CS & IT Engineering

Finite Automata:
NFA with epsilon moves

Lecture no-05

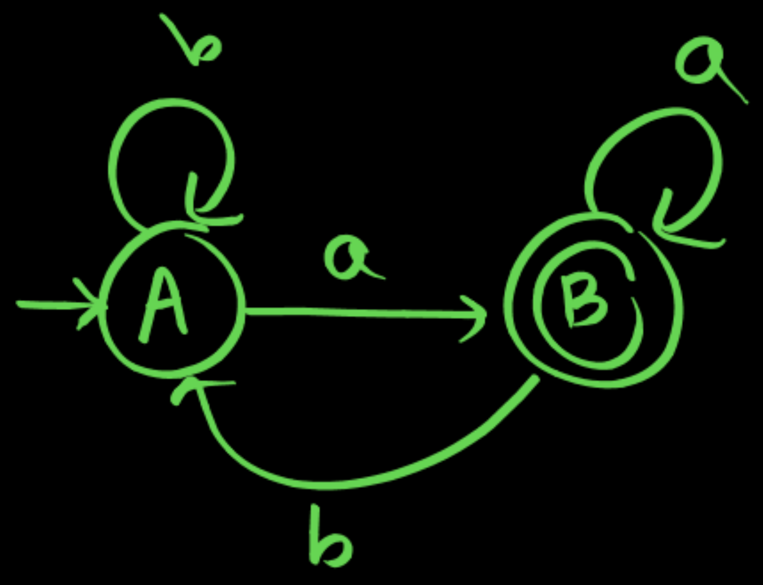
Deva sir

Topics:

- NFA with ε-moves

What is a regular language ?

- It is a set represented by
  reg Exp / DFA / NFA / Reg grammar
- It is simple,
  takes constant space



Ends with 'a'

```
char ch;
main()
{
  ch=getchar();
  if(ch=='a')  B();
  else if (ch=='b')  main();
  else if (ch==' ')
        printf("Invalid");
}
B()
{
    ch=getchar();
    if (ch=='a')  B();
    else if ch=='b'  main();
}
```

Examples of Regular Sets:

X

aX

Xa

XaX

$|w| = 2$

$|w| \leq 2$

$|w| \geq 2$

$n_a(w) = 2$

$n_a(w) \geq 2$

$n_a(w) \leq 2$

$|w| \% 2 == 0$

$|w| \% 3 = 0$

$n_a(w) \% 2 = 0$

xaX

Xax

$X(aa+bb)$

$(aa+bb)X$

$X(aa+bb)X$

$(aa)^*$

$a(aa)^*$

$(ab)^*$

⋮

Assume

$x = a+b$

$X = (a+b)^*$

$w \in \{a,b\}^*$

Examples of $\underbrace{\text{non-regular sets}}_{\text{we do not have DFA, regExp, NFA, reg gram.}}$:

$a^n b^n$

$a^n b^{2n}$

$a^{2n} b^n$

nonregular

$a^* b^*$ is regular

$a^n b^n$ is nonregular

$\varepsilon, ab, a^2b^2, \ldots$

NFA with ε-moves:

$\boxed{\text{move} = \text{Transition}}$

DFA $\cong$ NFA

Algo 1: DFA → NFA

Algo 2: NFA → DFA

X iff Y

$X \Leftrightarrow Y$

$\boxed{X \cong Y}$

1) $X \Rightarrow Y$ and
2) $Y \Rightarrow X$

DFA	$\cong$ NFA without ε transitions	$\cong$ NFA with ε transitions
$\delta : Q \times \Sigma \rightarrow Q$	$\delta : Q \times \Sigma \rightarrow 2^{Q}$	$\delta : Q \times \Sigma_{\varepsilon} \rightarrow 2^{Q}$

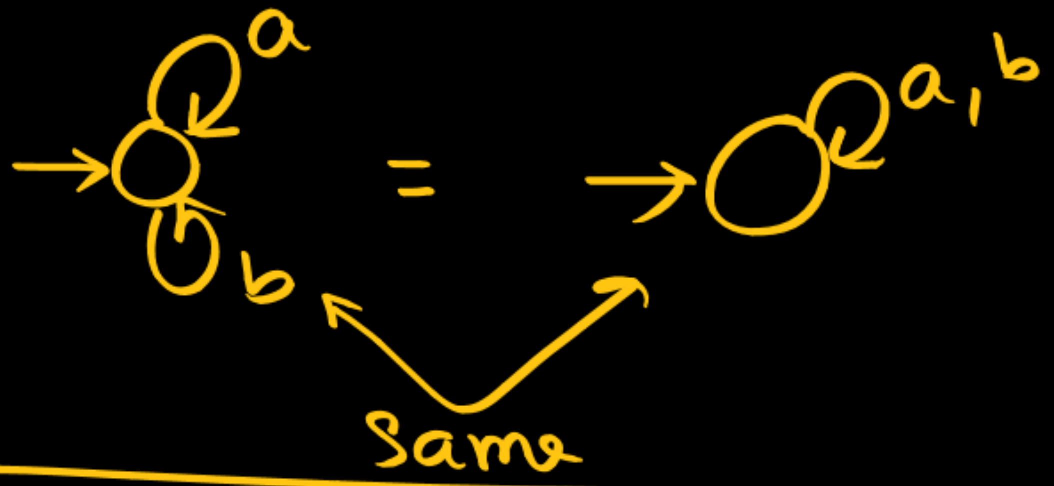
DFA

$$\text{DFA} \xrightarrow{\text{By definition}} \text{NFA} \xrightarrow{\text{By def}} \varepsilon\text{-NFA}$$

