

CS & IT Engineering



Deva sir

Topics to be covered:

↳ practice on TOC

Topics Covered in Previous Session:

↳ Undecidability ✓

Which Two of the following four regular expressions are equivalent?

~~(i)~~ $(00)^*(\epsilon + 0) = 0^*$

(ii) $(00)^*$
even no. of 0's

~~(iii)~~ 0^*

(iv) $0(00)^*$
odd no. of 0's

(a) (i) and (ii)

(b) (ii) and (iii)

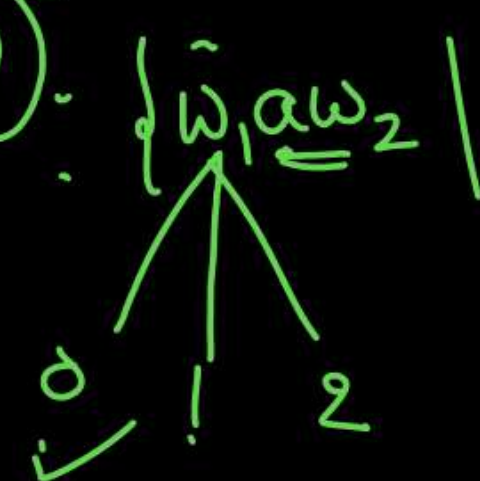
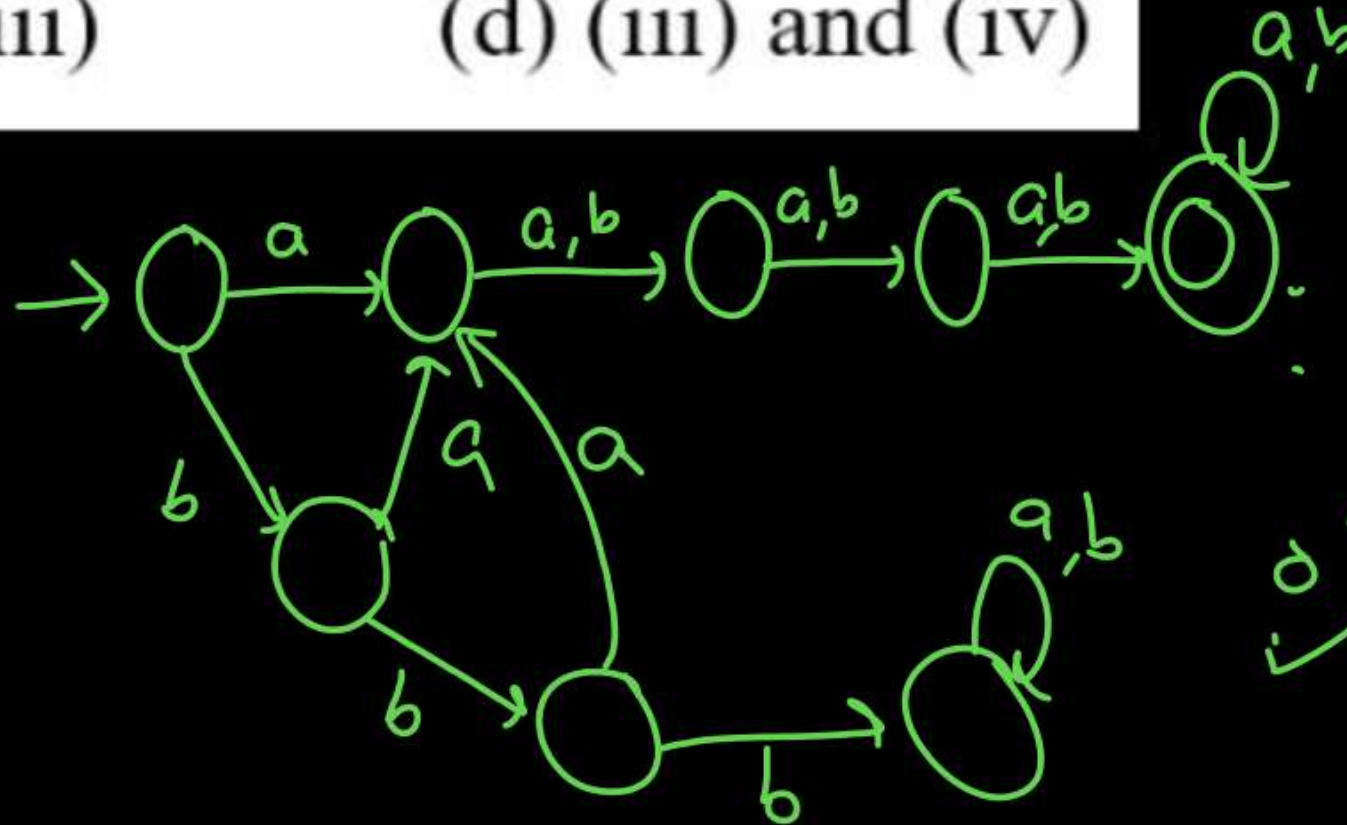
~~(c)~~ (i) and (iii)

(d) (iii) and (iv)

(GATE - 96)

$|w_1| = 0$
 $a w_2$
 ≥ 3

$|w_1| = 1$
 $|w_1| = 2$



$\{w_1 a w_2 \mid w_1, w_2 \in \{a, b\}^*, |w_1| \leq 2, |w_2| \geq 3\}$

Let $L \subseteq \Sigma^*$ where $\Sigma = \{a, b\}$ which of the following is true? (GATE - 96)

DCFL but not regular

- (a) $L = \{x \mid x \text{ has an equal number of } a\text{'s and } b\text{'s}\}$ is regular ✗
- (b) $L = \{a^n b^n \mid n \geq 1\}$ is regular ✗
- (c) $L = \{x \mid x \text{ has more } a\text{'s than } b\text{'s}\}$ is regular ✗
- ✓ (d) $L = \{a^m b^n \mid m \geq 1, n \geq 1\}$ is regular

$\overbrace{a}^+ \overbrace{b}^+$

If the regular set A is represented by $A = (01+1)^*$ and the regular set 'B' is represented by $B = ((01)^*1^*)^*$, which of the following is true? **(GATE - 98)**

