COMPUTER SCIENCE

Database Management System

File Org. & Indexing

Lecture_5

Vijay Agarwal sir

TOPICS TO BE COVERED

01 Multi level Indexing

02 B & B+ Tree

B Tree :

ORDER : P [Maximum Number of Block | Child | Tree Pointer]

$Bp = P$

$key = P-1$

$Rp = P-1$

ORDER : P

Max keys $= P-1$

Min keys $= \lceil P/2 \rceil - 1$

Max BP $= P$

Min BP $= \lceil P/2 \rceil$

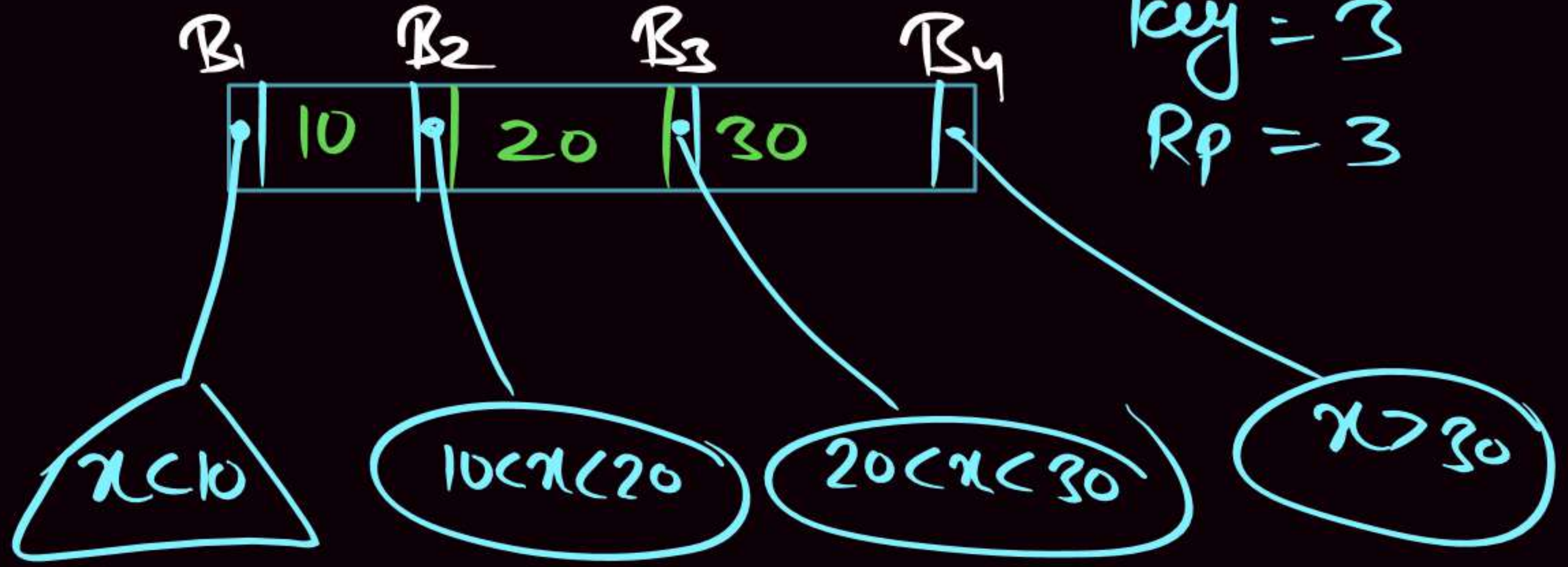
B Tree Definition

ORDER : P.

① Internal Node.

B_1	$K_1 R_1$	B_2	$K_2 R_2$		B_{P-1}	K_{P-1}, R_{P-1}	B_P

ORDER : 4



$B_p = 4$

$key = 3$

$R_p = 3$

ORDER : 5

Max Bp = 5

keys = 4

Rp = 4

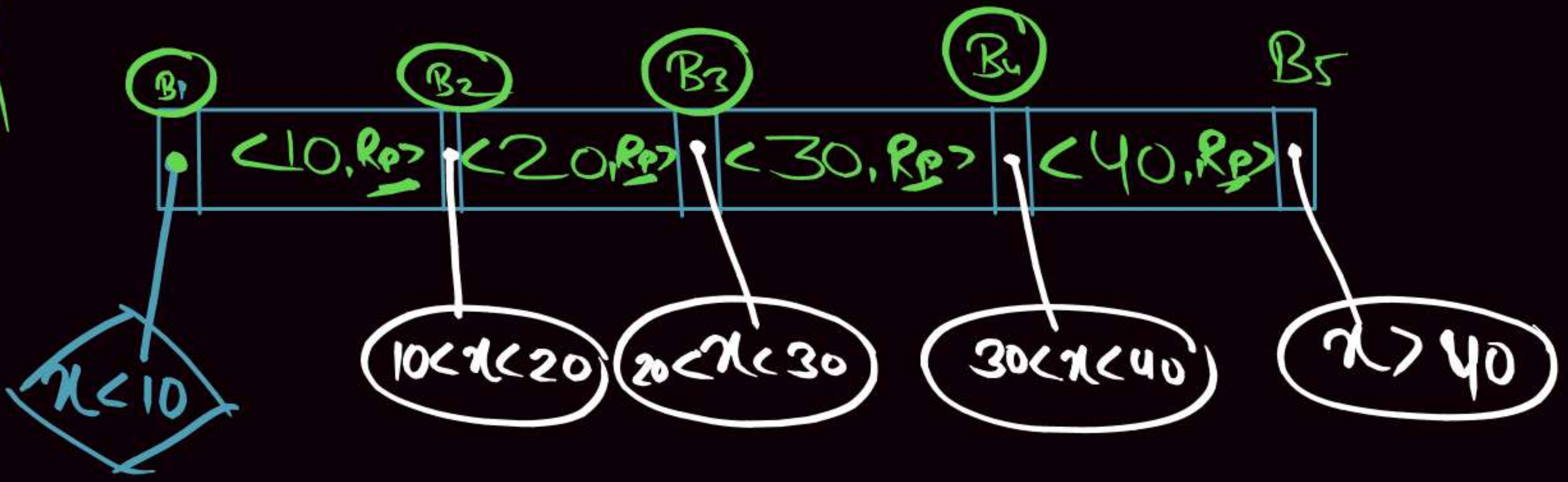
ORDER:5

Bp = 5

key = 4

Rp = 4

B_1	$K_1 R_1$	B_2	$K_2 P_2$	B_3	$K_3 R_3$	R_4	$K_4 R_4$	B_5



B Tree Definition

(2) Structure of Leaf Node:

(3) Every Internal Node except the Root Node Contain atleast min. $\lceil P/2 \rceil$ Block Pointer ($\lceil P/2 \rceil - 1$ keys) & Maximum P Block Pointer ($P-1$ keys)

min $\geq 50\%$ Filled except Root

B Tree Definition

(4) Root Contain at least 2 Block Pointer (min 1 key) & at most (Maximum) P Block Pointer (P-1 keys).

(5) keys within the Node Should be in ascending order.

(6) Every Leaf Node Should be at Same Level.

---

if all 6 Condition Satisfy then its Correct B Tree ✓ of order P.

B Tree

Bp : Block | Tree | Index | Child Pointer.

K : Search key | key.

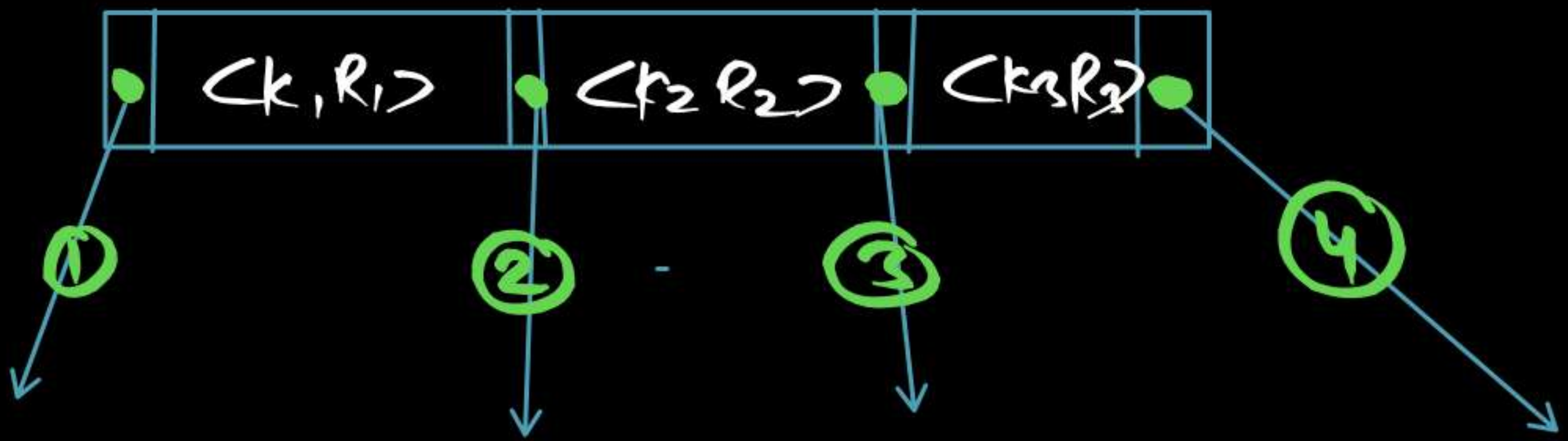
Rp : Data | Record | Read order.

P : ORDER P.

B Tree Definition

ORDER : P.

ORDER : $\underline{4}$



Max $Bp = 4$

keys $= 3$

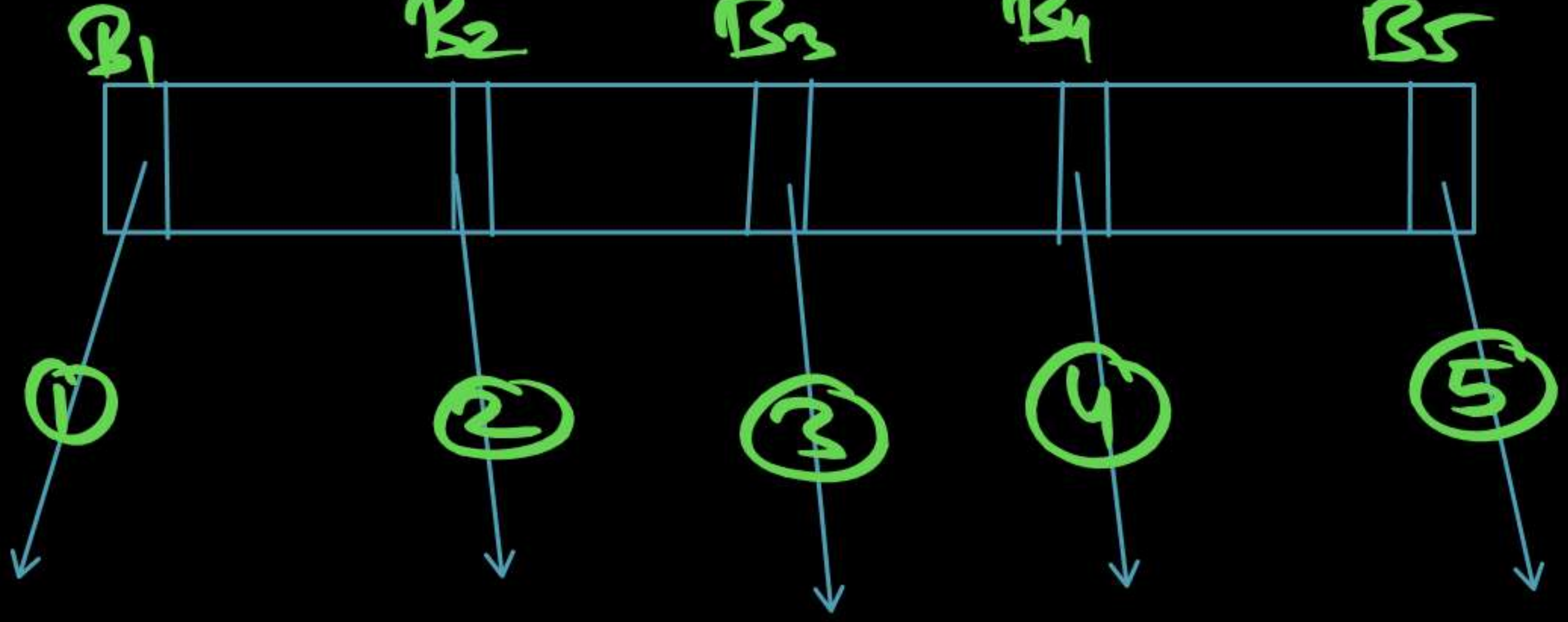
$Rp = 3$

Minimum keys $= \lceil P/2 \rceil - 1 \Rightarrow \lceil 4/2 \rceil - 1 = 1$

Maximum keys $= P - 1 = \underline{3}$

B Tree Definition

ORDER: 5.



