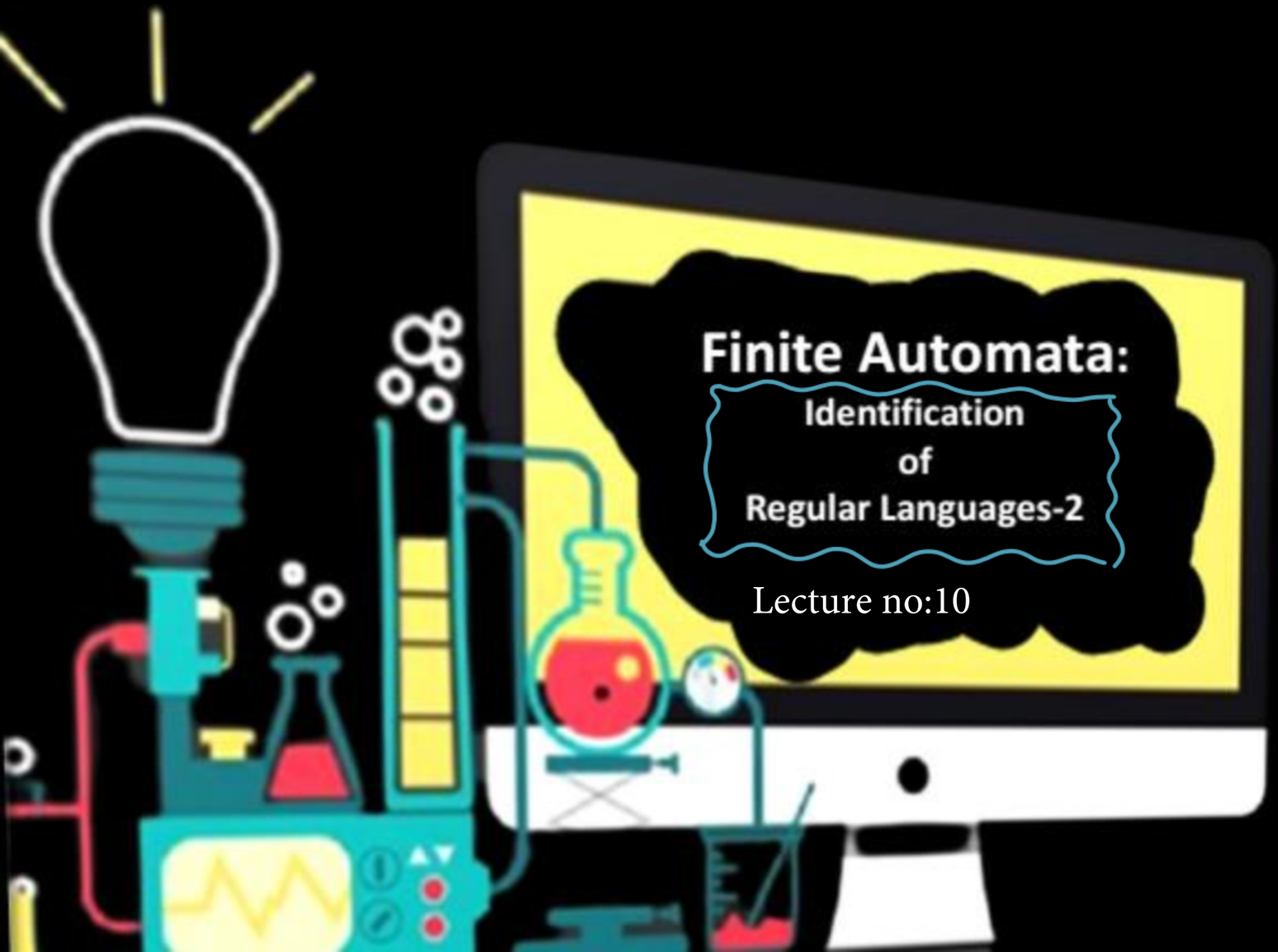
CS & IT Engineering



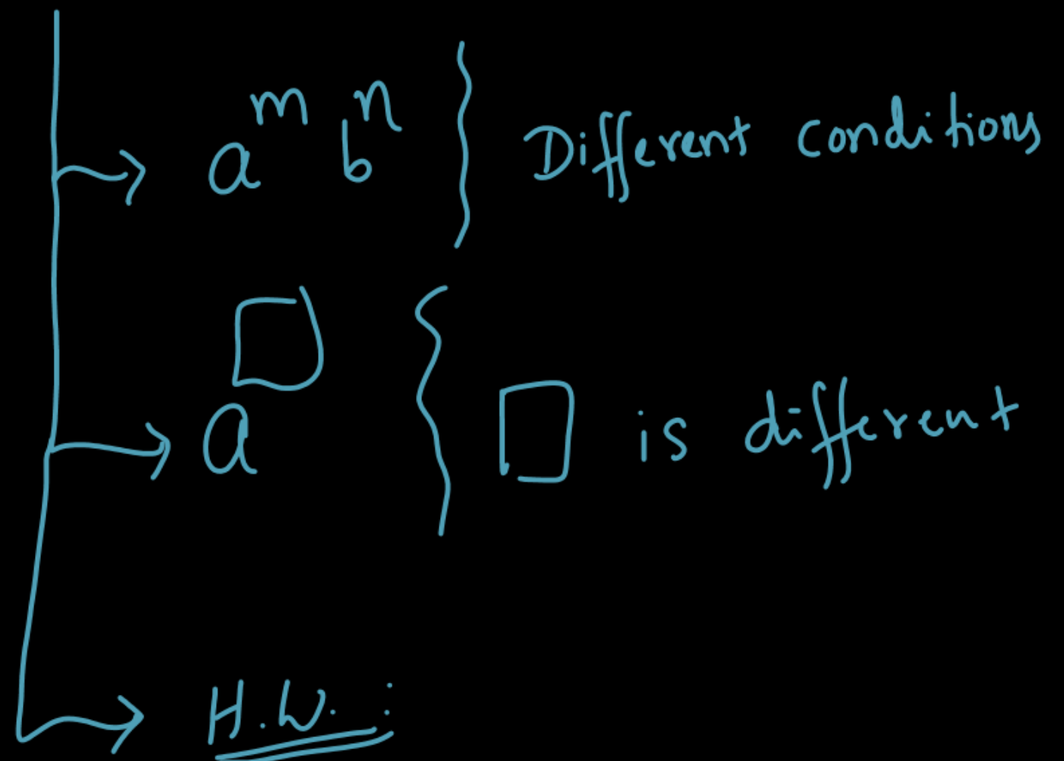
Finite Automata:

Identification of Regular Languages-2

Lecture no:10

Deva sir

Previous Class Summary:



Topics to be covered Today:

- other models

Languages Identification:

H.W.:

(S1) $\Rightarrow (aa)^*(bb)^*$ ⇒ Reg, Inf

(S2) ⇒ $a(aa)^*(bb)^*$ ⇒ Reg, Inf

(S3) ⇒ $(aa)^* b(bb)^*$ ⇒ "

(S4) ⇒ $a(aa)^* b(bb)^*$ ⇒ "

(S5) ⇒ $(aa)^*(bb)^* + a(aa)^* b(bb)^*$ ⇒ Reg, Inf

(S6) ⇒ $a(aa)^*(bb)^* + (aa)^* b(bb)^*$ ⇒ Reg, Inf

(S7) ⇒ $b^*(b^* a b^* a b^*)^* b^*$ ⇒ Reg, Inf

(S8) ⇒ $[(a+b)^2]^*$ ⇒ Reg, Inf

Languages Identification:

(59) Reg, Inf $\Rightarrow$ A is final

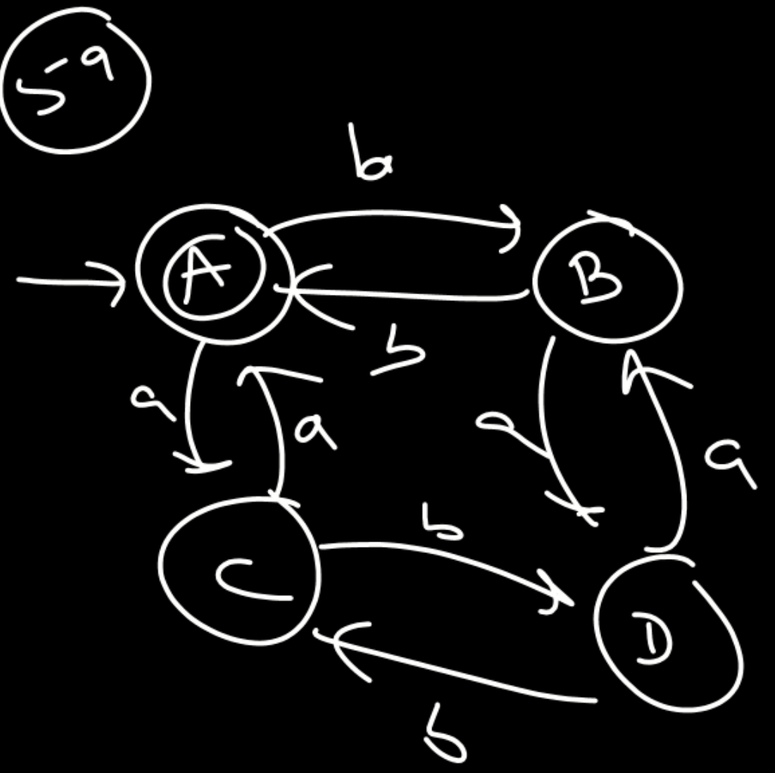
(60) Reg, Inf $\Rightarrow$ A & D are final

(61) " $\Rightarrow$ B & C are final

(62) $\{w_1 w_2 \mid w_1, w_2 \in \{a,b\}^*\}$

$= (a+b)^*$

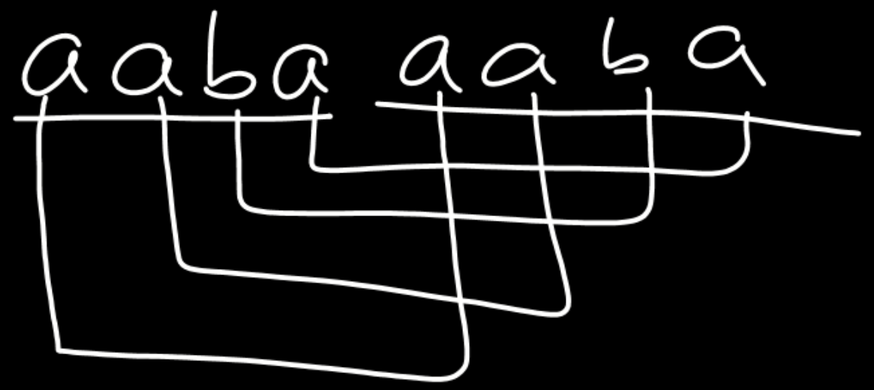
$\rightarrow$ Reg, Inf



Languages Identification:

(63) $\{w_1 w_2 \mid w_1, w_2 \in \{a,b\}^*, w_1 = w_2\}$

$= \{ww \mid w \in \{a,b\}^*\} = \{\underbrace{\varepsilon\varepsilon}_{\varepsilon}, aa, bb, aaaa, abab, baba, \cdots\}$



→ Inf, Not reg

Languages Identification:

(64) $\{w_1 w_2 \mid w_1, w_2 \in \{a,b\}^*, |w_1| = |w_2|\}$

$= \{w \mid |w| = \text{even}, w \in \{a,b\}^*\}$

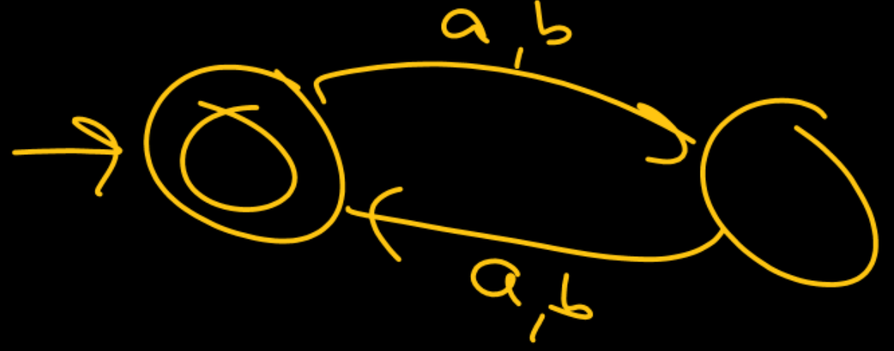
$= \{\varepsilon, aa, ab, ba, bb, \ldots\} \Rightarrow$ All even length

