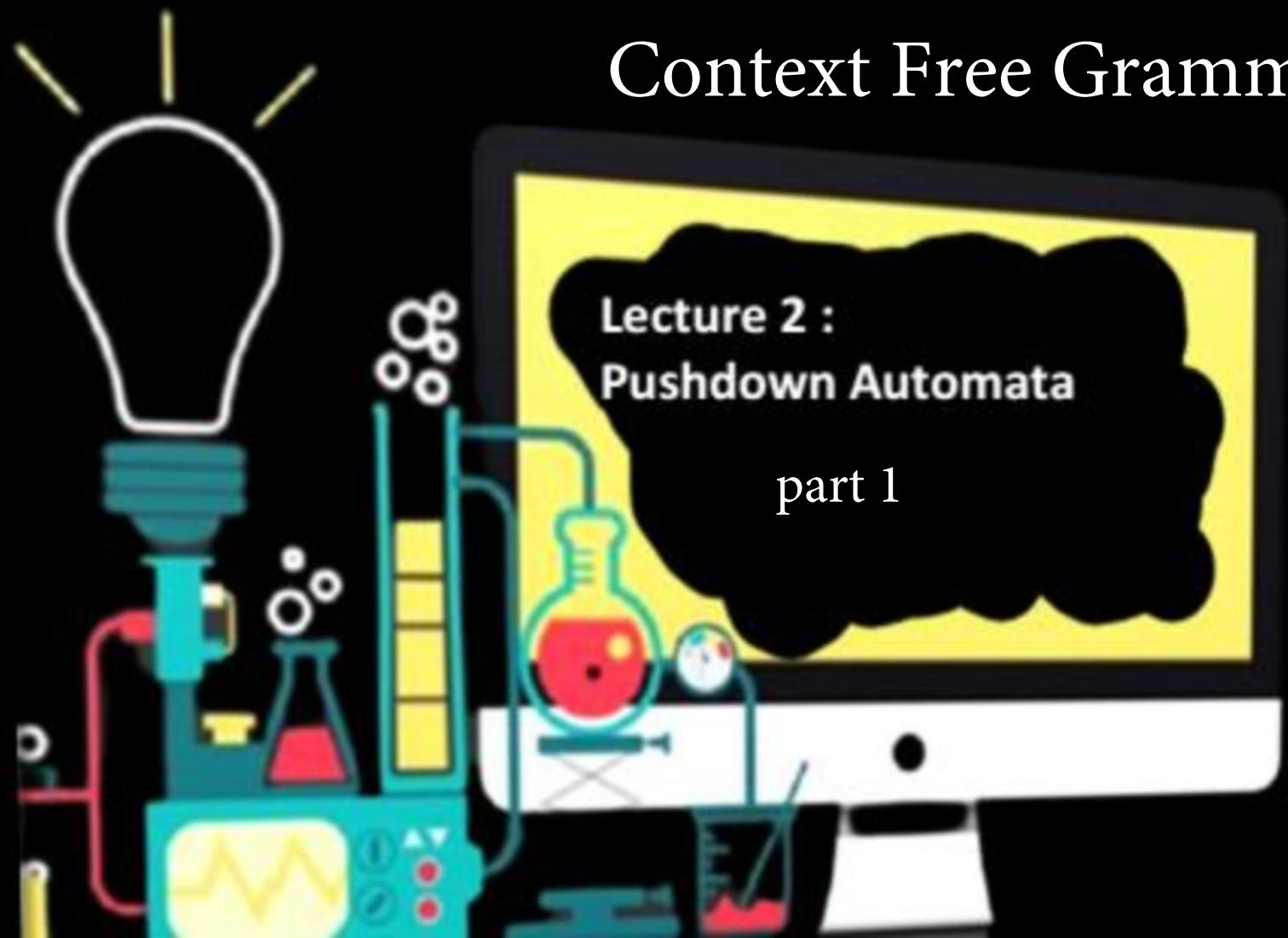


CS & IT Engineering

Context Free Grammar



Deva sir

Topics Covered in Previous Session:

↳ CFG ✓

- CFG Vs CFL
- CNF & GNF
- Simplifications of CFG

Topics to be covered:

↳ Push Down Automata

- ① What is PDA?
- ② PDA Configuration
- ③ PDA Acceptance mechanism
- ④ PDA Vs DPDA
- ⑤ PDA Construction

Pushdown Automata

- Accepts CFL
- also called Non-deterministic PDA
(NPDA) (NDPDA) (PDA)
- PDA \equiv FA + 1 stack

Expressive power:

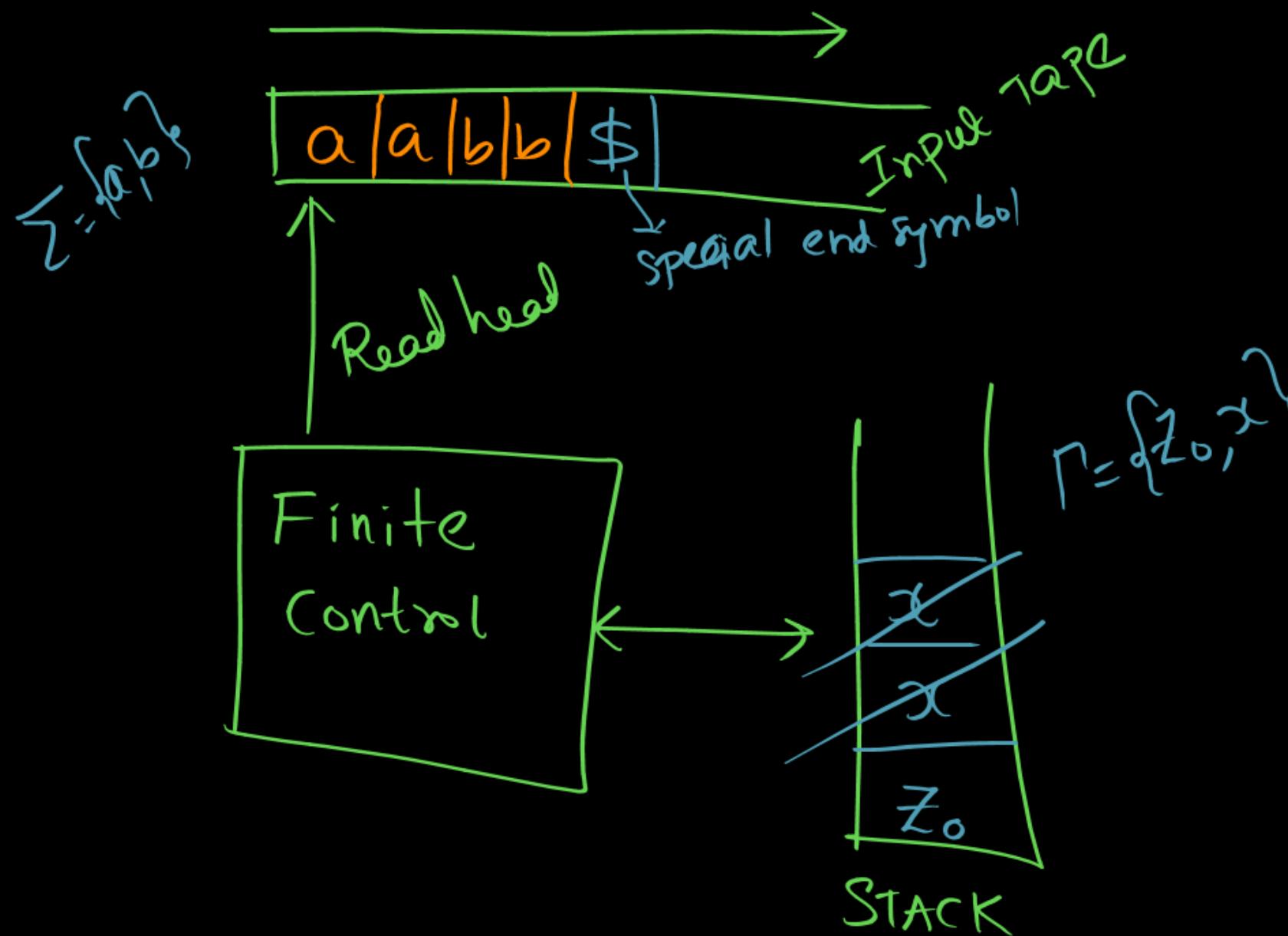
DPDA < PDA

↓
Accepts
CFLs

A(^{NP}DA)
D(CFLs)

Set of DCFLs \subset Set of all CFLs

DPDA < PDA



$$FA = (Q, \Sigma, \delta, q_0, F)$$

$$PDA = (Q, \Sigma, \delta, q_0, F, z_0, \Gamma)$$

(\perp) Initial top of stack(\perp)
 Bottom of stack symbol

Set of stack Symbols
 (Stack Alphabet)

Transition Function (δ)

For DPDA

$$\delta : Q \times \Sigma \times \Gamma \rightarrow Q \times \Gamma^*$$

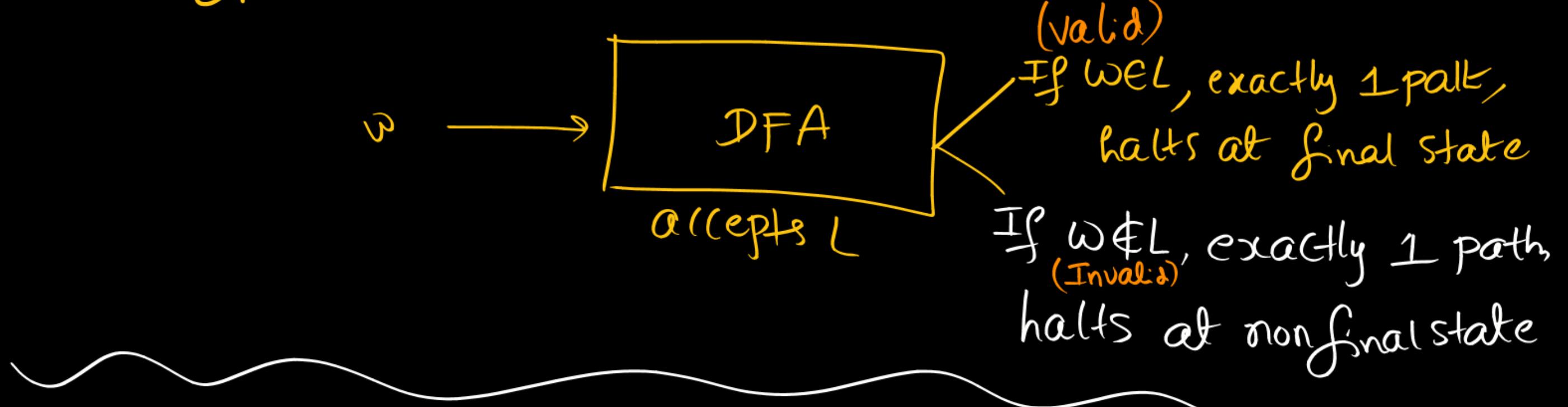
$$\delta(A, a, \alpha) = (B, [\alpha x])$$

