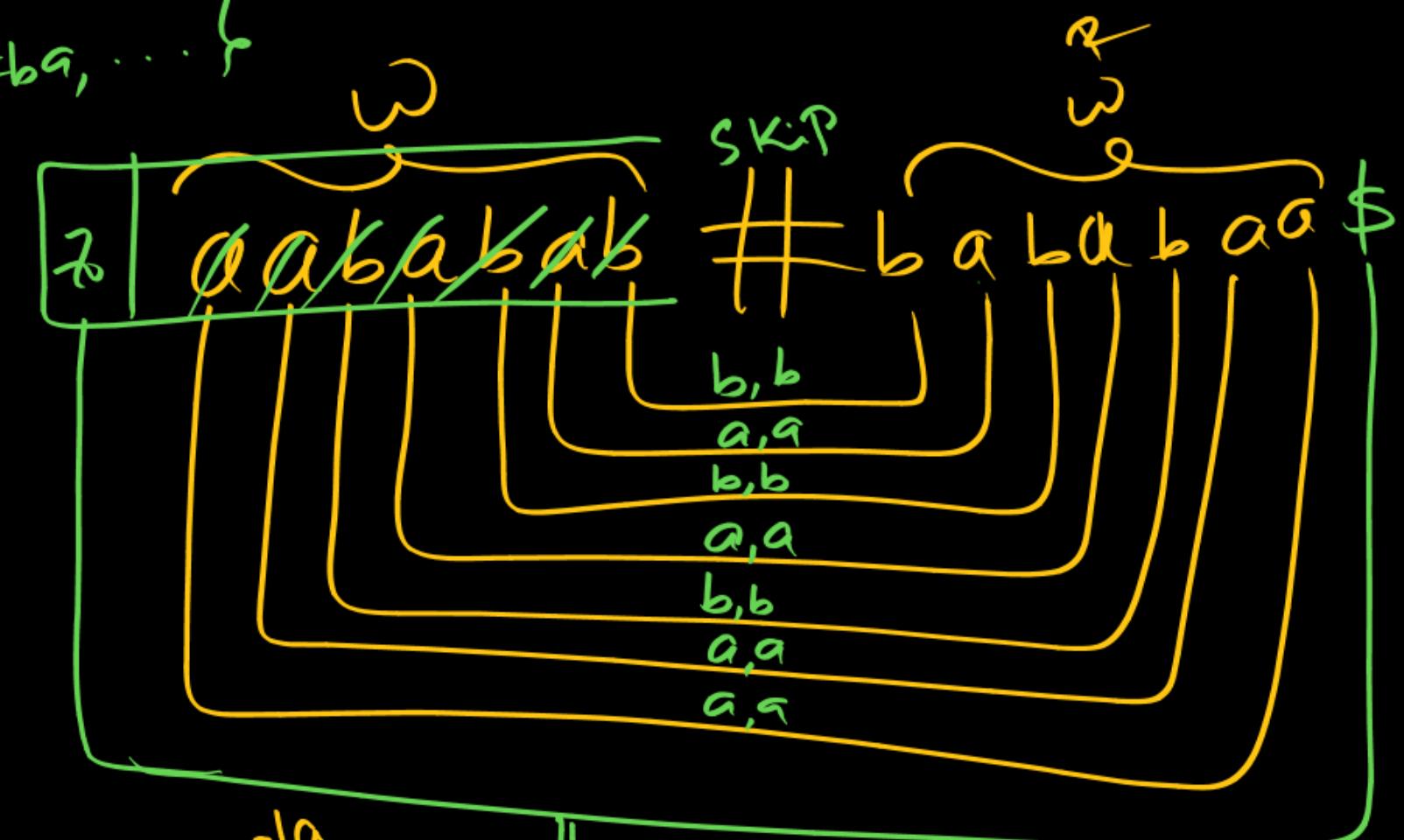
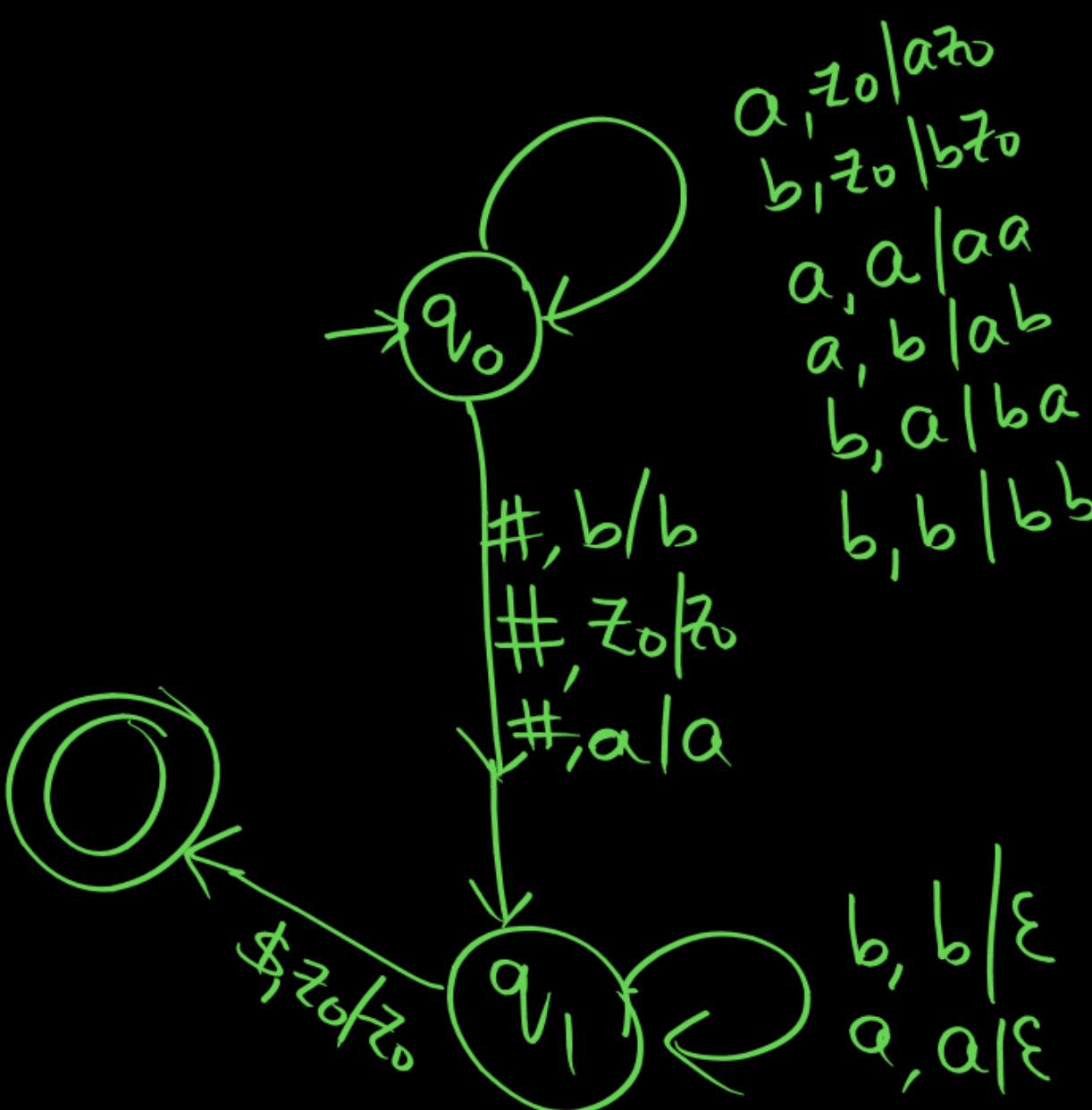
12

