# CS & IT ENGINEERING

## Digital Logic

## Minimization

**Lecture No. 1**

By- CHANDAN SIR


01 THEOREM

02 D-MORGAN'S Law

03 QUESTION PRACTICE

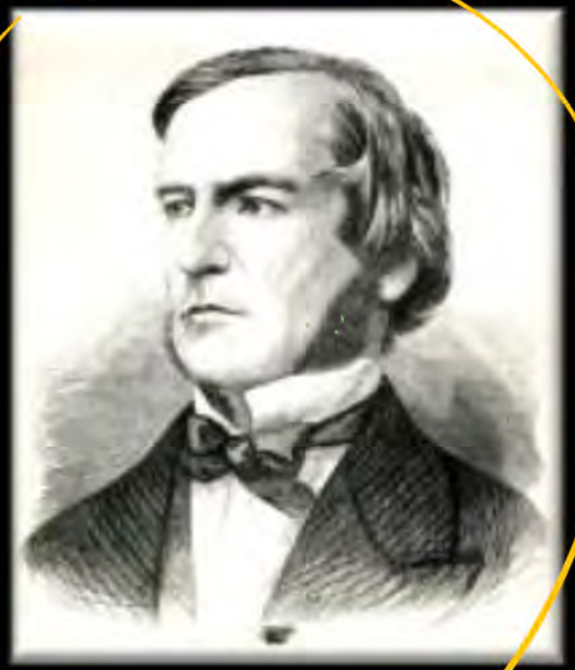
04 DUAL & SELF DUAL

05 DISCUSSION

# Boolean Algebra

- 1854- George Boole

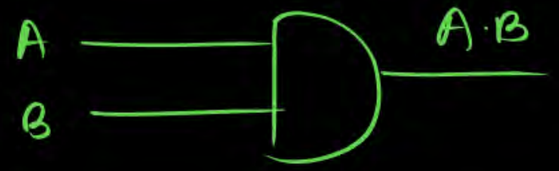
"An Investigation of Law of Thoughts"



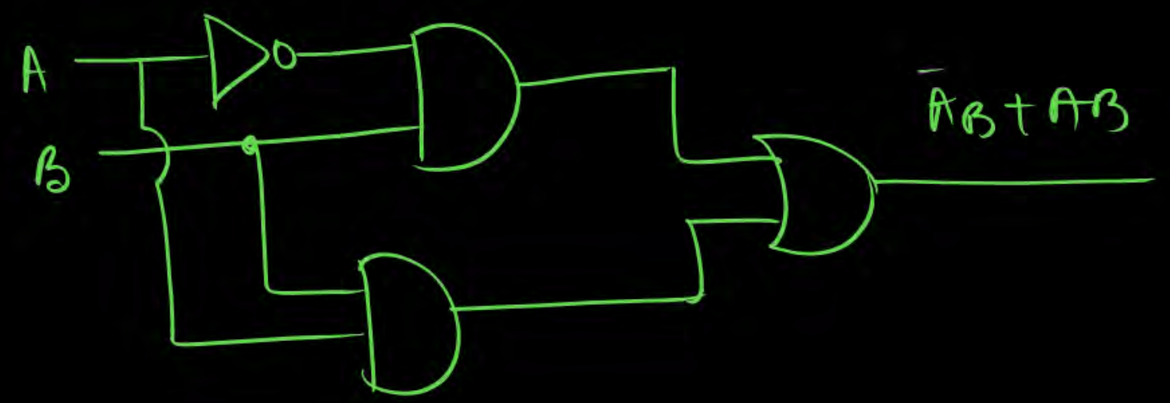
"Gareeb"

