

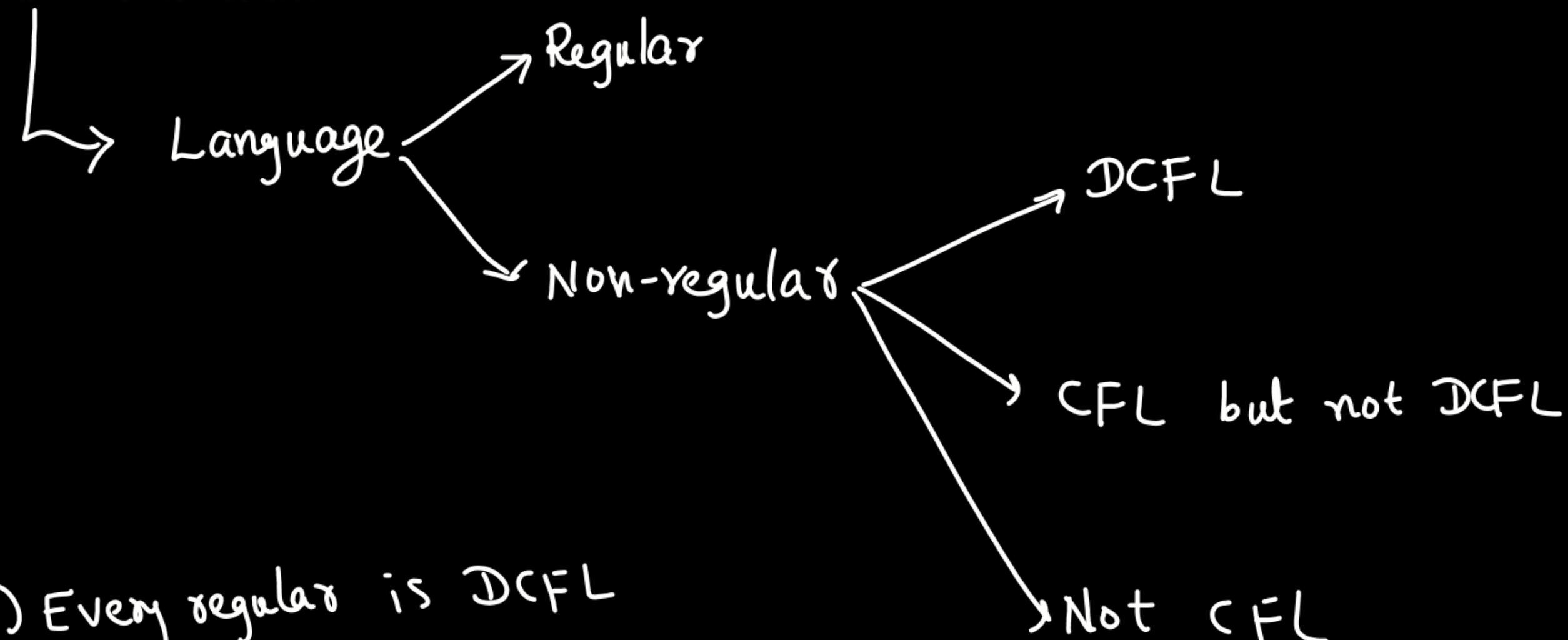
# CS & IT Engineering

Context Free Grammar



Deva sir

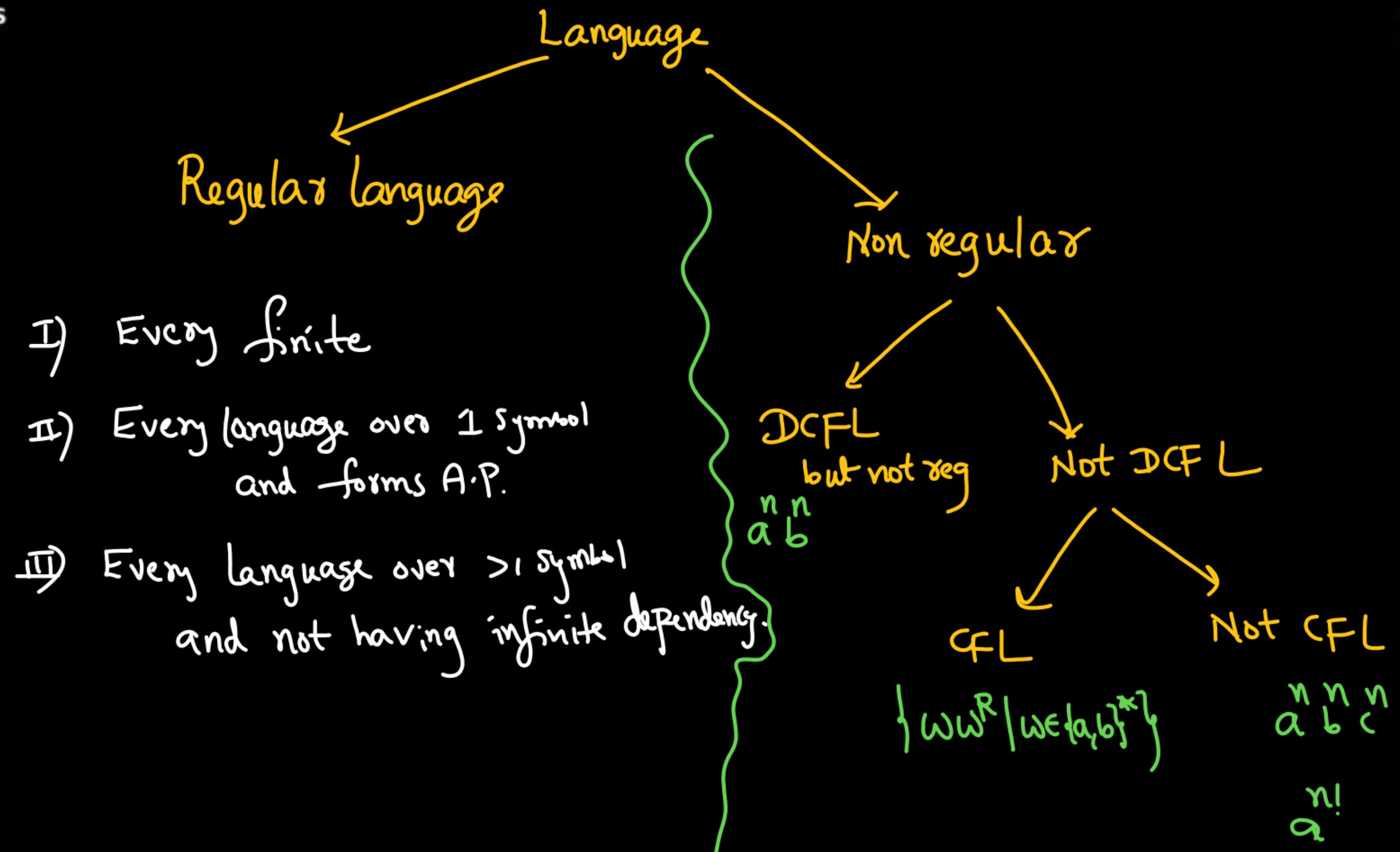
## Topics to be covered:



