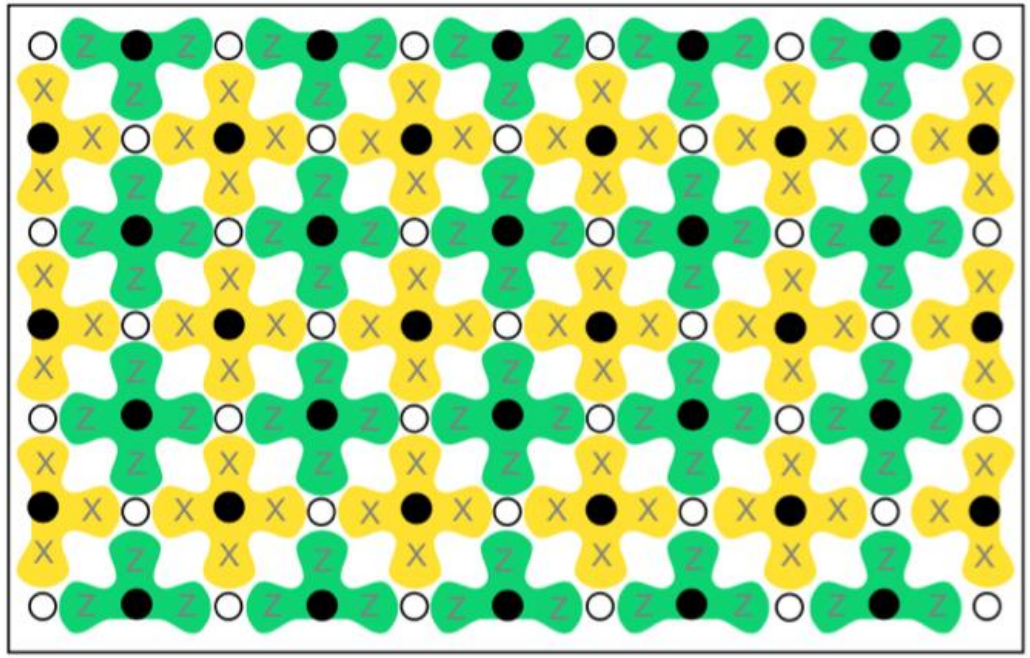
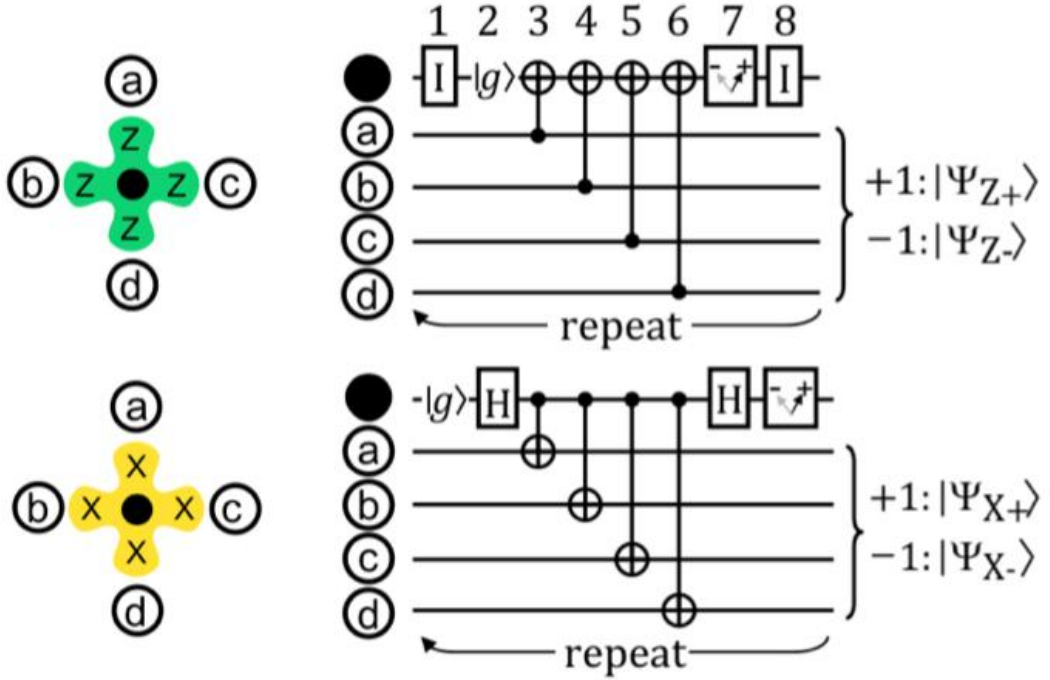
surface code

