COMPUTER SCIENCE

Database Management System

FD's & Normalization

Lecture_09

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TOPICS TO BE COVERED

01 Closure of FD set

02 Lossless Join Decomposition

Minimal Cover

Minimal Cover may/may Not be unique.

Redundant FD

Check for Minimal Cover.

Procedure to Find Minimal Cover

Finding Number of Super Keys.

Super keys

5 Attribute

(1) R(ABCDE)

C.K : A $\Rightarrow$ # Super key $= 2^{5-1} = 2^4 =$ 16 Ans

(2) CK A, D $\Rightarrow$ SK $= 2^{5-1} + 2^{5-1} - 2^{5-2} \Rightarrow 2^4 + 2^4 - 2^3 =$ 24 Ans

(3) CK A, BC S.K $= 2^{5-1} + 2^{5-2} - 2^{5-3} \Rightarrow 2^4 + 2^3 - 2^2 =$ 20 Ans

(4) CK AB, CD S.K : $2^{5-2} + 2^{5-2} - 2^{5-4} \Rightarrow 2^3 + 2^3 - 2^1 \Rightarrow$ 14 Ans

(5) CK AB, AD S.K $= 2^{5-2} + 2^{5-2} - 2^{5-3} \Rightarrow 2^3 + 2^3 - 2^2 =$ 12 Ans

$48 - 24 + 4$

(6) CK ABC, DE SK $= 2^{5-3} + 2^{5-2} - 2^{5-5} \Rightarrow 2^2 + 2^3 - 2^0 \Rightarrow 4+8-1 =$ 11 Ans

CK ABC, CDE SK $= 2^{5-3} + 2^{5-3} - 2^{5-5} \Rightarrow 2^2 + 2^2 - 2^0 \Rightarrow$ 7 Ans

