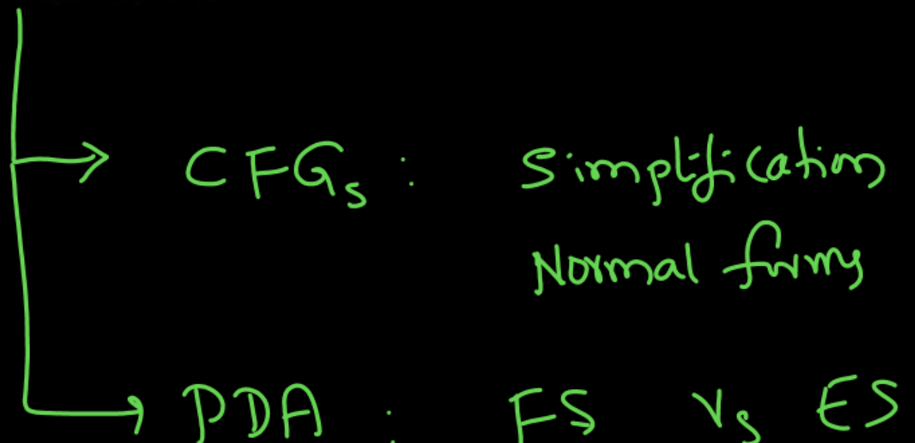
CS & IT Engineering

Context Free Grammar

Deva sir

Topics to be covered:



Topics Covered in Previous Session:

↳ Closure properties :

For DCFLs : $\subset$ PIF closed

For CFLs : ICDS $\underline{\cap}$ F_2 I_6 not closed

not CFL ⟵ $\{(a^n b^n)^* \mid n \geq 0\}$ ⟵ abaabb ✗

ε, $a^n b^n$, $\underbrace{a^n b^n a^n b^n}$, $a^n b^n a^n b^n a^n b^n$

Impossible to understand with 1 stack

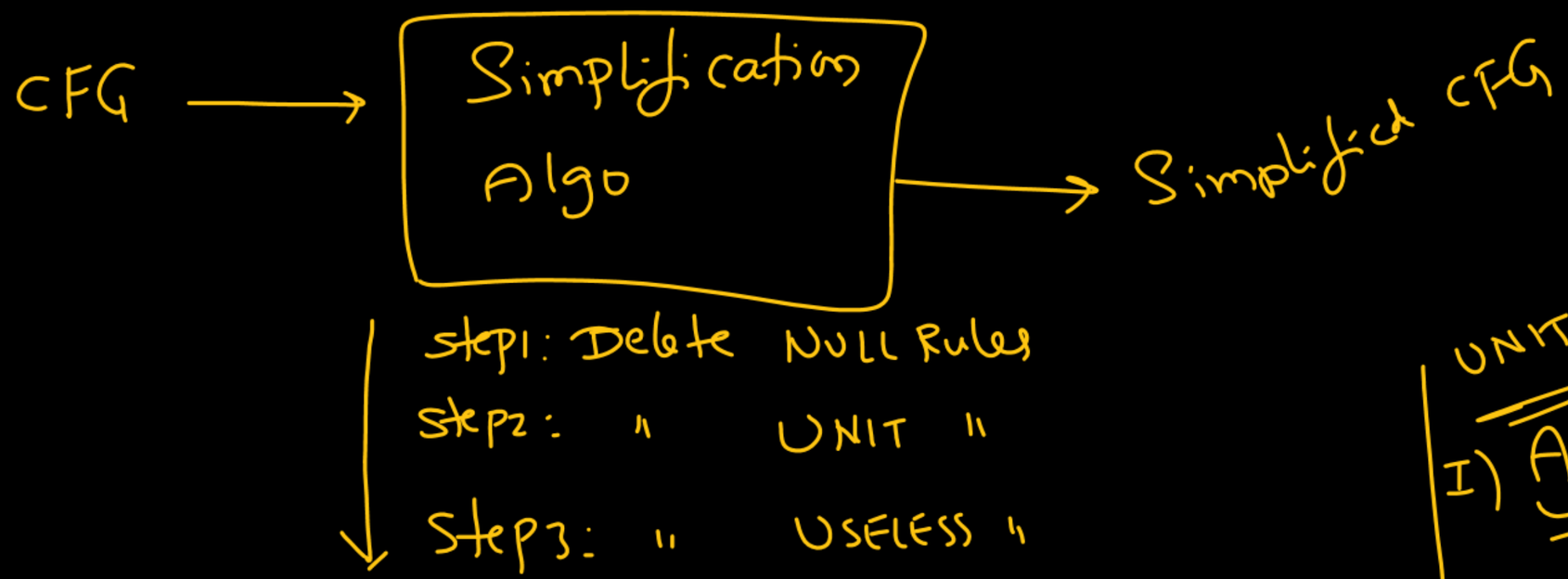
DCFL ⟵ $\{a^n b^n \mid n \geq 0\}^*$ abaabb ✓