- I) Every regular is DCFL
- II) Every DCFL is CFL

## Topics Covered in Previous Session:

- ↳ PDA construction
- DPDA vs PDA



①  $\{a^n b^n \mid n < 10\} \Rightarrow \text{Finite} \Rightarrow \text{Reg} \Rightarrow \text{DCFL} \Rightarrow \text{CFL}$

②  $\{a^n b^n\} \Rightarrow \text{DCFL but not regular} \Rightarrow \text{CFL but not regular}$

③  $\{a^m b^n\} = a^* b^* \Rightarrow \text{Regular but not finite}$

④  $\{a^m b^n \mid m=n\} = ②$

⑤  $\{a^m b^n \mid \underbrace{m < n}\} \Rightarrow \text{DCFL but not regular}$

⑥  $\{a^m b^n \mid m > n\}$

⑦  $\{a^m b^n \mid \underbrace{m < n} \text{ or } \underbrace{m > n}\}$

⑧  $\{a^m b^n \mid m < n < 100\} \Rightarrow \text{Finite} \Rightarrow \text{Reg} \Rightarrow \text{DCFL} \Rightarrow \text{CFL}$

⑨  $\{a^m b^n c^k\} = a^* b^* c^* \Rightarrow$  Regular but not finite

⑩  $\{a^m b^n c^k \mid m < n < k\} \Rightarrow$  Not CFL

⑪  $\{a^m b^n c^k \mid m = n = k\} = \{a^n b^n c^n\} \Rightarrow$  Not CFL

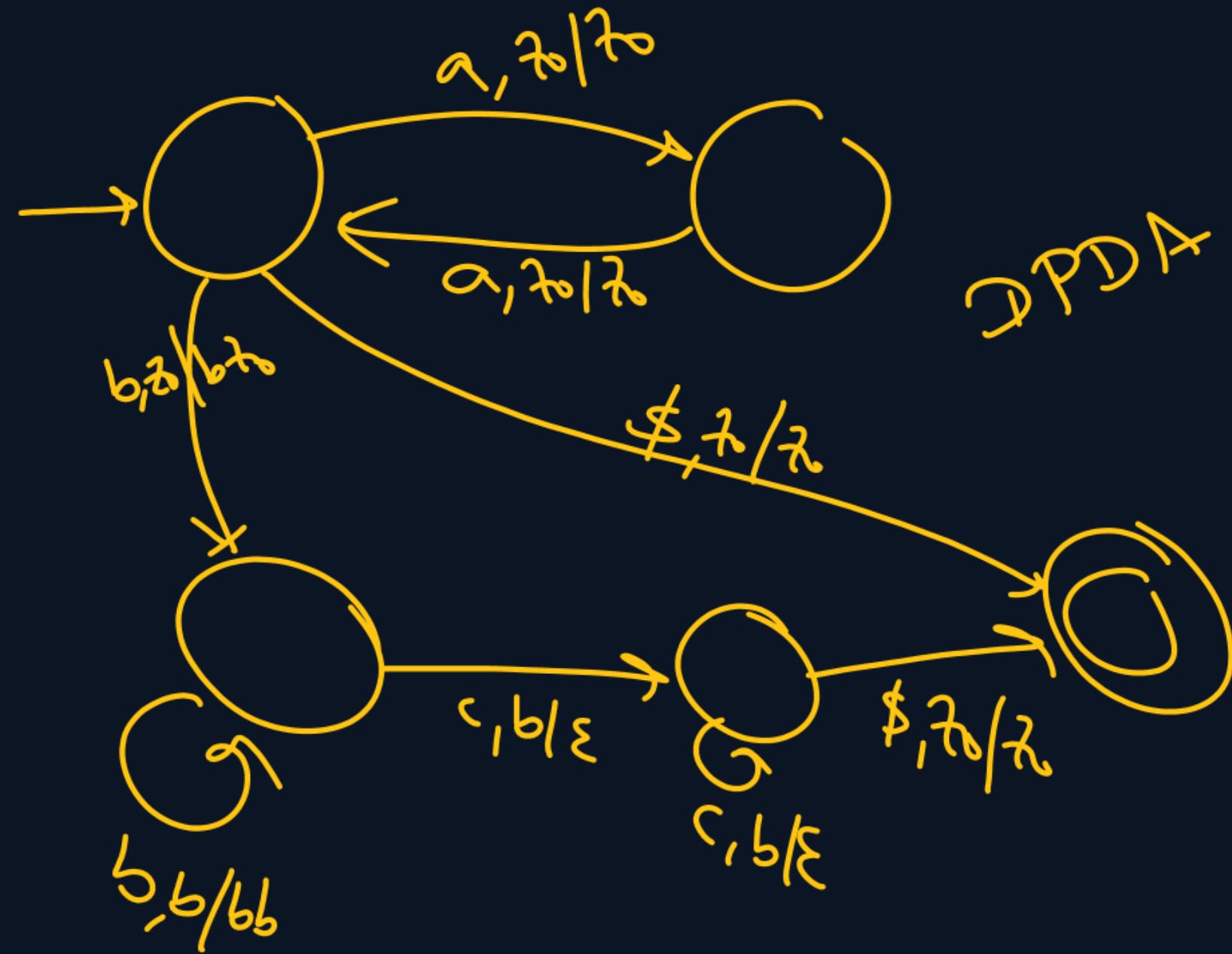
⑫  $\{a^m b^n c^k \mid m > n > k\} \Rightarrow$  Not CFL

