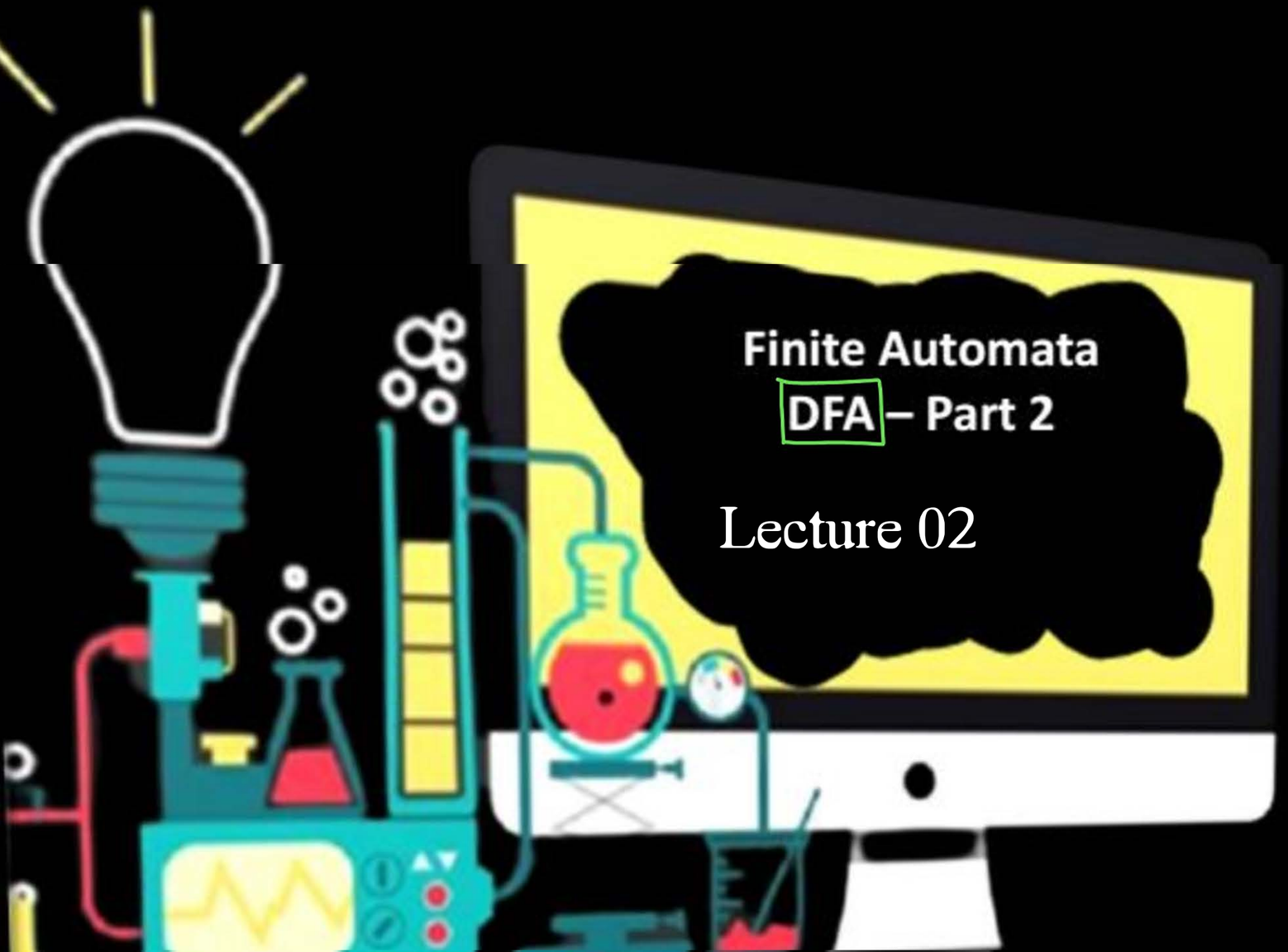
CS & IT Engineering



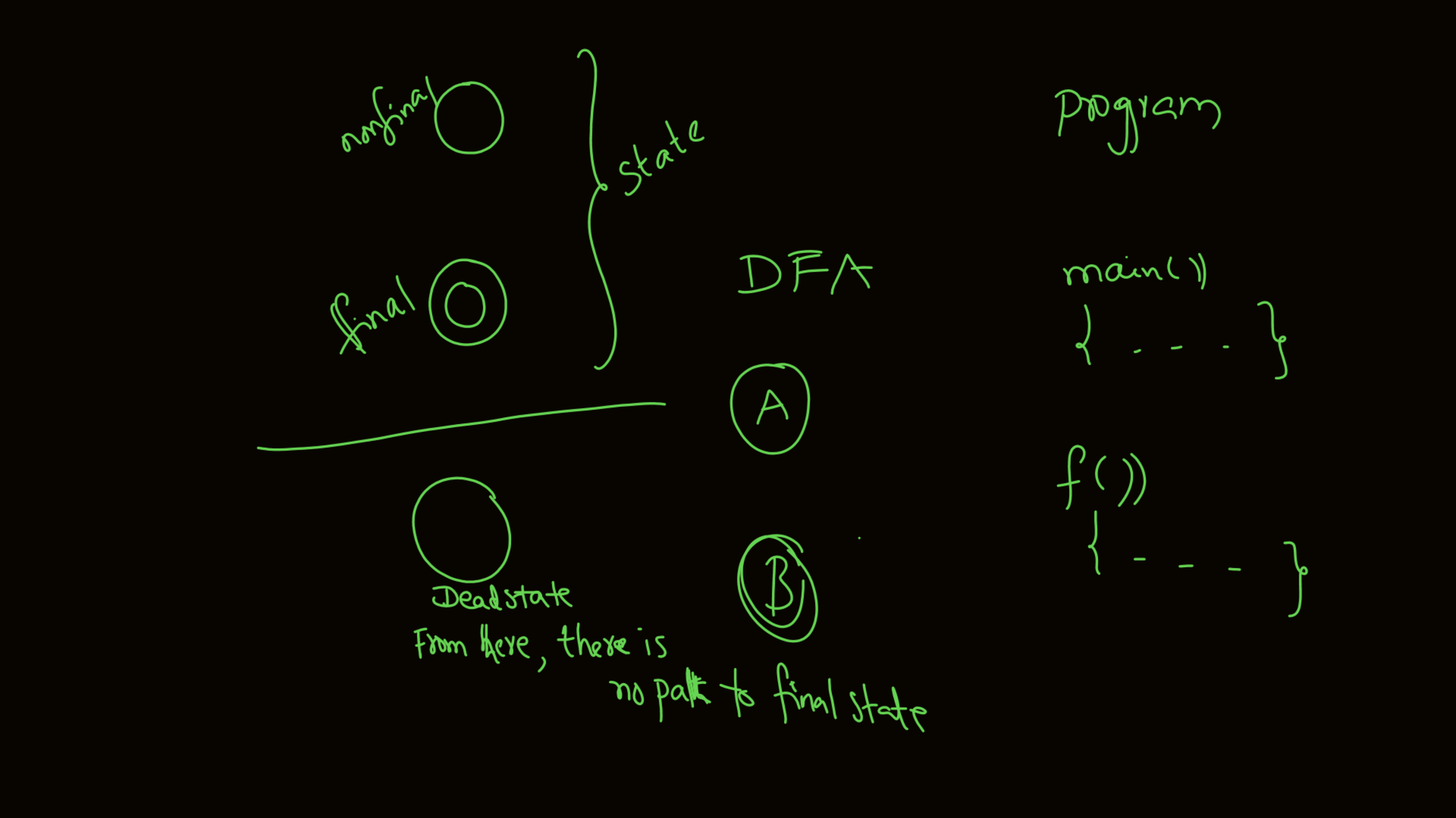
Deva sir

Topics: To be covered

- 4 to 5 Models in DFA [easy]
- Advanced Models

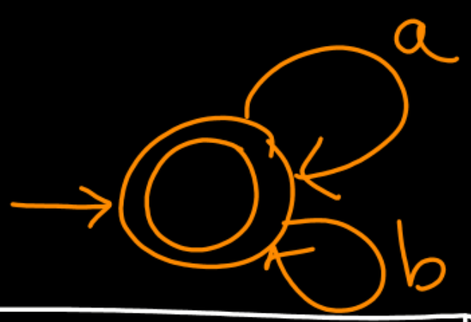
Reg Exp ✓

DFA


nonfinal
final
state
DFA
A
B
Deadstate
From here, there is
no path to final state
Program
main()
{ - - - }
f()
{ - - - }

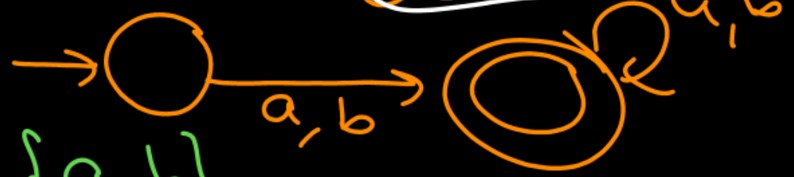
Model-I :

① $L = (a+b)^* = \Sigma^*$

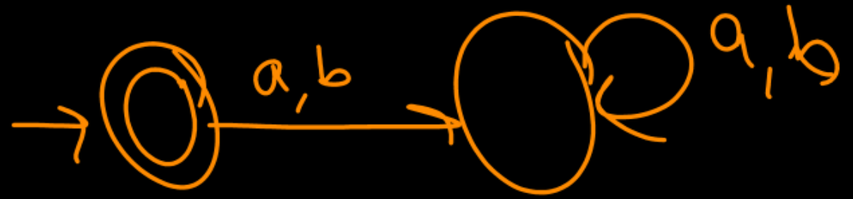


② $L = \phi$ over $\Sigma = \{a, b\}$

③ $L = (a+b)^+ = \Sigma^+$



④ $L = \{\varepsilon\}$ over $\Sigma = \{a, b\}$



Model-II : Start / end / contain [focus on min string]

