CS & IT ENGINEERING

DISCRETE MATHS
SET THEORY

Lecture No. 05

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TOPICS

01 TYPES OF RELATION

02 NUMBER OF RELATION

3 OPERATION ON RELATION

Relation:

$\left\{ \begin{array}{l} \underline{P}_1 \ R \ \underline{P}_2 \end{array} \right.$

$\left\{ \begin{array}{l} e_1 \ R \ e_2 \\ 5 > 3 \end{array} \right.$

$G_1 \ R \ G_2$

$G_1 \subset G_2$.

$S_1 \ R \ S_2$.

Set1 R set2

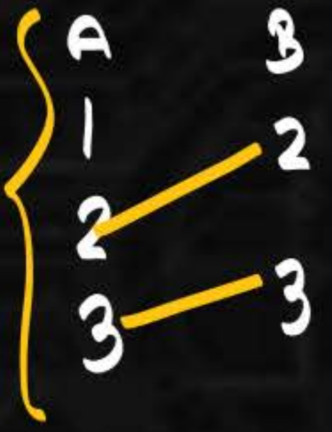
$A \subseteq B$

$011 \ R \ 0111$

$A = \{1, 2, 3\}$ $B = \{2, 3\}$

$A \times B = \{(1,2)\ (1,3)\ (2,2)\ (2,3)\ (3,2)\ (3,3)\}$

$a < b$



Relation::

Relation:

Result = $\{(2,2)\ (3,3)\}$

Result: $\{(1,2)(1,3)\ (2,3)\}$ $(a<b)$

2 2

$A = \{1, 2\}$ $B = \{1\}$

$A \times B = \{ (1,1)(2,1) \}$

$$2^{AB} \begin{cases} R_1 = \{\ \} - \phi \\ R_2 = \{(1\,1)\} \\ R_3 = \{(2\,1)\} \\ R_4 = \{(1\,1)(2\,1)\} - A \times B \end{cases}$$

Relation: subset of crossproduct of A, B

$|A| = m \quad |B| = n.$ Total Relations = Total no of subsets

$|A \times B| = m \cdot n.$

$= 2^{m \cdot n}$

$|A| = n \quad |A \times A| = n^2$ Total relations $= 2^{n^2}$

Symmetric Relation :

$$\forall a \forall b \, [\, (a,b) \in R \rightarrow (b,a) \in R \,]$$

if $(a,b) \in R$ then $(b,a) \in R$.

$A = \{ \quad \}$

$A \times A = \{ \ldots\ldots\ldots \}$

Symmetric

Result :

$R_1 = \{ \quad \}$ Symmetric :

$$\underbrace{\underbrace{(a,b) \in R}_{F} \rightarrow (b,a) \in R}_{T}$$

Symmetric

$R_2 = \{ (\overset{H}{1}, 2)(\overset{W}{2}, 1) \} \rightarrow$

$(a, b) \in R \rightarrow (b, a) \in R$.

$\underbrace{(1, 2) \in R}_{T} \rightarrow \underbrace{(2, 1) \in R}_{T}$ ✓

$a = 1 \quad b = 2$

$R_3 = \{ (1, 2) \}$ not symmetric

$(a, b) \in R \longrightarrow (b, a) \in R$.

$\underbrace{(1, 2) \in R}_{T} \longrightarrow \underbrace{(2, 1) \in R}_{F}$.

$T \rightarrow F$

$a = 1$

$b = 2$

$R_4 = \{(1,1)\}$ Symmetric

$(a,b) \in R \rightarrow (b,a) \in R.$

$\underbrace{(1,1) \in R}_{T.} \rightarrow \underbrace{(1,1) \in R}_{T.}$

$a=1 \quad b=1$

$$R_1 = \{ \underbrace{(2\,3)}_{\times} \underbrace{(2\,2)}_{\checkmark} \} \times$$

$\forall a \neq b$

$$R_2 = \{ \underbrace{(1\,1)}_{\checkmark} \underbrace{(2\,2)}_{\checkmark} \underbrace{(3\,3)}_{\checkmark} \underbrace{(1\,3)}_{\times} \} \times$$

$$R_3 = \{ \underbrace{(1\,1)}_{\checkmark} \underbrace{(2\,2)}_{\checkmark} \underbrace{(3\,3)}_{\checkmark} \} \checkmark$$

2 cases. 2 cases.

$$\begin{array}{c|cc} & \omega_1 & \omega_2 \\ \hline & 0 & 0 \\ & 0 & 1 \\ 4 & 1 & 0 \\ & 1 & 1 \end{array}$$

$\{ \underbrace{(1\,1)}_{w_1} \ \underbrace{(1\,2)}_{w_2} \}$

0 1 0 1.

