



# Realized jumps on financial markets and predicting credit spreads<sup>☆</sup>

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## ABSTRACT

This paper extends the jump detection method based on bipower variation to identify realized jumps on financial markets and to estimate parametrically the jump intensity, mean, and variance. Finite sample evidence suggests that the jump parameters can be accurately estimated and that the statistical inferences are reliable under the assumption that jumps are rare and large. Applications to equity market, treasury bond, and exchange rate data reveal important differences in jump frequencies and volatilities across asset classes over time. For investment grade bond spread indices, the estimated jump volatility has more forecasting power than interest rate factors and volatility factors including option-implied volatility, with control for systematic risk factors. The jump volatility risk factor seems to capture the low frequency movements in credit spreads and comoves countercyclically with the price–dividend ratio and corporate default rate.

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## 1. Introduction

The relatively large credit spread on high grade investment bonds has long been an anomaly in financial economics. Historically, firms that issue such bonds appear to entail very little default risk yet their credit spreads are sizable and positive (Huang and Huang, 2003). A natural explanation is that these firms are exposed to large sudden and unforeseen movements in the financial markets. In other words, the spread accounts for exposure to market jump risk. Jump risk has been proposed before as a possible source of the credit premium puzzle (Delianedis and Geske, 2001; Zhou, 2001; Huang and Huang, 2003), but the empirical validation in the literature has met with mixed and inconclusive results (Collin-Dufresne et al., 2001, 2003; Cremers et al., 2004, in press). In this paper, we develop a jump risk measure based on identified

realized jumps (as opposed to latent or implied jumps) as an explanatory variable for high investment grade credit spread indices.

The continuous-time jump–diffusion modeling of asset return processes has a long history in finance, dating back to at least Merton (1976). However, the empirical estimation of the jump–diffusion processes has always been a challenge to econometricians. In particular, the identification of actual jumps is not readily available from the time series data of underlying asset returns. Most of the econometric work relies on some combination of numerical methods, computationally intensive simulation-based procedures, and possibly joint identification schemes from both the underlying asset and the derivative prices (see, e.g., Bates, 2000; Andersen et al., 2002; Pan, 2002; Chernov et al., 2003; Eraker et al., 2003, among others).

This paper takes a different and direct approach to identify the realized jumps based on the seminal work by Barndorff-Nielsen and Shephard (2004, 2006). Recent literature suggests that the realized variance measure from high frequency data provides an accurate measure of the true variance of the underlying continuous-time process (Barndorff-Nielsen and Shephard, 2002a; Meddahi, 2002; Andersen et al., 2003b). Within the realized variance framework, the continuous and jump part contributions can be separated by comparing the difference between realized variance and bipower variation (see Barndorff-Nielsen and Shephard, 2004; Andersen et al., 2004; Huang and Tauchen, 2005). Other jump detection methods have been proposed in the literature based on the swap variance contract (Jiang and Oomen, 2005), the range statistics (Christensen and Podolski, 2006; Dobrslav, 2006),

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and the local volatility estimate (Lee and Mykland, forthcoming). Under the reasonable presumption that jumps on financial markets are usually rare and large, we assume that there is at most one jump per day and that the jump dominates the daily return when it occurs. This allows us to filter out the realized jumps, and further to directly estimate the jump distributions (intensity, mean, and variance). Such an estimation strategy based on identified realized jumps stands in contrast with existing literature that generate noisy parameter estimates based on daily returns.

Aït-Sahalia and Yacine (2004) examines how to estimate the Brownian motion component by maximum likelihood, while treating the Poisson or Lévy jump component as a nuisance or noise. Our approach is exactly the opposite – we estimate the jump component directly and then use the results for further economic analysis. The advantages of this approach include that we do not require the specification and estimation of the underlying drift and diffusion functions and that the jump process can be flexible. Such a jump detection and estimation strategy could be invalid for a certain highly active Lévy process with infinite small jumps in a finite time period (Bertoin, 1996; Barndorff-Nielsen and Shephard, 2001; Carr and Wu, 2004). A recent paper by Aït-Sahalia and Jacod (2006) develops a method for detecting the infinite activity Lévy type jumps from Brownian motions. The approach here is more applicable to the compound Poisson jump process, where rare and potentially large jumps in financial markets are presumably the responses to significant economic news arrivals (Merton, 1976). It should be pointed out that bipower variation also works for the infinite activity jumps (Barndorff-Nielsen et al., 2006; Jacod, in press), although we focus solely on the case of rare and large jumps.

In Monte Carlo work, we examine two main settings where the jump contribution to the total variance is 10% and 80%. In these situations, our realized jump identification approach performs well, in that the parameter estimates are accurate and converge as the sample size increases (long-span asymptotics). One important caveat is that these convergence results depend on choosing appropriately the level of the jump detection test. The significance level needs to be set rather loosely at 0.99 when the jump contribution to the total variance is low (10%), but set rather tightly at 0.999 when the jump contribution is high (80%). Note that a smaller jump contribution like 10% seems to be the main empirical finding in the literature (see Andersen et al., 2004; Huang and Tauchen, 2005, for example).

The proposed jump detection mechanism is implemented for the S&P 500 market index, the ten-year US treasury bond, and the dollar/yen exchange rate, to cover a representative set of asset classes. The jump intensity is estimated to be the smallest for the equity index (13%), but larger for the government bond (18%) and the exchange rate (20%), while the jump mean estimates are insignificantly different from zero. The jump volatility estimates are for the stock market (0.53%), the bond market (0.65%), and the currency market (0.39%). Rolling estimates reveal interesting jump dynamics. The jump probabilities are quite variable for equity index and treasury bond (from 5% to 25%), but relatively stable for dollar/yen currency (20%). Although the jump means are mostly statistically indistinguishable from zero for all assets considered here, there are obvious positive deviations from zero for the S&P 500 index in the late 1990s. Finally, the jump volatilities have not changed much for government bonds, except for a hike in 1994, and the exchange rate, but have increased significantly for the US equity market from 2000 to 2004.

It turns out that the capability of identifying realized jumps has important implications for estimating financial market risk adjustments. For the Moody's AAA and BAA credit spread monthly indices, we find that the rolling estimates of stock market jump volatility can predict the spread variation with  $R^2$ 's of 0.62–0.66, which are considerably higher than those obtained with the

standard interest rate factors, volatility factors including the option-implied volatility, and the systematic Fama-French factors. This result is important, since explaining high investment grade credit spreads has not been very successful and the empirical role of jumps in explaining these credit spreads has largely not been confirmed in the literature so far. Jump volatility remains statistically significant even when the lagged credit spread is controlled for. The market jump risk factor constructed from high frequency data seems to be able to capture the low frequency movements in credit spreads in terms of long-run trend and business cycles. Jump volatility also comoves countercyclically with the price-dividend ratio and corporate default rate, with correlations of 0.67 and 0.65, which has important asset pricing implications along the lines of Bansal and Yaron (2004).

The rest of the paper is organized as follows. The next section introduces the jump identification mechanism based on high frequency intra-day data, then Section 3 provides some Monte Carlo evidence on the small sample performance of such an estimation strategy. Section 4 illustrates the approach with four financial market assets, Section 5 discusses the implications for predicting credit risk spreads, and Section 6 concludes.

## 2. Identifying realized jumps

Jumps are important for asset pricing (Merton, 1976), yet the estimation of jump distribution is very difficult, especially when only low frequency daily data are employed (Bates, 2000; Andersen et al., 2002; Pan, 2002; Chernov et al., 2003; Eraker et al., 2003; Aït-Sahalia and Yacine, 2004). In recent years, Andersen et al. (1998), Andersen et al. (2001, in press), Barndorff-Nielsen and Shephard (2002a,b), and Meddahi (2002) have advocated the use of so-called realized variance measures by utilizing the information in the intra-day data for measuring and forecasting volatilities. More recent work on bipower variation measures developed in a series of papers by Barndorff-Nielsen and Shephard (2003, 2004, 2006) allows for the use of high frequency data to disentangle realized volatility into separate continuous and jump components (see Andersen et al., 2004; Huang and Tauchen, 2005, as well). In this paper, we rely on the presumption that jumps on financial markets are rare and large in order to extract the realized jumps and to explicitly estimate the jump intensity, mean, and volatility parameters. Empirical evidence presented by Lee and Mykland (forthcoming, Table V) is generally supportive of the notion of very rare jumps.

### 2.1. Filtering jumps from bipower variation

Let  $p_t = \log(P_t)$  denote the time  $t$  logarithmic price of the asset, and assume that it evolves in continuous time as a jump-diffusion process:

$$dp_t = \mu_t dt + \sigma_t dW_t + J_t dq_t \quad (1)$$

where  $\mu_t$  and  $\sigma_t$  are the instantaneous drift and diffusion functions that are completely general and may be stochastic (subject to the regularity conditions),  $W_t$  is the standard Brownian motion,  $dq_t$  is a Poisson jump process with intensity  $\lambda_j$ , and  $J_t$  refers to the corresponding (log) jump size distributed as  $\text{Normal}(\mu_j, \sigma_j)$ . Note that this approach can be extended to allow for time variation in jump rates  $\lambda_{j,t}$ , jump means  $\mu_{j,t}$ , and jump volatilities  $\sigma_{j,t}$ , which can be implemented empirically once the actual jumps are filtered out. Time is measured in daily units and the intra-daily returns are defined as follows:

$$r_{t,j} \equiv p_{t,j,\Delta} - p_{t,(j-1)\Delta} \quad (2)$$

where  $r_{t,j}$  refers to the  $j$ th within-day return on day  $t$ , and  $\Delta$  is the sampling frequency within each day.

Barndorff-Nielsen and Shephard (2004) propose two general measures for the quadratic variation process – realized variance and realized bipower variation – which converge uniformly (as  $\Delta \rightarrow 0$  or  $m = 1/\Delta \rightarrow \infty$ ) to different quantities of the underlying jump–diffusion process,

$$RV_t \equiv \sum_{j=1}^m r_{t,j}^2 \rightarrow \int_{t-1}^t \sigma_s^2 ds + \int_{t-1}^t J_s^2 dq_s \quad (3)$$

$$BV_t \equiv \frac{\pi}{2} \frac{m}{m-1} \sum_{j=2}^m |r_{t,j}| |r_{t,j-1}| \rightarrow \int_{t-1}^t \sigma_s^2 ds. \quad (4)$$

Therefore the difference between the realized variance and bipower variation is zero when there is no jump and strictly positive when there is a jump (asymptotically).

