

CS & IT Engineering

2-3M



Deva sir

Topics to be covered:

2-3 marks → Decision properties Table

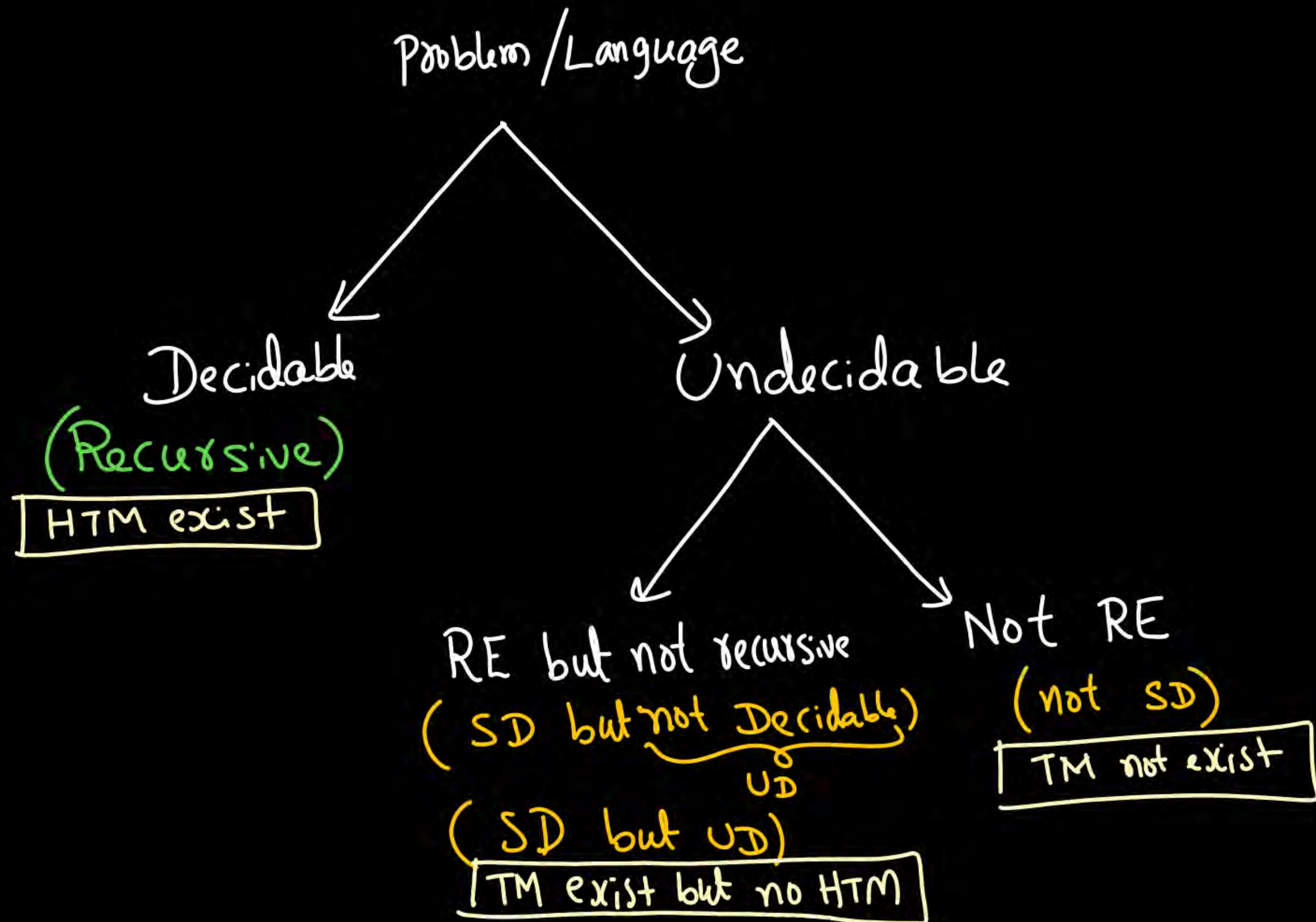
Topics Covered in Previous Session:

→ Turing Machine
closure properties

Undecidability



		Regulars FA	DCFLs DPDA	CFLs PDA	Recursive, HTM	RECs TM
H	① Halting problem	D	D	D	D	UD
M	② Membership problem	D	D	D	D	UD
E	③ Emptiness problem	D	D	D	UD	UD
F	④ Finiteness problem	D	D	D	UD	UD
T	⑤ Totality problem	D	D	UD	UD	UD
E ₂	⑥ Equivalence problem	D	D	UD	UD	UD
D	⑦ Disjointness problem	D	UD	UD	UD	UD
S	⑧ Set containment problem	D	UD	UD	UD	UD



All problems

All RE Problems
(All SD Problems)

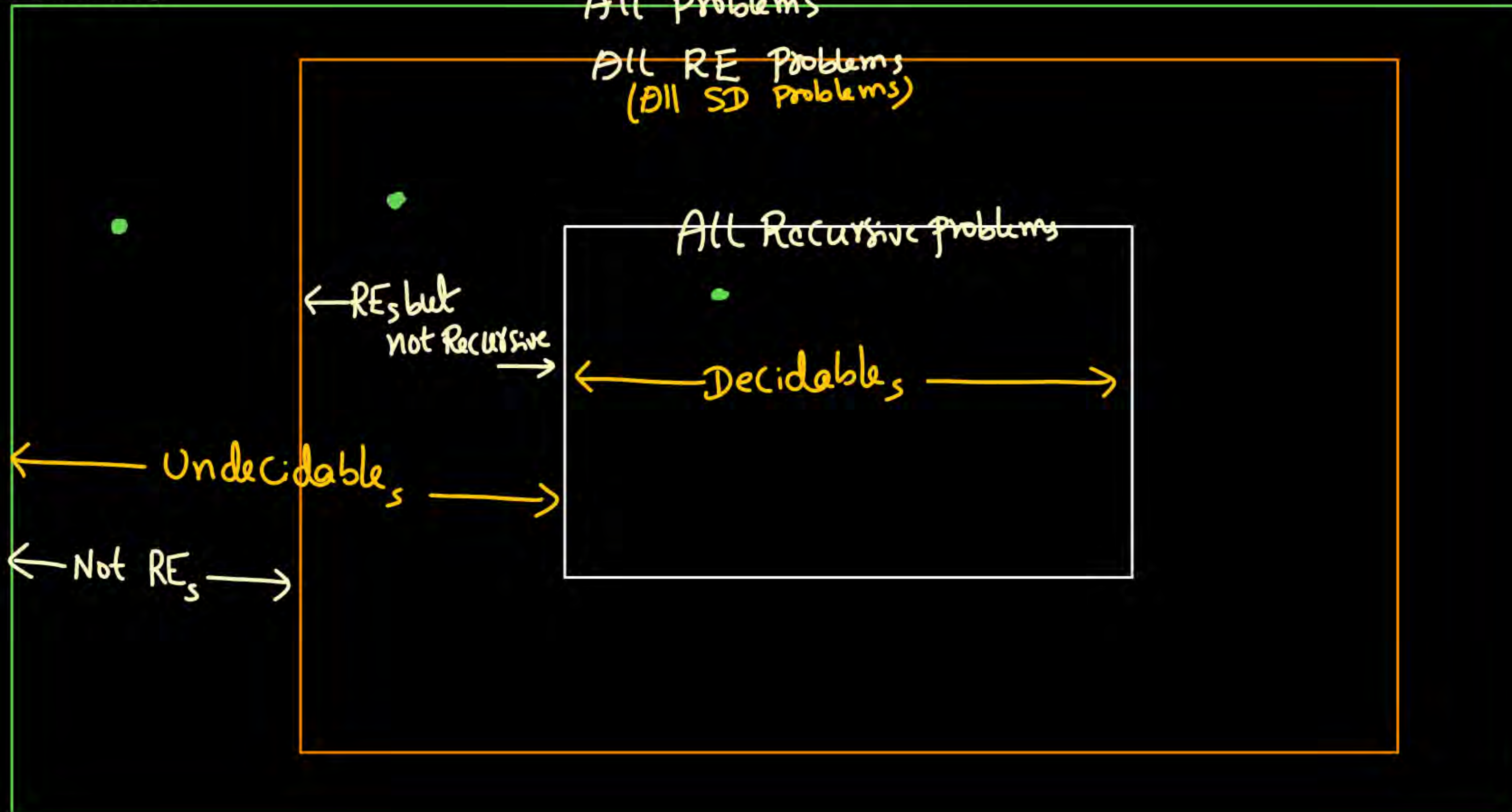
All Recursive problems