Austin G. Fowler

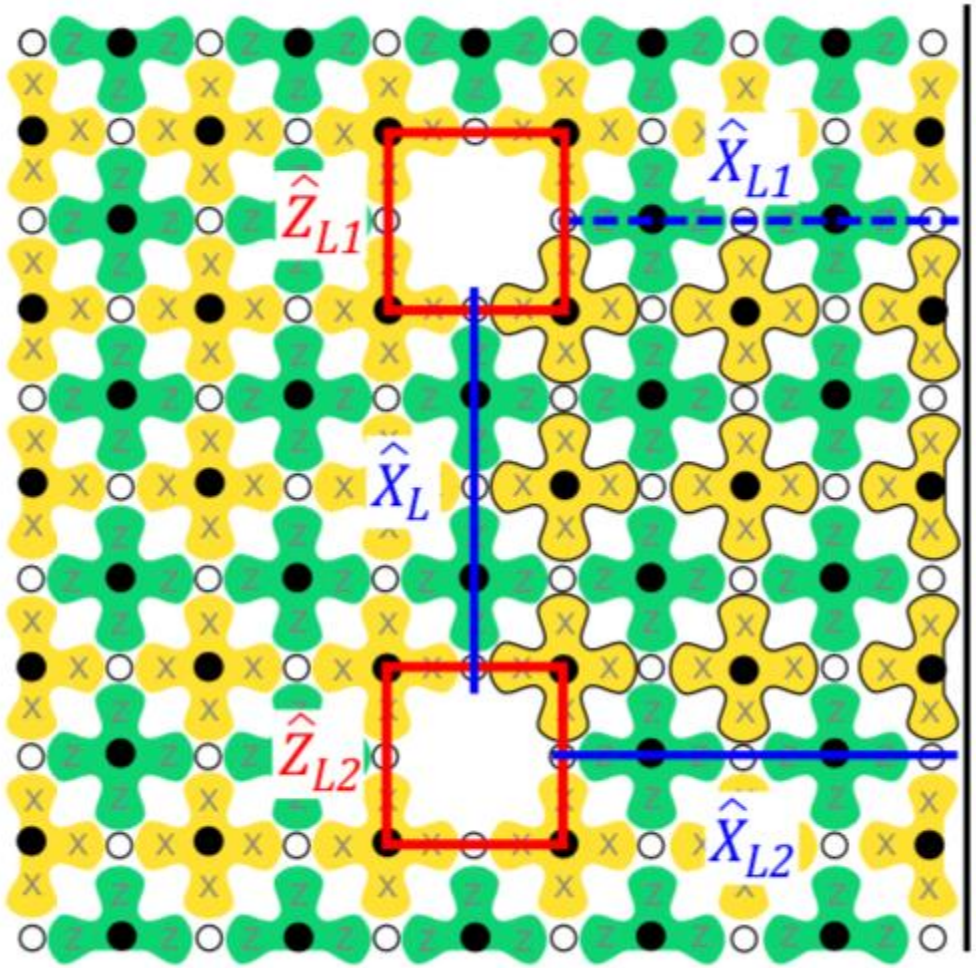
- 1-logical qubit sureface code

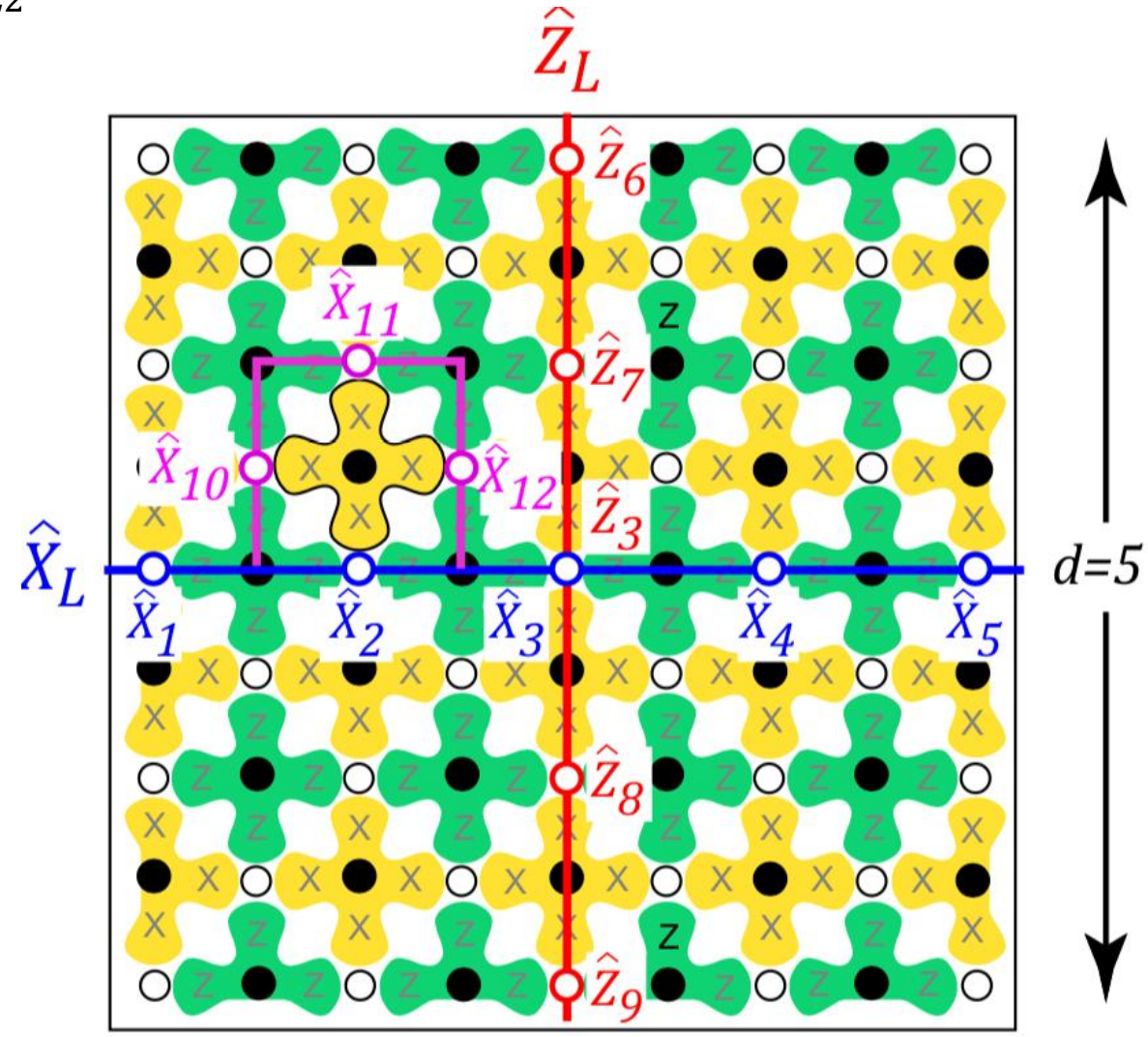


- 每个测量比特对应一个电路



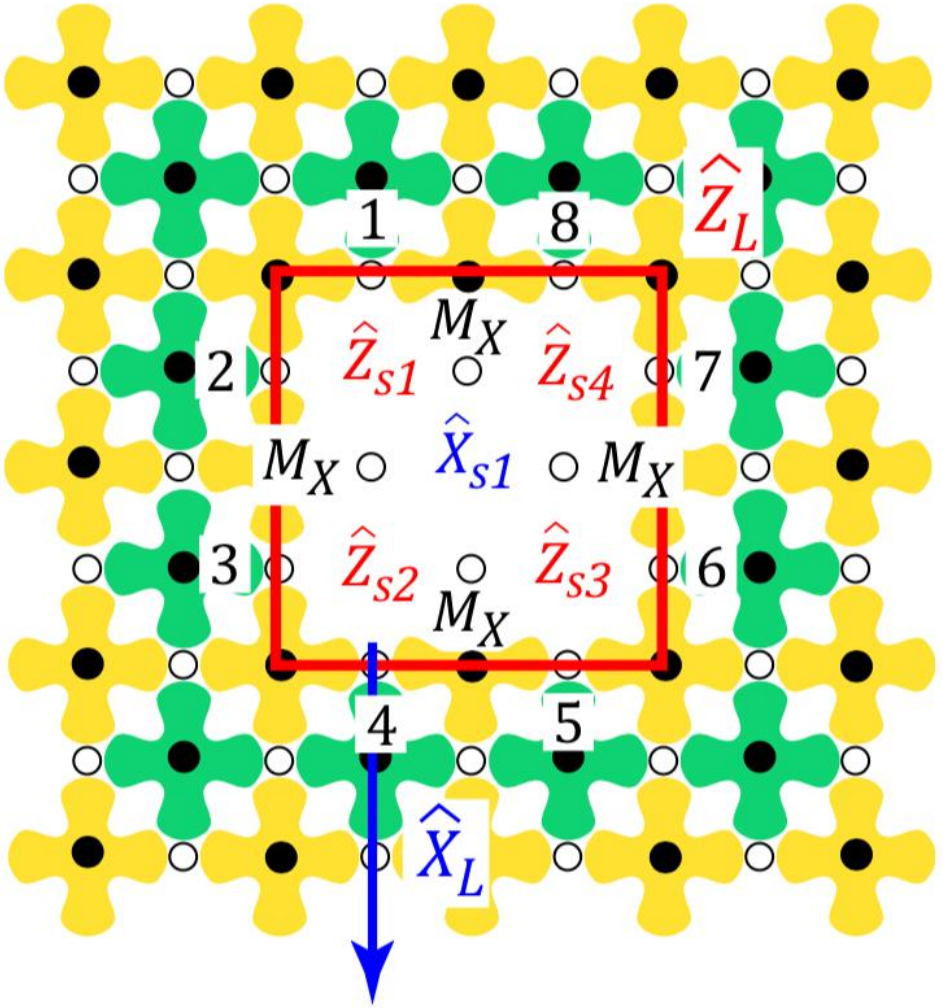
两种X_L, Z_L形式（贯穿surface边界和cut）
做cut本质上是形成边界， 取$Z_L = Z_{L1}$或Z_{L2}



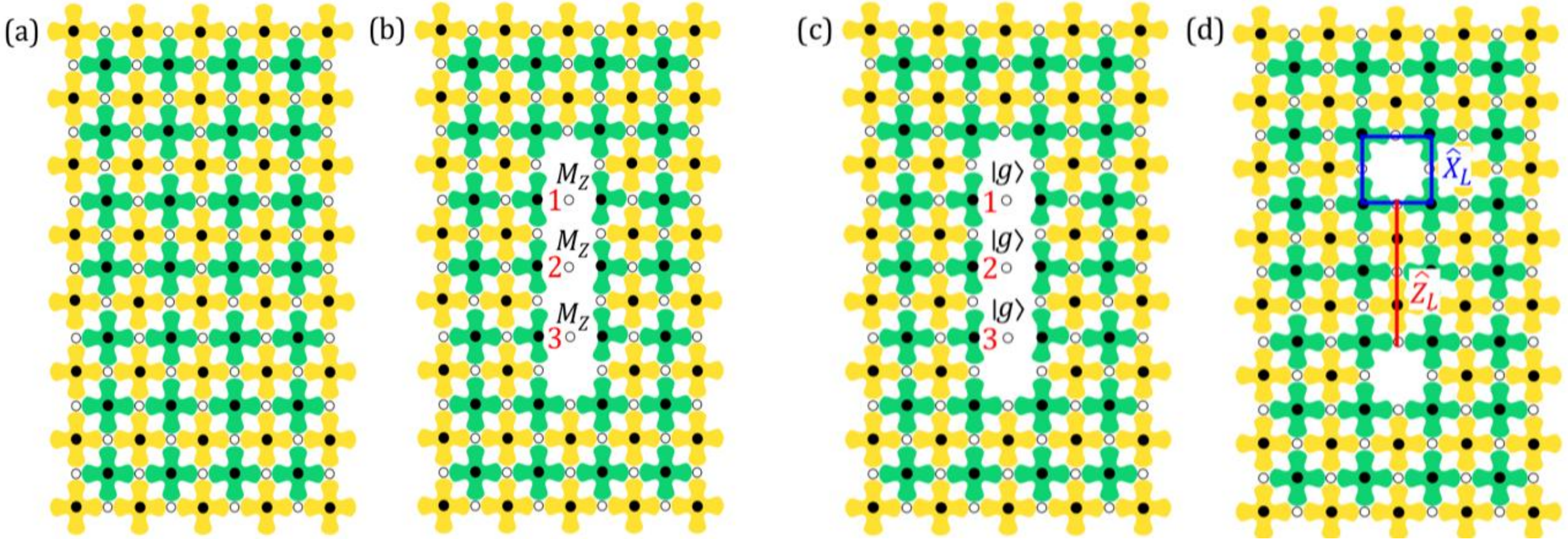


更大的 cut，目的是增加d，提高纠错能力(图示为scale = 2 的cut)

- 在代码中，给定cut中心的坐标，人为取定非loop的operator如(X_L)为两cut之间data qubit的连线中最靠边界的一列（如示意图所示）
- 在braid操作中，选其为使得编织形成的loop最小的一列
- 在 Hardamard门中，选它为图示位置
- turn off 内部的measure qubit之后，测量从4-terminal变为3-terminal的measure qubit测量的stabilizer所缺失的data qubit(即为产生的孤立data qubit最外圈的)，并将测量结果与现在的3-terminal measure qubit的测量结果相乘，作为syndrome measurement.



- 初始化操作，两次error tracking
- a->b:测量孤立1, 2, 3 qubit in z basis，并将测量结果与相应measure qubit测量结果相乘，作为syndrome measurement。
- b->c:将1, 2, 3重置为|g>.
- c->d:turn on，获取测量结果进行error tracking.