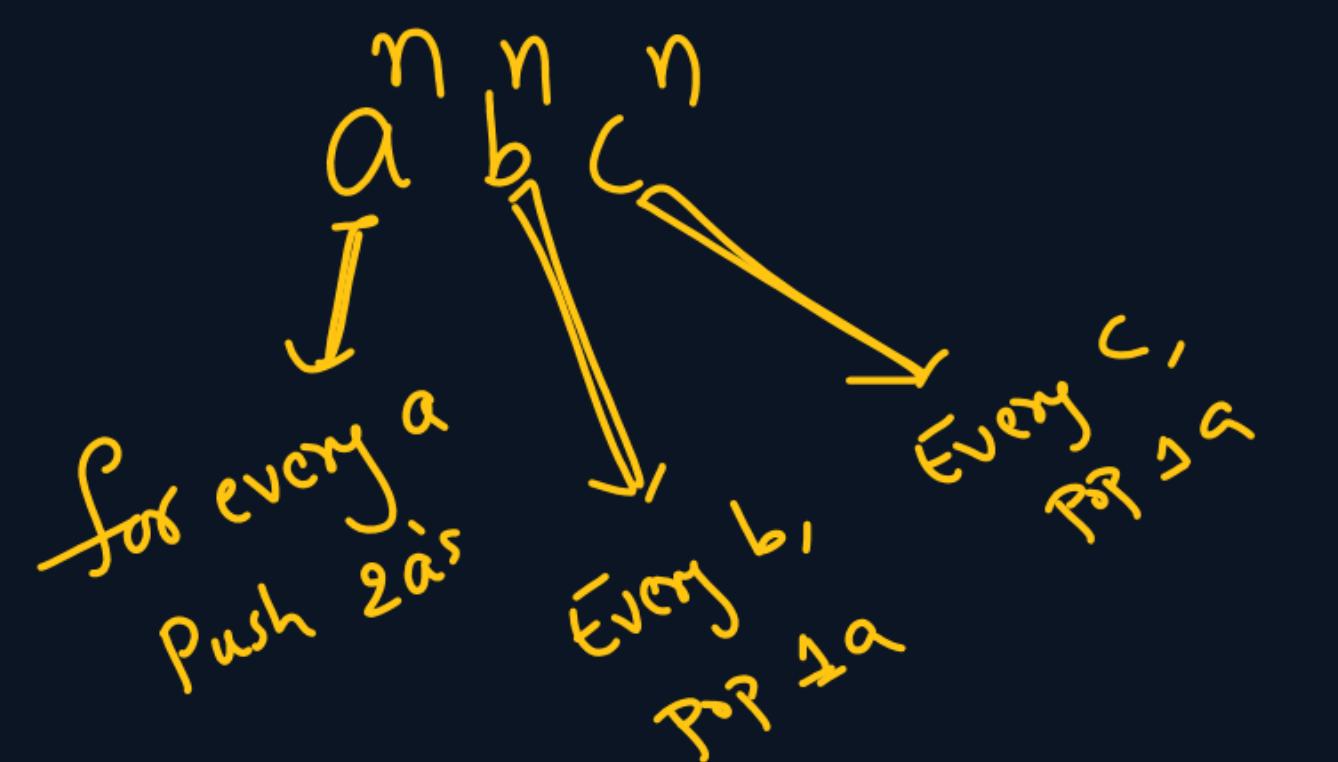
⑬  $\{a^m b^n c^k \mid m < n < k < \underline{1000}\} \Rightarrow$  Finite lang (Reg, DCFL, CFL)

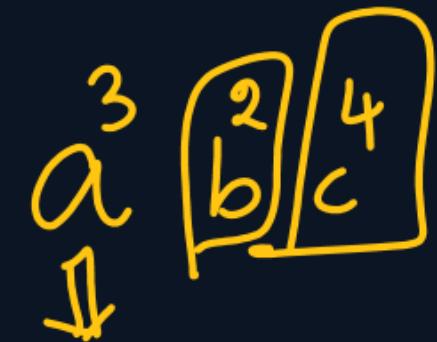
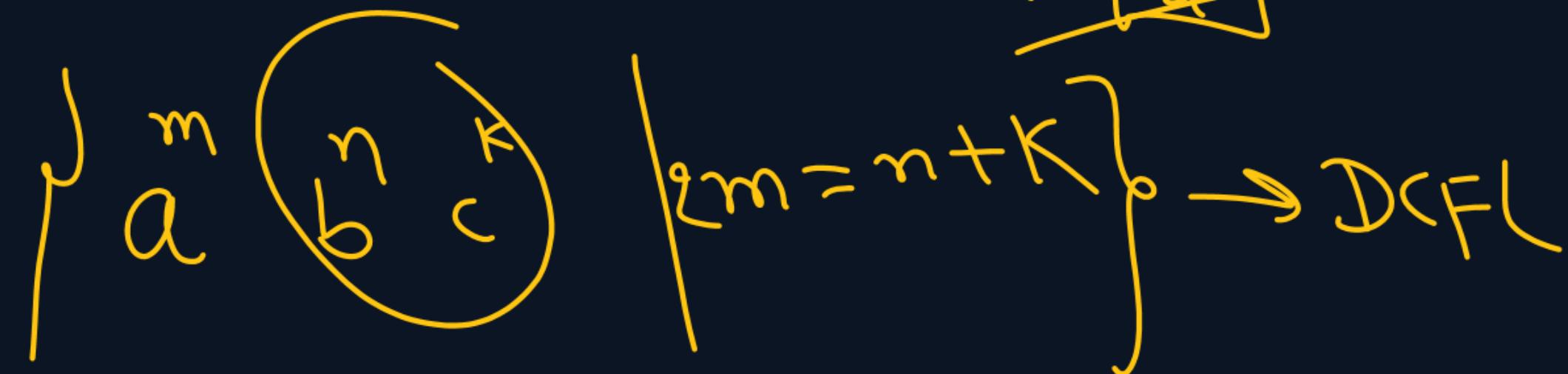
⑭  $\{a^m b^n c^k \mid m = n\} = \{a^n b^n c^*\} \Rightarrow$  DCFL but not regular

⑮  $\{a^m b^n c^k \mid m \neq n\}$

⑯  $\{a^m b^n c^k \mid m = \text{even}, n = k\} = (aa)^* b^n c^n$

$$(aa)^* b^n c^n$$


$a^n b^n c^n$  
  
 For every  $a$   
 Push  $2a$ s  
 Every  $b$ ,  
 Pop  $2a$ s  
 Every  $c$ ,  
 Pop  $2a$ s



