CS & IT ENGINEERING

DISCRETE MATHS
SET THEORY

Lecture No. 02

By- SATISH YADAV SIR

01 Basics of Functions

02 Terms in Functions

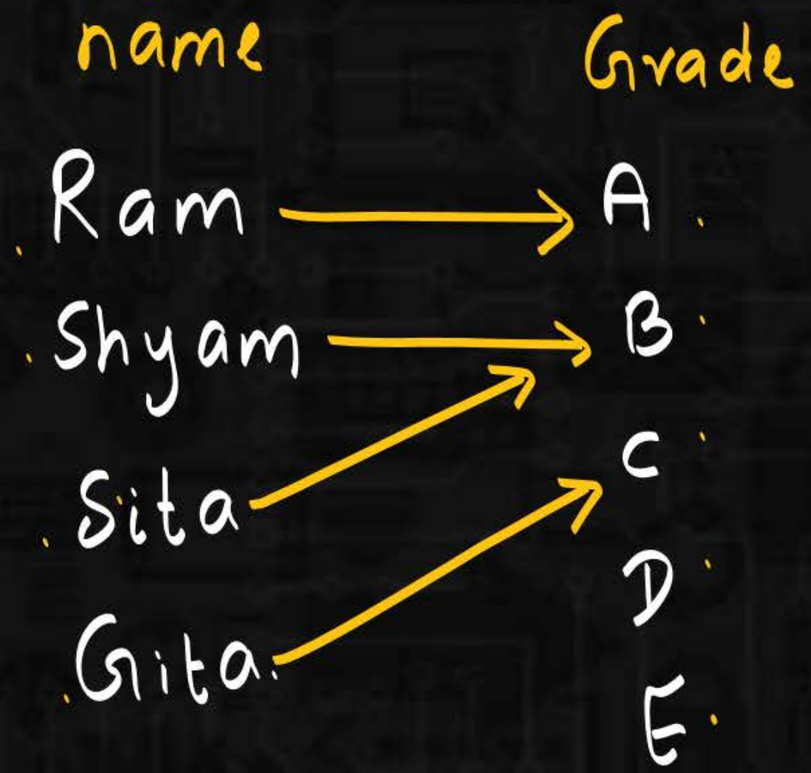
03 Number of Functions

04 Types of Functions

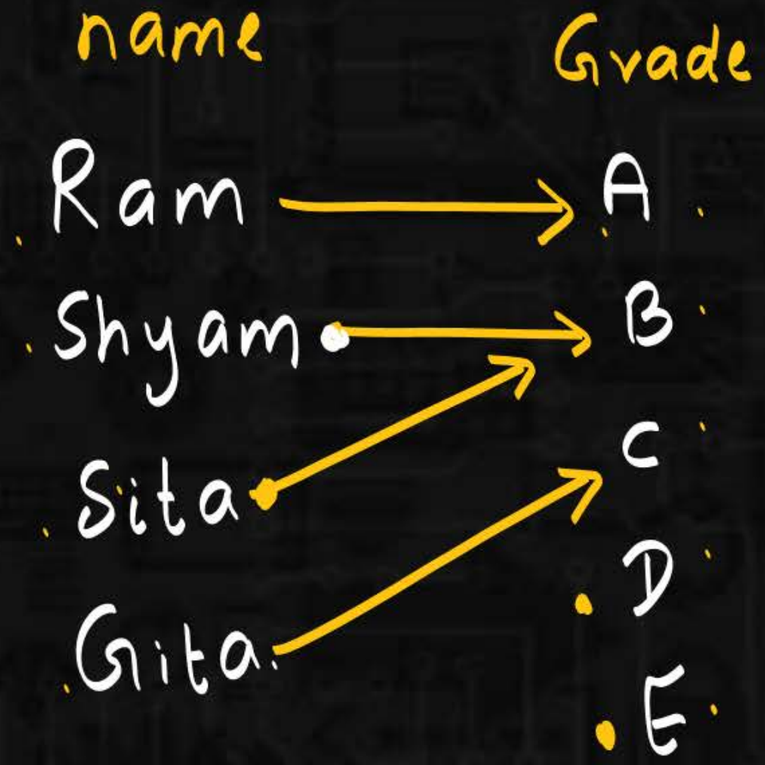
05 Various Examples in Functions

