CS & IT ENGINEERING

CJ Sir

DIGITAL LOGIC

MINIMIZATION

Lecture No. 2

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TOPICS TO BE COVERED

01 Minimization

02 Question Practice

03 Discussion

- ✓ SOP
- ✓ POS
- ✓ Standard canonical

\# $A+BC = (A+B)(A+C)$

$A\cdot(B+C) = AB+AC$

\# $AB+\bar{A}C+BC = AB+\bar{A}C$

\# $(A+B)(\bar{A}+C) = AC+\bar{A}B$

\# $\overline{ABC} = \bar{A}+\bar{B}+\bar{C}$

$\overline{A+B+C} = \bar{A}\cdot\bar{B}\cdot\bar{C}$

Q.5

Minimize the expression.

$$f(A, B) = \overline{A}\,\overline{B} + \overline{A}\,B + A\,\overline{B} + AB$$

$$= 1$$

$n = 1$ (A)

A

$\bar{A}$

0

1

$\} \; 4$

$n = 2$ (A, B)

$\bar{A}\bar{B}$	$\bar{A}+\bar{B}$	A	0
$\bar{A}B$	$\bar{A}+B$	$\bar{A}$	1
$A\bar{B}$	$A+\bar{B}$	B	$\bar{A}B+A\bar{B}$
AB	$A+B$	$\bar{B}$	$\bar{A}\bar{B}+AB$

$\} \; 16$

n. variable $\rightarrow$ 2^{2^n} distinct function

Q.6

Minimize the expression.

$f(A, B) = \overline{A}B + \overline{A}\,B$

$= \bar{A}B$

Ans

$f(A,B) = \bar{A}B + A\bar{B}$ $\longrightarrow$ already minimize.

$= A \oplus B$

$f(A,B) = \bar{A}\bar{B} + AB$

$= A \odot B$

Q.7

Minimize the expression.

$f(A, B) = AB + \overline{A}\,C + BC$

$\cancel{AB + \overline{A}C}$

function written in wrong format.

$f(B, A) = \overline{A}\,\overline{B} + \overline{A}B + A\overline{B}$

$= \overline{B}\,\overline{A} + B\overline{A} + \overline{B}A$

$$f(\underbrace{A,C,B}) = \bar{A}\bar{B}\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + ABC \Rightarrow \sum m(0,5,4,7)$$

POS of the function given below -

(A) $\prod M(2,5,6)$

(B) $\prod M(3,4,5,6)$

(C) $\prod M(0,5,4,7)$

(D) None

$$\begin{array}{l} \quad\;\; 0\,0\,0 \\ f(A,C,B) = \bar{A}\bar{C}\bar{B} + AC\bar{B} + A\bar{C}\bar{B} + ACB \end{array}$$

$$= \sum m(0,6,4,7)$$

$$= \sum m(0,4,6,7)$$

$$\prod M(1,2,3,5)$$

Q.8

Minimize the expression.

$$f(\underbrace{A, B, C}) = \bar{A}\,\bar{B} + \underbrace{\bar{A}\,C} + \underbrace{\bar{B}\,\bar{C}}$$

$$= \bar{A}C + \bar{B}\,\bar{C}$$

Redundant $= \bar{A}\bar{B}$

Q.9

Minimize the expression.

$$f(A, B, C) = (A + B)(A + C)(\bar{B} + C)$$

$$= (A+B)(\bar{B}+C)$$

Q.10

Write the function for truth table and minimize it.

	A	B	Y(0/p)
$A+B$	0	0	1
$A+\bar{B}$	0	1	0 ✓
$\bar{A}+B$	1	0	1
$\bar{A}+\bar{B}$	1	1	1

In POS.

$$Y(A,B) = (\cancel{A+B}+1)\cdot(A+\bar{B}+0)\cdot(\cancel{\bar{A}+B}+1)(\cancel{\bar{A}+\bar{B}}+1)$$

$$= 1\cdot(A+\bar{B})\cdot 1\cdot 1$$

$$= A+\bar{B}$$ ✓

Ans

$$Y = A+\bar{B}$$

In SOP

$$Y(A,B) = \bar{A}\bar{B}\cdot 1 + \bar{A}B\cdot 0 + A\bar{B}\cdot 1 + AB\cdot 1$$

$$= \bar{A}\bar{B} + A\bar{B} + AB$$

$$\Rightarrow \bar{B}[\bar{A}+A] + AB$$

$$\Rightarrow \bar{B} + AB$$

$$= (A+\bar{B})(B+\bar{B}) = A+\bar{B}$$

Ans

Q.11

Write the function for truth table and minimize it.