A variety of jump detection techniques are proposed and studied by Barndorff-Nielsen and Shephard (2004), Andersen et al. (2004), and Huang and Tauchen (2005). Here we adopted the ratio statistics favored by their findings,

$$RJ_t \equiv \frac{RV_t - BV_t}{RV_t} \quad (5)$$

which converges to a standard normal distribution with appropriate scaling

$$ZJ_t \equiv \frac{RJ_t}{\sqrt{\left[\left(\frac{\pi}{2}\right)^2 + \pi - 5\right] \frac{1}{m} \max\left(1, \frac{TP_t}{BV_t^2}\right)}} \xrightarrow{d} \mathcal{N}(0, 1) \quad (6)$$

where  $TP_t$  is the tripower quarticity robust to jumps, and as shown by Barndorff-Nielsen and Shephard (2004),

$$TP_t \equiv m \mu_{4/3}^{-3} \frac{m}{m-2} \sum_{j=3}^m |r_{t,j-2}|^{4/3} |r_{t,j-1}|^{4/3} |r_{t,j}|^{4/3} \rightarrow \int_{t-1}^t \sigma_s^4 ds \quad (7)$$

with  $\mu_k \equiv 2^{k/2} \Gamma((k+1)/2) / \Gamma(1/2)$  for  $k > 0$ . This test has excellent size and power properties and is quite accurate at detecting jumps as documented in Monte Carlo work (Huang and Tauchen, 2005).

On the basis of the economic intuition regarding the nature and source of jumps on financial market (Merton, 1976), we further assume that there is at most one jump per day and that the jump size dominates the return when a jump occurs. These assumptions allow us to filter out the daily realized jumps as

$$\hat{J}_t = \text{sign}(r_t) \times \sqrt{(RV_t - BV_t) \times I_{(ZJ_t \geq \Phi_\alpha^{-1})}} \quad (8)$$

where  $\Phi$  is the cumulative distribution function of a standard Normal,  $\alpha$  is the significance level of the z-test, and  $I_{(ZJ_t \geq \Phi_\alpha^{-1})}$  is the resulting indicator function on whether there is a jump during the day. Our approach of filtering out the realized jumps is a simple extension to the concept of a “significant jump” in Andersen et al. (2004), the signed square root of which is equivalent to our  $J_t$ .

Of course, the accuracy of Eq. (8) in filtering out realized jumps hinges critically on the assumption that there is only one jump per day. Although this is a reasonable description for the price jumps caused by macroeconomic new announcements, in general, it is at best a close approximation. If there are multiple jumps per day, especially those not driven by news announcement but driven by market sentiment, a more refined approach for sequentially detecting each jump within a day could be preferable (Andersen et al., 2006a). Another very promising approach is to use a trailing bipower estimate of volatility to scale high frequency returns and to detect jumps accordingly (Lee and Mykland, forthcoming). In on-going research (by students supervised by the first author),

the Lee and Mykland (forthcoming) approach appears to give quite sensible results when applied to high frequency individual stock returns; the findings generally support the notion of about one jump per day. However, the Lee and Mykland (forthcoming) approach requires an appropriate choice for the lag length, or lag window, used in computing the trailing bipower variation, and the optimal choice is still an open research question. Also, the approach has yet to be extensively investigated in Monte Carlo work like that carried out by Huang and Tauchen (2005) for the Barndorff-Nielsen and Shephard (2004, 2006) approach, so we defer use of this statistic to future work.

## 2.2. Estimating the jump distribution

Once the individual jump size is filtered out, we can further estimate the jump intensity, mean, and variance, by imposing a simple model of Poisson-mixing-Normal jump specification,

$$\hat{\lambda}_J = \frac{\text{Number of Realized Jump Days}}{\text{Number of Total Trading Days}}$$

$$\hat{\mu}_J = \text{Mean of Realized Jumps}$$

$$\hat{\sigma}_J = \text{Standard Deviation of Realized Jumps}$$

with appropriate formulas for the standard error estimates. Such an approach for estimating jumps is robust to the specifications of time-varying or even stochastic drift and diffusion functions, as long as the diffusion volatility noise is not too large (to be made more precise in the Monte Carlo study of the next section). It also allows us to specify more flexible dynamic structures of the underlying jump arrival rate and/or jump size distribution (see, for example, Andersen et al., 2006b). Realized jumps therefore can help us to avoid those estimation methods such as EMM or MCMC that rely heavily on numerical simulations.

## 3. Finite sample experiment

It is important to evaluate whether the proposed jump filtering and estimation procedure works well under the assumptions of large and rare jumps. In particular, we want to know whether the jump parameters can be accurately estimated and whether the correct inferences can be made, as both the sample size increases and the sampling interval decreases.

### 3.1. Experimental design

Here we adopt the following benchmark specification of a stochastic volatility jump–diffusion process:

$$dp_t = \mu_t dt + \sigma_t dW_{1t} + J_t dq_t \quad (9)$$

$$d\sigma_t^2 = \beta(\theta - \sigma_t^2)dt + \gamma \sqrt{\sigma_t^2} dW_{2t} \quad (10)$$

with log price drift  $\mu_t = 0$ ; volatility mean reversion  $\beta = 0.10$  and volatility-of-volatility  $\gamma = 0.05$ ; jump parameters  $\lambda_J = 0.05$ ,  $\mu_J = 0.20$ ,  $\sigma_J = 1.40$ ; and leverage coefficient  $\rho \equiv \text{corr}(dW_{1t}, dW_{2t}) = -0.50$ . The volatility long-run mean parameter  $\theta$  is chosen for two scenarios to cover a possible range of financial asset classes. Scenario (a) has  $\theta = 0.9$  such that the discontinuous part contribution to the total variance is 10%. Such a scenario applies more likely to the US equity market, major currencies, and blue chip stocks. In fact, 10% is about the average empirical finding in Andersen et al. (2004) and Huang and Tauchen (2005). Scenario (b) with  $\theta = 0.025$  and 80% jump contribution to variance resembles the illiquid and infrequently traded assets, like corporate bonds, small stocks, and emerging market equities or currencies. The choice of jump parameters also reflects the empirical findings in the literature that (1) jumps are rare, (2)

**Table 1**

Monte Carlo experiment with scenario (a). This table reports the Monte Carlo evidence for estimating the jump rate, mean, and volatility parameters. Scenario (a) has the jump contribution to the total variance as 10%. The results are organized across two sample sizes (1000 days versus 4000 days), two sampling frequencies (5 min versus 1 min), and two jump test significance levels (0.99 versus 0.999).

	Mean bias		Medium bias		RMSE	
	$T = 1000$	$T = 4000$	$T = 1000$	$T = 4000$	$T = 1000$	$T = 4000$
Benchmark maximum likelihood estimation						
$\lambda_j = 0.05$	0.0006	0.0004	0.0005	0.0004	0.0068	0.0035
$\mu_j = 0.2$	−0.0020	0.0017	−0.0060	0.0046	0.1989	0.1033
$\sigma_j = 1.2$	−0.0094	−0.0005	−0.0042	−0.0030	0.1443	0.0690
Sampling frequency $\Delta = 5$ min, level of significance $\alpha = 0.99$						
$\lambda_j = 0.05$	−0.0065	−0.0061	−0.0070	−0.0060	0.0092	0.0068
$\mu_j = 0.2$	−0.0131	−0.0079	−0.0147	−0.0082	0.2152	0.1116
$\sigma_j = 1.2$	−0.0116	−0.0006	−0.0126	0.0023	0.1443	0.0706
Sampling frequency $\Delta = 1$ min, level of significance $\alpha = 0.99$						
$\lambda_j = 0.05$	−0.0006	−0.0001	0.0000	0.0000	0.0067	0.0035
$\mu_j = 0.2$	−0.0239	−0.0194	−0.0233	−0.0238	0.1965	0.1039
$\sigma_j = 1.2$	−0.0464	−0.0363	−0.0550	−0.0348	0.1504	0.0766
Sampling frequency $\Delta = 5$ min, level of significance $\alpha = 0.999$						
$\lambda_j = 0.05$	−0.0204	−0.0199	−0.0210	−0.0200	0.0211	0.0201
$\mu_j = 0.2$	0.0719	0.0791	0.0699	0.0778	0.3199	0.1772
$\sigma_j = 1.2$	0.2415	0.2492	0.2475	0.2514	0.2959	0.2616
Sampling frequency $\Delta = 1$ min, level of significance $\alpha = 0.999$						
$\lambda_j = 0.05$	−0.0116	−0.0111	−0.0120	−0.0113	0.0131	0.0115
$\mu_j = 0.2$	0.0267	0.0315	0.0289	0.0311	0.2529	0.1330
$\sigma_j = 1.2$	0.1236	0.1323	0.1261	0.1331	0.1976	0.1510

jumps are large in terms of standard deviation, and (3) the jump mean is hard to distinguish from zero.

The Monte Carlo experiment is designed as follows. Each day one simulates the jump–diffusion process, using one second as a tick size totaling six and a half trading hours, imitating the US equity market in recent years. The diffusion process with stochastic volatility is simulated by the Euler scheme, the jump timing is simulated from an Exponential distribution, and the jump size is simulated from a Normal distribution. Then the realized jumps are combined with the realized diffusion, and sampled by an econometrician at both one-minute and five-minute intervals, illustrating the infill asymptotics. To contrast the long-span asymptotics of sample sizes, we use both  $T = 1000$  days and  $T = 4000$  days. Further, the choice of significance level in the jump detection test is also compared between  $\alpha = 0.99$  and  $\alpha = 0.999$ . The appropriate choice of the pre-test level seems to be relevant for achieving consistent parameter estimates, given varying degree of jump contribution to the total variance. In addition, the simulation provides us with the exact jump timing (Exponential) and jump size (Normal); therefore a maximum likelihood estimator (MLE) can be used as a benchmark for judging the relative efficiency of the jump filtering approach examined in this paper.

### 3.2. Parameter estimation

The finite sample results on various jump parameter estimates are presented in Tables 1 and 2. The first column of each table gives the true parameter values, and the first row gives the mean bias, median bias, and root mean squared error (RMSE) of the maximum likelihood estimator (MLE). Note that the MLE results do not vary across the two scenarios (since only the diffusion variance level is altered), nor across the pre-test  $\alpha = 0.99$  and  $\alpha = 0.999$  levels (since no pre-estimation filtering is involved), nor across the five-minute and one-minute sampling intervals (since jumps are observed exactly in simulations). For MLE, the estimation biases at both 1000 and 4000 days are negligible for all three parameters, relative to their true values. In terms of the estimation efficiency, both jump rate  $\lambda_j$  and jump volatility  $\sigma_j$  can be very accurately estimated with RMSE's much smaller than the parameter values.

However, for the jump mean parameter  $\mu_j$ , the estimate is not accurate at 1000 days (RMSE about the size of the parameter value), but can be accurate at 4000 days (RMSE about half the size of the parameter value). In addition, all the RMSE's decrease almost exactly at the rate of  $\sqrt{4}$ , as predicted by the asymptotic theory.