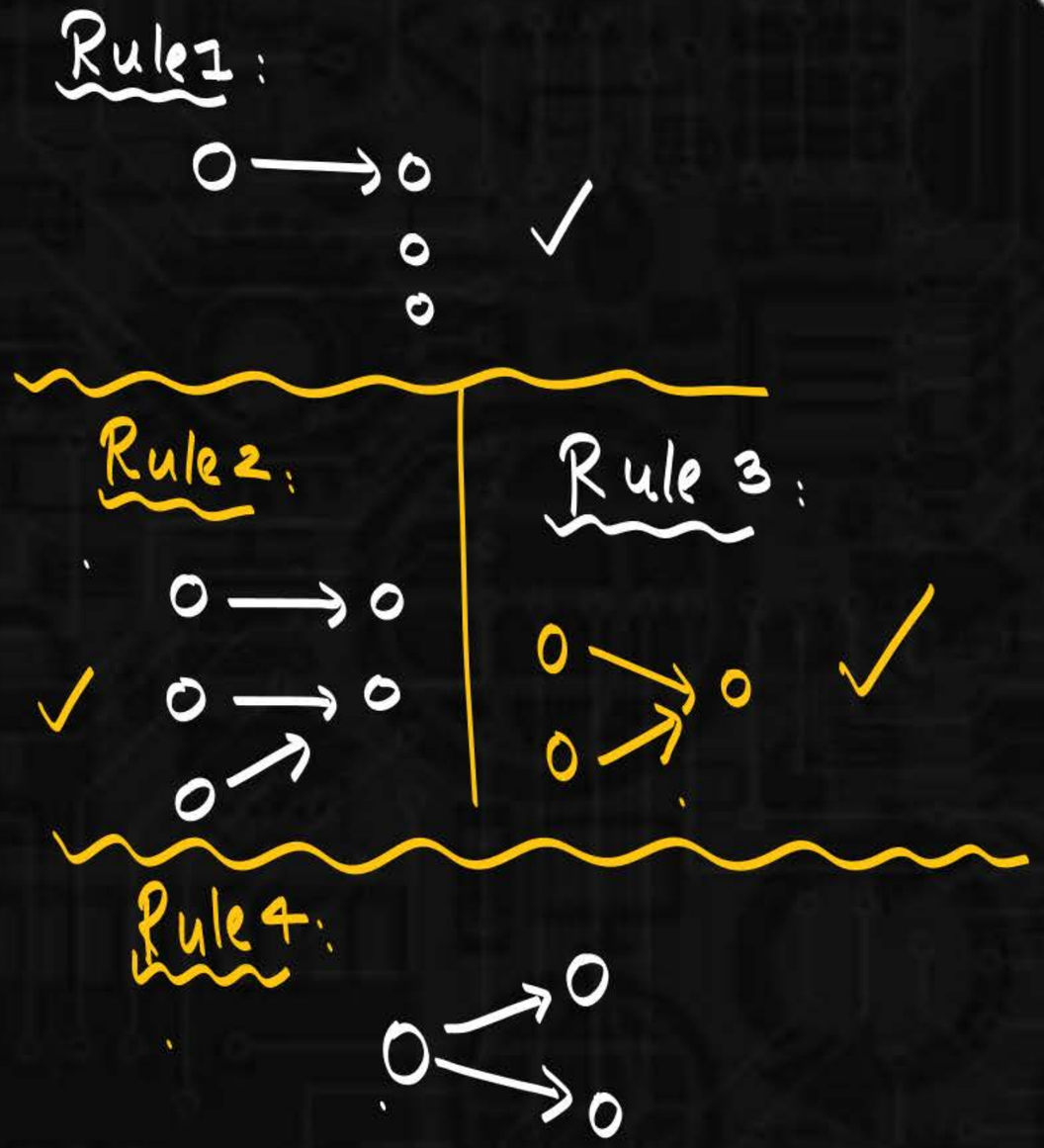
Functions

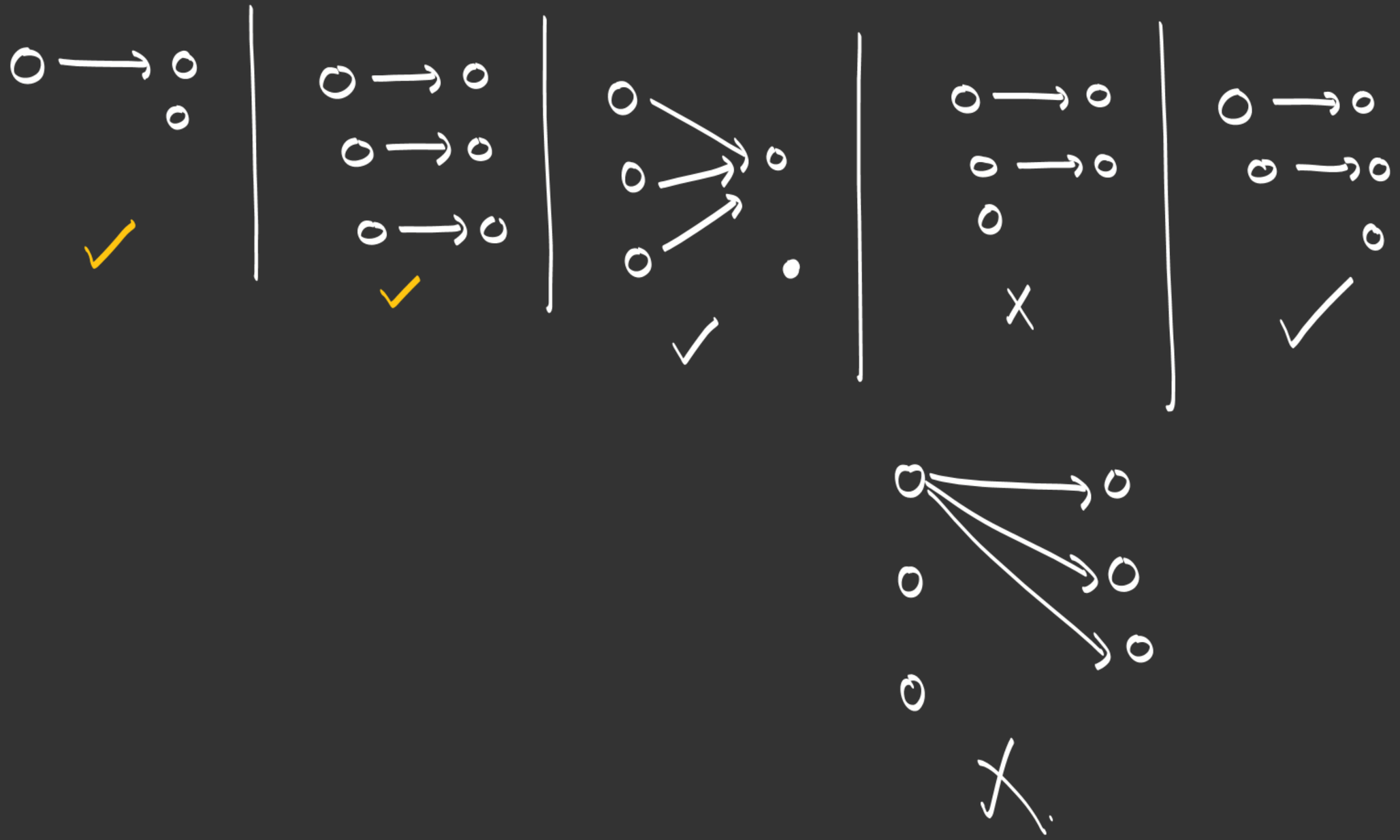
Function / assignment / mapping / transformation.