- 测量操作，两次error tracking，初始化操作的逆
- a->b:测量孤立1, 2, 3 qubit in z basis，并将测量结果与相应measure qubit测量结果相乘，作为syndrome measurement。
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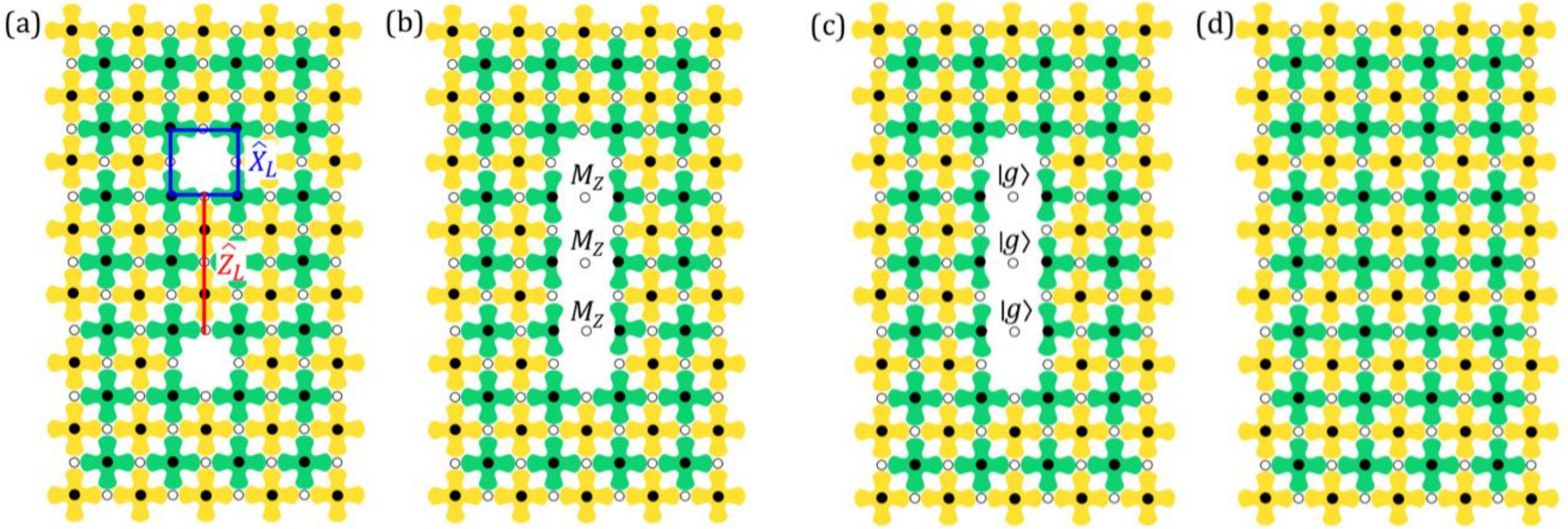
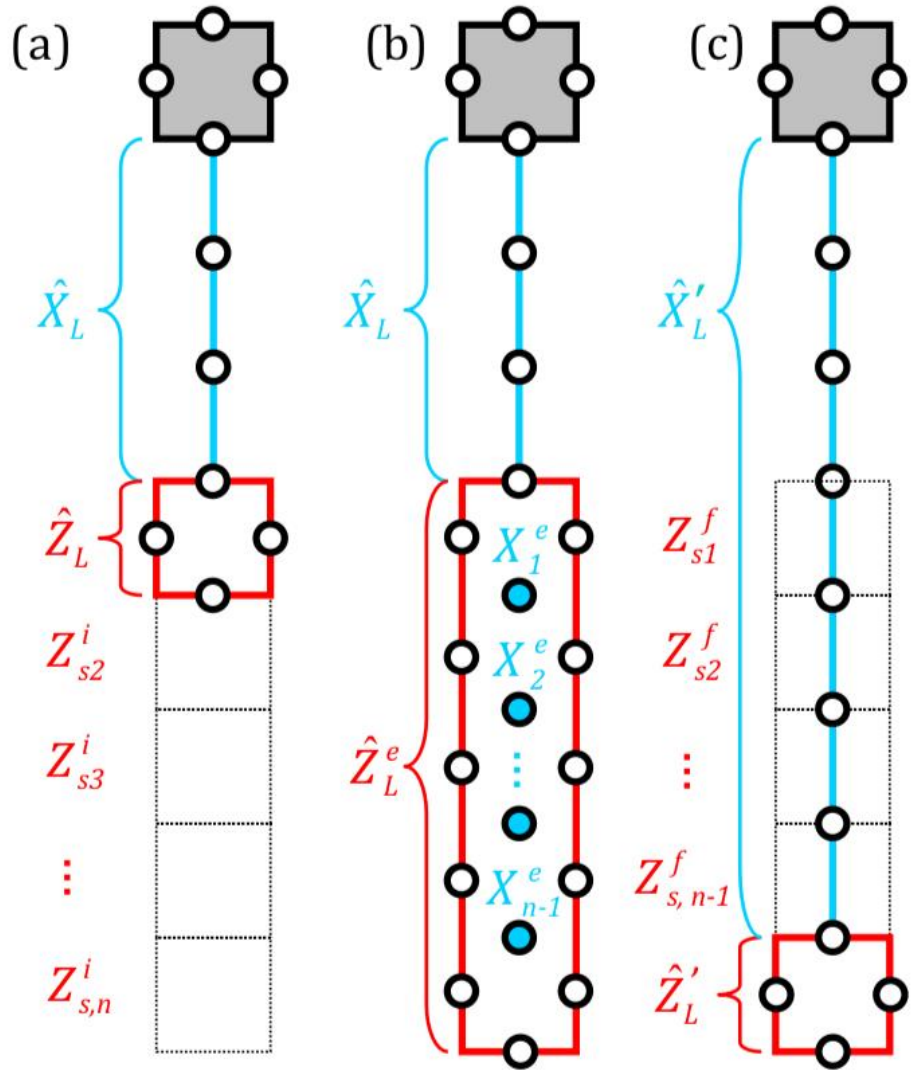




- move cut操作，两次error tracking
- a->b:测量孤立1, 2, ..., n-1 qubit in x basis，并将测量结果与相应measure qubit测量结果相乘，作为syndrome measurement
- b->c:turn on，获取stabilizer($Z^f_{s,1}$,...,$Z^f_{s,n-1}$)测量结果进行error tracking.

- byproduct:

pz = $Z^i_{s,1} * \cdots * Z^i_{s,n-1} * Z^f_{s,1} * \cdots Z^f_{s,n-1}$，若结果为-1，相当于在1,...,n-1stabilizer涵盖的data qubit上任取某一个引入x错误

px = $X^e_1 * \cdots * X^e_{n-1}$，相当于在1,...,n-1stabilizer涵盖的data qubit上任取某一个引入z错误