For the jump filtering mechanism based on the bipower variation measure (Tables 1, 2), the parameter estimation efficiency approaches that of MLE very differently, depending upon whether the jump contribution to the total variance is small or large. In Scenario (a) where the jump contribution to the total variance is as small as 10%, the RMSE's of parameter estimates are all closer to those of MLE and the convergence rates are closer to  $\sqrt{4}$ , as the sample size increases from  $T = 1000$  to  $T = 4000$ , when we set the pre-test level  $\alpha = 0.99$  but not  $\alpha = 0.999$ . In other words, when the jump contribution is relatively small as is typical in observed data, the asymptotic filtering scheme seems to work better when the pre-test level is less stringent. In contrast, for Scenario (b) where the jump contribution to the total variance is as large as 80%, the scheme seems to work much better when we set  $\alpha = 0.999$  rather than  $\alpha = 0.99$ , where the RMSE's can almost match those of MLE. These findings are intuitive in the following sense. It is clearly more difficult to detect jumps when they are relatively small; therefore loosening the jump detection standard can reveal more jumps that otherwise would have been missed (minimizing the type-I error). On the other hand, when jumps are large they are easier to detect, so we want a more stringent jump filtering standard, such that false revelation of jumps can be avoided as much as possible (minimizing the type-II error). In short, the jump filtering approach based on the bipower variation measure can bring us efficient parameter estimates relative to MLE, provided that we appropriately choose the significance level  $\alpha$  according to the relative contributions of jumps to the total variance.

### 3.3. Statistical inference

In addition to the parameter estimation efficiency, we also need to know whether the asymptotic standard error estimated in finite samples can provide a reliable statistical inference about the true parameter value. To set the right benchmark, Fig. 1 plots



**Table 2**  
Monte Carlo experiment with scenario (b). This table reports the Monte Carlo evidence for estimating the jump rate, mean, and volatility parameters. Scenario (b) has the jump contribution to the total variance as 80%. The results are organized across two sample sizes (1000 days versus 4000 days), two sampling frequencies (5 min versus 1 min), and two jump test significance levels (0.99 versus 0.999).

	Mean bias		Medium bias		RMSE	
	$T = 1000$	$T = 4000$	$T = 1000$	$T = 4000$	$T = 1000$	$T = 4000$
Benchmark maximum likelihood estimation						
$\lambda_j = 0.05$	0.0006	0.0004	0.0005	0.0004	0.0068	0.0035
$\mu_j = 0.2$	−0.0020	0.0017	−0.0060	0.0046	0.1989	0.1033
$\sigma_j = 1.2$	−0.0094	−0.0005	−0.0042	−0.0030	0.1443	0.0690
Sampling frequency $\Delta = 5$ min, level of significance $\alpha = 0.99$						
$\lambda_j = 0.05$	0.0090	0.0094	0.0090	0.0095	0.0116	0.0101
$\mu_j = 0.2$	−0.0374	−0.0337	−0.0391	−0.0313	0.1690	0.0920
$\sigma_j = 1.2$	−0.1388	−0.1272	−0.1332	−0.1284	0.1926	0.1436
Sampling frequency $\Delta = 1$ min, level of significance $\alpha = 0.99$						
$\lambda_j = 0.05$	0.0081	0.0087	0.0080	0.0085	0.0107	0.0094
$\mu_j = 0.2$	−0.0336	−0.0304	−0.0321	−0.0290	0.1719	0.0918
$\sigma_j = 1.2$	−0.1214	−0.1109	−0.1228	−0.1113	0.1842	0.1291
Sampling frequency $\Delta = 5$ min, level of significance $\alpha = 0.999$						
$\lambda_j = 0.05$	−0.0033	−0.0026	−0.0040	−0.0025	0.0073	0.0042
$\mu_j = 0.2$	0.0059	0.0083	−0.0029	0.0084	0.2099	0.1075
$\sigma_j = 1.2$	0.0136	0.0214	0.0188	0.0206	0.1475	0.0734
Sampling frequency $\Delta = 1$ min, level of significance $\alpha = 0.999$						
$\lambda_j = 0.05$	−0.0020	−0.0015	−0.0020	−0.0015	0.0067	0.0036
$\mu_j = 0.2$	0.0013	0.0053	−0.0007	0.0089	0.2038	0.1053
$\sigma_j = 1.2$	0.0042	0.0149	0.0037	0.0148	0.1457	0.0718

the finite sample rejection rates from the Monte Carlo replications against the asymptotic test size. The rejection rate is based on the chi-square (1) test statistics of each parameter. The deviation between the dashed line (Monte Carlo finite sample result) and dotted diagonal line (asymptotic result), indicates how big the size distortion is. It is clear from Fig. 1 that the MLE asymptotic variance estimated in a finite sample behaves extremely well, so there is effectively no size distortion at all.

The Wald test statistics based on a bipower variation approach are reported in Figs. 2 and 3. In general, the  $t$ -test for the jump mean  $\mu_j$  is well behaved, while the result for jump rate  $\lambda_j$  and jump volatility  $\sigma_j$  varies with the setting. In Scenario (a) where jumps contribute 10% to the total variance, the chi-square statistics under the choice of  $\alpha = 0.999$  have a much higher over-rejection bias compared to the choice of  $\alpha = 0.99$ . In Scenario (b) with the relative jump contribution being 80%, there is almost no over-rejection bias at the  $\alpha = 0.999$  level, while the chi-square test does not converge at all for  $\alpha = 0.99$ . In short, if jumps are small then a less stringent jump detection test generates more reliable inferences about the true parameters, while if jumps are large then a more stringent test generates more reliable inferences.

### 3.4. Further discussion on econometric issues

There appears to be no conclusive agreement about the optimal significance level  $\alpha$  of the jump detection  $z$ -test in various empirical settings (Barndorff-Nielsen and Shephard, 2004; Andersen et al., 2004; Huang and Tauchen, 2005). However, our finite sample evidence presented below suggests that when the relative jump contribution to the total variance is small (10%) a more generous test level ( $\alpha = 0.99$ ) performs better, while for a large jump contribution (80%) a more stringent test level ( $\alpha = 0.999$ ) is preferred. This relationship may be formalized as determined by a “noise-to-signal” in identifying jumps, with the presence of diffusion as a measurement error. More precisely, the ratio of unconditional expectations of  $\int_{t-1}^t J_s^2 dq_s$  over  $RV_t$ ,

$$\frac{E\left(\int_{t-1}^t J_s^2 dq_s\right)}{E\left(\int_{t-1}^t J_s^2 dq_s\right) + E\left(\int_{t-1}^t \sigma_s^2 ds\right)} \equiv \frac{\text{Signal}}{\text{Signal} + \text{Noise}}, \quad (11)$$

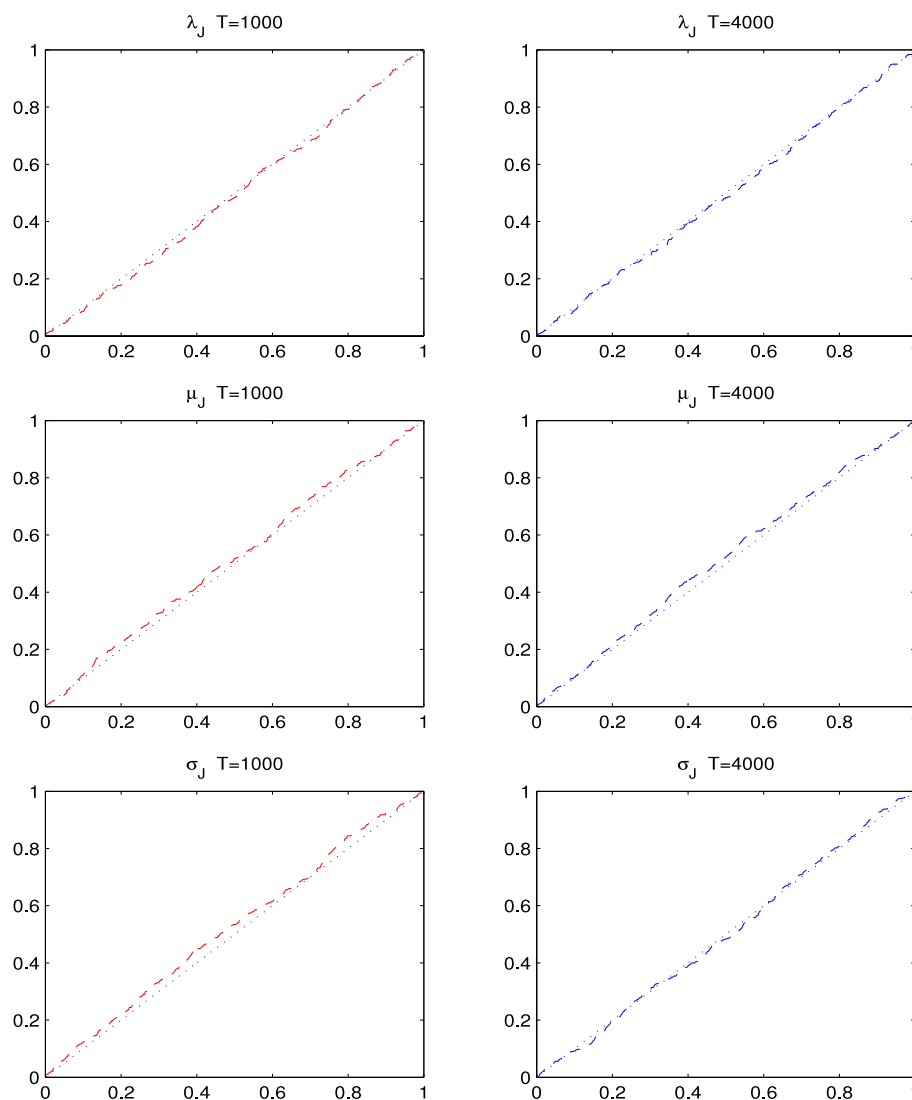
appears to indicate the optimal choice of the test level  $\alpha$ . The choice of  $\alpha$  is similar to the case of setting up a threshold for daily data in terms of a number of standard deviations as suggested in Clewlow and Strickland (2000).

The assumption of large jumps is rather innocuous, as long as jumps remain discretely distinct from the continuous diffusion, and therefore a higher sampling frequency can eventually capture the jumps. The resulting estimates of jump parameters become more noisy but have no asymptotic bias as both sample size increases and sampling interval decreases. However, more frequent jumps might distort the finite sample properties and cause bias. Because several jumps may be misclassified as one jump in a particular day, the jump filtering and estimation strategy will underestimate the jump rate  $\lambda_j$ , overestimate (or underestimate) the jump mean  $\mu_j$  if its true value is positive (or negative), and overestimate the jump variance  $\sigma_j$ . These are asymptotic biases that persist even as sample size increases but are mitigated as sampling interval decreases.

The ability to detect and filter out jumps has an important implication for estimating jump–diffusion processes. The existing method, like the GMM approach of Bollerslev and Zhou (2002), estimate the volatility and jump parameters jointly with the conditional moments of the total return and total realized variance. The limitation of the estimation approach based on “latent” jumps is that the jump distribution assumption has to be very simple such that the joint moments are in closed form. In contrast, here the jumps are “observable” and can be filtered out first before the estimation of volatility parameters, as implemented in Bollerslev et al. (2006). Therefore the jump distribution assumptions can be more general and the volatility parameter estimates are more robust.