(a) $A \subset B$

(b) $B \subset A$

(c) A and B are incomparable

☒ (d) $A = B$

$$A = (01+1)^*$$

$$B = ((01)^*1^*)^*$$

$$\begin{array}{l} x = 01 \\ y = 1 \end{array}$$

$$A = (x+y)^*$$

$$B = (x^*y^*)^*$$

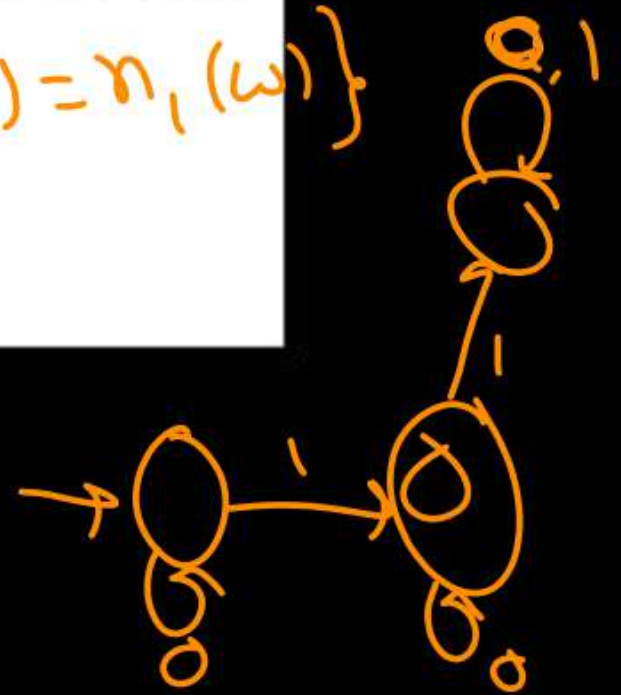
$$A = B$$

Which of the following sets can be recognized by a Deterministic Finite-state Automaton? (GATE - 98)

DFA ? $\xrightarrow{0} 2 \xrightarrow{1} 2 \xrightarrow{2} 2 \xrightarrow{3} 2$

- (a) The numbers 1, 2, 4, 8, 2^n , written in binary. = $0^* 1 0^*$
- (b) The numbers 1, 2, 4, ..., 2^n , written in unary. = a^{2^n} not regular
- (c) The set of binary strings in which the number of zeros is the same as the number of ones. $\{w \mid w \in \{0,1\}^*, n_0(w) = n_1(w)\}$
- (d) The set $\{1, 101, 11011, 1110111, \dots\}$

$\{1\} \cup \{1^n 0 1^n \mid n \geq 1\}$ $b_4 b_3 b_2 b_1 b_0$ \rightarrow not regular



The string 1101 does not belong to the set represented by (GATE - 98)

1101 \notin (a) 110^{*} (0+1)

1101 \notin (b) 1(0+1)^{*} 101

1101 \notin (c) (10)^{*} (01)^{*} (00+11)^{*}

1101 \notin (d) (00+(11)^{*} 0)^{*}

Whether TM accepts Some string

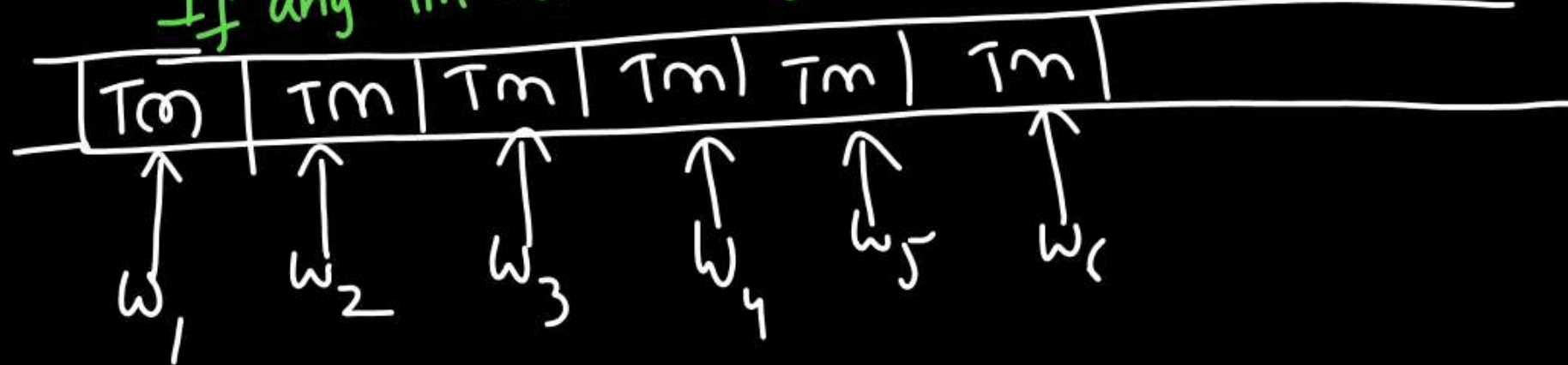
Yes \Rightarrow logic exist

No \Rightarrow logic not exist

\hookrightarrow UD

REL but not rec

If any TM halts at final



Whether T_m accepts finite language

Yes \Rightarrow no log.

No

Not RE

Is $L(T_m) = \text{Finite}$?

Is $L(T_m) = \{ab\}$? \Rightarrow Not RE

Is $L(T_m) = \emptyset$? \Rightarrow

How many maximum substrings of different lengths (non-zero) can be formed from a character string of length n ? (GATE - 98)

~~(a) n~~

(b) n^2

(c) 2^n

(d) $n(n+1)/2$

$w = abc$

↓

Substrings:

← ϵ
 ← a, b, c
 ← ab, bc
 ← abc

3 different lengths
 2 different lengths
 1 different length

$|w| = n$

Different lengths ✓

Substrings ✓

Maximum ✓

non-zero length

Consider the regular expression $(0+1)^n$. The minimum state finite automaton that recognizes the language represented by this regular expression contains: **(GATE - 99)**

(a) n states

(b) $n+1$ states \Rightarrow NFA

(c) $n+2$ states \Rightarrow DFA

(d) None of the above

Set of all n length strings

Case I: DFA $\Rightarrow n+2$

Case II: NFA $\Rightarrow n+1$

Let S and T be languages over $\Sigma = \{a, b\}$ represented by the regular expressions $(a + b^*)^*$ and $(a + b)^*$, respectively.

Which of the following is true? (GATE - 2000)

(a) $S \subset T$

(b) $T \subset S$

(c) $S = T$

(d) $S \cap T = \phi$

$S \subseteq T$ ✓

$T \subseteq S$ ✓

$T = S$ ✓

$S \cap T = S$ ✓

$S \cap T = T$ ✓

Consider the following two statements:

~~S₁~~: $\{0^{2n} \mid n \geq 1\}$ is a regular language $= (00)^+$

~~S₂~~: $\{0^m 1^n 0^{m+n} \mid m \geq 1 \text{ and } n \geq 1\}$ is a regular language

Which of the following statements is correct? (GATE - 01)