distance:衡量了被编织的两个cut中心measure qubit的下标之差，

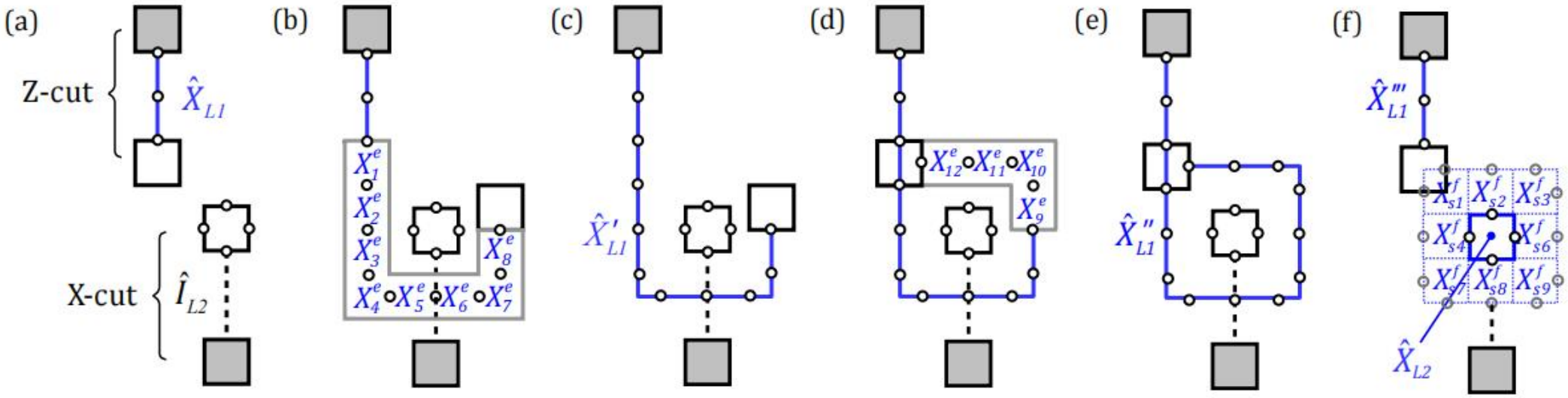
e.g. distance = k 表示 (z2-x1)/wideth = k.

a->e: 流程原理同move操作

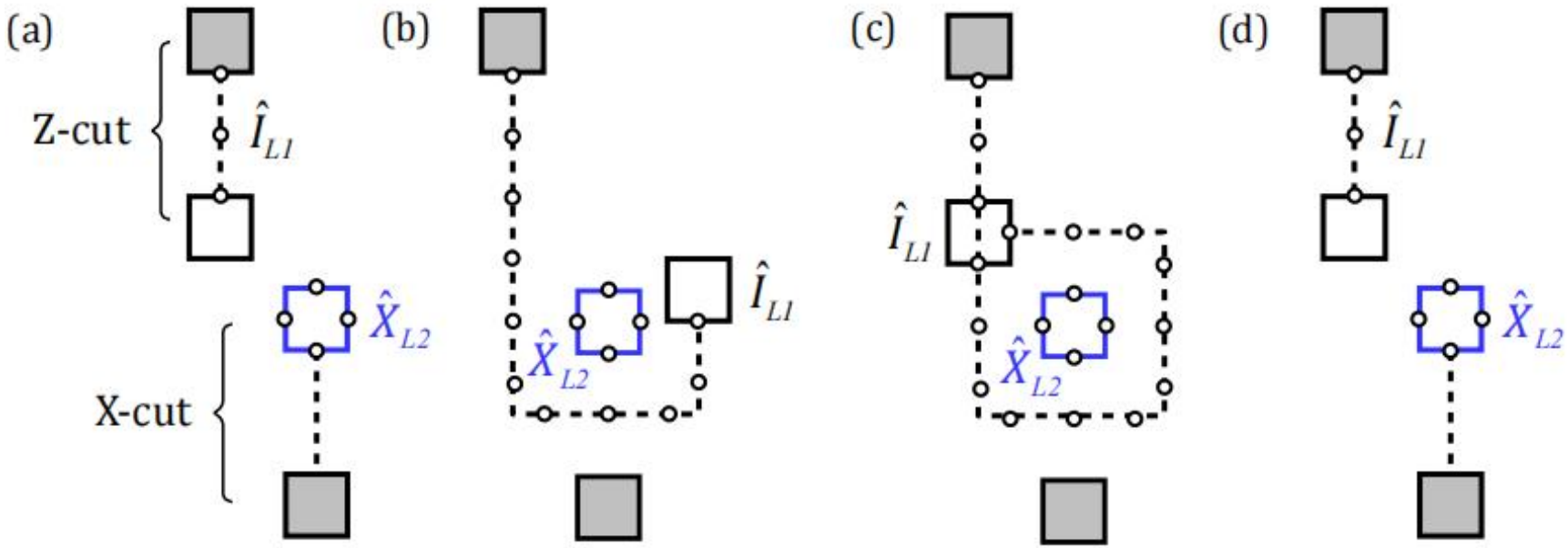
e->f: $X'''_{L1}I_{L2} = X_{Loop}X'''_{L1}I_{L2} = (X_{s1}\cdots X_{s9})X'''_{L1} -> X'''_{L1}X_{L2}$

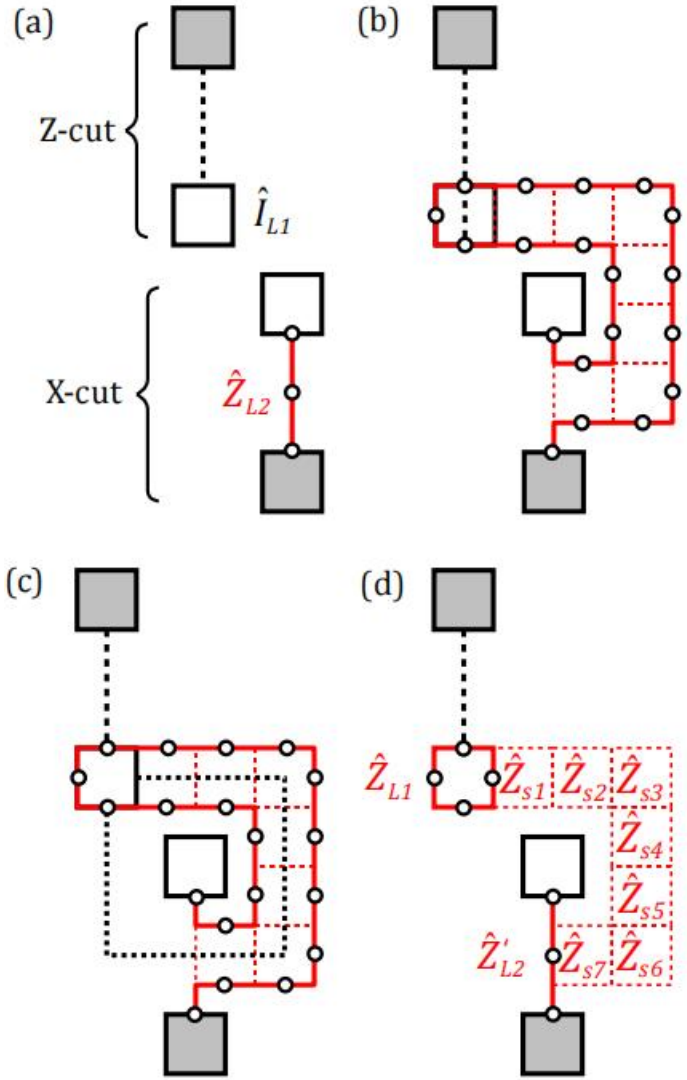
byproduct: px1 = $X^e_{s,1} * \cdots * X^e_{s,12}$