Present State input symbol current top of stack symbol Next state operation

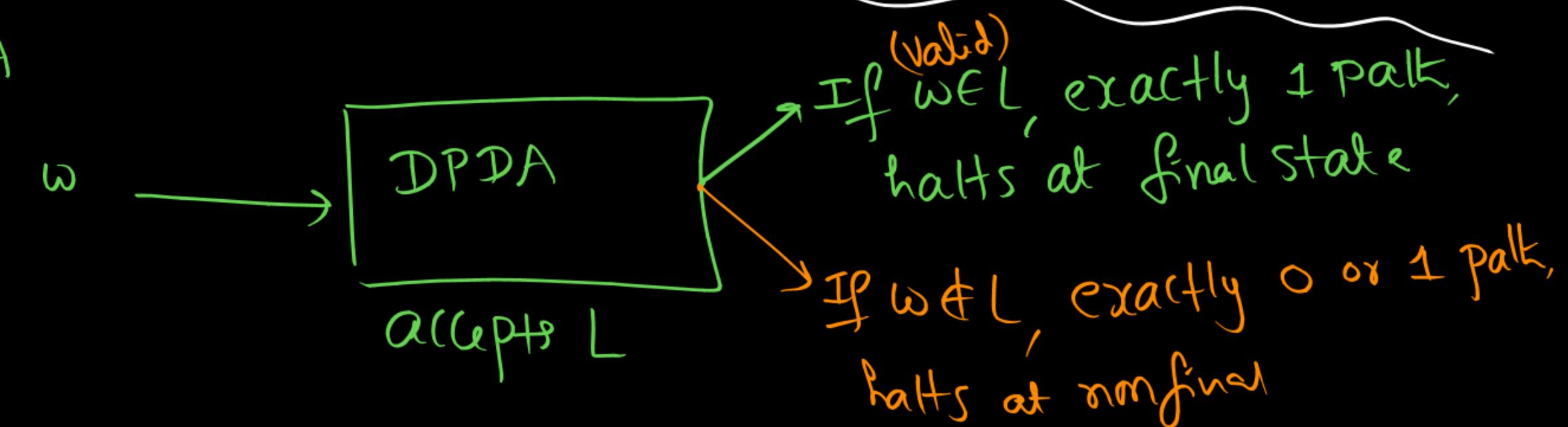
For PDA

$$\delta : Q \times \Sigma \times \Gamma^* \rightarrow 2^{Q \times \Gamma^*}$$

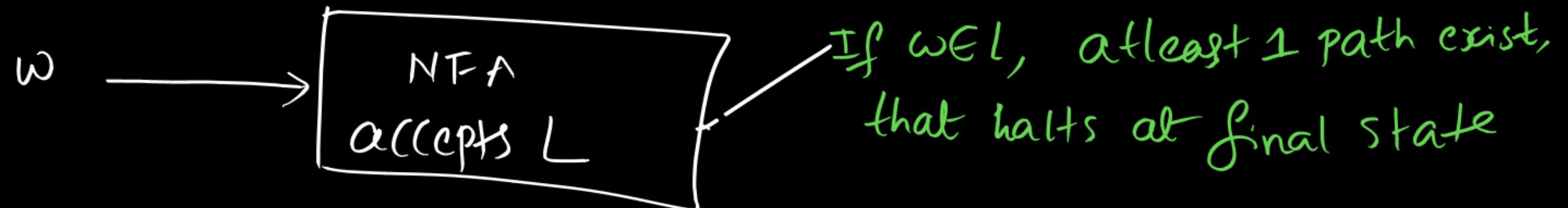
DFA



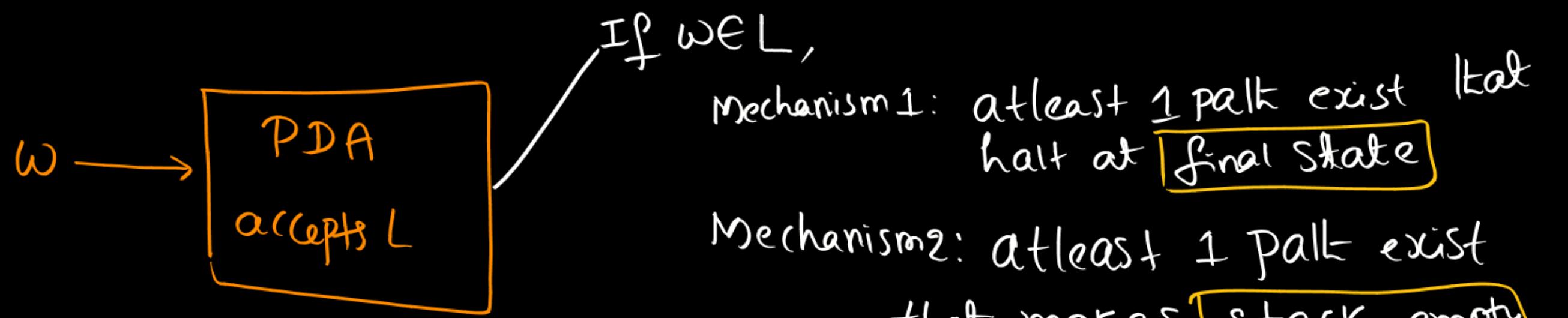
DPDA



NFA:



PDA (NDPDA)(NPDA)



Mechanism 3: at least 1 path exist that makes stack empty and halts at final

PDA Acceptance Mechanisms:

① Acceptance Using Final State



② Acceptance Using Empty Stack



③ Acceptance Using both Final state & Empty stack

L_1 = Set of all languages accepted by PDA with final state
acceptance mechanism

L_2 = " " " " " with Empty stack mech.

$L_1 = L_2 = \text{Set of all CFLs}$

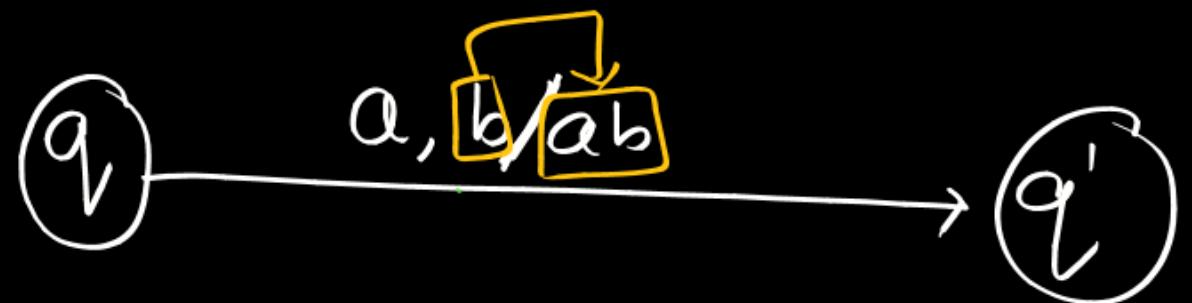
\mathcal{DPDA}

Acceptance $\underset{-}{\approx}$ Acceptance using bolt Final state & Empty stack

using Final state

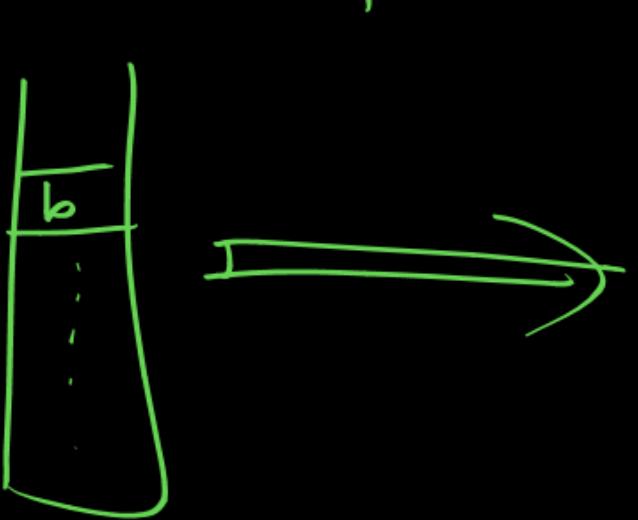
Note: DPDA Acceptance using Empty stack $<$ DPDA

Pushdown Automata

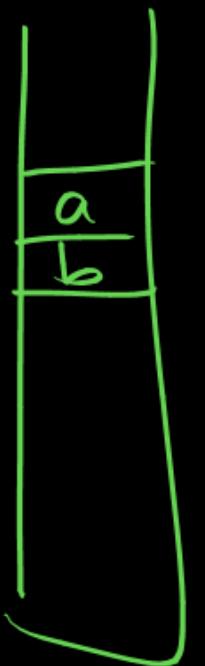


$\frac{a}{\uparrow}$

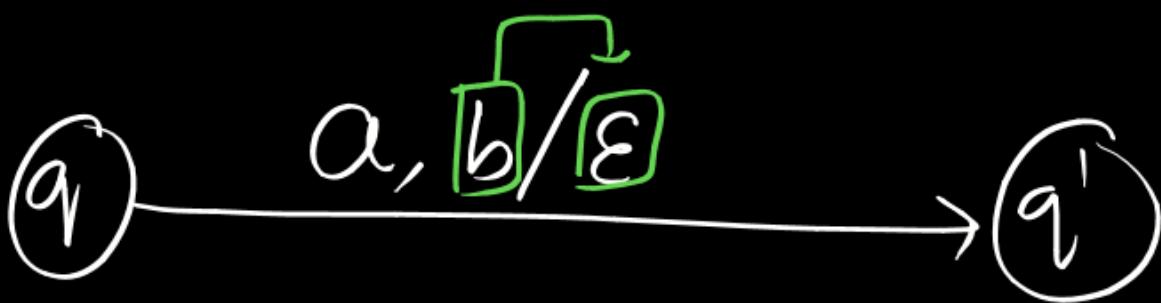
PUSH
 $I/P = a$
 $tos = b$
 \Downarrow push a'