# Boolean Algebra

Ex 1 $f(A,B) = A \cdot B$




Ex.2 $f(A,B) = \bar{A}B + AB$



# Boolean Algebra

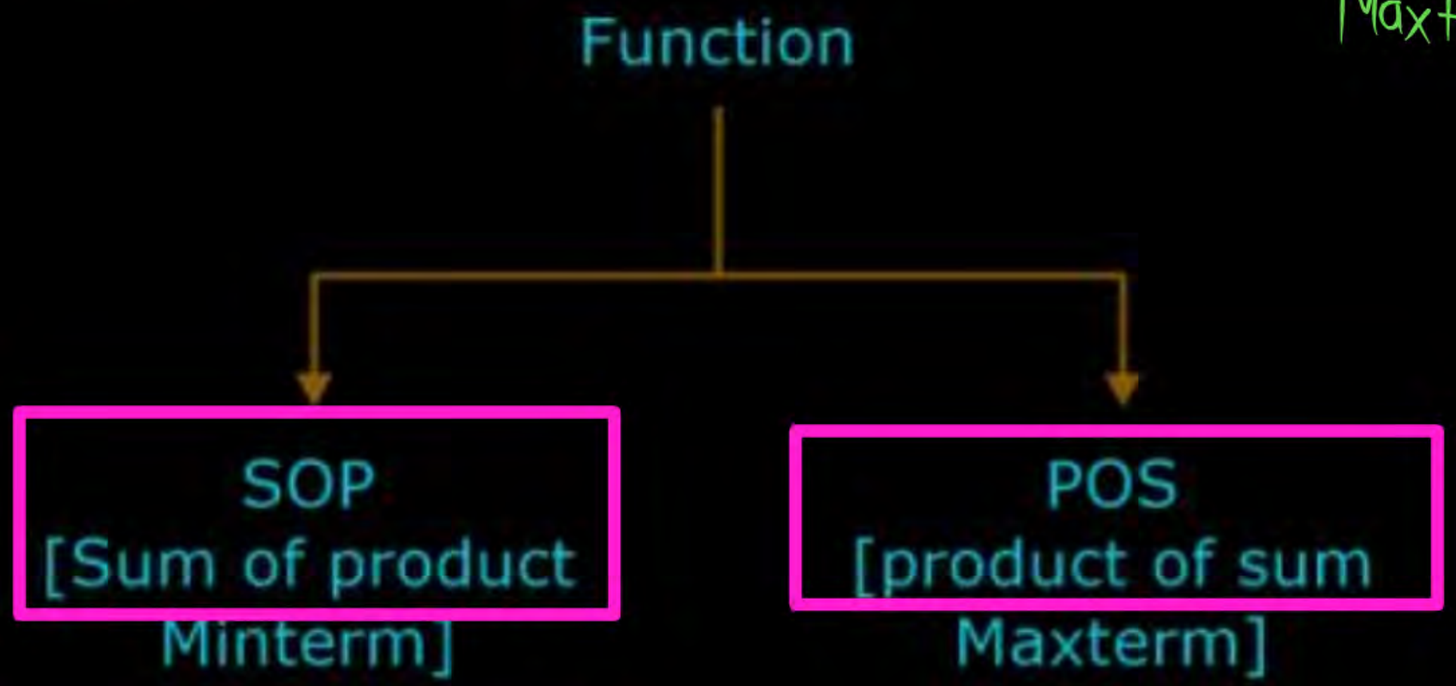
Boolean Function- It is the combination of inputs on which output is depends.

$f(A, B) = AB$

inputs

# Boolean Algebra

- **BOOLEAN ALGEBRA**




$\text{Maxterm} = \overline{\text{Minterm}}$

# Boolean Algebra

$\overline{A\bar{B}\bar{C}} = \bar{\bar{A}} + \bar{\bar{B}} + \bar{\bar{C}}$

$A + B + C$

Ye aise hi likha hai example ke liye.

| Decimal | A B C | Min Term | Max Term | Function |
|---|---|---|---|---|
| 0 | 0 0 0 | $\bar{A}\bar{B}\bar{C}$ | $A+B+C$ | 1 |
| 1 | 0 0 1 | $\bar{A}\bar{B}C$ | $A+B+\bar{C}$ | 0 |
| 2 | 0 1 0 | $\bar{A}B\bar{C}$ | $A+\bar{B}+C$ | 0 |
| 3 | 0 1 1 | $\bar{A}BC$ | $A+\bar{B}+\bar{C}$ | 1 |
| 4 | 1 0 0 | $A\bar{B}\bar{C}$ | $\bar{A}+B+C$ | 0 |
| 5 | 1 0 1 | $A\bar{B}C$ | $\bar{A}+B+\bar{C}$ | 1 |
| 6 | 1 1 0 | $AB\bar{C}$ | $\bar{A}+\bar{B}+C$ | 0 |
| 7 | 1 1 1 | $ABC$ | $\bar{A}+\bar{B}+\bar{C}$ | 1 |