### 4. Application to financial markets

We apply the jump detecting and filtering scheme to three financial markets: stocks, bonds, and foreign exchange. The intra-day high frequency data for the S&P 500 index (1986–2005) are obtained from the Institute of Financial Markets, the ten-year US treasury bond (1991–2005) from the Federal Reserve Board, and the dollar/yen exchange rate (1997–2004) from Olsen & Asso-



**Fig. 1.** Wald test for realized jumps with a maximum likelihood estimator. The estimates are based on simulated jump timing and jump sizes. The dotted line is the reference Uniform distribution; the dashed line is for 500 Monte Carlo replications and for a sample size of 1000 days or 4000 days.

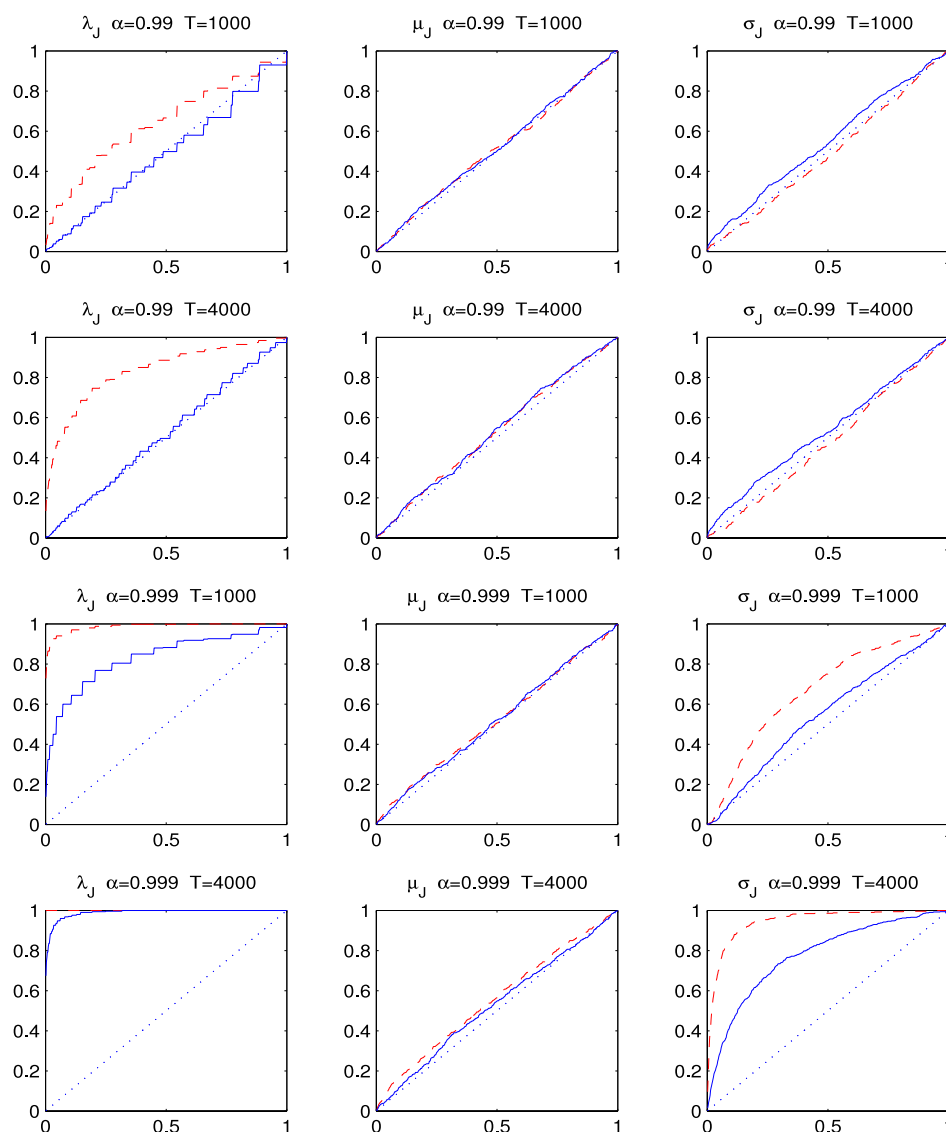
ciates. These choices are meant to give a representative view of the available major asset classes. All the data are transformed to five-minute log returns, which are generally known to be quite robust to market microstructure noise. We eliminate days with less than 60 trades or quotes. We also drop the after-hour tradings and exclude the overnight returns due to the liquidity concern, except for the dollar/yen exchange rate, which opens for 24 h.

Summary statistics for daily percentage returns and realized volatility (square root of realized variance) are reported in Table 3. The sample means suggest annualized returns of 8.4% for the S&P 500, 4.1% for the *t*-bond, and 2.4% for yen currency (six trading days per week). The average realized volatilities are for the stock market index 0.73%, the *t*-bond 0.56%, and the exchange rate 0.62%. The return skewness is negative for the S&P 500 index and government bond, while it is positive for exchange rate. The kurtosis statistics suggests that all three returns deviate from the Normal distribution, as is expected. The returns are approximately serially uncorrelated, while the volatility series exhibit pronounced serial dependencies. In fact, the first ten autocorrelations reported in the bottom part of the table are all highly significant with the gradual, but very slow, decay suggestive of long-memory type features. This is also evident from the time series plots of realized volatility series given in the top panels of Figs. 4–6.

#### 4.1. Unconditional jump parameter estimates

As shown in the top panel of Table 4, the jump contribution to the total variance is about 5.35% for S&P 500, 19.11% for the *t*-bond, and 6.47% for the exchange rate. These numbers are very close to the findings in Andersen et al. (2004) and Huang and Tauchen (2005), and quite similar to those for Scenario (a) of our Monte Carlo section. We expect the jump filtering and estimation method based on the bipower variation approach to work reasonably well. The realized jumps filtered by our method are plotted in the second panels of Figs. 4–6. Jumps in the S&P 500 index clearly have jump sizes between  $-2\%$  and  $+2\%$ . The treasury bond has less frequent jumps with a range  $-2\%$  and  $+4\%$ . The yen currency has more frequent jumps with jump size between  $-1.2\%$  and  $+1.6\%$ .

The bottom panel of Table 4 reports the parametric distribution estimates based on the filtered realized jumps. Except for the S&P 500 index ( $\mu_J = 0.06$  with s.e. = 0.02), all the jump mean estimates are statistically indistinguishable from zero. The jump intensity estimates are highly significant and vary across assets, with the S&P 500 index being the lowest (0.13 with s.e. 0.01), the treasury bond moderate (0.18 with s.e. 0.02), and the exchange rate the highest (0.20 with s.e. 0.01). The standard deviations of jumps are estimated the most accurately and very close to each other (0.53



**Fig. 2.** Asymptotic Wald test for scenario (a). The estimates are based on filtered jumps with the bipower variation approach. The relative contribution of the diffusion and jump to the variance is 90% versus 10%. The dotted line is the reference Uniform distribution, the dashed line is for sampling interval  $\Delta = 5$  min, and the solid line is for sampling interval  $\Delta = 1$  min.

with s.e. 0.01 for the stock index, 0.65 with s.e. 0.02 for bond, and 0.39 with s.e. 0.01 for currency).

These results differ from the usual jump estimation results in empirical finance that use latent variable simulation-based methods on daily data. Our findings regarding the jump frequency and jump size can be reconciled with the notion that significant jumps on financial markets are related to market responses to fundamental economic news (Andersen et al., 2003a, 2005).

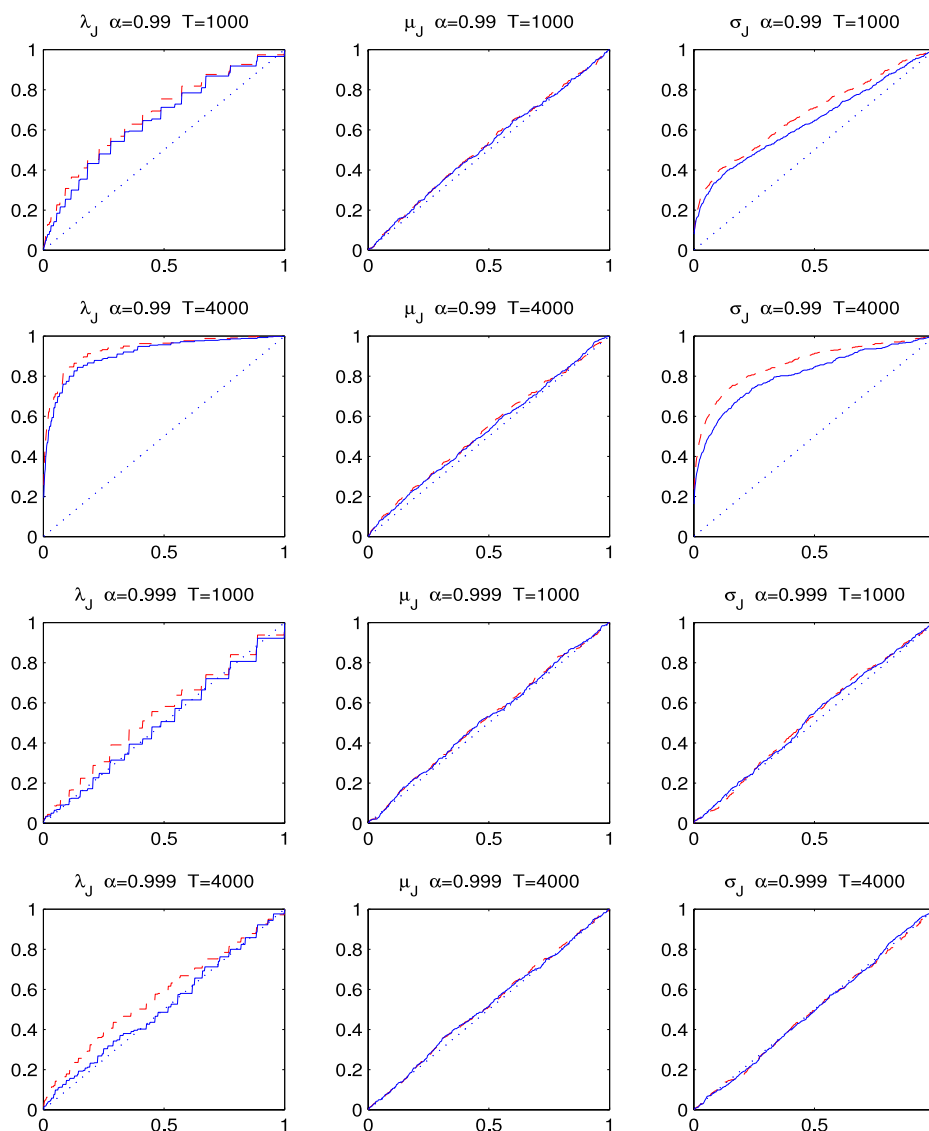
#### 4.2. Time-varying jump distribution estimates

Another interesting feature that can be seen from the second panels of Figs. 4–6 is that the clustering and amplitude of jumps change over time, which leads to the usual conjecture of time-varying jump rate and jump size distribution. To get an initial handle on such a possibility, we perform a two-year rolling estimation of the jump parameters  $\lambda_{j,t}$ ,  $\mu_{j,t}$ , and  $\sigma_{j,t}$ , with corresponding 95% standard error bands.