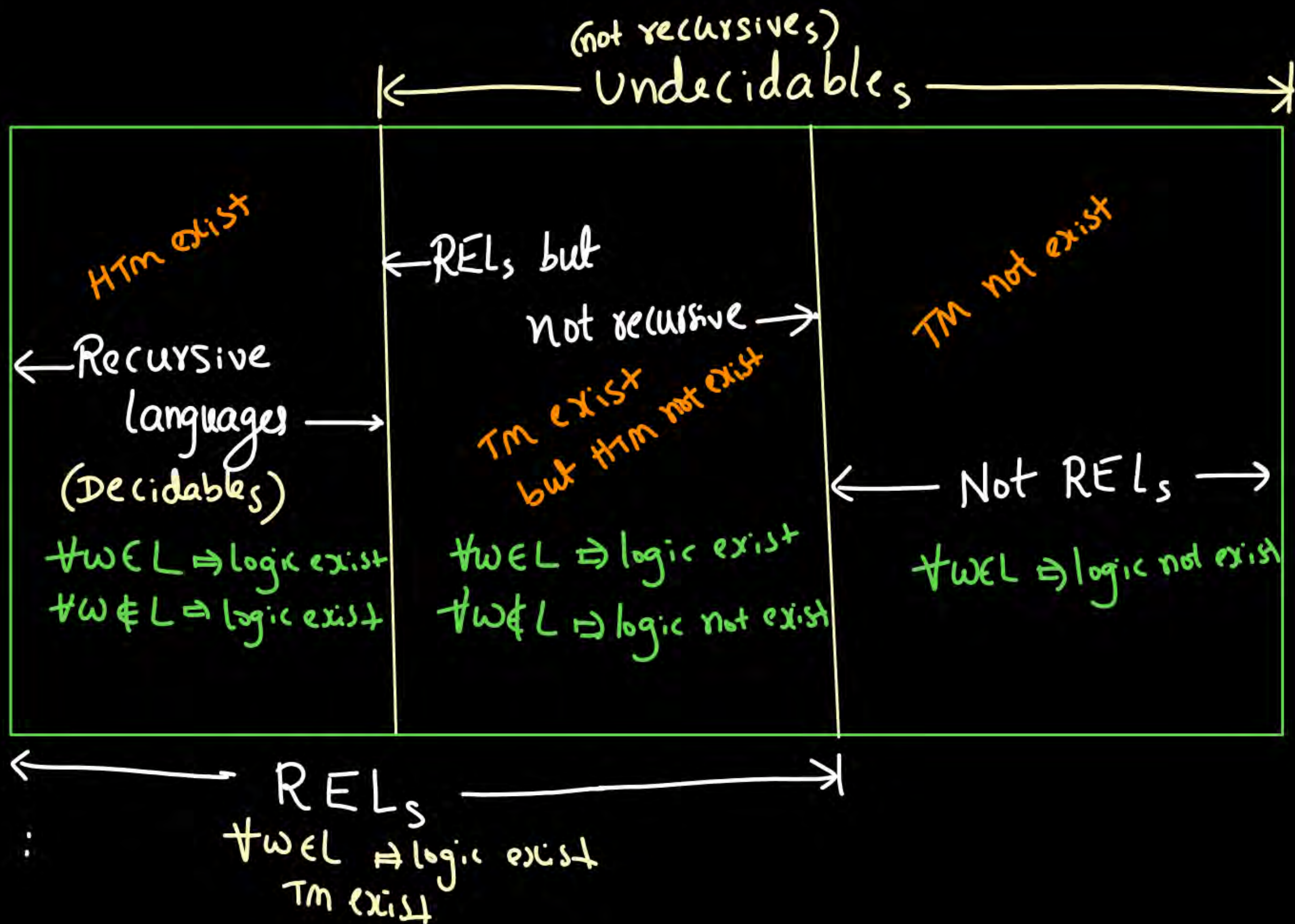
← REs but
not Recursive →

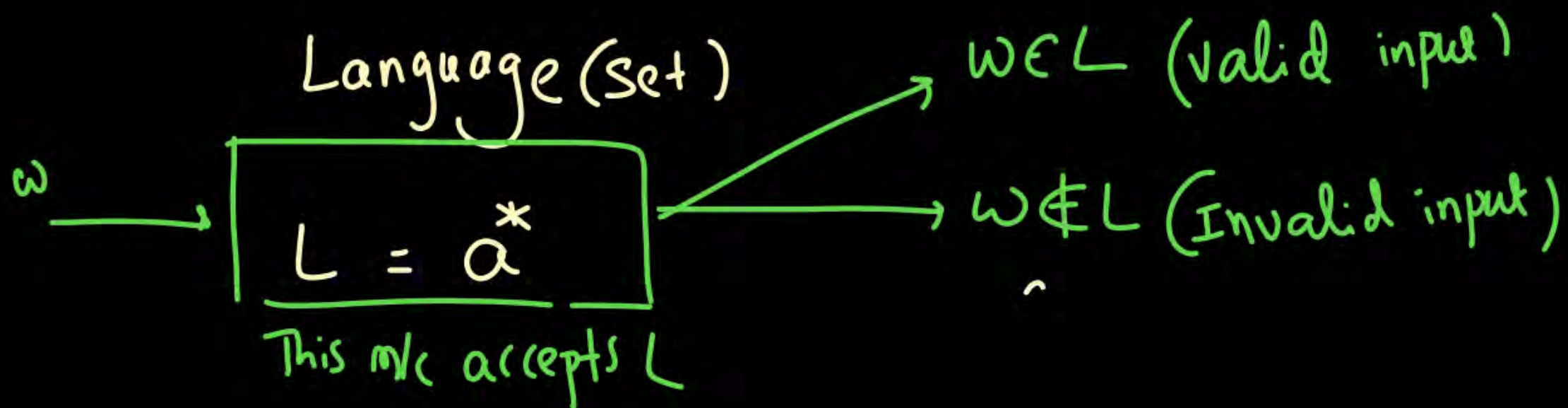
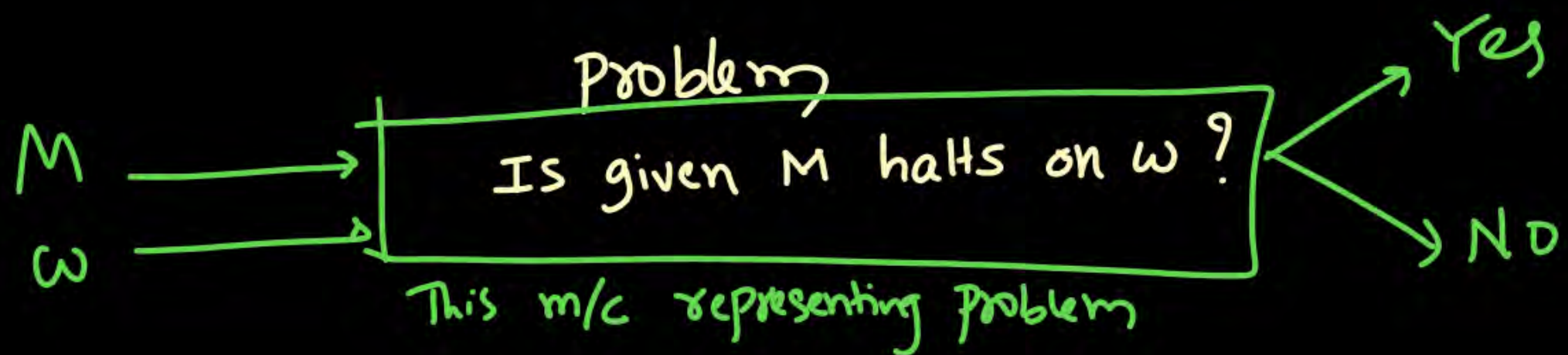
← Decidable →

← Undecidable →

← Not REs →







Problem

Yes \Rightarrow ?No \Rightarrow ?