DCFL

$$\{a^m b^n c^k \mid m=n+k\}$$

Push  $a$ , Pop  $a$ , Push  $b$ , Pop  $b$ , Push  $c$ , Pop  $c$

DCFL

$$\{a^m b^n c^k \mid n=m+k\} = a^m b^{m+k} c^k$$

Push  $a$ , Pop  $a$ , Push  $b$ , Pop  $b$ , Push  $c$ , Pop  $c$

DCFL

$$\{a^m b^n c^k \mid k=m+n\}$$

Push  $a$ , Pop  $a$ , Push  $b$ , Pop  $b$ , Push  $c$ , Pop  $c$

$$\{a^m b^n c^k \mid \text{if } (m=\text{even}) \text{ then } (n=k)\} \Rightarrow \text{DCFL}$$

Same

$$\{a^m b^n c^k \mid \text{if } (m=\text{odd}) \text{ then } (n=k)\} \Rightarrow \text{DCFL}$$

20

$$\{a^m b^n c^k \mid \text{if } (m=\text{even}) \text{ then } (n \neq k)\} \Rightarrow \text{DCFL}$$

21

$$\{a^m b^n c^k \mid (m=\text{odd}) \text{ OR } (n=k)\} = 20$$

22

$$\{a^m b^n c^k \mid (m=\text{even}) \text{ OR } (n=k)\} = 21$$

$P \Rightarrow Q$

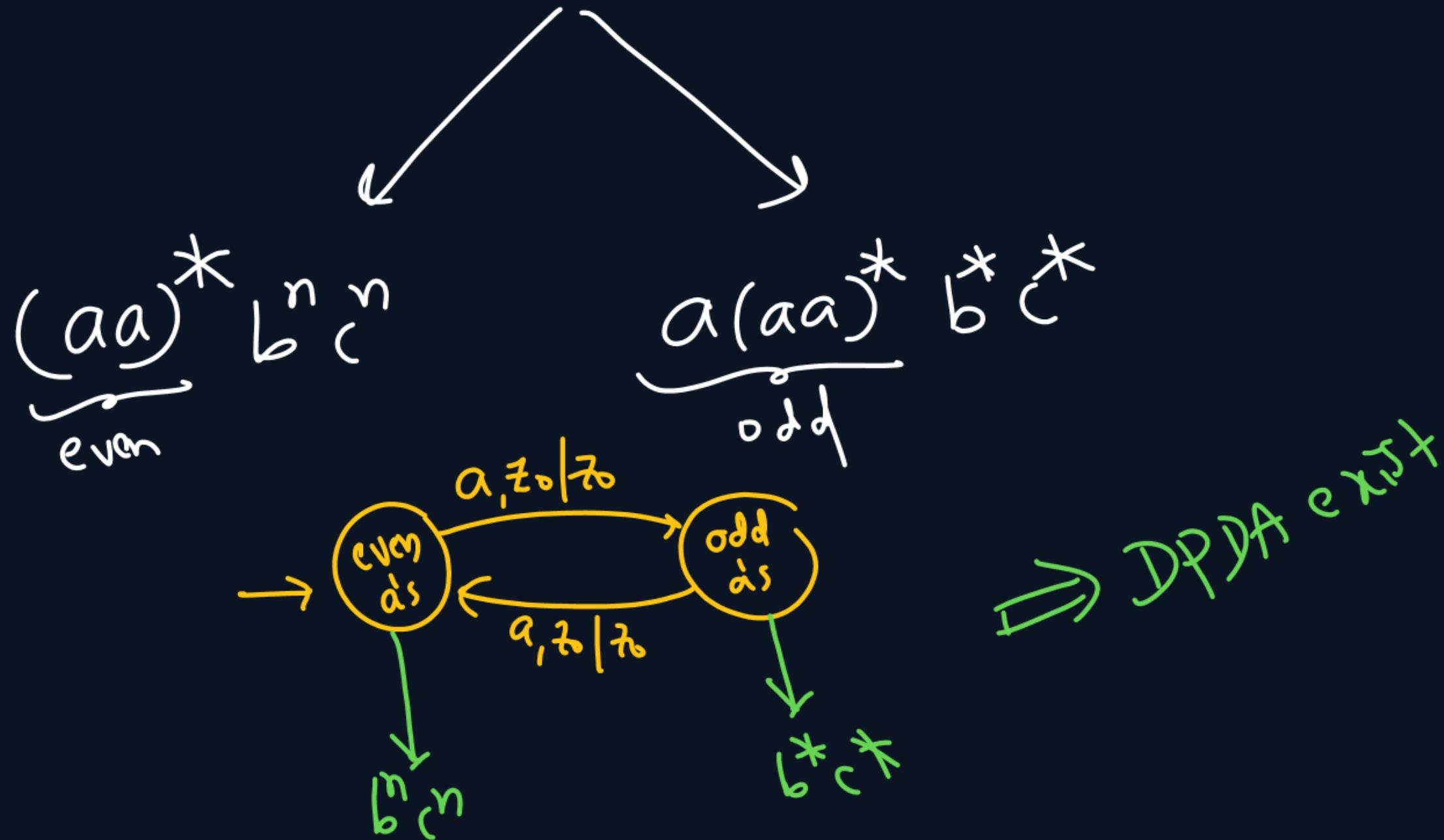
If  $P$  then  $Q$

$\sim P \vee Q$

If  $(m = \text{even})$  then  $(n = k)$

$(m = \text{odd}) \text{ OR } (n = k)$

$$\left\{ a^m b^n c^k \mid \text{if } (m = \text{even}) \text{ then } (n = k) \right\}$$



②5  $\{a^m b^n c^k \mid \text{if } (n=\text{even}) \text{ then } (\underline{m < k})\} \Rightarrow \text{DCFL}$

②6  $\{a^m b^n c^k \mid \text{if } (n=\text{odd}) \text{ then } (\underline{m \neq k})\}$