$$\{ \omega \mid \omega \in a,b^*, n_a(\omega) \neq n_b(\omega) \}$$



⑯ $\{ w \# w^R \mid w \in \{a, b\}^* \}$

$\{ \#, a\#a, b\#b, aa\#aa, ab\#ba, \dots \}$



Pushdown Automata

P
W

⑯

$$\{ a w \# w^R \mid w \in \{a, b\}^* \}$$

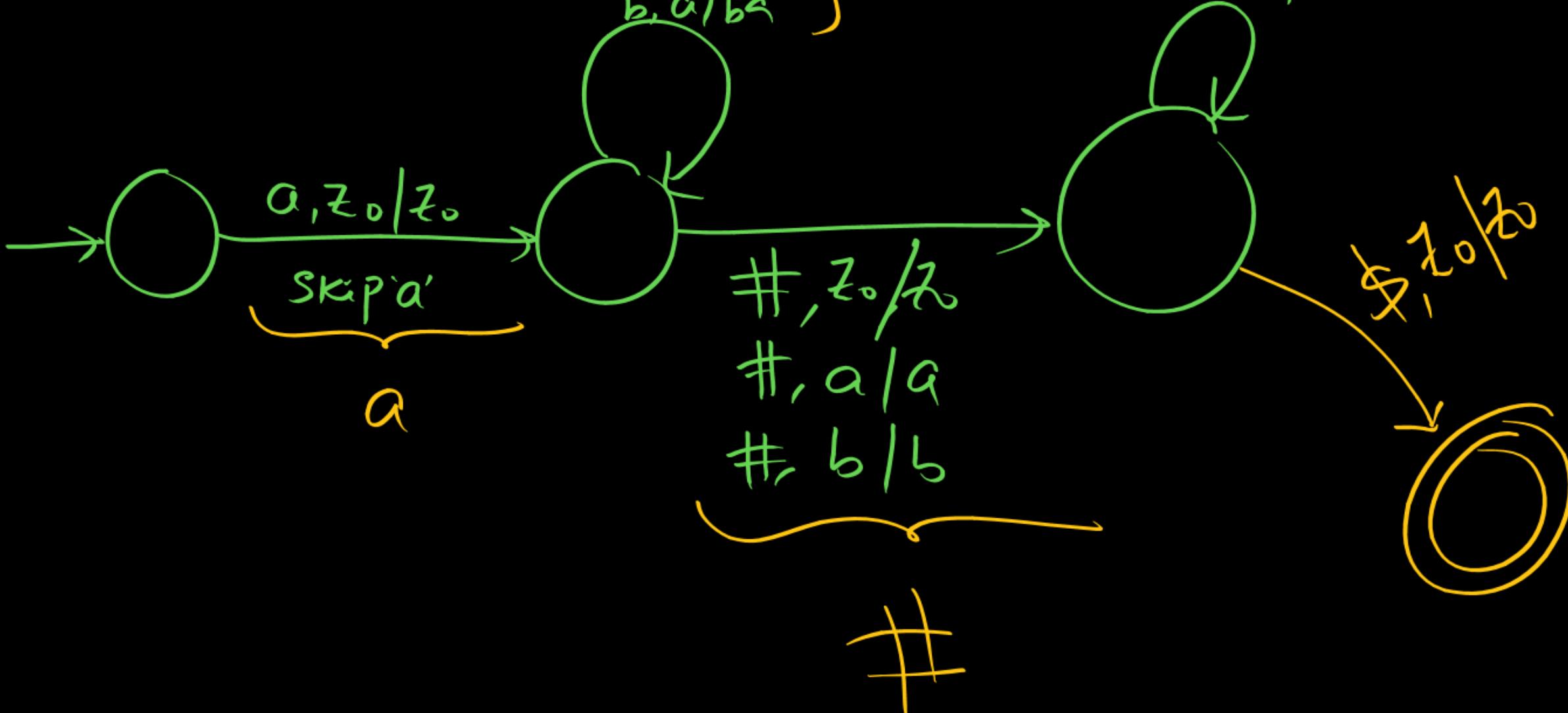
$\Downarrow \Downarrow \Downarrow \Downarrow$
skip push skip pop