As seen from Fig. 4, the jump intensity of the S&P 500 index was fairly high during the early 1990s (above 20%), then dropped considerably during the late 1990s (around 5%), and has started to

rise again since 2002. The jump size mean is usually close to zero, except for the during late 1990s, where positive jump means are statistically significant and coincide with the stock market run-up. Jump volatility had been largely stable from the late 1980s to the late 1990s, around 40%, but has been elevated since 1999 and peaked around 2002 at a high of 100%. As Fig. 5 shows, the jump intensity of the bond market was high in the early 1990s and around 2001–2002, then kept falling until 2004 to around 10%, while the jump mean is mostly zero and the jump volatility is little changed around its unconditional level of 60% (except for the early 1990s when it is around 100%). For the dollar/yen exchange rate in Fig. 6, the jump intensity is mostly stable around 20%, the jump mean is statistically indistinguishable from zero, and the jump volatility is somewhat elevated during 1991–1992 and 1998–2000.

Time-varying jump intensity and jump volatility are very important risk factors in asset pricing, but until recently most of the evidence has been drawn from the option-implied or latent jump specifications (see, for example, Duffie et al., 2000; Eraker et al., 2003, among others). A recent paper by Andersen et al. (2006b) used the realized jump timing to examine the temporal dependency in jump durations.



**Fig. 3.** Asymptotic Wald test for scenario (b). The estimates are based on filtered jumps with the bipower variation approach. The relative contribution of the diffusion and jump to the variance is 20% versus 80%. The dotted line is the reference Uniform distribution, the dashed line is for sampling interval  $\Delta = 5$  min, and the solid line is for sampling interval  $\Delta = 1$  min.

## 5. Jump risks and credit spreads

Direct identification of realized jumps and the characterizations of time-varying jump distributions make it straightforward to study the relationship between jumps and risk adjustments. The reason is that jump parameters are generally very hard to pin down even with both underlying and derivative asset prices, due to the fact that jumps are latent in daily return data and are rare events in financial markets. Inaccurate estimates of the underlying jump dynamics make the jump risk premia even harder to quantify. However, as seen below, a reliable estimate of stock market jump volatility based on identified realized jumps can have a superior predicting power for the bond market risk premia.

### 5.1. Predicting corporate bond spread indices

Here we examine the monthly forecasting powers for Moody's AAA and BAA bond spreads, using the estimated S&P 500 jump volatility from the identified realized jumps, which is illustrated in Section 4. A longstanding puzzle has been how to explain the credit spreads of high investment grade bonds, since those firms

entertain very little default risk historically, yet their credit spreads are sizable and positive (Huang and Huang, 2003). Although jump risk has been proposed as a possible source of such a credit premium puzzle (Delianedis and Geske, 2001; Zhou, 2001; Huang and Huang, 2003), the empirical validation in the literature has met with mixed and unsatisfactory results (Collin-Dufresne et al., 2001, 2003; Cremers et al., 2004, in press).

Here we use an alternative jump risk measure, based on identified realized jumps as opposed to latent or implied jumps, to provide some contrasting positive evidence in explaining high investment grade credit spread indices. For comparison purposes, we also include standard predictors like the short rate and term spread in Longstaff and Schwartz (1995), long-run historical volatility (Campbell and Taksler, 2003) and short-run realized volatility (Zhang et al., 2006), and option-implied volatility (Cremers et al., 2004; Cao et al., 2006), with a control for market return, book-to-market, and size risk factors (Fama and French, 1993). Typically rising short rate coincides with economic expansion and lowers the credit spreads, while steep term spread indicates high inflation risk and increases credit spreads. Various volatility variables proxy for risks of different horizons and usually push up the credit spreads.



**Table 3**

Summary statistics for daily returns and realized variances. This table reports summary statistics of moments, percentiles, and autocorrelations for daily returns and volatilities aggregated from high frequency intra-day five-minute data. The three assets are the S&P 500 index (1986–2005), the ten-year US treasury bond (1991–2005), and the dollar/yen exchange rate (1997–2004).

Asset type statistics	S&P 500 index (%)		T-bond (%)		Dollar/yen (%)	
	Return <sub>t</sub>	$\sqrt{RV_t}$	Return <sub>t</sub>	$\sqrt{RV_t}$	Return <sub>t</sub>	$\sqrt{RV_t}$
Mean	0.0336	0.7259	0.0164	0.5598	0.0076	0.6227
Std. dev.	1.0694	0.4086	0.5995	0.2894	0.6153	0.2882
Skewness	-2.1019	2.3130	-0.3418	3.3803	0.6392	2.3871
Kurtosis	48.8873	13.7525	4.3559	24.9531	10.7761	24.9966
Minimum	-22.8867	0.1309	-3.3200	0.1327	-4.7029	0.0027
5% qntl.	-1.6161	0.2982	-0.9900	0.2755	-0.9336	0.2407
25% qntl.	-0.4507	0.4488	-0.3300	0.3861	-0.3214	0.4481
50% qntl.	0.0524	0.6226	0.0380	0.4932	-0.0083	0.5844
75% qntl.	0.5607	0.8941	0.3900	0.6550	0.3186	0.7444
95% qntl.	1.5850	1.5030	0.9488	1.0361	0.9962	1.1309
Maximum	8.3795	5.4363	2.2200	4.0919	7.1117	5.6396
$\rho_1$	0.0126	0.7536	0.0348	0.2415	0.0329	0.5474
$\rho_2$	-0.0468	0.7020	-0.0138	0.2011	0.0562	0.3740
$\rho_3$	-0.0085	0.6676	-0.0446	0.1568	-0.0261	0.3255
$\rho_4$	-0.0211	0.6475	-0.0421	0.1651	-0.0259	0.3075
$\rho_5$	-0.0176	0.6397	0.0002	0.1959	-0.0226	0.3716
$\rho_6$	-0.0056	0.6175	-0.0079	0.1576	0.0335	0.5173
$\rho_7$	-0.0436	0.6072	0.0213	0.1260	-0.0222	0.3370
$\rho_8$	0.0130	0.6051	0.0013	0.1767	0.0246	0.2380
$\rho_9$	0.0291	0.5928	0.0020	0.1502	-0.0013	0.2243
$\rho_{10}$	0.0222	0.5832	0.0189	0.1453	-0.0057	0.2155

Table 5 presents the univariate forecasting regressions for Moody's AAA and BAA bond spread monthly indices. The OLS coefficients show remarkable similarity between the two rating grades. To be more precise, an increase of one per cent in the short rate lowers the credit spread by 20–21 basis points; positive term spread shock increases the default premium by 21–26 basis points. Usually rising short rate coincides with economic expansion, while rising term spread reflects higher inflation expectation. The short rate predicts 59% and 47% of spread variation, while the term spread predicts only 18% and 20%. Short-run volatility (one month) has  $R^2$  around 26%–28% with marginal impact around 9–11 basis points, while long-run volatility (two years) has a slightly higher  $R^2$ , about 27%–29%, and a similar impact coefficient of 10 to 11 basis points. It is worth pointing out that option-implied volatility (VIX index) has a lower predicting power (14%–20%) and a smaller marginal effect (3–4 basis points); however, the magnitude of slope coefficient is comparable to the result on individual companies given by Cao et al. (2006) (5–6 basis points). In comparison, the S&P 500 jump volatility not only has a larger impact on credit spreads – an increase of one percentage point raises spreads by about 175–216 basis points, but also has the highest forecasting power – with  $R^2$ s of 62% for the AAA bond spread and 66% for the BAA bond spread. Higher equity (jump) volatility often implied higher asset volatility; therefore, the firm value is more likely to hit below the default boundary, which requires a higher credit spread to compensate for the default risk.

Given the common finding that typical default risk factors can only account for a very small fraction of the corporate bond spreads, recent effort has been directed more to the role of systematic risk premia in the economy (see Elton et al., 2001; Huang and Huang, 2003; Chen et al., 2005, e.g.). However, those business cycle effects usually explain only the spread variations of low investment grade or speculative grade credit spreads, but have very little or no explanatory power for the high investment grade credit spreads. As Table 6 shows, the systematic risk factors – market return, SMB, and HML Fama-French variables – have about one per cent or less predictive ability for the high investment grade credit spread at the monthly frequency. The fact that these bonds have little default risk yet command a sizable risk premium constitutes a major challenge in the credit risk pricing literature. In comparison, the jump volatility risk measure stands out as the

**Table 4**

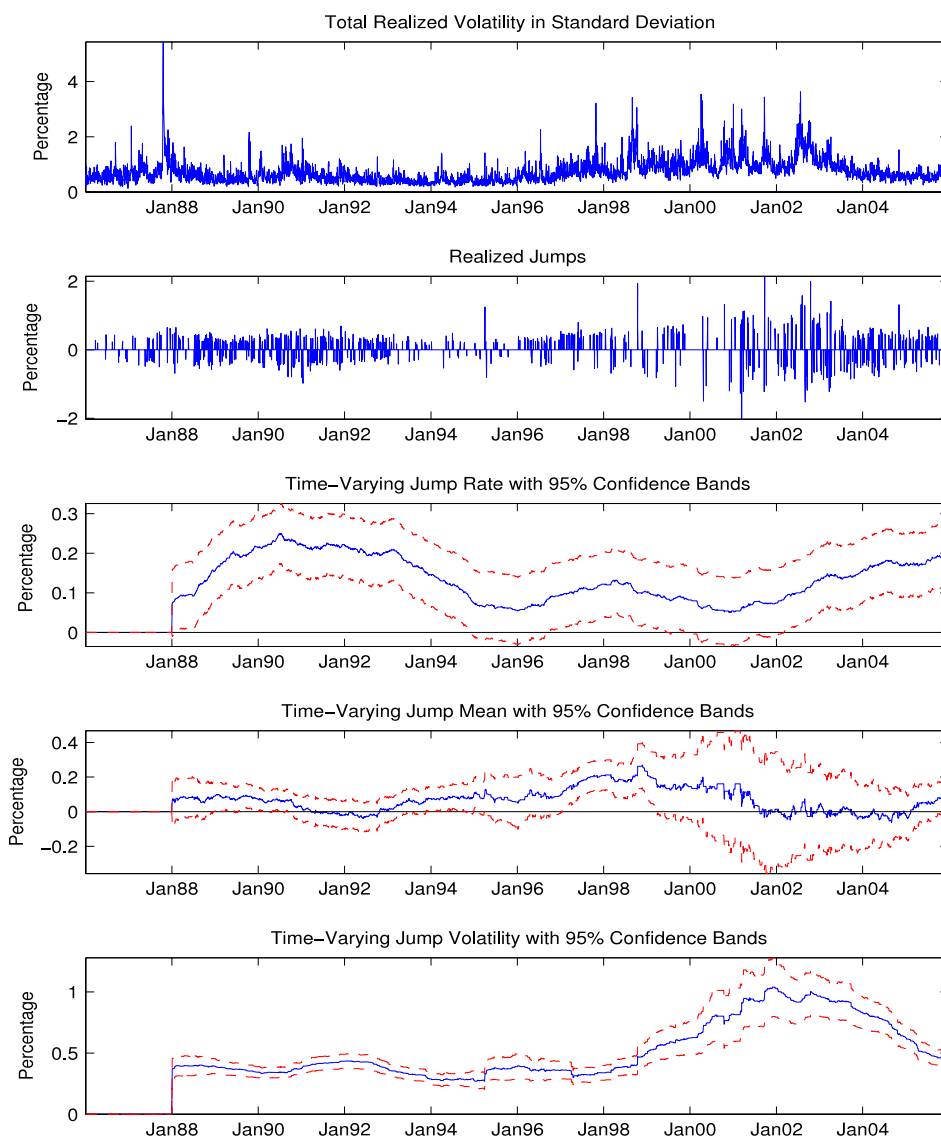
Jump parameter estimation for three assets. The top panel summarizes the average realized volatility, average jump contribution to the total variance, average jump contribution to the standard deviation, and total trading days – five days per week for the S&P 500 index and the ten-year treasury bond, and six days per week for the dollar/yen exchange rate. The bottom panel gives the parameter estimates of the jump intensity, jump mean, and jump standard deviation, based on the realized jump identification procedure discussed in Section 2.