Functions

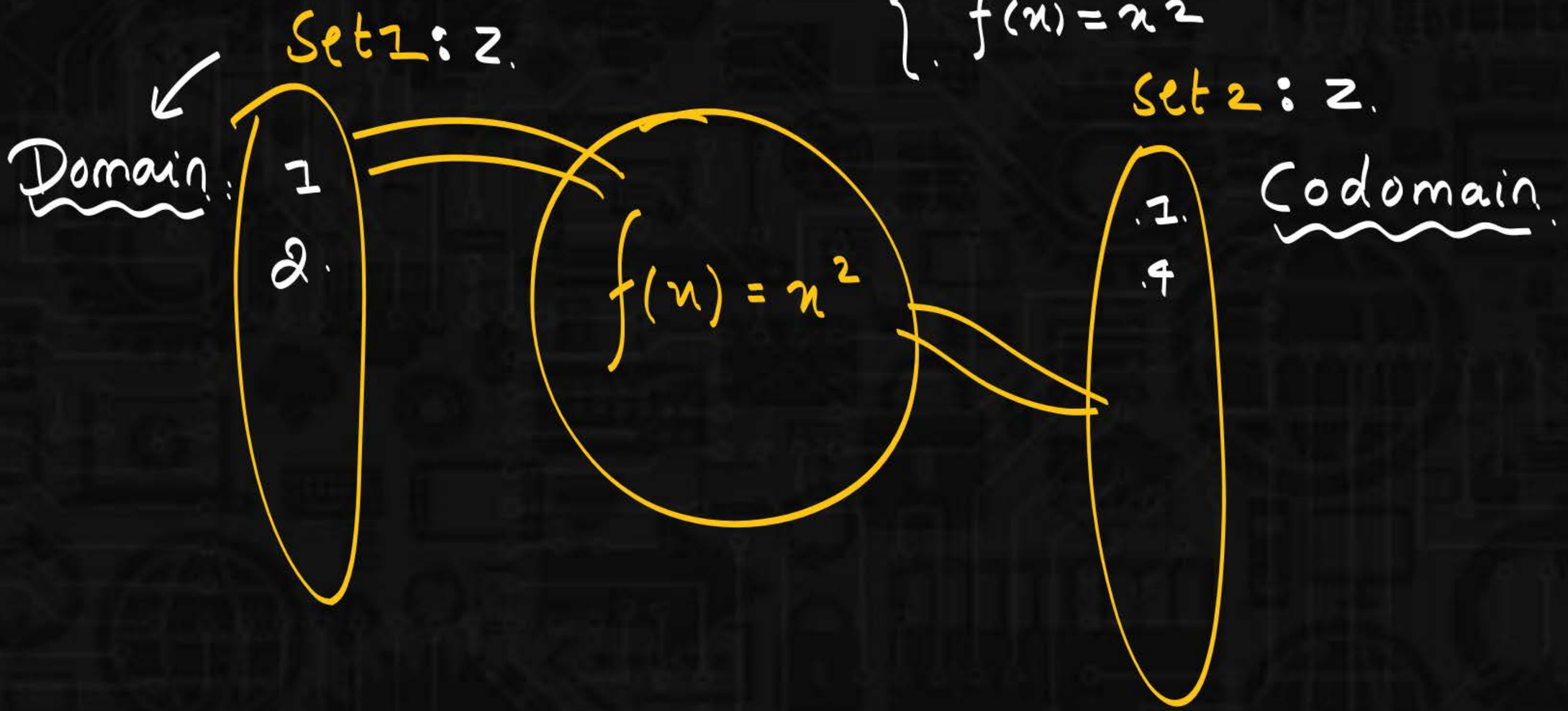

name
Grade
Ram
Shyam
Sita
Gita
A
B
C
D
E


Rule1 :
Rule2 :
Rule 3 :
Rule4 :

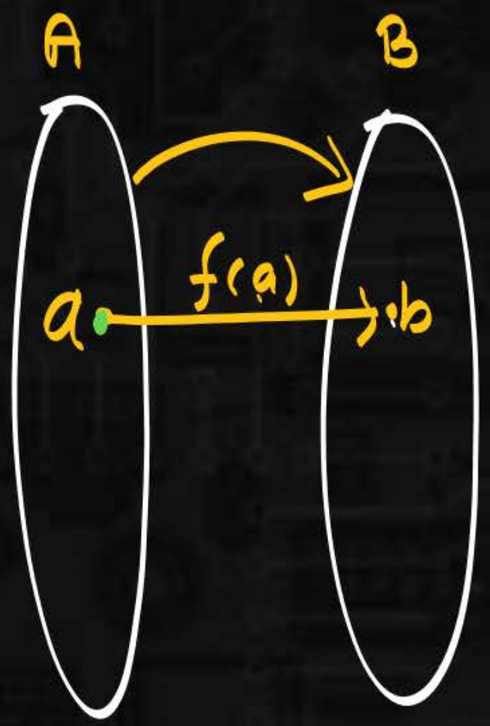


Functions

$$\begin{cases} f : z \to z \\ f(x) = x^2 \end{cases}$$



Functions



Domain

Codomain.

