CS & IT ENGINEERING

DIGITAL LOGIC

Number System

Lecture No. 1

By- CHANDAN SIR

01 Types of Number System

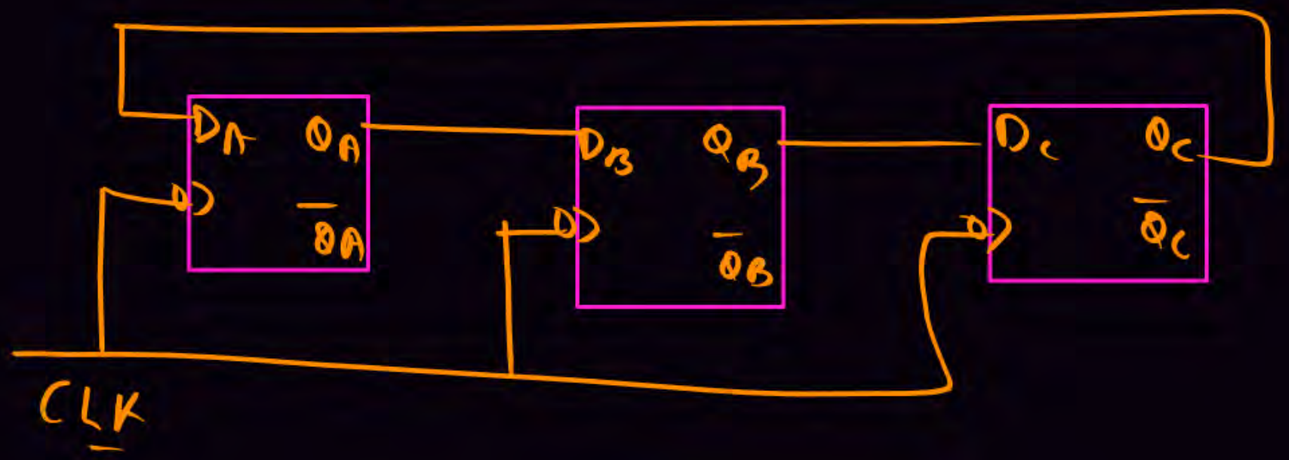
02 Practice

03 Discussion

Synchronous Counter

① RING COUNTER

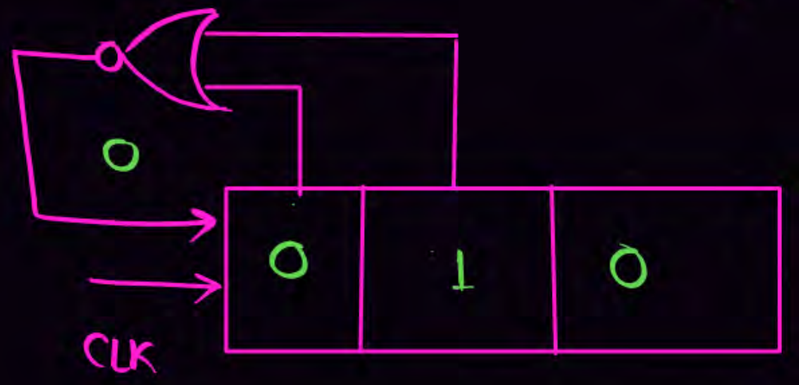
n bit Ring counter ⇒ MOD 'n'



4 bit Ring Counter

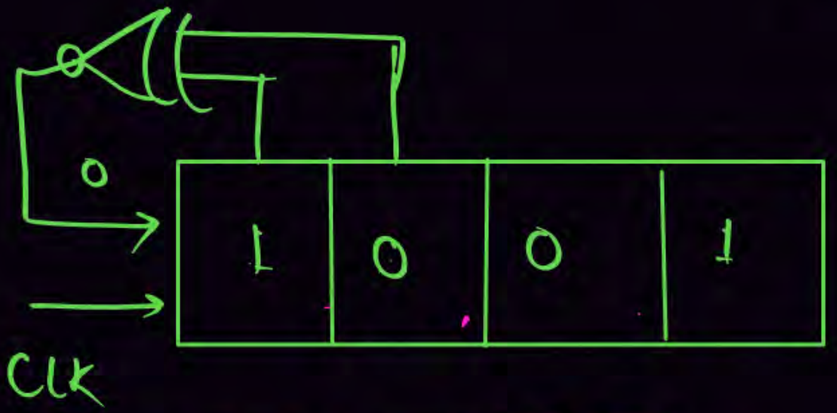
1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

