CS & IT Engineering

Finite Automata:
Regular Grammar
and
Pumping Lemma

Lecture No:08

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Previous Class Summary:

✓ FA with O/P

Topics to be covered Today:

1. Regular Grammar

2. pumping Lemma for regular languages

Grammar $(G) = (V, T, P, S)$

- S → start symbol, $S \in V$
- P → Set of rules (productions)
- T → Set of terminals
- V → Set of variables (non-terminals)

Grammar:

$\boxed{S} \rightarrow AaB$

Start

$A \rightarrow bc \mid \varepsilon$

$B \rightarrow a \mid bS$

$V = \{S, A, B\}$ $T = \{a, b, c\}$

Regular Grammar

- → It represents a regular set
- → It is $\underbrace{\text{LLG}}_{\text{Left Linear}}$ or $\underbrace{\text{RLG}}_{\text{Right Linear}}$

LLG

① Rule :

$$\underbrace{V}_{\text{exactly one non-terminal}} \rightarrow \underbrace{V\,T^*}_{\text{exactly one non-terminal followed by any sequence of terminals}} \;\Big|\; \underbrace{T^*}_{\text{any sequence of terminals}}$$

② Examples :

$$S \rightarrow Sab \mid S \mid a \mid \varepsilon \mid bc$$

$$V \rightarrow VTT \mid VT^0 \mid T^1 \mid T^0 \mid T^2$$

$T = \{a, b, c\}$ $V = \{S\}$

RLG

① Rule :

$$V \rightarrow T^*V \mid T^*$$

② Examples :

i) $S \rightarrow \underbrace{abcd}_{T^*} \mid \underbrace{aS}_{T^*V} \mid \underbrace{bS}_{T^*V} \mid \underbrace{\varepsilon}_{T^*}$

ii) $S \rightarrow abA$
$A \rightarrow \varepsilon \mid bS$

Identify Regular Grammars

① $S \to \varepsilon$ — LLG ✓, RLG ✓, RG ✓

② $S \to ab \mid \varepsilon$ — LLG ✓, RLG ✓, RG ✓

③ $S \to S \mid a \mid abc$ — LLG ✓, RLG ✓, RG ✓

VT^*

T^*V

④ $S \to Aa$, $A \to bcd$ — LLG ✓, RLG ✗, RG ✓

⑤ $S \to \underline{S}a \mid \underline{S}bc \mid \varepsilon$ — LLG ✓, RLG ✗, RG ✓

*⑥ $S \to abS \mid Sab \mid \varepsilon$ — LLG ✗, RLG ✗, RG ✗

⑦ $S \to aS \mid bS \mid \varepsilon$ — LLG ✗, RLG ✓, RG ✓

⑧ $S \to AaB \mid \varepsilon$, $A \to a$, $B \to b$ — not LLG, not RLG, not RG

⑨ $S \to SS \mid a$ — not LLG, not RLG, not RG

$$LLG \cong RLG$$

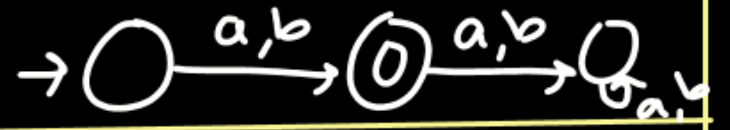
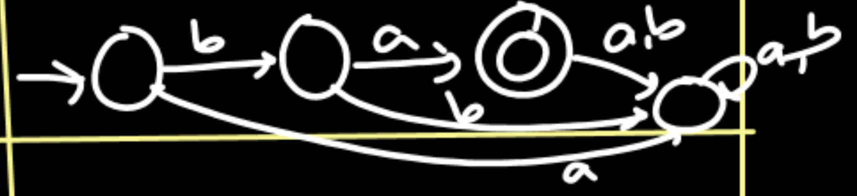
Algo