Language (L)

 $\forall w \in L \Rightarrow$? $\forall w \notin L \Rightarrow$?

Decidable

Yes \Rightarrow logic exist
No \Rightarrow logic exist

HTM exist

RE but not rec

Yes \Rightarrow logic exist
No \Rightarrow logic not exist

TM exist
but HTM not exist

Not RE

Yes \Rightarrow logic not exist
TM not exist

$\forall w \in L \Rightarrow$ logic exist
 $\forall w \notin L \Rightarrow$ logic exist
Decidable language
HTM exist

$\forall w \in L \Rightarrow$ logic exist
 $\forall w \notin L \Rightarrow$ logic not exist
REL but not recursive
TM exist but HTM not exist

$\forall w \in L \Rightarrow$ logic not exist
Not REL
TM not exist

I)

 $\forall w \in L \Rightarrow \text{logic exist}$ $\forall w \in \bar{L} \Rightarrow \text{logic exist}$ L is REL and \bar{L} is REL

iff

 L is Recursive language ^(decidable) $\forall w \in L \Rightarrow \text{logic exist}$ $\forall w \notin L \Rightarrow \text{logic exist}$

II)

 L has TM and \bar{L} has TM

iff

 L is Recursive

$\forall w \in L$	iff	$\forall w \notin \bar{L}$
$\forall w \notin L$	iff	$\forall w \in \bar{L}$

L valid
 Invalid

\bar{L} valid
 Invalid

① Halting problem

IS given M halts on given w ?

D i) IS FA halts on w ? Yes
No

D ii) IS DPDA halts on w ? Yes
No

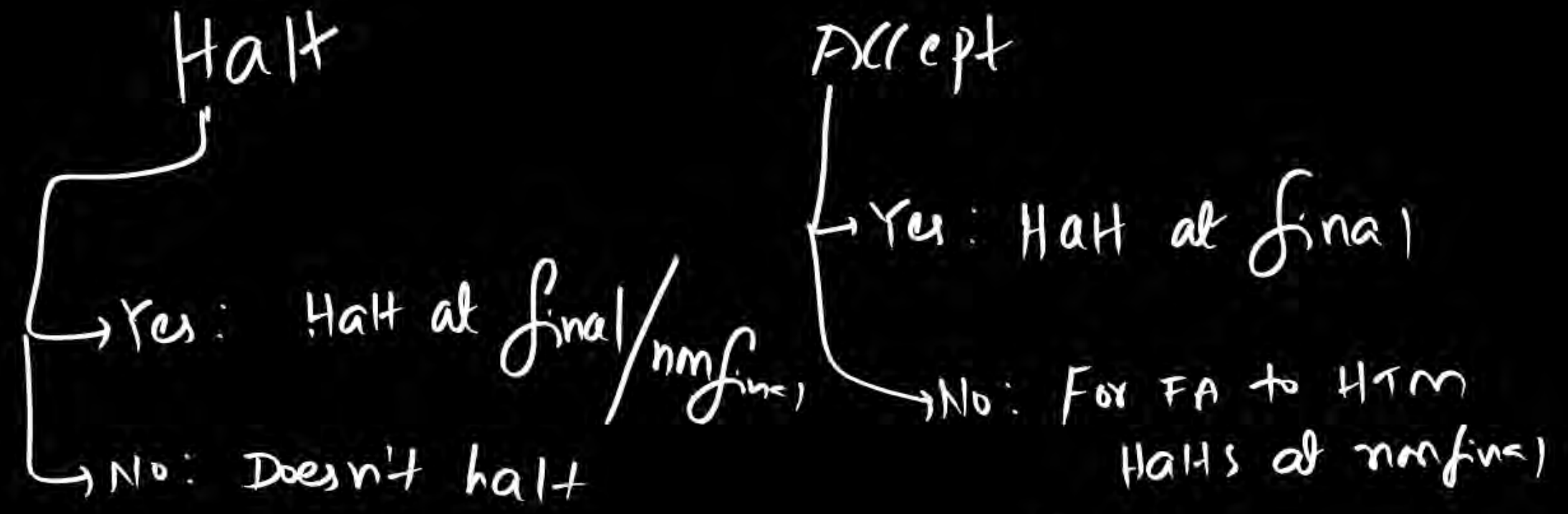
D iii) IS PDA halts on w ? Yes
No

D iv) IS Hrr. halts on w ? Yes
No

UD v) IS Tm halts on w ? Yes \Rightarrow logic exist
No \Rightarrow logic not exist!