$e_1 \quad e_2$

$\downarrow \quad \downarrow$

$3 \quad 3 = 3 \times 3 = 3^2.$

2^2

$\phi \rightarrow 00$

$\{(11)\} \rightarrow 10$

$\{(12)\} \rightarrow 01$

$\{(11)(12)\} \rightarrow 11.$

$A = \{1, 2, 3\}$ $|A| = n$

$n^2 = A \times A =$

(1,1)	(1,2)	(1,3)
(2,1)	(2,2)	(2,3)
(3,1)	(3,2)	(3,3)

→ 2 choices

2^n

$\to 2^n \times 2^{\frac{n^2-n}{2}}$

A :

$A \times A$

$\{ \underbrace{11 \; 22 \; 33}_{2^n} \; (12)(21) \; 13 \; 31 \; 23 \; 32 \}$

2 choices 2 choices 2 choices 2 choices

2 choices

2 choices

Total elements $= n^2$.

Total diag $= n$.

non diag $= n^2 - n$.

$A = \{1, 2, 3\}$ $|A| = n$

boxes $= \frac{n^2 - n}{2}$

$A \times A = n^2$

$$\{ \underbrace{(11)\ (22)\ (33)}_{n} \ \underbrace{\boxed{(12)(21)}\ \boxed{(13)(31)}\ \boxed{(23)(32)}}_{n^2 - n} \}$$

2 2 2

n^2

2 choices

$$2^n \times 2^{\frac{n^2 - n}{2}}$$

$$\begin{bmatrix} (1\,1) & (1\,2) & (1,3) \\ (2\,1) & (2\,2) & (2\,3) \\ (3\,1) & (3\,2) & (3\,3) \end{bmatrix}$$

Reflexive. $A \to$ non empty set.

$A = \{1, 2, 3\}$

$\forall a \in A \; (a,a) \in R$.

$A = \{1, 2, 3\}$

$\forall a \in A \; (a,a) \in R$

$R_1 = \{(11)(22)\}$ not reflexive.

$A = \{1, 2, 3\}$

$\forall a \in A (a,a) \in R$

$R_2 = \{(11)\}$ ✗

✓ $R_3 = \{(11)(22)(33)\}$

$R_3 = \{(11)(22)(33)\}$ ✓

$R_4 = \{\boxed{(11)(22)(33)}\,\underline{(12)}\}$

↓ present

$$\left\{\begin{array}{ll} \boxed{11 \quad 22 \quad 33} & \underline{(12)} - P \\ \boxed{11 \quad 22 \quad 33} & (21) \\ \boxed{11 \quad 22 \quad 33} & \\ \boxed{11 \quad 22 \quad 33} & (31) \\ \vdots & \end{array}\right.$$

$|A| = n$

$|A \times A| = n^2$

(1 1)	(1 2)	(1, 3)
(2, 1)	(2, 2)	(2, 3)
(3, 1)	(3, 2)	(3, 3)

$\rightarrow$ 1 choice present

1^n

Total elements $= n^2$.

Diagonal elem $= n$.

non diagonal $= n^2 - n$.

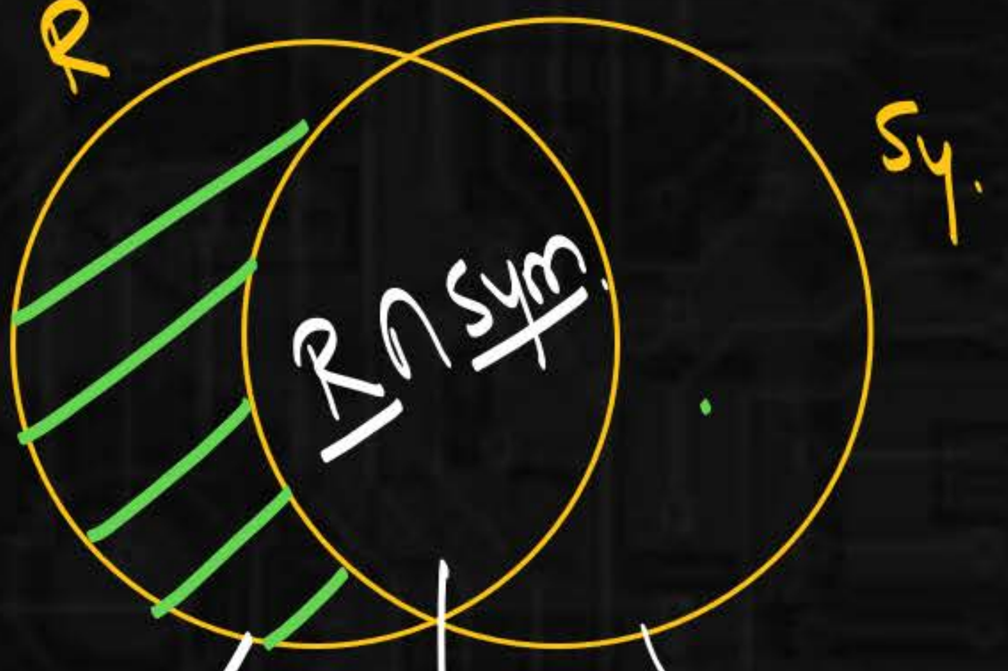
2 choices for non diagonal.