$f : A \to B$

$f(a) = b$.

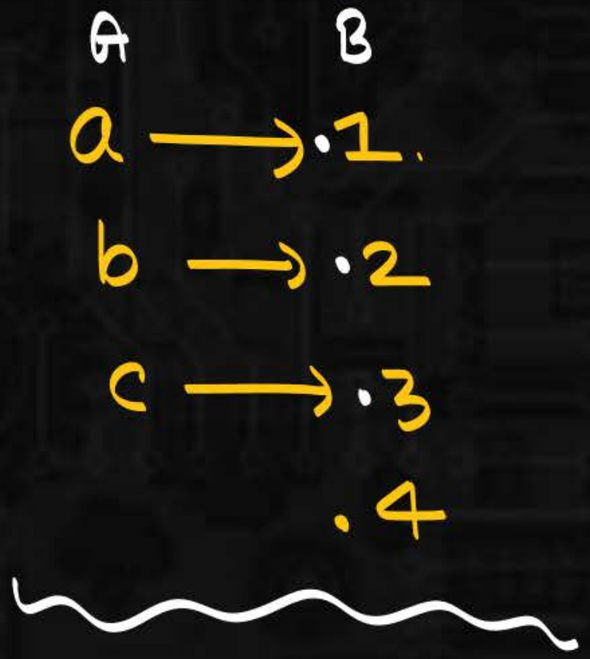
b is image of a.

a is called preimage of b.

Range = set of images.

Range $\subseteq$ Codomain.

Functions



$f: A \to B$

Range = {1, 2, 3}

codomain = {1, 2, 3, 4}

$a \to x$

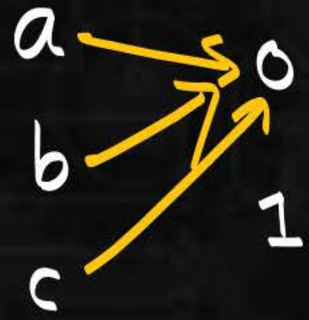
$b \to y$

Range = {x, y} codomain = {x, y}

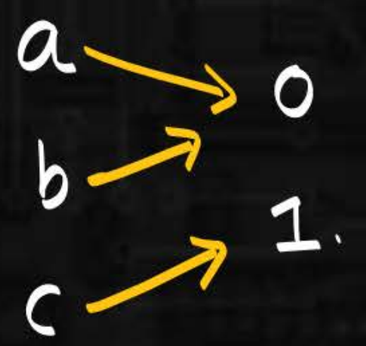
Functions

$f : A \to B \quad |A| = 3 \quad |B| = 2.$

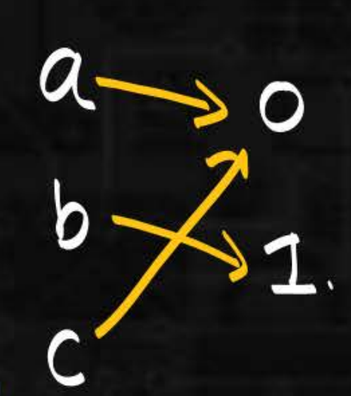
Total no. of functions
= Total diff. arrows repvestn.



000



001



010

$= 8$

$= 2^3 \quad = (R \cdot S)^{L \cdot S}$

011 ..... 111.

Functions

GATE.

y^x

$(R \cdot S)^{L \cdot S} = y^x$

$f: X \to Y$ Total no. of functions = 97.