RE but not Rec

By definition,
 FA, DPDA, PDA, LBA, & TM
 always halts



For TM:
either Halt at nonfinal
or
never halts

Non-halting problem [Is M does not halt on w ?]

i) Is FA doesn't halt on w ?

ii) " DPDA " " " ?

iii) " PDA " " " ?

iv) " LBA " " " ?

v) " HTM " " " ?

} Decidable

UD vi) Is TM doesn't halt on w ? $\begin{cases} \text{Yes} \Rightarrow \text{logic not exist} \\ \text{No} \end{cases}$
Not RE

② Membership problem [Is M accepts w ?] [Is $w \in L(M)$?]

Decidable {

- i) IS FA accepts w ?
- ii) IS DPDA accepts w ?
- iii) IS PDA accepts w ?
- iv) IS HTM accepts w ?

If they halt at final $\Rightarrow w$ accepted
 If they halt at nonfinal $\Rightarrow w$ not accepted

UD \Leftarrow v) Is TM accepts w ?

RE but not recursive

Yes: halts at final \Rightarrow logic exist

No: Halts
or
never halt \Rightarrow logic not exist

RE but not recursive \implies Not RE

Recursive \implies Recursive

Not REL \implies Either "not REL"
OR "RE but not recursive"

Decidable \implies Decidable

Undecidable \implies Undecidable

Non membership problem [Is M not accepting w ?] ($Is w \notin L(M)?$)

→ For FA, DPDA, PDA, HTM \Rightarrow Decidable

→ For TM \Rightarrow Undecidable
(not REL)

③_a Emptiness $\left[\begin{array}{l} \text{Is } M \text{ accepts } \phi ? \\ \text{Is } M \text{ accepts nothing?} \end{array} \right] \left[\text{Is } L(M) = \{ \} \right]$

③_b Non emptiness $\left[\text{Is } M \text{ accepts something?} \right] \left[\text{Is } L(M) \neq \phi ? \right]$

④_a Finiteness $\left[\text{Is } M \text{ accepts finite language?} \right]$
 $\left[\text{Is } L(M) = \text{finite set?} \right]$

④_b Non finiteness $\left[\text{Is } M \text{ accepts infinite language?} \right]$

⑤_a totality [Is M accepts everything?] [Is $L(M) = \Sigma^*$?]

⑤_b Not totality [Is M doesn't accepts everything?]
[Is $L(M) \neq \Sigma^*$?]

⑥_a Equivalence [Is $M_1 \cong M_2$?] [Is $L(M_1) = L(M_2)$?]

⑥_b Non Equivalence [Is $M_1 \neq M_2$?]
[Is $L(G_1) = L(G_2)$?]

⑦_a Disjointness problem $[Is L_1 \cap L_2 = \emptyset?]$
 $[Is L(M_1) \cap L(M_2) = \emptyset?]$

⑦_b Non-disjointness $[Is L_1 \cap L_2 \neq \emptyset?]$

⑧_a Set containment $[Is L_1 \subseteq L_2?]$
 $[Is L_1 \cap \bar{L}_2 = \emptyset?]$

⑧_b Non Set Containment $[Is L_1 \not\subseteq L_2?]$
 $[Is L_1 \cap \bar{L}_2 \neq \emptyset?]$

Is $\underbrace{Reg_1 \cap Reg_2}_{\text{}} = \emptyset$?

Is $\underbrace{Reg}_{\text{}} = \emptyset$?