Statistics	S&P 500	T-bond	Dollar/yen
Mean $\sqrt{RV_t}$	0.7259	0.5598	0.6227
Sum $(J_t^2)/\text{sum}(RV_t)$	0.0535	0.1911	0.0647
Sum $(\sqrt{J_t^2})/\text{sum}(\sqrt{RV_t})$	0.0837	0.1551	0.1083
Total trading days	4999	3376	5345
Parameter	S&P 500	T-bond	Dollar/yen
$\lambda_j$	0.1330	0.1795	0.1989
(s.e.)	(0.0132)	(0.0156)	(0.0122)
$\mu_j$	0.0605	-0.0002	0.0024
(s.e.)	(0.0204)	(0.0264)	(0.0120)
$\sigma_j$	0.5251	0.6498	0.3916
(s.e.)	(0.0144)	(0.0187)	(0.0085)

most powerful instrument in forecasting the credit spread indices, suggesting that a systematic jump risk factor may be important in pricing the top quality corporate credit.

Table 7 presents multiple regressions in forecasting the bond spreads. It seems that two interest rate factors are redundant, and the signs are now negative. Intuitively, when the economy is in expansion, the short rate and term spread tend to be rising, and the credit default condition is also improving. Note that on combining short- and long-run volatilities, there is marginal improvement in predictability but the long-run volatility is somewhat crowded out. The more interesting combination is the option-implied and jump volatilities, where the former is completely driven out by the latter. However, the best multivariate combination seems to be the one with two interest rate factors and two volatility factors (implied and jump), where the  $R^2$  is around 80%.

It is well known that the change of credit spread is much harder to predict (Collin-Dufresne et al., 2001). However, credit spreads are constrained to be stationary under both structural and reduced form models. Therefore the pricing relationship connects the spread levels with underlying risk factors. Nevertheless, we control for the lagged spread level in Table 8. The results suggest that the



**Fig. 4.** S&P 500 realized variance and jump dynamics. The realized variance is from intra-day five-minute returns, the realized jumps are filtered by the bipower variation method, and the jump parameters are estimated with a two-year rolling sample.

overwhelming explaining power is attributed to the lag spread, and the interest rate factors become statistically insignificant. Also notice that both short-run and long-run volatilities have impacts not significantly different from zero. Even the option-implied volatility loses its explaining power for the high investment grade (AAA rating), but only remains significant for the low investment grade (BAA rating). In contrast, it is the jump volatility variable that retains its statistical significance for both bond spread indices, with the powerful lag controls.

In short, contrary to the negative finding in the empirical literature about the jump impact for credit spread, our measure of market realized jump volatility has strong predictability for high investment grade credit spreads. The forecasting power is higher than the interest rate factors, the short-run and long-run volatility factors, and even the option-implied volatility factor, with controlling for the systematic risks and lagged spreads.

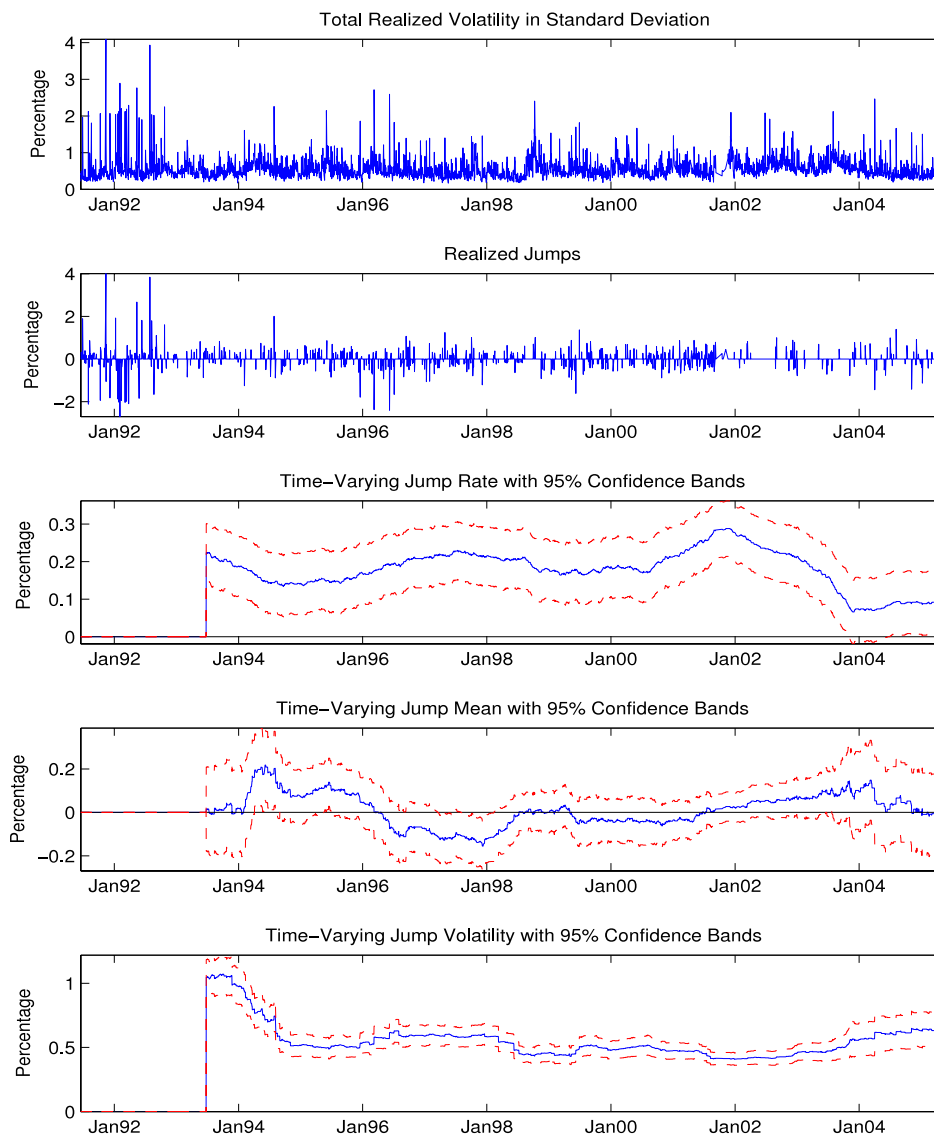
## 5.2. Further economic interpretation

The close association between the credit risk premium and market jump volatility can be more clearly seen in Fig. 7. Although the monthly credit spread is very noisy, there clearly exist certain

long-term trends and short-term cycles from 1988 to 2005. It is obvious that the time-varying jump volatility traces these trends and cycles closely, while discarding the month-to-month fluctuations in credit spread indices. This suggests that jump volatility may help to capture some long-run macroeconomic and financial risk similarly examined by Bansal and Yaron (2004).

To further appreciate the underlying economic reasons for the strong forecasting ability of market jump volatility risk, Fig. 8, upper panel, plots the S&P 500 price–dividend ratio, which clearly has a slight run-up in the late 1990s and has been falling since 2000. The realized jump volatility constructed here clearly mimics this long-run trend, and indeed the correlation between the log P/D ratio and jump volatility is as high as 67%. On the other hand, the recent popular market liquidity index (Pástor and Stambaugh, 2003), plotted in the lower panel of Fig. 8, seems to capture the short-run fluctuations in the equity market, and its correlation with the jump volatility is –26%. In other words, the jump volatility variable seems to proxy more for the long-run macroeconomic risk rather than the short-run financial market risk.

It is also well known that credit spreads are fundamentally driven by the underlying changes in expected future default probability and loss-given-default (recovery rate), although their



**Fig. 5.** Treasury bond realized variance and jump dynamics. The realized variance is from intra-day five-minute returns, the realized jumps are filtered by the bipower variation method, and the jump parameters are estimated with a two-year rolling sample.