Algo

L(LLG) is Regular language

L(RLG) is Regular language

Regular Grammar

Regular Grammar	Regular Language	DFA
① $S \to \varepsilon$	$L = \{\varepsilon\}$	
② $S \to a \mid b$	$L = a+b$	
③ $S \to Aa$, $A \to b$	$L = \{ba\}$	
④ $S \to aB$, $B \to a \mid b$	$L = a(a+b)$	
⑤ $S \to aA \mid bB$, $A \to a$, $B \to b$	$L = aa+bb$	
⑥ $S \to a \mid ab \mid aab \mid \varepsilon$	$L = \varepsilon + a + ab + aab$	
⑦ $S \to S \mid a \mid \varepsilon$	$L = a + \varepsilon$	
⑧ $S \to Aa$	$L = \phi$	

(9)

$S \to Aa \mid Bb$

$A \to a \mid b \mid \varepsilon$

$B \to a \mid \varepsilon$

$L(A) = a + b + \varepsilon$

$L(B) = a + \varepsilon$

$$L = L(S) = A.a + B.b$$

$$= \underbrace{(a+b+\varepsilon).a} + \underbrace{(a+\varepsilon).b}$$

$$= \underline{aa + ba + a + ab + b}$$

(10)

$S \to Sa$

$L = \phi$

(11)

$S \to Sa \mid Sb$

$L = \phi$

(12) $S \rightarrow Sa \mid b$ $\quad L = ba^*$

(13) $S \rightarrow Sa \mid a$ $\quad L = aa^* = a^+ = a^*a$

(14) $S \rightarrow Sa \mid \varepsilon$ $\quad L = \varepsilon a^* = a^*$

(15) $S \rightarrow Saa \mid \varepsilon$ $\quad L = \varepsilon (aa)^* = (aa)^*$

(16) $S \rightarrow Sab \mid \varepsilon$ $\quad L = \varepsilon.(ab)^* = (ab)^*$

(17) $S \rightarrow aS \mid b$ $\quad L = a^*b$

(18) $S \rightarrow aS \mid a$ $\quad L = a^*a = a^+ = aa^*$

(19) $S \rightarrow aS \mid \varepsilon$ $\quad L = a^*$

(20) $S \rightarrow aaS \mid \varepsilon$ $\quad L = (aa)^*$

(21) $S \rightarrow abS \mid \varepsilon$ $\quad L = (ab)^*$

$S \to Sa \mid b$

repeats

$S \Rightarrow \boxed{S}a \Rightarrow ba$

$S \Rightarrow \boxed{S}a \Rightarrow Saa \Rightarrow baa.$

b ✓ $\longrightarrow ba^0$

ba ✓ $\longrightarrow ba^1$

baa ✓ $\longrightarrow ba^2$

baaa ✓ ba^3

baaaa ✓ ba^4

⋮

$= ba^*$

I) $X \to X\alpha \mid \beta$

$L(X) = \beta\alpha^*$

II) $X \to \alpha X \mid \beta$

$\boxed{L = \alpha^*\beta}$

22) $S \rightarrow Sa \mid Sb \mid \varepsilon$

$= S(a+b)^*$

$$\boxed{L = (a+b)^*}$$

ε ✓
a ✓
b ✓
aa ✓
ab ✓
ba ✓
bb ✓
⋮

III) $X \rightarrow X\alpha_1 \mid X\alpha_2 \mid X\alpha_3 \mid \beta_1 \mid \beta_2$

$$\boxed{L = (\beta_1+\beta_2)(\alpha_1+\alpha_2+\alpha_3)^*}$$

(23) $S \rightarrow Sa \mid Sb \mid c$ $\quad L = S(a+b)^* = c(a+b)^*$

(24) $S \rightarrow \underbrace{Sa \mid Sb}_{} \mid a$ $\quad L = S(a+b)^* = a(a+b)^*$

(25) $S \rightarrow \underbrace{Sa \mid Sb}_{} \mid \underbrace{a \mid b}_{}$

$$L = S\,(a+b)^*$$

$$= (a+b)(a+b)^* = (a+b)^+$$

26. $S \rightarrow aS \mid bS \mid \varepsilon$ $\qquad L = (a+b)^*$

27. $S \rightarrow aS \mid bS \mid a$ $\qquad L = (a+b)^* a$

28. $S \rightarrow aS \mid bS \mid a \mid b$ $\qquad L = (a+b)^* S = (a+b)^*(a+b)$

$$= (a+b)^+$$

29. $S \rightarrow aaS \mid bbS \mid \varepsilon$ $\qquad L = (aa+bb)^*$

30. $S \rightarrow aaS \mid bbS \mid a \mid b \mid c$ $\qquad L = (aa+bb)^* (a+b+c)$