String $= \overbrace{w_1 w_2}^{|w_1| = |w_2|}$ $= (a+b)^2)^*$

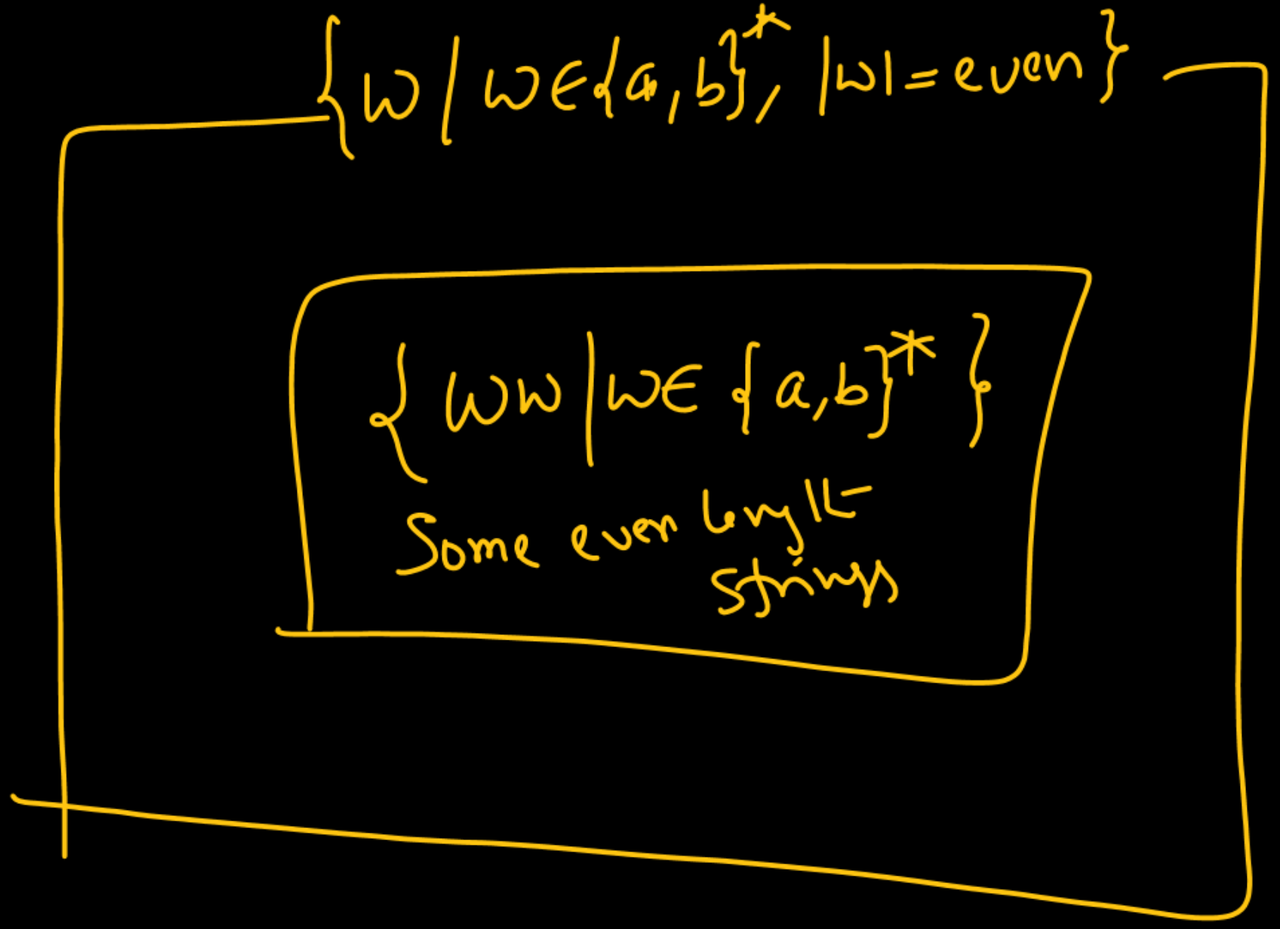
0 ← ε ε = ε

2 ← a–a, a–b, b–a, b–b

4 ← aa aa, aa ab, aa ba, aa bb |



Languages Identification:



Languages Identification:

65 $\{w \mid w \in \{a,b\}^*,\ n_a(w) = n_b(w)\}$

$= \{\varepsilon, \cancel{a}, \cancel{b}, \cancel{aa}, ab, ba, \cancel{bb}, \ldots\}$

→ Not regular

ε ✓
ab ✓
ba ✓
aabb ✓
abab ✓
baba ✓
bbaa ✓
abba ✓
baab ✓
:

Languages Identification:

(66) = (63) $= \{ww \mid w \in \{a,b\}^*\}$ ⟶ Not regular

(67) $\{ww \mid w \in a^*\} = \{\varepsilon\varepsilon, aa, aaaa, a^3a^3, \ldots\}$

$= \{\varepsilon, a^2, a^4, a^6 \ldots\} = a^{2n}$

$= \{a^{2n}\}^*$

$= (aa)^*$

$= (aa^*)^*$

→ Regular, Infinite

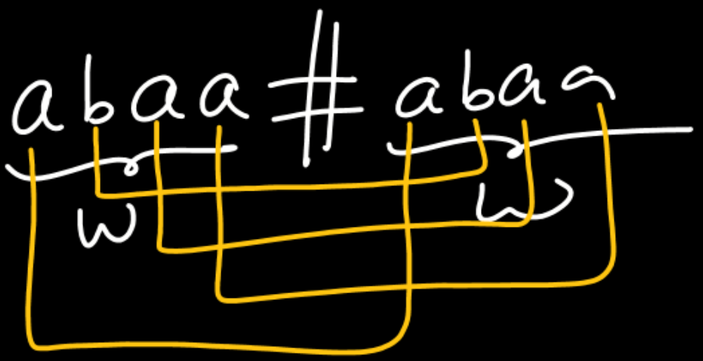
Note: 1) $((aa)^*)^* = (aa)^*$

2) $(a(aa)^*)^* = a^*$

Languages Identification:

(68) $\{w\#w \mid w \in \{a,b\}^*\}$ ⟶ Not regular

$= \{wcw \mid w \in \{a,b\}^*\}$



(69) $\{w\#w \mid w \in a^*\}$

$\Sigma = \{a, \#\}$

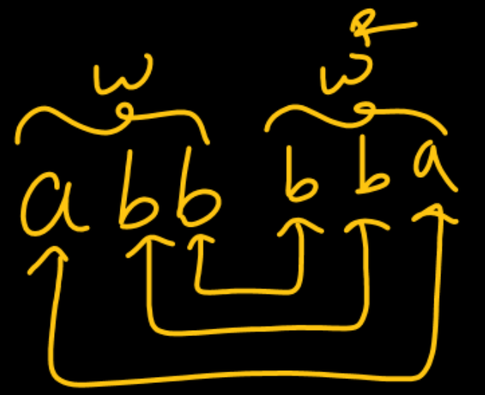
$= \{\#, a\#a, a^2\#a^2, \cdots\}$

$= \{a^n\#a^n\}$

⟶ Not regular

Languages Identification:

(70) $\{ww^R \mid w \in \{a,b\}^*\}$ → Not regular



(71) $\{ww^R \mid w \in a^*\} =$ (67)

$= \{ww \mid w \in a^*\}$

→ Regular, Inf $= (aa)^*$

over 1 symbol
$w = w^R$

(72) $\{w\#w^R \mid w \in \{a,b\}^*\}$

(73) $\{w\#w^R \mid w \in a^*\} = \{w\#w \mid w \in a^*\} = \{a^n\#a^n\}$ → Not reg

$=$ (69)

Languages Identification:

Regular

(74) $\{wwx \mid w, x \in \{a,b\}^*\} = (a+b)^*$

ε ε You can generate everything using x

$w = ab$, $x = aa$, string $= ababaa$

$w = \varepsilon$, $x = ababaa$

(75) $\{wxw \mid$ " $\} = (a+b)^*$

(76) $\{xww \mid$ " $\} = (a+b)^*$

(77) $\{ww^R x \mid$ " $\} = (a+b)^*$

(78) $\{wxw^R \mid$ " $\} = (a+b)^*$

(79) $\{xww^R \mid$ " $\} = (a+b)^*$

Languages Identification:

(80) $\{ww x \mid w, x \in \{a,b\}^{+}\}$ ⇨ Not regular

$|w|=1$ → Take min w

$w = a/b$

$aax + bbx$

$aa(a+b)^{+} + bb(a+b)^{+}$ → (I)

$|w|=2$ → 2nd min of w

$w = aa/ab/ba/bb$

String form $= wwx$

✓ $aaaax$
✗ $ababx$
✗ $baabx$
✓ $bbbbx$

If all these string forms covered in (I), we can say given language is regular. otherwise it is not regular.