b / a b
Push a



Pushdown Automata



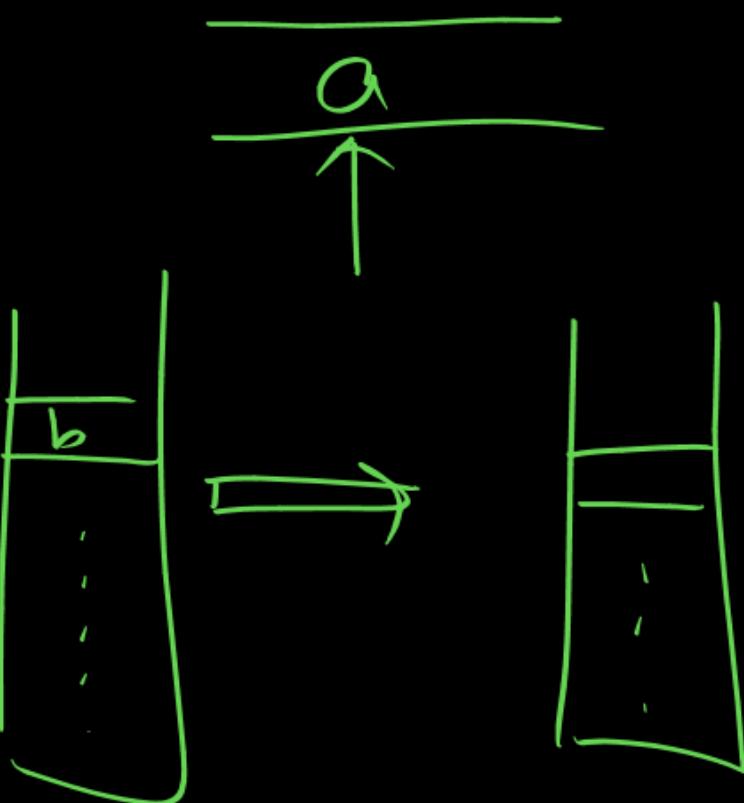
POP

$I/k = a$

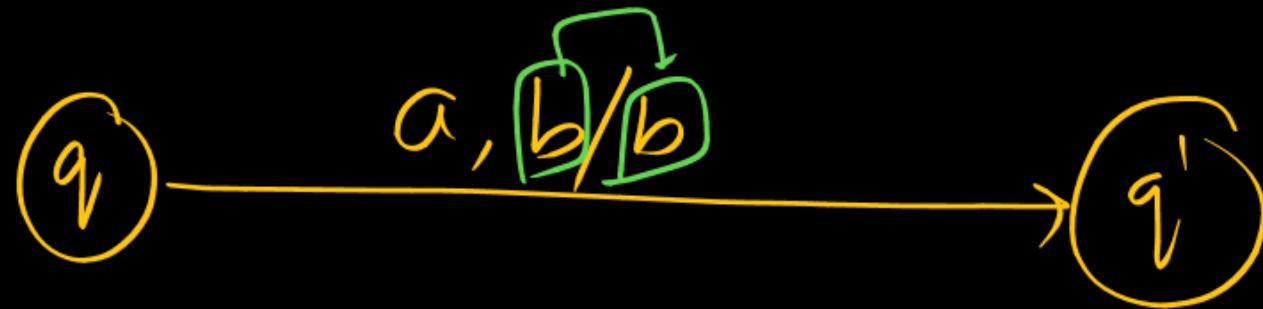
$\text{tos} = b$

\Downarrow

POP



Pushdown Automata

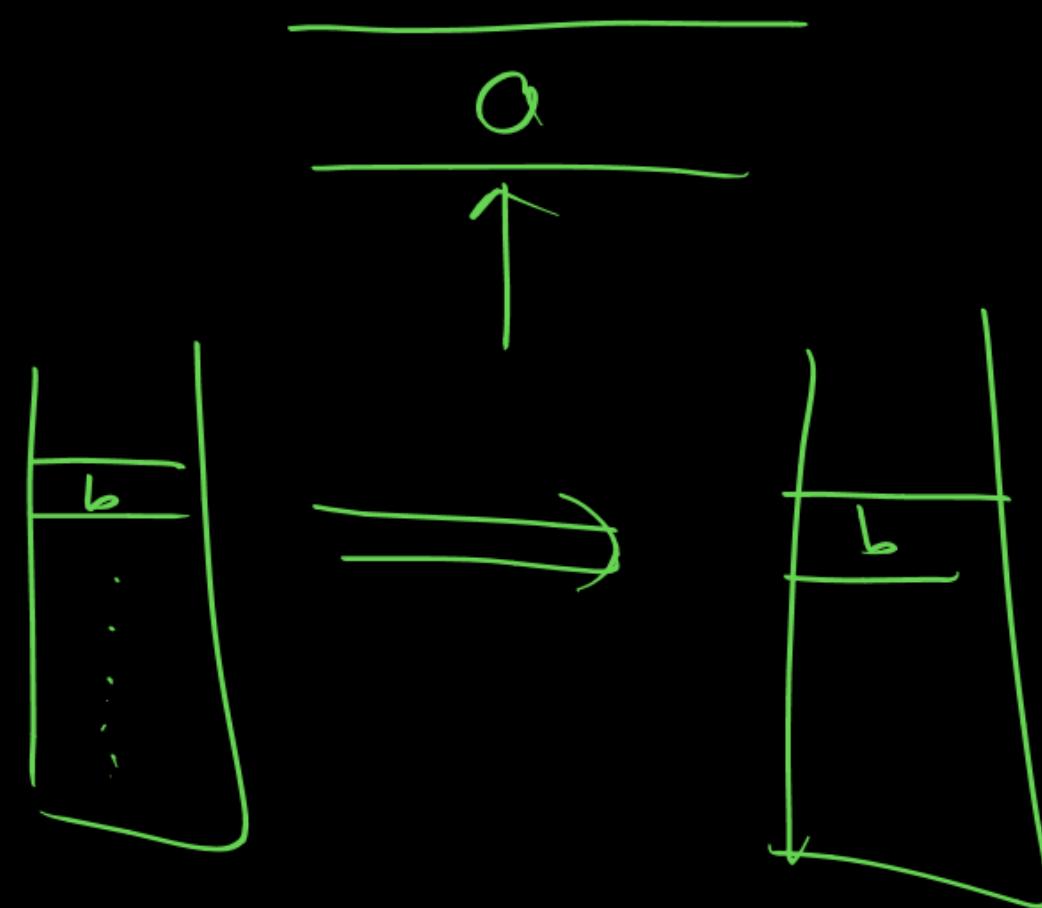


No operation

$I|_P = a$

$tos = b$

No operation



Pushdown Automata



tos / stack sequence
 $\sqcap \quad \sqcap^*$

$a/a \Rightarrow$ no operation

$a/\boxed{\epsilon} \Rightarrow a \text{ is popped}$

$a/\boxed{aa} \Rightarrow a \text{ is pushed}$

$a/\boxed{\underline{aaa}} \Rightarrow 2 a's \text{ pushed}$

Pushdown Automata

$$\textcircled{1} \quad \delta(A, \underline{\Sigma}, \underline{\epsilon}) = (B, \epsilon)$$

$$\textcircled{2} \quad \delta(A, \underline{\Sigma}, a) = (B, \epsilon)$$

$$\textcircled{3} \quad \delta(A, \underline{a}, a) = (B, \epsilon)$$

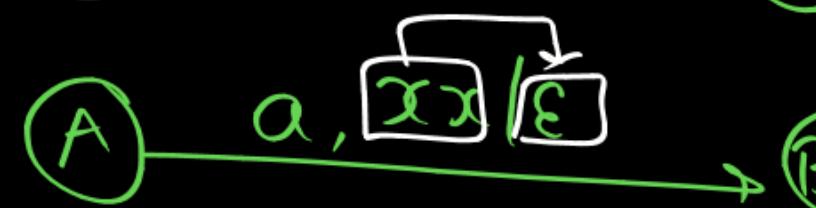
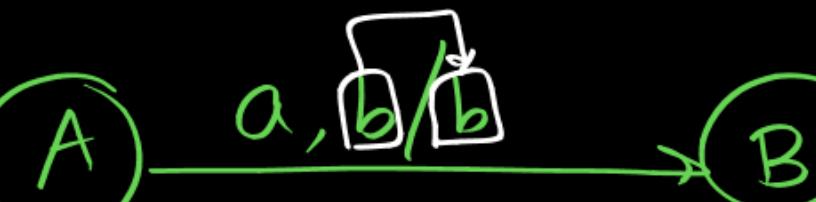
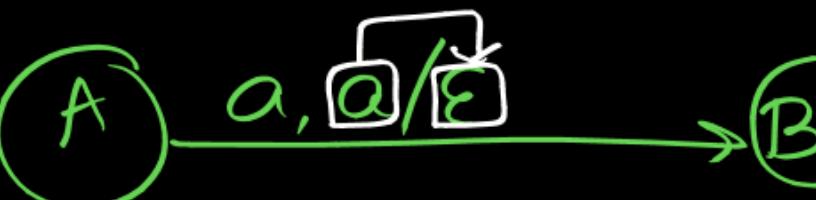
$$\textcircled{4} \quad \delta(A, \underline{a}, b) = (B, b)$$