~~(a)~~ Only S₁ is correct

(b) Only S₂ is correct

(c) Both S₁ and S₂ are correct

(d) None of S₁ and S₂ is correct

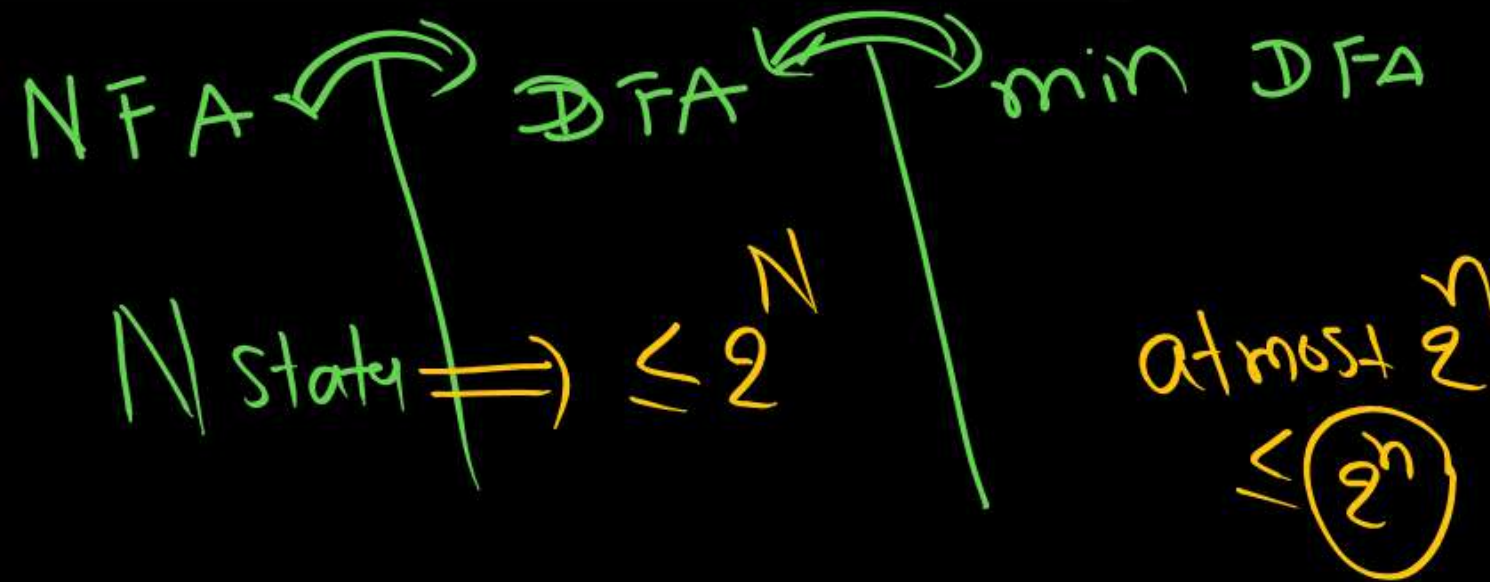
Given an arbitrary non-deterministic finite automaton (NFA) with N states, the maximum number of states in an equivalent minimized DFA is at least **(GATE - 01)**