(Emptiness)

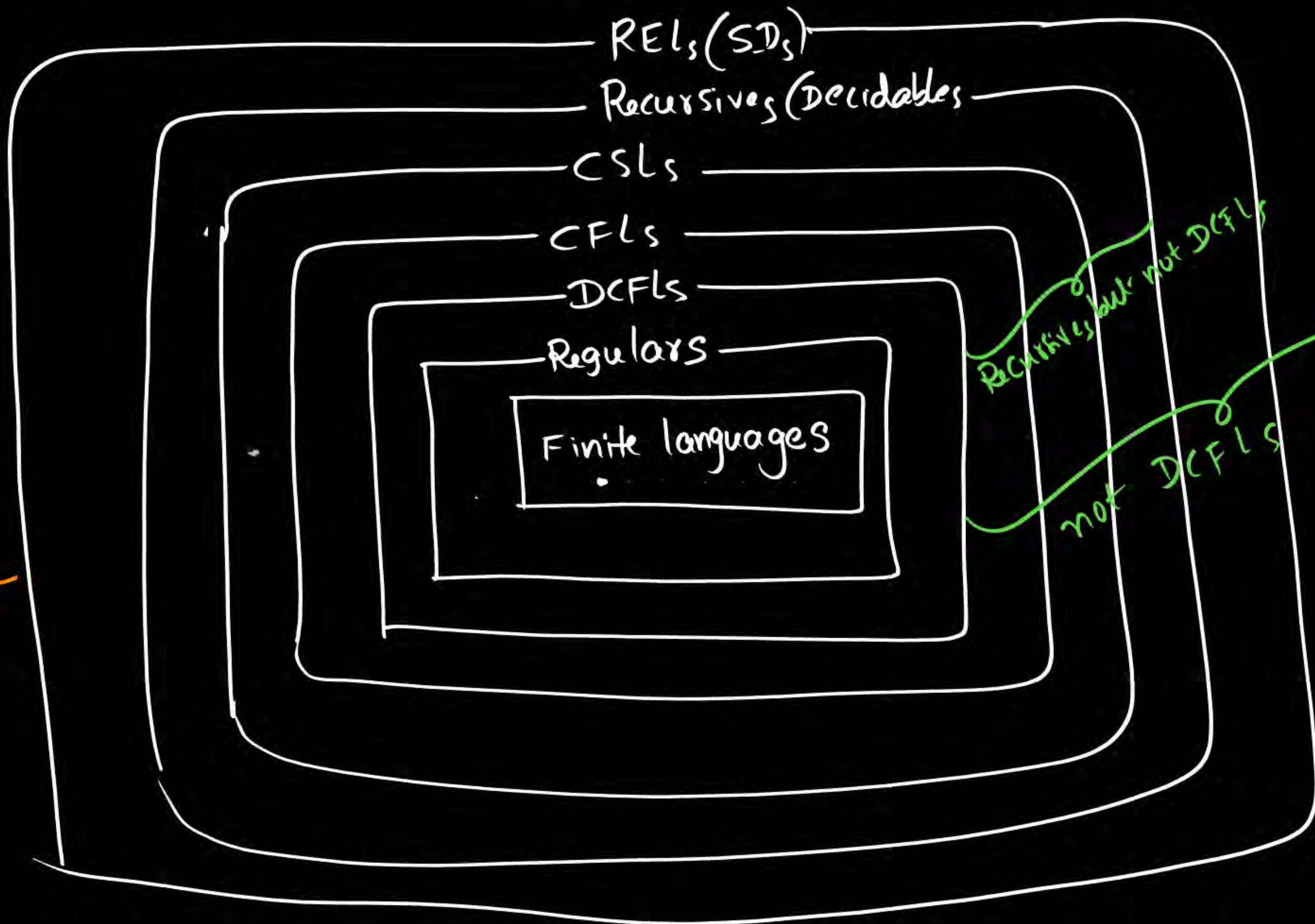
FA



min DFA

If no final $\Rightarrow L = \emptyset$
If final $\Rightarrow L \neq \emptyset$

Not RELs



Undecidability Identify Decidable language, RE but not rec, and Not REL



- ✓ ① Finite language
- ✓ ② $\{a^n b^n \mid n \leq 10\}$
- ✓ ③ $a^* b^*$
- ✓ ④ Regular language
- ✓ ⑤ DCFL
- ✓ ⑥ $a^n b^n$
- ✓ ⑦ CFL
- ✓ ⑧ CFL but not DCFL
- ✓ ⑨ CSL

All are recursive languages

$a^n b^n c^n$
It is decidable

$a^n b^n c^n$
It is decidable

Summary

Decision properties table

Thank you