ε, $a^n b^n$, $a^{n_1} b^{n_1} a^{n_2} b^{n_2}$, $\{\ \}^2 = \{a^1 b^1\}^1 \{a^2 b^2\}^1$

$a^{n_1} b^{n_1}$, $a^{n_2} b^{n_2} a^{n_3} b^{n_3}$

Simplified CFG : It is a CFG without NULL productions,
without UNIT productions,
and without USELESS productions.

Reduced CFG : It is a CFG without USELESS productions



UNIT Productions

I) $A \rightarrow a$ — This is also unit rule

II) $A \rightarrow B$ — We only delete this UNIT Rule

Elimination of NULL Rules:

$S \to Aa \mid Bb \mid \varepsilon$

$A \to SB \mid a$

$B \to aB \mid \varepsilon$

I) Delete $S \to \varepsilon$

$S \to Aa \mid Bb$

$A \to SB \mid a \mid B$

$B \to aB \mid \varepsilon$

II) Delete $B \to \varepsilon$

$S \to Aa \mid Bb \mid b$

$A \to SB \mid a \mid B \mid S \mid \varepsilon$

$B \to aB \mid a$

III) Delete $A \to \varepsilon$

$S \to Aa \mid Bb \mid b \mid a$

$A \to SB \mid a \mid B \mid S$

$B \to aB \mid a$

$$L(G') = L(G) - \{\varepsilon\}$$

Elimination of UNIT productions:

$(V \to V)$

$S \to Aa \mid Bb \mid B$

$A \to Sab \mid S$

$B \to B \mid Aa \mid A$

no null rules

I) Delete $S \to B$

$S \to Aa \mid Bb \mid Aa \mid A$

$A \to Sab \mid S$

$B \to Aa \mid A$

II) Delete $S \to A$

$S \to Aa \mid Bb \mid Aa \mid Sab \mid S$

$A \to Sab \mid S$

$B \to Aa \mid A$

III) Delete $A \to S$

$S \to Aa \mid Bb \mid Aa \mid Sab$

$A \to Sab \mid Aa \mid Bb$

$B \to Aa \mid A$

IV) Delete $B \to A$

$S \to Aa \mid Bb \mid Sab$

$A \to Sab \mid Aa \mid Bb$

$B \to Aa \mid Sab \mid Bb$

Elimination of USELESS productions :

I) Find non-terminals which derives some string, so that remaining non-terminals will be USELESS