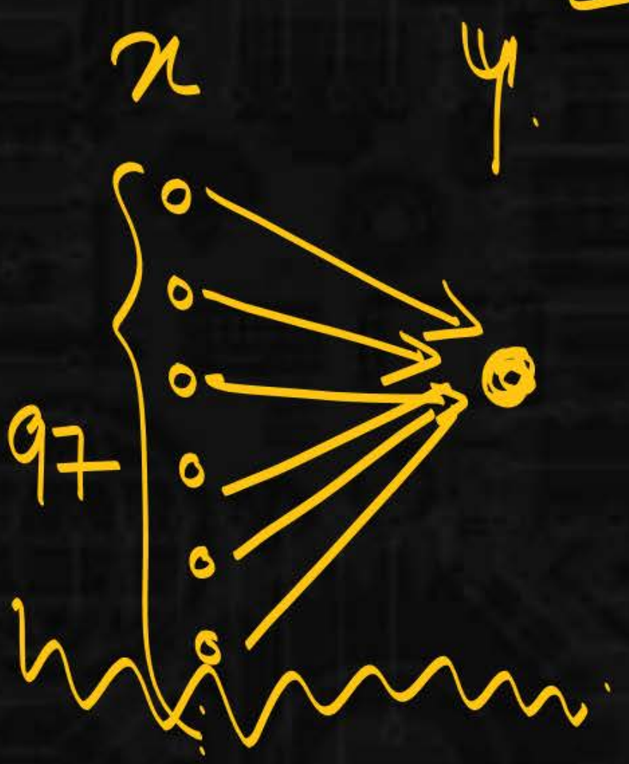
$(97)^1 = 97$.

a) $|X| = 1 \wedge |Y| = 97$. $(97)^1$

b) $|X| = 97 \wedge |Y| = 1$. $(1)^{97} = 1$

c) $|X| = 97 \wedge |Y| = 97$.

d) nota.

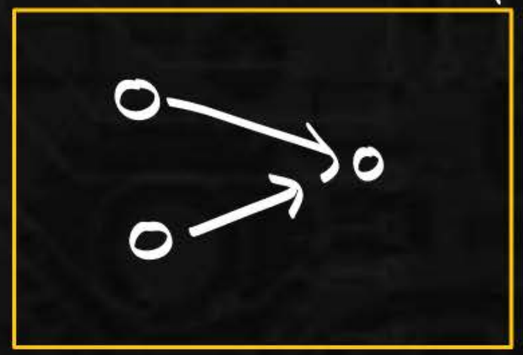


Functions

1:1 (injective):

$(f(a) = f(b) \rightarrow a = b)$

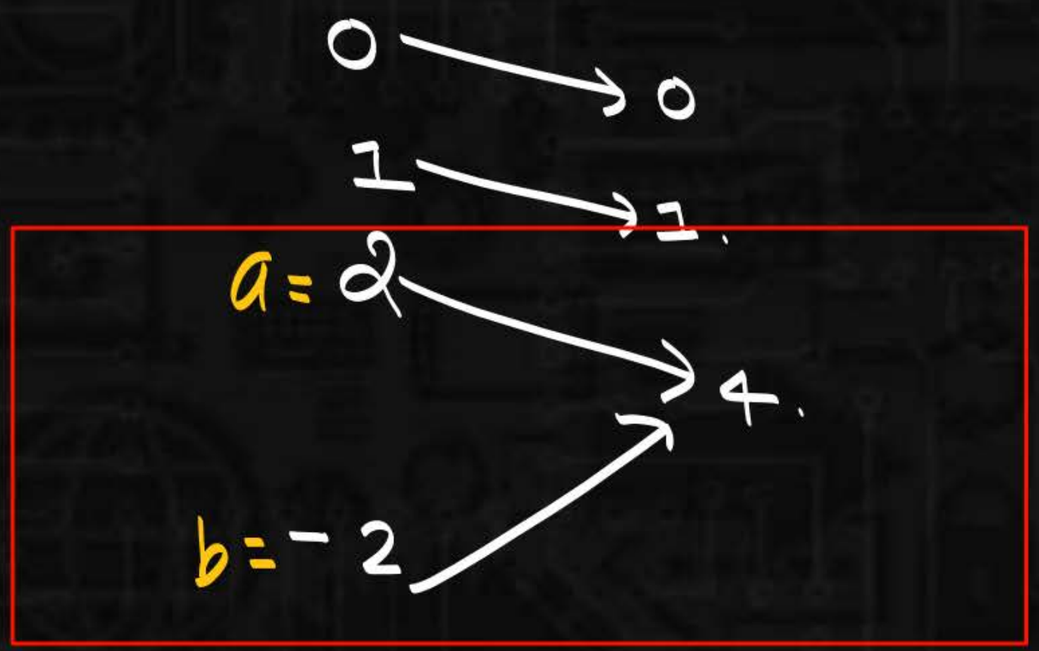
1:1 ✗



Functions

$f : Z \to Z$.