<u>Max</u> $B_p = 5$

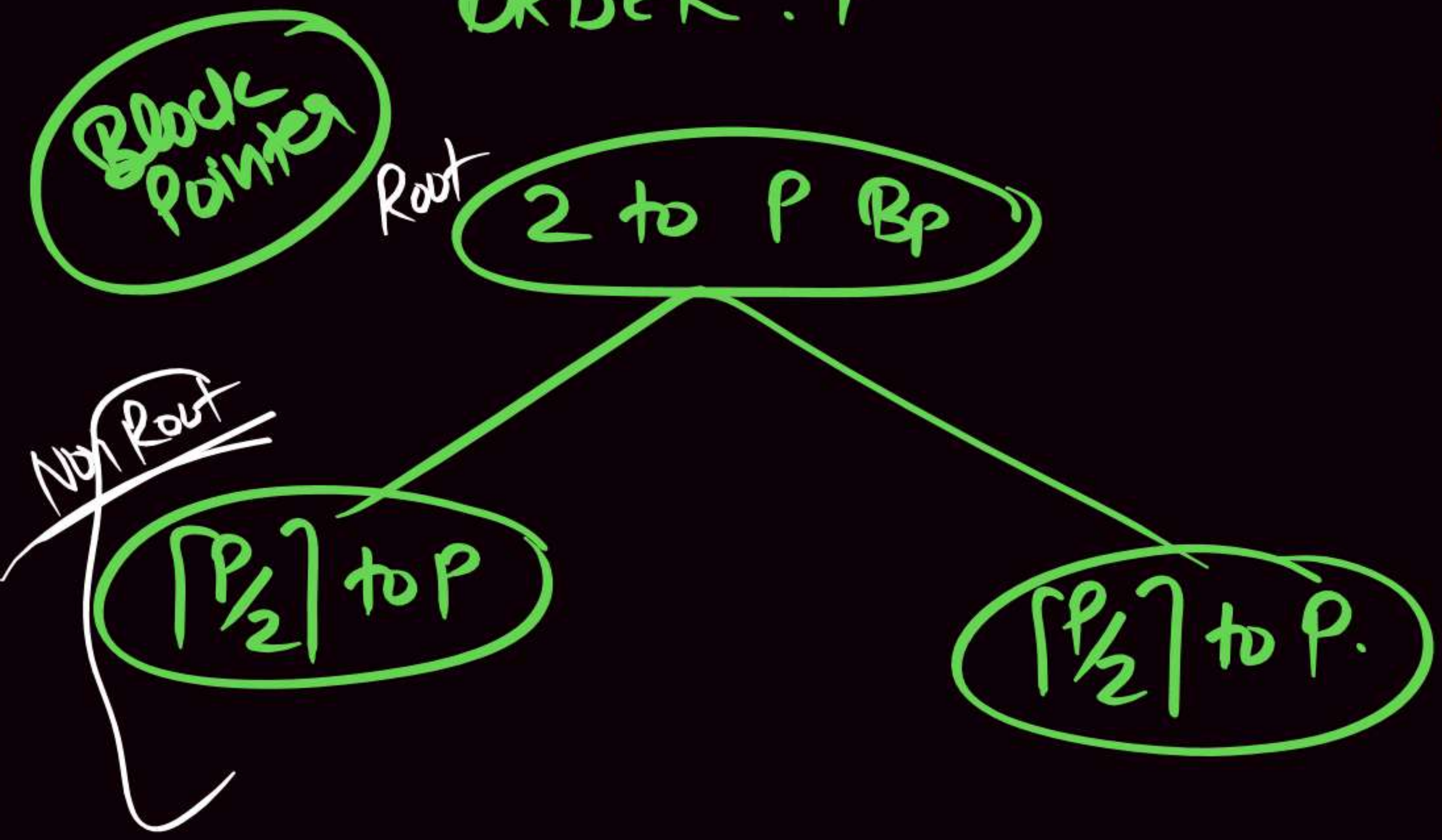
keys $= 4$

$R_p = 4$

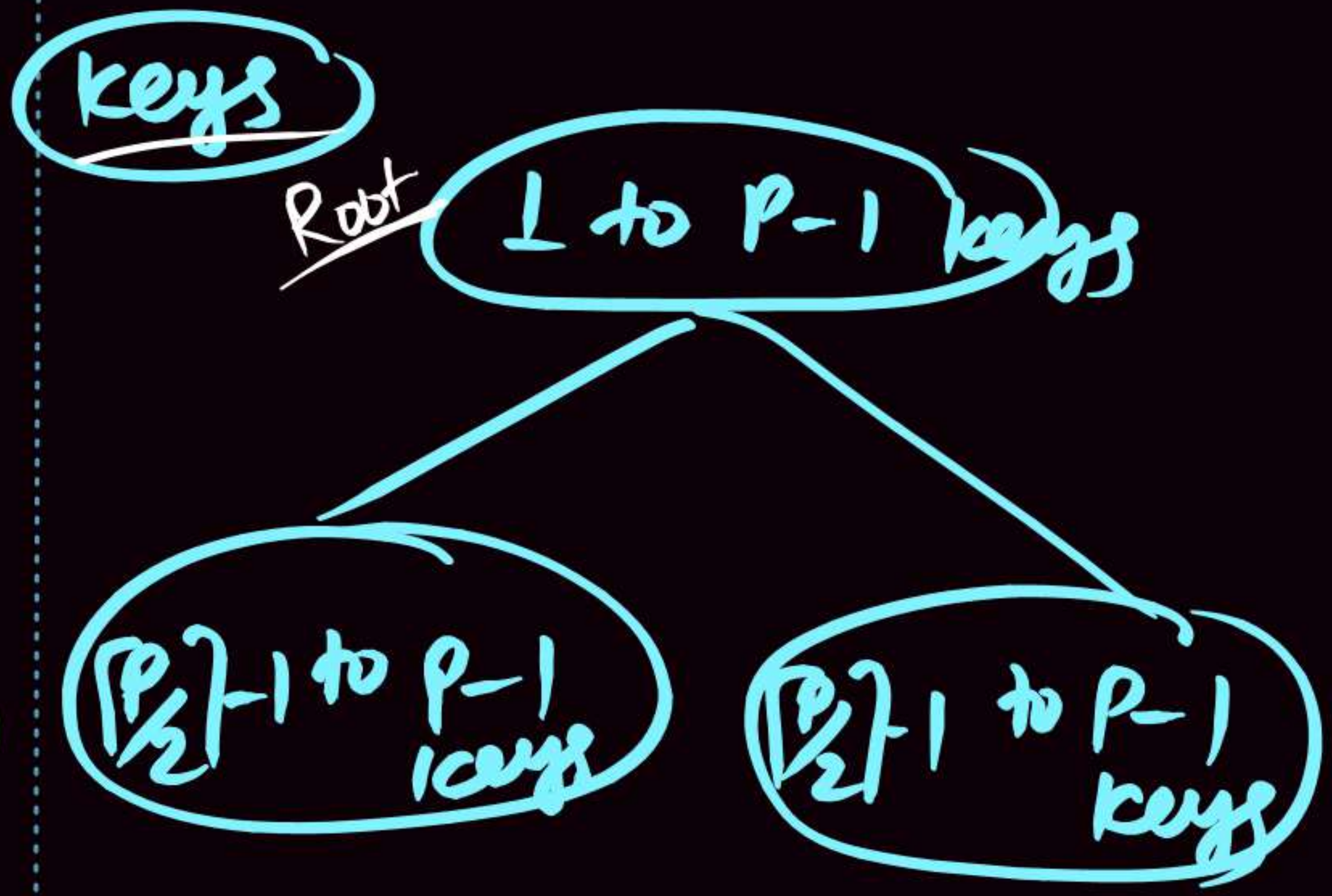
Minimum keys $= \lceil 5/2 \rceil - 1 \Rightarrow 3 - 1 = 2$ keys.
except Root Node!

Max $= p - 1 = 5 - 1 = 4$.

ORDER : P



ORDER : P



Q) Can Root have Only One Block Pointer?

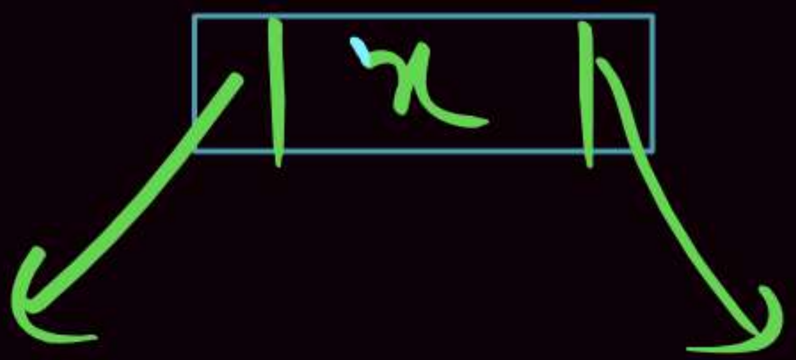
Sol^n Never / ~~

NO

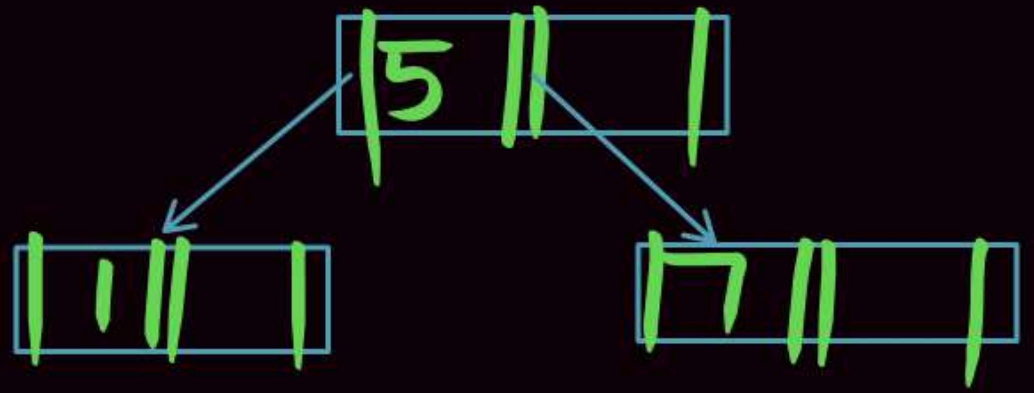
1, 5, 7, 9

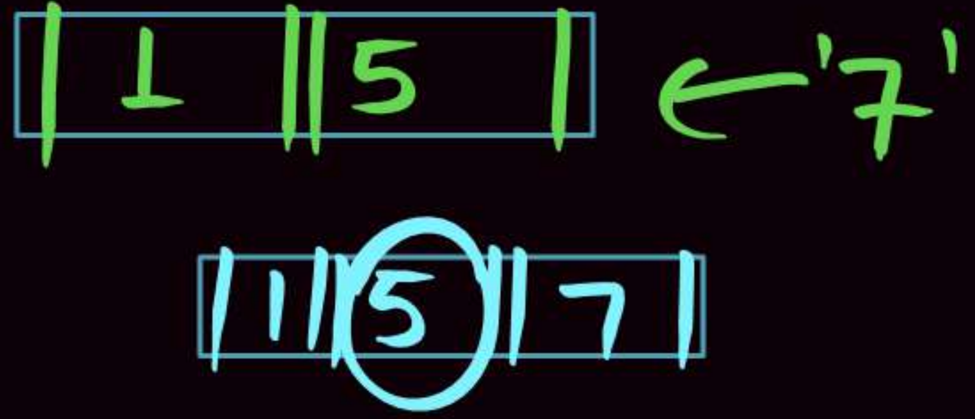
ORDER: 3

Max keys = 2



eg





ORDER : P.