b, b/bb
a, z₀/az₀
a, a/aa
a, b/bb
b, z₀/bz₀
b, a/ba

w

a, a/ε
b, b/ε

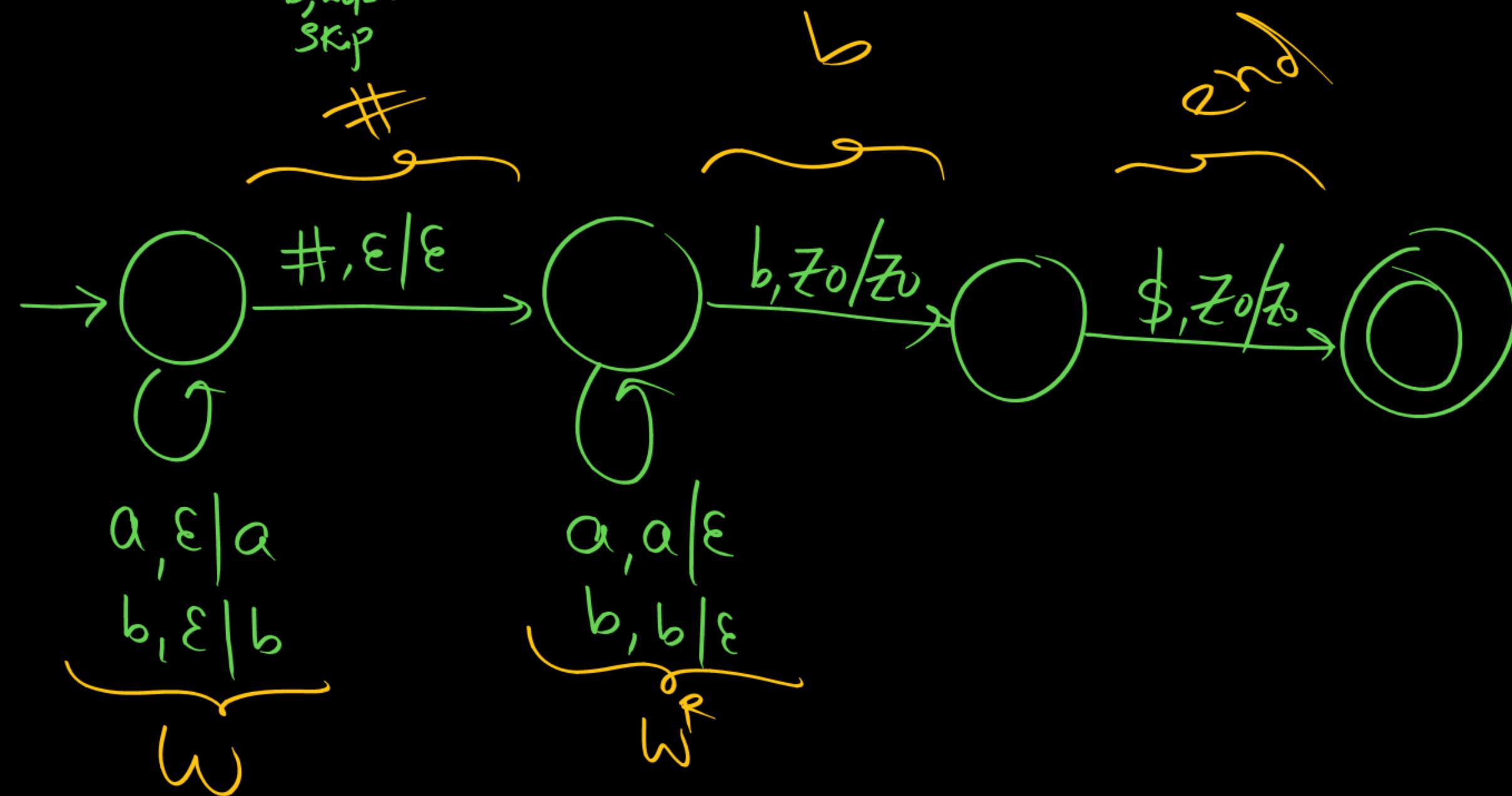
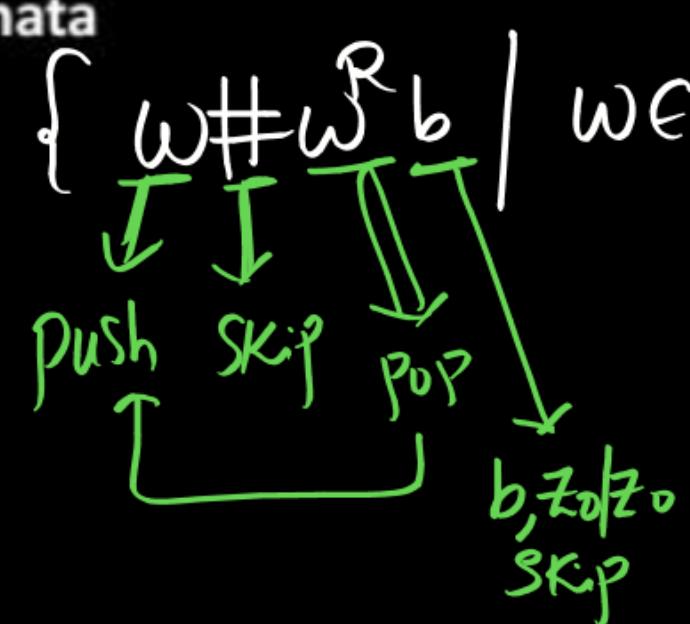
w^R