$$\textcircled{5} \quad \delta(A, \underline{a}, b) = (B, a \dot{a} b)$$

$$\textcircled{6} \quad \delta(A, \underline{a}, xx) = (B, \epsilon)$$

$$\textcircled{7} \quad \delta(A, \underline{\epsilon}, xx) = (B, x)$$

$$\textcircled{8} \quad \delta(A, \underline{\epsilon}, x) = (B, x)$$



What happened n stack?

No operation

Pop 'a'

Pop 'a'

No operation

push 2a's

pop 2 x's

Pop x

No operation



Pushdown Automata

PUSH

$$\textcircled{1} \quad \delta(-, -, \boxed{x}) = (-, \boxed{xx})$$

push x

$$\textcircled{2} \quad \delta(p, a, \boxed{X}) = (q, \boxed{aaX})$$

push aa

$$\textcircled{3} \quad \delta(p, \epsilon, \boxed{X}) = (q, \boxed{aX})$$

push a

$$\textcircled{4} \quad \delta(p, \epsilon, \boxed{\epsilon}) = (q, \boxed{a})$$

push a

$$\textcircled{5} \quad \delta(p, a, \boxed{b}) = (q, \boxed{ab})$$

push a

$$\textcircled{6} \quad \delta(p, a, \boxed{a}) = (q, \boxed{aaaa})$$

push 3a's

POP

$$\textcircled{1} \quad \delta(-, -, \boxed{x}) = (-, \boxed{\epsilon})$$

$$\textcircled{2} \quad \delta(p, a, \boxed{x}) = (q, \boxed{\epsilon})$$

$$\textcircled{3} \quad \delta(p, a, \boxed{xx}) = (q, \boxed{\epsilon})$$

$$\textcircled{4} \quad \delta(p, a, \boxed{xx}) = (q, \boxed{\epsilon})$$

$$\textcircled{5} \quad \delta(p, a, \boxed{xx}) = (q, \boxed{X})$$

$$\textcircled{6} \quad \delta(p, \epsilon, \boxed{xy}) = (q, \boxed{y})$$

No operation

$$\textcircled{1} \quad \delta(-, -, \boxed{x}) = (-, \boxed{x})$$

$$\textcircled{2} \quad \delta(p, \epsilon, \boxed{X}) = (q, \boxed{X})$$

$$\textcircled{3} \quad \delta(p, a, \boxed{xy}) = (q, \boxed{xy})$$

$$\textcircled{4} \quad \delta(p, \epsilon, \boxed{aaa}) = (q, \boxed{aaa})$$

$$\textcircled{5} \quad \delta(p, \epsilon, \boxed{\epsilon}) = (q, \boxed{\epsilon})$$

$$\textcircled{6} \quad \delta(p, a, \boxed{\epsilon}) = (q, \boxed{\epsilon})$$

Pushdown Automata

What i/p ?
Will a \Rightarrow b
Will ϵ

What tos ?

What operation ?

Push
Pop
No operation

- one symbol
- 2 symbols
- one symbol
- 2 symbols
- :

Any state change ?

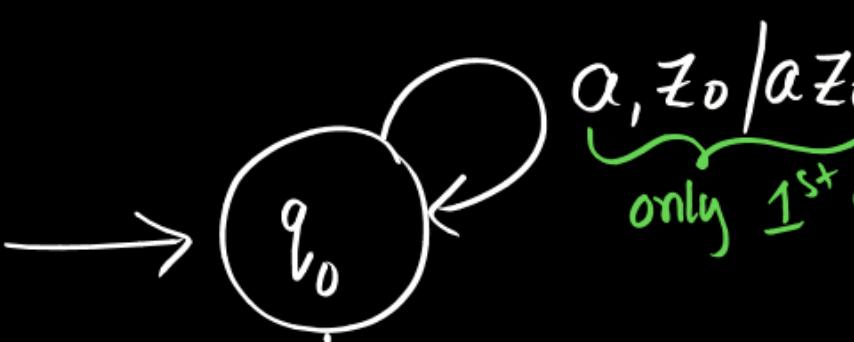
Yes

No
(self loop)

Pushdown Automata

P
W

①



$a, z_0/z_0$
only 1st a is pushed



$q_0 \xrightarrow{a, z_0/z_0} q_0 \xrightarrow{\epsilon, a} \text{no push}$



$q_0 \xrightarrow{a, z_0/z_0} q_0 \xrightarrow{a, a} \text{no transition}$

ϵ
 a
 aa
 aaa



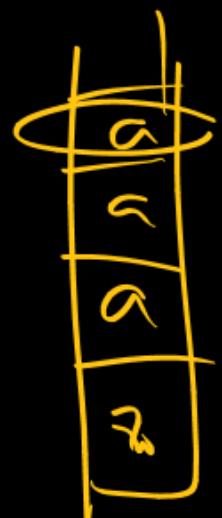
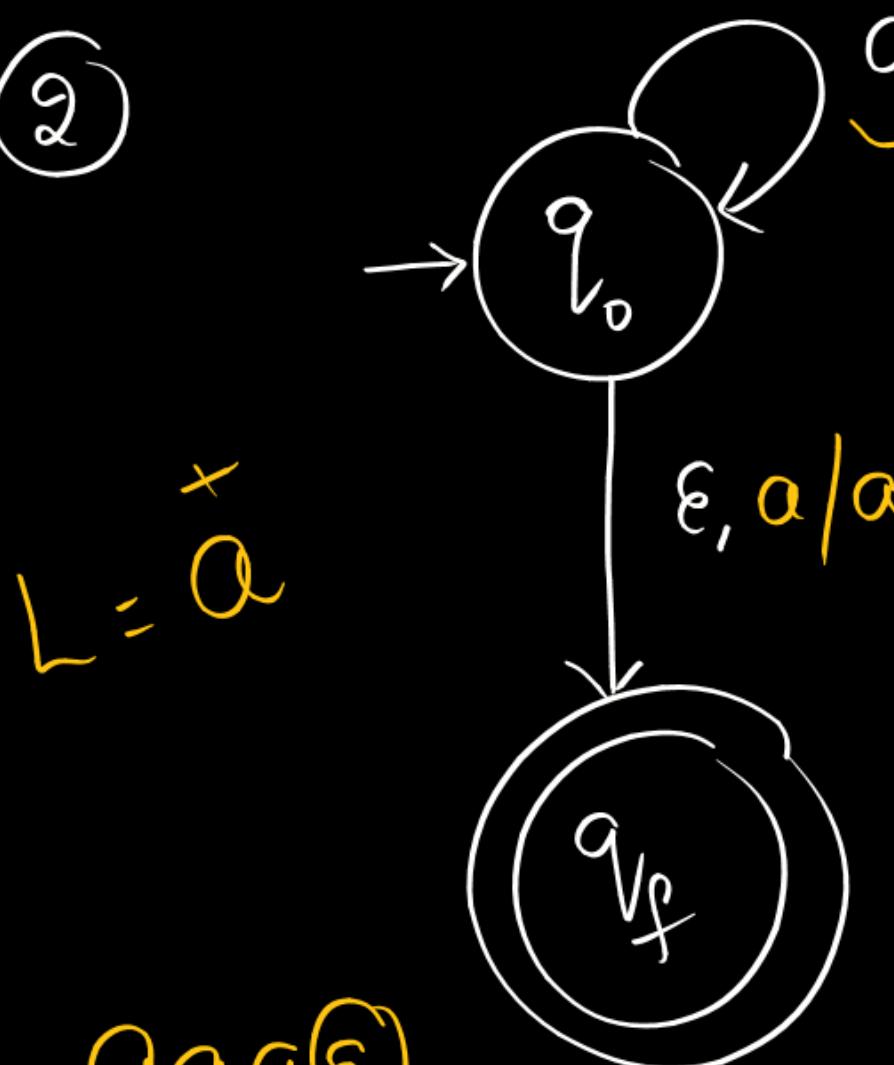
$L = a^*$

