CS & IT Engineering

Deva sir

Topics to be covered:

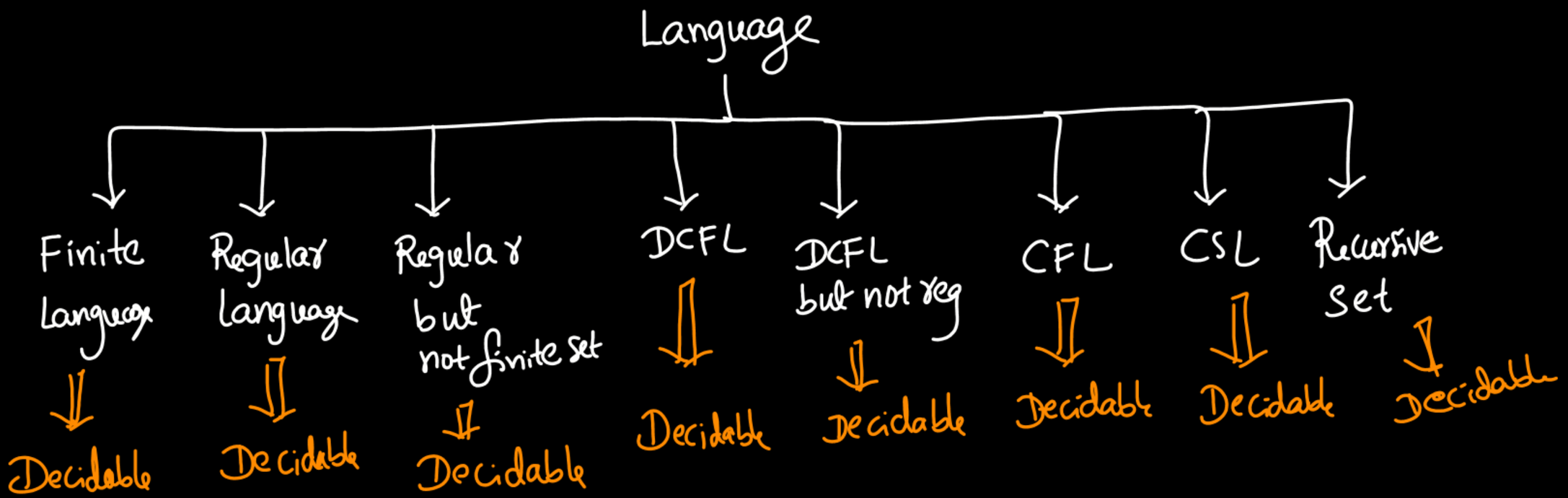
- Languages
- Reducibility

Topics Covered in Previous Session:

- Decision properties Table

Identify Decidable language, REL but not recursive, and Not REL.

(I)



(1) $\{a^n b^n \mid n < 100\}$ ⇒ D

(2) $a^* b^*$ ⇒ D

(3) $a^n b^n$ ⇒ D

(4) $\{ww^R \mid w \in \{a,b\}^*\}$ ⇒ D

(5) $a^n b^n c^n$ ⇒ D

(6) $\{\langle r_1, r_2 \rangle \mid L(r_1) = L(r_2),\ r_1 \text{ and } r_2 \text{ are regular Expressions}\}$ ⇒ D

(II)

1. Complement of finite set over Σ $\Rightarrow$ D
2. Σ = Set of all symbols $\Rightarrow$ Finite $\Rightarrow$ D
3. Σ^* = Set of all strings over Σ $\Rightarrow$ Reg $\Rightarrow$ D
4. 2^{Σ^*} = Set of all languages over Σ $\Rightarrow$ Not REL (UD)
5. $\overline{\text{Reg}}$ $\Rightarrow$ Reg $\Rightarrow$ D
6. $\overline{\text{DCFL}}$ $\Rightarrow$ DCFL $\Rightarrow$ D
7. $\overline{\text{CFL}}$ $\Rightarrow$ CSL $\Rightarrow$ D
8. $\overline{\text{CSL}}$ $\Rightarrow$ CSL $\Rightarrow$ D
9. $\overline{\{a^n b^n c^n\}}$ $\Rightarrow$ D
10. $\overline{\text{Recursive set}}$ $\Rightarrow$?

Undecidability

IV

UD ① Set of all languages $= 2^{\Sigma^*}$ ⇨ Not REL

UD ② Set of all finite sets ⇨ RE but not Rec

UD ③ Set of all regular sets ⇨ RE but not Rec

UD ④ Set of all DCFLs

UD ⑤ Set of all CFLs

UD ⑥ Set of all CSLs

UD ⑦ Set of all recursive sets (Decidable sets)

UD ⑧ Set of all RELs

UD ⑨ Set of all not RELs ⇨ Not REL

I

UD ⑩ Set of all infinite sets ⇨ Not REL

UD ⑪ Set of all non regular sets

UD ⑫ Set of all not DCFLs

UD ⑬ Set of all not CFLs

UD ⑭ Set of all undecidable sets

$$L = \{ w_1, w_2, w_3, w_4, \ldots \}$$

If w_i is string $\Rightarrow$ L is set of strings $\left[L = a^* = \{ \varepsilon, a, a^2, \ldots \} \right]$

If w_i is language $\Rightarrow$ L is set of languages $\left[L = \{ L_1, L_2, L_3, L_4, \ldots \} = \{ a^*, (a+b)^*, a^n b^n \ldots \} \right]$

If w_i is grammars $\Rightarrow$ L is set of grammars $\left[L = \{ G_1, G_2, G_3, G_4, \ldots \} \right]$

If w_i is DFA $\Rightarrow$ L is set of DFAs $\left[L = \{ D \mid D \text{ is DFA} \} \right]$