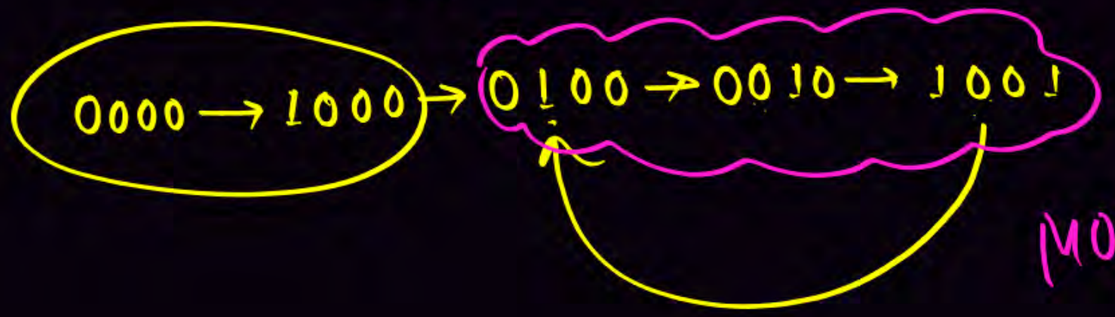
Self starting Ring counter



CLK	Q_1	Q_2	Q_3	
0	0	0	0	X
1	1	0	0	
2	0	1	0	
3	0	0	1	
	1	0	0	

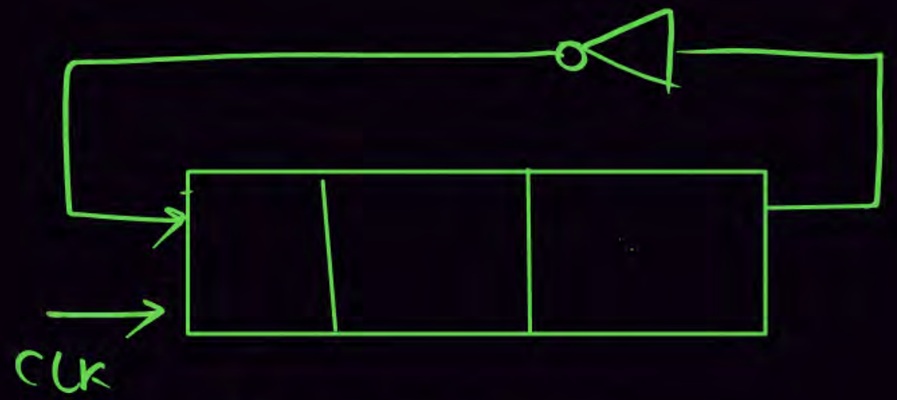
Q Write all the states of the circuit given below?





Clk	Q_1	Q_2	Q_3	Q_4
0	0	0	0	0
1	1	0	0	0
2.	0	1	0	0
3.	0	0	1	0
4.	1	0	0	1
5.	0	1	0	0
6.	0	0	1	0
7	1	0	0	1

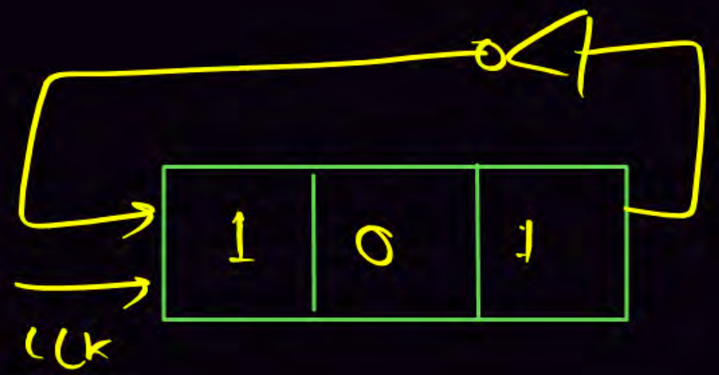
Johnson counter



MOD = 2n ← used state.