A	B	Y(0/p)
0	0	C
0	1	$\bar{C}$
1	0	1
1	1	1

SOP

$$Y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B} + AB$$

$$Y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A(\bar{B} + B)$$

$$Y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A$$

$$Y = \bar{A}[\bar{B}C + B\bar{C}] + A$$

$$Y = \bar{A}Z + A \qquad \bar{B}C + B\bar{C} : Z$$

$$Y = (\bar{A} + A)(A + Z)$$

$$Y = A + Z$$

$$= A + \bar{B}C + B\bar{C}$$

Ans

Q

$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC.$$

$$Y = \underbrace{\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C} + \bar{A}BC + \bar{A}\bar{B}C$$

$$Y = \bar{A}\bar{B}(\bar{C}+C) + \bar{A}C(B+\bar{B})$$

$$Y = \bar{A}\bar{B} + \bar{A}C$$

Ans

Two way switch

↓

X-OR

A	B	Y
0	0	0
1	0	1
0	1	1
1	1	0

Q.13

If we have '4' variable, then total different expression will be?

$$2^{2^n} \longrightarrow 2^{2^4} \Rightarrow 2^{16} \Rightarrow 2^6 \cdot 2^{10}$$

$1K \rightarrow \underline{1024}$

$$\Rightarrow 2^6 K$$

$$\Rightarrow \underline{64K}$$

$2^4 \rightarrow 16$

$2^5 \rightarrow 32$

$2^6 \rightarrow 64$

$2^7 \rightarrow 128$

$2^8 \rightarrow 256$

$2^9 \rightarrow 512$

$2^{10} \rightarrow 1024$

$2^{10} = K$

$2^{20} = M$

$2^{30} = G$

$2^{40} = T$

Laws of Boolean Algebra

1 → −5V
0 → −10V

1 → 0V
0 → 5V

Logic

Positive Logic ⟷ Dual ⟷ Negative Logic

Positive Logic

1 → HIGH

0 → LOW

Ex 1 → 5V
0 → 0V

1 → 0V
0 → −5V

Negative Logic

1 → LOW
0 → HIGH

Ex 1 → 5V

0 → 10V

AND GATE

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

OR GATE

A	B	Y
1	1	1
1	0	1
0	1	1
0	0	0

AND $\xleftrightarrow{\text{Dual}}$ OR

$A \cdot B \xleftrightarrow{\text{Dual}} A+B$

$\cdot \xleftrightarrow{\text{Dual}} +$

$$f = A \cdot B + C\bar{D} + \bar{E}F$$

$$f^D = (A+B) \cdot (C+\bar{D}) \cdot (\bar{E}+F)$$

$$f^{D^D} = AB + C\bar{D} + \bar{E}F$$

$$f^{D^D} = f$$

Dual

$f = \overline{A \cdot B}$ NAND

$f^D = \overline{A + B}$ NOR

Laws of Boolean Algebra

NPN

Dual

(i) Positive logic $\xrightarrow{\text{Dual}}$ negative logic ✓

(ii) NPN trans $\xrightarrow{D}$ PNP trans

(iii) N-MOS $\rightarrow$ P-MOS

(iv) $0 \overset{D}{\leftrightarrow} 1$

(v) $. \overset{D}{\leftrightarrow} +$

NPN → Transistor

PNP → Transistor

MOS → MOSFET.