$f(x) = x^2$.



$a = 2 \quad b = -2$.

$f(a) = 4 \quad f(b) = 4$.

$f(a) = f(b) \longrightarrow a = b$.

$\underline{4 = 4} \longrightarrow \underline{2 = -2}$.

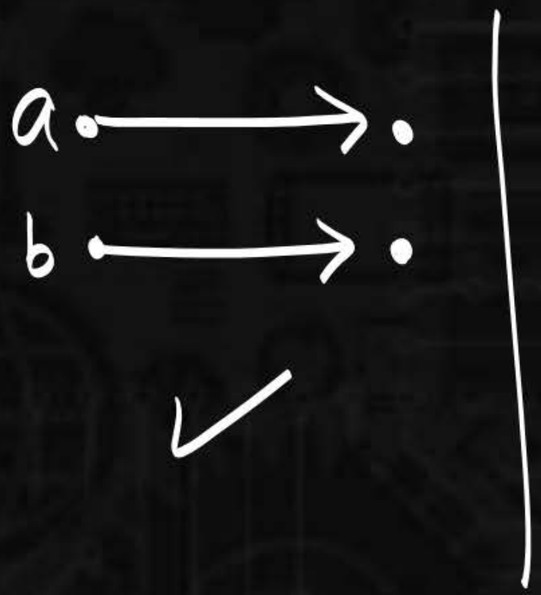
T F

Functions

$f(a) = f(b) \longrightarrow a = b.$

OR

$a \neq b \longrightarrow f(a) \neq f(b)$



Functions

$f: \mathbb{Z} \to \mathbb{Z}$. $f(x) = x+1$.

$1 \longrightarrow 2$

$2 \longrightarrow 3$

$3 \longrightarrow 4$

$a \circ \longrightarrow \circ f(a)$

$b \circ \longrightarrow \circ f(b)$

$(f(a) = f(b) \longrightarrow a = b)$

$f(x) = x+1$.

$f(w) = w+1$

$f(p) = p+1$.

$f(w) = f(p)$

$w+1 = p+1$.

$\downarrow$

$w = p$.