L = REL but not rec

⇓

$\overline{L}$ is Not REL

$L_1 = \{ F \mid F \text{ is finite set} \}$

$= \text{Set of all finite sets}$

$L_2 = \{ R \mid R \text{ is Regular set} \}$

$= \text{Set of all regular sets}$

$L_3 = \{ D \mid D \text{ is Decidable language} \}$

$= \text{Set of all decidable sets}$

Undecidability

(1) $\{G \mid G \text{ is CFG}\}$ = Set of all CFGs ⇒ Decidable

(2) $\{G \mid G \text{ is CSG}\}$ = Set of all CSGs ⇒

(3) $\{G \mid G \text{ is UG}\}$ = Set of all UGs ⇒

(4) $\{\text{CFG} \mid \text{CFG is ambiguous}\}$ = Set of all Ambiguous CFGs ⇒ RE but not rec

(5) $\{\text{CFG} \mid \text{CFG is unambiguous}\}$ = Set of all Unambiguous CFGs ⇒ Not REL

} UDs

(6) $\{G \mid G \text{ is Regular Grammar}\}$ = Set of all regular grammars ⇒ Decidable

(VII)

(1) Whether given G is CFG ⇒ Decidable

(2) " " " is CSG ⇒ Decidable

(3) " " " is UG ⇒ Decidable

(4) " " " is Ambiguous ⇒ Undecidable (RE but not rec)

(5) " " " is unambiguous ⇒ (Not RE) Undecidable

(6) " " " is Regular ⇒ Decidable

VIII

① $\{M \mid M \text{ is DFA, } \underbrace{M \text{ accepts } \varepsilon}_{\text{membership}}\}$ = Set of DFAs accepting ε ⇒ Decidable language

② $\{\langle M, w\rangle \mid M \text{ is DFA, } \underbrace{M \text{ accepts } w}_{\text{membership}}\}$ = Set of pairs where DFA accepts w ⇒ Decidable language

③ $\{TM \mid TM \text{ accepts } \varepsilon\}$ = Set of all TMs accepting ε ⇒ RE but not rec (Undecidable)

④ $\{Tm \mid Tm \text{ doesn't accept } \varepsilon\}$ ⇒ Not REL

⑤ $\{DFA \mid L(DFA) = \text{finite}\}$ ⇒ Decidable language

⑥ $\{DFA \mid L(DFA) = \text{Infinite}\}$

(7) $\{\langle DFA_1, DFA_2\rangle \mid L(DFA_1) = L(DFA_2)\}$ ⇨ Decidable language

Equivalence

(8) $\{\langle LLG, RLG\rangle \mid L(LLG) = L(RLG)\}$

(9) $\{\langle DFA, NFA\rangle \mid L(DFA) = L(NFA)\}$

(10) $\{\langle DPDA_1, DPDA_2\rangle \mid L(DPDA_1) = L(DPDA_2)\}$

(11) $\{\langle PDA_1, PDA_2\rangle \mid L(PDA_1) = L(PDA_2)\}$ ⇒ Undecidable

Not REL

Equivalence

(12) $\{\langle PDA_1, PDA_2\rangle \mid L(PDA_1) \neq L(PDA_2)\}$ ⇒ Undecidable

(RE but not REL)