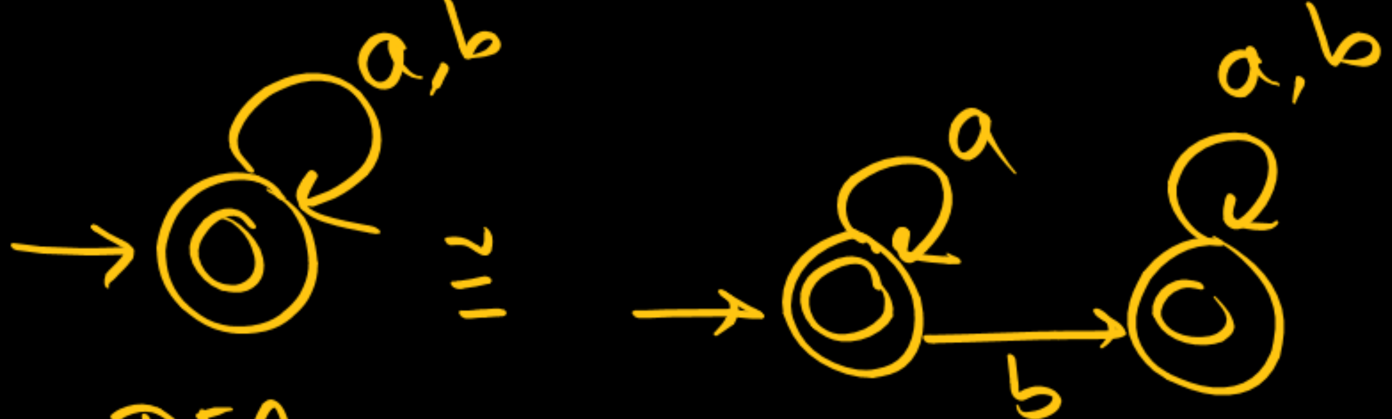
$$\text{DFA} \xrightarrow{\text{By def}} \varepsilon\text{-NFA}$$

DFA	NFA without ε moves	ε- NFA
$Q \times \Sigma \rightarrow Q$	$Q \times \Sigma \rightarrow 2^Q$	$Q \times \Sigma_\varepsilon \rightarrow 2^Q$
$\delta(A, a) = A$	$\delta(A, a) = \{A\}$	$\delta(A, a) = \{A\}$
$\delta(A, b) = A$	$\delta(A, b) = \{A\}$	$\delta(A, b) = \{A\}$
		$\delta(A, \varepsilon) = \{\}$

Same



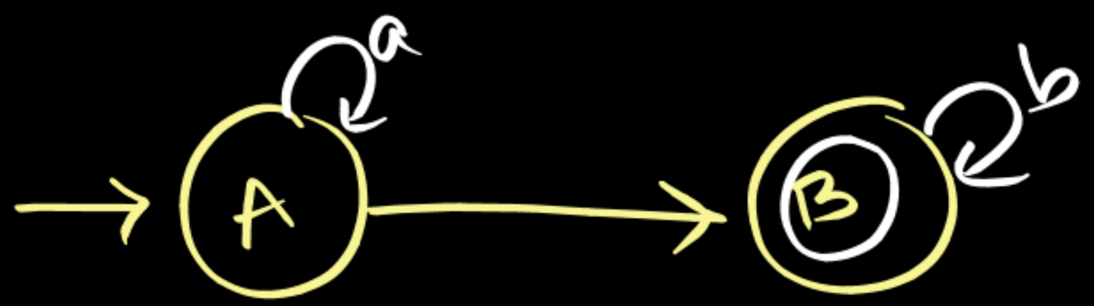
equivalent

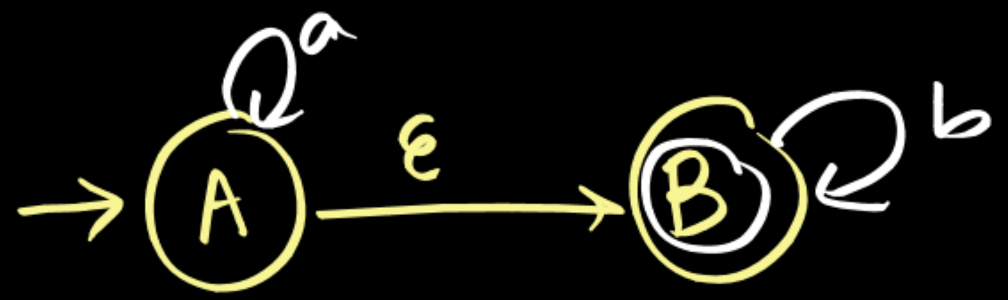


DFA$_1$ DFA$_2$

$DFA_1 \cong DFA_2$

$L(DFA_1) = L(DFA_2)$





DFA ✗

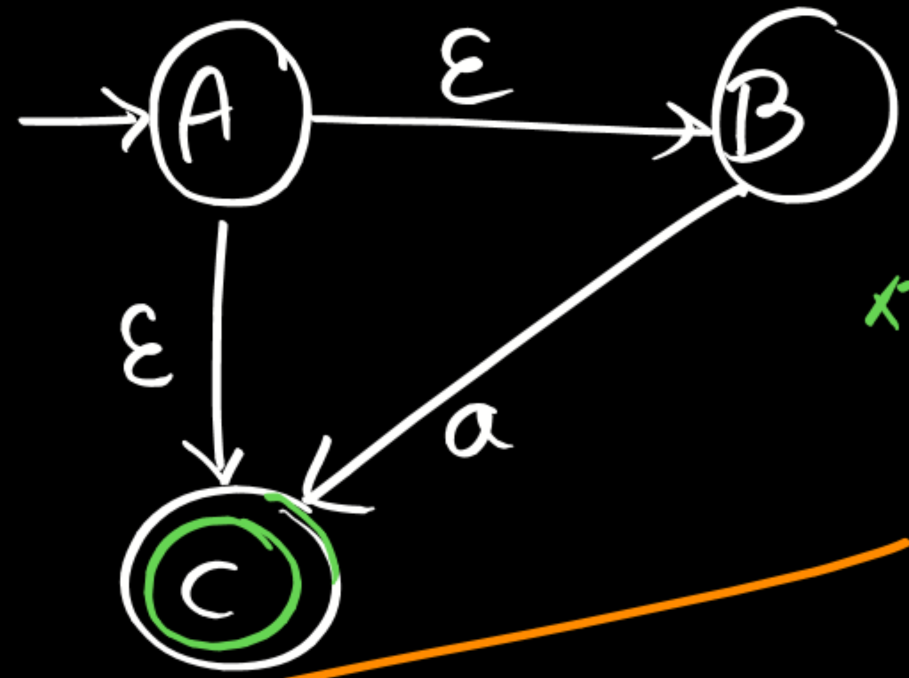
NFA without ε moves ✗

NFA with ε moves

$\delta_{\text{NFA with } \varepsilon \text{ moves}}$	a	b	ε
→ A	{A}	{ }	{B}
* B	{ }	{B}	{ }