(a) N^2

~~(b) 2^N~~

(c) $2N$

(d) $N!$

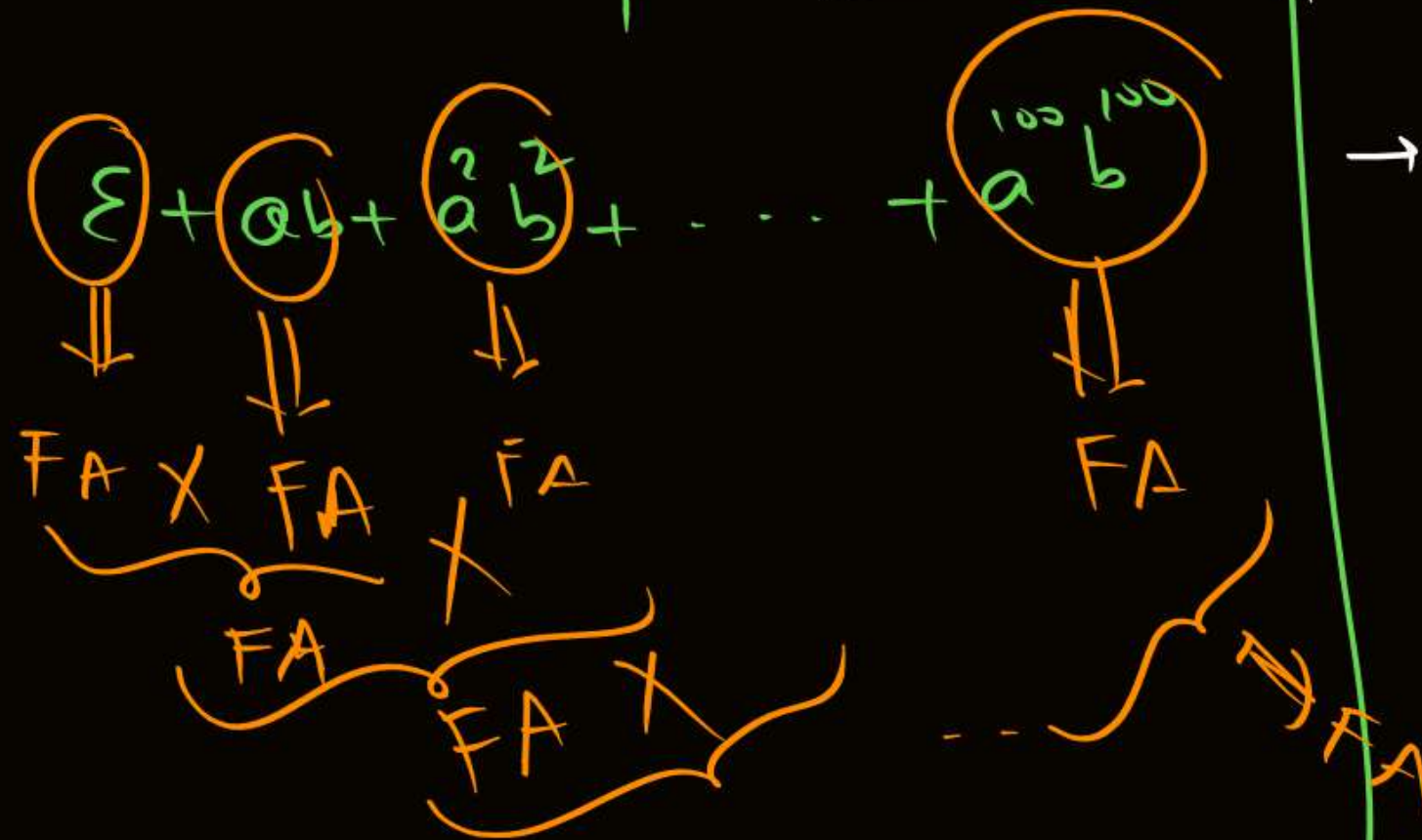


Every finite language over Σ is Regular

Understand

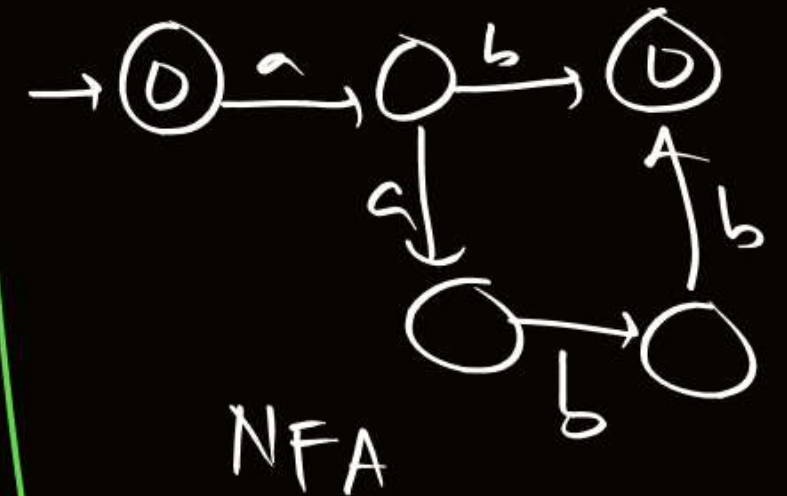
Don't try
to design FA

$$\{a^n b^n \mid n \leq \boxed{100}\}$$



$$\{a^n b^n \mid n \leq 10^{100}\}$$

$$\{a^n b^n \mid n \leq \boxed{2}\}$$



Consider the set Σ^* of all strings over the alphabet $\Sigma = \{0, 1\}$.
 Σ^* with the concatenation operator for strings **(GATE - 03)**

$$w \cdot \bar{w} = \bar{w} \cdot w = \epsilon$$

Discrete mathematics
TOL

- ~~(a) Does not form a group~~
- ~~(b) Forms a non-commutative group~~
- ~~(c) Does not have a right identity element~~
- ~~(d) Forms a group if the empty string is removed from Σ^*~~

closed ✓
 Associative
 Identity
 Inverse

$$\forall w_1 \in \Sigma^*$$

$$\forall w_2 \in \Sigma^*$$

$$w_1 w_2 \in \Sigma^*$$

$$(\Sigma^*, \cdot)$$

$$w \cdot \epsilon = \epsilon \cdot w = w$$

- closed ✓
- Associative ✓
- Identity = ϵ ✓
- Inverse ✗

$$(w_1 w_2) w_3 = w_1 (w_2 w_3)$$

The regular expression $0^*(10^*)^*$ denotes the same set as (GATE - 03)

~~(a)~~ $(1^*0)^*1^* = (0+1)^*$

(b) $0 + (0+10)^*$

(c) $(0+1)^*10(0+1)^*$

(d) None of the above

$$0^*(10^*)^* = (0+1)^*$$

Which of the following is TRUE? (GATE – 07)

- ☒ (a) Every subset of a regular set is regular
- ☒ (b) Every finite subset of a non-regular set is regular
- ☒ (c) The union of two non-regular sets is not regular
- ☒ (d) Infinite union of finite sets is regular

$\{ \epsilon \}$
 \cup
 $\{ ab^4 \}$
 \cup
 $\{ a^2b^2 \}$
 \cup
 $\{ b^2 \}$
 \vdots
 $a^n b^n$
 $a^n b^n \cup a^n b^n$
 $= \Sigma^*$

Finite Subset of Any Set
is finite set
regular

$(a+b)^*$ regular

$\phi, a^*, b^*, a^*b^*, \underbrace{a^n b^n}_{\text{non reg}}$

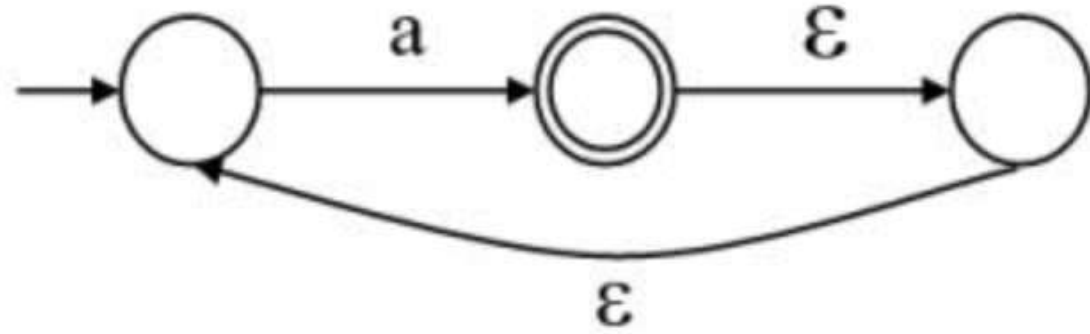
Some subsets of reg are regular

X Every Subset of reg is reg

Which one of the following languages over the alphabet $\{0, 1\}$ is described by the regular expression $(0+1)^*0(0+1)^*0(0+1)^*$ (**GATE - 09**)

- (a) The set of all strings containing the substring 00
- (b) The set of all strings containing at most two 0's
- (c) The set of all strings containing at least two 0's
- (d) The set of all strings that begin and end with either 0 or 1

What is the complement of the language accepted by the NFA shown below? Assume $\Sigma = \{a\}$ and ε is the empty string. **(GATE - 12)**



- (a) ϕ (b) $\{\varepsilon\}$ (c) a^* (d) $\{a, \varepsilon\}$

Consider the following two statements:

S₁: $\{0^{2n} \mid n \geq 1\}$ is a regular language

S₂: $\{0^m 1^n 0^{m+n} \mid m \geq 1 \text{ and } n \geq 1\}$ is a regular language

Which of the following statements is correct? **(GATE - 01)**

- (a) Only S_1 is correct
- (b) Only S_2 is correct
- (c) Both S_1 and S_2 are correct
- (d) None of S_1 and S_2 is correct

Consider the languages $L_1 = \phi$ and $L_2 = \{a\}$. Which one of the following represents $L_1 L_2^* \cup L_1^*$? (**GATE - 13**)

(a) $\{\epsilon\}$

(b) ϕ

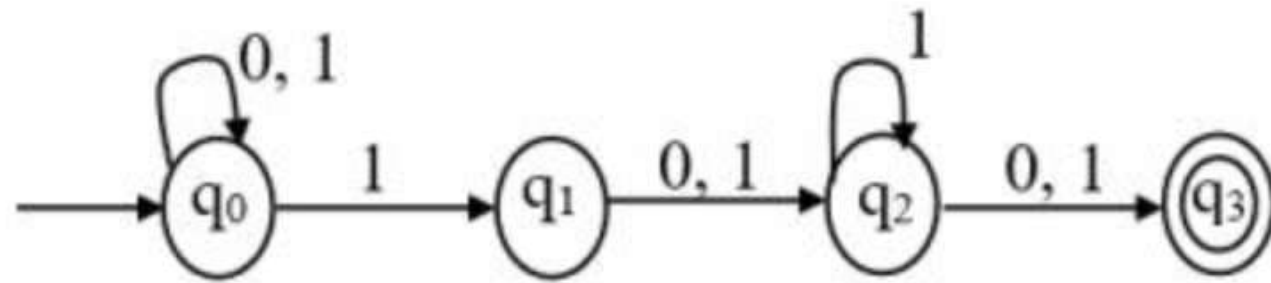
(c) a^*

(d) $\{\epsilon, a\}$

Which one of the following is **TRUE**? (GATE – 14-SET1)

- (a) The language $L = \{a^n b^n \mid n \geq 0\}$ is regular.
- (b) The language $L = \{a^n \mid n \text{ is prime}\}$ is regular.
- (c) The language $L = \{w \mid w \text{ has } 3k + 1 \text{ b's for some } k \in \mathbb{N} \text{ with } \Sigma = \{a, b\}\}$ is regular.
- (d) The language $L = \{ww \mid w \in \Sigma^* \text{ with } \Sigma = \{0, 1\}\}$ is regular

Consider the finite automaton in the following figure. (GATE – 14-SET1)



What is the set of reachable states for the input string 0011?