(13) $\{\langle TM_1, TM_2\rangle \mid L(TM_1) = L(TM_2)\}$ ⇒ Not REL

(14) $\{\langle TM_1, TM_2\rangle \mid L(TM_1) \neq L(TM_2)\}$ ⇒ Not REL

} Undecidable languages

UD

(15) $\{TM \mid$ TM accepts particular string $\}$ ⇒ RE but not rec

Membership

(16) $\{TM \mid$ TM accepts Some string $\}$ ⇒ RE but not rec

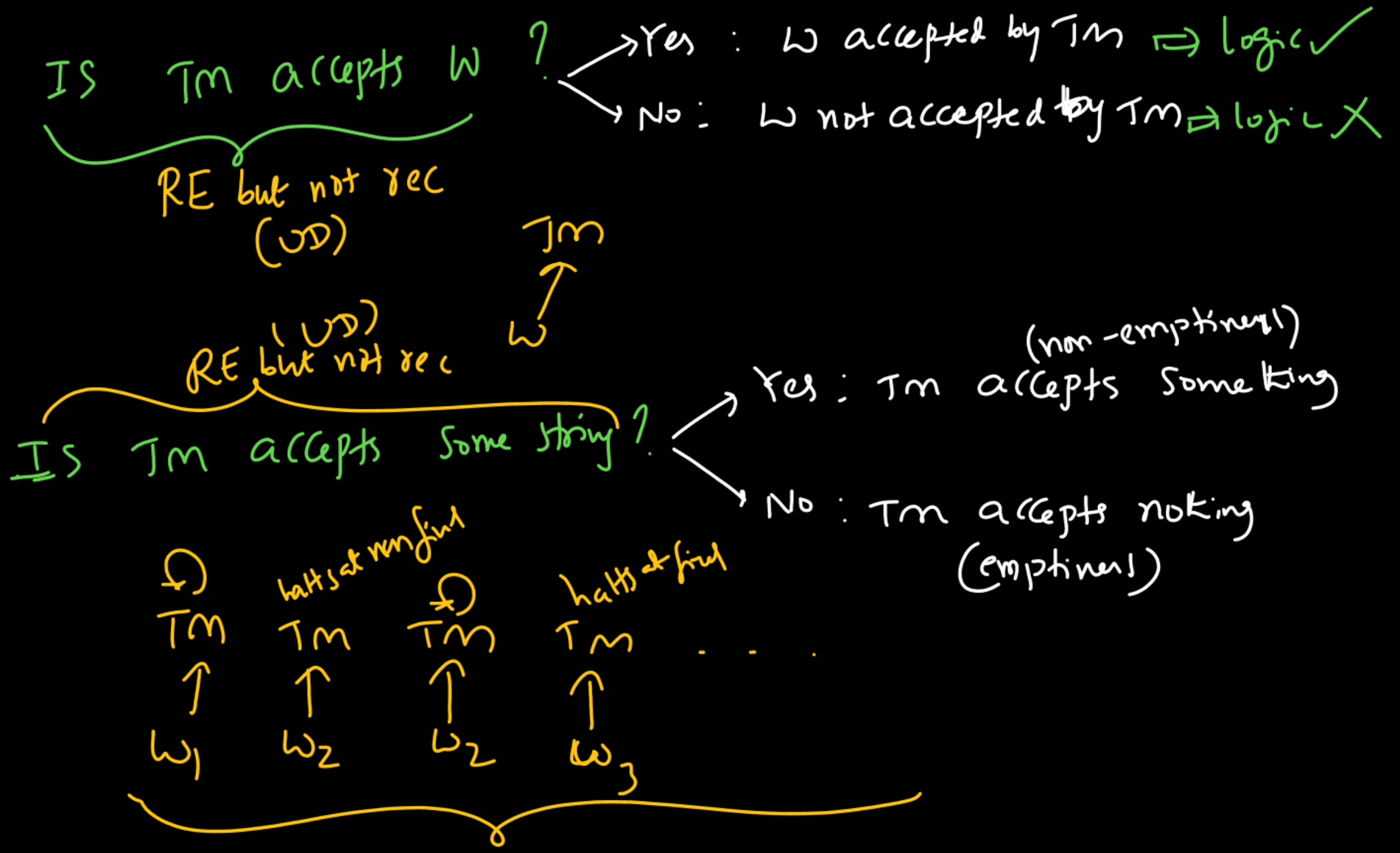
Non emptiness

(17) $\{TM \mid$ TM accepts no string $\}$ ⇒ Not REL

Emptiness

(18) $\{TM \mid$ TM accepts every string $\}$ ⇒ Not REL

Totality


IS TM accepts W ?
Yes : W accepted by TM ⇒ logic ✓
No : W not accepted by TM ⇒ logic ✗
RE but not rec
(UD)
TM
W
(UD)
RE but not rec
IS TM accepts Some String ?
(non-emptiness)
Yes : TM accepts Something
No : TM accepts nothing
(emptiness)
halts at some final
halts at final
TM
TM
TM
TM
W1
W2
W2
W3

(19) { TM | TM accepts REL } ⇒ Decidable

Trivial

(20) { TM | TM doesn't accepts REL } = { } ⇒ Decidable

(21) { TM | TM has 10 states } ⇒ Decidable

(22) { TM | No. of states(TM) > 10 }

(23) { TM | TM reaches state q within K (100) steps } ⇒ Decidable

(24) { TM | TM reaches state q } ⇒ RE but not rec (UD)

(25) { TM | TM halts on w } ⇒ RE but not rec (UD)

Reducibility :