NFA

M:



$L(M) = \{\varepsilon, a\}$

- ε ✓
- a ✓
- aa ✗
- aaa ✗
- ⋮

$A \overset{\varepsilon}{\Longrightarrow} ?$

$A \overset{a}{\Longrightarrow} ?$

transitions

① $\delta(A, \varepsilon) = \{B, C\}$ $\quad A \xrightarrow{\varepsilon} B$, $A \xrightarrow{\varepsilon} C$

② $\delta(A, a) = \phi = \{\ \}$

Extended transition

Paths

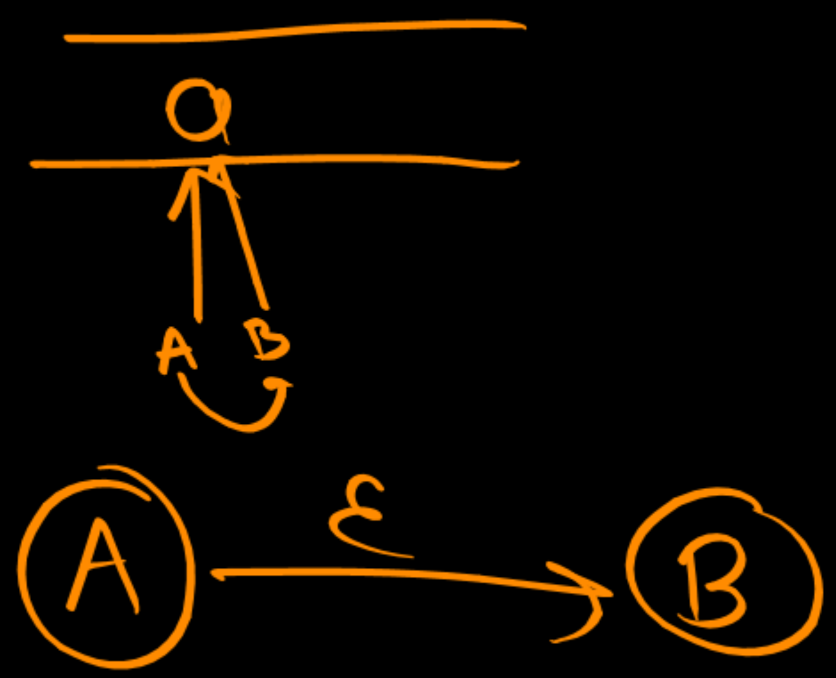
③ $\hat{\delta}(A, \varepsilon) = \{A, B, C\}$ $\quad A \overset{\varepsilon}{\Longrightarrow} A, B, C$ $\quad \{A,\ A \xrightarrow{\varepsilon} B,\ A \xrightarrow{\varepsilon} C$

④ $\hat{\delta}(A, a) = \{C\}$ $\quad \{A \xrightarrow{\varepsilon} B \xrightarrow{a} C$

$\varepsilon a = a$

Read ε

Read nothing



O/p = ϕ

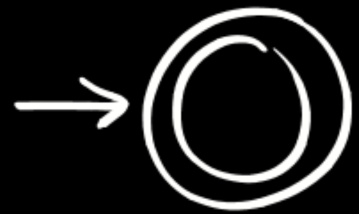
```
main()
{
    main();
}
```

```
main()
{
}
```

O/p = ε

Identify language:

1.

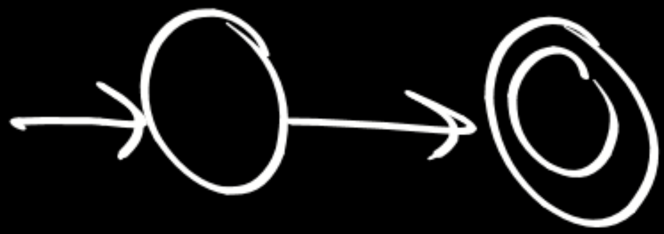


$L = \{\varepsilon\}$

2.

$L = \phi$

3.

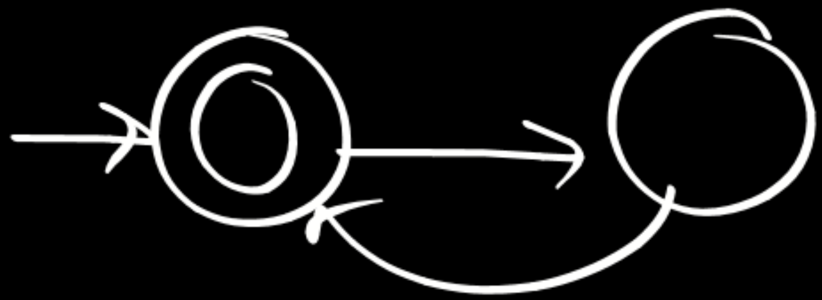


$L = \{\varepsilon\}$

4.

$L = \{\varepsilon\}$

5.