Laws of Boolean Algebra

Dual

(i) AND $\xleftrightarrow{D}$ OR

(ii) NAND $\xrightarrow{D}$ NOR

(iii) EX-OR $\xleftrightarrow{D}$ EX - NOR

(iv) BUFFER ↔ BUFFER

(v) INVERTER ↔ INVERTER

$$A \xleftrightarrow{\text{Dual}} A$$

$$\bar{A} \xrightarrow{\text{Dual}} \bar{A}$$

$\underline{\underline{n=1}}$

$$\boxed{\begin{array}{ccc} A & \xleftrightarrow{\text{Dual}} & A \\ \bar{A} & \xleftrightarrow{\text{Dual}} & \bar{A} \end{array}}$$

$$0 \xleftrightarrow{\text{Dual}} 1$$

$$1 \xleftrightarrow{\text{Dual}} 0$$

$n=2,\ (A, B)$

$\bar{A}\bar{B}$	$\bar{A}+\bar{B}$	A	0
$\bar{A}B$	$\bar{A}+B$	$\bar{A}$	1
$A\bar{B}$	$A+\bar{B}$	B	$\bar{A}B+A\bar{B}$
AB	$A+B$	$\bar{B}$	$\bar{A}\bar{B}+AB$

$$\boxed{f^D = f}$$

"Self Dual"

n Variables $\longrightarrow 2^{2^n} \longrightarrow \boxed{2^{2^{n-1}}}$ → Self Dual n Variable $\rightarrow 2^{2^n} \rightarrow 2^{2^{n-1}}$

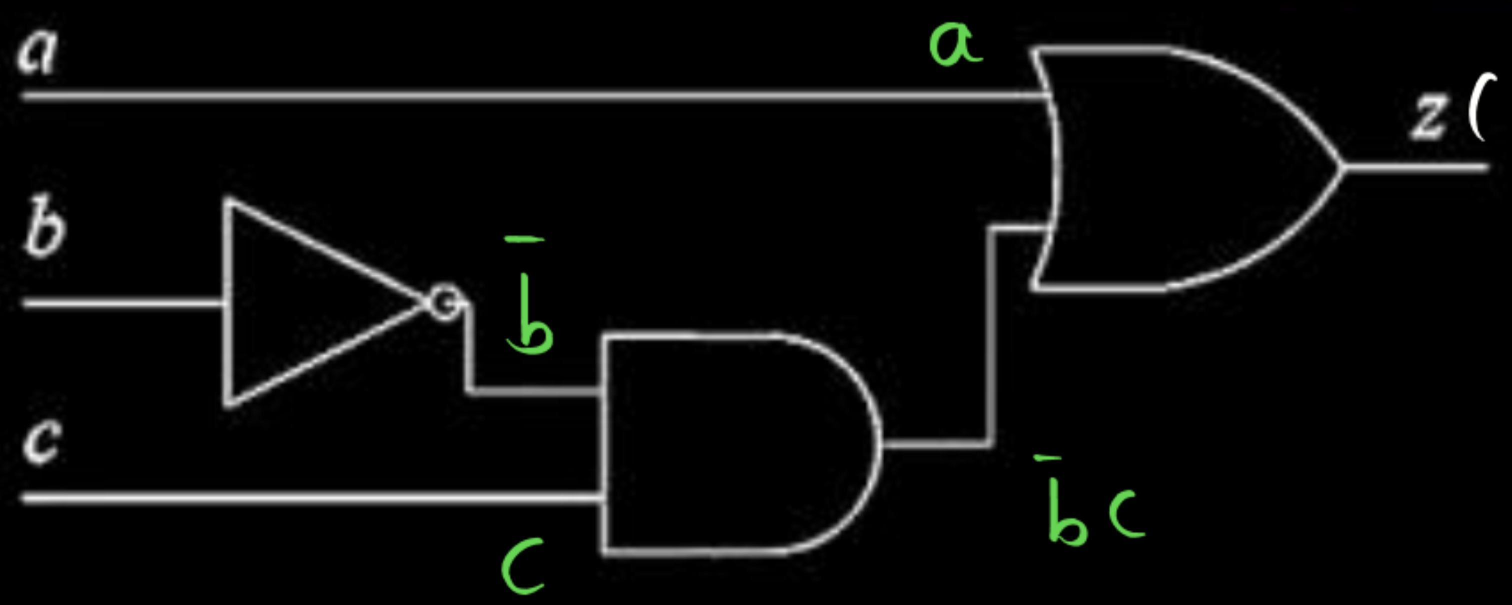
$2^{2^{n-1}} \rightarrow$ Self Dual

$2^{2^n} \swarrow$ 2^n Minterm

$2^{2^n} \searrow$ 2^n Maxterm

MCQ

Consider the Boolean function z(a,b,c).