⑤ $L = \{aw \mid w \in \{a,b\}^*\}$

$= a(a+b)^*$

Min valid string = a

1st : Try to accept only min string

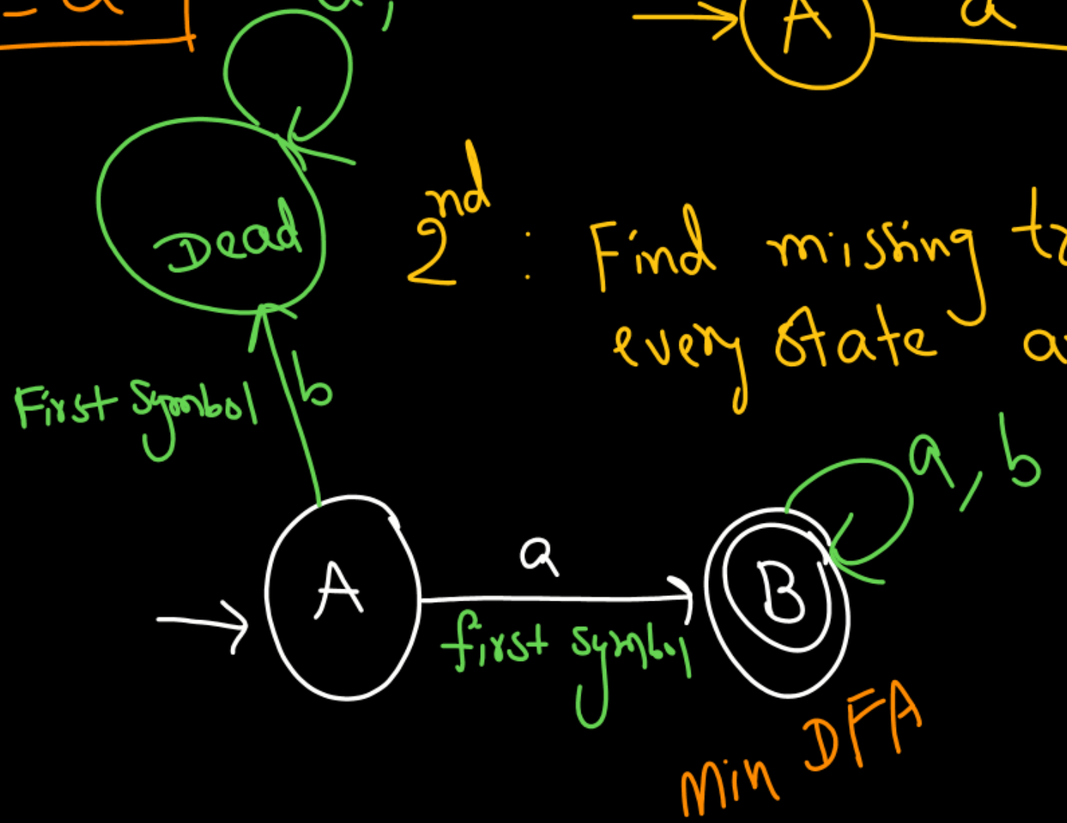
2nd : Find missing transitions from every state and make it DFA

A → b → ?

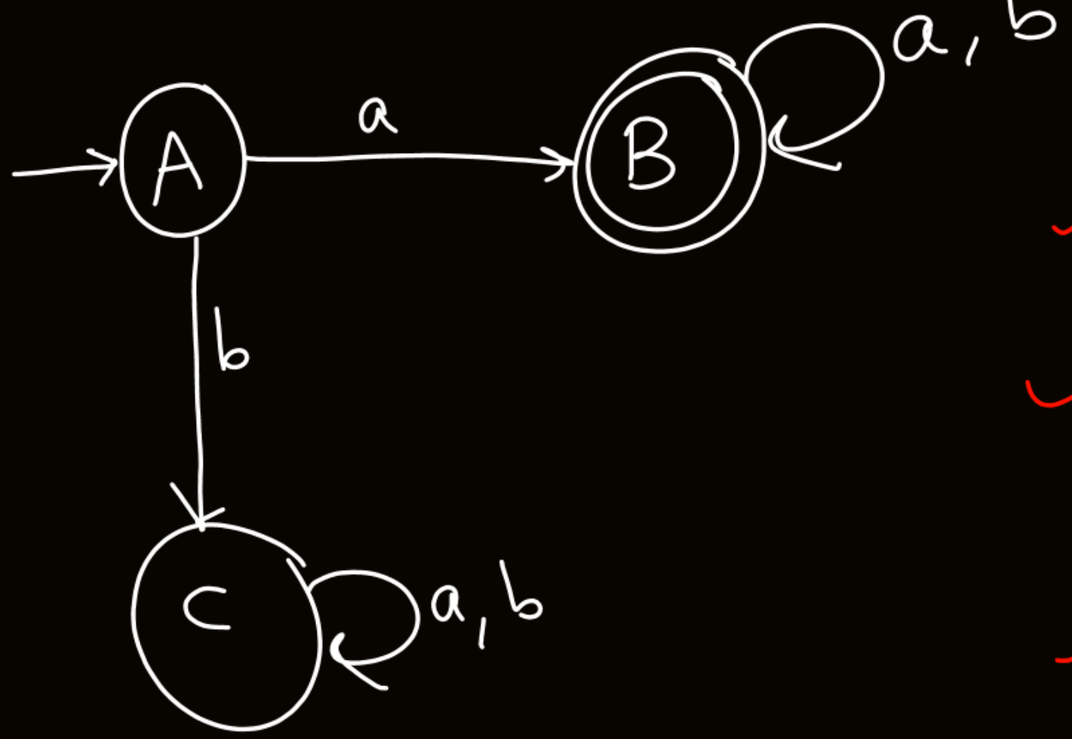
B → a → ?

B → b → ?

3 states



NFA :



Begin

① ε — A_{Halt}

② a — $A \xrightarrow{a} B_{Halt}$

③ b — $A \xrightarrow{b} C_{Halt}$

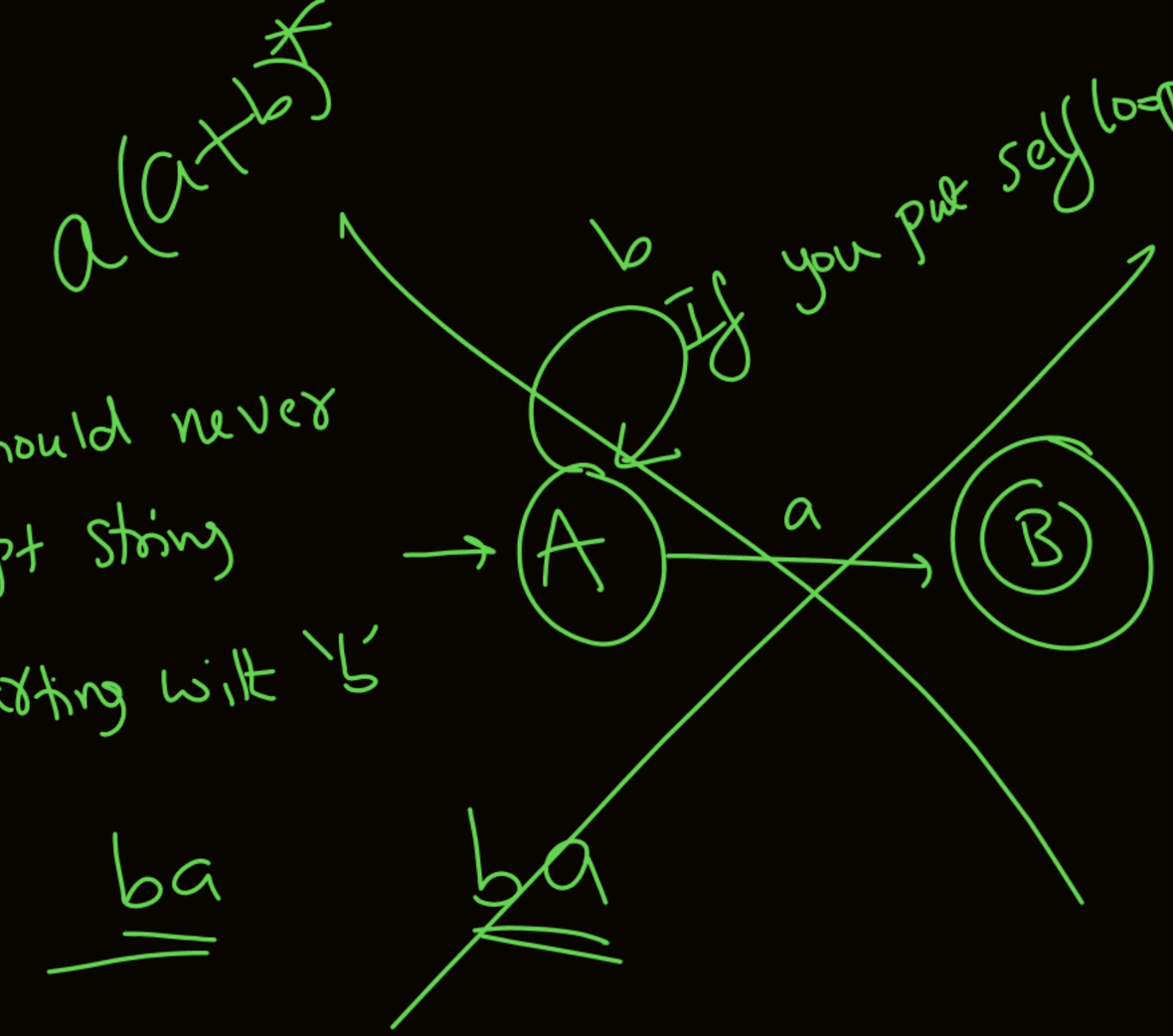
④ ǎǎ — $A \xrightarrow{a} B \xrightarrow{a} B_{Halt}$