$L = \{\varepsilon\}$

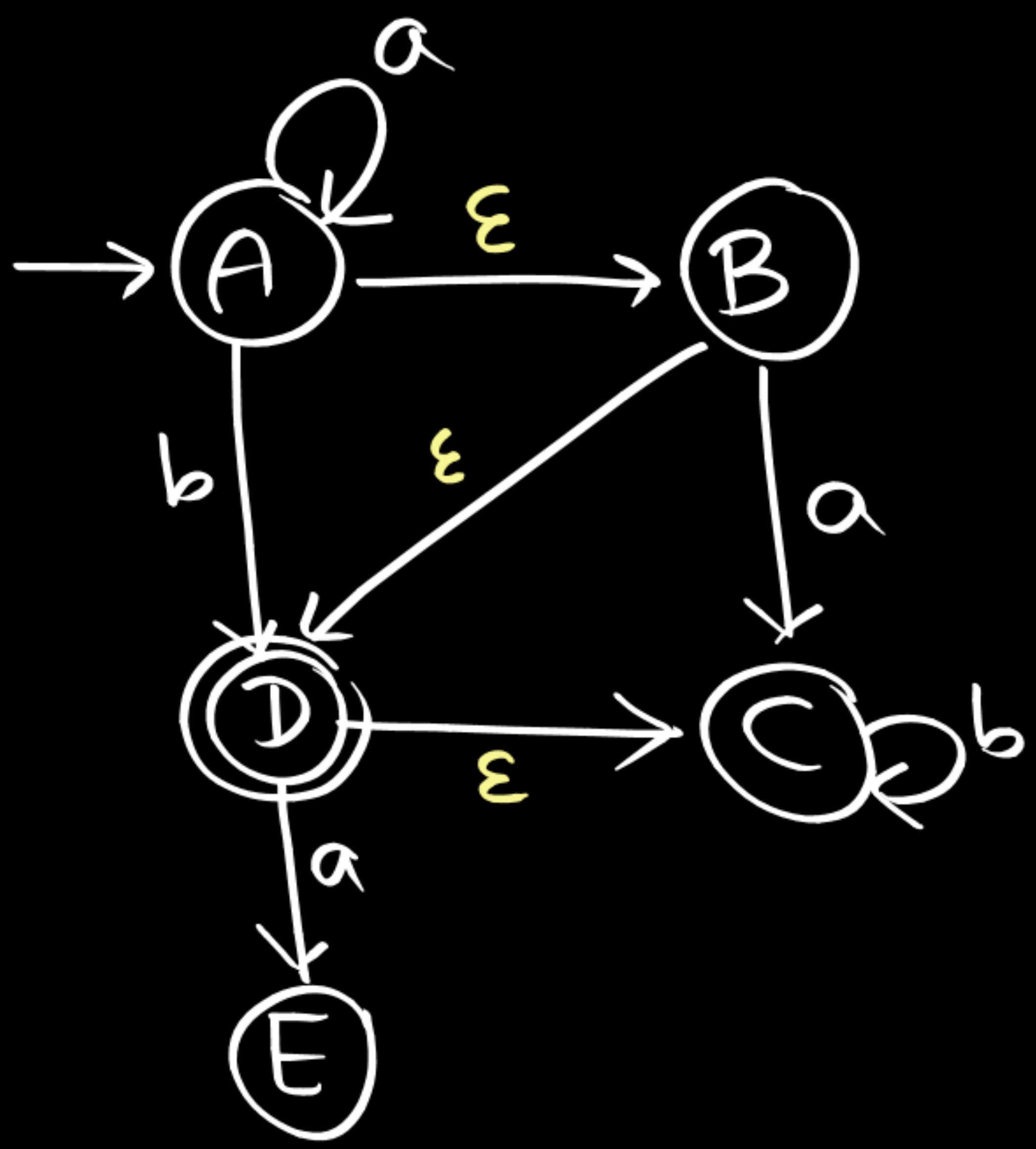
6.

$L = \phi$

Note: Eventhough initial state is not final, ε-NFA can accept "empty string" (ε).

ε-closure of a state:

$\boxed{\varepsilon = \varepsilon\varepsilon = \varepsilon\varepsilon\varepsilon}$

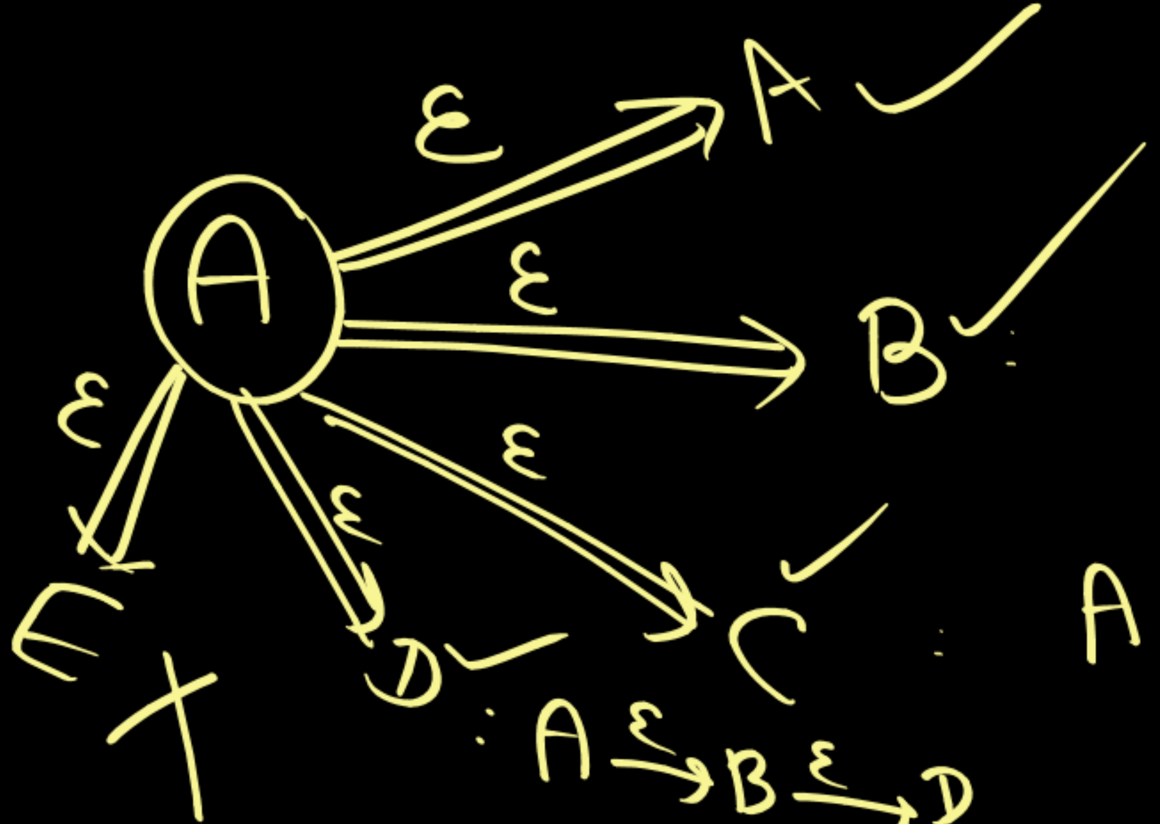


$|\varepsilon\text{-closure}(A)| = 4$

1. $\varepsilon\text{-closure}(A) = \{A, B, C, D\}$
2. $\varepsilon\text{-closure}(B) = \{B, C, D\}$
3. $\varepsilon\text{-closure}(C) = \{C\}$
4. $\varepsilon\text{-closure}(D) = \{D, C\}$
5. $\varepsilon\text{-closure}(E) = \{E\}$

$\underbrace{\hat{\delta}(A, \varepsilon)}_{\varepsilon\text{-closure}(A)}$ = set of states reachable from A without reading any symbol.