Pushdown Automata

P
W

$$\textcircled{17} \quad \left\{ w \# w^R b \mid w \in \{a,b\}^* \right\}$$



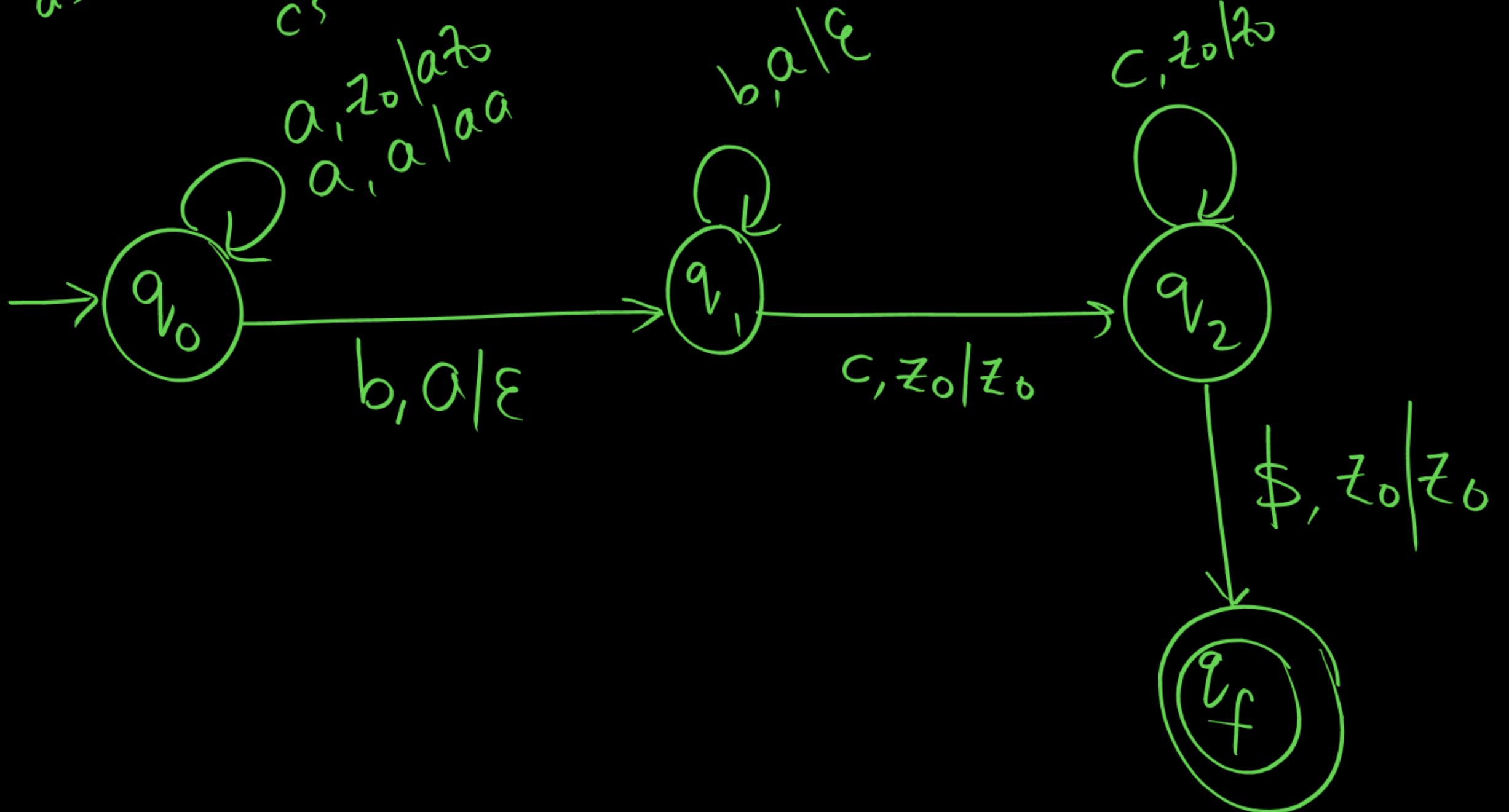
Pushdown Automata

P
W

⑯

$\{a^n b^n c^m \mid n, m \geq 1\}$

$\overbrace{a}^{\text{PUSH}} \quad \overbrace{b}^{\text{POP}} \quad \overbrace{c}^{\text{SKIP}}$