$z(a,b,c) = a + \bar{b}c$

$= 100 \rightarrow 4 \quad 001 \rightarrow 1$

$101 \rightarrow 5 \quad 101 \rightarrow 5$

$110 \rightarrow 6$

$111 \rightarrow 7$

Which one of the following minterm lists represents the circuit given above?

[GATE-2020-CS: M]

(A) $Z=\Sigma (0, 1, 3, 7)$

(B) $Z=\Sigma (2, 4, 5, 6, 7)$

(C) $Z=\Sigma (1, 4, 5, 6, 7)$

(D) $Z=\Sigma (2, 3, 5)$

MCQ

HW

If w, x, y, z are Boolean variables, then which one of the following is INCORRECT?

[GATE-2017-CS: M]

(A) $wx + w(x+y)+x(x+y) = x + wy$

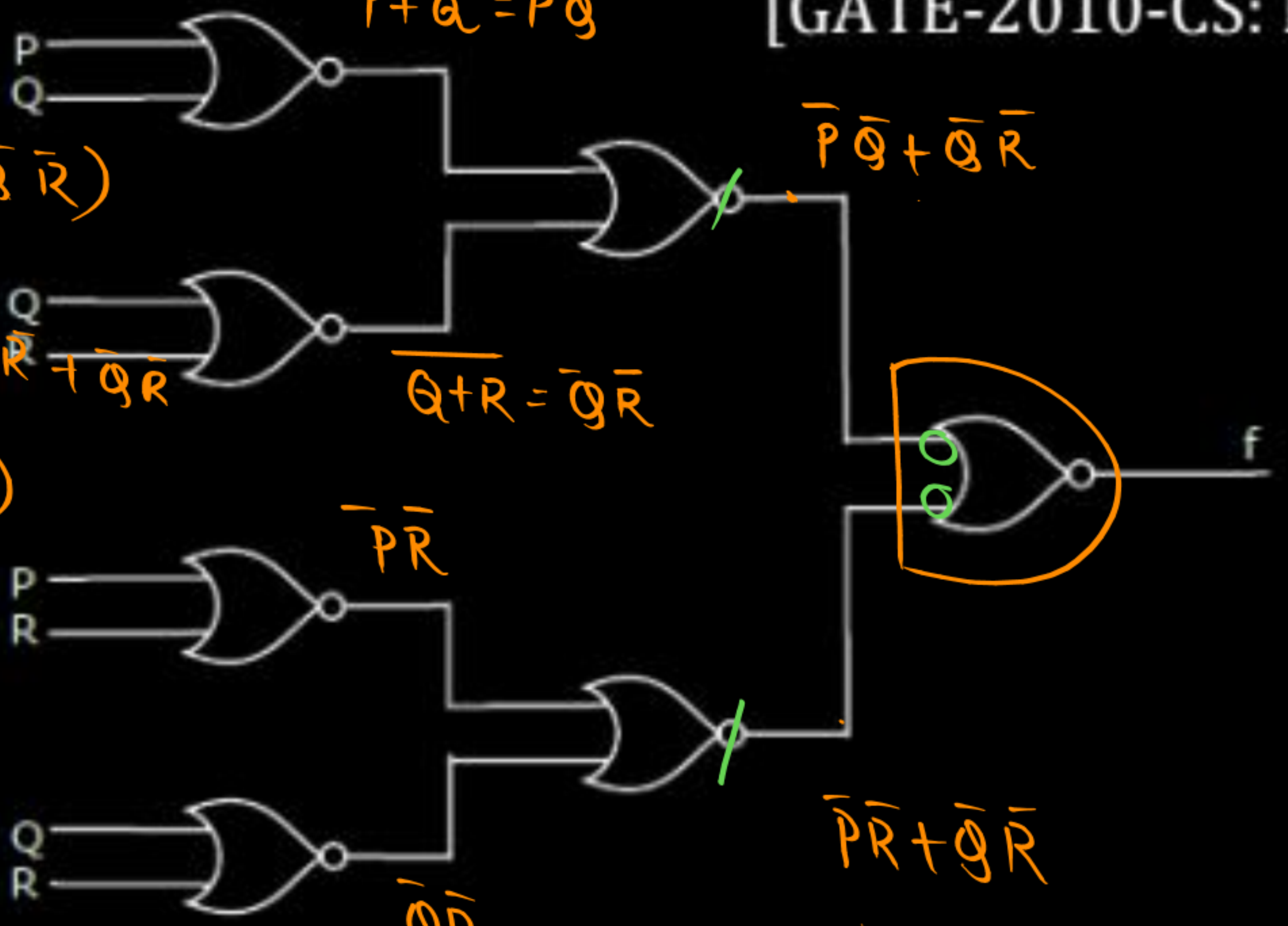
(B) $\overline{w\bar{x}(y+\bar{z})}+\bar{w}x = \bar{w} + x + \bar{y}z$