II) Find unreachable non-terminals,

USELESS

$S \to aA \mid bC \mid bG$

$A \to a \mid bS \mid Eb$

$C \to dC$

$D \to eA \mid f$

$E \to g$

$F \to e$

Step 1: Find nonterminals which derives some string

S ✓, A ✓, C ✗, D ✓, E ✓, F ✓, G ✗

Delete C and G productions

$S \to aA$

$A \to a \mid bS \mid Eb$

$D \to eA \mid f$

$E \to g$

$F \to e$

Step 2: Find unreachables

$\{S, A, E\}$ is reachable from S

D and F are unreachable from S

$S \to aA,\ A \to a \mid bS \mid Eb,\ E \to g$

CFG Normal Forms

CNF CFG

No null production

$V \rightarrow VV \mid T$

Chomsky Normal Form

$S \rightarrow AS \mid AB \mid a$

$A \rightarrow b$

$B \rightarrow AB \mid a$

GNF CFG

$V \rightarrow TV^*$

No null production

Greibach normal form

$S \rightarrow aSSA \mid b$

$A \rightarrow a$

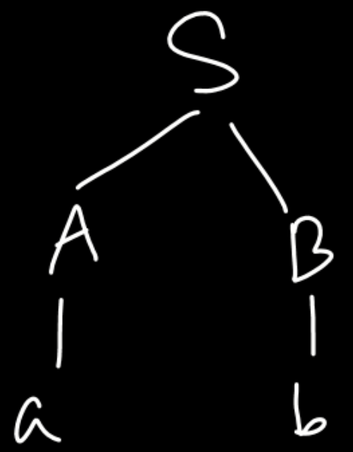
CNF CFG:

$S \rightarrow AB \mid a$

$A \rightarrow a$

$B \rightarrow b$

$w = ab$



Parse tree is Binary tree

How many steps (Length of derivation) required to derive n length string using CNF CFG ?

$\Downarrow$

$2n-1$ steps

$n=1$ (one length string)

$w = a$

No. of steps = 1

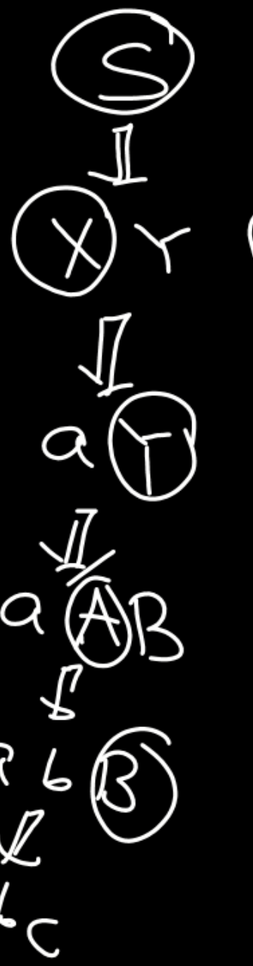
$n=2$ (2 length string)

$w = ab$

3 steps

$n=3$

$w = abc$



5 steps

How many steps required to derive n length string using GNF CFG ?

= n steps

n = 1

$w = a$

S ⇓ $S \to a$ a

1 Step

n = 2

$w = ab$

S ⇓ $S \to aA$

aA ⇓ $A \to b$

ab

2 steps

n = 3

$w = aab$

S ⇓ $S \to aA$

aA ⇓ $A \to aB$

aaB ⇓ $B \to b$

aab

3 Steps

How to Convert CFG to CNF CFG and GNF CFG ?

$S \rightarrow abC$

CFG

CNF CFG :