⑤ ǎbǎ — $A \xrightarrow{a} B \xrightarrow{b} B \xrightarrow{a} B_{Halt}$

⑥ ǎbba — $A \xrightarrow{a} B \xrightarrow{b} B \xrightarrow{b} B \xrightarrow{a} B_{Halt}$

$a(a+b)^*$

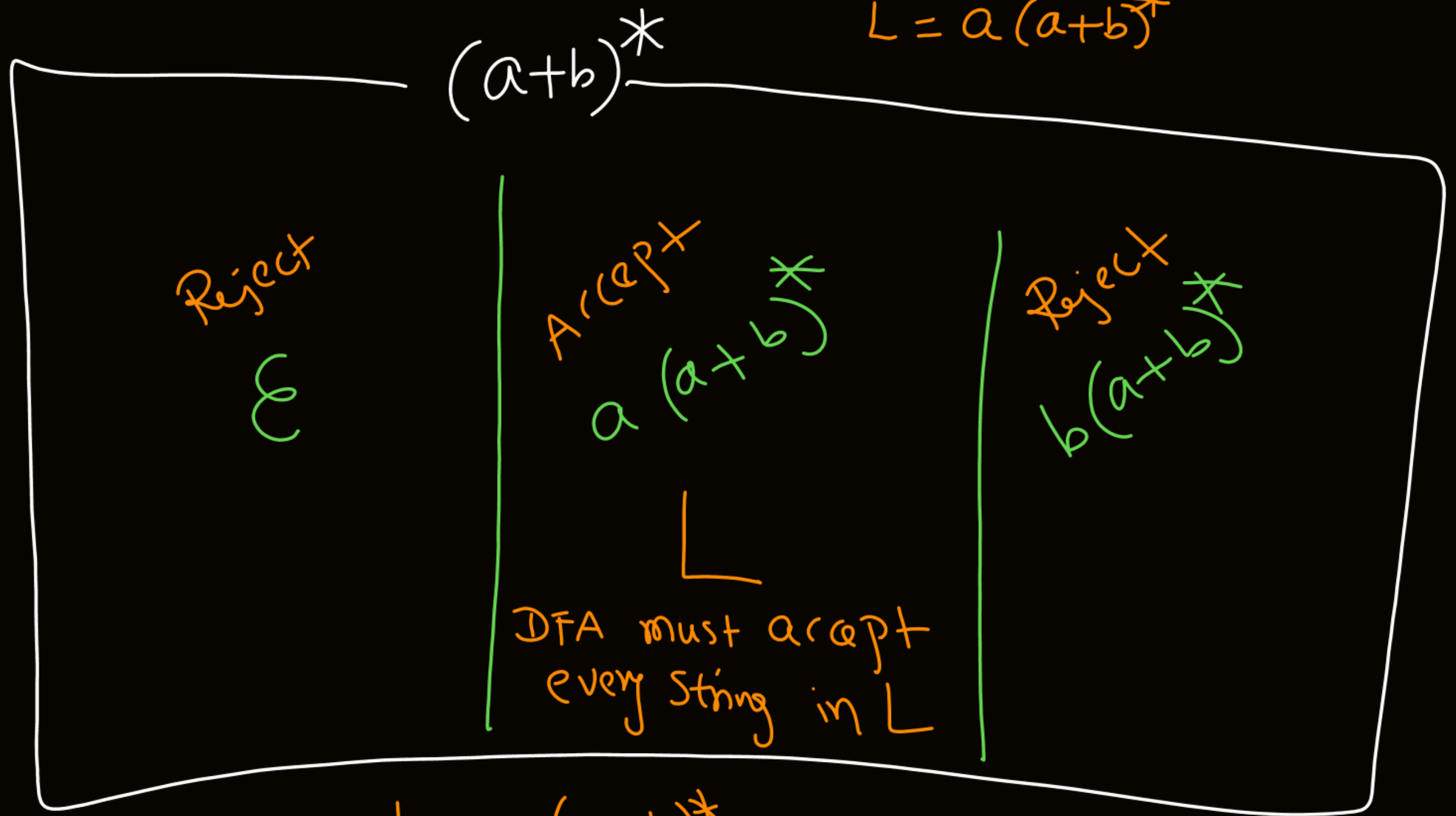
We should never accept string Starting with 'b'



ba

ba

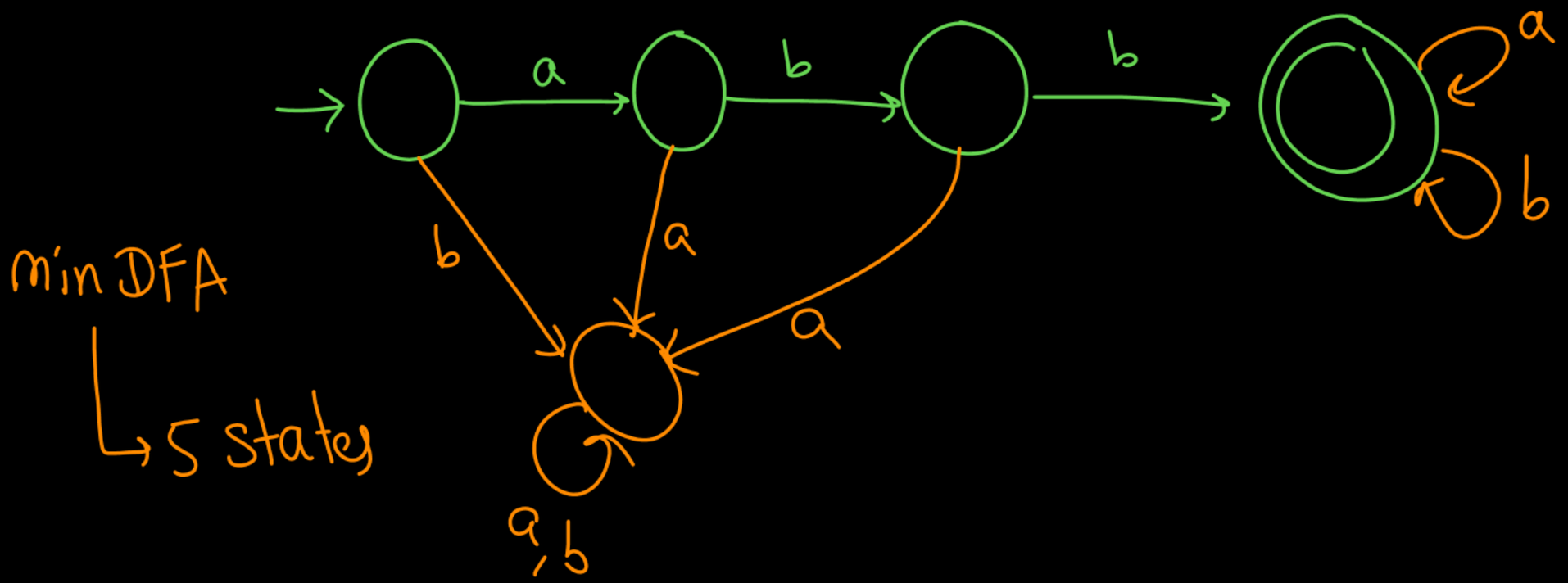
$L = a(a+b)^*$



$L = a(a+b)^* \Rightarrow \bar{L} = \epsilon + b(a+b)^*$

⑥ $L = \{ abbw \mid w \in \{a,b\}^* \}$

Min String = abb



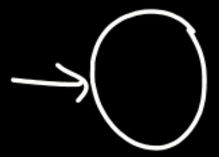
Min DFA

↳ 5 states

(7) L = ababba (a+b)*

Length 2

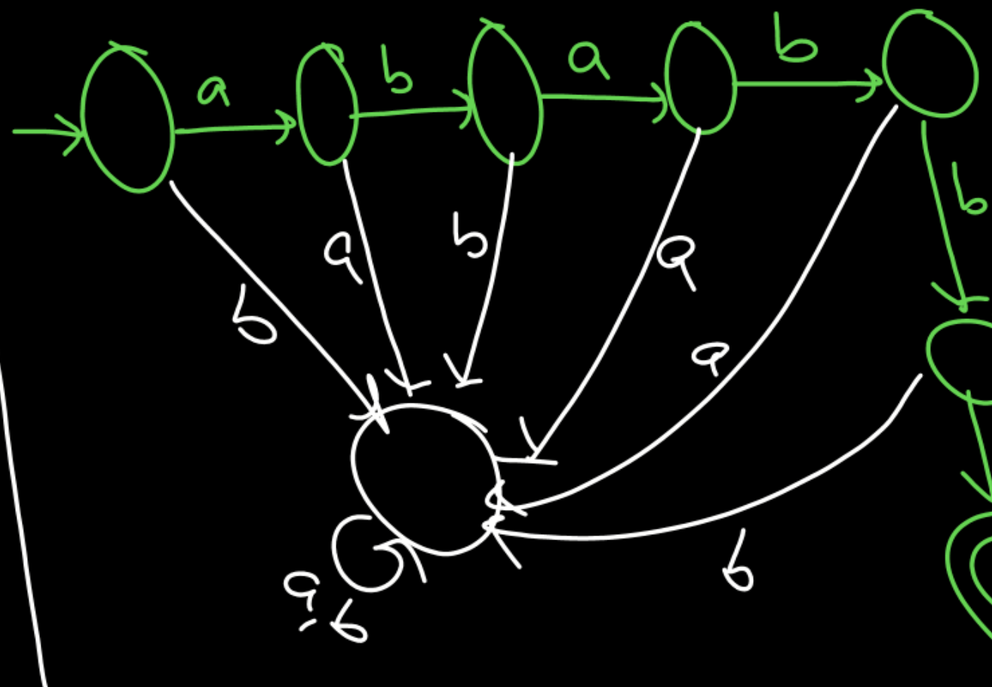
Length 0



Length 1

How many states in min DFA ?