(C) $(w\bar{x}(y + x\bar{z}) + \bar{w}\bar{x})y = x\bar{y}$

(D) $(w + y)(wxy + wyz) = wxy + wyz$

MCQ

What is the Boolean expression for the output f of the combinational logic circuit of NOR gates given below? [GATE-2010-CS: M]



(A) $\overline{Q+R}$

(B) $\overline{P+Q}$

(C) $\overline{P+R}$

(D) $\overline{P+Q+R}$

$f = (\bar{P}\bar{Q} + \bar{Q}\bar{R})(\bar{P}\bar{R} + \bar{Q}\bar{R})$

$f = \bar{P}\bar{Q}\bar{R} + \bar{P}\bar{Q}\bar{R} + \bar{P}\bar{Q}\bar{R} + \bar{Q}\bar{R}$

$f = \bar{Q}\bar{R}(\bar{P} + \bar{P} + \bar{P} + 1)$

$f = \bar{Q}\bar{R}$

$f = \overline{Q+R}$

Modified Veitched Diagram

→ also known as K-MAP

Karnaugh MAP

K-MAP

→ It is based on $\boxed{\text{Gray code.}}$

Binary → B_3 B_2 B_1 B_0

Gray code → G_3 G_2 G_1 G_0

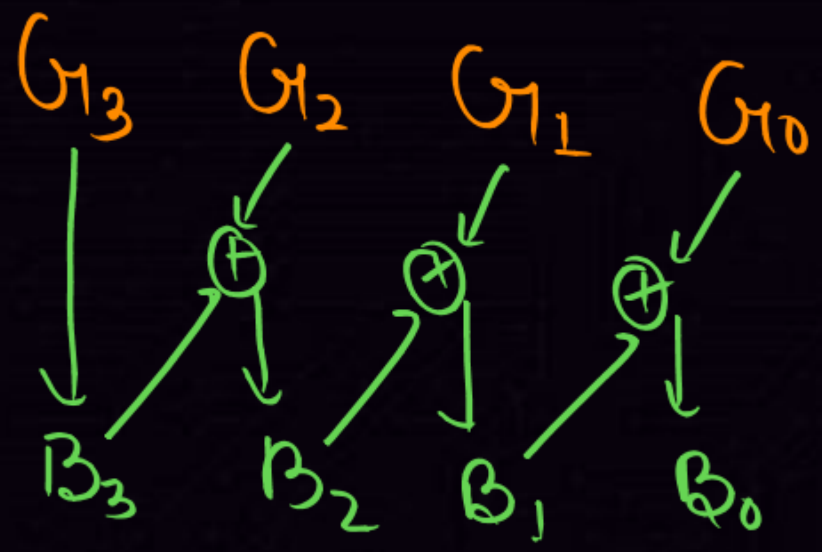
Ex

Binary → 1 1 0 0 1 0

Gray → 1 0 1 0 1 1

Gray code $\longrightarrow$

Binary code $\longrightarrow$



Ex Gray code $\rightarrow$ 1 0 0 1 0 1 1 0 1

Binary $\longrightarrow$ 1 1 1 0 0 1 0 0 1 ✓

Karnaugh MAP

Decimal	Binary	Gray
0	0 0	0 0
1	0 1	0 1
2	1 0	1 1
3	1 1	1 0

Gray code.

↳ successive numbers are differ by one bit.

↳ unity hamming distance code.

↳ cyclic code

↳ Reflecting code

Karnaugh MAP

Decimal	Binary	Graycode
0	000	000
1	001	001
2	010	011
3	011	010
4	100	110
5	101	111
6	110	101
7	111	100

Thank you

GW

Soldiers !