# Boolean Algebra

- **Standard Canonical SOP Form**

  → Each term should contain all the variable.

$$f(A,B,C) = \underset{①}{\bar{A}\bar{B}\bar{C}} + \underset{②}{\bar{A}BC} + \underset{③}{A\bar{B}C} + \underset{④}{ABC}$$

$$= m_0 + m_3 + m_5 + m_7$$

$$= \Sigma m(0,3,5,7)$$

# Boolean Algebra

- **Standard Canonical POS Form**

$$F(A,B,C) = \underset{001}{(A+B+\bar{C})} \cdot \underset{010}{(A+\bar{B}+C)} \cdot \underset{100}{(\bar{A}+B+C)} \cdot \underset{110}{(\bar{A}+\bar{B}+C)}$$

$$= M_1 \cdot M_2 \cdot M_4 \cdot M_6$$

$$= \Pi M(1,2,4,6)$$

Ex $f(A,B,C) = \Sigma m(0,1,3) \rightarrow$ SOP

$F(A,B,C) = \Pi(2,4,5,6,7) \rightarrow$ POS

| | 8 | 4 | 2 | 1 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 0 |
| 3 | 0 | 0 | 1 | 1 |
| 4 | 0 | 1 | 0 | 0 |
| 5 | 0 | 1 | 0 | 1 |
| 6 | 0 | 1 | 1 | 0 |
| 7 | 0 | 1 | 1 | 1 |

| | 8 | 4 | 2 | 1 |
|---|---|---|---|---|
| 8 | 1 | 0 | 0 | 0 |
| 9 | 1 | 0 | 0 | 1 |
| 10 | 1 | 0 | 1 | 0 |
| 11 | 1 | 0 | 1 | 1 |
| 12 | 1 | 1 | 0 | 0 |
| 13 | 1 | 1 | 0 | 1 |
| 14 | 1 | 1 | 1 | 0 |
| 15 | 1 | 1 | 1 | 1 |

'n' digit

↳ $2^n$ combination

# Boolean Algebra

**Question- For the the function given below how many terms are present in the standard canonical SOP form?**

$$f(A, B, C) = A + BC$$

$$= A(\bar{B}+B)(\bar{C}+C) + (\bar{A}+A)BC$$

$$= A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC + \bar{A}BC + ABC$$

$$= A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + \bar{A}BC + ABC$$

$$= \Sigma m(4, 5, 6, 3, 7)$$

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

(F) Sir Mujhe nahi ata, Mai Gajni hu.

$f(A,B,C) = A + BC$ (B, C)

$= A \cdot (\bar{B}+B) \cdot (\bar{C}+C) + (\bar{A}+A)BC$

1 0 0 → 4
1 0 1 → 5
1 1 0 → 6
1 1 1 → 7

0 1 1 → 3
1 1 1 → 7

$= \Sigma m(3,4,5,6,7)$

(B,C) (A)

Ex $f(A,B,C) = \bar{A} + \bar{B}C$ $(\bar{A}+A)\bar{B}C$ $\bar{A}\bar{B}C \to 001 \to 1$, $A\bar{B}C \to 101 \to 5$