unused state = $2^n - 2n$

0	0	0	
1	0	0	
1	1	0	MOD 6
1	1	1	
0	1	1	
0	0	1	

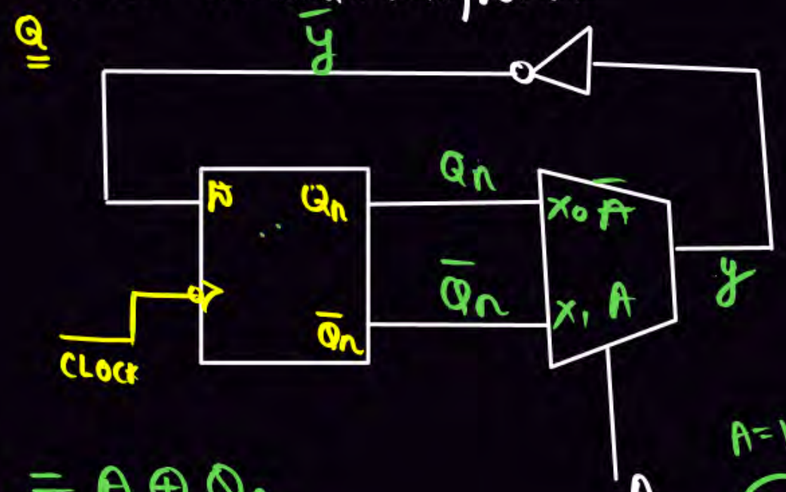


CLK	Q_1	Q_2	Q_3
0	1	0	1
1	0	1	0
2	1	0	1
3	0	1	0

} MOD = 2

Lockout problem

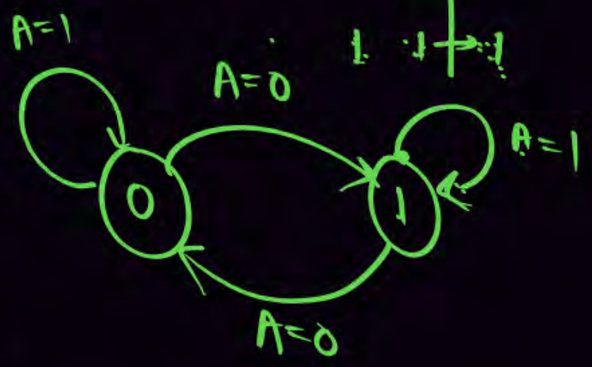
Draw the state Diagram?-



$$y = \bar{A}Q_n + A\bar{Q}_n = A \oplus Q_n$$

$$D = \bar{y} = \overline{A \oplus Q_n} = A \odot Q_n$$

A	Q_n	Q_{n+1}
0	0	1
0	1	0
1	0	0
1	1	1



D-FF

$$Q_{n+1} = D$$

$$Q_{n+1} = A \odot Q_n$$

$$\boxed{A = Q_n \odot Q_{n+1}}$$

Q_n	Q_{n+1}	A
0	0	1
0	1	0
1	0	0
1	1	1

Synchronous counter design

Q Design a synchronous counter by using T-FF which count

{000 → 100 → 110 → 111 → 011 → 001 → 000}

Method ① With lockout problem

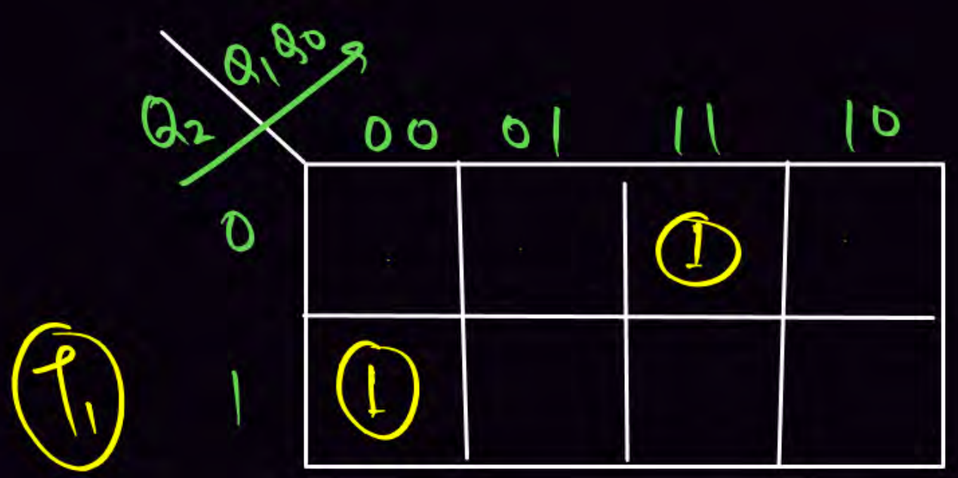
	Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0^+	T_2	T_1	T_0
0 →	0	0	0	1	0	0	1	0	0
4 →	1	0	0	1	1	0	0	1	0
6 →	1	1	0	1	1	1	0	0	1
7 →	1	1	1	0	1	1	1	0	0
3 →	0	1	1	0	0	1	0	1	0
1 →	0	0	1	0	0	0	0	0	1

010

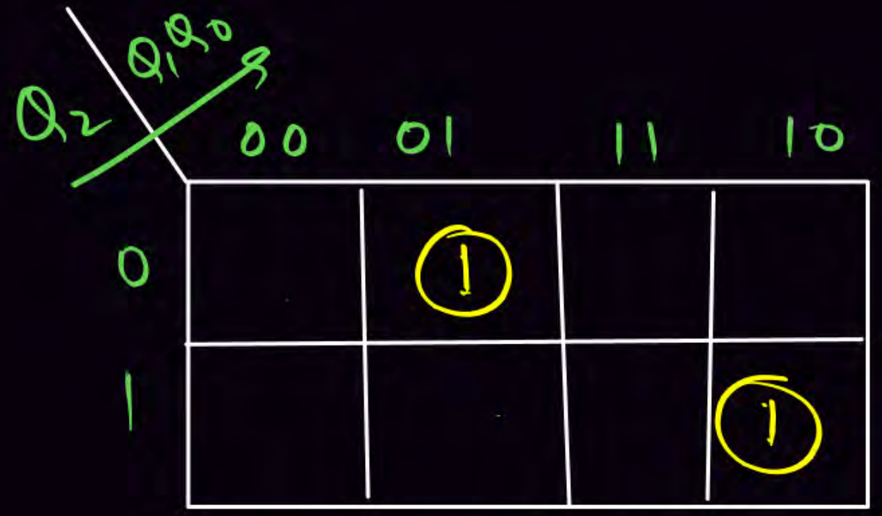
T_2 map (rows Q_2, columns Q_1Q_0 in order 00, 01, 11, 10):

Q_2 \ Q_1Q_0	00	01	11	10
0	1			
1			1	

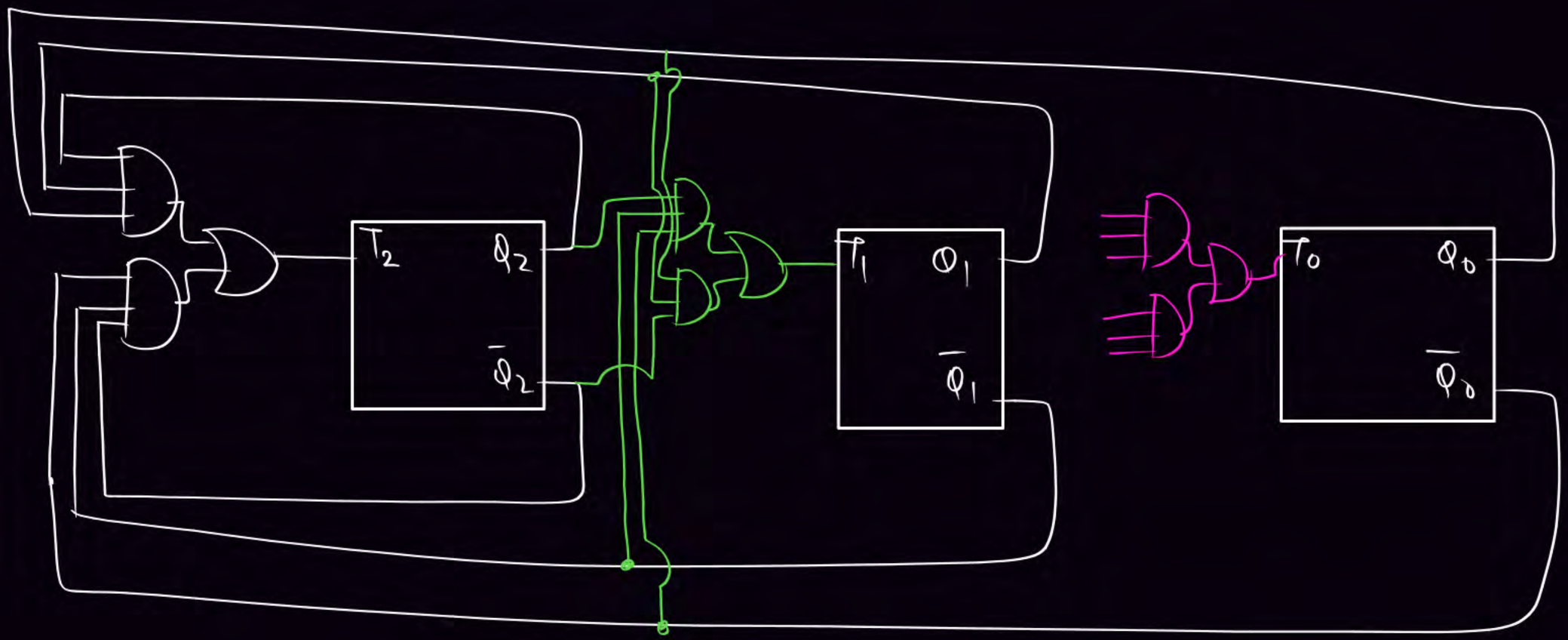
$$T_2 = \bar{Q}_2\bar{Q}_1\bar{Q}_0 + Q_2Q_1Q_0$$