ε-closure(A) : From state A by reading $\underbrace{\text{nothing}}_{(\varepsilon)}$, find all states that are reachable.



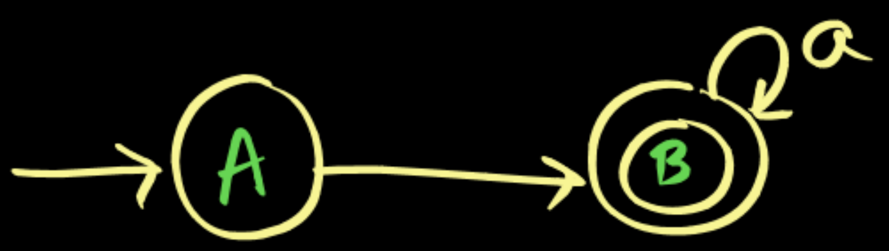
$A \xrightarrow{\varepsilon} B$

$A \xrightarrow{\varepsilon} B \xrightarrow{\varepsilon} D \xrightarrow{\varepsilon} C$

$A \xrightarrow{\varepsilon} B \xrightarrow{\varepsilon} D$

NFA

7



$L = a^*$

ε ✓ $A \xrightarrow{\varepsilon} B$

a ✓ $A \longrightarrow B \xrightarrow{a} B$

aa ✓

aaa ✓

⋮

8

$L = a^*$

ε ✓

a ✓

aa ✓

⋮

NFA

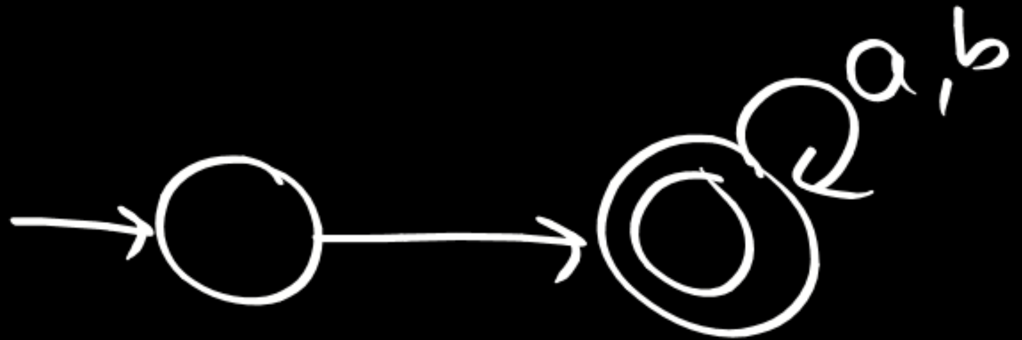
9

$L = (a+b)^*$

10

$L = (a+b)^*$

11

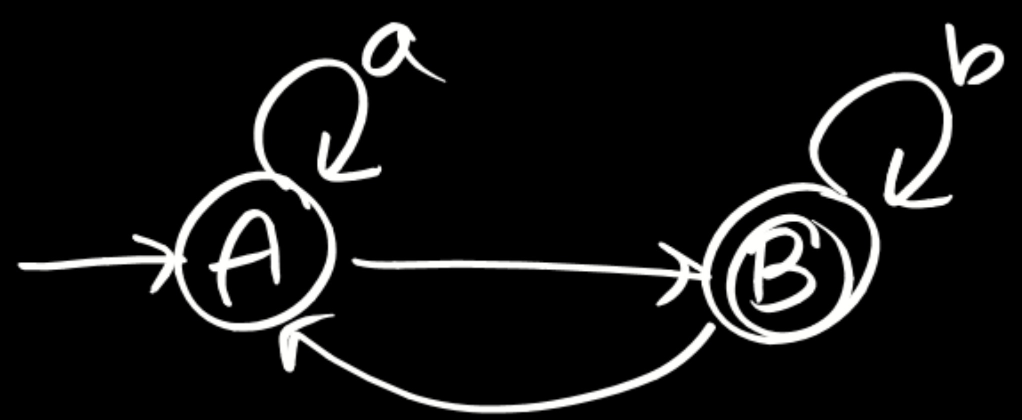


$L = (a+b)^*$

12

$L = (a+b)^*$

***(13)



✓ ε : $A \xrightarrow{\varepsilon} B$

✓ a : $A \xrightarrow{a} A \xrightarrow{\varepsilon} B$

✓ b : $A \longrightarrow B \xrightarrow{b} B$

$= (a+b)^*$

✓ aa : $A \xrightarrow{a} A \xrightarrow{a} A \longrightarrow B$

✓ ab : $A \xrightarrow{a} A \longrightarrow B \xrightarrow{b} B$

✓ ba : $A \rightarrow B \xrightarrow{b} B \rightarrow A \xrightarrow{a} A \rightarrow B$

Start from Initial state

$\Downarrow$ after reading complete string

Can I halt at final state

14

ε ✓

✓ a, aa, ...