$S \rightarrow C_a C_b C_c$

$S \rightarrow X C_c$

$X \rightarrow C_a C_b$

$C_a \rightarrow a$

$C_b \rightarrow b$

$C_c \rightarrow c$

GNF CFG

$S \rightarrow a C_b C_c$

$C_b \rightarrow b$

$C_c \rightarrow c$

CFG Vs CFL

① $S \to SA \mid \varepsilon$

$A \to aAb \mid \varepsilon$

$L = \{a^n b^n \mid n \geq 0\}^*$

$= \{a^{n_1} b^{n_1} a^{n_2} b^{n_2} a^{n_3} b^{n_3} \cdots a^{n_k} b^{n_k}\}$

② $S \to SA \mid \varepsilon$

$A \to aAa \mid bAb \mid \varepsilon$

$L = \{ww^R \mid w \in \{a,b\}^*\}^*$

$= \{\varepsilon, aa, bb, aaaa, abba, \ldots\}^*$

*** (3) $\overline{\{a^n b^n\}} = X\,ba\,X \cup \{a^i b^j \mid i \neq j\}$

$i > j$ or $i < j$

$a^i b^j \mid i > j$

$S \rightarrow A \mid B \mid C$

$A \rightarrow X\,ba\,X$

$X \rightarrow aX \mid bX \mid \epsilon$

$B \rightarrow aBb \mid aB \mid a$

$C \rightarrow aCb \mid Cb \mid b$

(4) $L = a^n b^n c^*$

$a^n b^n$ → A, c^* → B

$S \rightarrow AB$

$A \rightarrow aAb \mid \varepsilon$

$B \rightarrow cB \mid \varepsilon$

(5) $L = a^n b^* c^n$

$S \rightarrow aSc \mid A$

$A \rightarrow bA \mid \varepsilon$

(6) $L = a^* b^n c^n$

a^* → A, $b^n c^n$ → B

$S \rightarrow AB$

$A \rightarrow aA \mid \varepsilon$

$B \rightarrow bBc \mid \varepsilon$

CFGs

(7) $L = \{\underbrace{a^n b^n}_{A}\ \underbrace{c^m d^m}_{B}\ \underbrace{e^k g^k}_{C}\}$

$S \rightarrow ABC$

$A \rightarrow aAb \mid \varepsilon$

$B \rightarrow cBd \mid \varepsilon$

$C \rightarrow eCg \mid \varepsilon$

(8) $L : \{\underbrace{c\,a^n b^n}_{X}\} \cup \{\underbrace{a^n b^{2n}}_{Y}\}$

$S \rightarrow X \mid Y$

$X \rightarrow cA$

$A \rightarrow aAb \mid \varepsilon$

$Y \rightarrow aYbb \mid \varepsilon$

(9) $L = \{a^m b^n \mid m = n \text{ or } m = 2n\}$

$S \rightarrow A \mid B$

$A \rightarrow aAb \mid \varepsilon$

$B \rightarrow aaBb \mid \varepsilon$

(10)

$S \to ABC$

$A \to aA \mid \varepsilon$

$B \to bB \mid \varepsilon$

$C \to cC \mid \varepsilon$

$L = a^* b^* c^*$

$= \{a^m b^n c^k\}$

(11)

$S \to AB$

$A \to aA \mid \varepsilon$

$B \to aB \mid bB \mid \varepsilon$

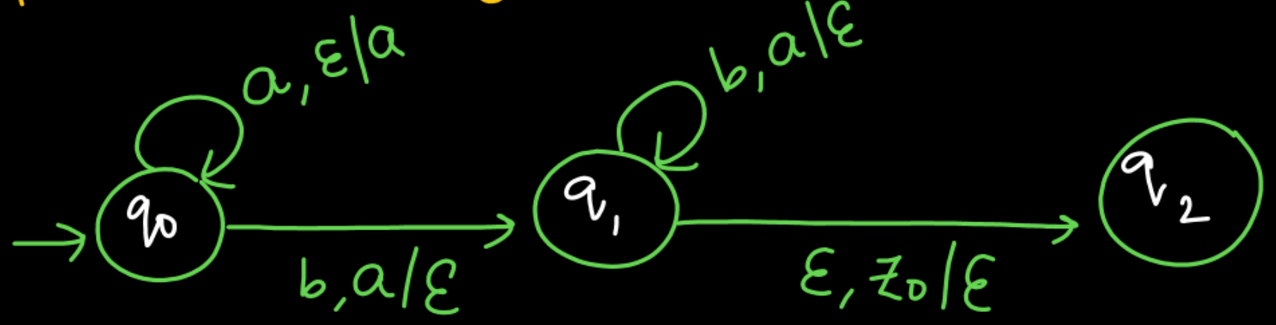
$L = a^* \cdot (a+b)^* = (a+b)^*$

(12)

$S \to SS \mid S \mid a \mid \varepsilon \Rightarrow L = a^*$

ab ✓
aabb ✓
aaabbb —

PDA Acceptance by Empty Stack



$L = \{a^n b^n \mid n \geq 1\}$

If String is accepted then Stack will be empty.