- | | |
|------------------------------|--------------------|
| (a) $\{q_0, q_1, q_2\}$ | (b) $\{q_0, q_1\}$ |
| (c) $\{q_0, q_1, q_2, q_3\}$ | (d) $\{q_3\}$ |

If $L_1 = \{a^n \mid n \geq 0\}$ and $L_2 = \{b^n \mid n \geq 0\}$, consider

I. $L_1 \cdot L_2$ is a regular language

II. $L_1 \cdot L_2 = \{a^n b^n \mid n \geq 0\}$

Which one of the following is CORRECT? **(GATE – 14-SET2)**

(a) Only (I)

(b) Only (II)

(c) Both (I) and (II)

(d) Neither (I) nor (II)

The length of the shortest string NOT in the language (over $\Sigma = \{a, b\}$) of the following regular expression is _____.

$$a^*b^*(ba)^*a^*$$

(GATE – 14-SET3)

Let L be the language represented by the regular expression $\Sigma^*0011\Sigma^*$ where $\Sigma=\{0,1\}$. What is the minimum number of states in a DFA that recognizes \bar{L} (complement of L)?
(GATE – 15 – SET3)

(a) 4

(b) 5

(c) 6

(d) 8

Which of the following languages is generated by the given grammar? **(GATE – 16 – SET1)**

$$S \rightarrow aS \mid bS \mid \varepsilon$$

- (a) $\{a^n b^m \mid n, m \geq 0\}$
- (b) $\{w \in \{a, b\}^* \mid w \text{ has equal number of } a\text{'s and } b\text{'s}\}$
- (c) $\{a^n \mid n \geq 0\} \cup \{b^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$
- (d) $\{a, b\}^*$

Which one of the following regular expressions represents the language: the set of all binary strings having two consecutive 0's and two consecutive 1's? **(GATE – 16 – SET1)**

- (a) $(0+1)^* 0011(0+1)^* + (0+1)^* 1100(0+1)^*$
- (b) $(0+1)^* (00(0+1)^* 11 + 11(0+1)^* 00)(0+1)^*$
- (c) $(0+1)^* 00(0+1)^* + (0+1)^* 11(0+1)^*$
- (d) $00(0+1)^* 11 + 11(0+1)^* 00$

Let $r = 1(1+0)^*$, $s = 11^*0$ and $t = 1^*0$ be three regular expressions. Which one of the following is true? **(GATE - 91)**

- (a) $L(s) \subseteq L(r)$ and $L(s) \subseteq L(t)$
- (b) $L(r) \subseteq L(s)$ and $L(s) \subseteq L(t)$
- (c) $L(s) \subseteq L(t)$ and $L(s) \subseteq L(r)$
- (d) $L(t) \subseteq L(s)$ and $L(s) \subseteq L(r)$.

Which of the following regular expression identities are true?
(GATE - 92)

(a) $r^* = r^*$

(b) $(r^*s^*)^* = (r+s)^*$

(c) $(r+s)^* = r^* + s^*$

(d) $r^*s^* = r^* + s^*$

Consider the language L given by the regular expression $(a+b)^*b(a+b)$ over the alphabet $\{a, b\}$. The smallest number of states needed in a deterministic finite-state automaton (DFA) accepting L is _____. **(GATE – 17 – SET1)**

The minimum possible number of states of a deterministic finite automaton that accepts the regular language **(GATE – 17 – SET2)**

$L = \{w_1aw_2 \mid w_1, w_2 \in \{a, b\}^*, |w_1| = 2, |w_2| \geq 3\}$ is _____.

The lexical analysis for a modern computer language such as java needs the power of which one of the following machine models in a necessary and sufficient sense? **(GATE - 11)**

- (a) Finite state automata
- (b) Deterministic pushdown automata
- (c) Non –deterministic pushdown automata
- (d) Turing machine

The number of substrings (of all lengths inclusive) that can be formed from a character string of length n is **(GATE – 89 & 94)**

(a) n

(b) n^2

(c) $\frac{n(n-1)}{2}$

(d) $\frac{n(n+1)}{2} + 1$

Let R_1 and R_2 be regular sets defined over the alphabet Σ then:
(GATE - 90)

- (a) $R_1 \cap R_2$ is not regular.
- (b) $R_1 \cup R_2$ is regular.
- (c) $\Sigma^* - R_1$ is regular.
- (d) R_1^* is not regular.

Consider the following language.

$$L = \{x \in \{a, b\}^* \mid \text{number of } a\text{'s in } x \text{ divisible by 2 but not divisible by 3}\}$$

The minimum number of states in DFA that accepts L is _____

Which one of the following regular expressions represents the set of all binary strings with an odd number of 1's?