$\bar{A}(\bar{B}+B).(C\bar{C}+C)$

$\bar{A}\bar{B}\bar{C} \to 000$

$\bar{A}\bar{B}C \to 001$

$\bar{A}B\bar{C} \to 010$

$\bar{A}BC \to 011$

$000 \to 0$

$001 \to 1$

$010 \to 2$

$011 \to 3$

$001 \to 1$

$101 \to 5$

$f(A,B,C) = \Sigma m(0,1,2,3,5)$

# Boolean Algebra

## ① DISTRIBUTION THEOREM

$$(A+B)(A+C)$$

$$\Rightarrow AA+AC+AB+BC$$

$$\Rightarrow A+AC+AB+BC$$

$$\Rightarrow A[1+C+B]+BC$$

$$\Rightarrow A+BC$$

$$A+BC = (A+B)(A+C)$$

$$A+BCD = (A+B)(A+C)(A+D)$$

$$A+BCDE = (A+B)(A+C)(A+D)(A+E)$$

Q $\bar{A} + AB$

$(\bar{A}+A)(\bar{A}+B)$

$1 \cdot (\bar{A}+B)$

$\Rightarrow \bar{A}+B$

Ans

Q $(A+B)(A+\bar{B})$

same

$A + B\bar{B}$

$A + 0 = A$

Ans

$A+BC = (A+B)(A+C)$

(2) (2)

(1) (1)

$A+BC$

# Boolean Algebra

## ② Concensus Theorem

$$AB+\bar{A}C+BC$$

$$\Rightarrow AB+\bar{A}C+(\bar{A}+A)BC$$

$$\Rightarrow \underline{AB}+\underline{\bar{A}C}+\underline{\bar{A}BC}+\underline{ABC}$$

$$\Rightarrow AB[1+C]+\bar{A}C[1+B]$$

$$\Rightarrow AB+\bar{A}C$$

① Three terms
② Each terms consist of two variables
③ Each variable repeated twice only one variable repeated in complement.

$AB + \bar{A}C + BC = AB + \bar{A}C$

Redundant term

$AB + BC + \bar{A}C = AB + \bar{A}C$

Q $\bar{A}\bar{B} + \bar{B}C + \bar{A}\bar{C} = \bar{B}C + \bar{A}\bar{C}$

↳ Redundant term

Q $(A+B)(\bar{A}+C)(B+C) = (A+B)(\bar{A}+C)$

# Boolean Algebra

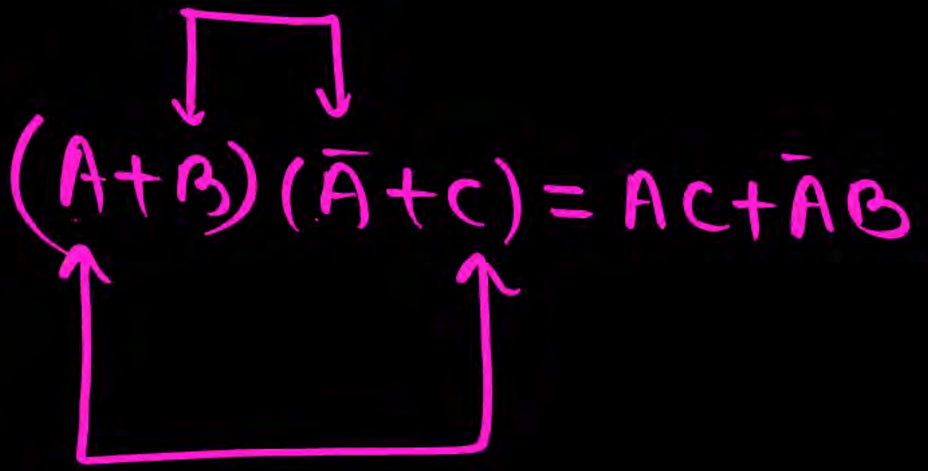
## ③ TRANSPOSE THEOREM

$$(A+B)(\bar{A}+C)$$

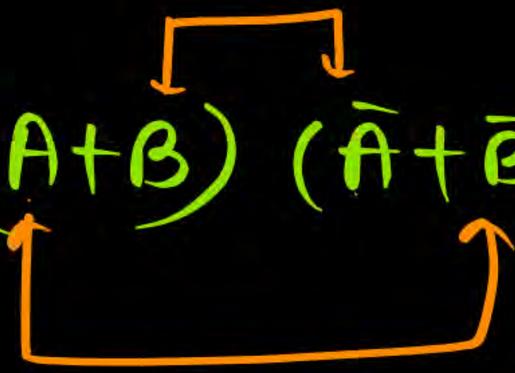
$$\Rightarrow A\cdot\bar{A}+AC+\bar{A}B+BC$$