✓ b, bb, ...

$$L = \underbrace{a^* b^*}$$

→ ε ✓

a^+ ✓

b^+ ✓

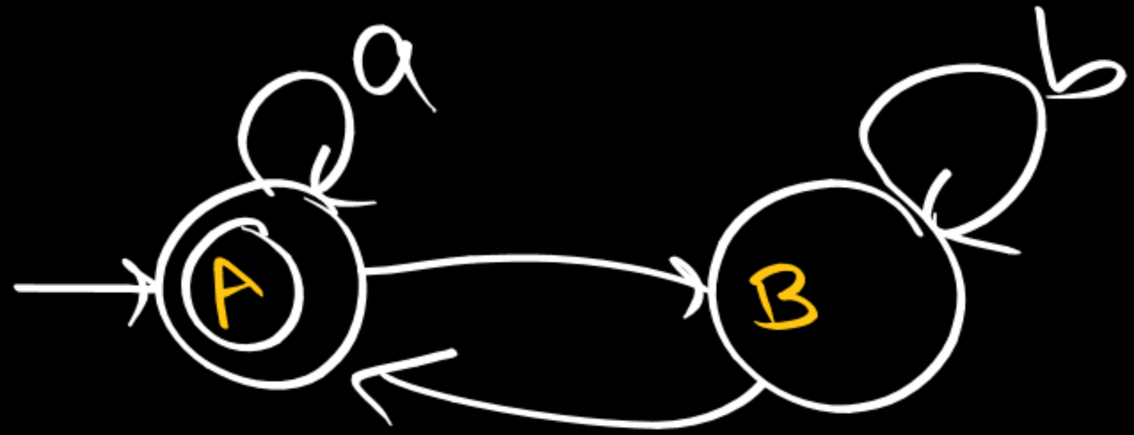
$a^+ b^+$ ✓

15

ε ✓	b ✗
a ✓	bb ✗
aa ✓	⋮
⋮	

$L = a^*$

16



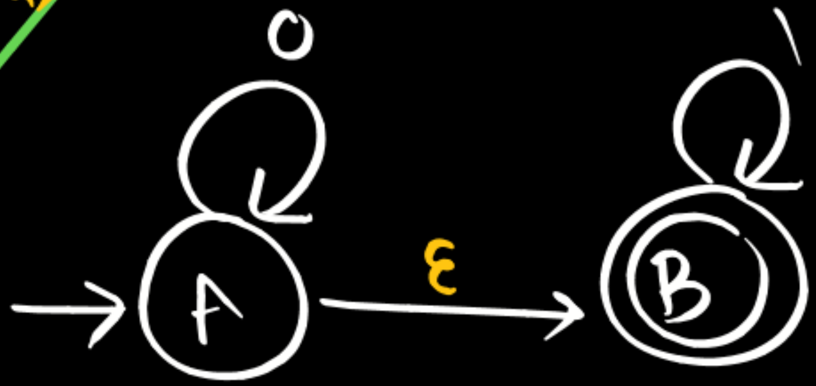
$L = (a+b)^*$

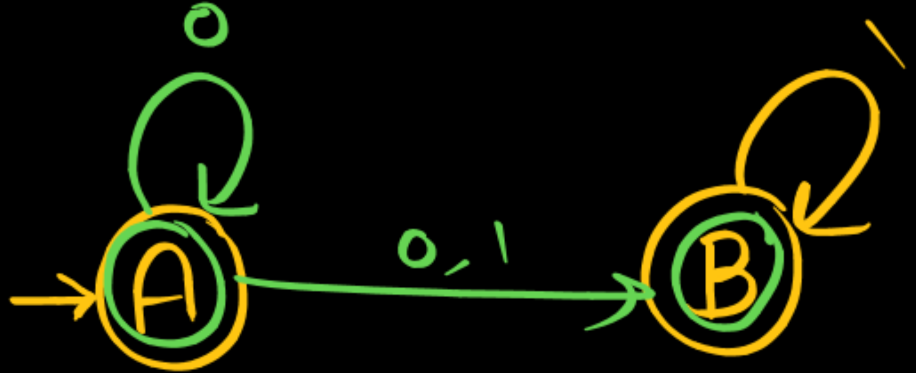
NFA

④ $\delta'(B,1) = \varepsilon\text{-clo}[\underbrace{\delta(\underbrace{\varepsilon\text{-clo}(B)}_{B},1)}_{B}] = \{B\}$

NFA with ε moves ⟹ NFA without ε moves

$\delta(\{A,B\},0) = \delta(A,0) \cup \delta(B,0)$





Final state in NFA without ε-moves:

If final of εNFA present in any ε-clo set X then make X as final

δ	0	1	ε
→A	{A}	φ	{B}
*B	φ	{B}	φ

ε-closure(A) = {A, B}

ε-closure(B) = {B}

⟹

① $\delta'(A,0) = \varepsilon\text{-clo}[\underbrace{\delta(\underbrace{\varepsilon\text{-clo}(A)}_{\{A,B\}},0)}_{A}] = \{A,B\}$

② $\delta'(A,1) = \varepsilon\text{-clo}[\underbrace{\delta(\underbrace{\varepsilon\text{-clo}(A)}_{\{A,B\}},1)}_{B}] = \{B\}$

③ $\delta'(B,0) = \varepsilon\text{-clo}[\underbrace{\delta(\underbrace{\varepsilon\text{-clo}(B)}_{B},0)}_{\phi}] = \phi$