A. $((0 + 1)^* 1 (0 + 1)^* 1)^* 10^*$

B. $(0^* 10^* 10^*)^* 0^* 1$

C. $10^* (0^* 10^* 10^*)^*$

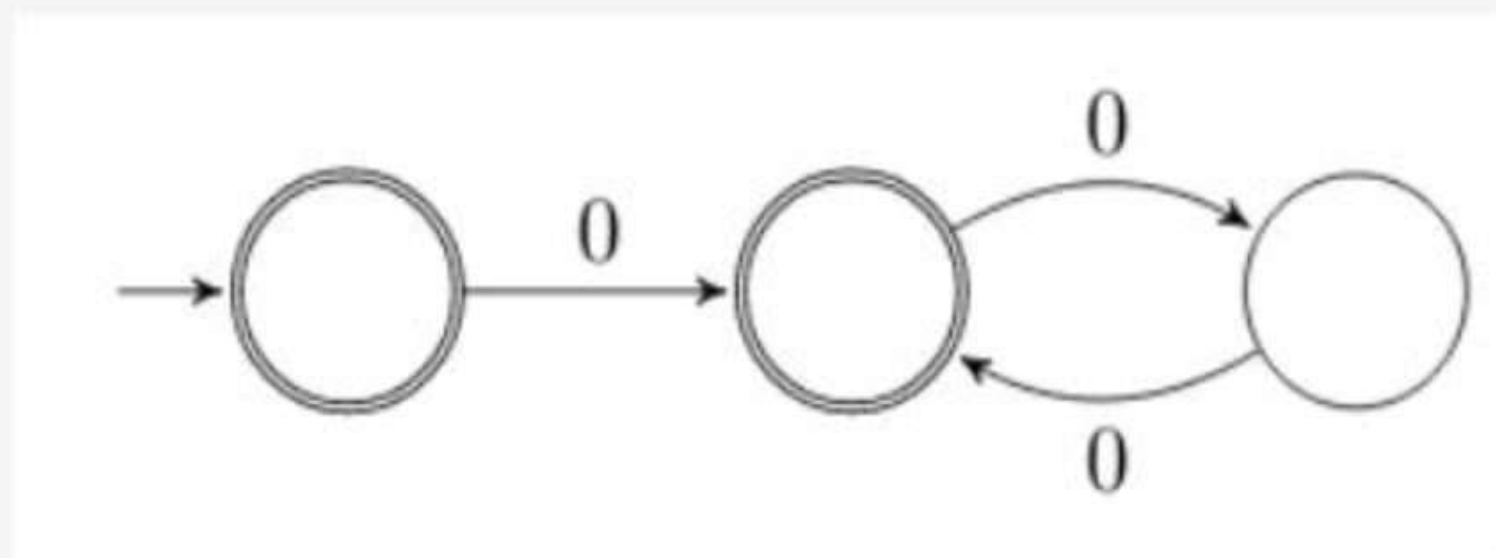
D. $(0^* 10^* 10^*)^* 10^*$

Given a language L , define L^i as follows:

$$L^0 = \{\varepsilon\}$$

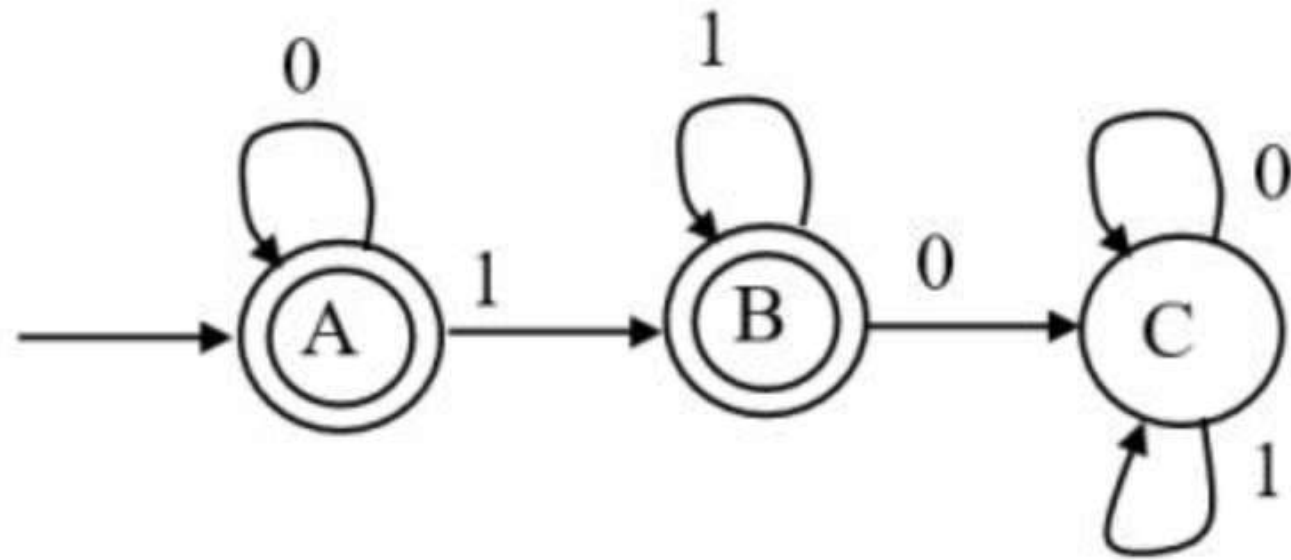
$$L^i = L^{i-1} \bullet L \text{ for all } i > 0$$

The order of a language L is defined as the smallest k such that $L^k = L^{k+1}$. Consider the language L_1 (over alphabet 0) accepted by the following automaton.



The order of L_1 is _____

The regular expression for the language recognized by the finite state automation of the below figure is _____ (**GATE - 94**)



A finite state machine with the following state table has a single input X and a single output Z .

Present state	Next state Z	
	$X=1$	$X=0$
A	D,0	B,0
B	B,1	C,1
C	B,0	D,1
D	B,1	C,0

If the initial state is unknown, then the shortest input sequence to reach the final state C is **(GATE - 95)**

- (a) 01 (b) 10 (c) 101 (d) 110

Which one of the following regular expressions over $\{0, 1\}$ denotes the set of all strings **not** containing 100 as a substring?

(GATE - 97)

(a) $0^*(1^+0)^*$

(b) 0^*1010^*

(c) $0^*1^*01^*$

(d) $0^*(10+1)^*$

Let L be the set of all binary strings whose last two symbols are the same. The number of states in the minimum state deterministic finite-state automaton accepting L is

(GATE - 98)

What can be said about a regular language L over $\{a\}$ whose minimal finite state automaton has two states? **(GATE - 2000)**

- (a) L must be $\{a^n \mid n \text{ is odd}\}$
- (b) L must be $\{a^n \mid n \text{ is even}\}$
- (c) L must be $\{a^n \mid n \geq 0\}$
- (d) Either L must be $\{a^n \mid n \text{ is odd}\}$ or L must be $\{a^n \mid n \text{ is even}\}$

Consider a DFA over $\Sigma = \{a, b\}$ accepting all strings which have number of a's divisible by 6 and number of b's divisible by 8. What is the minimum number of states that the DFA will have?

(GATE - 01)

- | | |
|--------|--------|
| (a) 8 | (b) 14 |
| (c) 15 | (d) 48 |

Consider the following languages:

$$L_1 = \{w w \mid w \in \{a, b\}^*\}$$

$$L_2 = \{w w^R \mid w \in \{a, b\}^*, w^R \text{ is the reverse of } w\}$$

$$L_3 = \{0^{2i} \mid i \text{ is an integer}\}$$

$$L_4 = \{0^{i^2} \mid i \text{ is an integer}\}$$

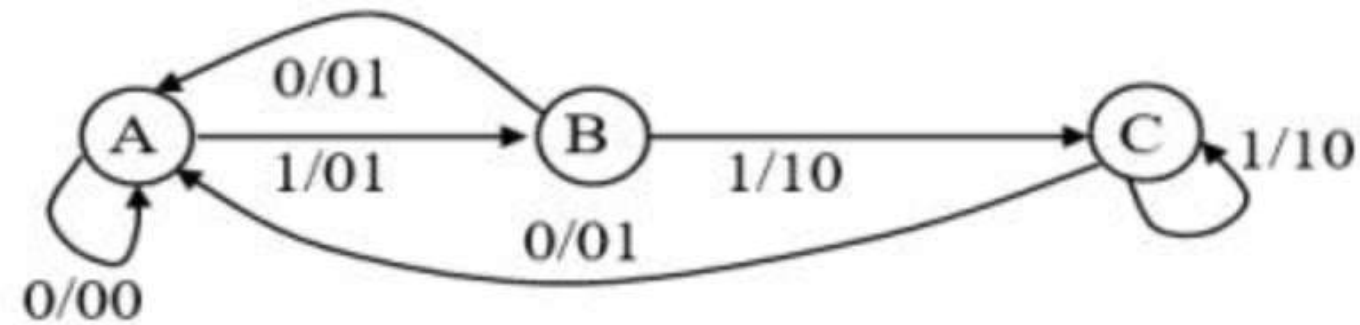
Which of the languages are regular?