Languages Identification:

(81) $\{wxw \mid w, x \in \{a,b\}^+\}$ — Not regular

- $|w|=1 \Rightarrow axa + bxb$
- $|w|=2 \Rightarrow aaxaa + abxab + baxba + bbxbb$
- ⋮

(82) $\{xww \mid$ " $\}$ — Not regular

- $|w|=1 \Rightarrow xaa + xbb$
- $|w|=2 \Rightarrow xaaaa + xabab + xbaba + xbbbb$

(83) $\{ww^Rx \mid$ " $\}$

- $|w|=1 \Rightarrow aax + bbx$
- $|w|=2 \Rightarrow aaaax + abbax + baabx + bbbbx$

(84) $\{wxw^R \mid$ " $\}$ — Regular — ***

$a(a+b)^+a + b(a+b)^+b$

- $|w|=1 \Rightarrow axa + bxb$
- $|w|=2 \Rightarrow aaxaa + abxba + baxab + bbxbb$

(85) $\{xww^R \mid$ " $\}$ — Not regular

Languages Identification:

(86) $\{wwx \mid w, x \in \{a,b\}^*, |x| < 10\}$

(87) $\{wxw \mid$ " , " $\}$

(88) $\{xww \mid$ " , " $\}$

(89) $\{ww^Rx \mid$ " , " $\}$

(90) $\{wxw^R \mid$ " , " $\}$

(91) $\{xww^R \mid$ " , " $\}$

x is finite
we can't eliminate dependency
$\therefore$ Not regular

Languages Identification:

(92) $\{w\varepsilon w\varepsilon x \mid w, x \in \{a,b\}^*, \ |w| < 10\}$

(93) $\{wxw \mid \ " \ , \ |w| < 10\}$

(94) $\{xww \mid \ " \ " \ \}$

(95) $\{ww^R x \mid \ " \ " \ \}$

(96) $\{wxw^R \mid \ " \ " \ \}$

(97) $\{xww^R \mid \ " \ " \ \}$

$|w| =$ finite
$\Rightarrow$ regular

Languages Identification:

(98) $\{wwx \mid w, x \in \{a,b\}^+ \quad |w| < 10, \ |x| < 50\}$

(99) $\{wxw \mid$ " " $\}$

(100) $\{xww \mid$ " " $\}$

(101) $\{ww^Rx \mid$ " " $\}$

(102) $\{wxw^R \mid$ " " $\}$

(103) $\{xww^R \mid$ " " $\}$

$|w| =$ finite
$|x| =$ finite

All languages are finite

$\Downarrow$

Regulars

Languages Identification:

$$ww x \mid w, x \in \{a, b\}^+$$

Case I : $|w| = \text{fin}$, $|x| = \text{fin}$

Language is finite

Case II : $|w| = \text{finite}$

Language is regular

Case III : $|x| = \text{finite}$

Language is nonregular

Languages Identification:

Regular

(104) $\{ wxw^R \mid w, x \in \{a,b\}^+ \}$

- $|w|=1 \Rightarrow axa+bxb$
- $|w|=2 \Rightarrow$ All forms covered in above

(105) $\{ xww^Ry \mid w, x, y \in \{a,b\}^+ \}$

$(a+b)^+aa(a+b)^+ + (a+b)^+bb(a+b)^+$

- $|w|=1 \Rightarrow xaay + xbby$
- $|w|=2 \Rightarrow$ xaaaay, xabbay, xbaaby, xbbbby

(106) $\{ wxwy \mid w, x, y \in \{a,b\}^+ \}$

$a(a+b)^+a(a+b)^+ + b(a+b)^+b(a+b)^+$

(107) $\{ xwyw \mid w, x, y \in \{a,b\}^+ \}$

$(a+b)^+a(a+b)^+a + (a+b)^+b(a+b)^+b$

(108) $\{ xwyw^Rz \mid w, x, y, z \in \{a,b\}^+ \}$

$(a+b)^+a(a+b)^+a(a+b)^+ + (a+b)^+b(a+b)^+b(a+b)^+$

(109) $\{ xwywz \mid w, x, y, z \in \{a,b\}^+ \}$

Languages Identification:

110 $\overline{\{a^n b^n \mid n \geq 0\}}$ is non regular

$L \cup \bar{L} = \Sigma^*$

111 $\left(\{a^n b^n\} \cup \overline{\{a^n b^n\}}\right)$ is $\Sigma^* = (a+b)^*$ regular