NFA with ε moves $\Longrightarrow$ NFA without ε moves

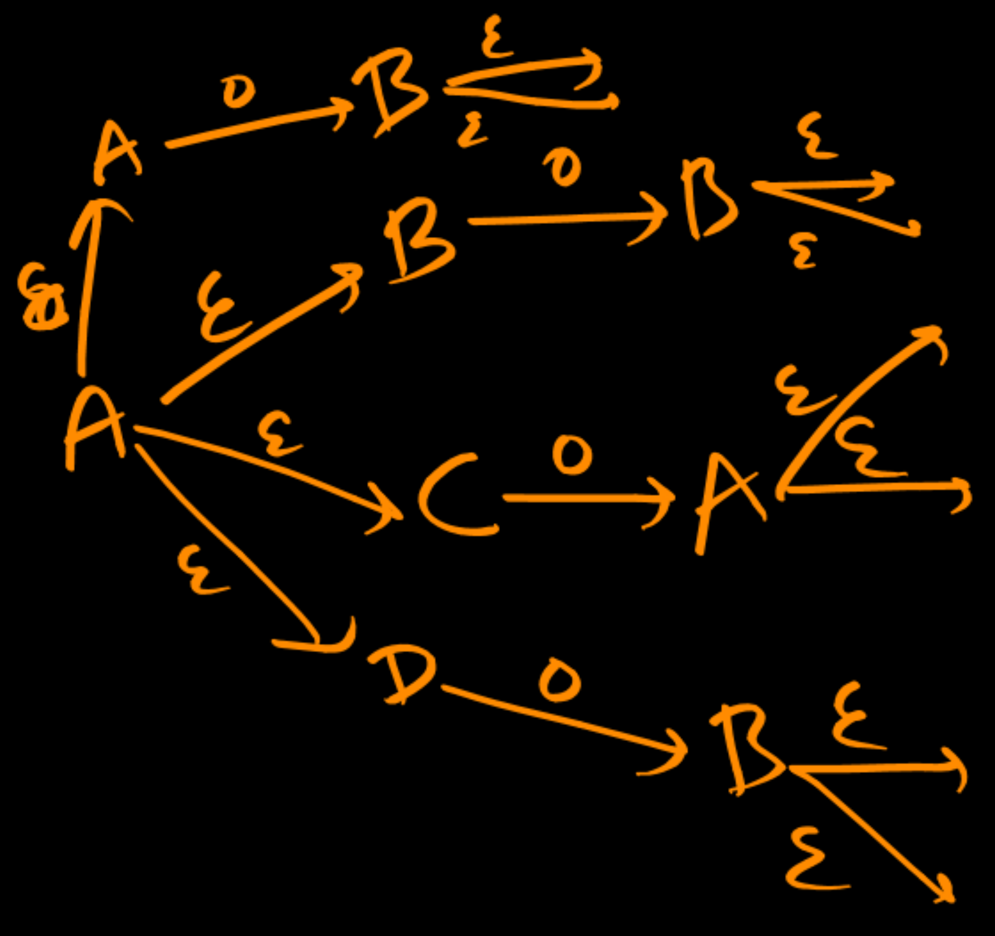
δ $\qquad$ δ'

given

$\forall q \in Q$

$\forall i \in \Sigma$

$$\delta'(\overset{\text{state}}{q}, \overset{\text{i/p Symbol}}{i}) = \underbrace{\varepsilon\text{-clo}\left(\underbrace{\delta\left(\underbrace{\varepsilon\text{-clo}(q)}_{1^{st}}, i\right)}_{2^{nd}}\right)}_{3^{rd}}$$



$$\delta' : A \xrightarrow{0} ?$$

$$\varepsilon\text{-clo}\left[\delta\left(\varepsilon\text{-clo}(A), 0\right)\right]$$

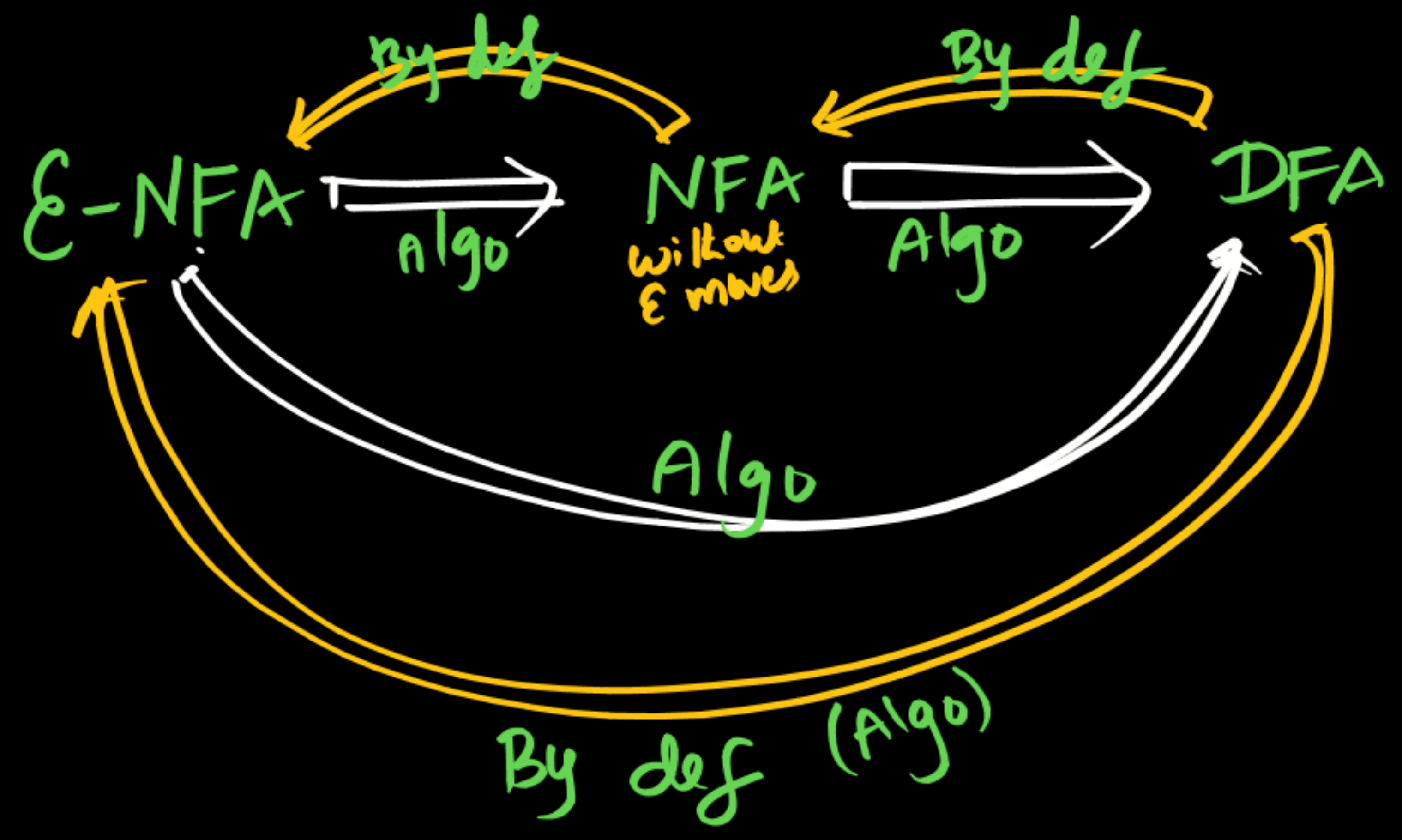
$$\varepsilon\text{-clo}\left(\underbrace{\delta\left(\{A, B, C, D\}, 0\right)}\right)$$