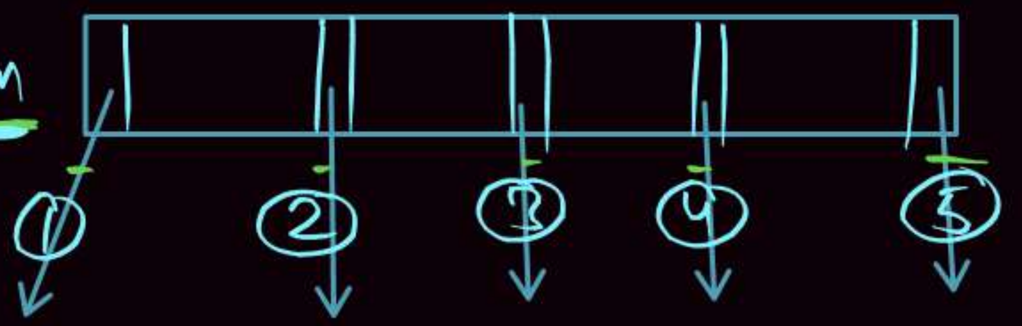
		Minimum	Maximum.
Block Pointer.	Root : (Bp)	2	P
	Non Root (Bp)	$\lceil P/2 \rceil$	P
keys	Root(keys)	1	P-1
	Non Root (keys)	$\lceil P/2 \rceil - 1$	P-1

ORDER : P. [5]

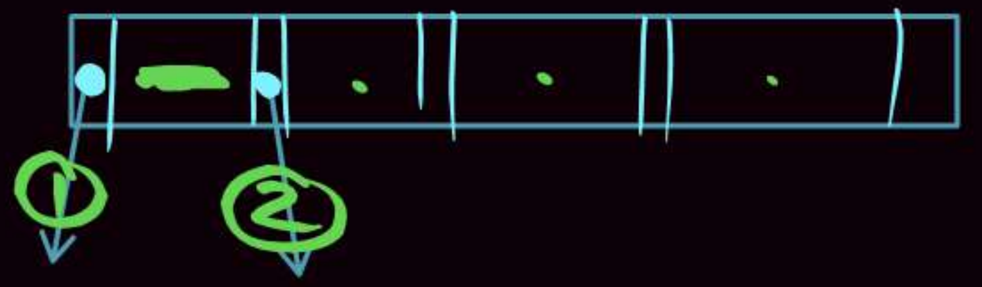
		Minimum	Maximum.
Block pointer	Root : (Bp)	2	5
	Non Root (Bp)	$\lceil \frac{5}{2} \rceil = 3$	5
keys	Root(keys)	1	4
	Non Root (keys)	$\lceil \frac{P}{2} \rceil - 1$ $\lceil \frac{5}{2} \rceil - 1 = 2$	4

ORDER : 5.

① Maximum

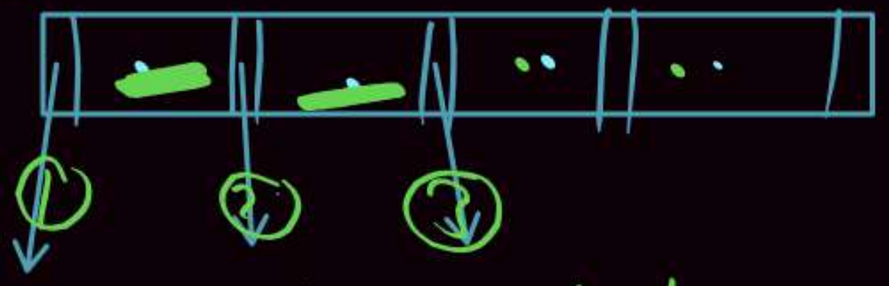


② Root



	min	Max
ORDER : 5 Root :	1 key to	4 keys
Non Root :	2 key to	4 keys.

Non Root :

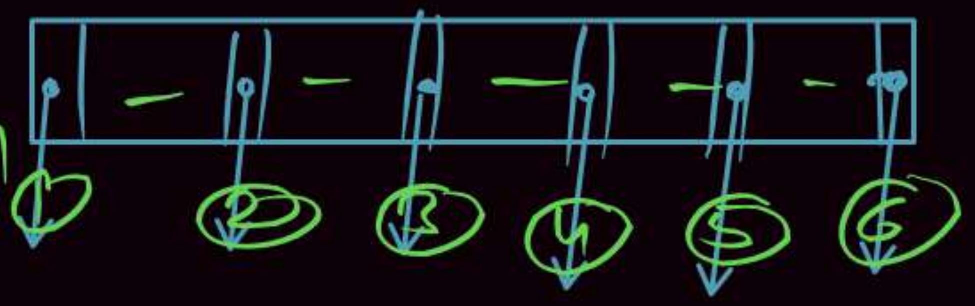


50% occupied

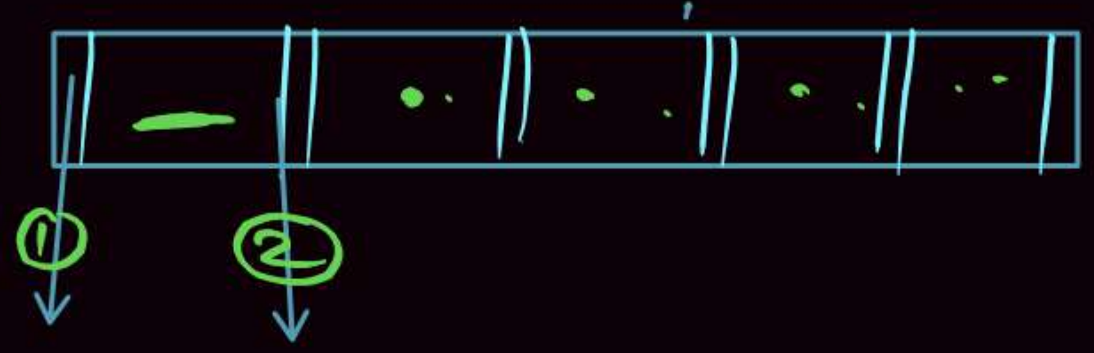
ORDER : P. [6]

		Minimum	Maximum.
Block Pointer	Root : (Bp)	2	6
	Non Root (Bp)	$\lceil 6/2 \rceil = 3$	6
Keys	Root(keys)	1	5
	Non Root (keys)	$\lceil 6/2 \rceil - 1 = 2$	5

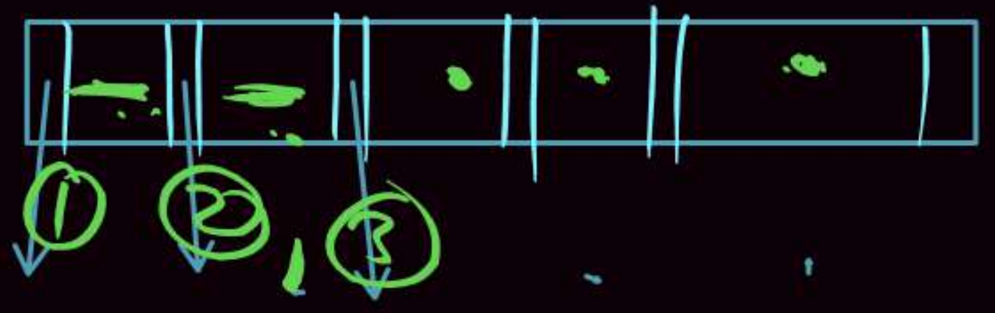
① Maximum



② ROOT



③ Non Root



ORDER:6

min

Root: 1 key to 5 keys.

Non Root: 2 key to 5 keys.

ORDER : P. [7]

		Minimum	Maximum.
Block Pointer	Root : (BP)	2	7
	Non Root (BP)	$\lceil 7/2 \rceil = 4$	7
keys.	Root(keys)	1	6
	Non Root (keys)	$\lceil 7/2 \rceil - 1 = 3.$	6.

ORDER : 7

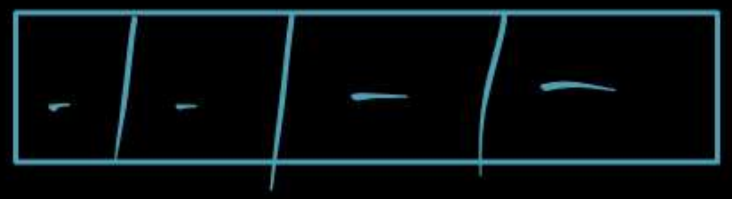
	min		max
Root :	1 key	to	6 keys.
Non Root :	3 key	to	6 keys.

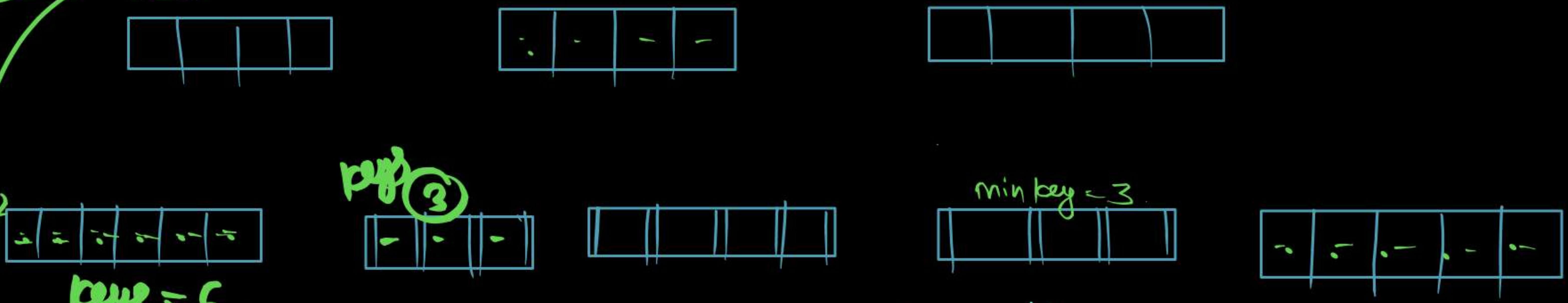
B Tree Definition

(Q) Find the order of below Diagram?

Soln 7

Q.1 Can we have order 8 in this Diagram?





keys = 6

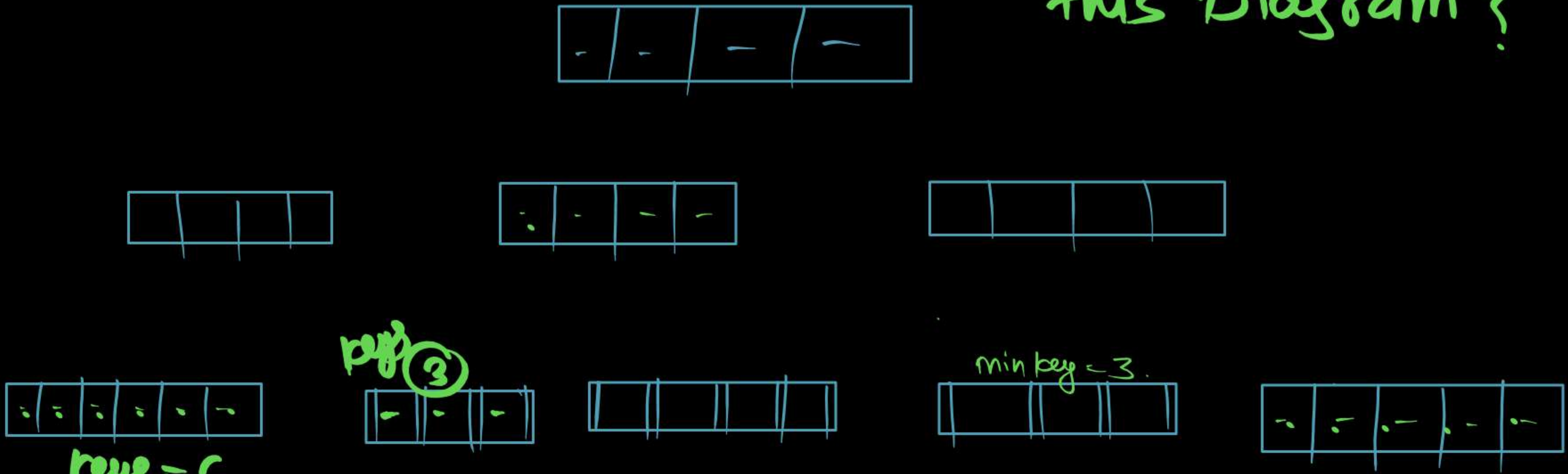
Bp = 7

YES Order 8 also Possible.

WHY?

B Tree Definition

Q.1 Can we have order 8 in this Diagram?



Bp = 7

YES order 8 also Possible.

WHY?