px2= $X^f_{s,1} * \cdots * X^f_{s,9}$



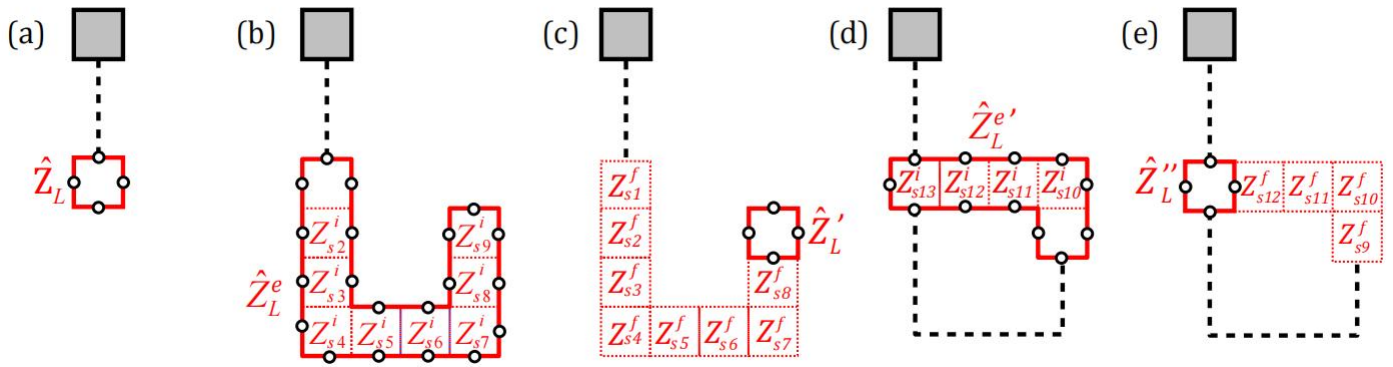
$$I_{L1}X_{L2} -> I_{L1}X_{L2}$$





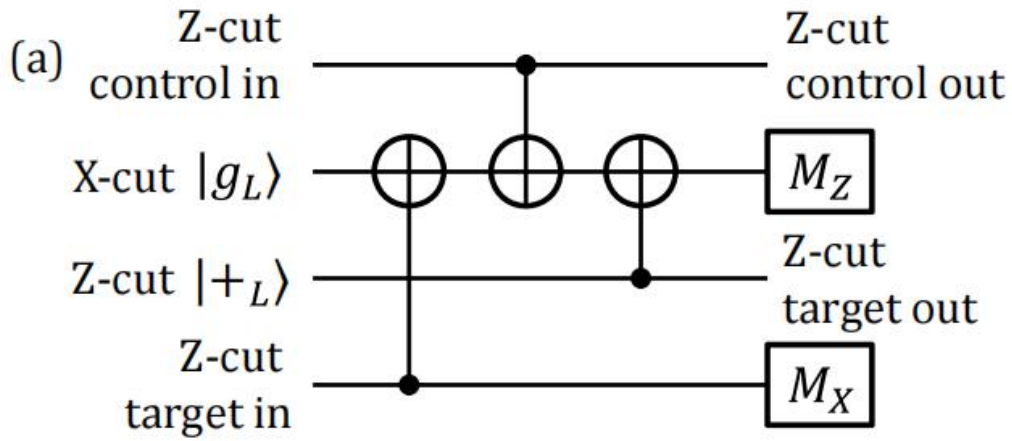
a -> b: 形变Z_{L2}

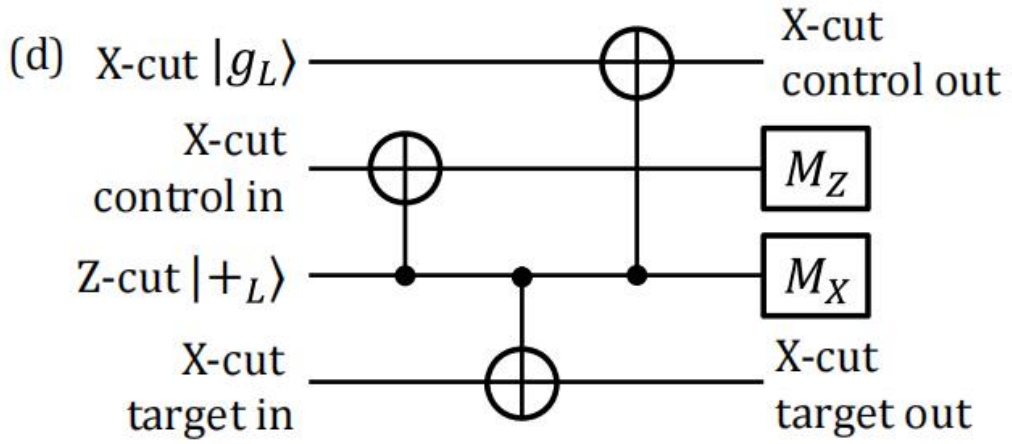
b ->c: 移动Z-cut下面的洞，按下图所示方式

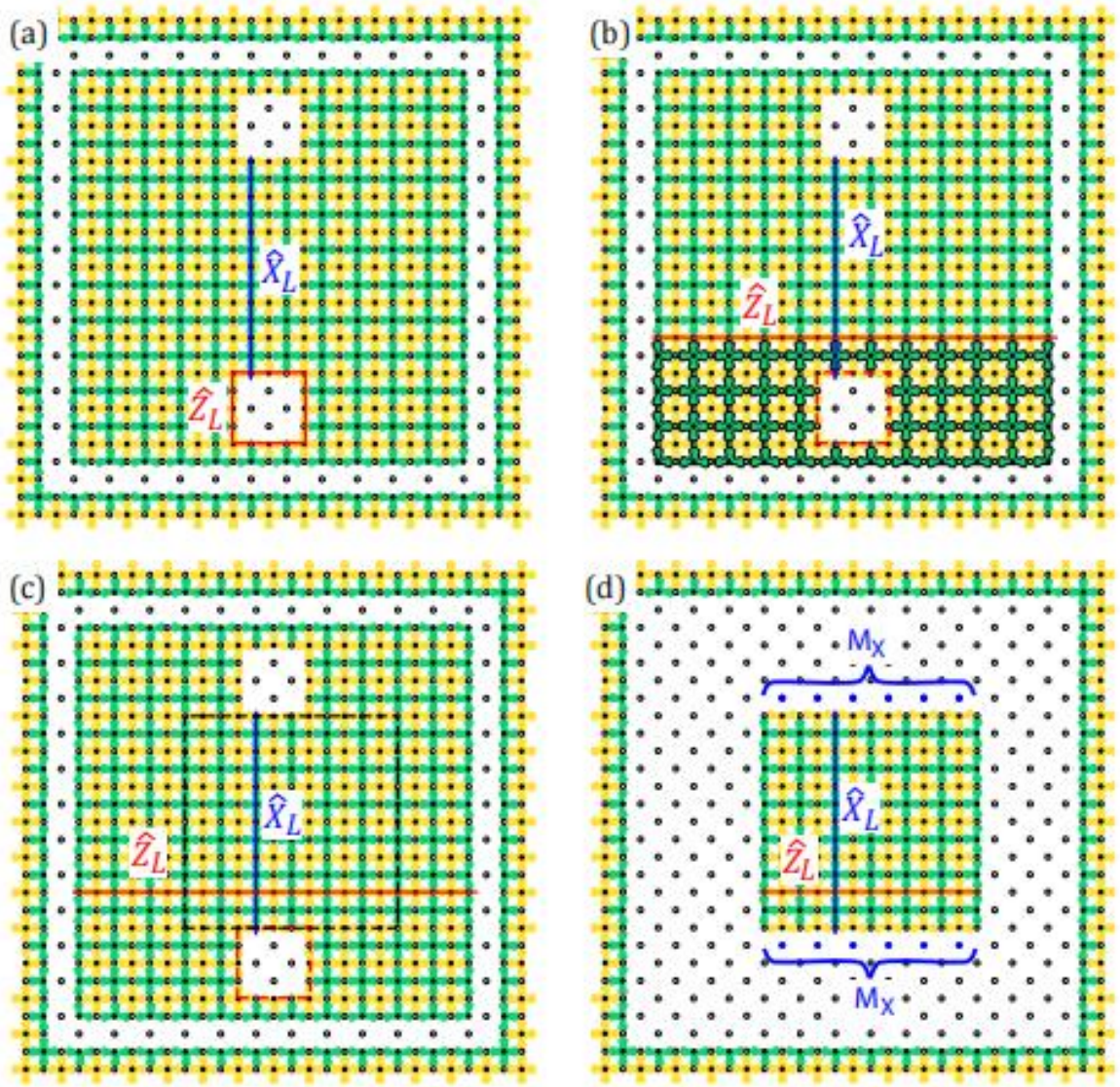


c ->d: byproduct operator处理相位差（乘红框里stabilizer测量结果）