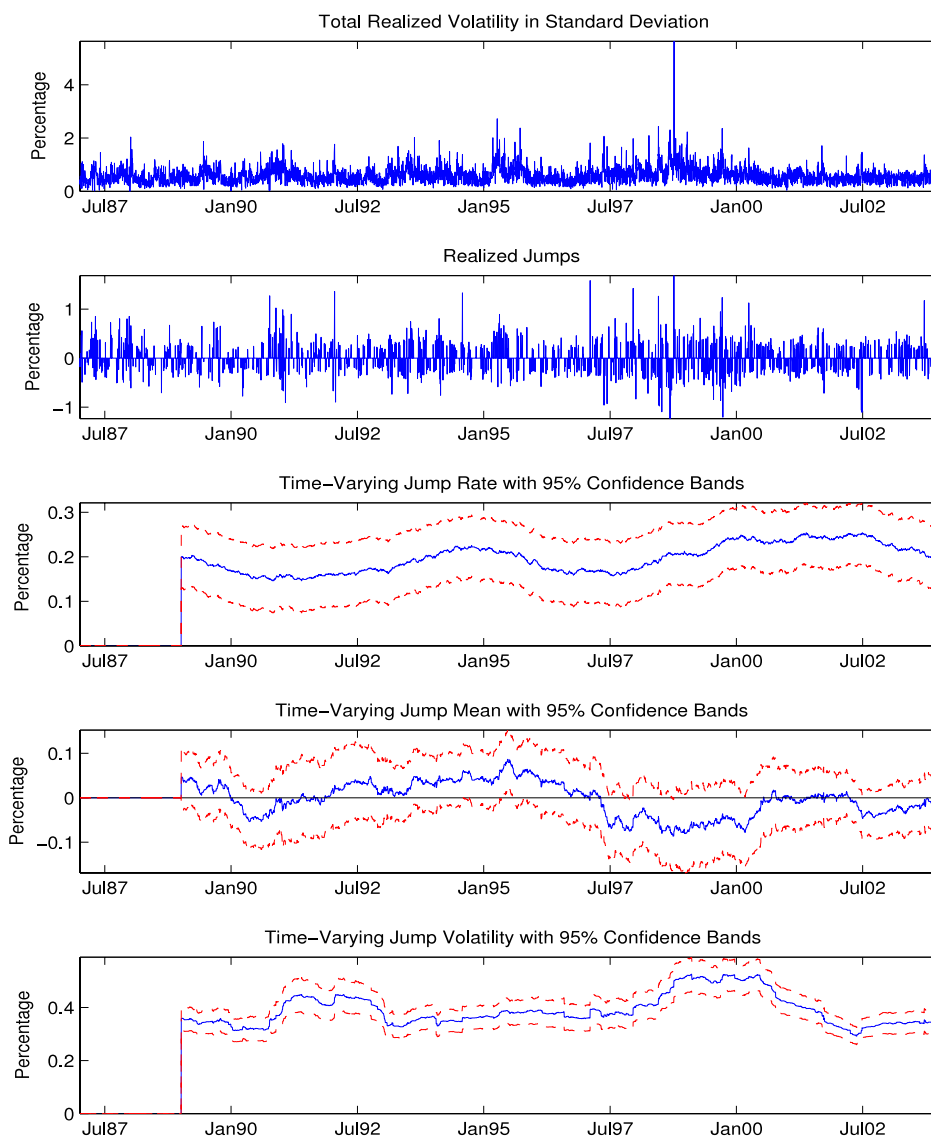
direct contribution to explaining the credit spread is rather weak empirically (see, e.g., [Elton et al., 2001](#); [Collin-Dufresne et al., 2001](#), among others). Nevertheless, our market jump volatility variable seems to predict credit spreads well on the one hand, and also tracks the historical default probability well on the other ([Fig. 9](#) top panel). In fact, the correlation between jump volatility and default rate at quarterly horizons is as high as 65%, while the correlation with the quarterly loss rate is merely 11% ([Fig. 9](#) bottom panel).

In a related paper, [Zhang et al. \(2006\)](#) apply the jump identification strategy of this paper to individual firms, and find that the realized jump risk measures (intensity, mean, and volatility) from firm level equity returns all have strong explanatory power for credit default swap (CDS) spreads. In particular, jump risk alone can be used to predict about 19% of the variation of the CDS spreads. By separating realized volatility and jump measures, they also strengthen the forecasting power of equity volatility measures as in [Campbell and Taksler \(2003\)](#), and increase the overall forecasting  $R^2$  to 77%. Furthermore, they find that the nonlinear effects of jump and volatility risk measures on credit spreads are largely consistent with a structural model with stochastic volatility and jumps. For individual equities, [Lee and Mykland \(forthcoming\)](#)

find that the majority of jumps occur with unscheduled news announcements and their magnitudes are comparable to those that occur with earnings announcements. In other words, jumps are driven by the information release of firms' fundamental profitability. That is why jump dynamics based on these realized jumps may have important implications for firms' credit default prices ([Zhang et al., 2006](#)) as well as for the equity option pricing of individual names.

Note that for individual firms' credit spreads examined by [Zhang et al. \(2006\)](#), most of the jump variation is driven by the firm specific news arrival, like earning announcements ([Jiang and Oomen, 2005](#); [Lee and Mykland, forthcoming](#)). However, for the aggregated credit spread indices studied here, the market jump volatility from S&P 500 is more of a proxy for the systematic (jump) risk factor. More research is desirable to further differentiate between systematic jumps and idiosyncratic jumps (as in [Bollerslev et al., 2007](#), for example).

In summary, the utility of realized jump volatility for predicting credit spreads well seems to reside in its ability to track down the long-term countercyclical movements in spreads. This feature is also reflected in the jump volatility's association with long-run macroeconomic risk embedded in the price-dividend ratio. As



**Fig. 6.** Dollar/yen realized variance and jump dynamics. The realized variance is from intra-day five-minute returns, the realized jumps are filtered by the bipower variation method, and the jump parameters are estimated with a two-year rolling sample.

a sanity check, jump volatility does match the historical default frequency, suggesting that its forecasting ability is not merely a statistical fluke.

## 6. Conclusion

Disentangling jumps from the diffusion has always been a challenge for pricing financial assets and for estimating the jump–diffusion processes. Building on the recent jump detection literature for separating realized variance and bipower variation (Barndorff-Nielsen and Shephard, 2003, 2004, 2006; Andersen et al., 2004; Huang and Tauchen, 2005), we extend the methodology to filter out the realized jumps, under two key assumptions typically adopted in financial economics: (1) jumps are rare and there is at most one jump per day, and (2) jumps are large and dominate return signs when they occur.

These approximations provide a powerful tool for identifying the realized jumps in financial markets. Our Monte Carlo experiments under realistic empirical settings suggest that accurate parameter estimates and properly sized inference tests can be obtained with an appropriate choice of the significance level of the jump detection pre-test.

The proposed jump identification method is applied to three financial markets – the S&P 500 index, a treasury bond, and the dollar/yen exchange rate. We find that the jump intensity varies among these asset classes from 13% to 20%. All of the jump mean estimates are insignificantly different from zero, except for the S&P 500 index driven by a positive run in late 1990s. Jump volatilities are similar for the exchange rate (0.39%), the equity market (0.53%), and the bond market (0.65%). Rolling estimates reveal that the jump probabilities are quite variable for the equity index and treasury bonds (from 5% to 25%), but relatively stable for the yen currency (20%). The jump volatility is little changed for government bonds (except for the run-up in 1992–1994), while it is elevated a great deal for the stock market from 2000 to 2004 and only moderately for the dollar/yen in the early and late 1990s.

The identification of realized jumps and direct estimation of jump distributions has important implications in assessing financial market risk adjustments. Given more reliable estimates of the jump dynamics, the impact on jump risk premia can be more precisely quantified. For example, Moody's AAA and BAA credit risk premia can be predicted using the realized jump volatility measure much better than using the usual variables such as interest rate factors, and volatility factors including option-implied volatility,

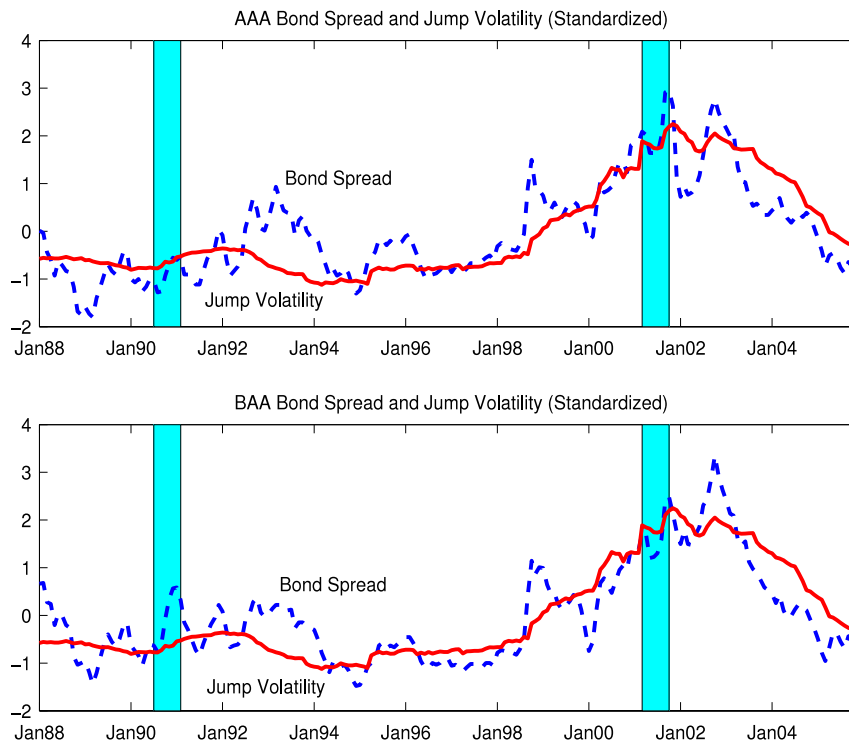
**Table 5**

Credit spreads with interest rate and volatility factors. The left-hand-side variable is the monthly Moody AAA or BAA bond spread index. The right-hand-side variables are the monthly six-month short rate and the term spread for ten years and six months (from the Federal Reserve H.15 release), the short-run volatility of one month and the long-run volatility of two years (from S&P 500 five-minute data), the option-implied volatility (VIX from CBOE), and the jump volatility from a two-year rolling estimation result discussed in Section 4.

Regressors	Moody's AAA bond yield spread					
Constant	2.2434	0.9957	0.1716	0.1214	0.7789	0.4349
(s.e.)	(0.1253)	(0.1352)	(0.2078)	(0.3260)	(0.2113)	(0.1198)
Short rate	−0.1979					
(s.e.)	(0.0218)					
Term spread		0.2051				
(s.e.)		(0.0651)				
Short-run volatility			0.0884			
(s.e.)			(0.0178)			
Long-run volatility				0.0961		
(s.e.)				(0.0269)		
Implied volatility					0.0277	
(s.e.)					(0.0109)	
Jump volatility						1.7495
(s.e.)						(0.1953)
Adj. $R^2$	0.5869	0.1792	0.2556	0.2927	0.1408	0.6152

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Regressors	Moody's BAA bond yield spread					
Constant	3.1544	1.7511	0.7285	0.7947	1.3851	1.0688
(s.e.)	(0.1947)	(0.1448)	(0.2901)	(0.4874)	(0.2397)	(0.1285)
Short rate	−0.2112					
(s.e.)	(0.0354)					
Term spread		0.2606				
(s.e.)		(0.0778)				
Short-run volatility			0.1105			
(s.e.)			(0.0248)			
Long-run volatility				0.1099		
(s.e.)				(0.0390)		
Implied volatility					0.0396	
(s.e.)					(0.0130)	
Jump volatility						2.1648
(s.e.)						(0.2373)
Adj. $R^2$	0.4681	0.2035	0.2804	0.2686	0.2039	0.6610



**Fig. 7.** Bond spread and jump volatility. This figure plots the monthly Moody AAA and BAA bond spread indices and the two-year rolling estimates of the S&P 500 index jump volatility. These series are standardized as mean 0 and variance 1.



**Table 6**

Credit spreads with Fama-French and jump risk factors. The left-hand-side variable is the monthly Moody AAA or BAA bond spread indices. The right-hand-side variables are the monthly market return, SMB and HML originally from Fama and French (1993) and updated at <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french> ; and the jump intensity, mean, and volatility from a two-year rolling estimation result discussed in Section 4.