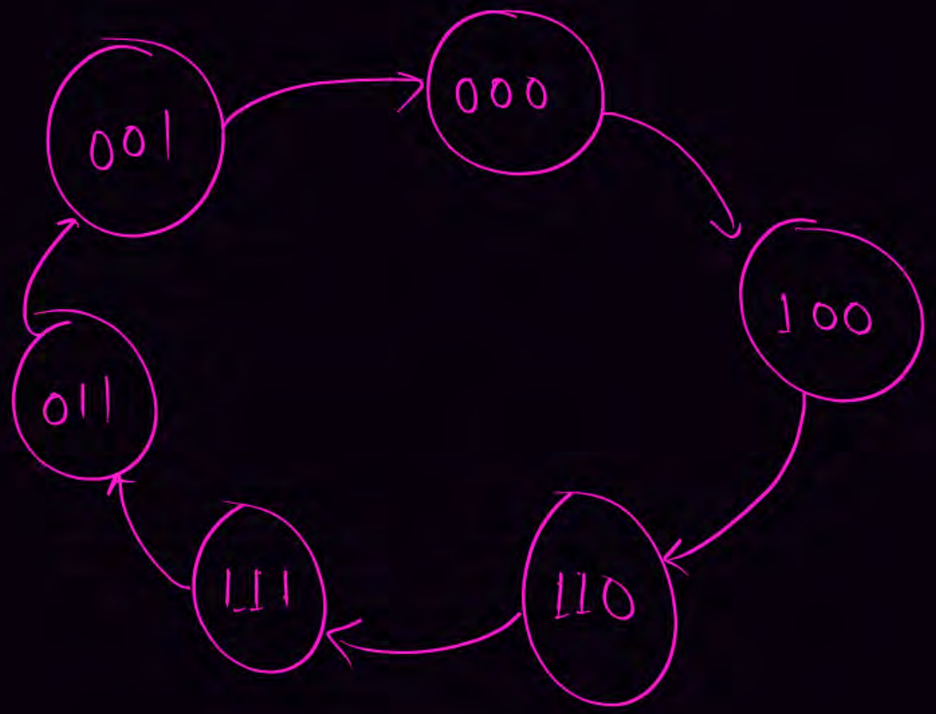
$$T_1 = Q_2\bar{Q}_1\bar{Q}_0 + \bar{Q}_2Q_1Q_0$$



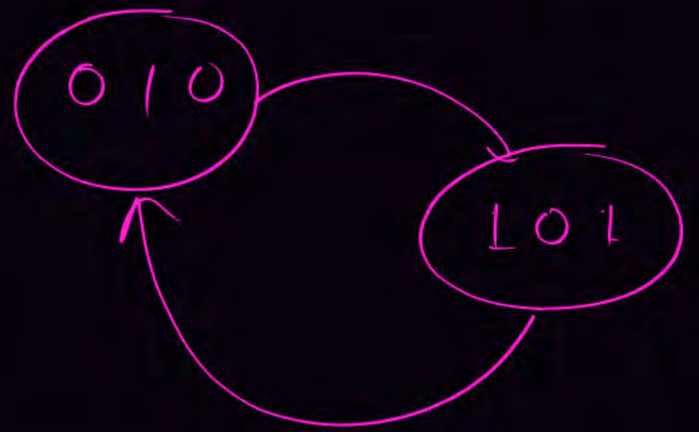
$$T_0 = \bar{Q}_2\bar{Q}_1Q_0 + Q_2Q_1\bar{Q}_0$$


T2
Q2
Q̄2
T1
Q1
Q̄1
T0
Q0
Q̄0

Used state:



Unused state:

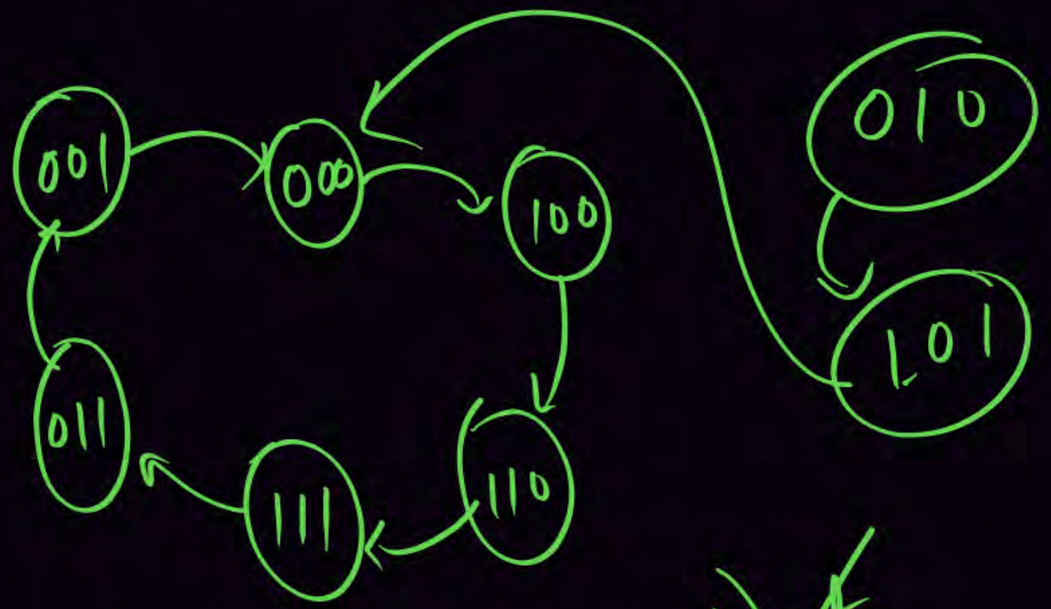


Lockout problem

Method ② Without Lockout

Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0^+	T_2	T_1	T_0
0	0	0	1	0	0	1	0	0
0	0	1	0	0	0	0	0	1
0	1	0	X	X	X	X	X	X
0	1	1	0	0	1	0	1	0
1	0	0	1	1	0	0	1	0
1	0	1	X	X	X	X	X	X
1	1	0	1	1	1	0	0	1
1	1	1	0	1	1	1	0	0