Pushdown Automata

P
W

19

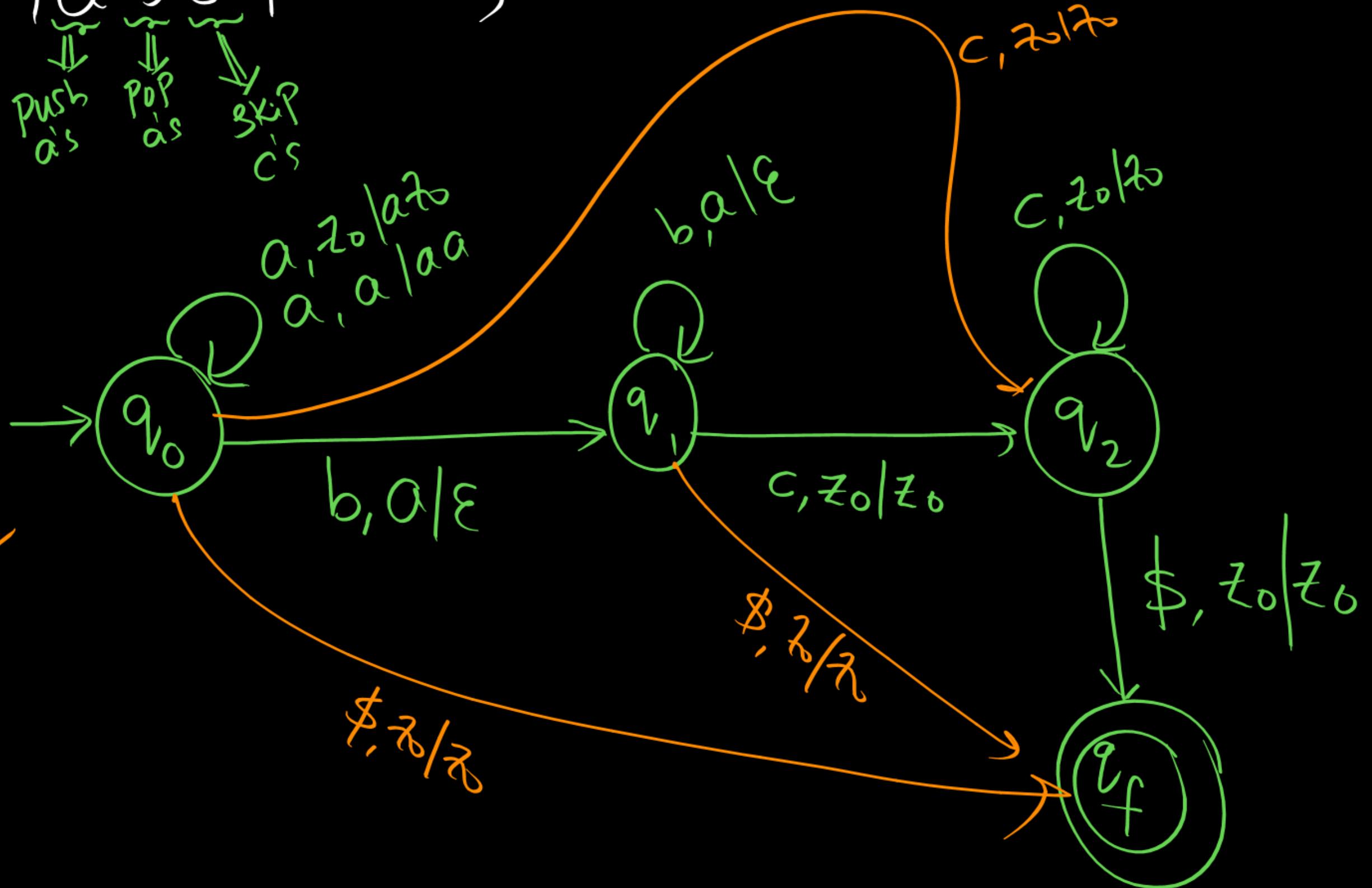
$\{a^n b^n c^m \mid n, m \geq 0\}$

↓
Push
a's

↓
POP
a's

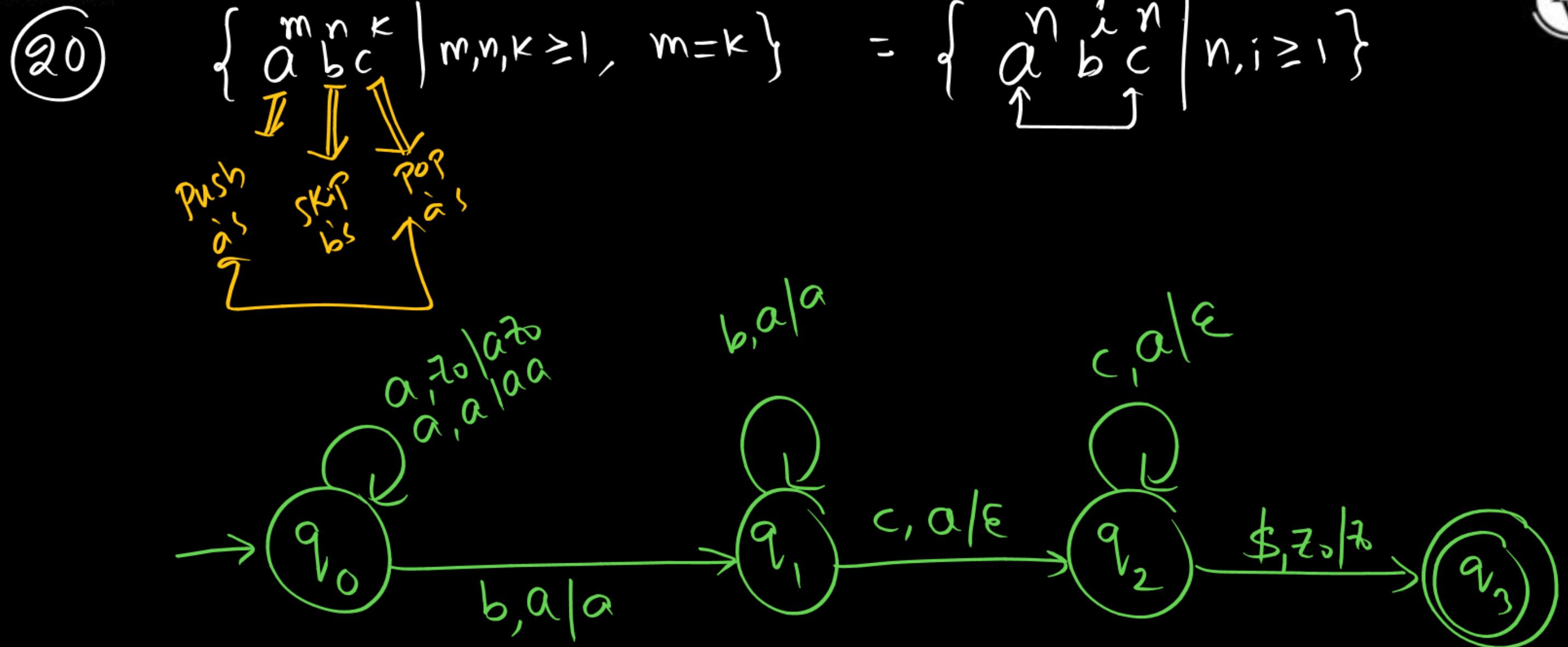
↓
Skip
c's

$\epsilon \checkmark$
 $a^n b^n \checkmark$
 $c^* \checkmark$
 $a^n b^n c^n \mid n \geq 1 \checkmark$