ϵ
 a
 aa
 \vdots

DFA or NFA

Pushdown Automata

②



$Qaaq(\epsilon)$

$q_0 \xrightarrow{\epsilon, a} \text{no pair}$

$a, \epsilon/a$
push 'a' for every 'a'

$a, \epsilon/\epsilon$

$a, a/\epsilon$
pop 'a'

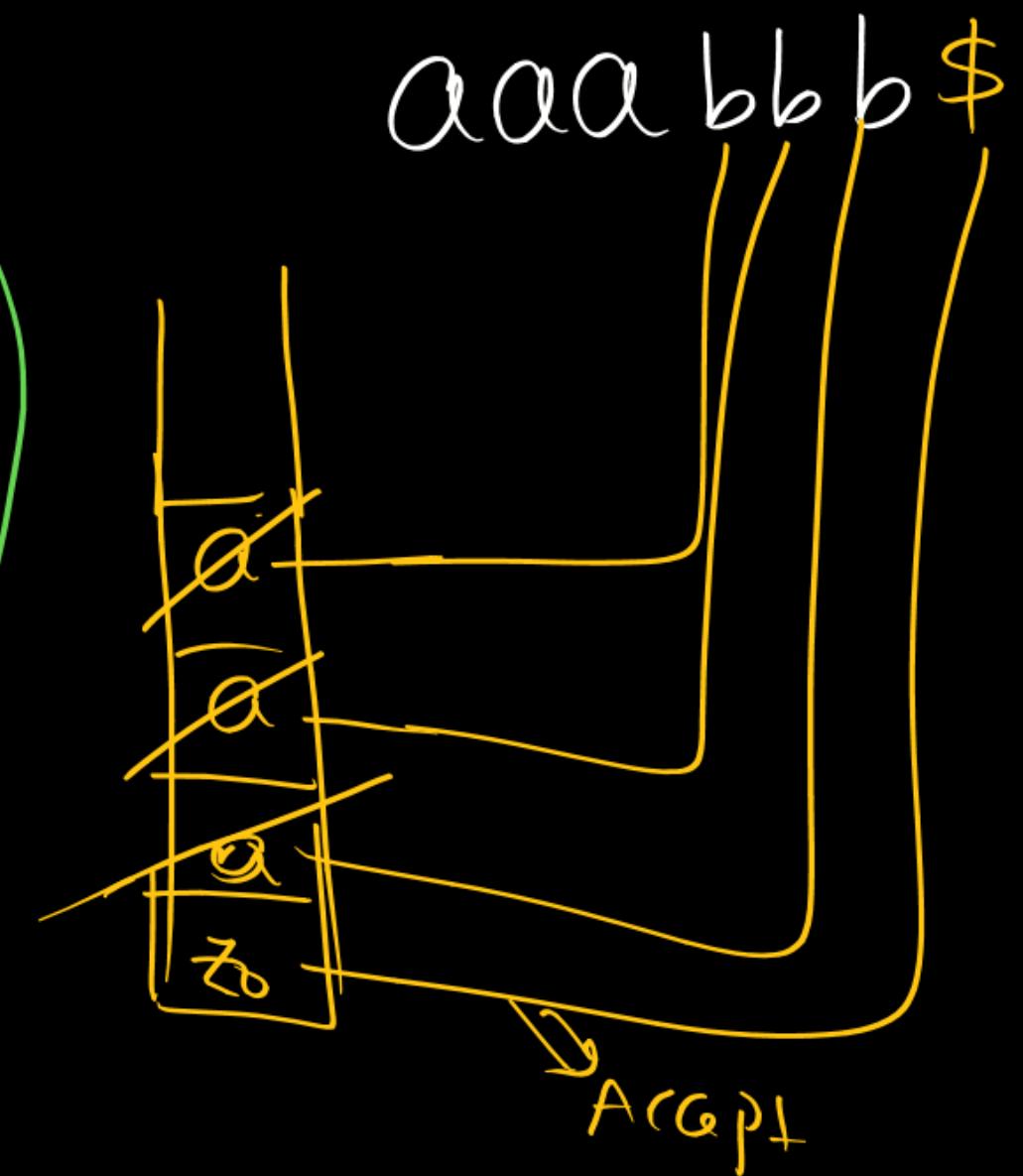
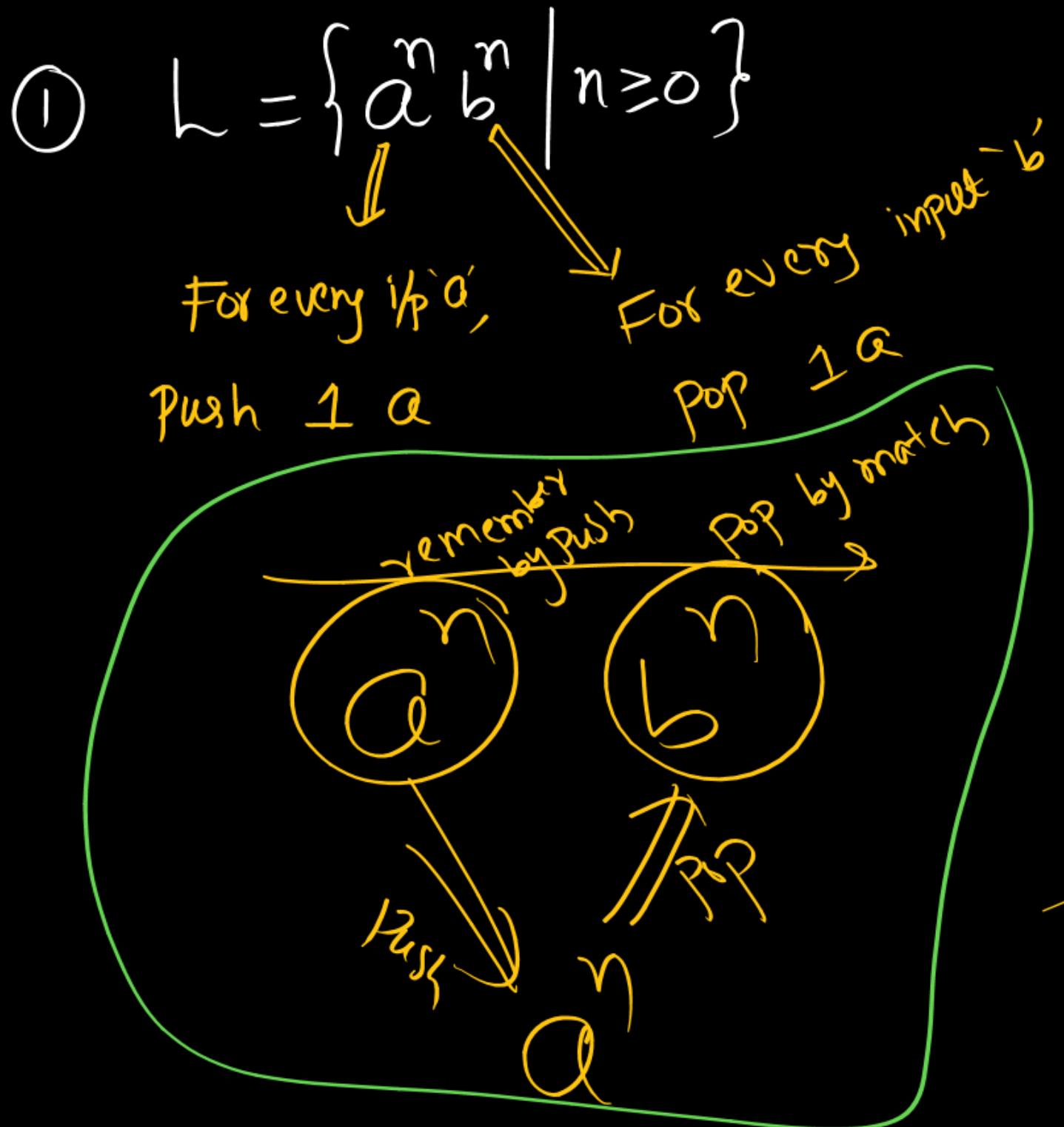
$\epsilon, a/a$