000 → 100 → 110 → 111 → 011 → 001



Lockout

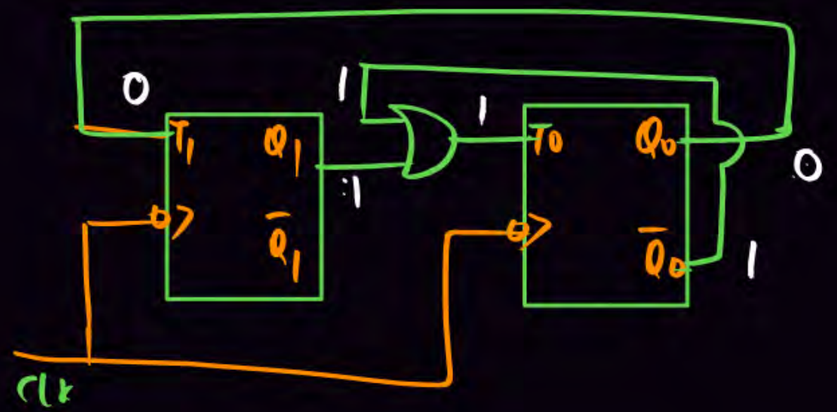
Q $0 \to 1 \to 3 \to 0$

without Lockout

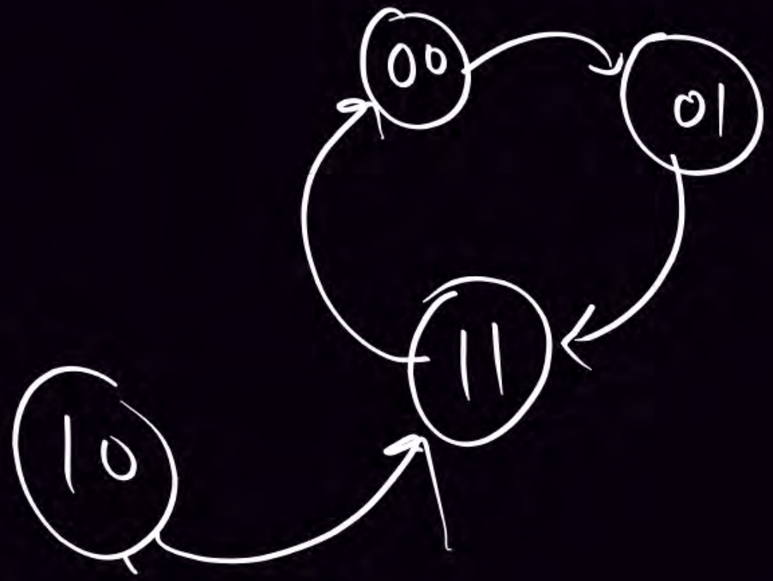
Q_1 Q_0	Q_1^+	Q_0^+	T_1	T_0
0 0	0	1	0	1
0 1	1	1	1	0
1 0	x	x	x	x
1 1	0	0	1	1