②7  $\{a^m b^n c^k \mid \text{if } (k=\text{even}) \text{ then } (m=n)\}$

②8  $\{a^m b^n c^k \mid \text{if } (k=\text{odd}) \text{ then } (m < n)\}$

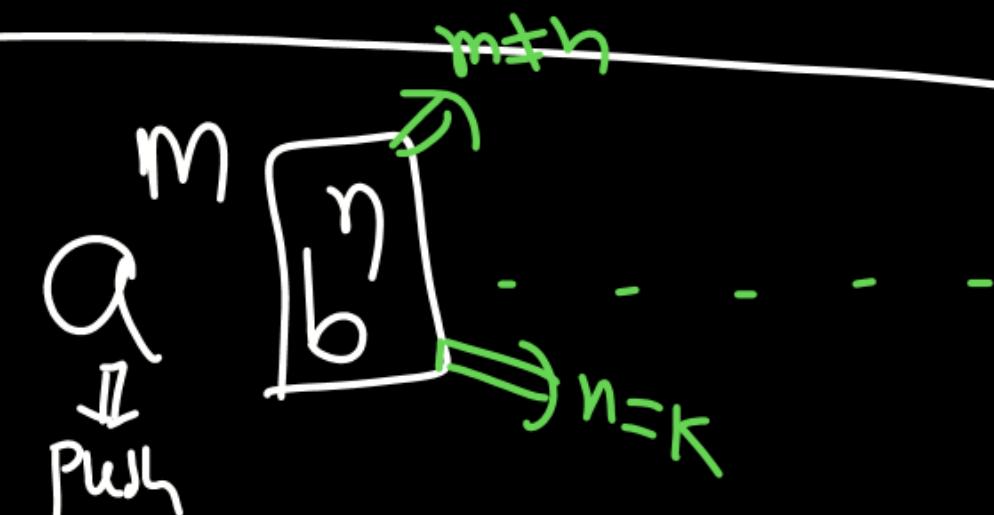
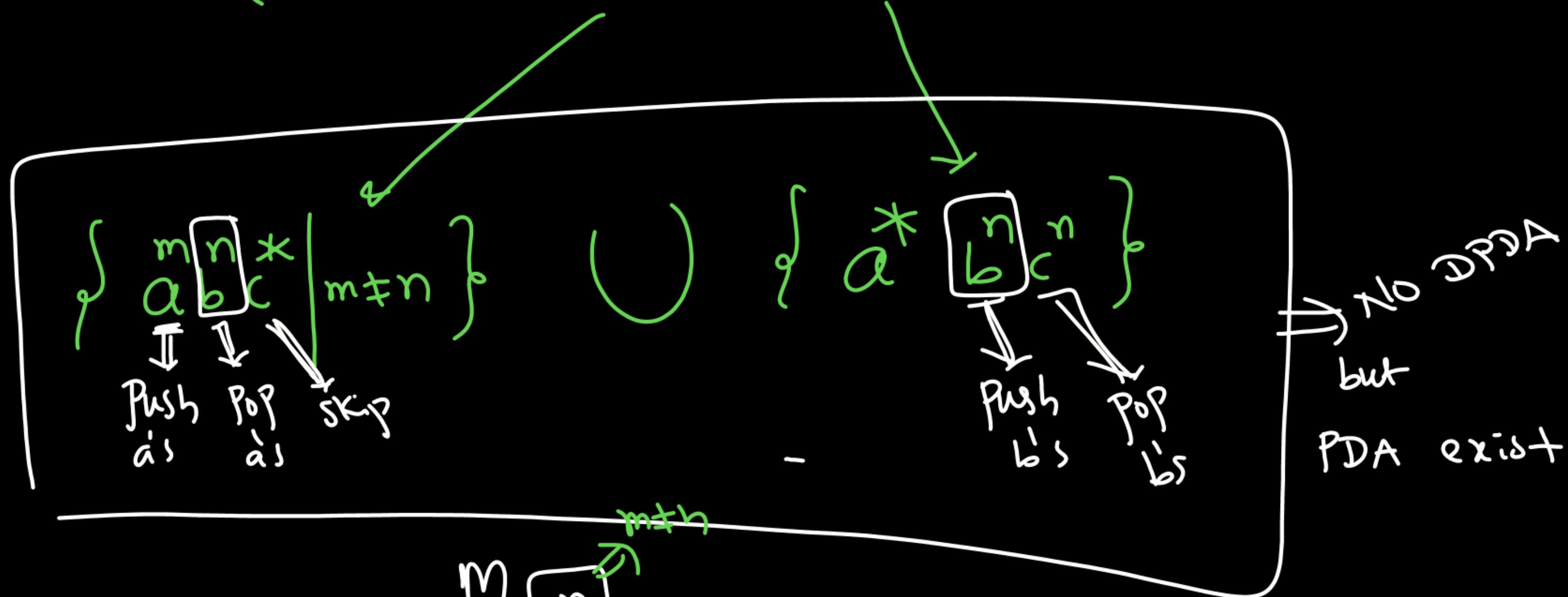
②9  $\{a^m b^n c^k \mid \text{if } (m=n) \text{ then } (n=k)\} \Rightarrow \text{CFL but not DCFL}$

③0  $\{a^m b^n c^k \mid (m \neq n) \text{ OR } (n=k)\} = ②9$

③1  $\{a^m b^n c^k \mid (m=n) \text{ OR } (n>k)\}$

③2  $\{a^m b^n c^k \mid \text{if } (m \neq n) \text{ then } (n>k)\} = ③1$

$\{ a^m b^n c^k \mid \underline{m \neq n} \text{ OR } \underline{n = k} \}$  CFL but not DCFL



- (33)  $\{a^m b^n c^k \mid m=n \text{ AND } n=k\} = a^n b^n c^n \Rightarrow \text{not CFL}$
- (34)  $\{a^m b^n c^k \mid m=n, n \neq k\}$
- (35)  $\{a^m b^n c^k \mid m < n, n > k\}$
- (36)  $\{a^m b^n c^k \mid n < m, m < k\}$
- (37)  $\{a^m b^n c^k \mid n \neq m, m \neq k\}$
- (38)  $\{a^m b^n c^k \mid m < n > k\}$
- (39)  $\{a^m b^n c^k \mid m > n < k\}$
- (40)  $\{a^m b^n c^k \mid m = n = k\}$

④1  $\{ ww \mid w \in \{a, b\}^*\} \Rightarrow \text{Not CFL}$

④2  $\{ w\#w \mid w \in \{a, b\}^*\}$

\* \* \* ④3  $\{ ww^R \mid w \in \{a, b\}^*\} \Rightarrow \text{CFL but not DCFL}$

④4  $\{ \underbrace{w}_{\text{push}} \# \underbrace{w^R}_{\text{skip}} \mid w \in \{a, b\}^* \} \Rightarrow \text{DCFL}$