Pushdown Automata

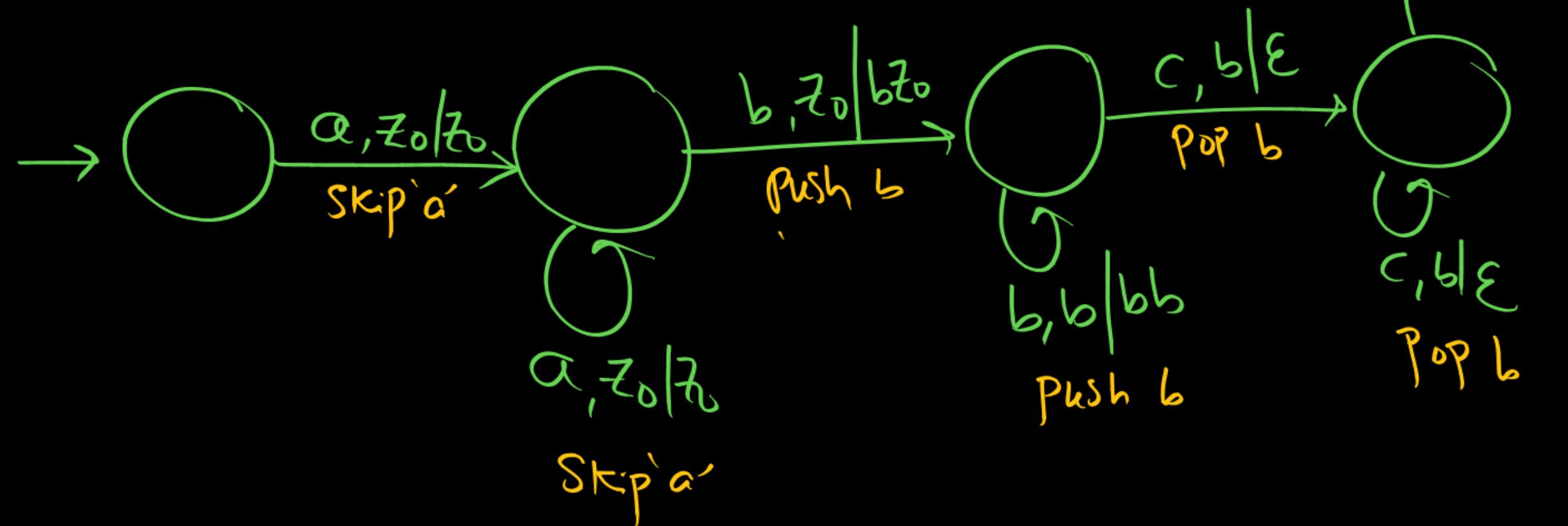
P
W



Pushdown Automata

P
W

Q1) $\{ a^m b^n c^k \mid m, n, k \geq 1, n = k \}$



Note: all 21 languages are DCFLs

Pushdown Automata

P
W

LFLs
not DFLs

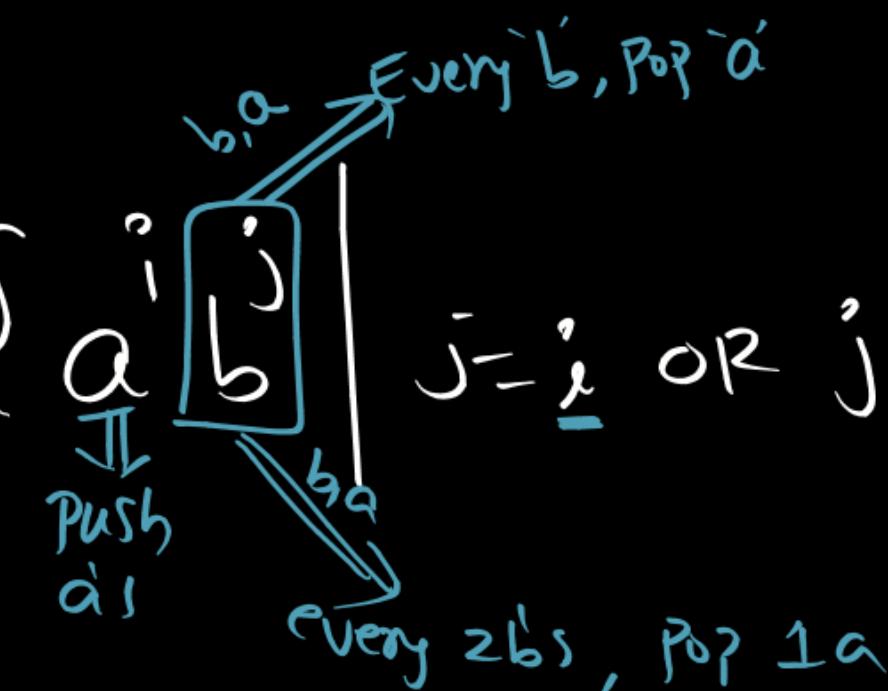
PDA exist
(no DPDA)