$\#R = 2^{n^2-n}$

$$A \times A = \{(1\,1)\,(2\,2)\,(3\,3)\,\underbrace{(1\,2)}_{1 \quad 0}\,(2\,1)\,(2\,3)\,(3\,2)\,(1\,3)\,(3\,1)\}$$

$$R_1 : \{\boxed{11 \quad 22 \quad 33}\ (1\,2)\}$$

$$R_2 = \{\boxed{11 \quad 22 \quad 33}\ (2\,1)\}$$



$$2^{n^2-n} \qquad 2^{\frac{n^2-n}{2}} \qquad 2^n \cdot 2^{\frac{n^2-n}{2}}$$

$$|A| = n$$

$$\# R = 2^{n^2-n}$$

$$\# Sy = 2^n \cdot 2^{\frac{n^2-n}{2}}$$

Diagonal must be present

$$\text{Ref} \cap \text{Sym} = 1^n \cdot 2^{\frac{n^2-n}{2}}$$

$$= 2^{\frac{n^2-n}{2}}$$

$\boxed{11\ 22\ 33}$

~~Ry~~

~~Sym~~

- → allows same elements
- → demand flipping.

2^n $\quad 2^{\frac{n^2-n}{2}}$

can present

Irreflexive: (hate same element)

A → non empty set

$A = \{1, 2, 3\}$.

$\forall a \in A \; (a,a) \notin R$.

$R_3 = \{(1\,2)\}$ IRR ✓

$R_4 = \{(2\,3)\}$ IRR ✓

$R_1 = \{(1\,1)\}$ IRR X. Ref X.

$R_2 = \{(1\,2)(2\,3)(2\,2)\}$ IRR X. REF X.

$R_1 = \{ \ \}$ ✓ IRR ✓

$R_2 = \{ (1\,2) \}$ ✓

$R_3 = \{ (2\,1) \}$ ✓

$R_4 = \{ (1\,1) \}$ ✗

$R_5 = \{ (1\,2) (2\,1) (2\,3) (3\,2) \}$ ✓

$R_6 = \{ (1\,3) (3\,3) \}$ IRR ✗

$A = \{1, 2, 3\}$.

(1, 1)	(1, 2)	(1, 3)
(2, 1)	(2, 2)	(2, 3)
(3, 1)	(3, 2)	(3, 3)

$\rightarrow$ 1 choice.
absent.

Total $= n^2$.
Diagonal $= n$.
non diagonal $= n^2 - n$.

$\hookrightarrow$ 2 choices $\rightarrow$ absent, present.

$\rightarrow \left\{ 2^{n^2 - n} \right.$

$\# R = 2^{n^2-n}$

$\boxed{11 \quad 22 \quad 33}$ (12)

$\boxed{11 \quad 22 \quad 33}$ (12) (21)

$\boxed{\quad}$

$\# IRR = 2^{n^2-n}$

$\boxed{\text{absent}}$ (12)

$\boxed{\text{absent}}$ (12) (21)

$A \rightarrow$ nonempty set.

$R = \{ \}$

not reflexive

Irreflexive ✓

HW discussion :

In a class of 3 students how many ways we can choose 2.

$\{s_1, s_2, s_3\} \to \{s_1, s_2\}$

$\{s_2, s_3\}$

$\{s_1, s_3\}$

${}^3C_2$

A =


THANK YOU GW
SOLDIERS !