④5  $\{ ww \mid w \in a^*\} = (aa)^* \Rightarrow \text{Regular}$

④6  $\{ w\#w \mid w \in a^* \} = \{ a^n \# a^n \} \Rightarrow \text{DCFL}$

④7  $\{ ww^R \mid w \in a^* \} = \textcircled{45} \Rightarrow \text{Regular}$

④8  $\{ w\#w^R \mid w \in a^* \} = \textcircled{46} \not\Rightarrow \text{DCFL}$

$$\{ WW \mid W \in \{a, b\}^*\}$$


Stack is helpless



Stack  
is helpful

④⁹  $\{ \underline{w} \underline{w} \underline{w} \mid w \in \{a, b\}^* \} \Rightarrow \text{Not CFL}$

⑤⁰  $\{ \underline{w} \# \underline{w} \# \underline{w} \mid w \in \{a, b\}^* \} \Rightarrow \text{Not CFL}$

⑤¹  $\{ \underline{w} w^R \underline{w} \mid w \in \{a, b\}^* \} \Rightarrow \text{Not CFL}$

⑤²  $\{ w \# w^R \# w \mid w \in \{a, b\}^* \} \Rightarrow \text{Not CFL}$

⑤³  $\{ www \mid w \in a^* \} = \{ \epsilon, \overset{3}{a}, \overset{6}{a}, \overset{1}{a}, \dots \} = (aaa)^* \Rightarrow \text{Regular}$

⑤⁴  $\{ w \# w \# w \mid w \in a^* \} = a^n \# a^n \# a^n \Rightarrow \text{Not CFL}$

⑤⁵  $\{ ww^R w \mid w \in a^* \} = (aaa)^* = \textcircled{53}$

⑤⁶  $\{ w \# w^R \# w \mid w \in a^* \} = \textcircled{54} \Rightarrow \text{Not CFL}$