Sol^n 1

if order = 8

Max Bp = 8 [p]

min Bp = $\lceil 8/2 \rceil$ = 4

Max keys = 8-1 = 7

min keys = $\lceil 8/2 \rceil - 1 \Rightarrow 4-1 = 3$

Non Root

min Bp

ORDER: 8

Min (Range) Max

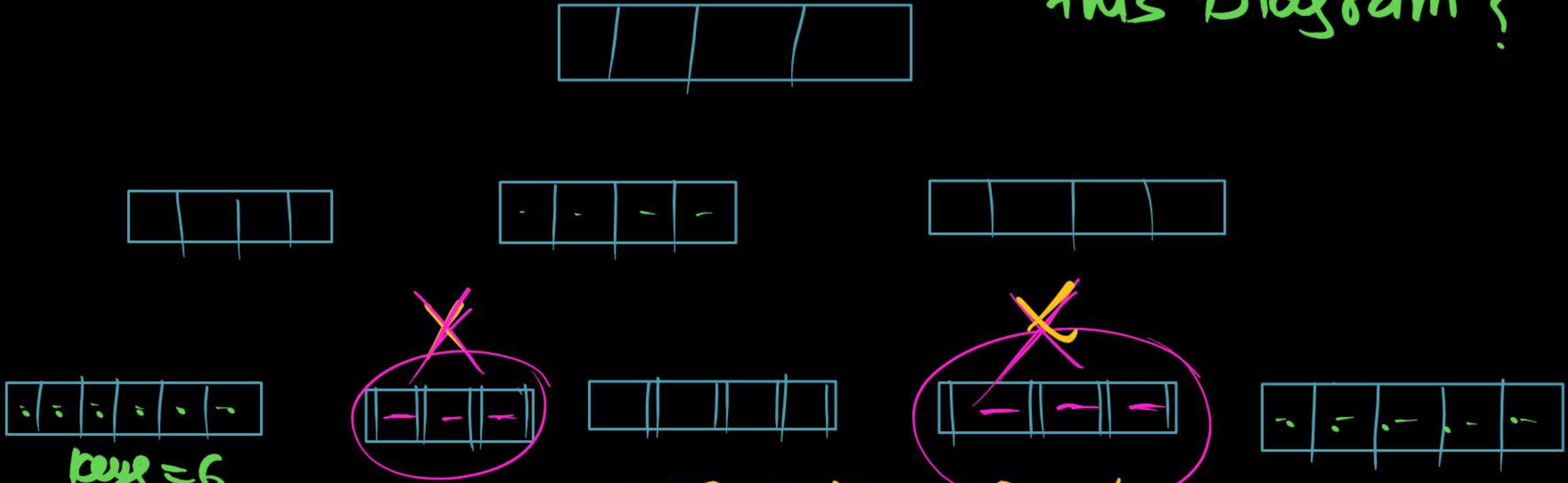
3 key to 7 keys.

4 Bp to 8 Bp

So order Can be 8 also

B Tree Definition

Q.2 Can we have order 9 in this Diagram?



keys = 6

Bp = 7

ORDER: 9 ✗ Not Possible

Bcz min keys = $\lceil 9/2 \rceil - 1 \Rightarrow 5 - 1 = 4$ min keys if order = 9

$$\text{Min Bp} = \left\lceil \frac{9}{2} \right\rceil = 5$$

$$\text{Max Bp} = 9$$

$$\text{Max keys} = 9 - 1 = 8.$$

$$\text{Min keys} = \left\lceil \frac{9}{2} \right\rceil - 1 = 5 - 1 = 4$$

ORDER: 9

Non Root node

	Min	Max
keys:	4 key	to 8 keys
Bp:	5 Bp	to 9 Bp.

B Tree Definition

ORDER : P.

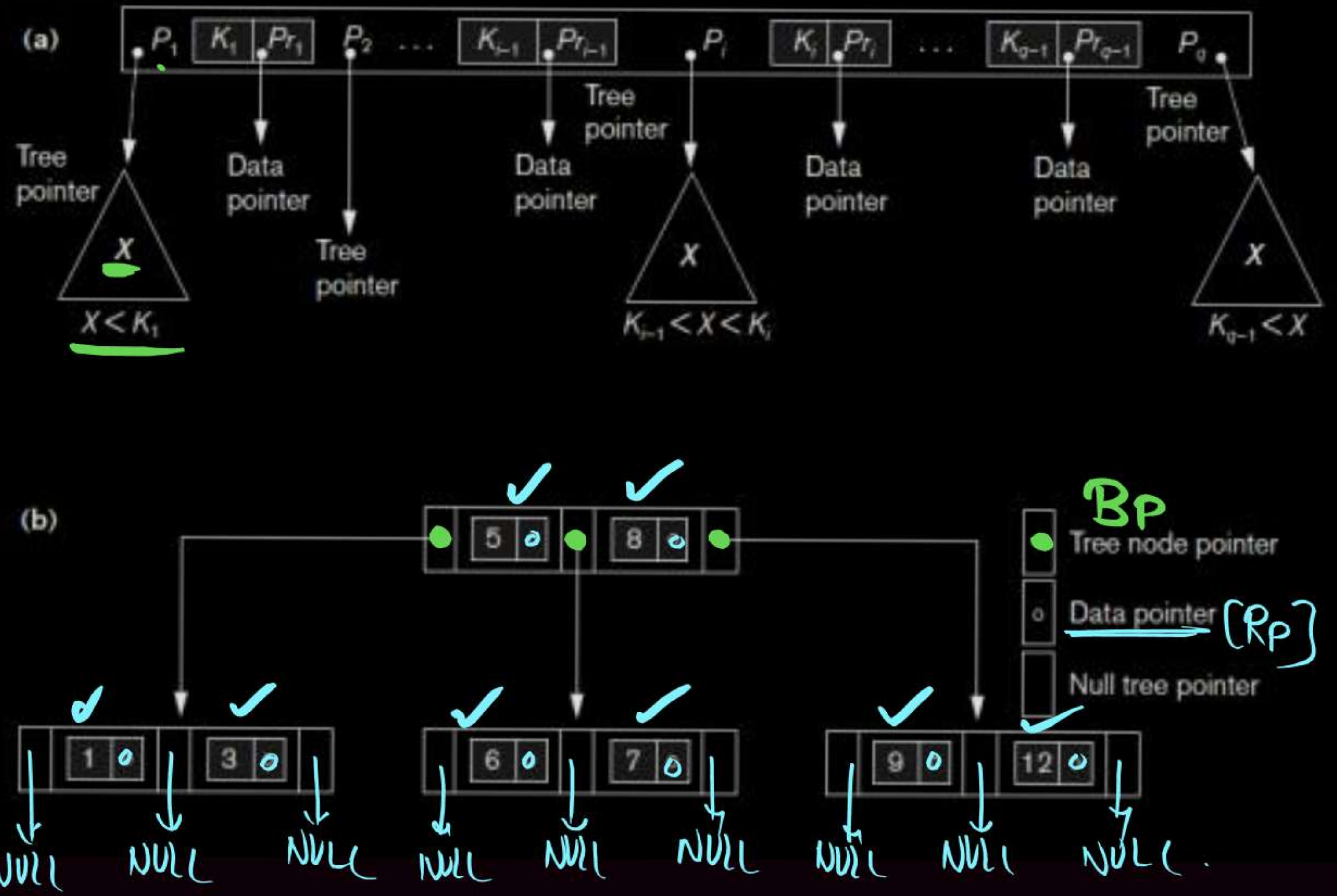
B_1	$K_1 R_1$	B_2	$K_2 R_2$	B_3	$K_3 R_3$	. . . . . .	B_{P-1}	$K_{P-1}\ R_{P-1}$	B_P

$$\boxed{P * B_P + (P-1)(\text{keys} + R_P) \le \text{Block size}}$$

B Tree Structure

ORDER : q.

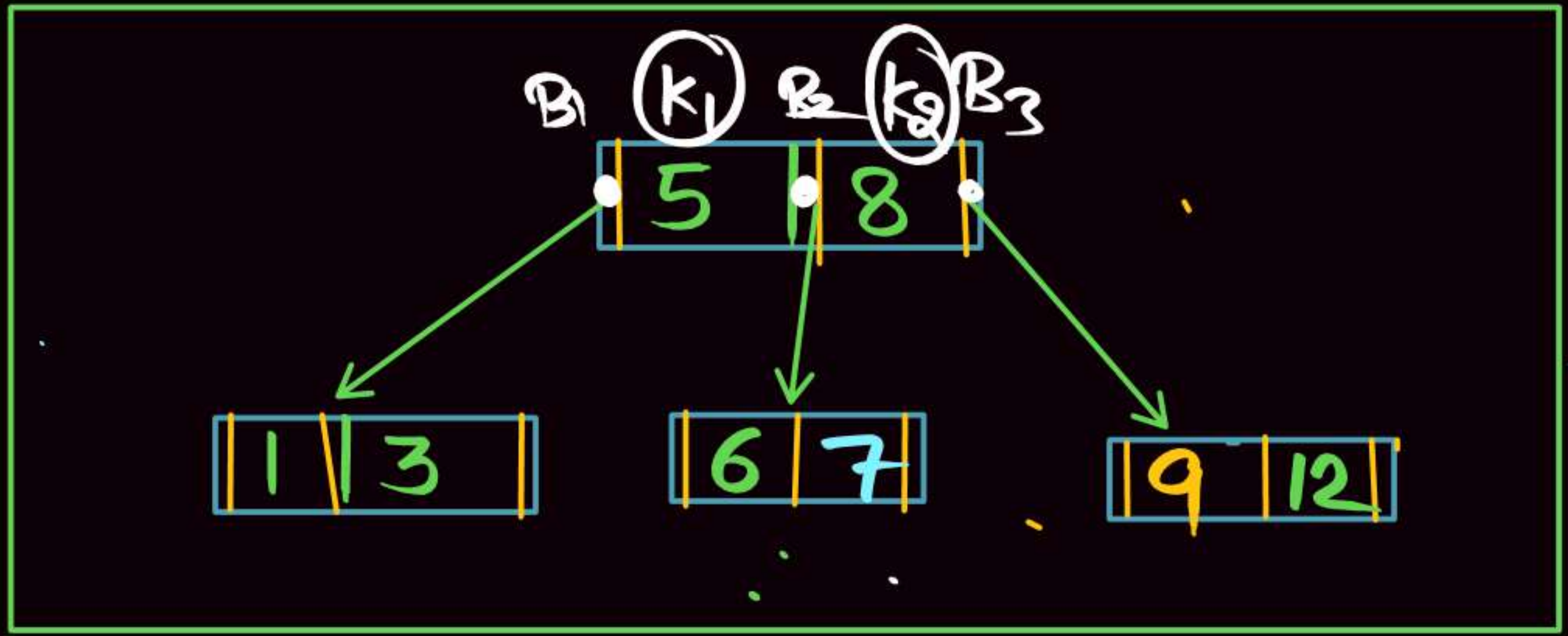
B-tree structures (a) A node in a B-tree with q−1 search values (b) A B-tree of order p=3. The values were inserted in the order 8, 5, 1, 7, 3, 12, 9, 6



8, 5, 1, 7, 3, 12, 9, 6.

ORDER: 3

max = 2 key.