(GATE - 01)

- | | |
|--------------------------|-------------------------------|
| (a) Only L_1 and L_2 | (b) Only L_2, L_3 and L_4 |
| (c) Only L_3 and L_4 | (d) Only L_3 |

The Finite state machine described by the following state diagram with A as starting state, where an arc label is x/y and x stands for 1-bit input and y stands for 2-bit output

(GATE - 02)



- (a) Outputs the sum of the present and the previous bits of the input.
- (b) Outputs 01 whenever the input sequence contains 11
- (c) Outputs 00 whenever the input sequence contains 10
- (d) None of the above

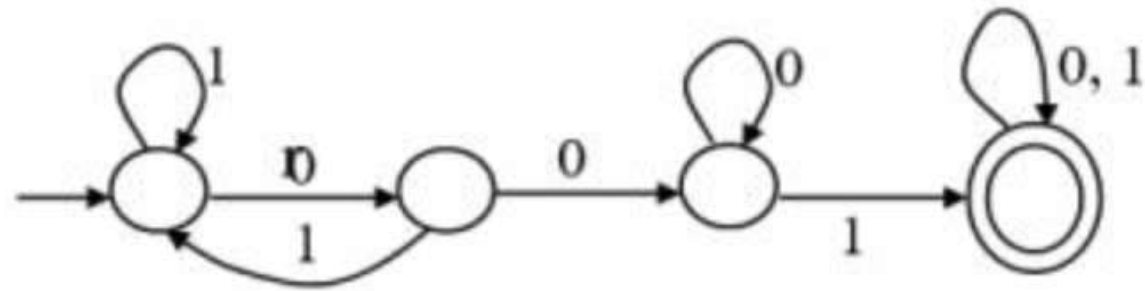
The smallest finite automation which accepts the language

$L = \{x \mid \text{length of } x \text{ is divisible by } 3\}$ has

(GATE - 02)

- | | |
|--------------|--------------|
| (a) 2 states | (b) 3 states |
| (c) 4 states | (d) 5states |

Consider the following deterministic finite state automation M.



Let S denote the set of seven bit binary strings in which the first, the fourth, and the last bits are 1. The number of strings in S that are accepted by M is **(GATE - 03)**

- (a) 1 (b) 5 (c) 7 (d) 8

Which of the following statements is true?

- A. If a language is context free it can always be accepted by a deterministic push-down automaton
- B. The union of two context free languages is context free
- C. The intersection of two context free languages is a context free
- D. The complement of a context free language is a context free

Consider the following problem X.

Given a Turing machine M over the input alphabet Σ , any state q of M and a word $w \in \Sigma^*$, does the computation of M on w visit the state of q ?

Which of the following statements about X is correct?

- A. X is decidable
- B. X is undecidable but partially decidable
- C. X is undecidable and not even partially decidable
- D. X is not a decision problem

Consider the following languages:

$$L1 = \{ww \mid w \in \{a,b\}^*\}$$

$$L2 = \{ww^R \mid w \in \{a,b\}^*, w^R \text{ is the reverse of } w\}$$

$$L3 = \{02^i \mid i \text{ is an integer}\}$$

$$L4 = \{0^i 2 \mid i \text{ is an integer}\}$$

Which of the languages are regular?

A. Only L1 and L2

B. Only L2, L3 and L4

C. Only L3 and L4

D. Only L3

- Which of the following is true?
 - A. The complement of a recursive language is recursive
 - B. The complement of a recursively enumerable language is recursively enumerable
 - C. The complement of a recursive language is either recursive or recursively enumerable
 - D. The complement of a context-free language is context-free

- The language accepted by a Pushdown Automaton in which the stack is limited to 10 items is best described as
 - A. Context free
 - B. Regular
 - C. Deterministic Context free
 - D. Recursive

Let $G = (\{S\}, \{a, b\}, R, S)$ be a context free grammar where the rule set R is $S \rightarrow aSb \mid SS \mid \epsilon$

Which of the following statements is true?

- A. G is not ambiguous
- B. There exist $x, y \in L(G)$ such that $xy \notin L(G)$
- C. There is a deterministic pushdown automaton that accepts $L(G)$
- D. We can find a deterministic finite state automaton that accepts $L(G)$

Define languages L_0 and L_1 as follows :

$L_0 = \{ \langle M, w, 0 \rangle \mid M \text{ halts on } w \}$

$L_1 = \{ \langle M, w, 1 \rangle \mid M \text{ does not halts on } w \}$

Here $\langle M, w, i \rangle$ is a triplet, whose first component M is an encoding of a Turing Machine, second component w is a string, and third component i is a bit.

Let $L = L_0 \cup L_1$. Which of the following is true?

- A. L is recursively enumerable, but L' is not
- B. L' is recursively enumerable, but L is not
- C. Both L and L' are recursive
- D. Neither L nor L' is recursively enumerable

• If the strings of a language L can be effectively enumerated in lexicographic (i.e., alphabetic) order, which of the following statements is true?

- A. L is necessarily finite
- B. L is regular but not necessarily finite
- C. L is context free but not necessarily regular
- D. L is recursive but not necessarily context-free

• L_1 is a recursively enumerable language over Σ . An algorithm A effectively enumerates its words as $\omega_1, \omega_2, \omega_3, \dots$. Define another language L_2 over $\Sigma \cup \{\#\}$ as $\{w_i \# w_j \mid w_i, w_j \in L_1, i < j\}$. Here $\#$ is new symbol. Consider the following assertions.

- S1: L_1 is recursive implies L_2 is recursive
- S2: L_2 is recursive implies L_1 is recursive
- Which of the following statements is true?