It is start condition
Starts with ababba
min
↳ length 6



⇓

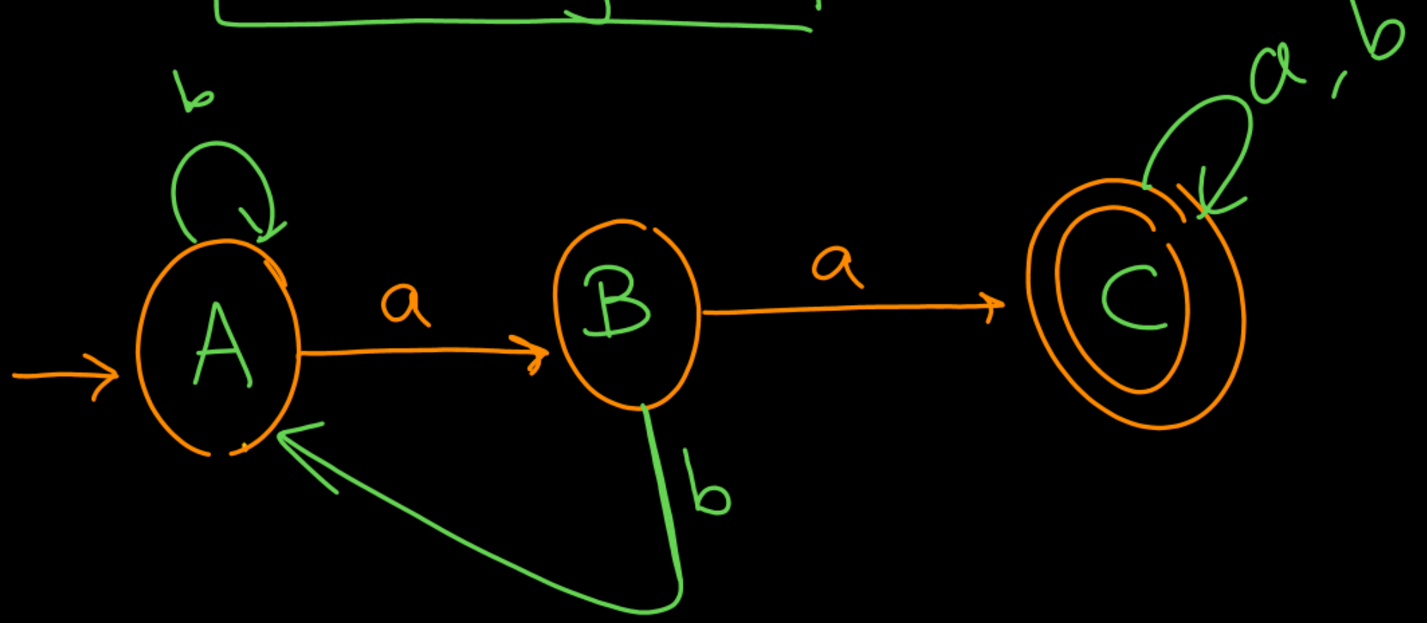
7 States to accept min + 1 Dead state

= 8 States

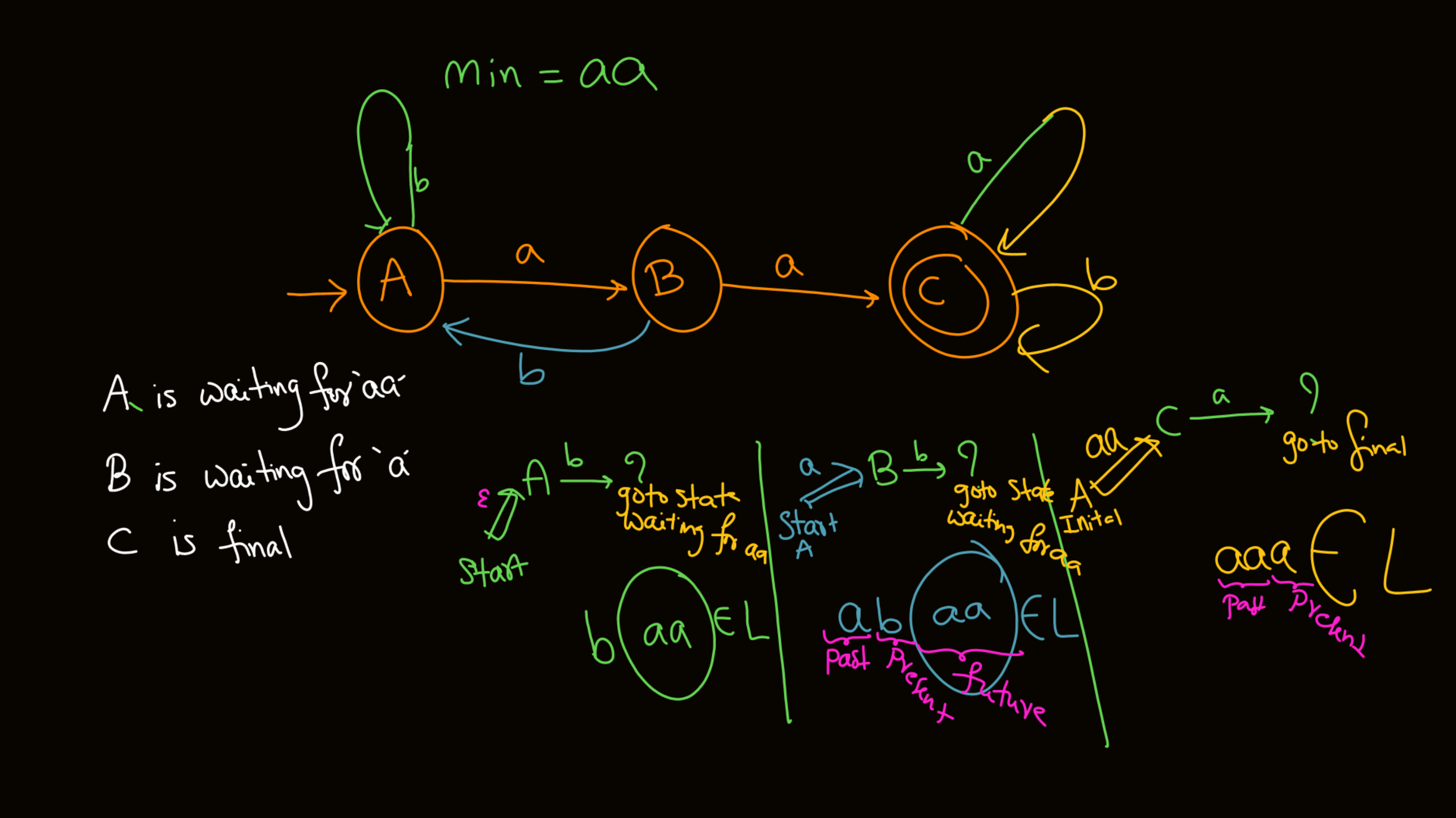
(8) $L = (a+b)^* \underbrace{aa}_{\text{contains}} (a+b)^*$

$= \{ w_1 aa w_2 \mid w_1, w_2 \in \{a,b\}^* \}$

Min string = aa



for this problem
Do you need dead state?
↳ No

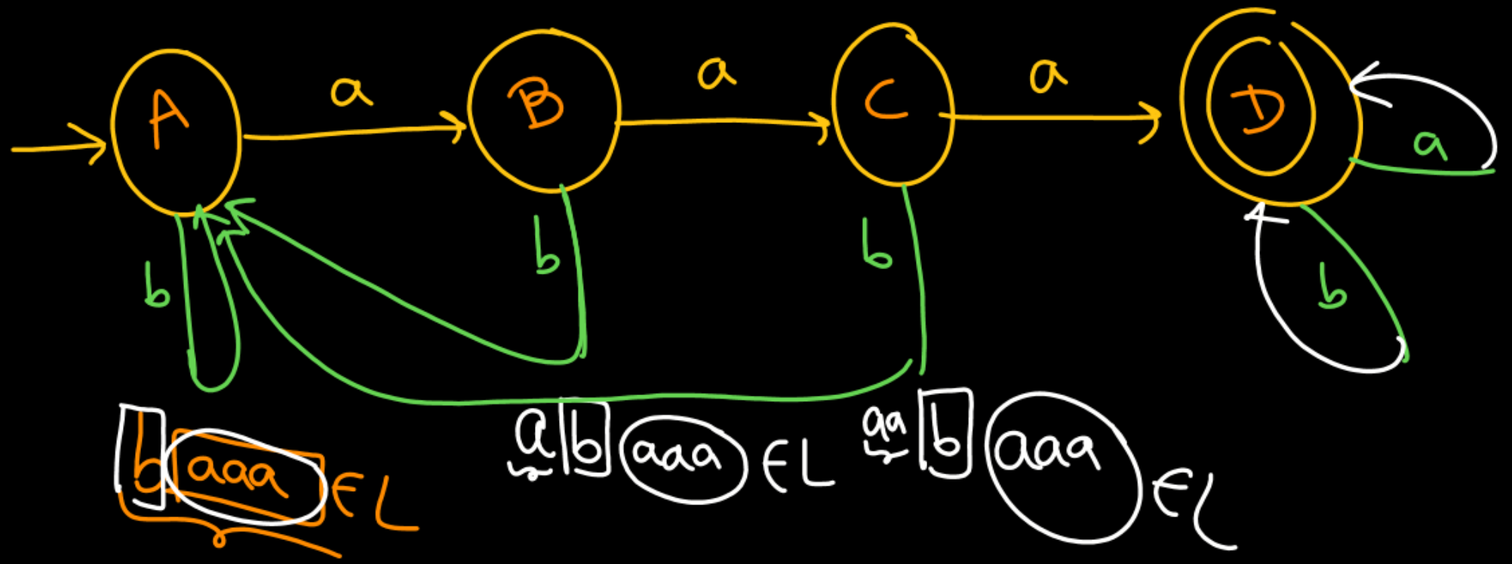

min = aa
A
B
C
a
a
a
b
b
A is waiting for 'aa'
B is waiting for 'a'
C is final
ε
Start
A
b
? goto state waiting for aa
b aa ∈ L
a
Start
A
B
b
? goto state waiting for a
ab aa ∈ L
Past
Present
Future
aa
Initial
A
C
a
? goto final
aaa ∈ L
Past
Present