$T_1 = Q_0$

$T_0 = Q_1 + \bar{Q}_0$

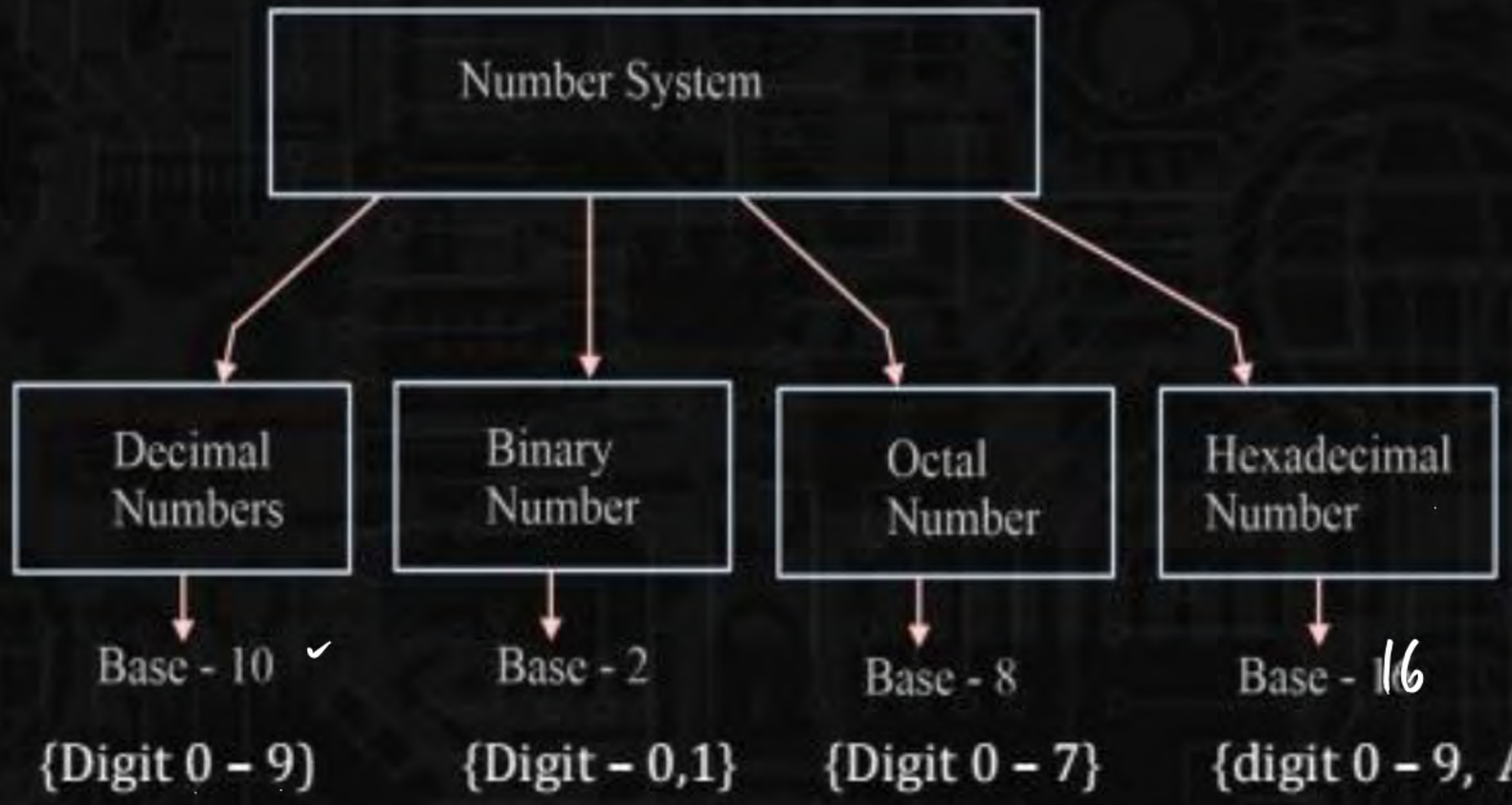


clk	Q_1	Q_0
0	0	0
1	0	1
2	1	1
3	0	0



Base (Radix)

Total number of digit used in the system



10 = A
11 = B
12 = C
13 = D
14 = E
15 = F

Decimal Number System

...	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3} ...
...	a_4	a_3	a_2	a_1	a_0	a_{-1}	a_{-2}	a_{-3} ...

$a^i \rightarrow$ Coefficient of decimal number system

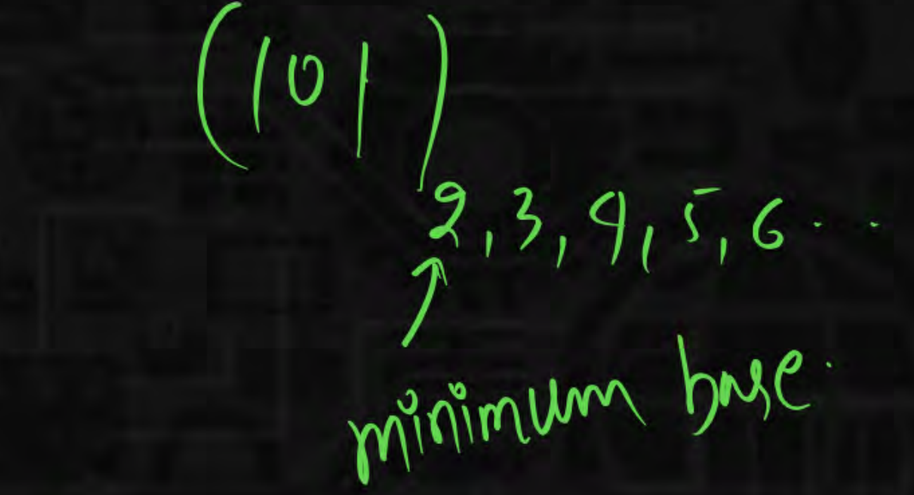
$10^i \rightarrow$ Weight of decimal number system

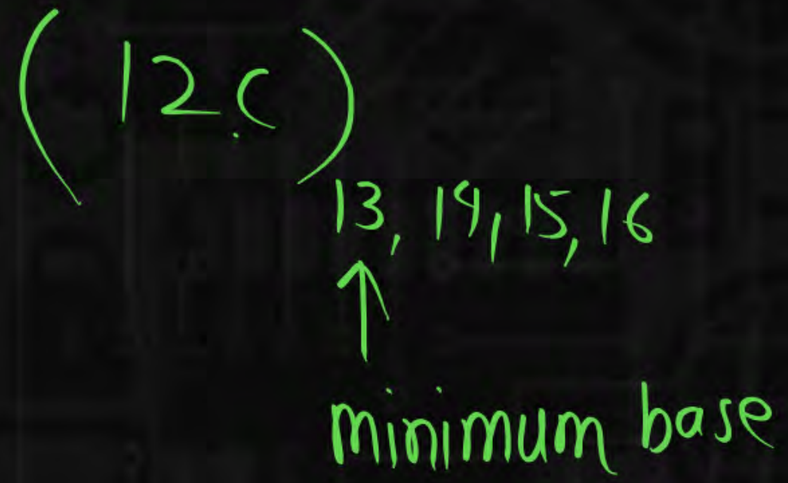
$10^2\ 10^1\ 10^0\ 10^{-1}\ 10^{-2}$

Example : $(501.23)_{10}$

10^2	10^1	10^0	10^{-1}	10^{-2}
5	0	1	2	3

Base	Digit
2	0, 1
3	0, 1, 2
4	0, 1, 2, 3
5	0, 1, 2, 3, 4
6	0, 1, 2, 3,4,5
7	0, 1, 2, 3, 4, 5, 6
8	0, 1, 2, 3, 4, 5, 6, 7
9	0, 1, 2, 3, 4, 5, 6, 7, 8
10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
11	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A
12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B
13	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C
14	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D
15	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E
16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F





Binary Number System (Base (Radix) = 2)

...	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3} ...
...	a_4	a_3	a_2	a_1	a_0	a_{-1}	a_{-2}	a_{-3} ...

$2^i \rightarrow$ Weight of Binary number system

$a_i \rightarrow$ Coefficient of Binary number system {0, 1}

Example:- $(101.11)_2$

2^2	2^1	2^0	2^{-1}	2^{-2}
1	0	1	1	1

Octal Number System (Base (Radix) = 8)

...	8^3	8^2	8^1	8^0	8^{-1}	8^{-2}	8^{-3}...
...	a_3	a_2	a_1	a_0	a_{-1}	a_{-2}	a_{-3}...

$8^i \rightarrow$ Weight of Octal number system

$a_i \rightarrow$ Coefficient of Octal number system {0 -7}

Example:- $(728.64)_8$

8^2	8^1	8^0	8^{-1}	8^{-2}
7	2	8	6	4

Hexadecimal Number System (Base (Radix) = 16)

...	16^3	16^2	16^1	16^0	16^{-1}	16^{-2}	16^{-3} ...
...	a_3	a_2	a_1	a_0	a_{-1}	a_{-2}	a_{-3} ...