$$\varepsilon\text{-clo}\left(\{A, B\}\right)$$

$$\boxed{\varepsilon}\ 0\ \boxed{\varepsilon} = 0$$

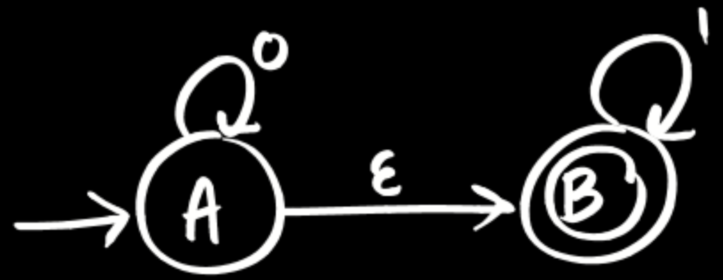
ε-clo 1st δ 2nd ε-clo 3rd

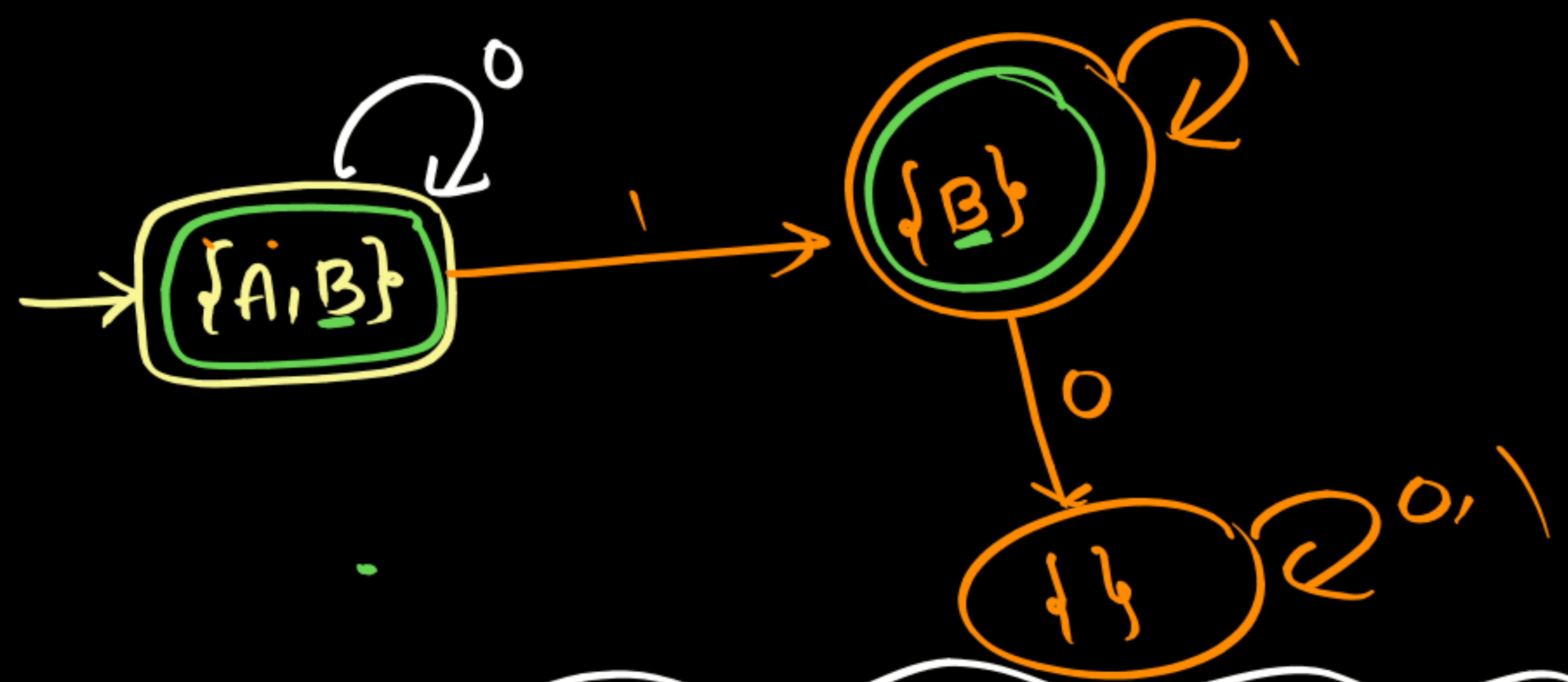
NFA



$$\boxed{NFA \cong DFA}$$

NFA





$\delta(A,0)=\{A\}$

$\delta(A,1)=\{\ \}$

$\delta(B,0)=\{\ \}$

$\delta(B,1)=\{B\}$

$\epsilon\text{-clo}(A)=\{A,B\}$

$\epsilon\text{-clo}(B)=\{B\}$

$\delta'(\{A,B\},0)=\underbrace{\epsilon\text{-clo}(\underbrace{\delta(\{A,B\},0)}_{\{A\}})}_{\{A,B\}}=\{A,B\}$

$\delta'(\{A,B\},1)=\{B\}$

$\delta'(\{B\},0)=\{\ \}$

$\delta'(\{B\},1)=\{B\}$

I) Initial state of DFA $=\epsilon\text{-clo}(\text{Initial of }\epsilon\text{-NFA})$
$=\epsilon\text{-clo}(A)=\{A,B\}$

II) $\delta'(\{A,B\},i)=\epsilon\text{-clo}[\delta(\{A,B\},i)]$

Summary

ε-NFA ✓

ε-NFA $\Longrightarrow$ NFA ✓

ε-NFA $\Longrightarrow$ DFA ✓

Don't apply in GATE

$\boxed{99\%}$

Thank you