CK A, D, E S.K = 28 Ans

$3(2^{n-1}) - 3(2)^{n-2} + 2^{n-3}$

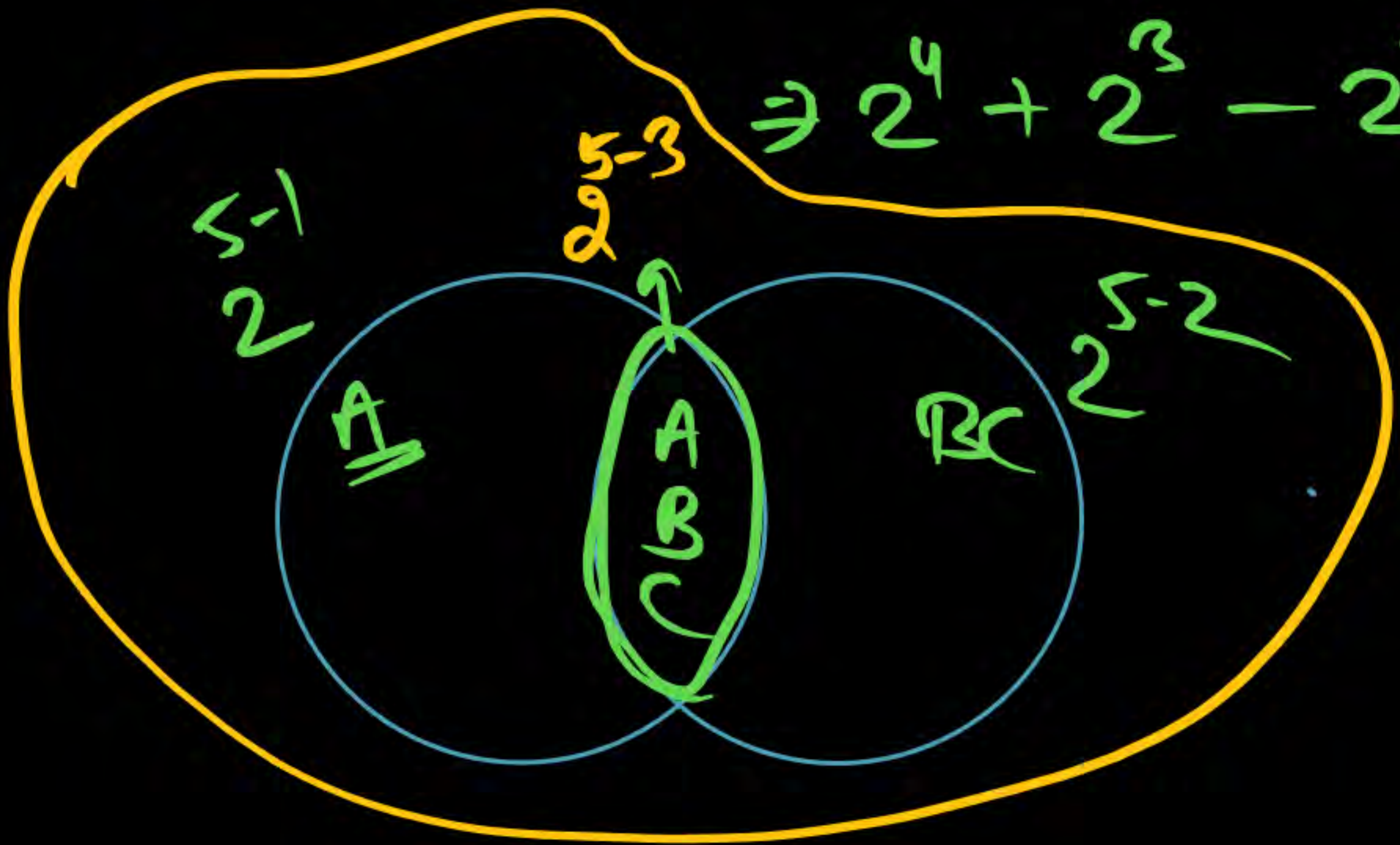
$3 \times 2^4 - 3 \times 2^3 + 2^{5-3}$

Q R(ABCDE) with C.K : A, BC

$$\# \text{Super keys} = 2^{5-1} + 2^{5-2} - 2^{5-3}$$

$$\Rightarrow 2^4 + 2^3 - 2^2 \Rightarrow 16 + 8 - 4 = 20 \text{ Super keys}$$

Ans



$\underline{\text{II}^{nd}\text{ Method}}$

Q) R(ABCDE) CK : [A, BC]

$\cancel{\underline{A}}\ BCDE = 2^4 = 16$

$BC\ DE = 2^2 = 4$

20 Super keys

R(A.BCDE) C.K [A, B, CD]

\# Super keys $= 2^{5-1} + 2^{5-1} + 2^{5-2} - 2^{5-2} - 2^{5-3} - 2^{5-3} + 2^{5-4}$

$= 2^4 + 2^4 + 2^3 - 2^3 - 2^2 - 2^2 + 2^1$

$= \underbrace{16 + 16} + \cancel{8} - \cancel{8} - 4 - 4 + 2$

$32 - 8 + 2$

26 Super keys Ans

$R(A, B, C, D, E)$ C.K $[A, B, CD]$

$\cancel{A}\ \underline{BCDE} = 2^4 = 16$

$\cancel{B}\ \underline{CDE} = 2^3 = 8$

$\underline{CD}\ \underline{E} = 2^1 = 2$

26 Super keys

Note

R(ABCD) with Candidate keys = [A, B, C, D]

Super keys ?

$$\cancel{A}\,BCD = 2^3 = 8$$

$$\cancel{B}\,CD = 2^2 = 4$$

$$\cancel{C}\,D = 2^1 = 2$$

$$D = 2^0 = 1$$

15 Super keys.

Note

R(ABCD) with Candidate keys = [A, B, C, D]

Super keys ?

$\cancel{A}\,\underbrace{BCD} = 2^3 = 8$ [A, AB, AC, AD, ABC, ABD, ACD, ABCD]

$\cancel{B}\,\underbrace{CD} = 2^2 = 4$ [B, BC, BD, BCD]

$\cancel{C}\,\underbrace{D} = 2^1 = 2$ [C, CD]

$\underline{D} = 2^0 = 1$ [D]

15 Super keys.

Note

n: is # Attribute.

$$\boxed{\text{Total Maximum Number of Super Keys} = 2^n - 1}$$

Under the Assumption if each single Attribute is candidate keys.

like in Previous example.