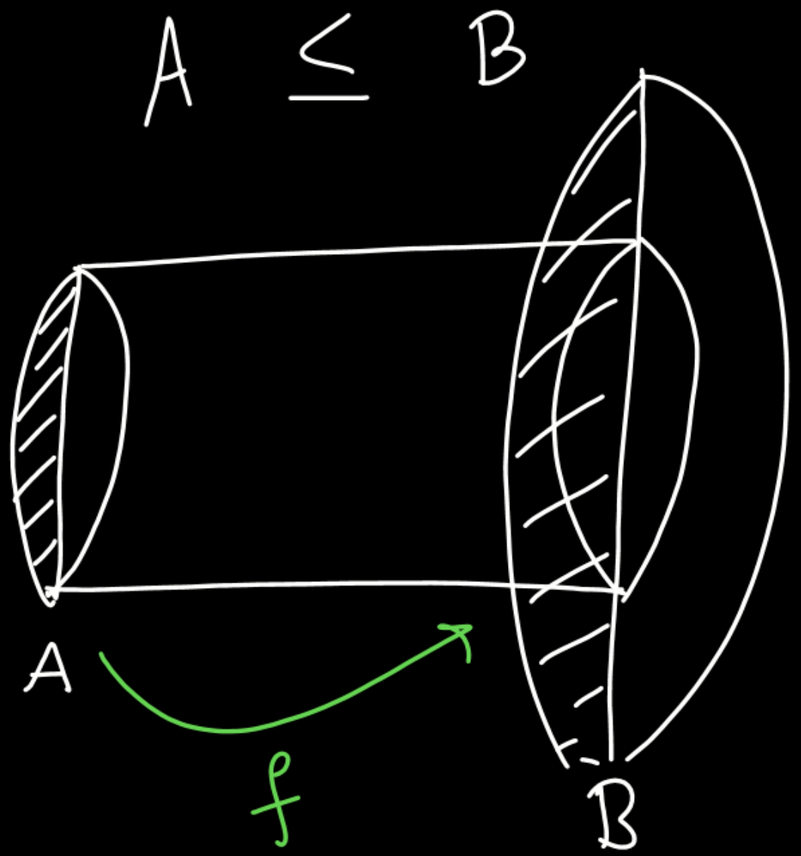
A is reducible to B

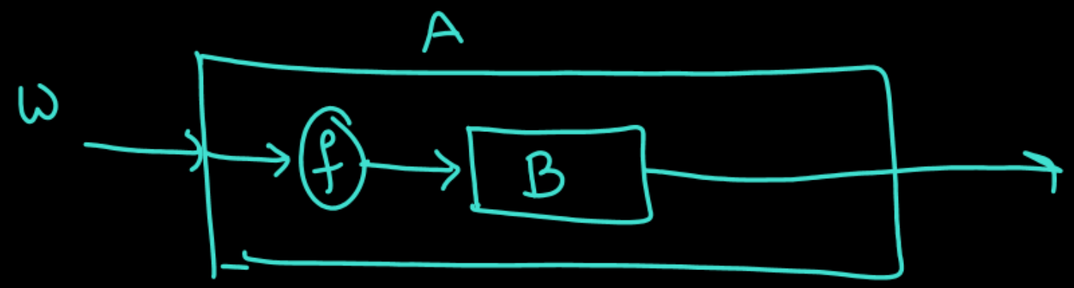
$$A \leq B$$

B is atleast as hard as A

$$A \propto_p B$$

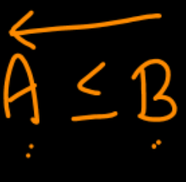
A is polynomially reducible to B


A ≤ B
A
f
B


A
w
f
B

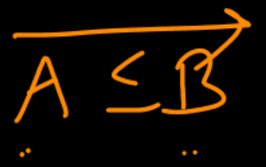
$A \leq B$: Every instance of A is mapped to Some instance of B

Let $A \leq B$.



① If B is decidable then A is Decidable

② If A is undecidable then B is undecidable



③ If B is REL then A is REL

④ If B is Recursive then A is Recursive

⑤ If A is Recursive then $\bar{B}$ is anything is possible (Decidable or undecidable)

⑥ If $\bar{A}$ is not REL then $\bar{B}$ is not REL

Let $A \leq B$.

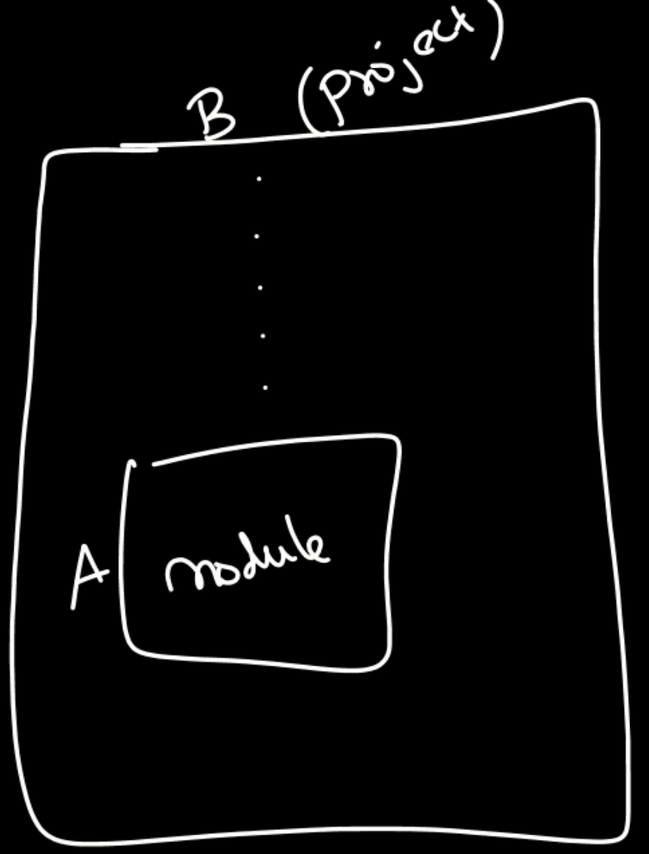
(7) If $B = \Theta(n^2)$ then $A = O(n^2)$ (A takes max n^2)

(8) If $A = \Theta(n^2)$ then $B = \Omega(n^2)$ (B takes min n^2)

(9) If A is infinite then B is Infinite

(10) If B is finite then A is finite

$A \leq B$



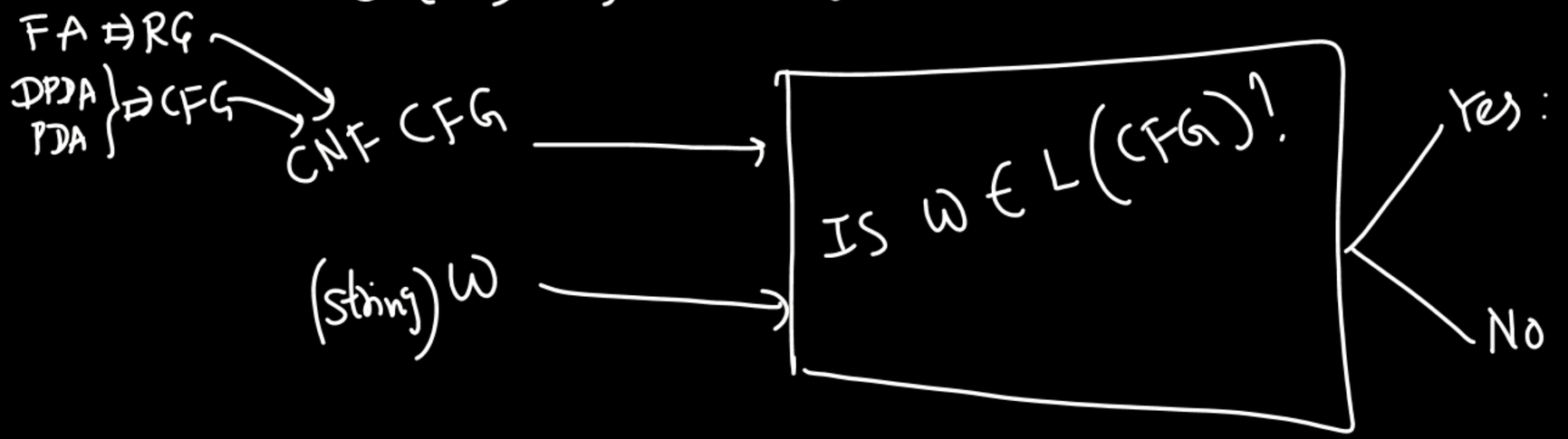
CYK Algorithm ⇨ membership for FA, DPDA, PDA