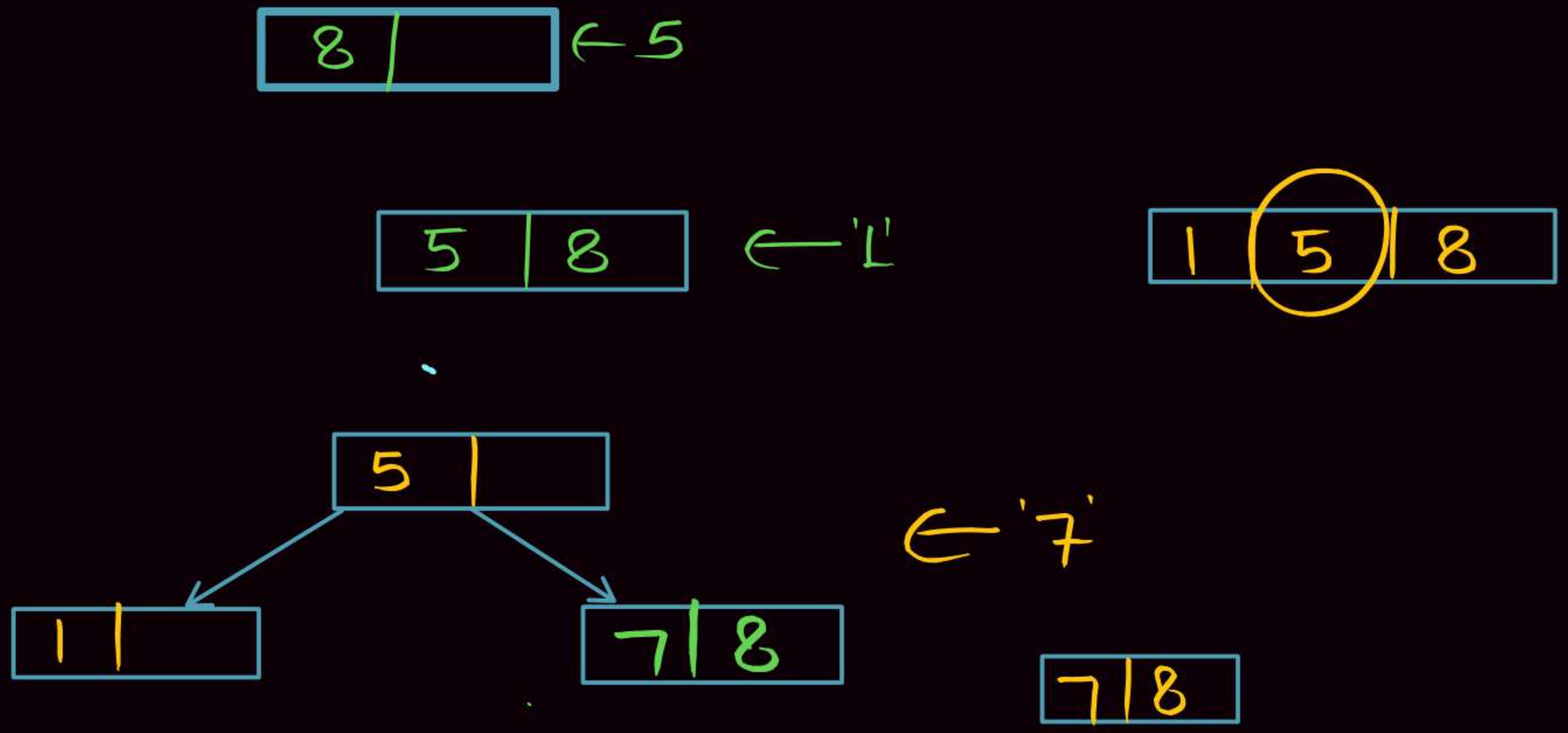
Avg

8, 5, 1, 7, 3, 12, 9, 6.

ORDER: 3

max = 2 key.

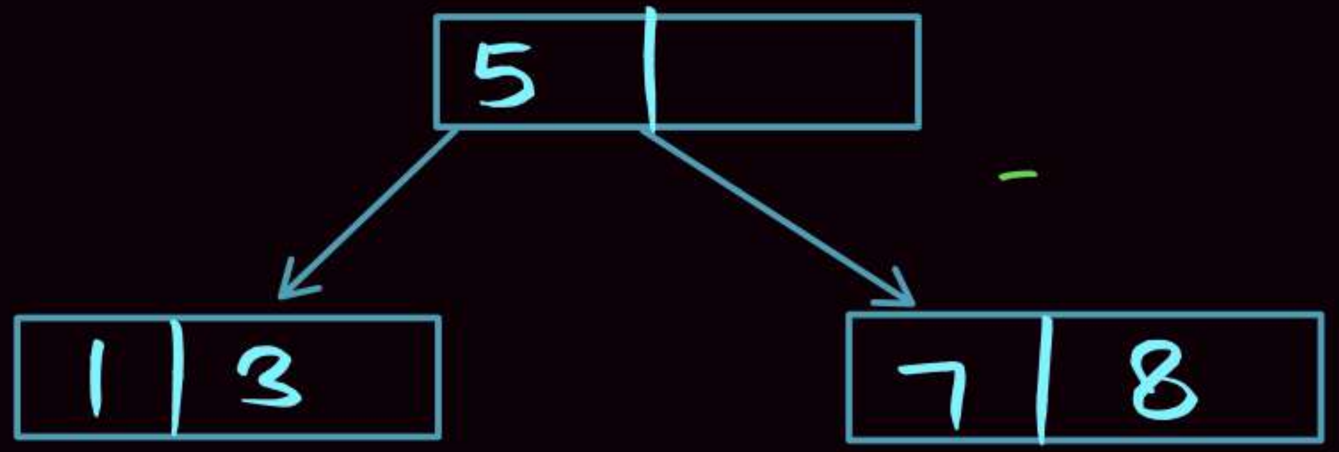
Draw B Tree

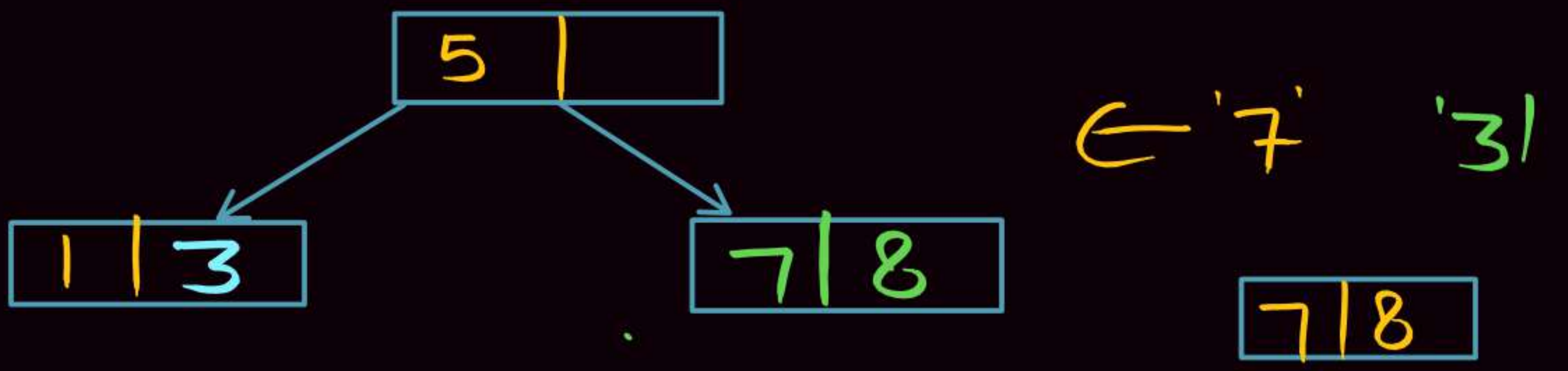


8, 5, 1, 7, 3, 12, 9, 6.

ORDER: 3

max = 2 key.

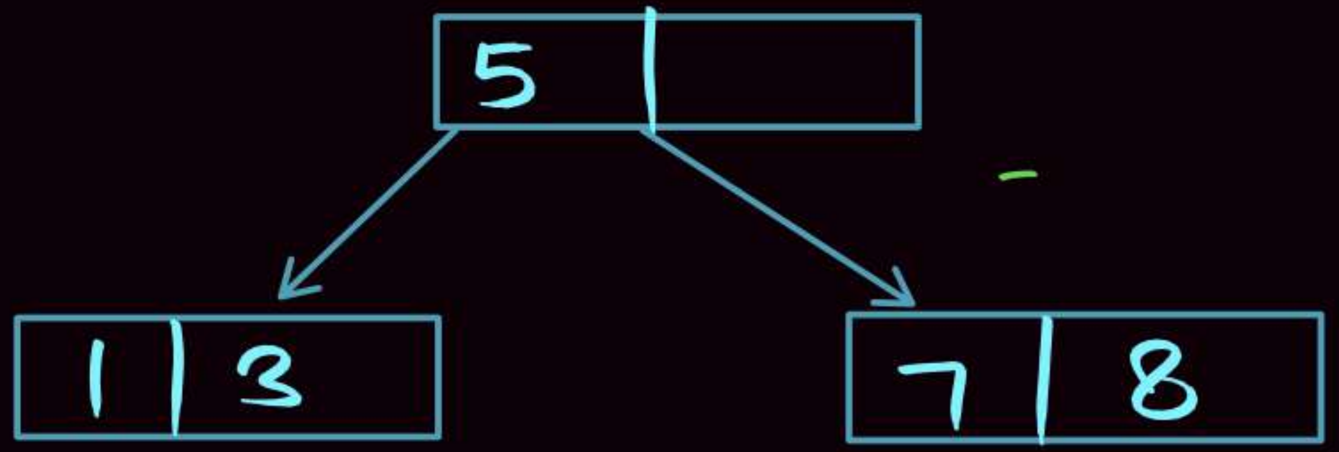


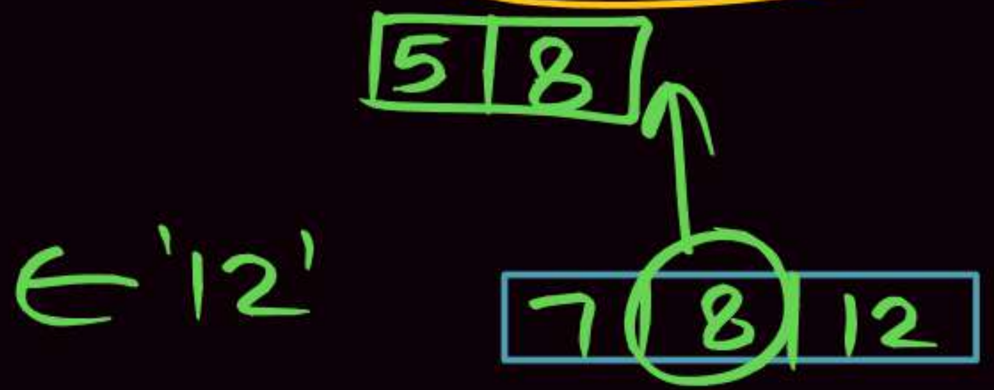


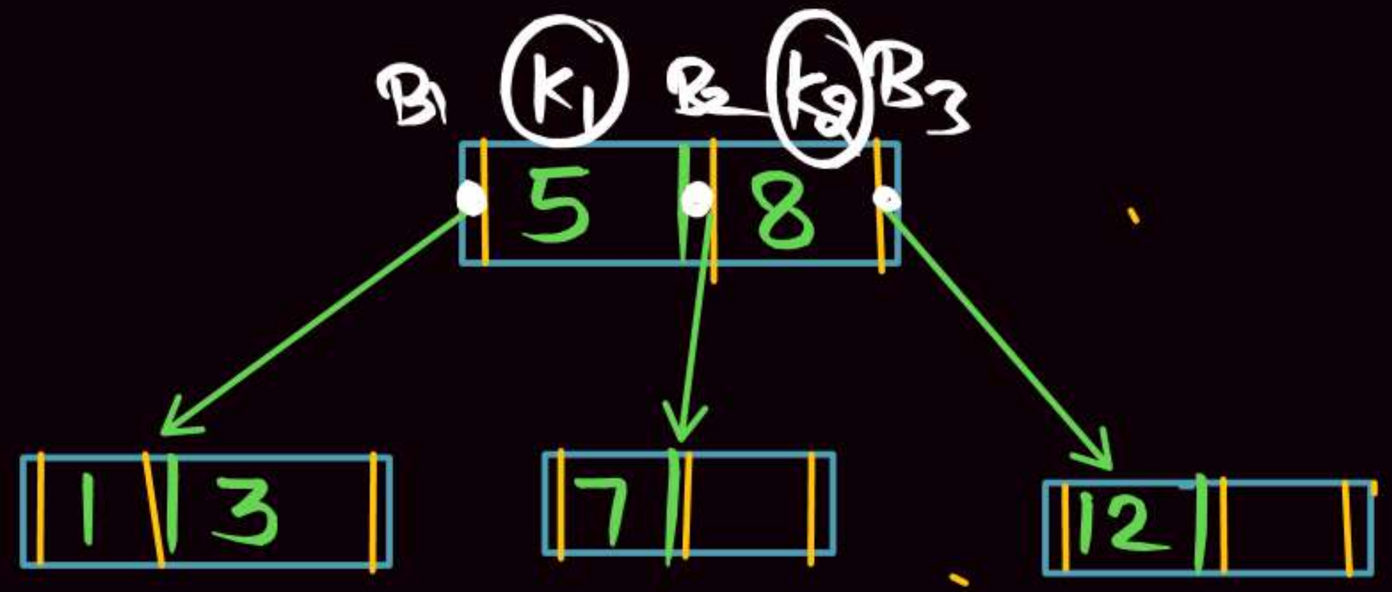
8, 5, 1, 7, 3, 12, 9, 6.

ORDER: 3

max = 2 key.