$16^i \rightarrow$ Weight of Hexadecimal number system

$a_i \rightarrow$ Coefficient of Hexadecimal number system {0 – 9, A–F}

Example: $(A2C.F)_{16}$

16^2	16^1	16^0	16^{-1}
A	2	C	F

- ✓ Base conversion
- ✓ Magnitude Representation

In base conversion 2 key points are there:

(A) Any base to Decimal conversion $(\quad)_x \longrightarrow (\quad)_{10}$

(B) Decimal to any other base conversion $(\quad)_{10} \longrightarrow (\quad)_x$

(A) Any base to Decimal conversion :

$$(a_3\ a_2\ a_1\ a_0\ .\ a_{-1}\ a_{-2})_{r} = (\ ?\ )_{10}$$

$$(\{a_3 \times r^3\} + \{a_2 \times r^2\} + \{a_1 \times r^1\} + \{a_0 \times r^0\} + \{a_{-1} \times r^{-1}\} + \{a_{-2} \times r^{-2}\})_{10}$$

Case (1) : Binary to Decimal conversion

$2^3\ 2^2\ 2^1\ 2^0\ 2^{-1}\ 2^{-2}$

Ex. $(1011.11)_2 = (\ ?\ )_{10}$

$$\Rightarrow \left[(1\times 2^3)+(0\times 2^2)+(1\times 2^1)+(1\times 2^0)+(1\times 2^{-1})+(1\times 2^{-2})\right]$$

$$\Rightarrow [8+0+2+1+0.5+0.25]_{10}$$

$$\Rightarrow (11.75)_{10}$$

Case (2) : Octal to Decimal conversion

$8^2\ 8^1\ 8^0\ 8^{-1}$

Ex. $(721.4)_8 = (\ ?\ )_{10}$

$$\Rightarrow \left[(7\times8^2)+(2\times8^1)+(1\times8^0)+(4\times8^{-1})\right]_{10}$$

$$\Rightarrow \left[448+16+1+0.5\right]_{10}$$

$$\Rightarrow (465.5)_{10}$$

✓

Case (3) : Hexadecimal to Decimal conversion

$16^2\ 16^1\ 16^0\ 16^{-1}$

Ex. $(A2B.C)_{16} = (\ ?\ )_{10}$

$$\Rightarrow \left[(A\times16^2)+(2\times16^1)+(B\times16^0)+(C\times16^{-1})\right]_{10}$$

$$\Rightarrow \left[(10\times256)+(2\times16)+(11\times1)+(12\times16^{-1})\right]_{10}$$

$$\Rightarrow \left[2560+32+11+0.75\right]_{10}$$

$$\Rightarrow (2603.75)_{10}$$

Case (4) : Base 5 to Decimal conversion

$5^2\ 5^1\ 5^0\ 5^{-1}\ 5^{-2}$

Ex. $(432.22)_5 = (\quad)_{10}$

$$\Rightarrow \left[(4\times5^2)+(3\times5^1)+(2\times5^0)+(2\times5^{-1})+(2\times5^{-2})\right]_{10}$$