without reading input

$a, \epsilon/a$

- By reading input 'a'
- without looking at top of stack
- Push 'a' onto stack

Pushdown Automata



Homework

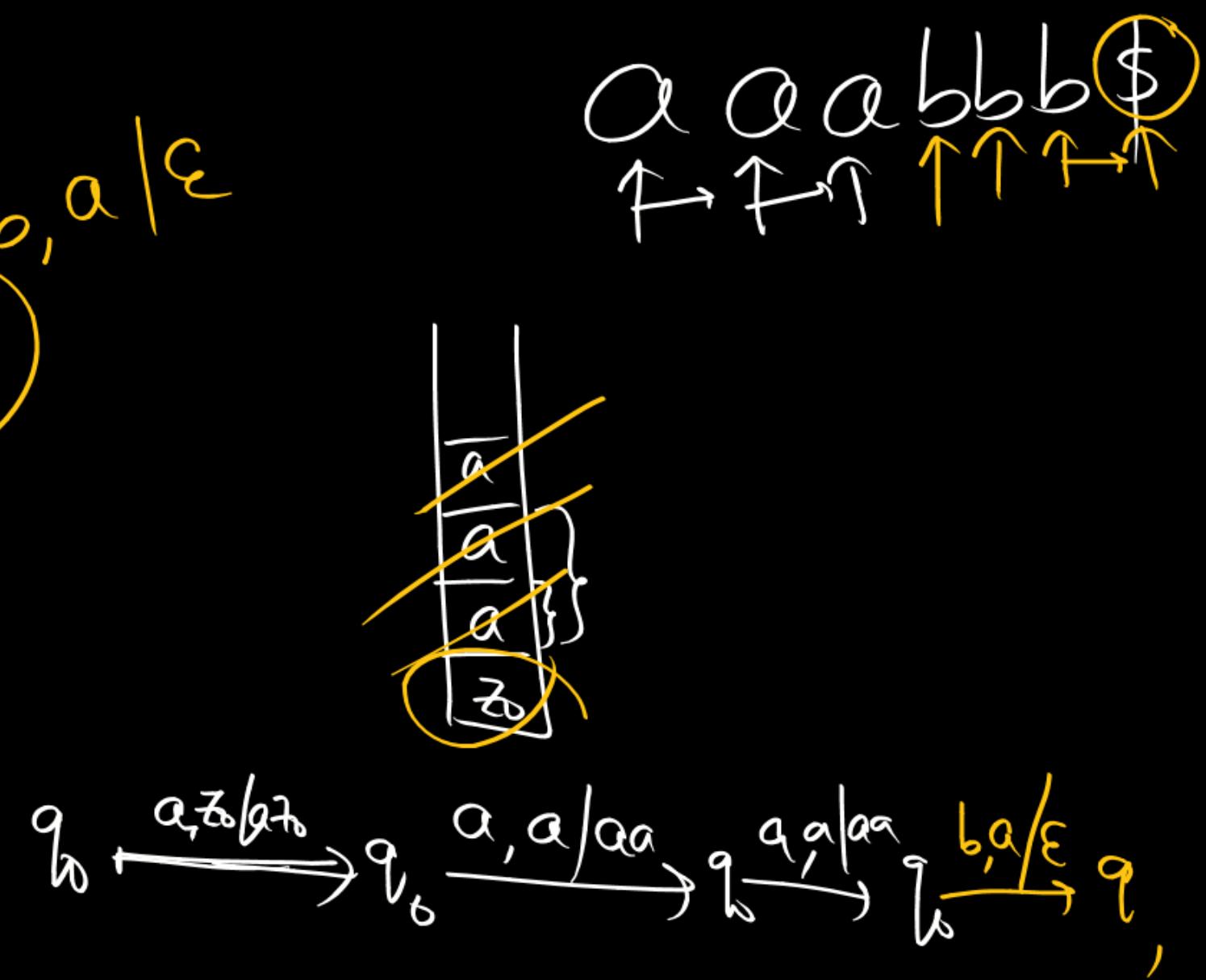
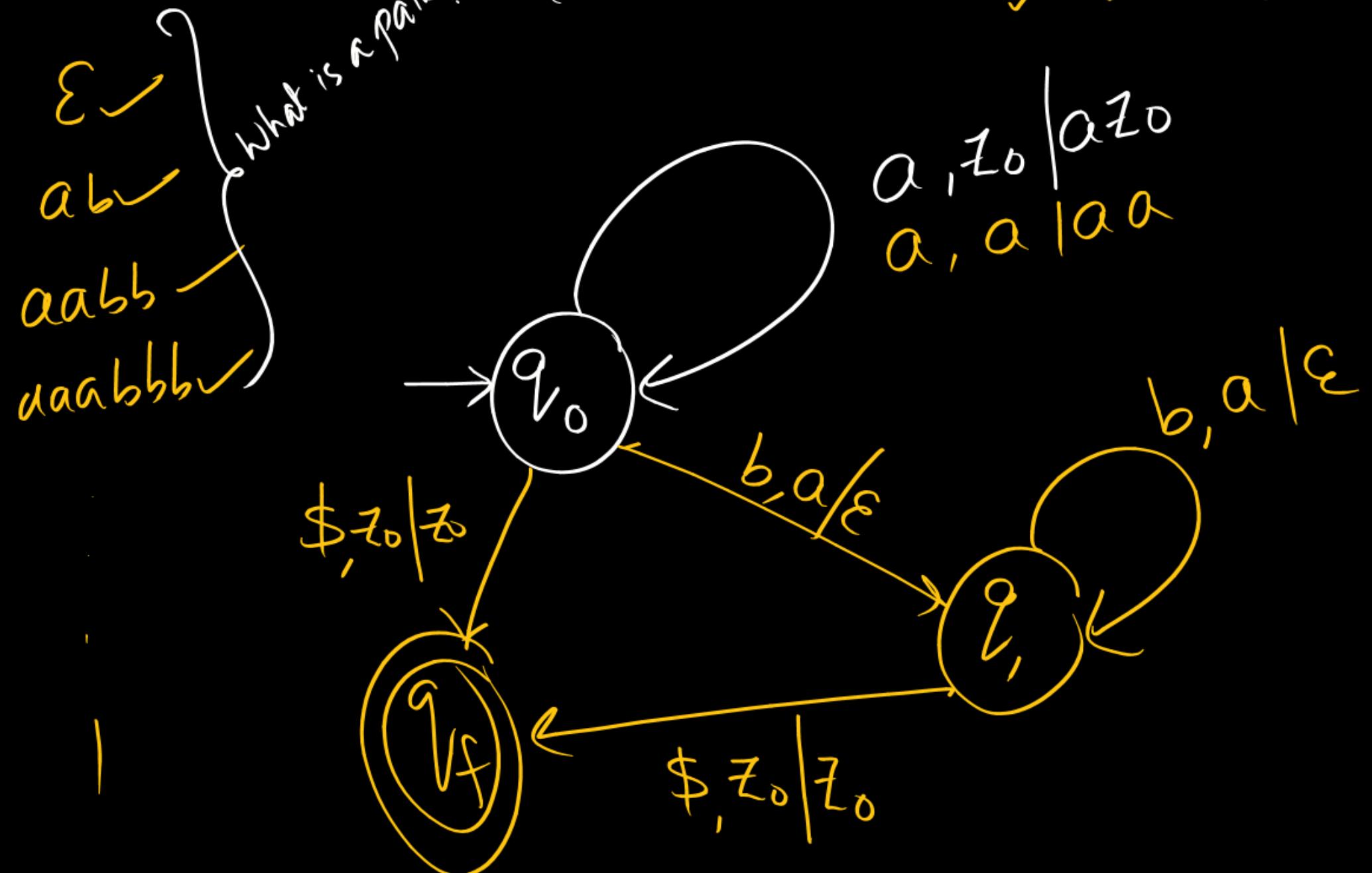
- ϵ
- ab
- aabb
- aaabb