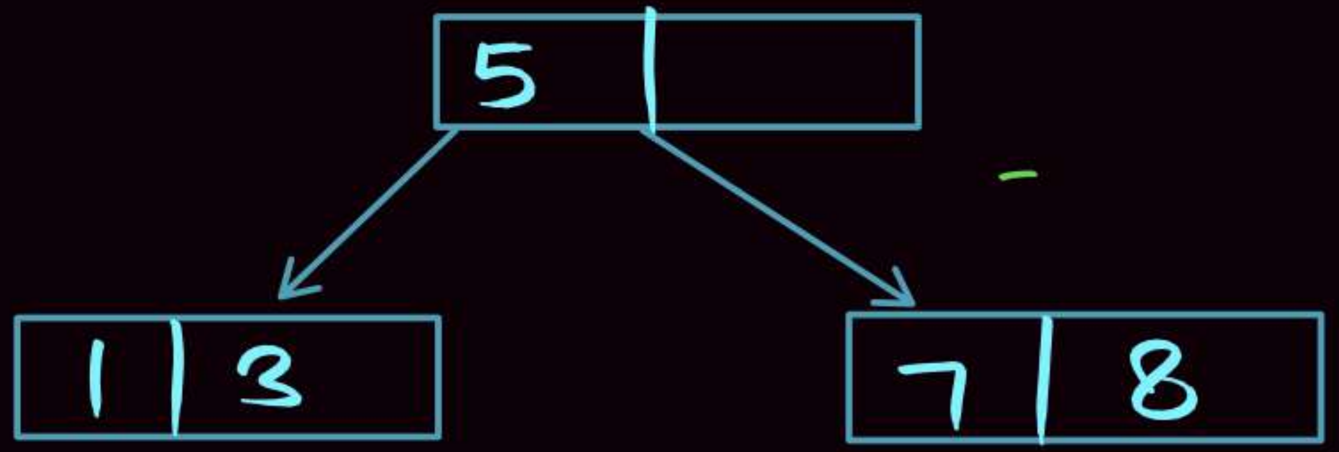




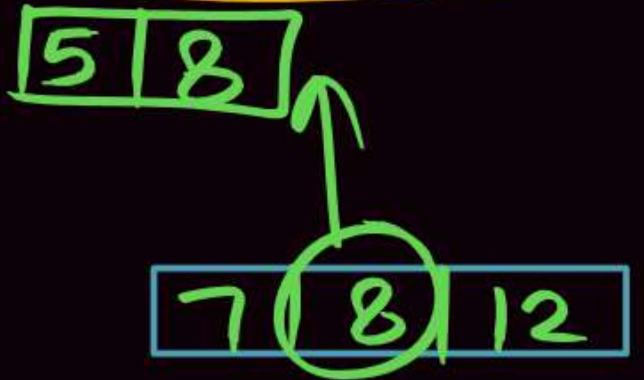
8, 5, 1, 7, 3, 12, 9, 6.

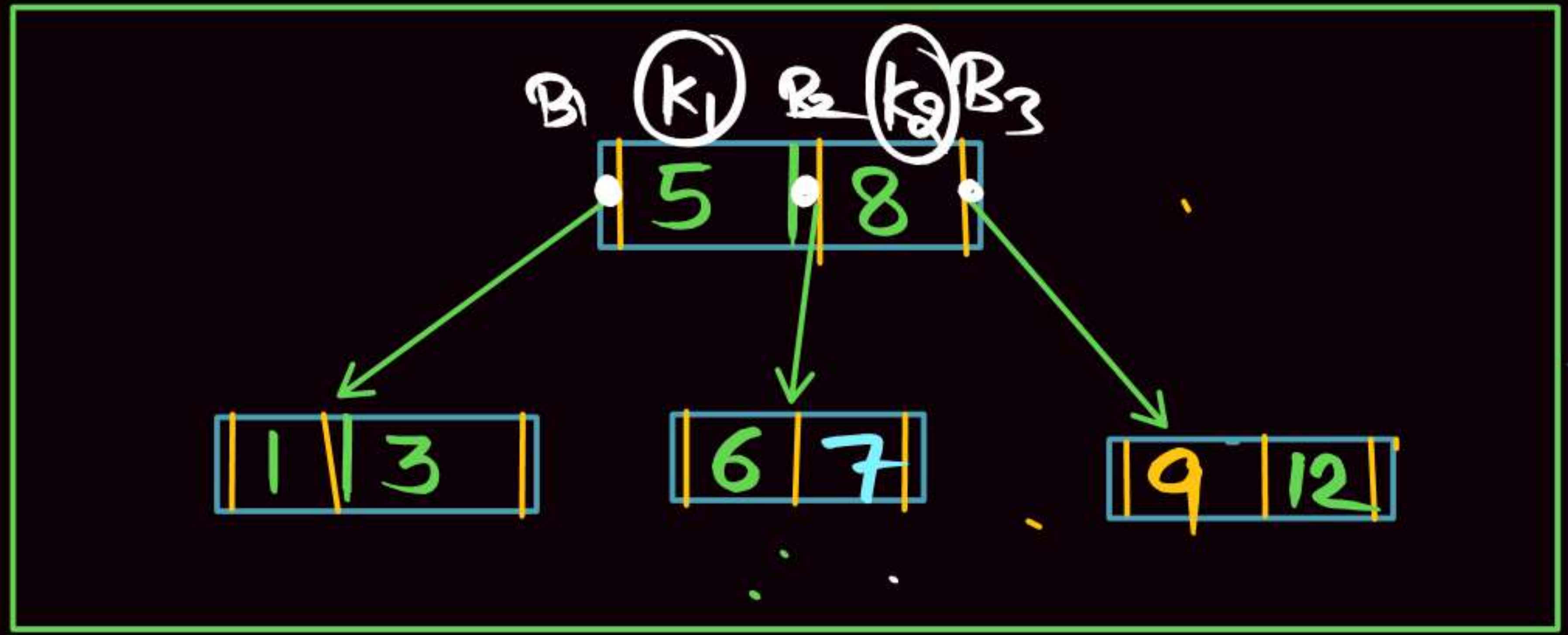
ORDER: 3

max = 2 key.



← '12'



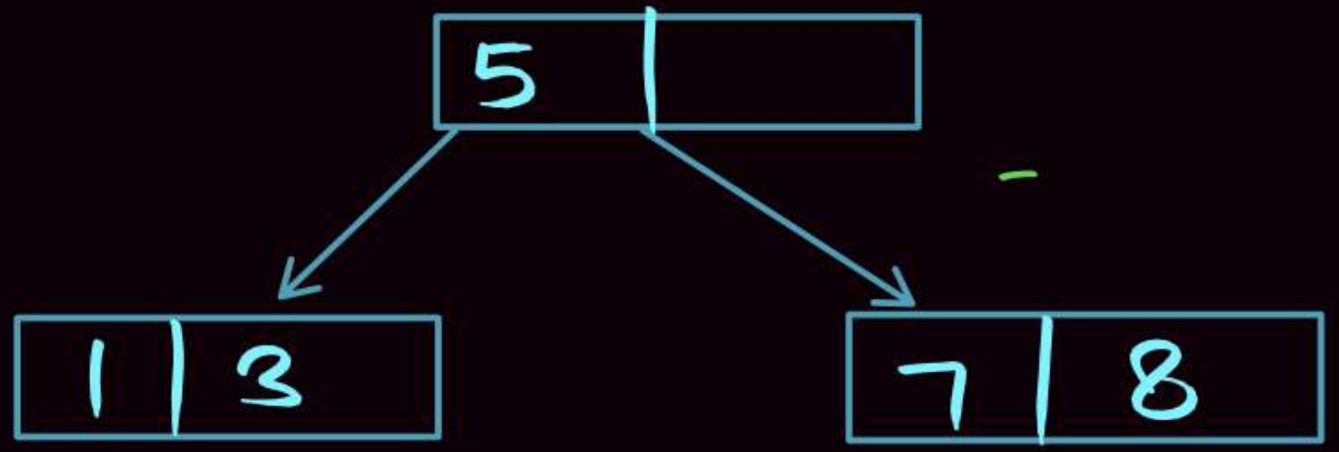


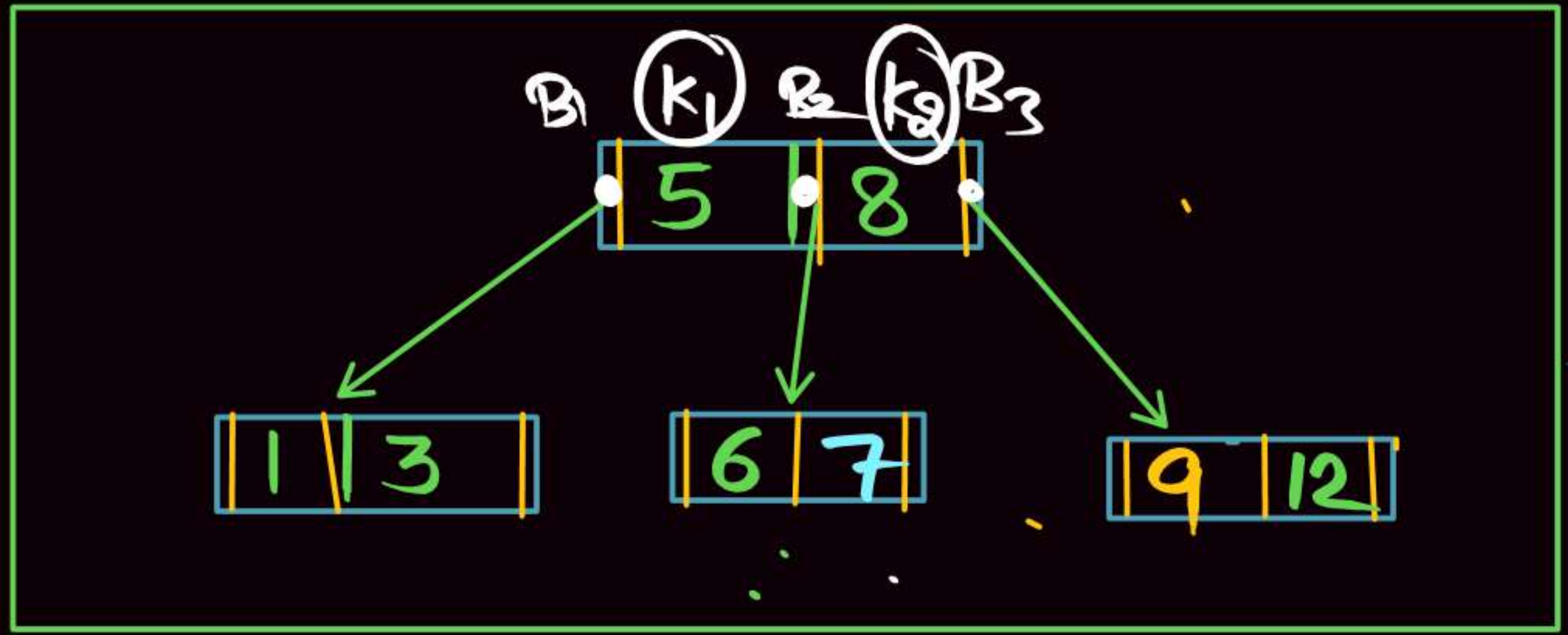
Ans

8, 5, 1, 7, 3, 12, 9, 6.