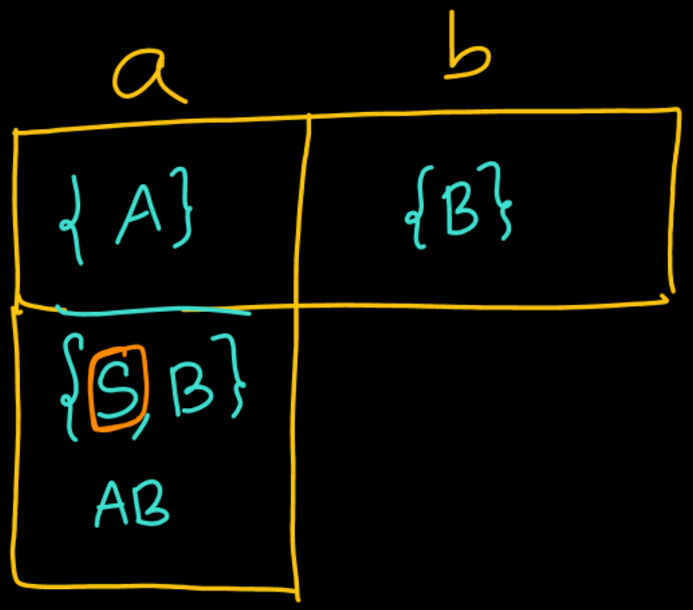
Simplification Algorithm ⇨ Emptiness for FA, DPDA, PDA

Dependency graph ⇨ Finiteness for FA, DPDA, PDA

CYK Algorithm:

- Membership Algorithm
- Bottom up parsing Algorithm
- Dynamic programming
- $O(n^3)$ for n-length input string