22 $\{WW^R \mid w \in \{a,b\}^*\}$

23 $\{a^m b^n c^k \mid m, n, k \geq 1, m=n \text{ or } m=k\}$

24 H.W. $\{a^n b^n\} \cup \{a^n b^{2n}\} = \{a^{\frac{j}{2}} b^j \mid j=2i \text{ OR } j=2i\}$

H.W. 25 $\{a^n \boxed{b}^n c \} \cup \{a^n \boxed{b}^{2n} d\}$



DCFL 26 $\{a^n c^n\} \cup \{a^n b^{2n}\}$

DCFL 27 $\{c a^n b^n\} \cup \{a^n b^{2n}\}$

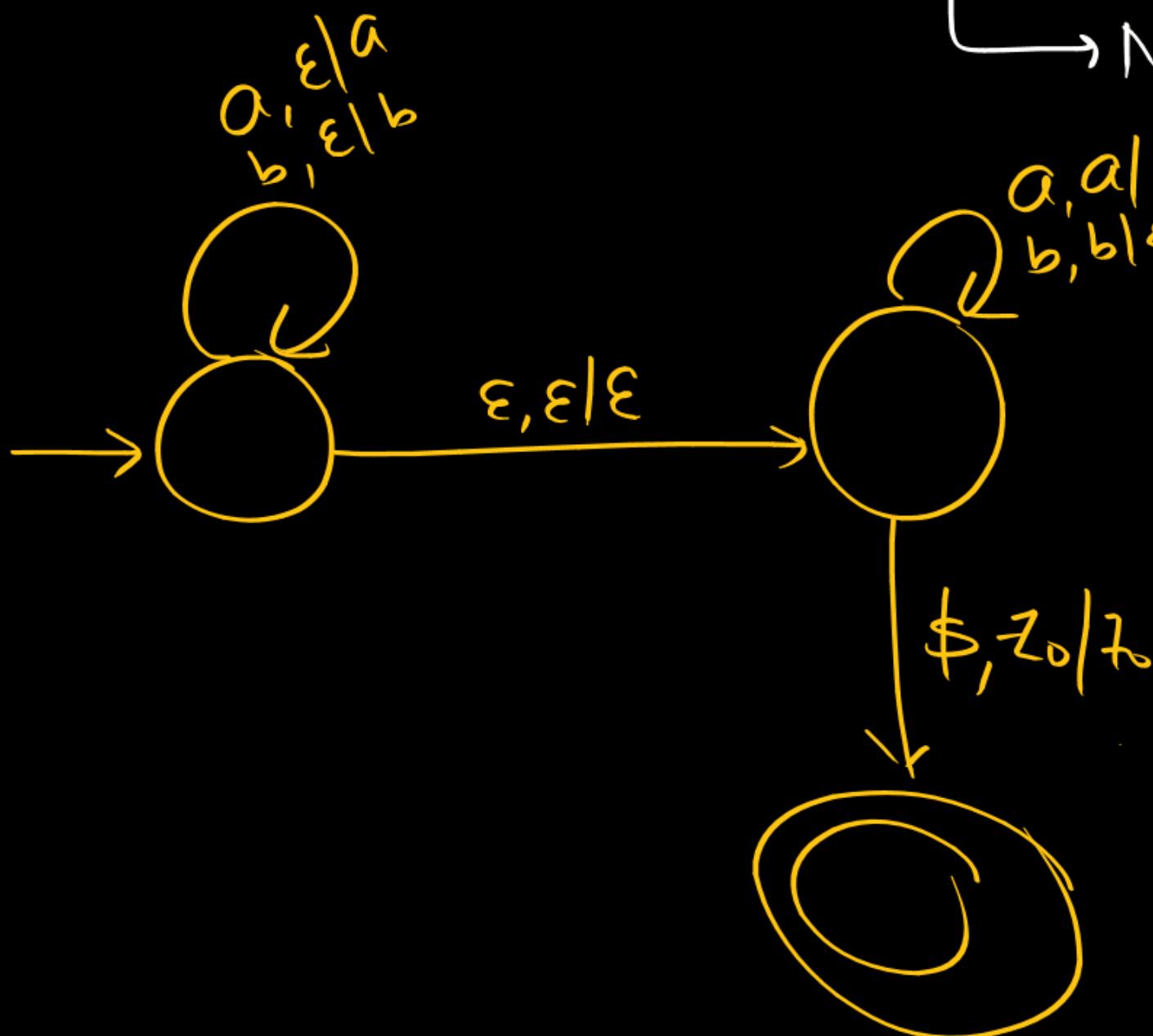
Pushdown Automata

P
W

Q2 $\{ww^R \mid w \in \{a,b\}^*\}$ → CFL but not DCFL

→ we can construct PDA

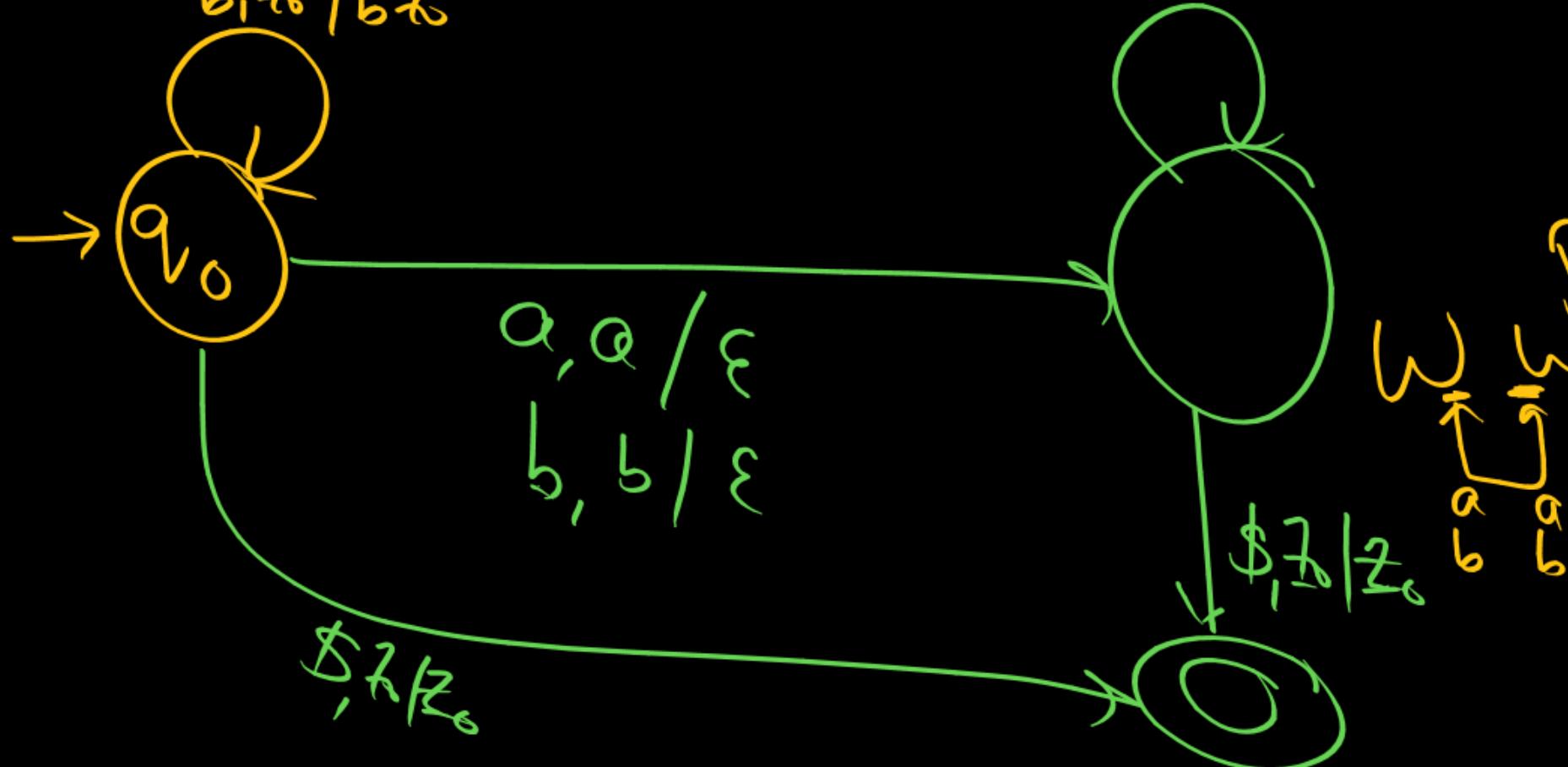
→ NO DPDA for this language



$S \rightarrow aSa \mid bSb \mid a \mid b \mid \epsilon$

CFG



$\{WW^R \mid w \in \{a,b\}^*\}$
 $b,b/bb$
 $a,a/aa$
 $b,a/ba$
 $a,b/lab$
 $a,z_0/az_0$
 $b,z_0/bz_0$


When w^R begins?