⑨ $L = (a+b)^* aaa (a+b)^*$

Contain

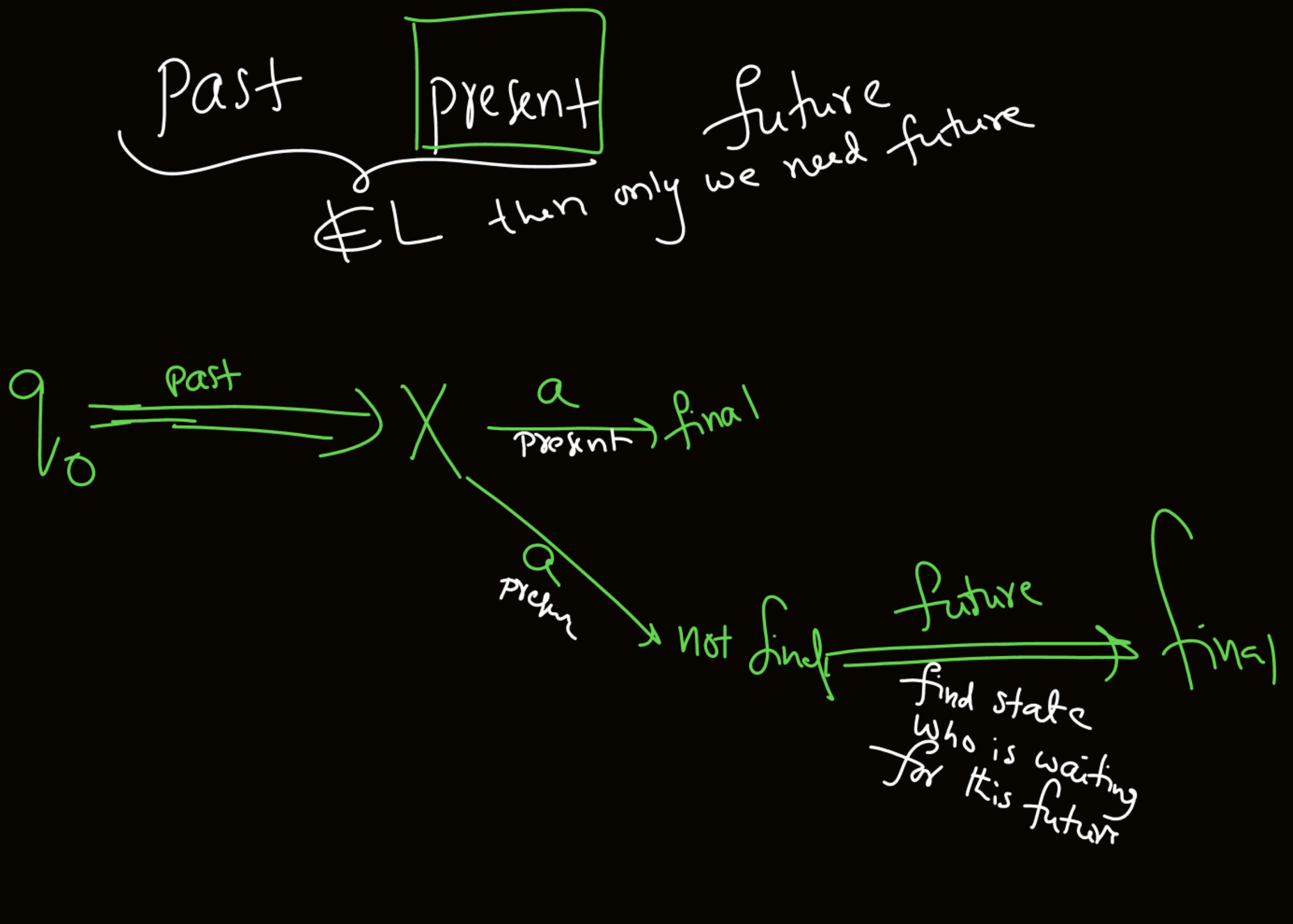
A : waiting for aaa
B : Waiting for aa
C : waiting for a
D : final

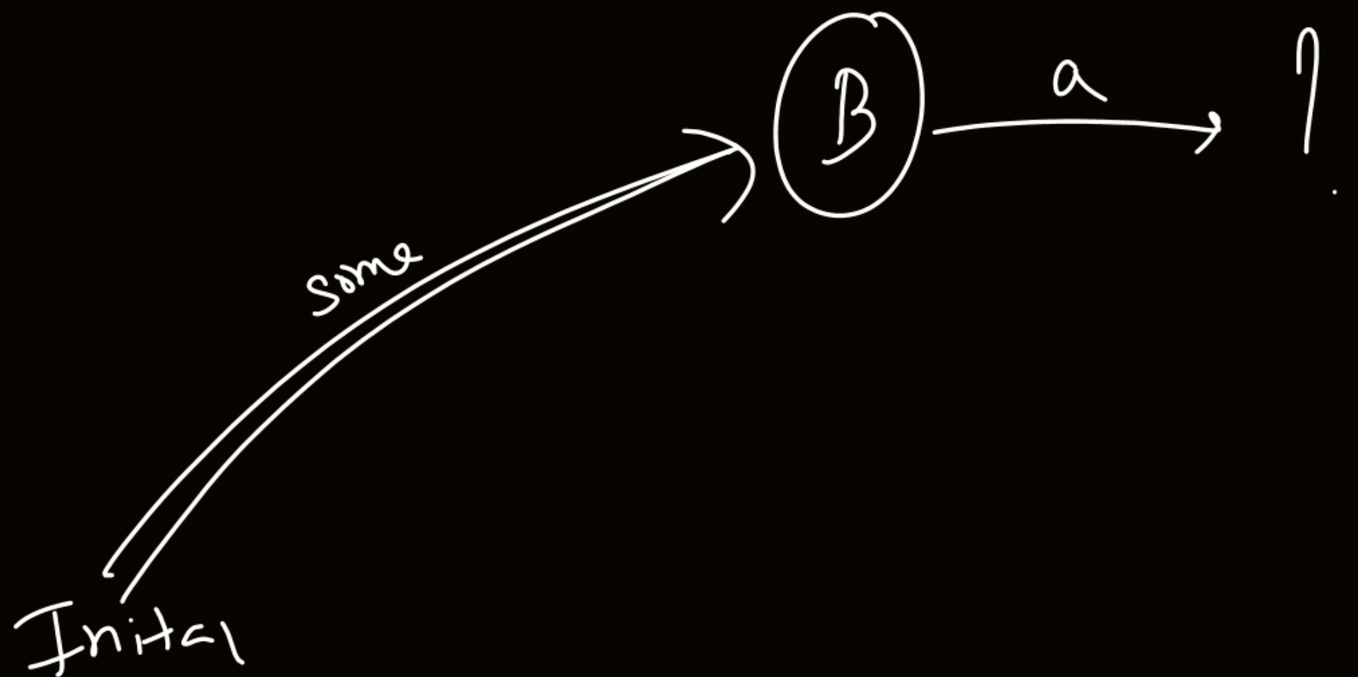
Min string = aaa
↳ length of min string + 1
= 4 states in min DFA



aaa a ∈ L

aaa b ∈ L


Past
present
future
∉L then only we need future
q_0
past
X
a
present
final
a
present
not final
future
final
find state
who is waiting
for this future


B
a
?
Some
Initial

(10) $L = (a+b)^* aba (a+b)^*$

H.W.