R(ABCD) Total Maximum # Super key $= 2^4 - 1 = 16 - 1 =$ 15 Super key Ans

(When A, B, C, D is CK)

Q $R(ABCDE)$ with CK : [AB, BC]

Super keys $= 2^{5-2} + 2^{5-2} - 2^{5-3}$

$\Rightarrow 2^3 + 2^3 - 2^2$

$= 8 + 8 - 4$

= 12 Super keys

$\underline{AB}\,CDE = 2^3 = 8$ Super key

$\underline{BC}\,DE = 2^2 = 4$ Super key

12 Super keys.

R(ABCDE) with C.K [AB, CD]

Venn Diagram

$$\#\text{Super keys} = 2^{5-2} + 2^{5-2} - 2^{5-4}$$

$$\Rightarrow 2^3 + 2^3 - 2^1 \Rightarrow 8+8-2$$

$$= \boxed{14 \text{ Super keys}} \text{ Ans}$$

14 Ans

IInd Method

$\cancel{AB}CDE \Rightarrow 2^3 = 8$

$CDE = 2^1 = 2$

~~10 Super keys~~

AB	CD
ABC	CDE
ABD	CDA
ABE	CDB
ABCD	CDEA
ABDE	CDEB
ABCE	:
ABCDE	

Other Approach :

Q. If in a Relation R with n Attribute, if every (n−1) attribute is a Candidate key then Number of Super keys ?

Solⁿ

(n−1) Attribute is C.K

$$\boxed{{}^{n}C_{n-1}} + \boxed{{}^{n}C_{n}} \Rightarrow \boxed{n+1} \text{ Ans}$$

+ Superset of ${}^{n}C_{n-1}$

${}^{n}C_{n-1}$ + Any Super set of ${}^{n}C_{n-1}$

Q. R(ABC) n=3

AB, BC, AC } is C.K.

#Super keys = AB, BC, AC, ABC } 4 Super keys

⇓

n+1

3+1 = 4 SK Ans

$$^{n}C_{r} = \frac{n!}{(n-r)!\ r!} \Rightarrow {}^{4}C_{3} \Rightarrow \frac{4!}{(4-3)!\ 3!}$$

$$= \frac{4 \times \cancel{3!}}{\cancel{3!}} = 4$$

$R(ABCD) \Rightarrow \boxed{n=4}$ every $(n-1)$ Attribute is C.K

every $(n-1)$

every 3 Attribute

is C.K

ABC
BCD
ACD
ABD

is C.K

S.K = $n+1$

$= 4+1 = 5$ Ans

C.K: ${}^4C_3 \Rightarrow \frac{4!}{(4-3)!\,3!}$

$= \frac{4 \times \cancel{3!}}{\cancel{3!}} = 4$ CK

#super keys

$\Rightarrow {}^4C_3 + {}^4C_4$

$\Rightarrow 4+1 = 5$ Ans

ABC
BCD
ACD
ABD
ABCD

Q) In Relation R with n Attribute if every (n-2) attribute is CK then Number of Super keys.

Sol^n

super keys ⇒ $^nC_{n-2}$ + Any Super Set of $^nC_{n-2}$

$^nC_{n-2}$ $\quad$ $^nC_{n-1}$ $\quad$ ^nC_n

$$\boxed{\#\text{Super keys} = {}^nC_{n-2} + {}^nC_{n-1} + {}^nC_n}$$

$5 \Rightarrow (n-2) \Rightarrow 3$

Q R(ABCDE) if every '3' Attribute is candidate key then ① Finding Total # C.K ② Find Total # Super keys ?

Soln Total # C.K $\Rightarrow {}^{n}C_{n-2} \Rightarrow {}^{5}C_{5-2} \Rightarrow {}^{5}C_{3} \Rightarrow$ $\boxed{\text{10 Candidate key}}$ Ans

Total # S.K $= {}^{n}C_{n-2} + {}^{n}C_{n-1} + {}^{n}C_{n}$

$\Rightarrow {}^{5}C_{3} + {}^{5}C_{4} + {}^{5}C_{5}$

$\Rightarrow 10 + 5 + 1 = 16$ Ans

Q If every $(n-3)$ Attribute form a C.K then No. of Super key?

$$\boxed{\#\text{Super keys} = {}^{n}C_{n-3} + {}^{n}C_{n-2} + {}^{n}C_{n-1} + {}^{n}C_{n}}$$

Q R(ABCDE) if every 2 Attribute is C.K then Total # C.K & # Superkey?

$n=5$ $(n-3)$ ${}^{n}C_{2}$

$\# C.K = {}^{n}C_{n-3} \Rightarrow {}^{5}C_{2} \Rightarrow$ 10 C.K

$\# SK = {}^{5}C_{2} + {}^{5}C_{3} + {}^{5}C_{4} + {}^{5}C_{5} \Rightarrow$ 26 Super keys

$R \Rightarrow n$ Attribute

Q) If every single Attribute is a Candidate key than Total # Super key?