$$\Rightarrow AC+\bar{A}B+BC$$

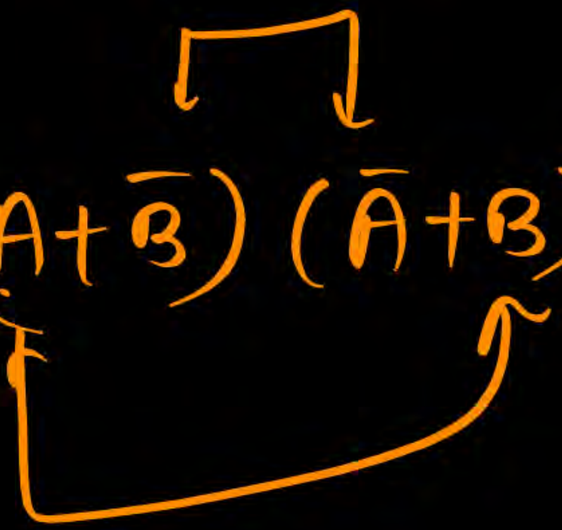
$$\Rightarrow AC+\bar{A}B$$



Ex $(A+B)(\bar{A}+\bar{B}) = A\bar{B}+\bar{A}B = A \oplus B$



Ex $(A+\bar{B})(\bar{A}+B) = AB+\bar{A}\bar{B} = A \odot B$



## ④ D·E·MORGANS LAW

$$\overline{ABC} = \bar{A}+\bar{B}+\bar{C}$$

$$\overline{A+B+C} = \bar{A}\cdot\bar{B}\cdot\bar{C}$$

① $A+BC=(A+B)(A+C)$

② $AB+\bar{A}C+BC=AB+\bar{A}C$

③ $(A+B)(\bar{A}+C)=AC+\bar{A}B$

④ $\overline{ABC}=\bar{A}+\bar{B}+\bar{C}$

$\overline{A+B+C}=\bar{A}\cdot\bar{B}\cdot\bar{C}$

# Boolean Algebra

1) Distribution theorem

$(A + B)\,(A + C)$ $\qquad A \cdot (B + C) = AB + AC$

2) Consensus theorem

$AB + \overline{A}C + BC$

3) Transpose theorem

$(A + B) + (\overline{A} + C)$

4) D-Morgan's Law

$\overline{ABC} = \overline{A} + \overline{B} + \overline{C}$

$\overline{A + B + C} -= \overline{A} \cdot \overline{B} \cdot \overline{C}$

# Boolean Algebra

- **Theorem**

5) Annulment Law

$A \cdot 0 = 0, \qquad A + 1 = 1$

6) Identity Law

$A + 0 = A, \qquad A \cdot 1 = A$

7) Idempotent Law

$A + A = A, \qquad A \cdot A = A$

8) Absorptive Law

$A + AB = A$

$A \cdot (A + B) = A$

$A + A = A$

$$A \cdot (A+B) = A$$

$$A \cdot A + AB$$

$$A + AB$$

$$A(1+B)$$

$$= A$$

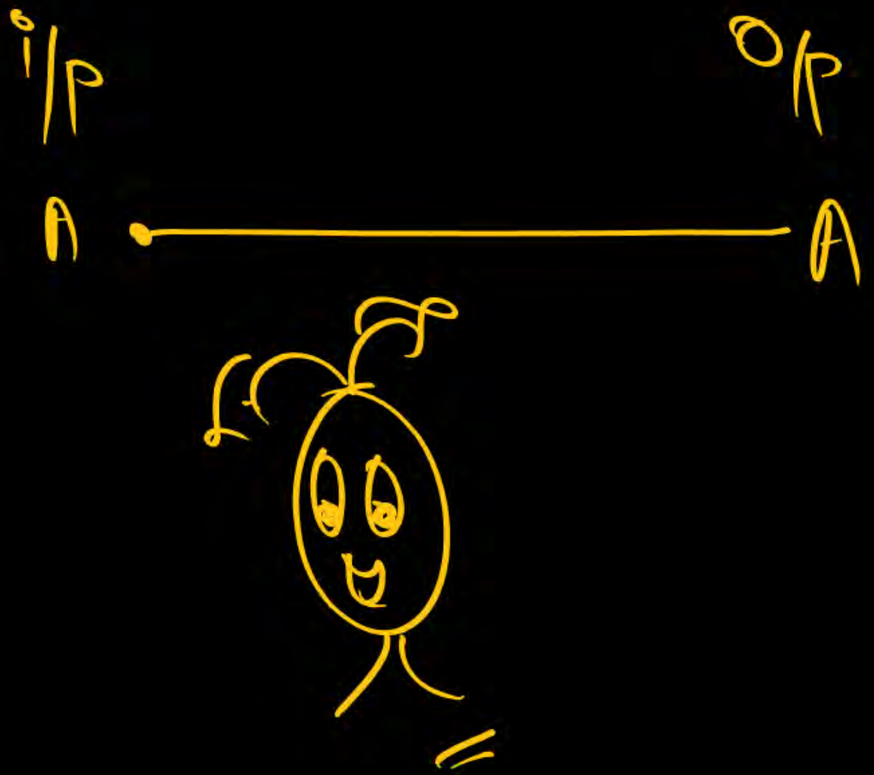
## Q.1

Find the minimum number of the NAND gate required to implement the Boolean function given below:

$f(A, B, C) = A + ABC + AB\bar{C}$

$\Rightarrow A[1 + BC + B\bar{C}]$

$\Rightarrow A$



## Q.2

Minimize the expression:

$f(A, B) = A + A\overline{B}$

$= A[1 + \overline{B}]$

$= A$