Z-cut braid Z-cut, X-cut braid X-cut

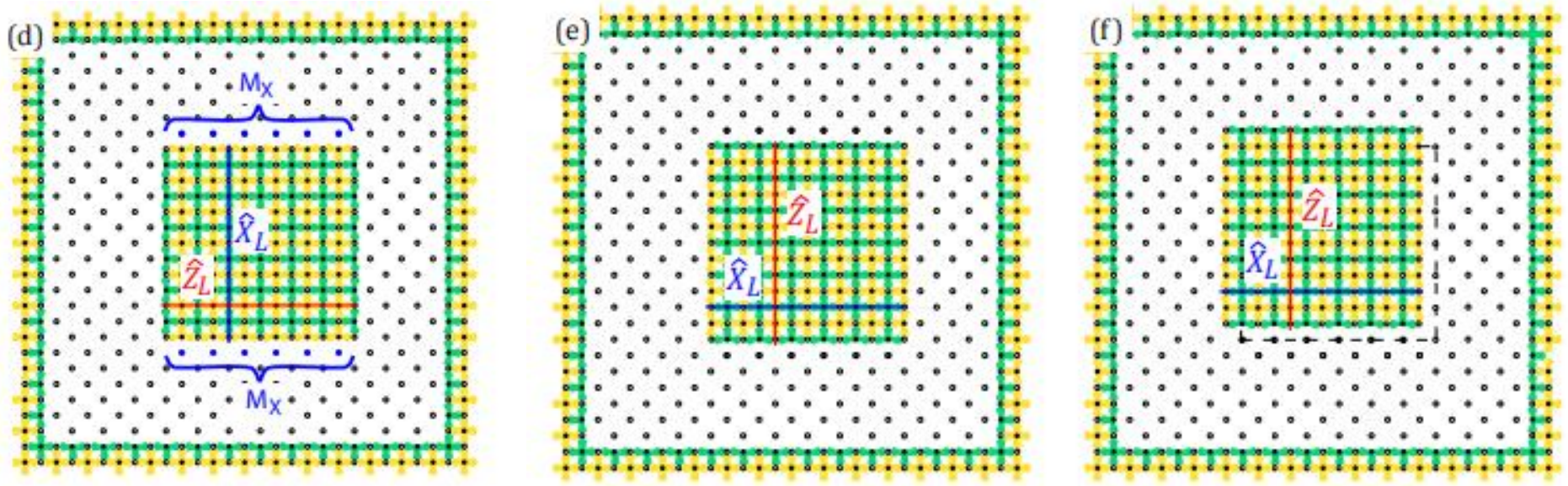






a->b: 形变Z, 乘绿色stabilizer测量结果变为横向Z

b->d: 缩小Z, X作用范围

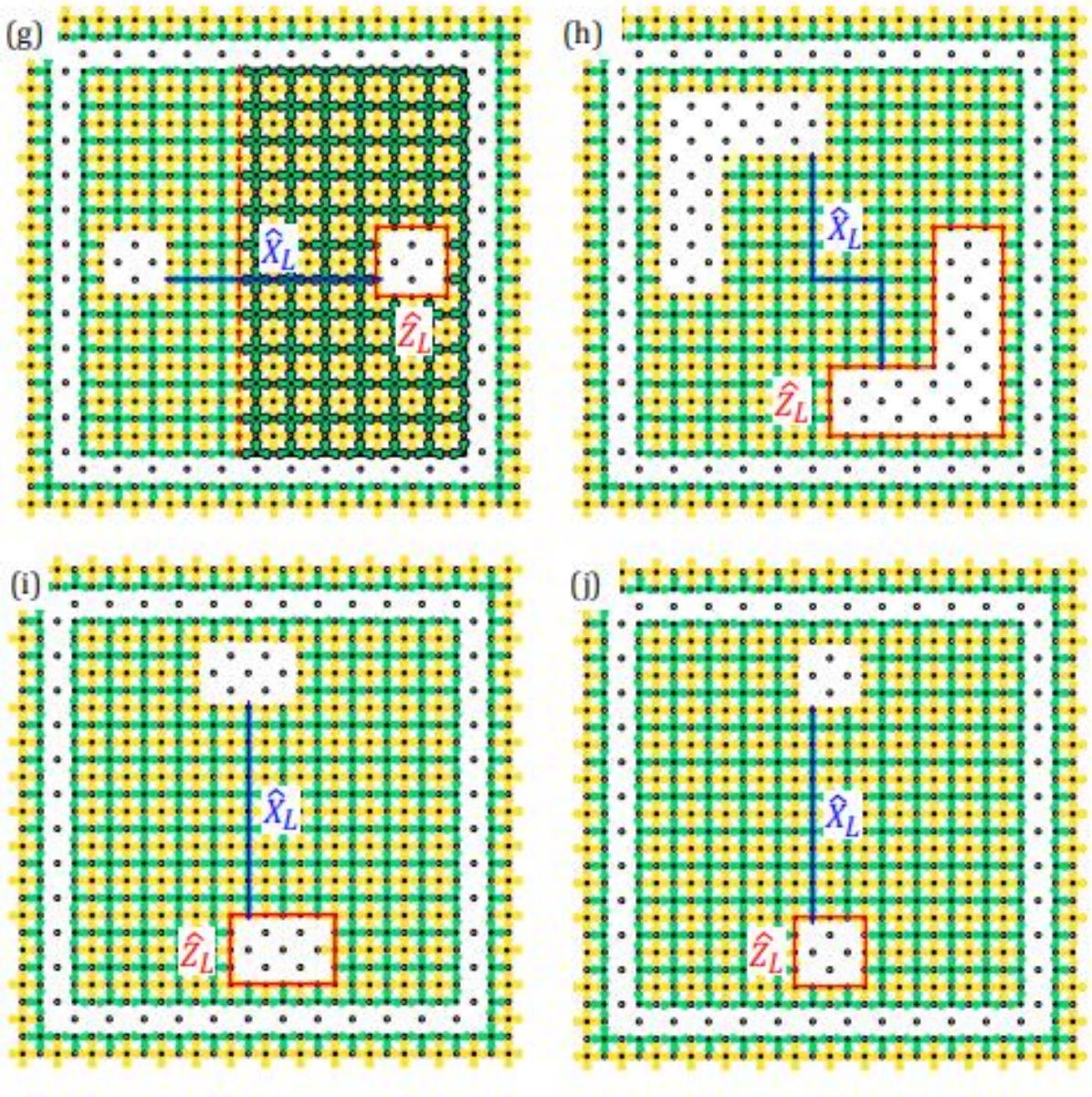


d->e:子块上所有数据比特作用H门

e->f: 作用SWAP门

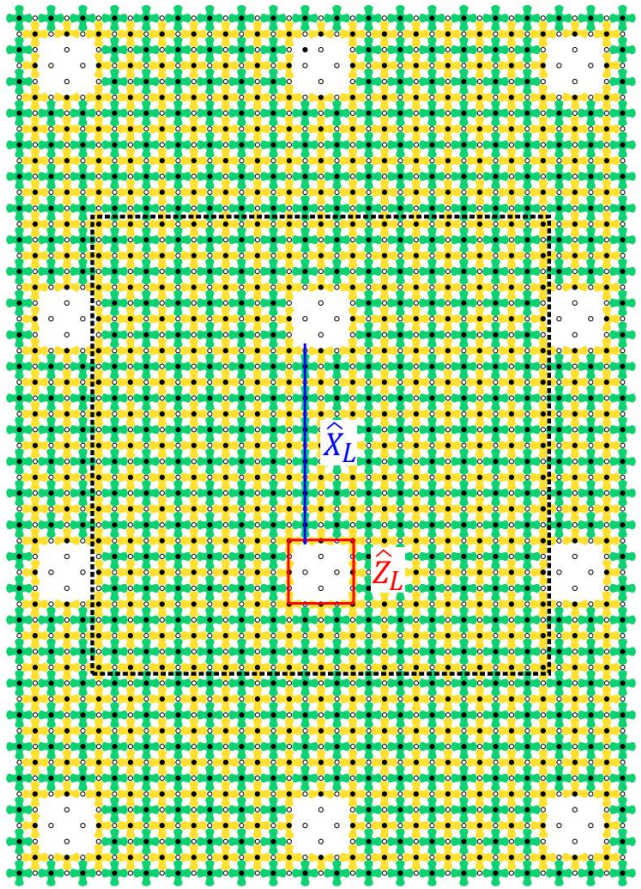
（1）between data qubit and the measure qubit above it