$2^n - 1$

$$\boxed{\text{Total \# super keys} = {}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4 + \ldots\ldots {}^nC_n}$$ Ans

$n = 4$

eg) R(ABCD) if every single Attribute is a Candidate key.

$$\text{\# super keys} = {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$$

$$\Rightarrow 4 + 6 + 4 + 1 = 15 \text{ Ans}$$

$$\text{\# super keys} = 2^n - 1 \Rightarrow 2^4 - 1$$

$= 15$ super keys

NAT

Consider a relation R(A, B, C, D, E) with the following three functional dependencies.

$(AB)^+ = (ABCDE)$

$\boxed{\text{AB is Candidate key}}$

$(A)^+ = (A)$

$(B)^+ = (B)$

$AB \to C$; $BC \to D$; $C \to E$;

CK : AB

The number of super keys in the relation R is ________.

[GATE-2022-CS: 1M]

$\underline{AB}\ \underbrace{CDE}$

$\Downarrow$

$\#Sk = 2^3$

$= 8$ Super keys

$\#Sk = 2^{n-2}$

$\Rightarrow 2^{5-2}$

$= 2^3 = 8$ Super key

AB	ABCD
ABC	ABDE
ABD	ABCE
ABE	ABCDE

NAT

The maximum number of superkeys for the relation schema R (E, F, G, H) with E as the key is ________.

[GATE-2014-CS: 1M]

Soln: R(EFGH) E is C.K

$$\#\text{Super keys} = 2^{n-1} \Rightarrow 2^{4-1} = 2^3 = 8 \text{ Superkeys}$$

R(EFGH) E is CK

E FGH

$$\#SK = 2^3 = 8 \text{ Superkey}$$

E F G H

0	0	0	= E
0	0	1	⇒ EH
0	1	0	= EG
0	1	1	= EGH
1	0	0	= EF
1	0	1	= EFH
1	1	0	= EFG
1	1	1	= EFGH

8 Super keys

E FGH   E is CK

E

EF

EG

EH

EFG

EGH

EFH

EFGH

$\boxed{EFG = EGF}$

Closure of FD Set $[F]^+$

Set of all possible FD's which can be derived from given FD set is called closure of FD set. $[F]^+$

$[F]^+$ Closure of FD

R(AB)

ϕ	$A \rightarrow \phi$	$B \rightarrow \phi$	$AB \rightarrow \phi$
A	$A \rightarrow A$	$B \rightarrow A$	$AB \rightarrow A$
B	$A \rightarrow B$	$B \rightarrow B$	$AB \rightarrow B$
AB	$A \rightarrow AB$	$B \rightarrow AB$	$AB \rightarrow AB$

R(ABC)

ϕ	$A \rightarrow \phi$	$B \rightarrow \phi$
A	$A \rightarrow A$	$B \rightarrow A$
B	$A \rightarrow B$	$B \rightarrow B$
C	$A \rightarrow C$	$B \rightarrow C$
AB	$A \rightarrow AB$	$B \rightarrow AB$
BC	$A \rightarrow BC$	$B \rightarrow AC$
AC	$A \rightarrow AC$	$B \rightarrow BC$
ABC	$A \rightarrow ABC$	$B \rightarrow ABC$

$R(ABC)$ $[A \rightarrow B, B \rightarrow C]$ $[F]^+ = 43$ Ans.

ϕ 0 attribute = $\phi \rightarrow \phi$

A 1Attribute = $[A]^+ = [ABC] = 2^3$

B $[B]^+ = [BC] = 2^2$

C $[C]^+ = [C] = 2^1$

AB 2Attribute = $[AB]^+ = [ABC] = 2^3$

BC $[BC]^+ = [BC] = 2^2$

AC $[AC]^+ = [ABC] = 2^3$

ABC 3 Attribute= $[ABC]^+ = [ABC] = 2^3$

$$[A]^+ = \begin{bmatrix} A\to\phi, A\to A, A\to B, A\to C \\ A\to AB, A\to BC, A\to AC, A\to ABC \end{bmatrix}$$

$$[B]^+ = [B \to \phi, B \to B, B \to C, B \to BC]$$

$$[C]^+ = [C \to \phi, C \to C]$$

$$[AB]^+ = \begin{bmatrix} AB\to\phi, AB\to A, AB\to B, AB\to C \\ AB\to AB, AB\to BC, AB\to AC, AB\to ABC \end{bmatrix}$$

$$[BC]^+ = [BC \to \phi, BC \to B, BC \to C, BC \to BC]$$

$$[AC]^+ = \begin{bmatrix} AC\to\phi, AC\to A, AC\to B, AC\to C \\ AC\to AB, AC\to BC, AC\to AC, AC\to ABC \end{bmatrix}$$

$$[ABC]^+ = \begin{bmatrix} ABC\to\phi, ABC\to A, ABC\to B, ABC\to C \\ ABC\to AB, ABC\to BC, ABC\to AC, ABC\to ABC \end{bmatrix}$$