Functions

$f: R \to R\left(\neq \frac{1}{2}\right)$ $\quad f(x) = \frac{4x}{2x-1}$ (1:1) ✓

$(f(a) = f(b) \to a = b)$

$f(a) = f(b)$

$$\frac{4a}{2a-1} = \frac{4b}{2b-1}$$

$a(2b-1) = b(2a-1)$

$2ab - a = 2ab - b$

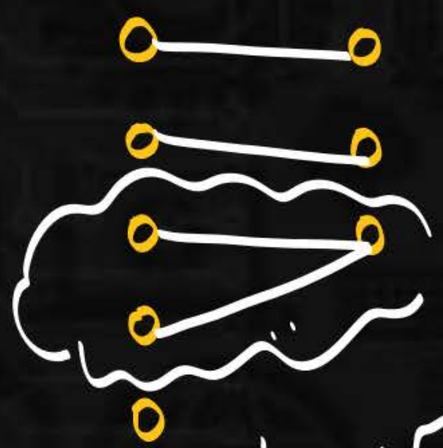
$-a = -b$

$a = b$

Functions

$f: A \to B$

$|A| = 5 \quad |B| = 3$

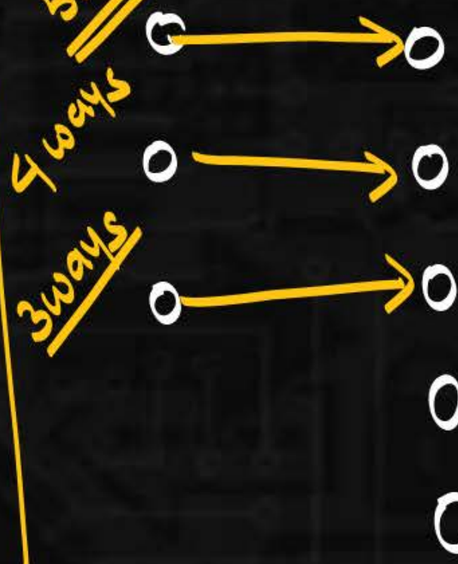


not 1:1 functions

$f: A \to B \quad |A| \leq |B|$

$|L \cdot S| \leq |R \cdot S|$

$|A| = 3 \quad |B| = 5$



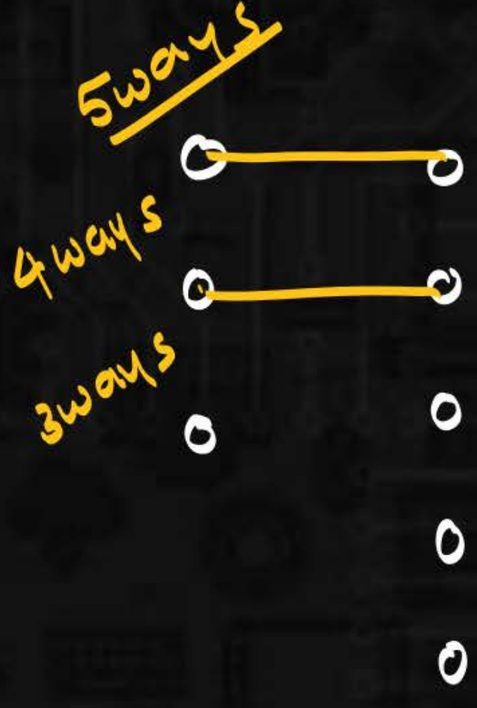
$= 5 \cdot 4 \cdot 3 \left(\frac{2!}{2!} \right)$

$= \frac{5!}{2!}$

$= \frac{5!}{(5-3)!} = {}^5P_3 = R \cdot S \; P_{L \cdot S}$

Functions

$${}^{n}P_{R} = \frac{n!}{(n-R)!}$$



$$5 \cdot 4 \cdot 3 \quad \frac{2!}{2!}$$

$$\frac{5!}{2!}$$

$$\frac{5!}{(5-3)!} = {}^{5}P_{3} = {}^{R.S}P_{L.S.}$$

Functions

$f : A \to B$

$|A| = 3 \quad |B| = 7$

What is the probability of 1:1 functions?

$$\frac{{}^{7}P_3}{7^3}$$

$\#\ 1:1 \to {}^{RS}P_{LS} = {}^{7}P_3$

$\#\ \text{Functions} = (R.S)^{L.S} = 7^3$

Functions

3. Let $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$. (a) List five functions from A to B. (b) How many functions $f: A \to B$ are there? (c) How many functions $f: A \to B$ are one-to-one? (d) How many functions $g: B \to A$ are there? (e) How many functions $g: B \to A$ are one-to-one? (f) How many functions $f: A \to B$ satisfy $f(1) = x$? (g) How many functions $f: A \to B$ satisfy $f(1) = f(2) = x$? (h) How many functions $f: A \to B$ satisfy $f(1) = x$ and $f(2) = y$?

4. If there are 2187 functions $f: A \to B$ and $|B| = 3$, what is $|A|$?

Functions

15. For each of the following functions, determine whether it is one-to-one and determine its range.

a) $f: \mathbf{Z} \to \mathbf{Z}$, $f(x) = 2x + 1$

b) $f: \mathbf{Q} \to \mathbf{Q}$, $f(x) = 2x + 1$

c) $f: \mathbf{Z} \to \mathbf{Z}$, $f(x) = x^3 - x$

d) $f: \mathbf{R} \to \mathbf{R}$, $f(x) = e^x$

e) $f: [-\pi/2, \pi/2] \to \mathbf{R}$, $f(x) = \sin x$

f) $f: [0, \pi] \to \mathbf{R}$, $f(x) = \sin x$

Functions

a) $f(x) = x + 7$ **b)** $f(x) = 2x - 3$

c) $f(x) = -x + 5$ **d)** $f(x) = x^2$

e) $f(x) = x^2 + x$ **f)** $f(x) = x^3$

Check 1:1 !


THANK YOU GW
SOLDIERS !