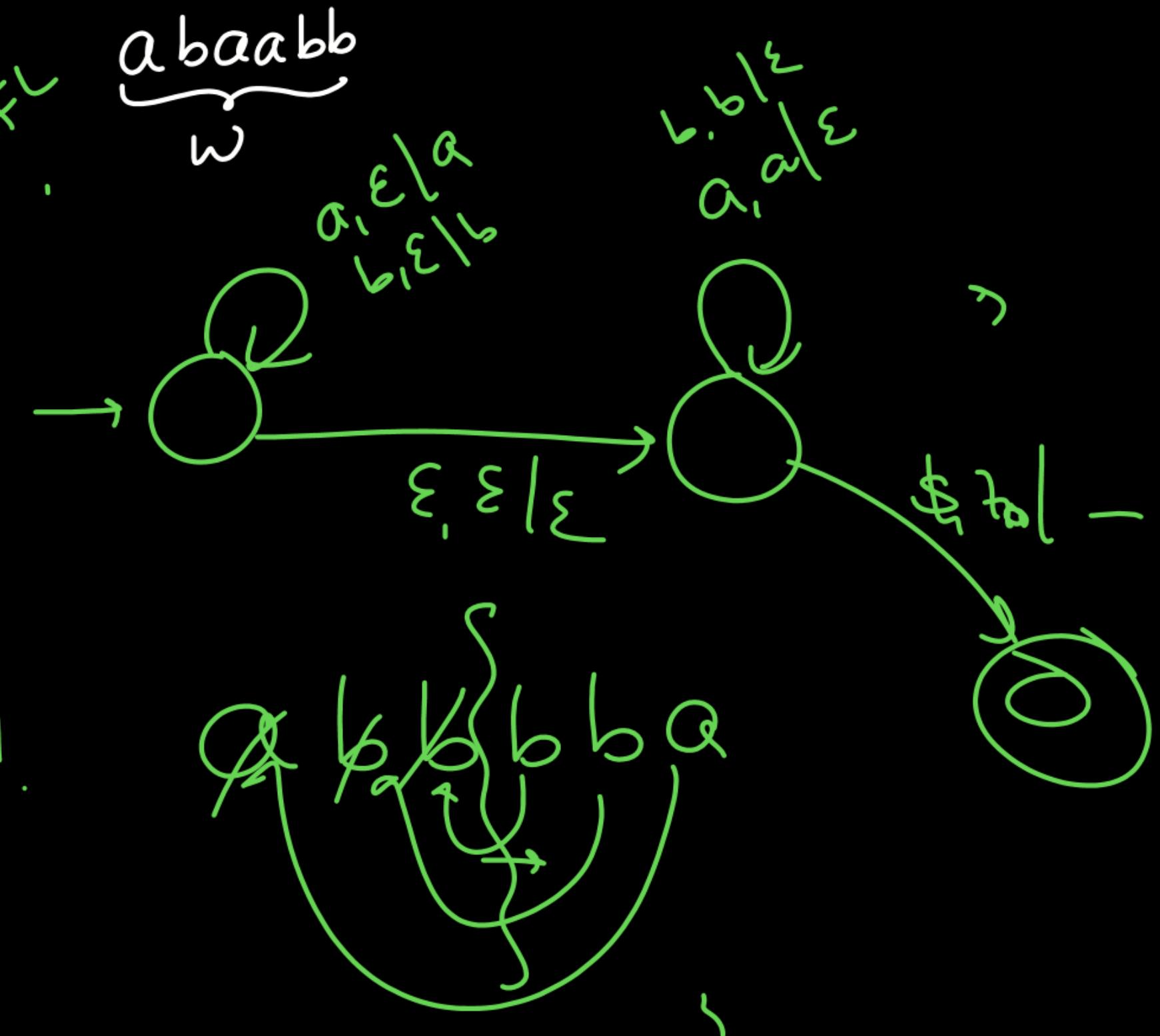
$$\{ww^R \mid w \in \{a,b\}^*\}$$

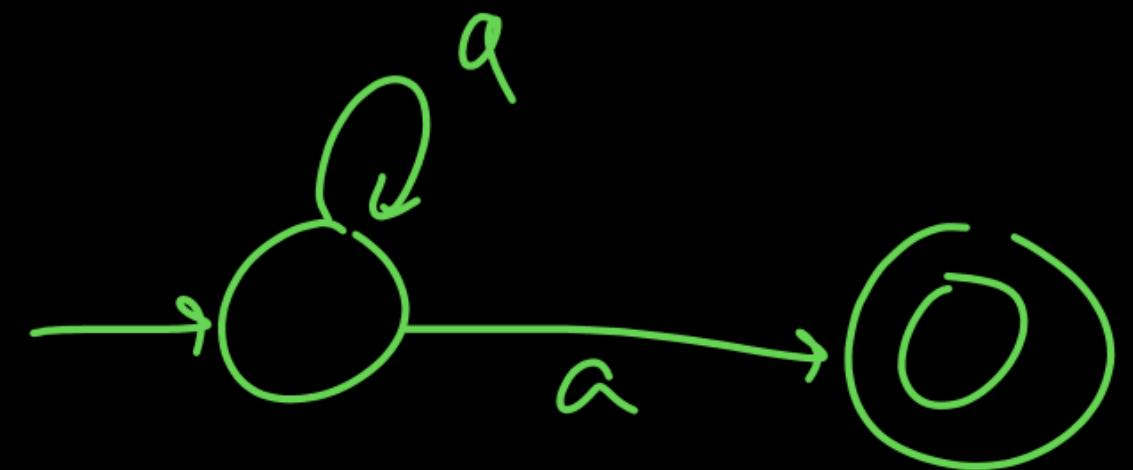
$\epsilon$  ✓  
 $a|a$  ✓  
 $b|b$  ✓  
 $a|aa$  ✓  
 $a|ba$  ✓  
 $b|ab$  ✓  
 $b|bb$  ✓

⋮ why  $w^R$  begins here?

CFL  
not DCFL

$abaabb$   
 $w$





$w = a$  valid

Is there any path that goes to final?

(57)  $\{ \underbrace{a^n b^n}_{\text{push } P \text{ of } \text{Push}} \underbrace{c^k d^l}_{\text{pop } P \text{ of } \text{Pop}} \} \Rightarrow \text{DCFL}$

(58)  $\{ \underbrace{a^n b^n}_{\text{Push } P} \underbrace{c^k d^l}_{\text{Pop } P} \}$

(59)  $\{ \underbrace{a^n b^k c^n d^l}_{\text{Push } P \text{ of } \text{Push}} \} \Rightarrow \text{Not CFL}$

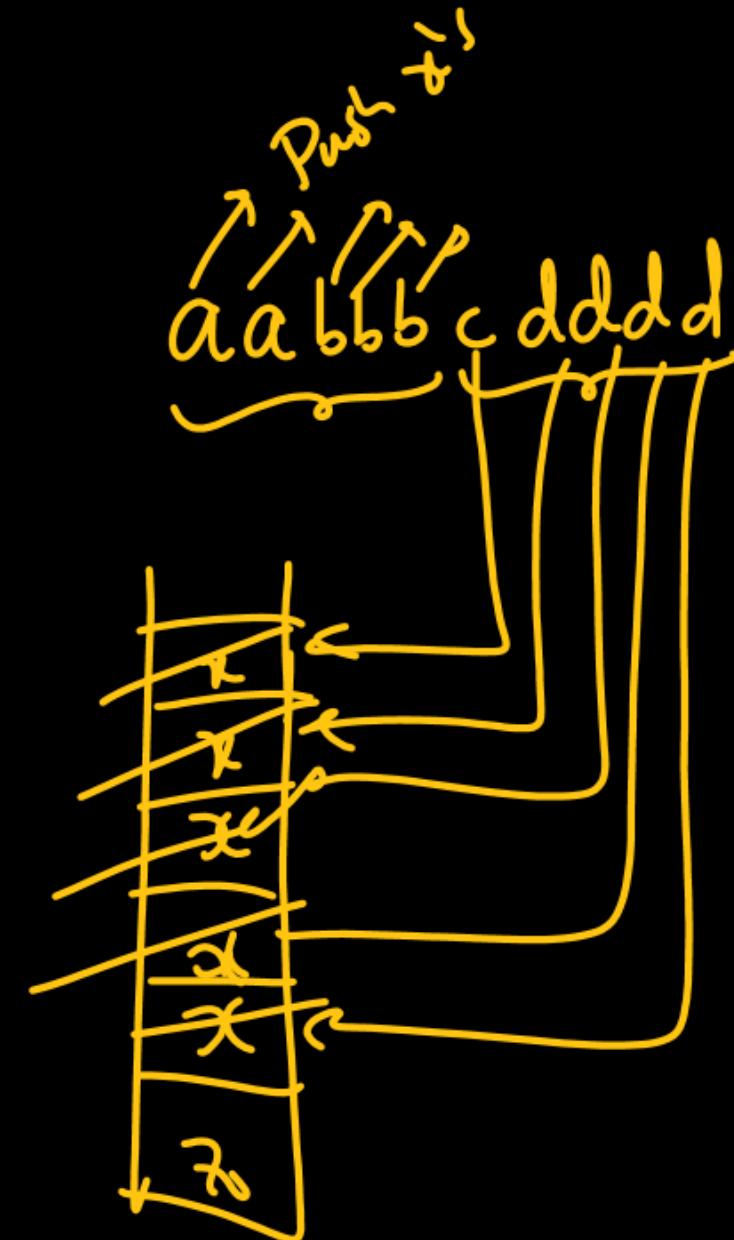
(60)  $\{ \underbrace{a^m b^n}_{\text{Push } P} \underbrace{c^k d^l}_{\text{Pop } P} \mid m+n=k+l \} \Rightarrow \text{DCFL}$

(61)  $\{ \underbrace{a^m b^n}_{\text{Push } P} \underbrace{c^k d^l}_{\text{Pop } P} \mid m \neq n, k=l \} \Rightarrow \text{DCFL}$

(62)  $\{ \underbrace{a^m b^n}_{\text{Push } P} \underbrace{c^k d^l}_{\text{Pop } P} \mid m < n, k < l \}$

(63)  $\{ \underbrace{a^m b^n}_{\text{Push } P} \underbrace{c^k d^l}_{\text{Pop } P} \mid m=n, k=l \} = (57)$

(64)  $\{ \underbrace{a^m b^n}_{\text{Push } P} \underbrace{c^k d^l}_{\text{Pop } P} \mid m=k, n=l \} = (59) \Rightarrow \text{Not CFL}$



## Languages

P  
W

65  $\{a^{\text{prime}}\}$

66  $\{a^{n!}\}$

67  $\{a^{2^n}\}$

68  $\{a^n\}$

69  $\{a^{n^2}\}$

70  $\{a^{\text{prime}}\}^*$

71  $\{a^{n!}\}^*$

Not regulars  
Not CFLs

Languages over 1 symbol  
Stack is helper

If  $L$  is over 1 symbol and  $L$  is not regular  
then  $L$  is not CFL

Regulars

72  $\{a^m b^n c^{m+n}\} = \underbrace{a^m}_{\text{Push}} \underbrace{b^n}_{\text{Push}} \underbrace{c^{m+n}}_{\text{Pop}} \Rightarrow \text{DCFL}$