$R(AB)$ $[A \to B]$

ϕ 0 attribute = 1

A 1 Attribute = $[A]^+ [AB] = 2^2$

B $[B]^+ = [B] = 2^1$

AB 2 Attribute = $[AB]^+ = [AB] = 2^2$

$$\boxed{\begin{array}{c} 1 \\ \\ 4 \\ \\ 2 \\ \\ 4 \end{array}} \begin{array}{l} \\ \\ (A \to \phi,\ A \to A,\ A \to B,\ A \to AB) \\ \\ (B \to \phi,\ B \to B) \\ \\ \begin{pmatrix} AB \to \phi, AB \to A \\ AB \to B, AB \to AB \end{pmatrix} \end{array}$$

11 Ans.

Properties of Decomposition

1. Lossless Join Decomposition.

2. Dependency Preserving Decomposition.

Lossless Join Decomposition

① BASIC CONCEPT

② Binary Method

③ CHASE TEST

Lossless Join Decomposition

Let R be the Relational Schema with Instance r,
is decomposed into Sub Relation $R_1, R_2, R_3 \ldots R_n$
with instance $r_1, r_2, r_3, \ldots r_n$ Respectivly.

If $R_1 \bowtie R_2 \bowtie R_3 \ldots \bowtie R_n \equiv R$
Lossless Join Decomposition

If $R_1 \bowtie R_2 \bowtie R_3 \ldots \bowtie R_n \supset R$
Lossy Join Decomposition

Natural JOIN ($\bowtie$) $R \bowtie S$

It is Performed into 3 steps.

Step1 : Cross Product of R & S.

R	S
n_1 Tuple	n_2 Tuple
C_1 Attribute	C_2 Attribute

$$\boxed{R \times S = \begin{array}{l} n_1 \times n_2 \text{ Tuple} \\ C_1 + C_2 \text{ Attribute} \end{array}}$$

Step2 : Select the tuples which Satisfy equality Condition on All Common Attribute (FROM R×S (From Step1))

Step3 Projection of Distinct Attribute.

Q.

R(ABC)

A	B	C
1	5	5
2	5	8
3	8	8

Decomposed into

Q.1 $R_1(AB)$ & $R_2(BC)$

3 Tuple
2 Attribute

3 Tuple
2 Attribute

Column

2 + 2 = 4 Attribute

R_1

A	B
1	5
2	5
3	8

R_2

B	C
5	5
5	8
8	8

Step1

$R_1 \times R_2 =$

$R_1.A$	$R_1.B$	$R_2.B$	$R_2.C$
1	5	5	5
1	5	5	8
1	5	8	8
2	5	5	5
2	5	5	8
2	5	8	8
3	8	5	5
3	8	5	8
3	8	8	8

$R \times S =$ $3 \times 3 = 9$ Tuple
$2+2 = 4$ Attribute

Q. R(ABC)

A	B	C
1	5	5
2	5	8
3	8	8

Decomposed into

Q.1 $R_1(AB)$ & $R_2(BC)$

Step2: select the tuple which satisfy equality condition on All Common Attribute from R X S

$R_1.B = R_2.B$

Step2

$R_1 X R_2 =$

R1.A	R1.B	R2.B	R2.C
1	5	5	5
1	5	5	8
1	5	8	8
2	5	5	5
2	5	5	8
2	5	8	8
3	8	5	5
3	8	5	8
3	8	8	8

Step3 (Distinct Attribute of step2 o/p)

A	B	C
1	5	5
1	5	8
2	5	5
2	5	8
3	8	8

Lossy Join..

Q.

R(ABC)

A	B	C
1	5	5
2	5	8
3	8	8

Decomposed into

Q.2 $R_1(AB)$ & $R_2(AC)$

Lossless - Join Decomposition

- For the case of $R = (R_1, R_2)$, we require that for all possible relations r on schema R

  $r = \pi_{R_1}(r) \bowtie \pi_{R_2}(r)$

- A decomposition of R into R_1 and R_2 is lossless join if at least one of the following dependencies is in F^+:
  - $R_1 \cap R_2 \rightarrow R_1$
  - $R_1 \cap R_2 \rightarrow R_2$

Any Doubt ?

THANK YOU!