Regressors	Moody's AAA bond yield spread					
Constant (s.e.)	1.3584 (0.0826)	1.3459 (0.0806)	1.3413 (0.0807)	1.5770 (0.1903)	1.5467 (0.1071)	0.4349 (0.1198)
Market return (s.e.)	−0.0163 (0.0101)					
SMB (s.e.)		0.0105 (0.0090)				
HML (s.e.)			0.0175 (0.0097)			
Jump intensity (s.e.)				−1.7047 (1.1901)		
Jump mean (s.e.)					−2.9095 (0.9541)	
Jump volatility (s.e.)						1.7495 (0.1953)
Adj. $R^2$	0.0127	0.0009	0.0081	0.0296	0.1521	0.6152
Regressors	Moody's BAA bond yield spread					
Constant (s.e.)	2.2108 (0.0989)	2.1959 (0.0966)	2.1915 (0.0976)	2.3070 (0.2402)	2.4848 (0.1301)	1.0688 (0.1285)
Market return (s.e.)	−0.0192 (0.0127)					
SMB (s.e.)		0.0135 (0.0124)				
HML (s.e.)			0.0184 (0.0114)			
Jump intensity (s.e.)				−0.8096 (1.3368)		
Jump mean (s.e.)					−4.1889 (1.2430)	
Jump volatility (s.e.)						2.1648 (0.2373)
Adj. $R^2$	0.0122	0.0017	0.0052	0.0007	0.2232	0.6610

**Table 7**

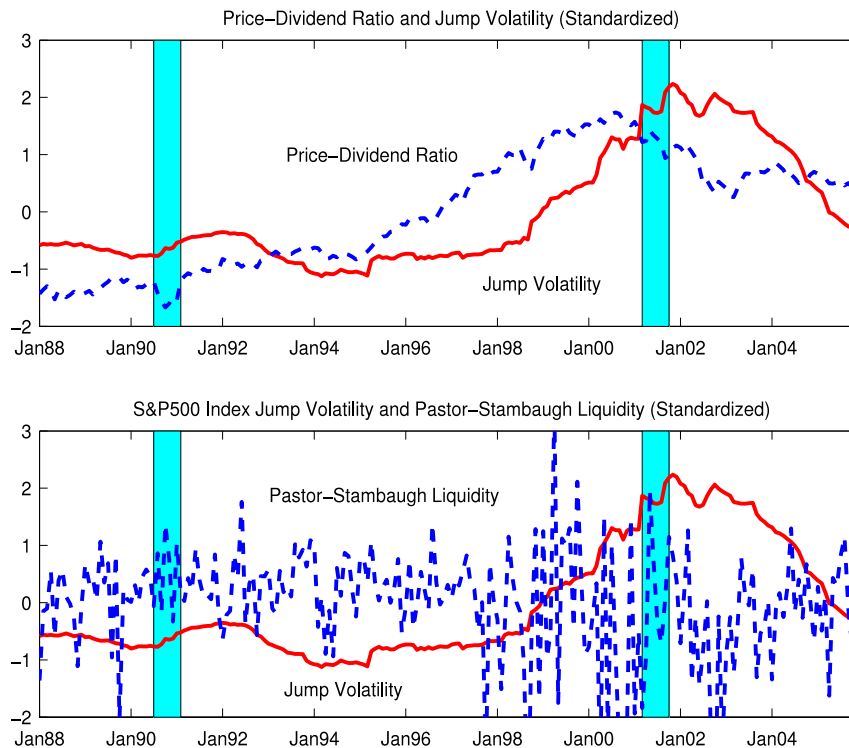
Multivariate prediction of credit spreads. The variable definitions are the same as those used in the univariate regressions in Table 5.

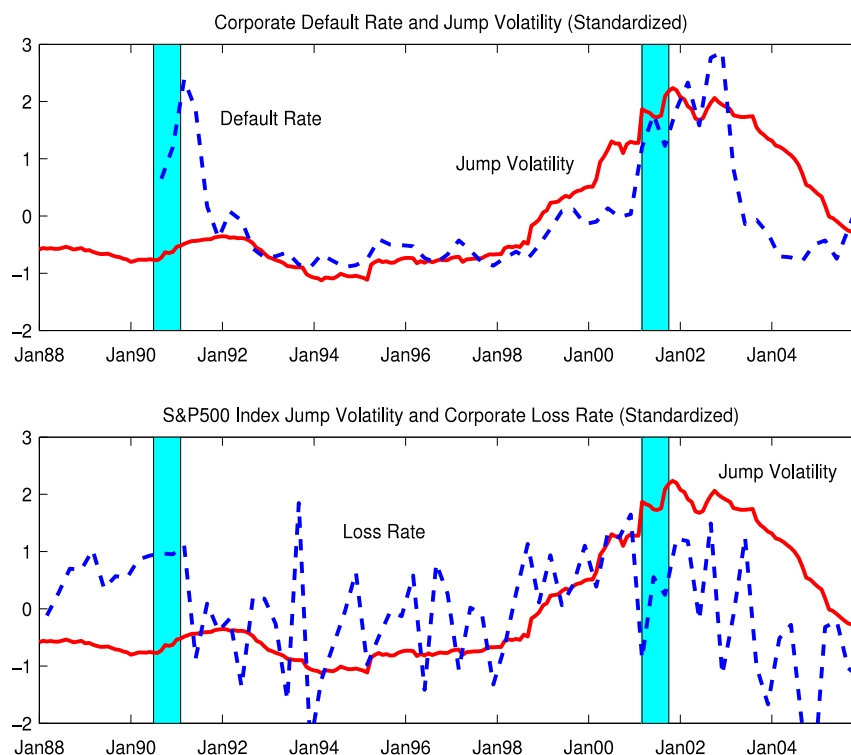
Regressors	Moody's AAA bond yield spread						
Constant	2.4011	−0.1266	0.4000	1.8677	1.0848	0.2849	1.1409
(s.e.)	(0.2783)	(0.2781)	(0.1881)	(0.2706)	(0.2544)	(0.2679)	(0.2811)
Short rate	−0.2144			−0.1913	−0.1266		−0.1304
(s.e.)	(0.0324)			(0.0236)	(0.0249)		(0.0237)
Term spread	−0.0483			−0.0294	0.0411		0.0377
(s.e.)	(0.0656)			(0.0509)	(0.0410)		(0.0394)
Short-run volatility		0.0481		0.0390		−0.0089	0.0041
(s.e.)		(0.0198)		(0.0130)		(0.0126)	(0.0110)
Long-run volatility		0.0654		−0.0096		0.0190	−0.0063
(s.e.)		(0.0295)		(0.0174)		(0.0223)	(0.0107)
Implied volatility			0.0026		0.0173	0.0051	0.0168
(s.e.)			(0.0072)		(0.0048)	(0.0080)	(0.0051)
Jump volatility			1.7138		0.7872	1.5997	0.7939
(s.e.)			(0.1633)		(0.2559)	(0.2863)	(0.3008)
Adj. $R^2$	0.5911	0.3362	0.6144	0.6208	0.7772	0.6160	0.7756
Regressors	Moody's BAA bond yield spread						
Constant	3.0946	0.4317	0.9365	1.9051	0.9793	0.8234	0.7671
(s.e.)	(0.3425)	(0.4007)	(0.1994)	(0.5069)	(0.2538)	(0.2808)	(0.4063)
Short rate	−0.2050			−0.1475	−0.0587		−0.0542
(s.e.)	(0.0459)			(0.0438)	(0.0283)		(0.0315)
Term spread	0.0183			0.0635	0.1604		0.1679
(s.e.)	(0.0761)			(0.0632)	(0.0376)		(0.0423)
Short-run volatility		0.0703		0.0635		−0.0018	0.0101
(s.e.)		(0.0258)		(0.0204)		(0.0136)	(0.0126)
Long-run volatility		0.0651		0.0006		0.0123	0.0067
(s.e.)		(0.0398)		(0.0307)		(0.0240)	(0.0154)
Implied volatility			0.0099		0.0248	0.0118	0.0266
(s.e.)			(0.0078)		(0.0062)	(0.0081)	(0.0065)
Jump volatility			2.0292		1.3449	1.9175	1.1936
(s.e.)			(0.1886)		(0.2547)	(0.2406)	(0.2574)
Adj. $R^2$	0.4662	0.3355	0.6698	0.5350	0.7958	0.6681	0.7959

**Table 8**

Credit spreads with lag controls. The variable definitions are the same as for Table 5.

Regressors	Moody's AAA bond yield spread					
Constant	−0.0183 (0.0730)	0.0426 (0.0303)	0.0264 (0.0391)	−0.0061 (0.0454)	0.0102 (0.0414)	0.0330 (0.0316)
Lag credit spread	0.9901 (0.0342)	0.9782 (0.0257)	0.9657 (0.0271)	0.9537 (0.0248)	0.9529 (0.0242)	0.9225 (0.0249)
Short rate	0.0065 (0.0079)					
Term spread		−0.0089 (0.0118)				
Short-run volatility			0.0013 (0.0032)			
Long-run volatility				0.0052 (0.0037)		
Implied volatility					0.0035 (0.0020)	
Jump volatility						0.1332 (0.0620)
Adj. $R^2$	0.9375	0.9375	0.9373	0.9379	0.9393	0.9386
Regressors	Moody's BAA bond yield spread					
Constant	−0.0174 (0.0847)	0.0507 (0.0430)	0.0373 (0.0526)	0.0081 (0.0552)	0.0012 (0.0476)	0.0800 (0.0426)
Lag credit spread	0.9913 (0.0273)	0.9896 (0.0222)	0.9685 (0.0237)	0.9591 (0.0211)	0.9489 (0.0224)	0.9209 (0.0252)
Short rate	0.0074 (0.0081)					
Term spread		−0.0180 (0.0118)				
Short-run volatility			0.0022 (0.0040)			
Long-run volatility				0.0062 (0.0038)		
Implied volatility					0.0053 (0.0024)	
Jump volatility						0.1745 (0.0657)
Adj. $R^2$	0.9495	0.9500	0.9493	0.9499	0.9523	0.9507

**Fig. 8.** Jump volatility and macro-financial risk. This figure plots the monthly series of price-dividend ratios for the S&P 500 index, the market liquidity measure of Pastor and Stambaugh (2003), and the two-year rolling estimates of the S&P 500 index jump volatility. These series are standardized as mean 0 and variance 1.



**Fig. 9.** Jump volatility and credit risk. This figure plots the quarterly series for the corporate sector default rate and loss rate (Federal Reserve internal source) and the two-year rolling estimates of the S&P 500 index jump volatility. These series are standardized as mean 0 and variance 1.

under appropriate controls for systematic risk factors and lagged credit spreads. Explaining the credit spreads of high investment grade entities has always been a challenge in credit risk pricing, and a systematic jump risk factor holds some promise in resolving such a puzzle. It can be shown that the market jump volatility factor comoves with the price–dividend ratio and corporate default rate, with a correlation of 67% and 65%. The fact that jump volatility captures low frequency movements of credit spreads with a countercyclical pattern has important asset pricing implications along the lines of long-run risks (Bansal and Yaron, 2004).

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