$= \{w \mid w \in \{a,b\}^*, w \text{ contains } aba\}$

4 States

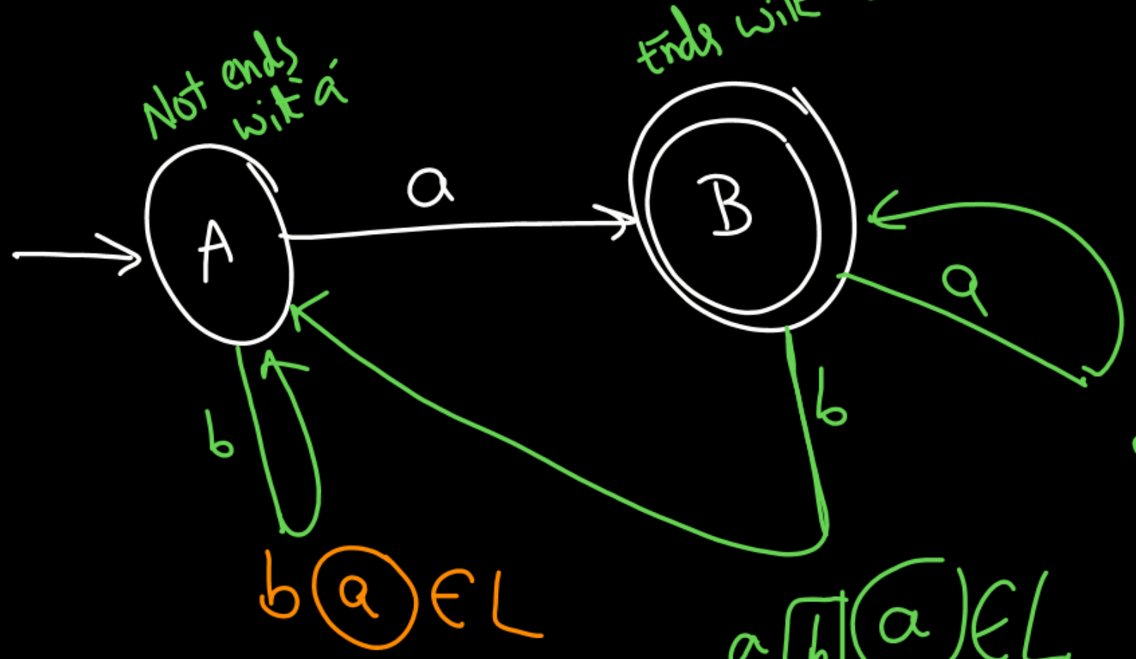
(11) $L = (a+b)^* a$

$= \{ w \mid w \in \{a,b\}^*, w \text{ ends with 'a'} \}$

A: waiting for 'a'

B: final

min = a



$b(a) \in L$

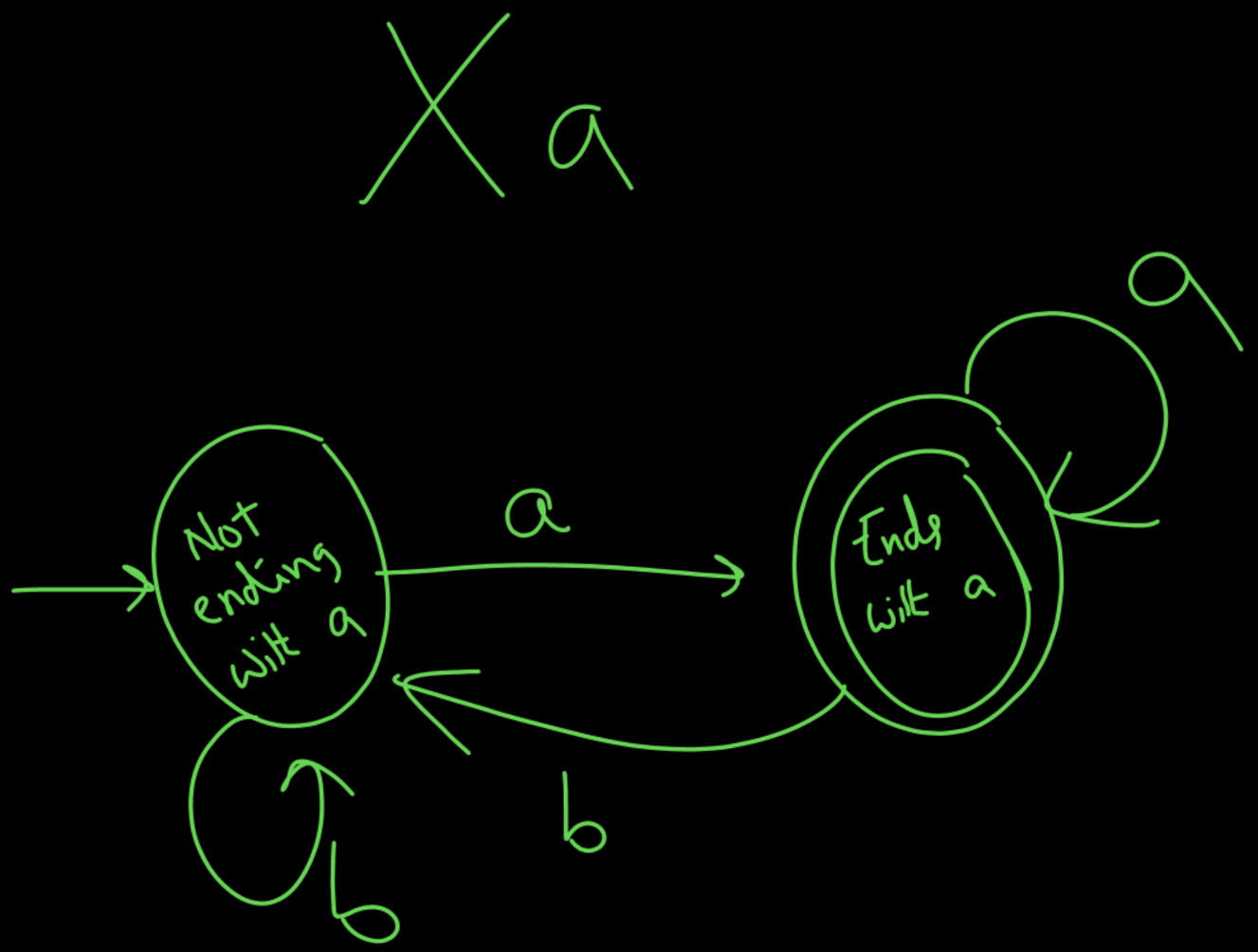
$a(b)(a) \in L$

$a(a) \in L$

Do you need dead state ?

for it is problem

NO


Not ending with a
a
Ends with a
a
b
b

(12) H.W.

$L = \underline{(a+b)^* aaa}$

→ min = aaa

→ 3 + 1 = 4 states

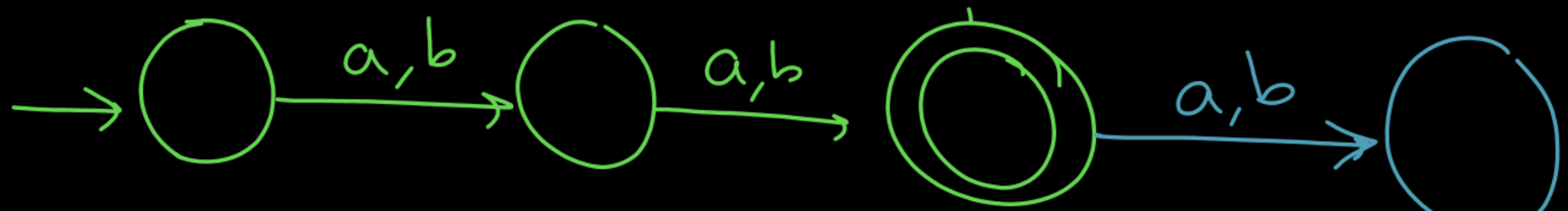
(13) H.W.

$L = \underline{(a+b)^* aba}$

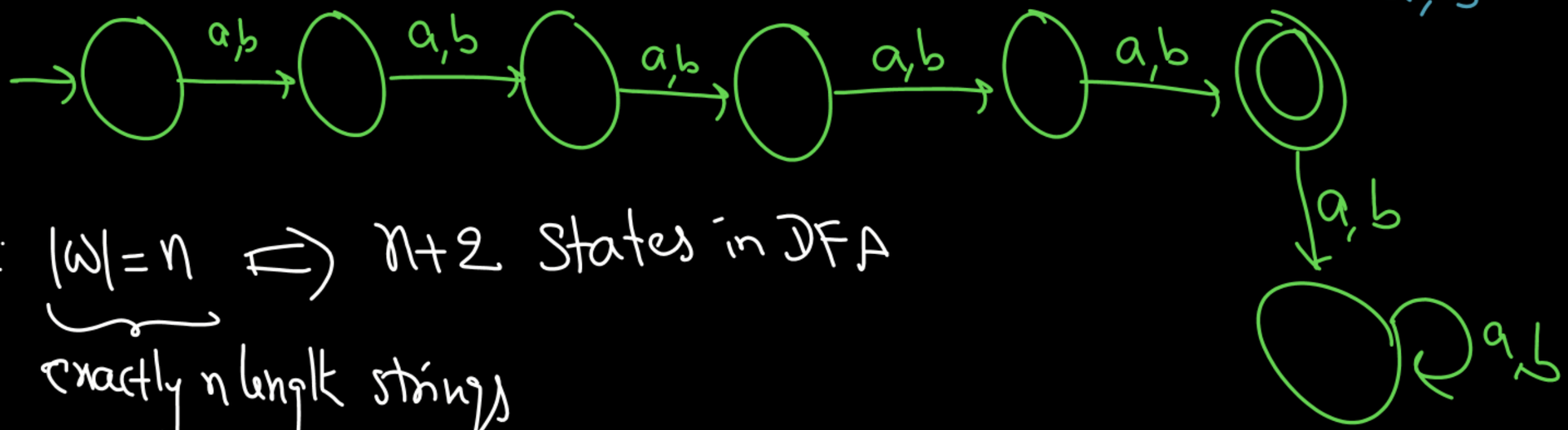
→ 4 States

Model-III : Length problems [focus on length]