$=$

## Q.3

Minimize the expression:

$f(A, B) = \bar{A} + A\bar{B}$

$= (\bar{A} + A)(\bar{A} + \bar{B})$

$= \bar{A} + \bar{B}$

Ans

## Q.4

Minimize the expression:

$f(A, B) = A + \bar{A}\bar{B}$

$= (A+\bar{A})(A+\bar{B})$

$= A+\bar{B}$

## Q.5

Minimize the expression.

$f(A, B) = \overline{A}\,\overline{B} + \overline{A}B + AB$

$= \bar{A}[\bar{B} + B] + AB$

$\Rightarrow \bar{A} + AB$

$= (\bar{A} + A)(\bar{A} + B)$

$= A + B$

## Q.6

Minimize the expression.

$f(A, B) = \overline{A}\,\overline{B} + \overline{A}B + A\overline{B} + AB$

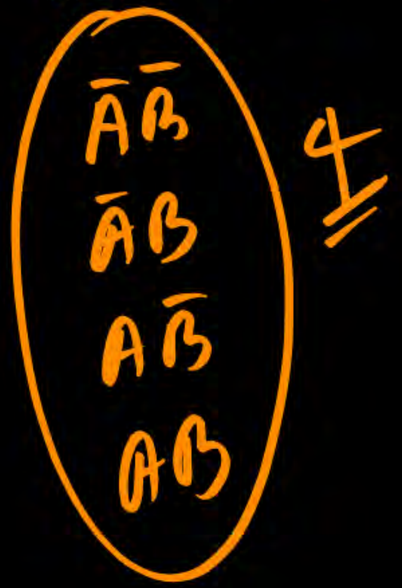
$= \overline{A}[\overline{B} + B] + A[\overline{B} + B]$

$= \overline{A} + A$

$= 1$

2 Variables

minterm



## Q.7

Minimize the expression.

$f(A, B) = \overline{A}B + A\overline{B}$

## Q.8

Minimize the expression.

$f(A, B) = AB + \overline{A}C + BC$

## Q.9

Minimize the expression.

$f(A, B, C) = \overline{A}\,\overline{B} + \overline{A}C + \overline{B}\,\overline{C}$

## Q.10

Minimize the expression.

$f(A, B, C) = (A + B)(A + C)(\overline{B} + C)$

## Q.11

Write the function for truth table and minimize it.

| A | B | Y(0/p) |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

## Q.12

Write the function for truth table and minimize it.

| A | B | Y(O/p) |
|---|---|---|
| 0 | 0 | C |
| 0 | 1 | $\overline{C}$ |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

Q.13

MCQ

Two way switch is a example of which logic?

A AND

B OR

C X-OR

D NAND

## Q.14

If we have ‘4’ variable, then total different expression will be?

Thank you

GW

*Soldiers !*