Q.L

for '9' search How Many Block Access

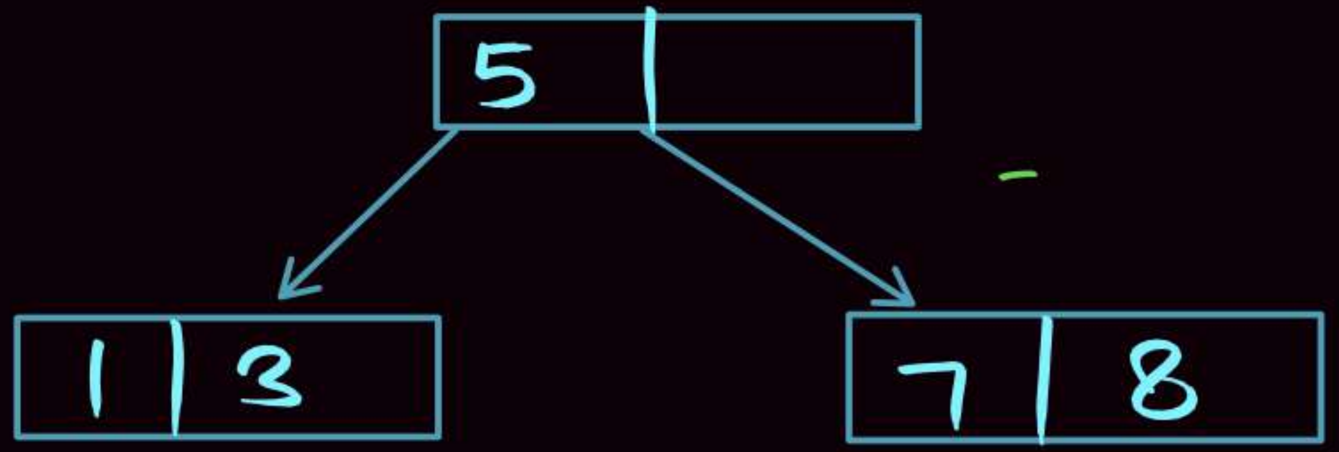


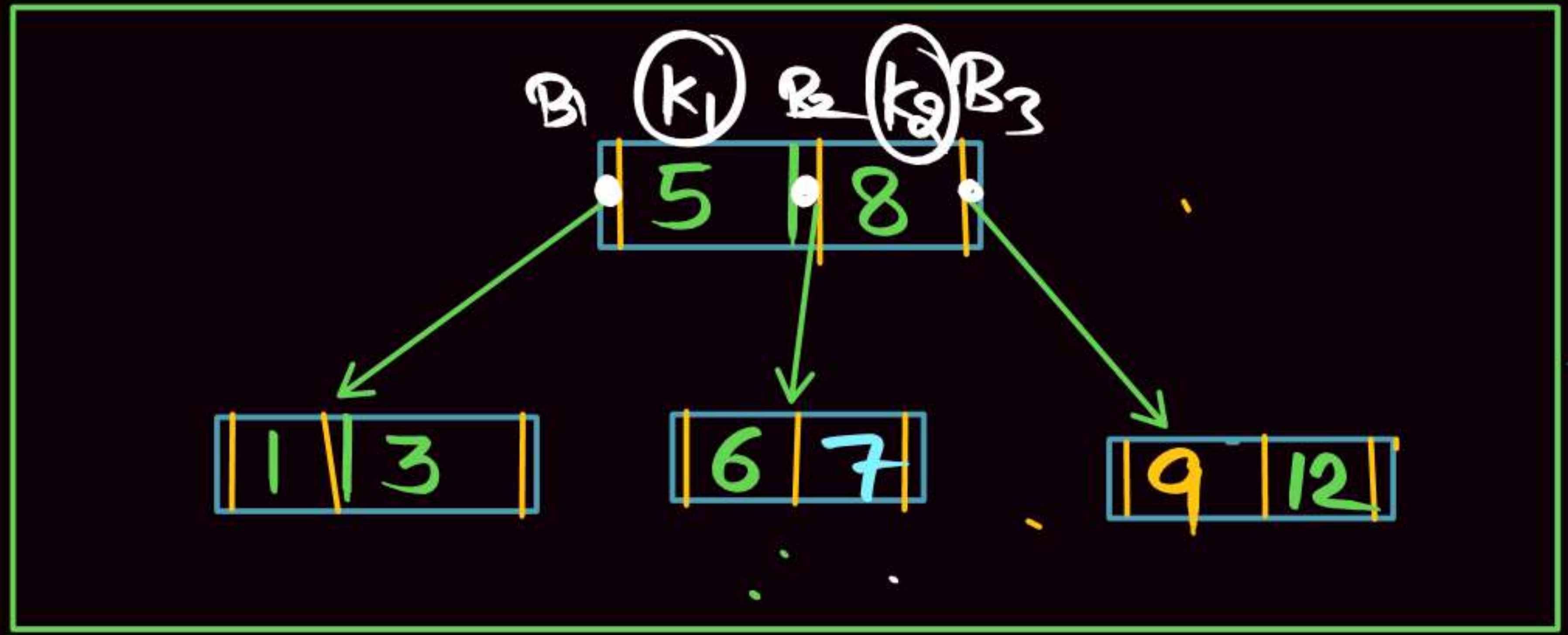


Soln 2.

A.

8, 5, 1, 7, 3, 12, 9, 6.





Q.L

for '13' search

How Many Block Access

Soln 2.

unsuccessful.

A.

2 marks.
GATE

Note If Unsuccessful Search that means we traverse from Root to till Leaf Node.

Note If Successfull Search that means we traverse from Root to till that key find (that Block in which key is Present).

GATE

ORDER P: By Default Max P Block Pointer.

Bp: P

By Default

$$\boxed{P * B_p + (P-1)(\text{keys} + R_p) \le \text{Block Size}.}$$

Ans

ORDER: P(key) Key order: P

key order: P.

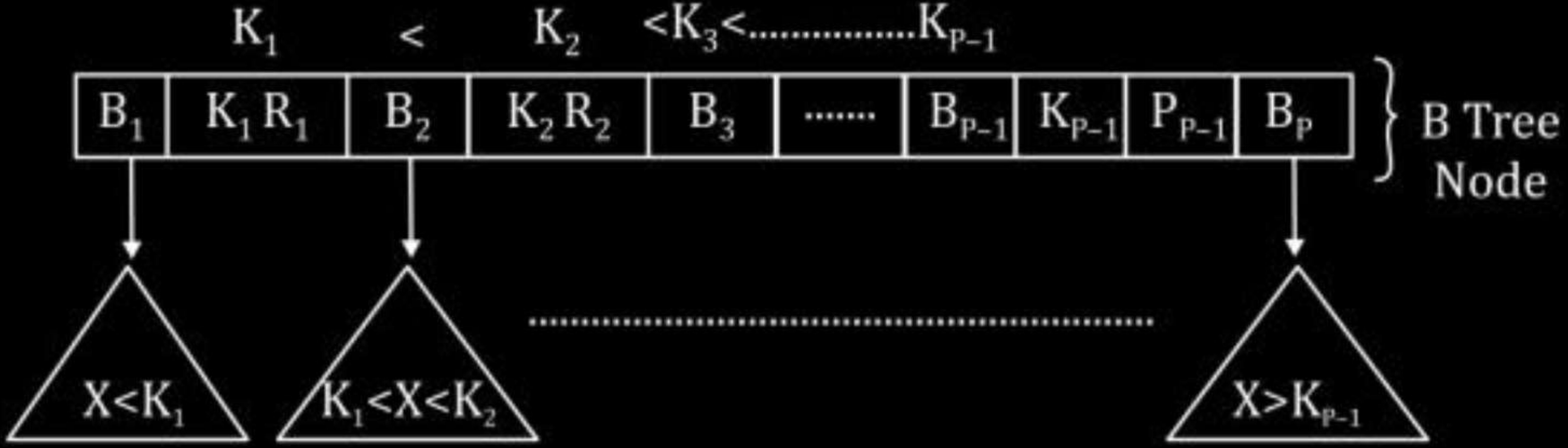
$$\boxed{(P+1)B_p + P(\text{keys} + R_p) \le \text{Block Size}.}$$

Ans

B Tree Definition

Order P: Max possible child pointers
[degree] can store in B Tree node.

(1) Node structure:



<P.Cp, (p-1)keys, (p-1)Rp>

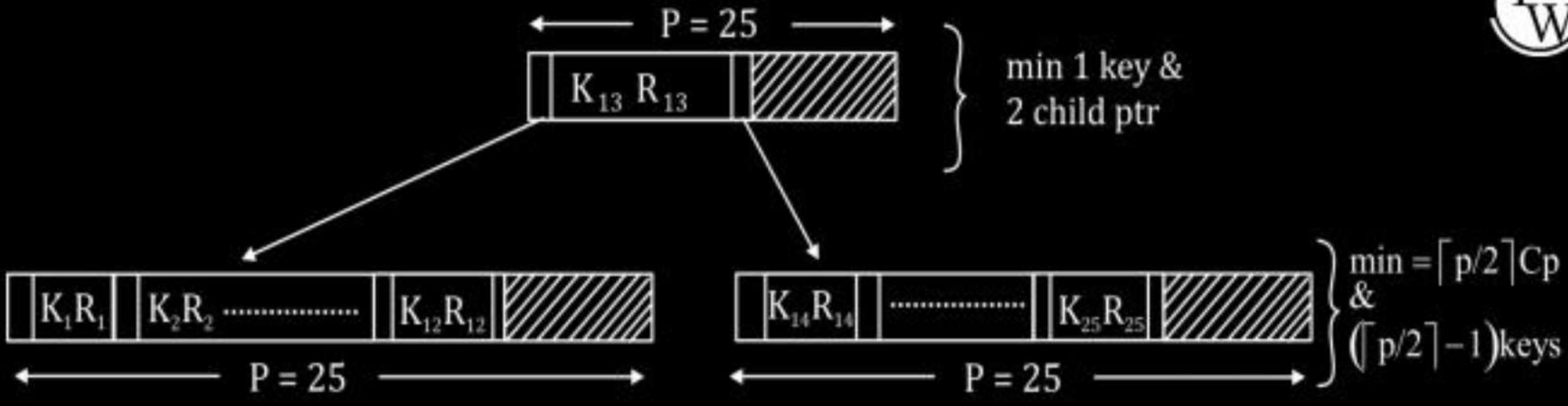
(2) Every internal node except root must be at least $\lceil P/2 \rceil$ child pointer with $(\lceil P/2 \rceil - 1)$ key's and at most P children pointer with (P - 1) key's must.

(3) Root node can be at least 2 children with 1 key, at most P children pointer and (P - 1) key's.

(4) Every leaf node must be at same level and keys with in node should be sequence order (Increasing order) Order P: 25 [Max Cp in node]

$\Downarrow$ Node Splitting

$$\underbrace{K_1 < K_2 < K_3 \ldots\ldots <}_{\text{left Subtree}} K_3 \underbrace{< K_{14} \ldots\ldots K_{25}}_{\text{Right Subtree}}$$



except Root Every
index node
atleast 50% occupied
in B/B$^+$ Tree index

Block B tree node
[Free]

78, 52, 81, 40, 33, 90, 85, 20, 38

ORDER: 3

Q.3 Consider a table T in a relational database with a key field K. A B-tree of order p is used as an access structure on K, where p denotes the maximum number of tree pointers in B-tree index node. Assume that K is 10 bytes long; disk block size is 512 bytes; each data pointer P_D is 8 bytes long and each block pointer P_B is 5 bytes long. In order for each B-tree node to fit in a single disk block, the maximum value of p is [GATE-2004 : 2 Marks]

(A) 20
(B) 22
(C) 23
(D) 32

key = 10B, Rp : 8B   Bp : 5B.   Block Size = 512 Byte.

ORDER : P.

$$P * B_p + (P-1)\,\text{keys} + (P-1)R_p \le \text{Block Size}$$

OR

$$P * B_p + (P-1)[\text{keys} + R_p] \le \text{Block Size}.$$

$$P*5 + (P-1)[10+8] \le 512$$

$$5P + (P-1)(18] \le 512$$

$$5P + 18P - 18 \le 512$$

$$23P \le 530$$

$$P = \left\lfloor \frac{530}{23} \right\rfloor = \lfloor 23.04 \rfloor = 23 \text{ Ans}$$

Note

if we take 24 then we put the value of order (P = 24) then its exceed by block size 512 Byte

P = 24 OR

it can not accomodate in a Block of Size 512 Byte

$p * B_p + (p-1)(key + R_p)$

$24 * B_p + (24-1)(key + R_p)$ > greater than Block Size (512 Byte).

$24 * 5 + 23(18)$

$120 + 414$

$= 534$ Byte > But given Block Size is 512 Byte

So Maximum order is 23 Not 24.