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$S \to aA$

$A \to aA \mid bA \mid \varepsilon \Rightarrow (a+b)^*$

$L = a(a+b)^*$

equivalent

32

$S \to Sa \mid Sb \mid a$

$L = a(a+b)^*$

33

$$\boxed{\begin{aligned} S &\to Aa \\ A &\to Aa \mid Ab \mid \varepsilon \end{aligned}} \Rightarrow L = (a+b)^* a$$

Equivalent

34

$S \to aS \mid bS \mid a \Rightarrow L = (a+b)^* a$

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$S \rightarrow aA \mid bA$

$A \rightarrow bB$

$B \rightarrow aB \mid bB \mid \varepsilon$

$L(A) = b(a+b)^*$

$L(B) = (a+b)^*$

$L = (a+b)\,A$

$= (a+b)\,\underline{b}(a+b)^*$

2nd

$= (ab+bb)(a+b)^*$

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$S \rightarrow Aa \mid Ab$

$A \rightarrow Ba$

$B \rightarrow Ba \mid Bb \mid \varepsilon$

$L(A) = (a+b)^* a$

$L(B) = (a+b)^*$

$L = A \cdot (a+b)$

$= (a+b)^* \underline{a}(a+b)$

2nd from end

$= (a+b)^*(aa+ab)$

recursi.

$S \rightarrow Sa \mid Sb$

$S \rightarrow \underbrace{Aa \mid Ab}_{A(a|b)}$

$$S \to Sa \mid Sb \mid c$$

$$S(a+b)^*$$

$$c(a+b)^*$$

$$S \to Aa \mid Ab$$

$$L(S) = Aa + Ab$$

$$= A(a+b)$$

*** (37)

$$S \rightarrow aA$$
$$A \rightarrow bS \mid c$$

$$L(A) = bS + c$$

$$S \rightarrow aA$$
$$\Downarrow$$
$$S \rightarrow a(bS + c)$$

$$S \rightarrow abS \mid ac$$

$$L = (ab)^* ac$$

(38)

$$S \rightarrow aS \mid bA$$
$$A \rightarrow bA \mid bS \mid c$$

$$\boxed{A} \rightarrow b\boxed{A} \mid bS \mid c$$

$$L(A) = b^*(bS + c)$$

$$S \rightarrow aS \mid bA$$
$$\Downarrow$$
$$S \rightarrow aS \mid bb^*(bS + c)$$
$$\Downarrow$$
$$S \rightarrow aS \mid bb^*bS \mid bb^*c$$

$$L = (a + bb^*b)^* \, bb^*c$$

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$S \rightarrow aA \mid bB$

$A \rightarrow aA \mid bA \mid \epsilon \Rightarrow (a+b)^*$

$B \rightarrow aB \mid bB \mid \epsilon \Rightarrow (a+b)^*$

$L = a(a+b)^* + b(a+b)^*$

$= (a+b)(a+b)^*$

$= (a+b)^+$

Regular Language (L) ⟹ $\boxed{\text{Pumping Lemma for regular languages}}$ ⟹ It gives a proof that helps to understand why L is regular.