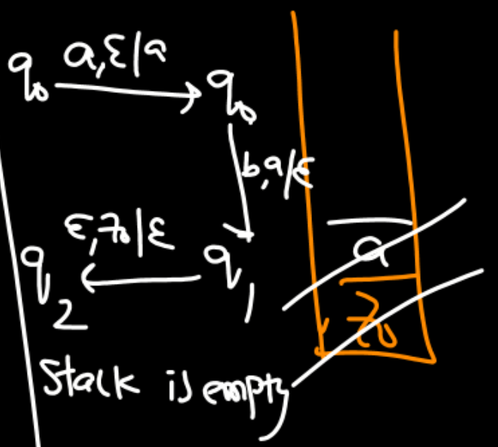
ε (not accepted)

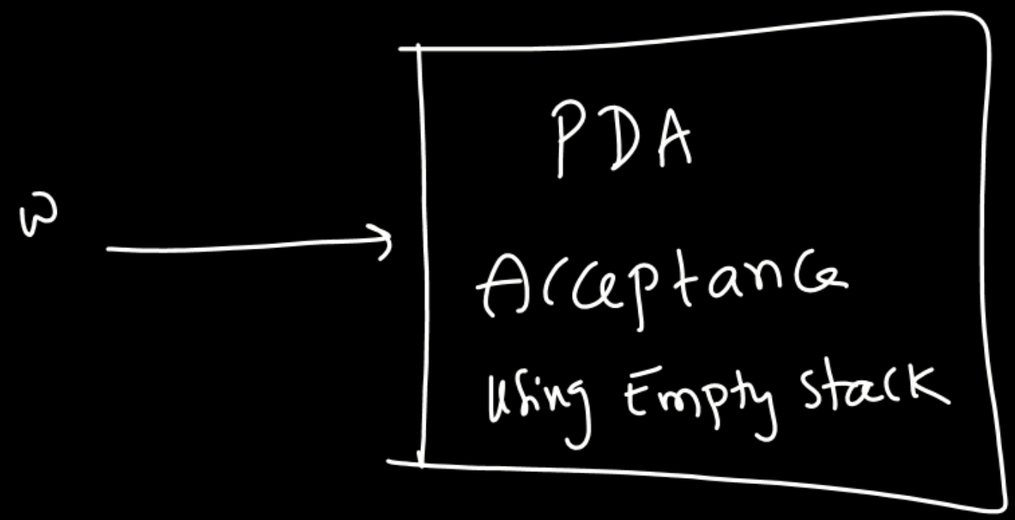
q_0 —ε, z0/ε→ we don't have transition

a (not accepted)

q_0 —a, ε/a→ q_0 —ε, a→ no transition

ab ε ✓





After reading whole string

If $w \in L$, atleast one path exist that makes empty stack

$L = a^*$

ε ✓

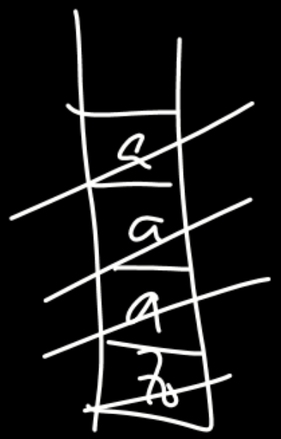
a ✓

aa ✓

aaa ✓

⋮

aaaεεεε



Summary

CNF CFG ✓

GNF CFG ✓

CFGs Vs CFLs ✓

PDA Using E.S. ✓

Thank you