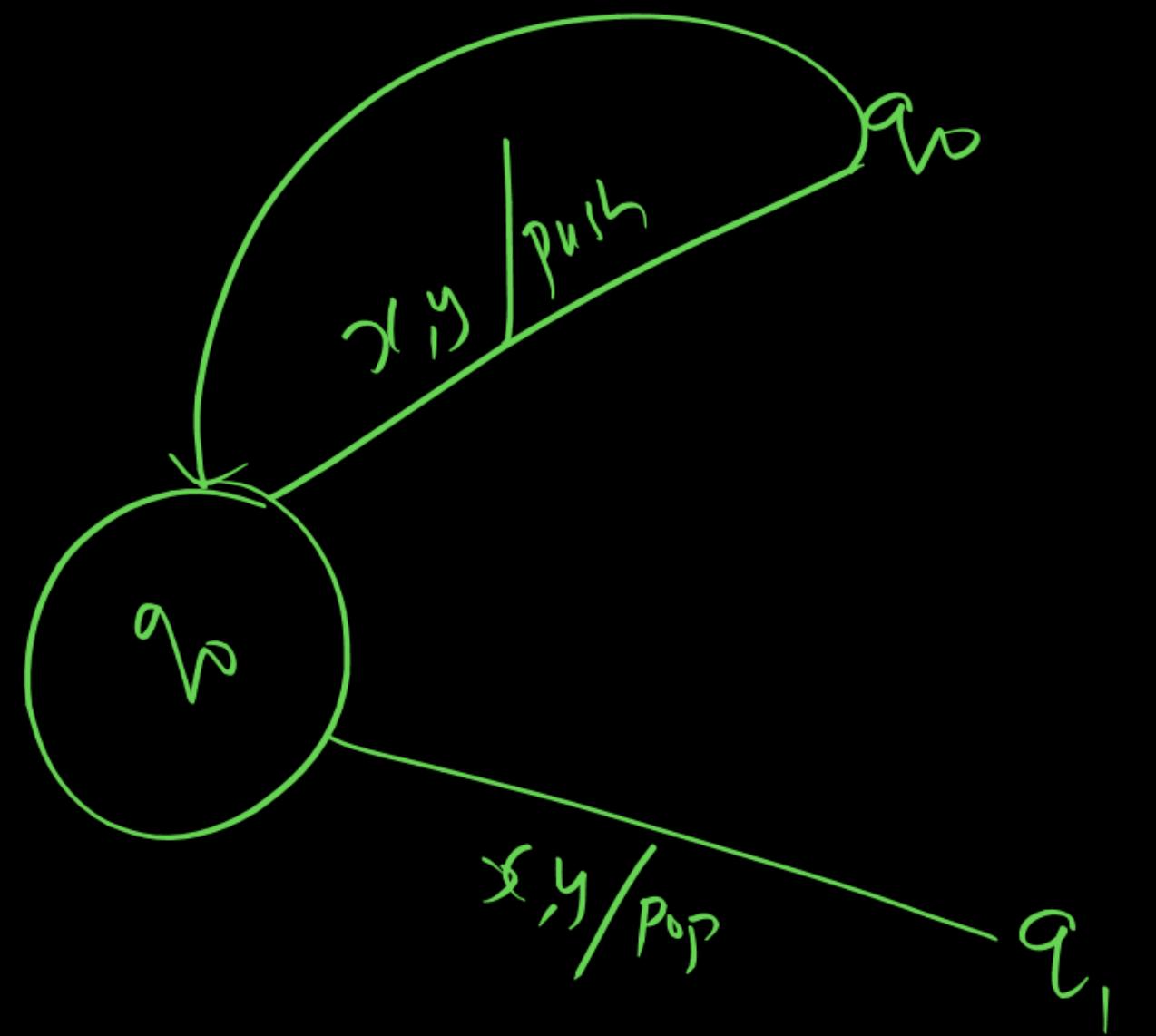
previous 'b'
present 'b'

a a

There is a chance of beginning of w^R

prev
b
b
→

Pushdown Automata

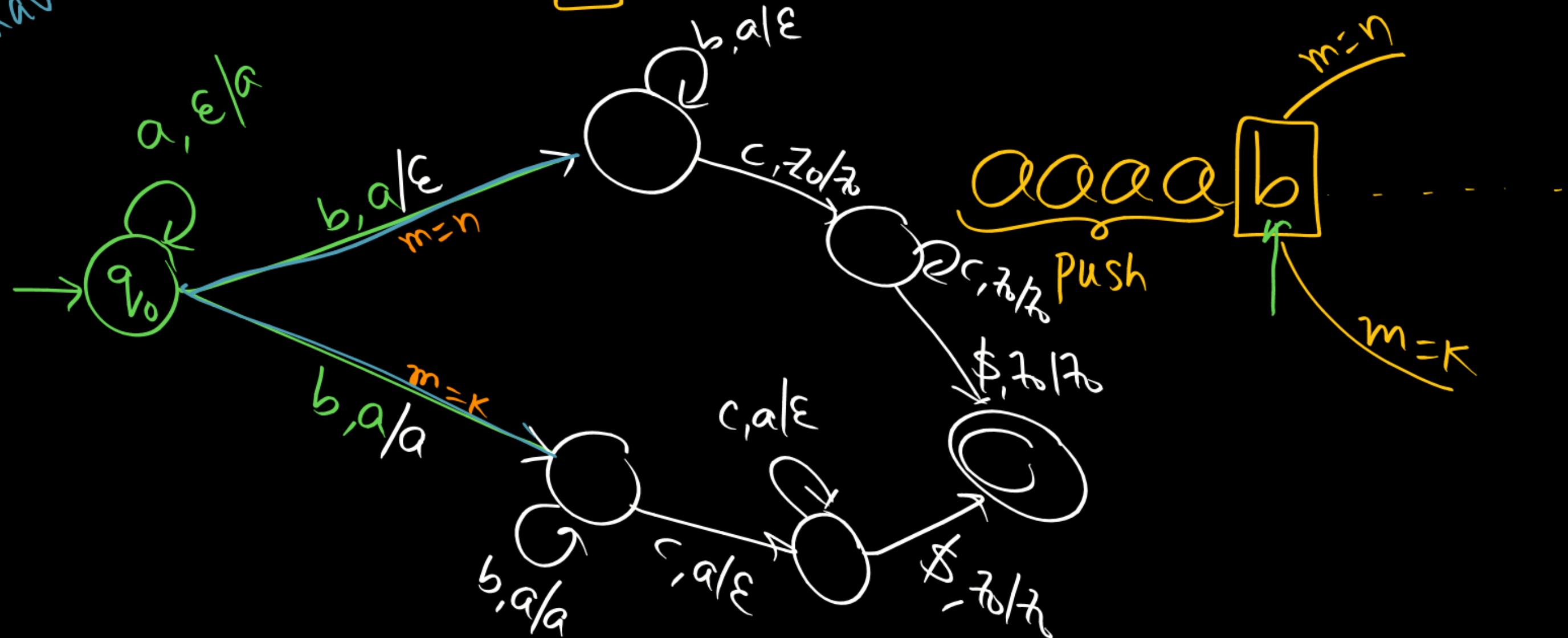


(23)

$$\{a^m b^n c^k \mid m, n, k \geq 1, \underline{m=n} \text{ OR } \underline{m=k}\}$$

no DPDA
we have PPA

$$= \{a^n b^n c^+ \mid n \geq 1\} \cup \{a^n b^+ c^n \mid n \geq 1\}$$



- not DFA
- CFLs
- ②8) $\{a^m b^n c^k \mid m, n, k \geq 1, n=m \text{ OR } n=k\}$
- ②9) $\{a^m b^n c^k \mid m, n, k \geq 1, k=m \text{ OR } k=n\}$
- ③0) $\{a^m b^n c^k \mid m, n, k \geq 1, m > n \text{ OR } m > k\}$

We have PPA
we don't have DPDA

Summary

→ We know PDA
DPDA

- I) Every DPDA is PDA
- II) PDA need not be DPDA



Thank you