Note:

I) $\boxed{\text{Pumping Lemma should not be used to check whether L is regular or not}}$

II) P.L. is used to prove regular languages

III) P.L. uses pigeon hole principle

Pumping Lemma for regular language (L) :

i) choose $\underbrace{\text{pumping constant}}$ (P)

It is greater or equal to no. of states in min DFA

ii) choose a string $w \in L \ni |w| \geq P$

iii) Divide w into 3 parts $\ni xyz = w$
(x, y, and z)

i) $y \neq \epsilon$

ii) $|xy| \leq P$

iv) $\forall_{i \geq 0}\ xy^iz \in L$ iff L is Regular

$w = xyz$

1 $L = (a+b)^* aa$

ignore dead state

No. of states in min DFA $= 3$

$P \geq 3$

Let $P = 3$

$w = baa$

x y z

ε b aa

$y \neq \varepsilon$

$|xy| \leq 3$

$\forall i \quad xy^iz \in L$

$\varepsilon (b)^i (aa)$

$i=0 \Rightarrow aa \in L$

$i=1 \Rightarrow baa \in L$

$i=2 \Rightarrow b^2aa \in L$

$\vdots$

Pumping Lemma

(2) $\{a^{3n+5} \mid n \geq 0\}$

$\boxed{\text{Pumping}^{(P)}\ \text{Constant} \geq 6}$

Let $P = 6$

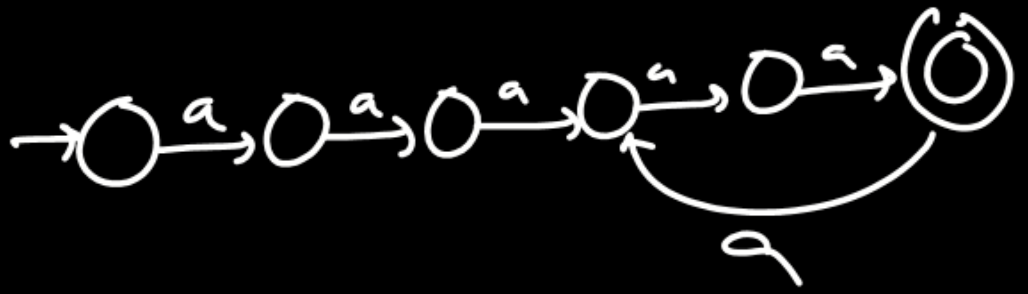
$w = a^8 \quad (|w| \geq P)$

$\varepsilon \quad a^3 \quad a^5$

$\forall i \geq 0 \quad \varepsilon (a^3)^i a^5 \in L$

$\{a^5, a^8, a^{11}, a^{14}, \dots\}$

↳ 6 States in min DFA



Pumping Lemma

↳ we can also use for proving non regulars with contradiction

L is non regular

L $\Longrightarrow$ Assume L is Regular $\Longrightarrow$ $\boxed{\text{Pumping Lemma for regulars}}$ $\Longrightarrow$ contradiction occurs, it means our assumption is wrong $\Longrightarrow$ L is nonreg

P.L. using contradiction

Pumping Lemma

$L = \{a^n b^n\}$ is non reg

1) Assume L is regular

2) Let constant is $2n$

3) $w = a^n b^n$ $\quad |w| \geq 2n$

$a^{n-1} \quad a \quad b^n$

4) $\forall_i \; a^{n-1}(a)^i b^n \in L$ iff L is Regular

5) But when $i=0 \Rightarrow a^{n-1} \varepsilon b^n \notin L$

Contradiction occurs, our assumption is wrong

6) L is nonregular

Pumping Lemma

Constant

$a^n b^n \Rightarrow 2n$

$a^{2n} b^{2n} \Rightarrow 4n$

$a^{2n} b^{n} \Rightarrow 3n$

given

$L = a^{2n} b^{2n}$

$w = a^{2n} b^{2n}$

$\underbrace{\varepsilon}_{x} \; \underbrace{a^{2n} b^{2n}}_{y} \; \underbrace{\varepsilon}_{z}$

What value of min i, $x y^i z \notin L$? $= 2$

$i=0 \Rightarrow \varepsilon \in L$

$i=1 \Rightarrow a^{2n} b^{2n} \in L$ $\quad \varepsilon \, (a^{2n} b^{2n})^i \, \varepsilon$

$i=2 \Rightarrow a^{2n} b^{2n} a^{2n} b^{2n} \notin L$

1st → You may remember words

2nd → You may understand definitions

3rd → You may understand meanings

4th → You may interpret/Apply

Summary

Regular Grammar ✓

P.L. ✓

Thank you