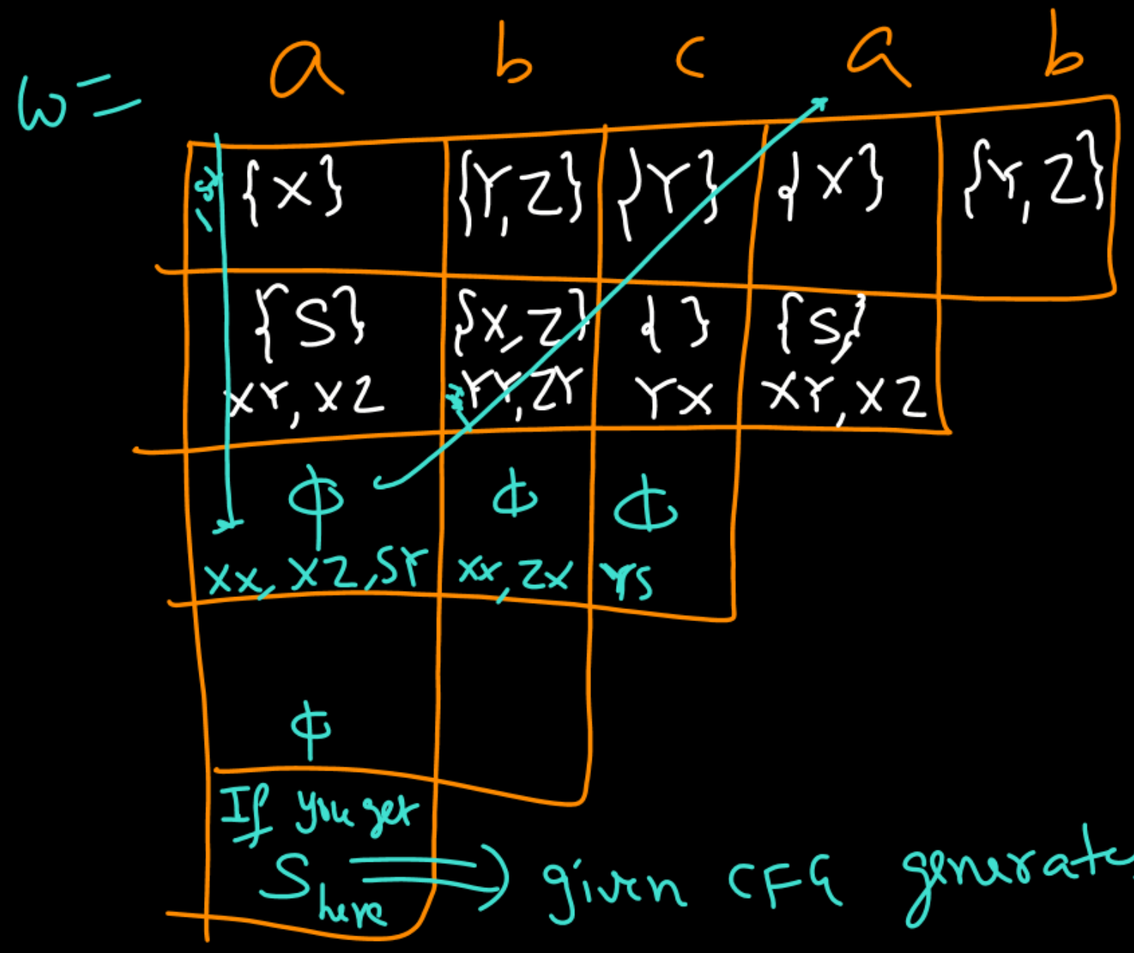
CNF CFG

$S \rightarrow AB \mid c$

$A \rightarrow a \mid SA$

$B \rightarrow b \mid SB \mid AB$

$w = ab$

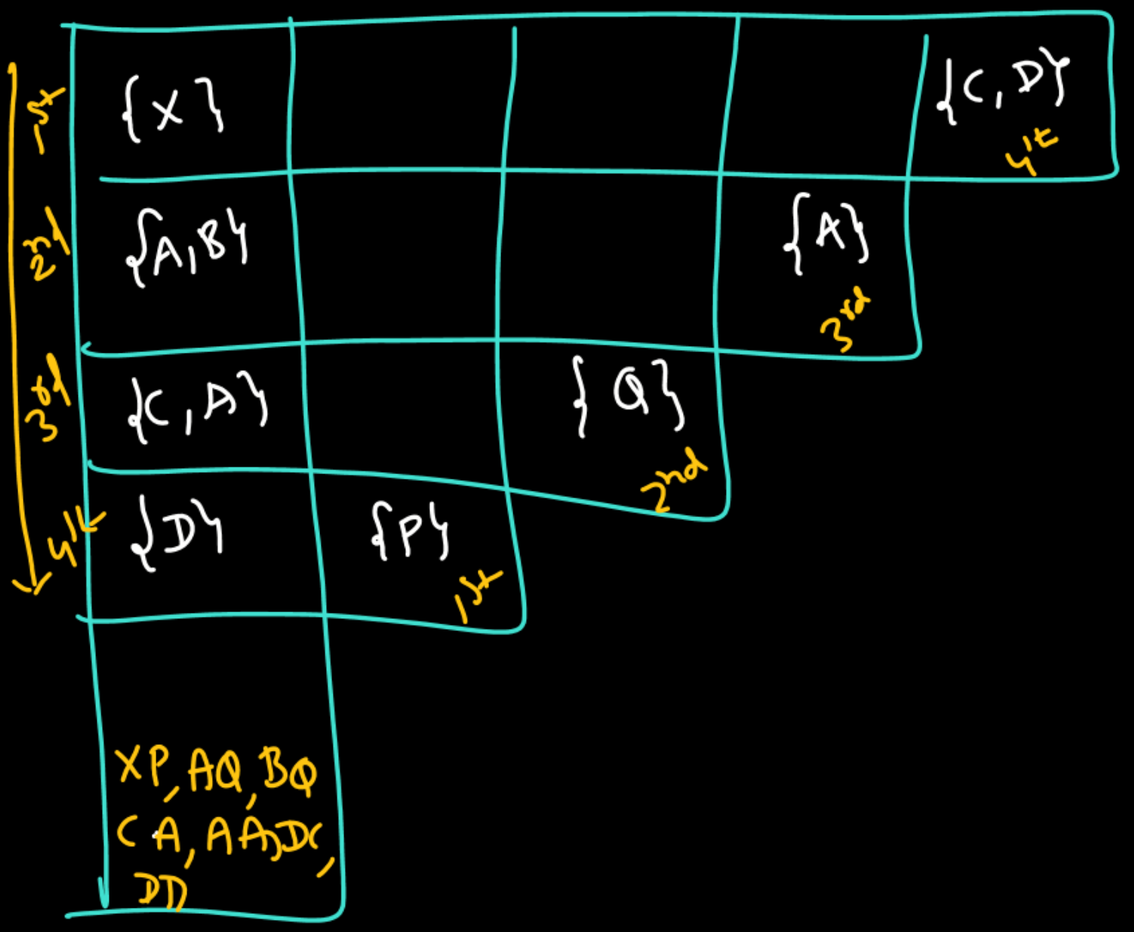


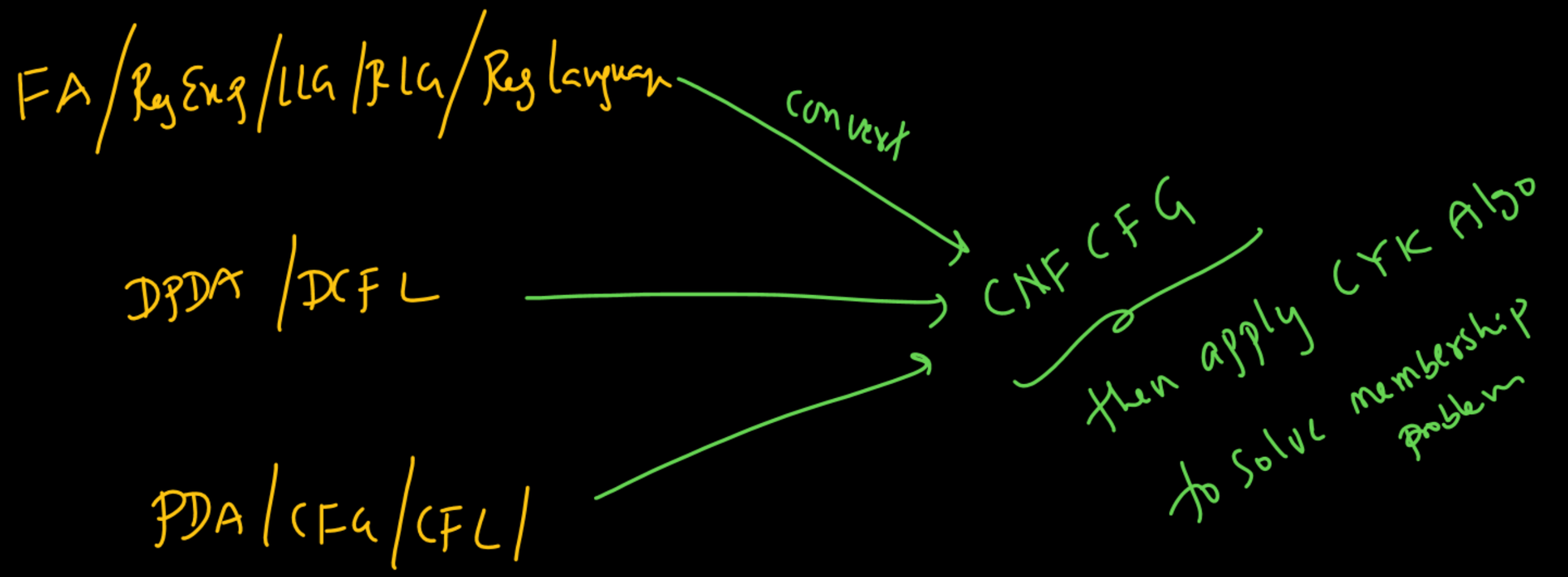
$S \rightarrow XY \mid YZ$

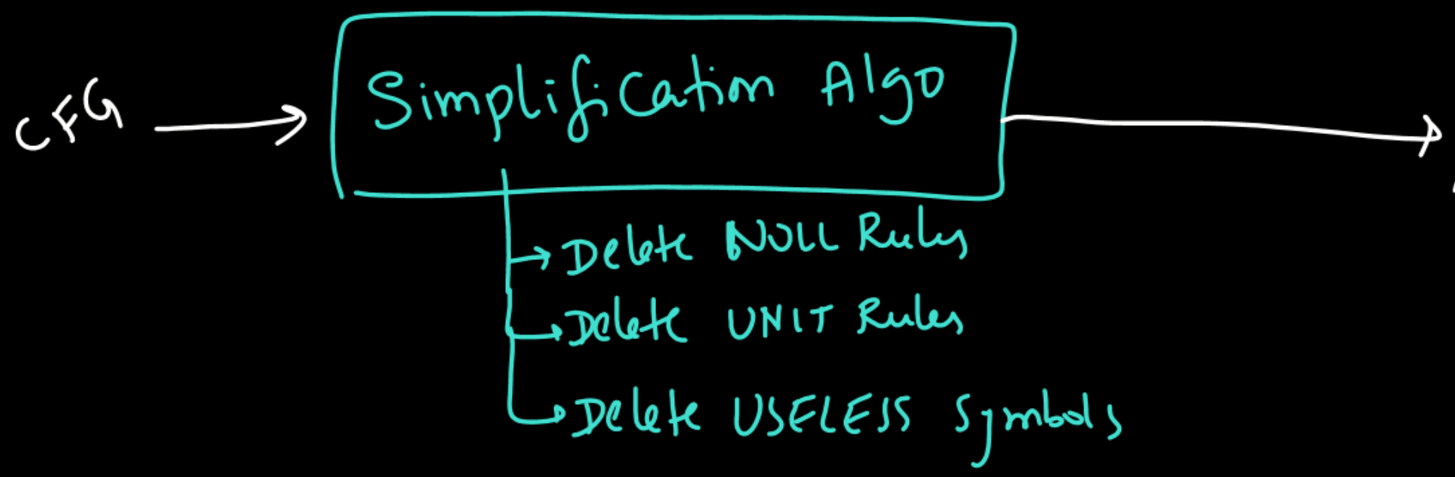
$X \rightarrow a \mid YY$

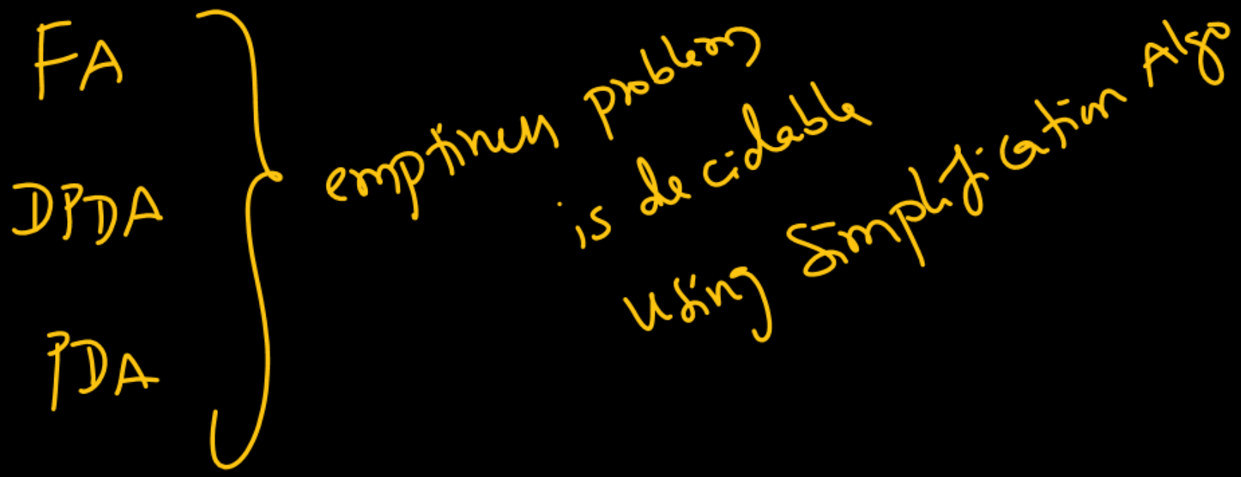
$Y \rightarrow b \mid c$

$Z \rightarrow b \mid ZY$


{X}
{C, D}
4th
{A, B}
{A}
2nd
3rd
{C, A}
{Q}
3rd
2nd
{D}
{P}
4th
1st
XP, AQ, BQ
CA, AA, DC,
DD


FA / RegExg / LLG / RLG / Reg Languages
convert
DPDA / DCFL
PDA / CFG / CFL
CNF CFG
then apply CYK Algo
to solve membership
problem


CFG
Simplification Algo
Delete NULL Rules
Delete UNIT Rules
Delete USELESS Symbols
If no production in result then L(CFG) = ϕ
otherwise L(CFG) ≠ ϕ