Advantage:

1) B Tree index best suitable for Random access of some record.

select *

FROM R

WHERE A = 24 ;

One record access

I/O cost: K + 1 blocks

$\approx [\log_P n] + 1 = \theta(\log_P n)$

Disadvantage:

1) B Tree index not best suitable for sequency access of range of records.

select *

FROM R

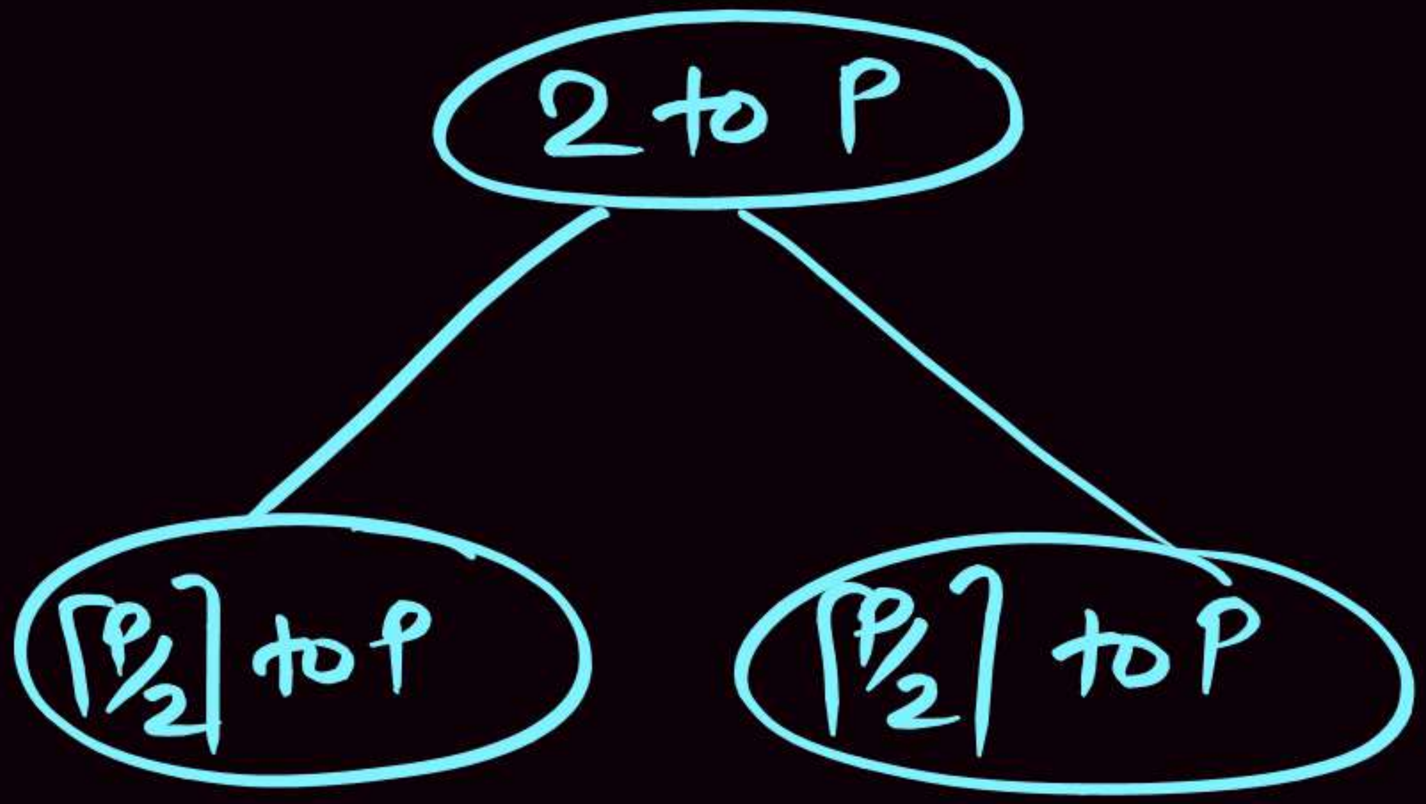
Range of record
Access required

WHERE $\boxed{A \geq 30 \text{ and } A \leq 85;}$

X blocks of DB

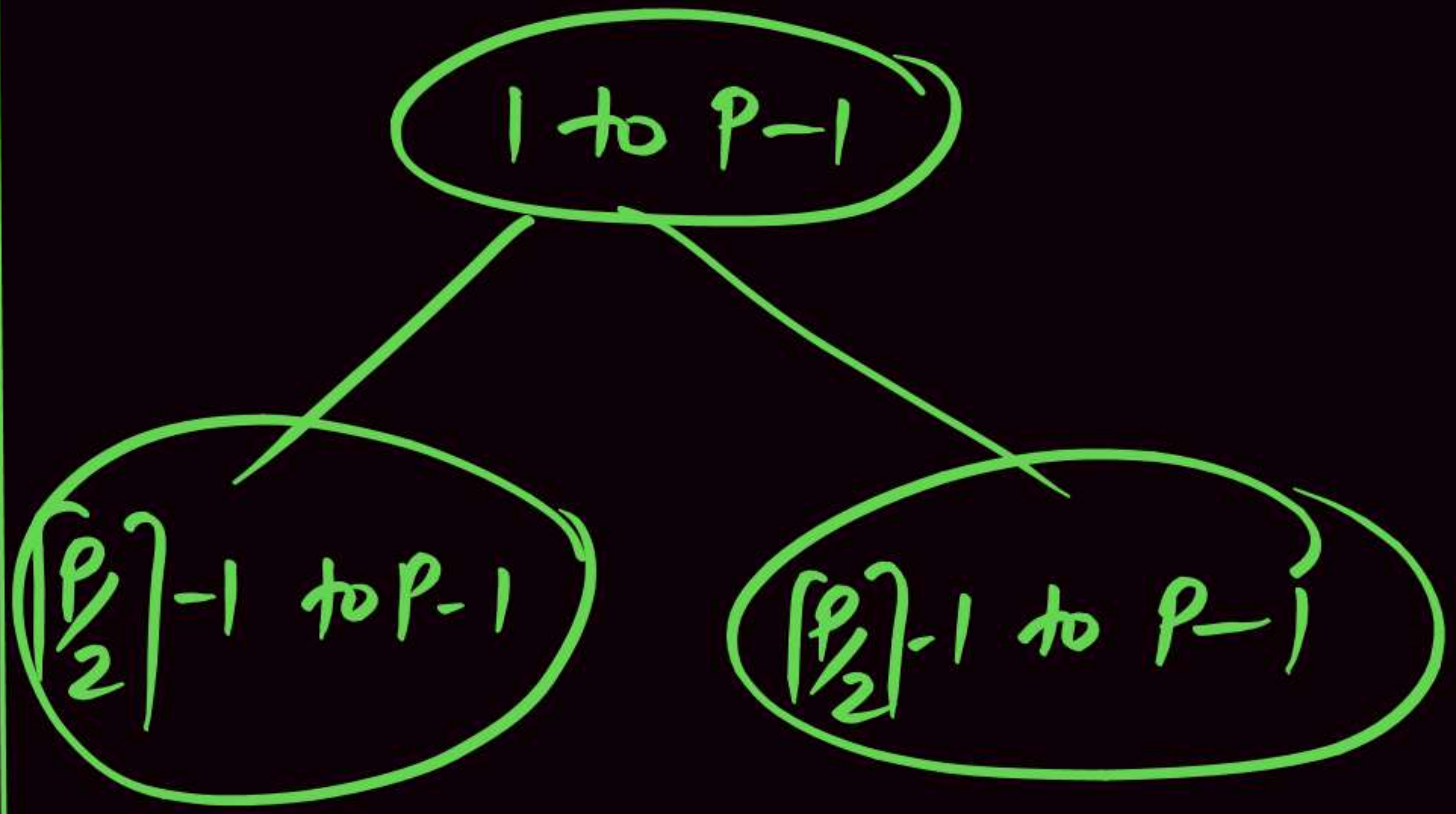
$$\underbrace{\text{I/O cost: } x[\log_p n + 1] + \text{cost of unsuccessful}}_{\text{More Access cost}}$$

[Unordered File DB]



Block Pointer

$\boxed{\text{Level} = \text{Height} + 1}$



Key = Bp − 1

B Tree Definition

ORDER: P. **Minimum**

ORDER[P]: 5

$\lceil P/2 \rceil = \lceil 5/2 \rceil = 3$

$\lceil P/2 \rceil = 3$

Height/Level	Min # Nodes	Min # BP	Min # keys	
0/1	1	2	1	1
1/2	2	$2\lceil P/2 \rceil$	$2[\lceil P/2 \rceil - 1]$	$2[3-1] \Rightarrow 2\times 2 = 4$
2/3	$2\lceil P/2 \rceil$	$2\lceil P/2 \rceil^2$	$2\lceil P/2 \rceil[\lceil P/2 \rceil - 1]$	$2\times 3 * 2 = 12$
3/4	$2\lceil P/2 \rceil^2$	$2\lceil P/2 \rceil^3$	$2\lceil P/2 \rceil^2[\lceil P/2 \rceil - 1]$	$2\times 3^2 \times 2 = 36$
4/5	$2\lceil P/2 \rceil^3$	$2\lceil P/2 \rceil^4$	$2\lceil P/2 \rceil^3[\lceil P/2 \rceil - 1]$	$2 * 3^3 \times 2 = 108$
⋮				
h/h+1	$2\lceil P/2 \rceil^{h-1}$	$2\lceil P/2 \rceil^{h}$	$2\lceil P/2 \rceil^{h-1}[\lceil P/2 \rceil - 1]$	161

B Tree Definition

ORDER: P.

Minimum

Height/Level	Min # Nodes	Min # Bp	Min # keys
0/1	1	2	1
1/2	2	$2\lceil \frac{P}{2} \rceil$	$2\left[\lceil \frac{P}{2} \rceil - 1\right]$
2/3	$2\lceil \frac{P}{2} \rceil$	$2\lceil \frac{P}{2} \rceil^2$	$2\lceil \frac{P}{2} \rceil\left[\lceil \frac{P}{2} \rceil - 1\right]$
3/4	$2\lceil \frac{P}{2} \rceil^2$	$2\lceil \frac{P}{2} \rceil^3$	$2\lceil \frac{P}{2} \rceil^2\left[\lceil \frac{P}{2} \rceil - 1\right]$
4/5	$2\lceil \frac{P}{2} \rceil^3$	$2\lceil \frac{P}{2} \rceil^4$	$2\lceil \frac{P}{2} \rceil^3\left[\lceil \frac{P}{2} \rceil - 1\right]$
⋮			
h/h+1	$2\lceil \frac{P}{2} \rceil^{h-1}$	$2\lceil \frac{P}{2} \rceil^{h}$	$2\lceil \frac{P}{2} \rceil^{h-1}\left[\lceil \frac{P}{2} \rceil - 1\right]$

Total Minimum # keys $= \left[1 + 2\left(\left\lceil \frac{p}{2} \right\rceil - 1\right) + 2\left\lceil \frac{p}{2} \right\rceil \left(\left\lceil \frac{p}{2} \right\rceil - 1\right) + \ldots\ldots + 2\left\lceil \frac{p}{2} \right\rceil^{h-1} \left(\left\lceil \frac{p}{2} \right\rceil - 1\right) \right]$

$\Rightarrow$ G.P formula then Calulate.

But Don't use formula.

For Maximum

Q ORDER : 5
Level : 5

Find Total Maximum # keys.

Soln 3124 Ans

B Tree Definition

ORDER: P.

Maximum

Height/Level	Maximum # Nodes	maximum # Bp	Maximum # keys	
0/1	1	P	(P-1)	(5-1) = 4
1/2	P	P^2	P(P-1)	5x4 = 20
2/3	P^2	P^3	$P^2(P-1)$	$5^2 \times 4 = 100$
3/4	P^3	P^4	$P^3(P-1)$	$5^3 \times 4 = 500$
4/5	P^4	P^5	$P^4(P-1)$	$5^4 \times 4 = 2500$
⋮				3124
h/h+1	P^h	P^{h+1}	$P^h(P-1)$	

B Tree Definition

ORDER: P.

Maximum

Height/Level	Maximum # Nodes	Maximum # Bp	Maximum # keys.
0/1	1	P	$(P-1)$
1/2	P	P^2	$P(P-1)$
2/3	P^2	P^3	$P^2(P-1)$
3/4	P^3	P^4	$P^3(P-1)$
4/5	P^4	P^5	$P^4(P-1)$
⋮			
h/h+1	P^h	P^{h+1}	$P^h(P-1)$.

Total maximum # keys $= (p-1) + p(p-1) + p^2(p-1) + p^3(p-1) \ldots\ldots + p^h(p-1)$

$$\Rightarrow (p-1)\left[1 + p + p^2 + p^3 + \ldots p^h\right].$$

Apply GP Series.

B Tree Definition

(Q) Consider a ORDER : 23, B Tree 69% Full.

then find Minimum # Keys.

(Soln) Number of BP Per Node $= 23 * .69$

$= 16$

$BP = 16$

(P) ORDER : 16

$(P-1)$ keys $= 15$

$\boxed{\text{Fanout} = 16}$

P=16

	max #Node	max #Bp	max #keys
Level 1			
Level 2			
Level 3			
Level 4			
.			
.			

B Tree Definition

& B^+ Tree.

ORDER : P

Note

Maximum Level : At each Level Minimum #key & Min #Bp.

$\left(\lceil \frac{P}{2} \rceil - 1\right)$ $\lceil \frac{P}{2} \rceil$

Note

Minimum level : At each Level Maximum #key & Max #Bp.

$(P-1)$ P

THANK YOU!