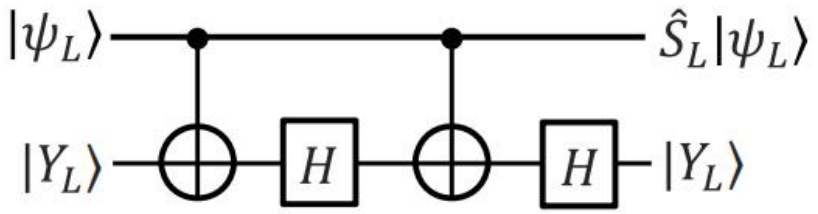
（2）between the measure qubit and data qubit to its left



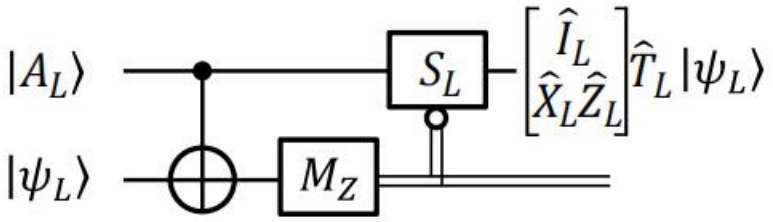
f->g: turn on，形变Z(与之前同理)

g->j:移动cut至原来的位置





$$Y_L = (|g_L> + i|e_L>)/\sqrt{2}$$



$$A_L = (|g_L> + e^{i\pi/4}|e_L>)/\sqrt{2}$$

若M_Z测量结果为1 时，得到
$T_L|\psi>$
若M_Z测量结果为 -1 时，得到
$X_L Z_L T_L|\psi>$