A. Both S1 and S2 are true

B. S1 is true but S2 is not necessarily true

C. S2 is true but S1 is not necessarily true

D. Neither is necessarily true

- The language $\{a^m b^n c^{m+n} | m, n \geq 1\}$ is
 - A. regular
 - B. context-free but not regular
 - C. context-sensitive but not context free
 - D. type-0 but not context sensitive

- Which one of the following statements is FALSE?
 - A. There exist context-free languages such that all the context-free grammars generating them are ambiguous
 - B. An unambiguous context-free grammar always has a unique parse tree for each string of the language generated by it
 - C. Both deterministic and non-deterministic pushdown automata always accept the same set of languages
 - D. A finite set of string from some alphabet is always a regular language

Let $M=(K,\Sigma,\Gamma,\Delta,s,F)$ be a pushdown automaton, where

$K=(s,f), F=\{f\}, \Sigma=\{a,b\}, \Gamma=\{a\}$ and

$\Delta=\{((s,a,\epsilon),(s,a)),((s,b,\epsilon),(s,a)),((s,a,a),(f,\epsilon)),((f,a,a),(f,\epsilon)),((f,b,a),(f,\epsilon))\}$

Which one of the following strings is not a member of $L(M)$?

A. aaa

B. aabab

C. baaba

D. bab

• Which one of the following regular expressions is NOT equivalent to the regular expression $(a+b+c)^*$?

A. $(a^*+b^*+c^*)^*$

B. $(a^*b^*c^*)^*$

C. $((ab)^*+c^*)^*$

D. $(a^*b^*+c^*)^*$

• Consider the languages:

- $L1 = \{ww^R \mid w \in \{0,1\}^*\}$
- $L2 = \{w\#w^R \mid w \in \{0,1\}^*\}$, where $\#$ is a special symbol
- $L3 = \{ww \mid w \in \{0,1\}^*\}$

Which one of the following is TRUE?

- A. $L1$ is a deterministic CFL
- B. $L2$ is a deterministic CFL
- C. $L3$ is a CFL, but not a deterministic CFL
- D. $L3$ is a deterministic CFL

- Consider three decision problems P_1 , P_2 and P_3 . It is known that P_1 is decidable and P_2 is undecidable. Which one of the following is TRUE?
 - A. P_3 is decidable if P_1 is reducible to P_3
 - B. P_3 is undecidable if P_3 is reducible to P_2
 - C. P_3 is undecidable if P_2 is reducible to P_3
 - D. P_3 is decidable if P_3 is reducible to P_2 's complement

• The language $\{0^n 1^n 2^n \mid 1 \leq n \leq 10^6\}$ is

A. regular

B. context-free but not regular

C. context-free but its complement is not context-free

D. not context-free

• Let L_1 be a recursive language, and let L_2 be a recursively enumerable but not a recursive language. Which one of the following is TRUE?

- A. L_1' is recursive and L_2' is recursively enumerable
- B. L_1' is recursive and L_2' is not recursively enumerable
- C. L_1' and L_2' are recursively enumerable
- D. L_1' is recursively enumerable and L_2' is recursive

- Which of the following statements is TRUE about the regular expression 01^*0 ?
 - A. It represents a finite set of finite strings.
 - B. It represents an infinite set of finite strings.
 - C. It represents a finite set of infinite strings.
 - D. It represents an infinite set of infinite strings.

Let X be a recursive language and Y be a recursively enumerable but not recursive language. Let W and Z be two languages such that \bar{Y} reduces to W , and Z reduces to \bar{X} (reduction means the standard many-one reduction). Which one of the following statements is TRUE? **(GATE – 16 – SET1)**

- (a) W can be recursively enumerable and Z is recursive.
- (b) W can be recursive and Z is recursively enumerable.
- (c) W is not recursively enumerable and Z is recursive.
- (d) W is not recursively enumerable and Z is not recursive.

Let A and B be finite alphabets and let $\#$ be a symbol outside both A and B . Let f be a total function from A^* to B^* . We say f is computable if there exists a Turing machine M which given an input x in A^* , always halts with $f(x)$ on its tape. Let L_f denote the language $\{x\#f(x) \mid x \in A^*\}$.

Which of the following statements is true:

(GATE – 17 – SET1)

- (a) f is computable if and only if L_f is recursive
- (b) f is computable if and only if L_f is recursively enumerable
- (c) If f is computable then L_f is recursive, but not conversely
- (d) If f is computable then L_f is recursively enumerable, but not conversely

Let $A \leq_m B$ denotes that language A is mapping reducible (also known as many-to-one reducible) to language B . Which one of the following is FALSE?

(GATE – 14-SET2)

- (a) If $A \leq_m B$ and B is recursive then A is recursive.
- (b) If $A \leq_m B$ and A is undecidable then B is un-decidable.
- (c) If $A \leq_m B$ and B is recursively enumerable then A is recursively enumerable.
- (d) If $A \leq_m B$ and B is not recursively enumerable then A is not recursively enumerable.

GATE2020

Which of the following languages are undecidable? Note that $\langle M \rangle \langle M \rangle$ indicates encoding of the Turing machine M .

$$L1 = \{ \langle M \rangle \mid L(M) = \emptyset \}$$

$$L2 = \{ \langle M, w, q \rangle \mid M \text{ on input } w \text{ reaches state } q \text{ in exactly 100 steps} \}$$

$$L3 = \{ \langle M \rangle \mid L(M) \text{ is not recursive} \}$$

$$L4 = \{ \langle M \rangle \mid L(M) \text{ contains at least 21 members} \}$$

- A. $L1$, $L3$, and $L4$ only
- B. $L1$ and $L3$ only
- C. $L2$ and $L3$ only
- D. $L2$, $L3$, and $L4$ only

GATE2019

Consider the following sets:

S1: Set of all recursively enumerable languages over the alphabet $\{0,1\}$

S2: Set of all syntactically valid C programs

S3: Set of all languages over the alphabet $\{0,1\}$

S4: Set of all non-regular languages over the alphabet $\{0,1\}$

Which of the above sets are uncountable?

- A. S1 and S2
- B. S3 and S4
- C. S2 and S3
- D. S1 and S4

GATE2018

Consider the following problems. $L(G)$ denotes the language generated by a grammar G . $L(M)$ denotes the language accepted by a machine M .

- I. For an unrestricted grammar G and a string w , whether $w \in L(G)$
- II. Given a Turing machine M , whether $L(M)$ is regular
- III. Given two grammar G_1 and G_2 , whether $L(G_1) = L(G_2)$
- IV. Given an NFA N , whether there is a deterministic PDA P such that N and P accept the same language

Which one of the following statement is correct?

- A. Only I and II are undecidable
- B. Only II is undecidable
- C. Only II and IV are undecidable
- D. Only I, II and III are undecidable

GATE2017

Let $L(R)$ be the language represented by regular expression R . Let $L(G)$ be the language generated by a context free grammar G . Let $L(M)$ be the language accepted by a Turing machine M . Which of the following decision problems are undecidable?

- I. Given a regular expression R and a string w , is $w \in L(R)$?
- II. Given a context-free grammar G , is $L(G) = \emptyset$?
- III. Given a context-free grammar G , is $L(G) = \Sigma^*$ for some alphabet Σ ?
- IV. Given a Turing machine M and a string w , is $w \in L(M)$?

- A. I and IV only
- B. II and III only
- C. II, III and IV only
- D. III and IV only

Summary

→ Practice all GATE PYQs

Thank you