FA
DPDA
PDA
emptiness problem is decidable using Simplification Algo

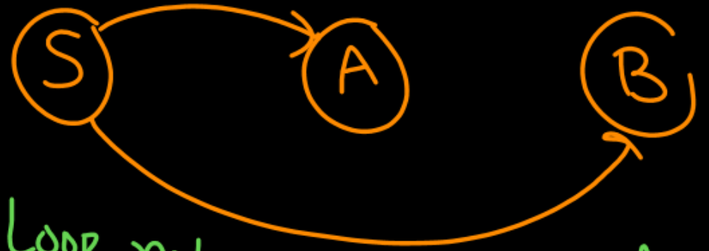
Dependency graph:

① $S \rightarrow Aa \mid Bba$

$A \rightarrow ab \mid \varepsilon$

$B \rightarrow a$

L = finite



Loop not exist in Dependency graph

② $S \rightarrow Sa \mid b$

Loop exist

Language is infinit

③ $S \rightarrow Aa \mid b$

$A \rightarrow bS \mid a$

Inf lang

Rice Theorem: Every non trivial property is Undecidable

$L = \{TM \mid TM \text{ accepts } a^*\} \Rightarrow$ Set of TM's accepts a^*

Not REL

UD

$\bar{L} = \{TM \mid TM \text{ not accepts } a^*\} \Rightarrow$ Set of TM's not accepts a^*

Not REL

$L \neq \phi$

$\bar{L} \neq \phi$

$\Rightarrow L$ is undecidable

$L = \{TM \mid TM \text{ accepts REL}\}$

$\bar{L} = \phi$

Trivial Case

Decidable

TM $\cong$ Program

HTM $\cong$ Algo $\cong$ Halting Program

Undecidability

Enumerable Set (REL)

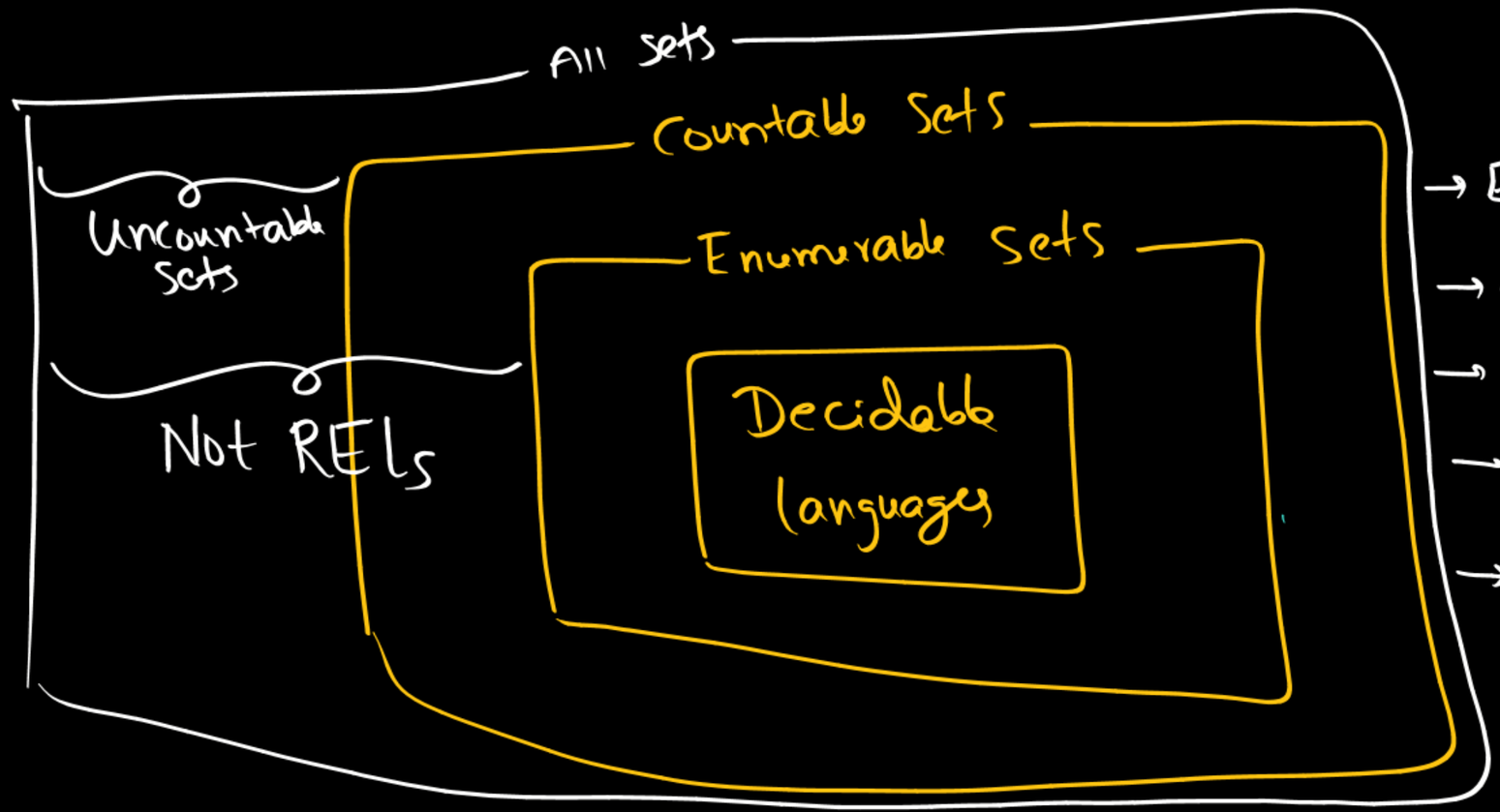
TM exist

Countable Set

X is countable set
iff
$f: X \to Y$ is bijective
and Y is known countable set

Decidable Set

HTM exist



→ Every Decidable is Countable
→ Countable need not be Decidable
→ Every REL is Countable
→ Countable need not be REL
→ Some not RELs are Countable

Summary

↳ Undecidability

Languages

Sat: 3pm-6pm
GATE PYQs

Friday: 4pm-6pm
GATE PYQs

Thank you