73  $\{a^{\boxed{n}} b^{\boxed{n+1}} c^{\boxed{n+2}}\} \Rightarrow \text{not CFL}$

74  $\{a^{n+i} b^{n+j} c^{n+k}\} = a^* b^* c^* = \underbrace{a^i b^j c^k}_{n=0} \Rightarrow \text{Regular}$

75  $\{a^{m+n} b^{n+k} c^{k+m}\} = \underbrace{a^m a^n b^k b^m c^k c^m}_{\text{Push}} \Rightarrow \text{CFL}$

76  $\{a^n b^{n^2} c^{n^3}\} \Rightarrow \text{Not CFL}$

77  $\{a^{\boxed{n}} b^{\boxed{n}} c^{\boxed{n}} d^{\boxed{n}} e^{\boxed{n}}\}$

78  $\{a^n b^n c^k d^k e^l\} \Rightarrow \text{DCFL}$

79  $\{ w w^R x \mid w, x \in \{a, b\}^* \} \rightarrow \text{Regular}$

80  $\{ \underbrace{w w^R}_{\substack{\text{Match} \\ \text{Pop}}} \underbrace{x}_{\substack{\text{Skip}} \downarrow} \mid w, x \in \{a, b\}^+ \} \rightarrow \text{CFL}$

81  $\{ w x w^R \mid w, x \in \{a, b\}^* \} \rightarrow \text{Regular}$

82  $\{ w x w^R \mid w, x \in \{a, b\}^+ \} \rightarrow \text{Regular}$

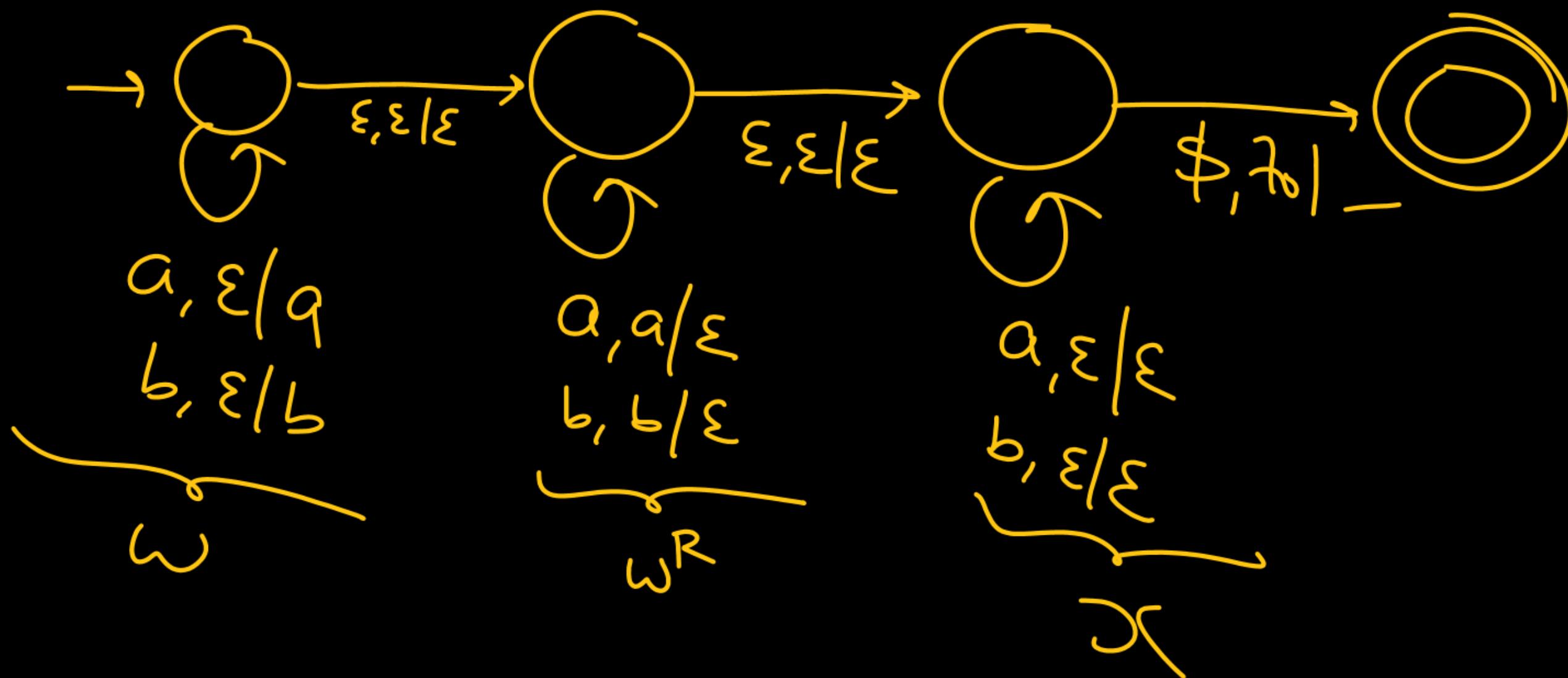
83  $\{ x w w^R \mid w, x \in \{a, b\}^* \} \rightarrow \text{Regular}$

84  $\{ \underbrace{x w w^R}_{\substack{\text{skip logic in PDA}}} \mid w, x \in \{a, b\}^+ \} \rightarrow \text{CFL}$

85  $\{ \underbrace{w_1 w_1^R}_{\substack{\text{skip} \\ \text{logic in PDA}}} \underbrace{w_2 w_2^R}_{\substack{\text{skip} \\ \text{logic in PDA}}} \underbrace{w_3 w_3^R}_{\substack{\text{skip} \\ \text{logic in PDA}}} \mid w_1, w_2, w_3 \in \{a, b\}^* \} \Rightarrow \text{CFL}$

86  $\{ \underbrace{w_1 \# w_1^R}_{\substack{\text{skip} \\ \text{logic in PDA}}} \# \underbrace{w_2 \# w_2^R}_{\substack{\text{skip} \\ \text{logic in PDA}}} \mid w_1, w_2 \in \{a, b\}^* \} \Rightarrow \text{DCFL}$

$$\omega \omega^R x \mid \omega, x \in \{a, b\}^+$$



# Summary

CFLs ✓

DCLFs ✓

not CFLs ✓

not DCLFs ✓

Res ✓

not Reg ✓

# Thank you