(14) $L = \{ w \mid w \in \{a,b\}^*, |w| = 2 \}$

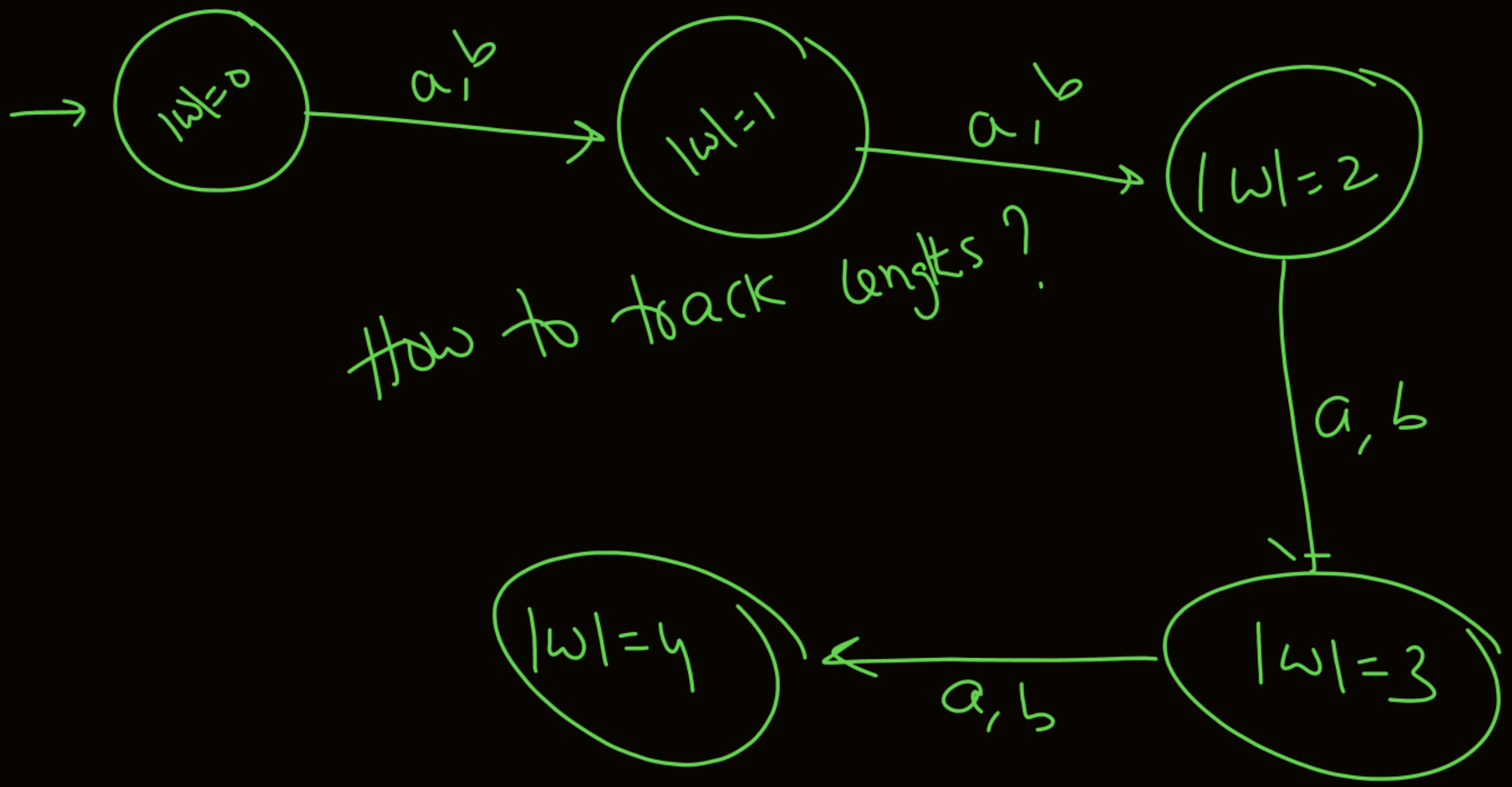


(15) $L = \{ w \mid w \in \{a,b\}^*, |w| = 5 \}$



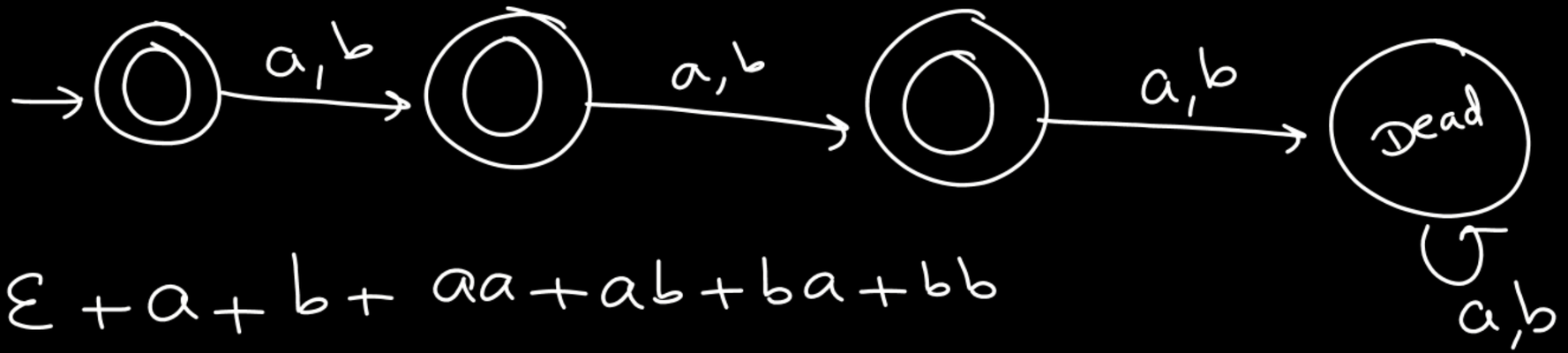
Note: $|w| = n \Rightarrow n+2$ States in DFA

exactly n length strings



How to track lengths?

(16) $\{w \mid w \in \{a,b\}^*, \ |w| \leq 2\}$



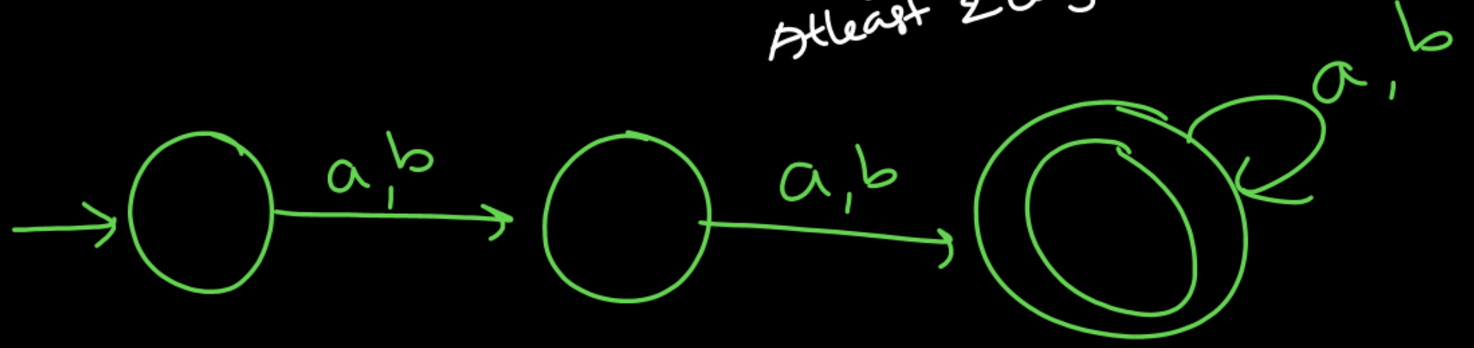
$= \varepsilon + a + b + aa + ab + ba + bb$

$= (\varepsilon + a + b)^2$

Note: If $|w| \leq n$ then $(n+2)$ states in DFA

(17) $\{w \mid w \in \{a,b\}^*, |w| \geq 2\}$

Atleast 2 length



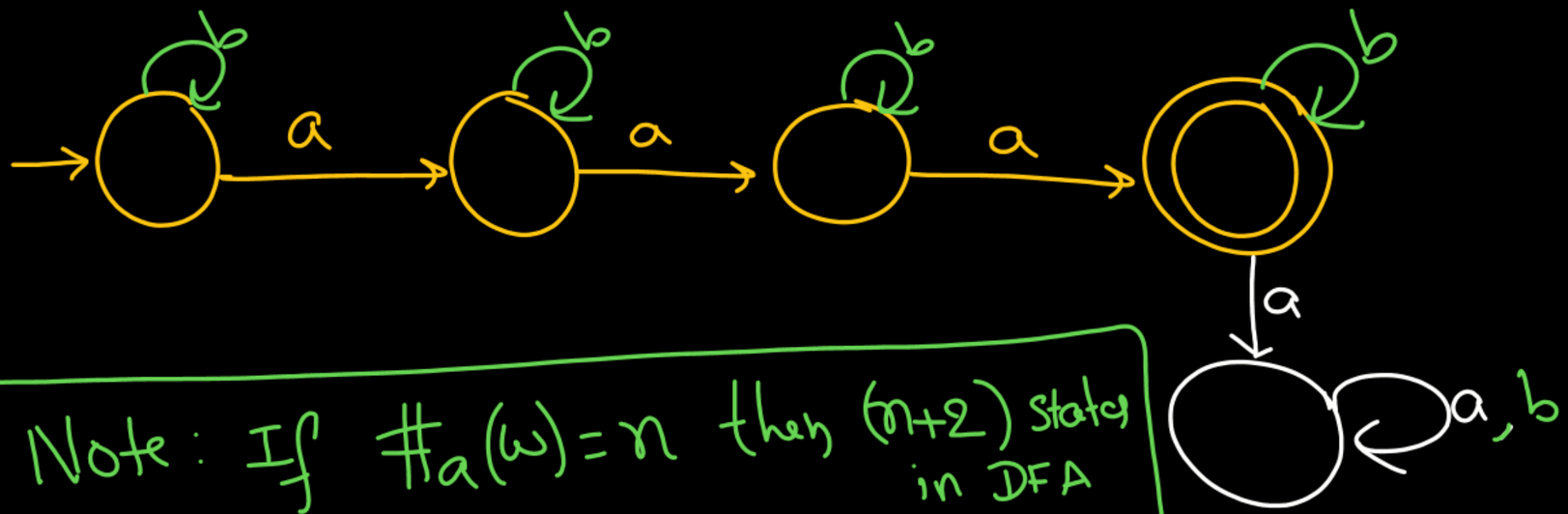
Note: If $|w| \geq n$ then $(n+1)$ states