$L \cap \bar{L} = \phi$

112 $\left(\{a^n b^n\} \cap \overline{\{a^n b^n\}}\right)$ is $\{\} = \phi$ is regular

113 $\{a^n b^n\}^*$ is not regular

Languages Identification:

$\overline{L}$ is Regular $\Rightarrow$ L is regular

L is Regular $\Rightarrow$ $\overline{L}$ is regular

$\overline{L}$ is nonregular $\Rightarrow$ L is non reg

L is nonregular $\Rightarrow$ $\overline{L}$ is non reg

Languages Identification:

$$\{a^n b^n\}^*$$

$$\{\varepsilon, ab, a^2b^2, a^3b^3, \ldots\}^*$$

$$= \{a^{n_1}b^{n_1}\ a^{n_2}b^{n_2}\ a^{n_3}b^{n_3}\ a^{n_4}b^{n_4} \ldots\ a^{n_k}b^{n_k}\}$$

Not regular

Languages Identification:

(114) $\{a^{\text{Prime}}\}^* \Rightarrow$ Regular

(115) $\{a^{2n}\}^* \Rightarrow$ Regular

$\{a^n b^n\}^*$ is not regular

$\{a^{\text{prime}}\}^*$ is regular

If L^* is Regular

then L may or may not regular

Languages Identification:

116. $\{w \mid w \in \{0,1\}^*,\ n_0(w) = n_1(w)\}$ ⇒ Not reg

117. $\{w \mid w \in \{0,1\}^*,\ n_{00}(w) = n_{11}(w)\}$

*118. $\{w \mid w \in \{0,1\}^*,\ n_{01}(w) = n_{10}(w)\}$ ⇒ Regular

reverse pattern

*119. $\{w \mid w \in \{0,1\}^*,\ n_{011}(w) = n_{110}(w)\}$ ⇒ Regular

Reverse pattern

120. $\{w \mid w \in \{0,1\}^*,\ n_{010}(w) = n_{110}(w)\}$

Languages Identification:

Regular (121) $\{0, 2, 4, 6, 8, \ldots, 2n, \ldots\}$ represented in unary

$\rightarrow \{a^{2n} \mid n \geq 0\} = \{\varepsilon, aa, aaaa, aaaaaa, \ldots\}$

Not regular (122) $\{1, 2, 4, 8, 16, \ldots, 2^n, \ldots\}$ represented in unary

$\rightarrow \{a^{2^n} \mid n \geq 0\}$

Languages Identification:

H.W. *** (123) $\{w \mid w \in \{0,1\}^*$, every prefix s of w satisfy $|n_0(s) - n_1(s)| \leq 2\}$

Regular

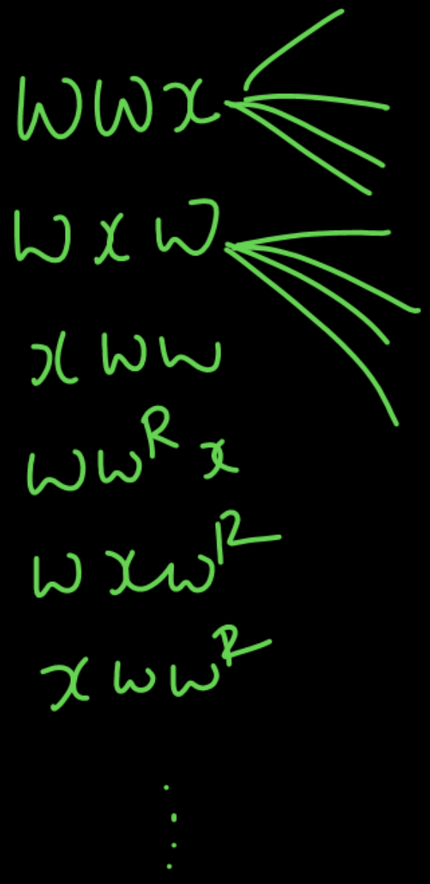
(124) $\{w \mid w \in \{0,1\}^*, |n_0(w) - n_1(w)| \leq 2\}$

Non-regular

Regular

(125) $\{w \mid w \in \{0,1\}^*$, every prefix s of w satisfy $|n_0(s) - n_1(s)| \leq 1\}$

Summary



Thank you

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Closure properties