wrote a note

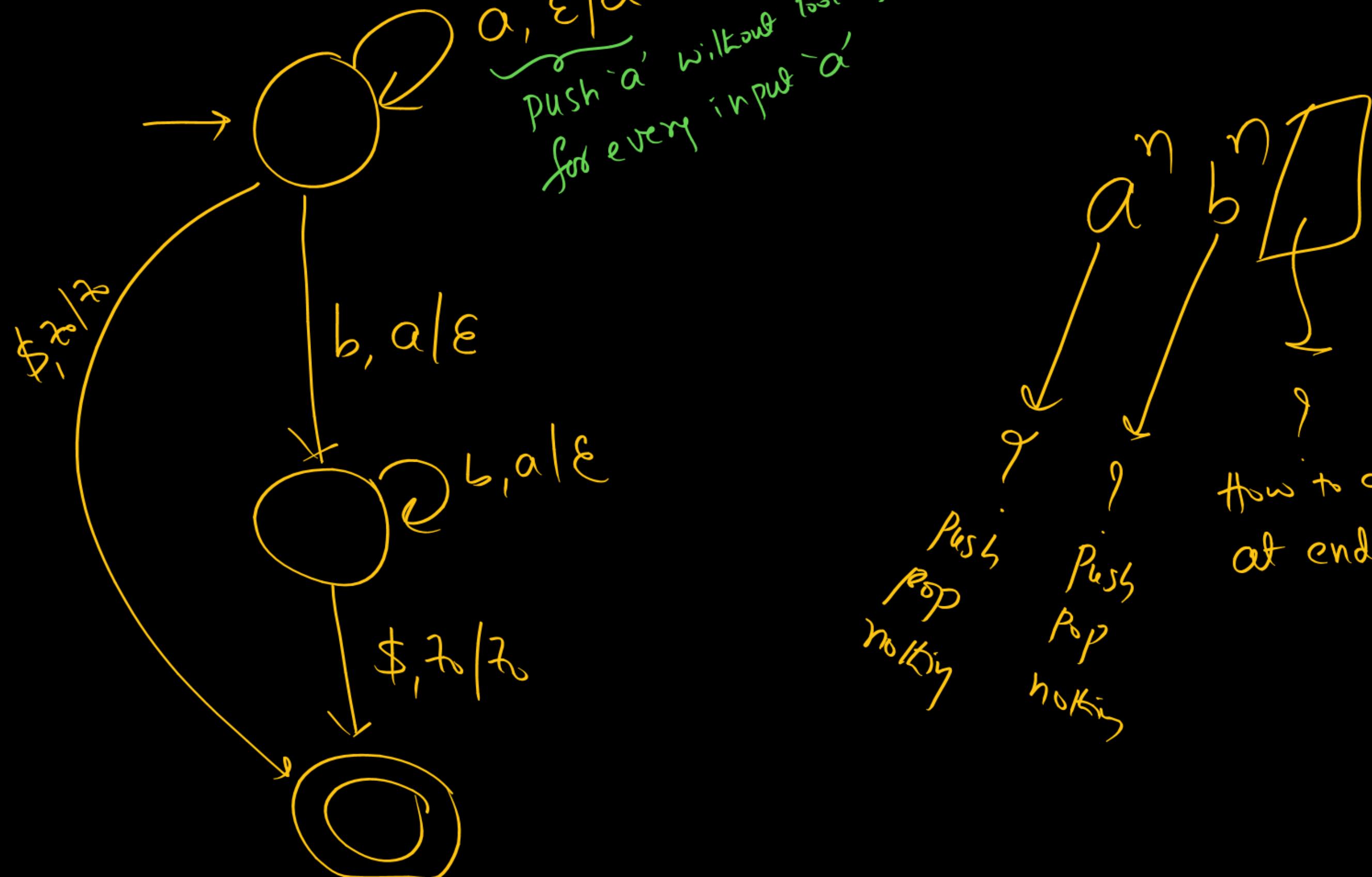
Pushdown Automata

P
W

$$\{ a^n b^n \mid n \geq 0 \} = \{ \epsilon, ab, aabb, aaabb, \dots \}$$



Pushdown Automata



Pushdown Automata

② $\{a^n b^{2n} \mid n \geq 0\}$

③ $\{a^{2n} b^n \mid n \geq 0\}$

~~Home work~~

Summary

What is PDA?

Stack operations

Transitions



Thank you