$$\Rightarrow \left[100+15+2+0.4+0.08\right]_{10}$$

$$\Rightarrow (117.48)_{10}$$

Q

$7^2\ 7^1\ 7^0$

$(2\ 6\ 3)_7 = (?)_{10}$

$[(2 \times 7^2) + (6 \times 7^1) + (3 \times 7^0)]$

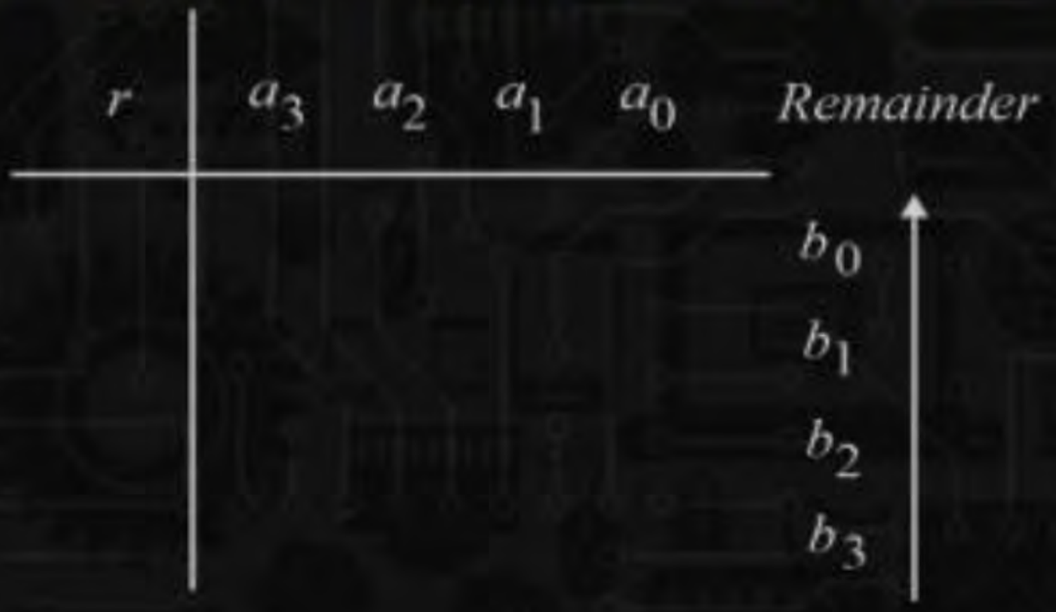
$2 \times 49 + 6 \times 7 + 3$

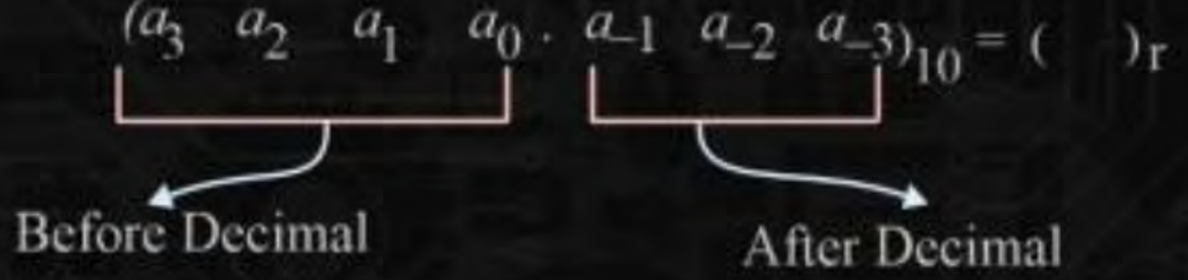
$98 + 42 + 3$

$(193)_{10}$ Ans

Saturday

(B) Decimal to any other Base conversion





$$0 \cdot a_{-1} a_{-2} a_{-3} \times r = x_0 \cdot x_{-1} x_{-2}$$

$$0 \cdot x_{-1} x_{-2} \times r = x_1 \cdot x_{-3} x_{-4}$$

$$0 \cdot x_{-3} x_{-4} \times r = x_2 \cdot x_{-5} x_{-6}$$

$$\left(a_3 a_2 a_1 a_0 \cdot a_{-1} a_{-2} a_{-3}\right)_{10} = \left(b_3 b_2 b_1 b_0 \cdot x_0 x_1 x_2\right)_r$$

Case (1) : Decimal to Binary Base conversion.

Ex. $(19.75)_{10} = (\quad)_2$

Before Decimal | R | After Decimal

2	19	1
2	9	1
2	4	0
2	2	0
	1	1

$0.75 \times 2 = 1.5$ 1

$0.5 \times 2 = 1.0$ 1

$(19.75)_{10} = (10011.11)_2$

Case (2) : Decimal to Octal Base conversion.

Ex. $(210.23)_{10} = (\quad)_8$

Before Decimal Remainder After Decimal

8	210	2
8	26	2
	3	3

$0.23 \times 8 = 1.84$ 1
$0.84 \times 8 = 6.72$ 6
$0.72 \times 8 = 5.76$ 5

$(210.23)_{10} = (322.165)_8$

Case (3): Decimal to Hexadecimal Base conversion.

Ex. $(1228.55)_{10} = (\quad)_{16}$

Before Decimal Remainder | After Decimal

16	1228	12 (C)
16	76	12 (C)
	4	4

$0.55 \times 16 = 8.8 \quad 8$

$0.8 \times 16 = 12.8 \quad 12(C)$

$(1228.56)_{10} = (4CC.8C)_{16}$

Some Special Case

Case (1): Binary to Octal base conversion

Ex. $(10110111)_2 = (\quad)_8$

Octal → means base 8

$8 = 2^3$

Every three digits of binary represent one digit of octal

010	110	111
2	6	7

Hence $(10110111)_2 = (267)_8$

Some Special Case

Case (2) : Binary to Hexadecimal base conversion

Ex. $(1011011)_2 = (\quad)_{16}$

Hexadecimal → means base 16

$16 = 2^4$

Every four digits of binary represent one digit of Hexadecimal.

0101 1011

5 11(B)

Hence $(1011011)_2 = (5B)_{16}$

BCD (Binary Coded Decimal)

In this each digit of the decimal number is represented by its four-bit binary equivalent. It is also called natural BCD or 8421 code. It is weighted code.

Excess – 3 Code: This is an non weighted binary code used for decimal digits. Its code assignment is obtained from the corresponding value of BCD after the addition of 3.

BCO (Binary Coded Octal): In this each digit of the Octal number is represented by its three-bit binary equivalent.

BCH (Binary Coded Hexadecimal): In this each digit of the hexadecimal number is represented by its four bit binary equivalent.

Decimal Digits	BCD 8421	Excess – 3	Octal digits	BCO	Hexadecimal Digits	BCH
0	0000	0011	0	000	0	0000
1	0001	0100	1	001	1	0001
2	0010	0101	2	010	2	0010
3	0011	0110	3	011	3	0011
4	0100	0111	4	100	4	0100
5	0101	1000	5	101	5	0101
6	0110	1001	6	110	6	0110
7	0111	1010	7	111	7	0111
8	1000	1011			8	1000
9	1001	1100			9	1001
					A	1010
					B	1011
					C	1100
					D	1101
					E	1110
					F	1111

Q. What is the possible base of the given number system?

$\sqrt{41} = 5$

A. 4

B. 5

C. 10

D. 6

Q. What is the base of the given number system?

$(292)_{10} = (1204)_{3}$

A. 5

B. 6

C. 6

D. None

Q. 21DA having minimum base conversion to decimal is-

A. 6079

B. 10173

C. 2067

D. None

Q.

Two 2's complement number having sign bits x and y are added and the sign bit of the result is z.

Then, the occurrence of overflow is indicated by the Boolean function.

A. $x\,y\,z$

B. $\bar{x}\,\bar{y}\,\bar{z}$

C. $\bar{x}\,\bar{y}\,z + x\,y\,\bar{z}$

D. $x\,\bar{y}\,\bar{z}$

Q. If $(12x)3 = (123)x$ then the value of x is

A. 3

B. 3 or 4

C. 2

D. None of these

Q.

Convert $(2724)_8$ to $(?)_5$

Q.

The base of the number system for the addition operation 24 + 14 = 41 to be true is

CHOICE (4)		Response
a.	8	
b.	6	
c.	5	
d.	7	

Q. $(1217)_8$ is equivalent to

A. $(1217)_{16}$

B. $(028F)_{16}$

C. $(2297)_{10}$

D. $(0B17)_{16}$

Q.

73_x (in base – x number system) is equal to 54_y (in base – y number system), the possible values of x and y are

A. 8, 16

B. 10,12

C. 9, 13

D. 8, 11

Q. Decimal 43 in Hexadecimal and BCD number system is respectively

A. B2, 0100 0011

B. 2B, 0100 0011

C. 2B, 0011 0100

D. B2, 0100 0100


THANK YOU GW
SOLDIERS !