Start	end
Exactly	Contain
Atmost	Atleast
Dead required	No dead state

Model-IV [No. of symbols] (focus on symbol tracking)

(18) $L = \{ w \mid w \in \{a,b\}^*, \; n_a(w) = 3 \}$

#a's = 3
#b's = any

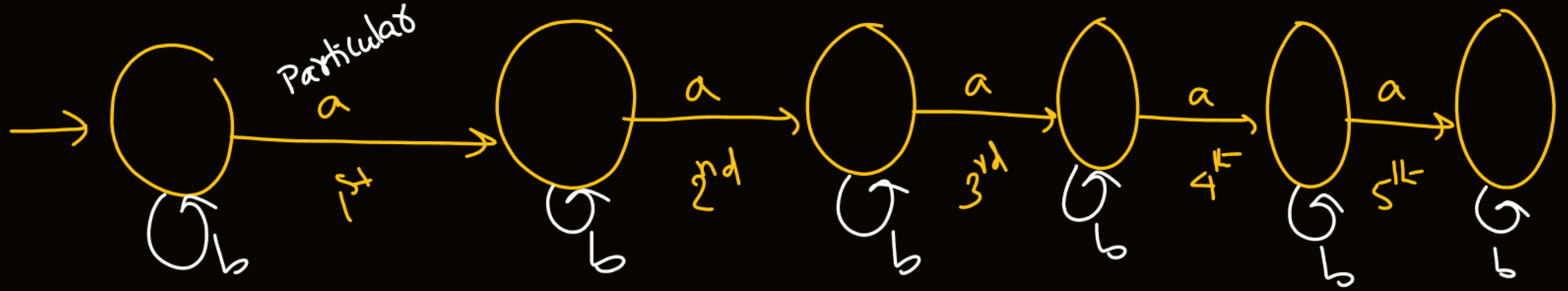


Note: If $\#_a(w) = n$ then $(n+2)$ states in DFA

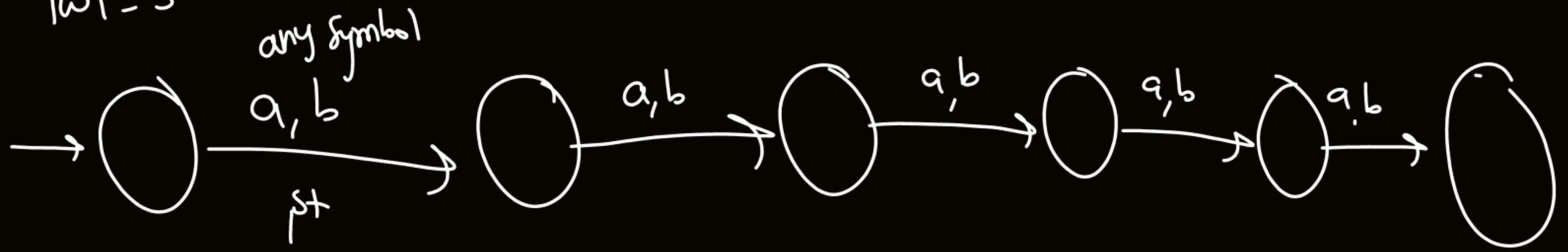
$b^* a b^* a b^* a b^*$

No. of a's = 5

How to track?

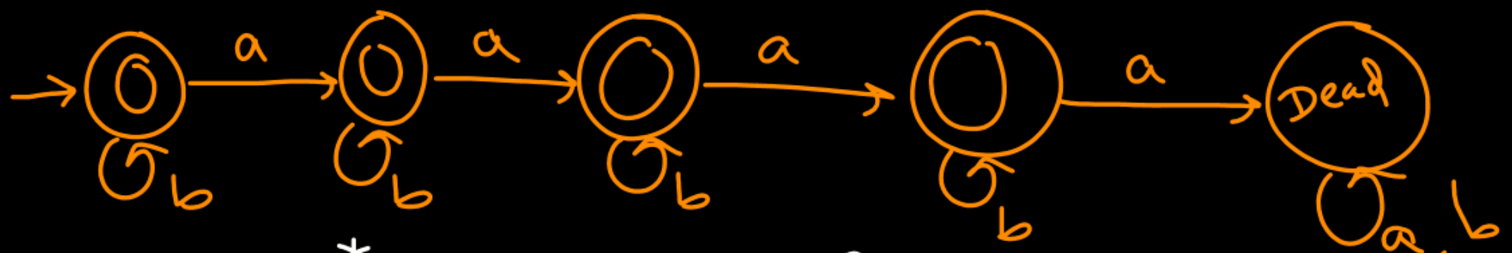


$|w| = 5$



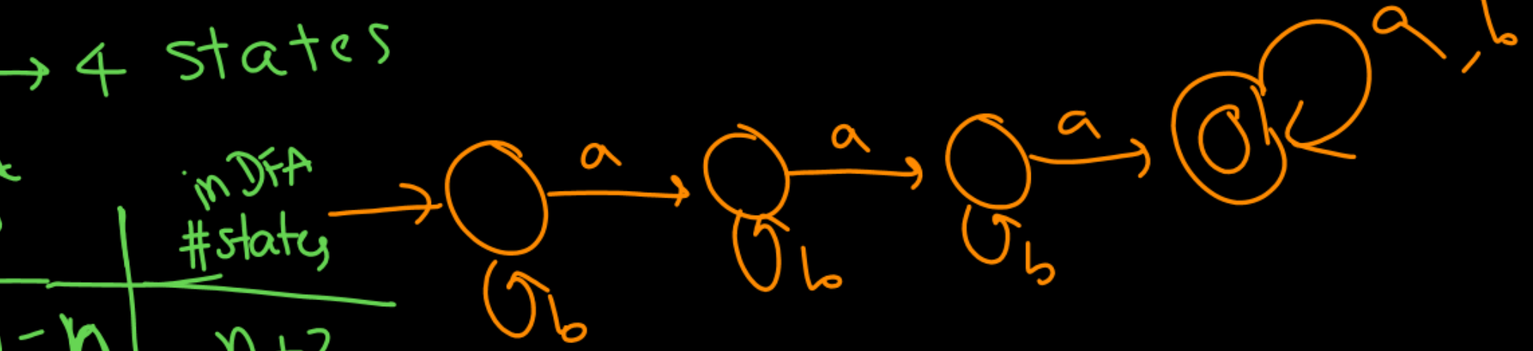
H.W. (19) $L = \{w \mid w \in \{a,b\}^*, \#a(w) \leq 3\}$

→ 5 states



H.W. (20) $L = \{w \mid w \in \{a,b\}^*, \#a(w) \geq 3\}$

→ 4 states



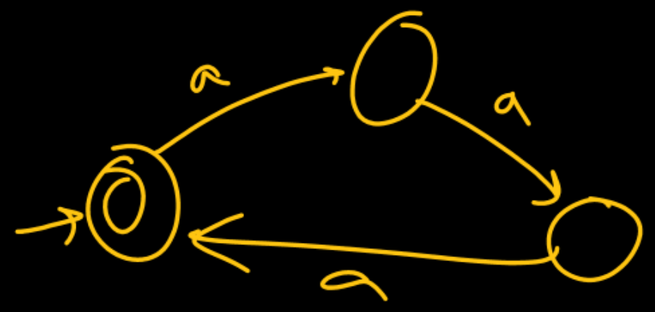
$w \in \{a,b\}^*$	in DFA # states
$\#a(w) = n$	$n+2$
$\#a(w) \leq n$	$n+2$
$\#a(w) \geq n$	$n+1$

DFA

Model-V [Language over 1 symbol] (focuses on Arithmetic progression)

$\Sigma = \{a\}$

to form a cycle



$a^{3n} \Rightarrow \varepsilon, a^3, a^6, a^9, \ldots$

$a^{2n} \Rightarrow \varepsilon, a^2, a^4, a^6, \ldots$

(21) $L = (aa)^*$

(22) $L = (aaa)^*$

(23) $L = (aa + aaa)^*$

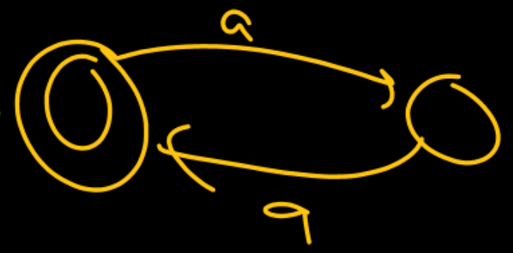
(24) $L = (aaa + aaaa)^*$

(25) $L = (aa + aaaaa)^*$

(26) $L = (aaa + aaaaa)^*$

(27) $L = \{a^{2n+3} \mid n \geq 0\}$

(28) $L = \{a^{3n+1} \mid n \geq 0\}$



(29) $L = \{a^{Kn+C} \mid n \geq 0,\ K \text{ is constant},\ C \text{ is constant}\}$

↳ Create a formula

(30) $\{a^{\text{prime}}\}^*$

Summary

	no. of states in DFA
start	$\lvert min\ str \rvert + 2$
end	$\lvert min\ str \rvert + 1$
Contain	$\lvert min\ str \rvert + 1$
$\lvert w \rvert = n$	$n+2$
$\lvert w \rvert \leq n$	$n+2$
$\lvert w \rvert \geq n$	$n+1$
$\#_a(w) = n$	$n+2$
$\#_a(w) \leq n$	$n+2$
$\#_a(w) \geq n$	$n+1$

Thank you