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Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads

HAYNE E. LELAND and KLAUS BJERRE TOFT*

ABSTRACT

This article examines the optimal capital structure of a firm that can choose both the amount and maturity of its debt. Bankruptcy is determined endogenously rather than by the imposition of a positive net worth condition or by a cash flow constraint. The results extend Leland's (1994a) closed-form results to a much richer class of possible debt structures and permit study of the optimal maturity of debt as well as the optimal amount of debt. The model predicts leverage, credit spreads, default rates, and writedowns, which accord quite closely with historical averages. While short term debt does not exploit tax benefits as completely as long term debt, it is more likely to provide incentive compatibility between debt holders and equity holders. Short term debt reduces or eliminates "asset substitution" agency costs. The tax advantage of debt must be balanced against bankruptcy and agency costs in determining the optimal maturity of the capital structure. The model predicts differently shaped term structures of credit spreads for different levels of risk. These term structures are similar to those found empirically by Sarig and Warga (1989). Our results have important implications for bond portfolio management. In general, Macaulay duration dramatically overstates true duration of risky debt, which may be negative for "junk" bonds. Furthermore, the "convexity" of bond prices can become "concavity."

IN AN EARLIER ARTICLE, LELAND (1994a) considered optimal capital structure and the pricing of debt with credit risk. His assumption of infinite life debt—consistent with Modigliani-Miller (1958)—permitted closed form solutions for debt values and equity values with endogenous bankruptcy. But the assumption of infinite life debt is clearly restrictive. Firms must choose the *maturity* as well as the *amount* of debt.¹

This article extends Leland's results to examine the effect of debt maturity on bond prices, credit spreads, and the optimal amount of debt. Our predictions

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¹ Another dimension of debt, priority, is not explicitly considered in this article. Section V discusses how our analysis could be extended to consider the priority structure of debt. See also Barclay and Smith (1995b).

are generally consistent with the empirical findings of Sarig and Warga (1989) on credit spreads, of Altman (1991) on bankruptcy rates and debt writedowns in default, and of Barclay and Smith (1995a) on the maturity structure of debt.

We show that longer term debt better exploits tax advantages because bankruptcy tends to occur at lower asset values. But longer term debt also creates greater agency costs by providing incentives for equity holders to increase firm risk through asset substitution. This potential agency cost can be substantially reduced or eliminated by using shorter term debt. Our results illuminate how the twin dimensions of optimal capital structure, amount and maturity, represent a tradeoff between tax advantages, bankruptcy costs, and agency costs.

Studies by Kim, Ramaswamy, and Sundaresan (1993), Longstaff and Schwartz (1995), and Nielsen, Saá-Requejo, and Santa-Clara (1993) have recently examined the pricing of bonds with credit risk and arbitrary maturity. There are three important differences between their results and ours.

First, they examine debt values only and do not consider optimal capital structure. In contrast, the optimal amount and maturity of debt is the central focus of our work.

Second, they assume either that bankruptcy is triggered at an exogenously specified asset value, such as debt principal value, or that it is triggered when cash flow fails to cover interest payments.² Yet neither approach correctly describes bankruptcy as an optimal decision by equity holders to surrender control to bond holders. We derive *endogenous* conditions under which bankruptcy will be declared, and contrast the comparative statics of our model with those in which bankruptcy is exogenous. There are substantial differences, particularly for high yield ("junk") bonds.

Third, they allow default-free short term interest rates to follow a stochastic process. Our work presumes a nonstochastic default-free interest rate. Kim, Ramaswamy, and Sundaresan (1993) and Longstaff and Schwartz (1995), however, show that introducing a stochastic default-free interest rate process has a relatively small effect on credit spreads, while significantly complicating the analysis.³ Our analysis provides relatively simple closed-form solutions that can be used to explore issues related to optimal capital structure.

² Nielsen, Saá-Requejo, and Santa-Clara (1993) assume that the bankruptcy-triggering asset value is an exogenously specified stochastic process.

³ If the correlation between asset value and short term default-free interest rates is -0.25 , the cited studies predict that credit spreads will be about 5 to 7 basis points less than when the default-free interest rate is nonstochastic. Previous authors (e.g. Jones, Mason, and Rosenfeld (1984)) have criticized contingent claim approaches for implying too small credit spreads. But a negative correlation between asset values and interest rates *reduces* credit spreads. A stochastic rate process therefore will not solve the problem of small spreads; other modifications (including payouts) do lead to credit spreads that appear in line with historical spreads. See also Anderson and Sundaresan (1995) and Mella-Barral (1995).

I. Finite Maturity Debt

As in Merton (1974), Black and Cox (1976), and Brennan and Schwartz (1978), the firm has productive assets whose unleveraged value V follows a continuous diffusion process with constant proportional volatility σ :

$$\frac{dV}{V} = [\mu(V, t) - \delta]dt + \sigma dz, \quad (1)$$

where $\mu(V, t)$ is the total expected rate of return on asset value V ; δ is the constant fraction of value paid out to security holders; and dz is the increment of a standard Brownian motion.⁴ The process continues without time limit unless V falls to a default-triggering value V_B .⁵ For the moment we shall assume that V_B is constant. Later, V_B will be determined endogenously and shown to be constant in a rational expectations equilibrium. We assume a default-free asset exists that pays a continuous interest rate r .

Consider a bond issue with maturity t periods from the present, which continuously pays a constant coupon flow $c(t)$ and has principal $p(t)$. Let $\rho(t)$ be the fraction of asset value V_B which debt of maturity t receives in the event of bankruptcy. Using risk-neutral valuation, and letting $f(s; V, V_B)$ denote the density of the first passage time s to V_B from V when the drift rate is $(r - \delta)$, gives debt with maturity t the value

$$\begin{aligned} d(V; V_B, t) = & \int_0^t e^{-rs} c(t) [1 - F(s; V, V_B)] ds + e^{-rt} p(t) [1 - F(t; V, V_B)] \\ & + \int_0^t e^{-rs} \rho(t) V_B f(s; V, V_B) ds. \end{aligned} \quad (2)$$

The first term in equation (2) represents the discounted expected value of the coupon flow (which will be paid at s with probability $(1 - F(s))$, where $F(s)$ is the cumulative distribution function of the first passage time to bankruptcy); the second term represents the expected discounted value of repayment of principal; and the third term represents the expected discounted value of the

⁴ For example, δ might represent a constant proportional cash flow generated by the assets and distributed to security holders. However, there is no requirement that δ be associated with the ongoing cash flow generated by assets—it is simply the cash flow paid out. As do other studies, we assume that δ is not affected by changes in leverage; otherwise, investment would change with capital structure, which in turn would raise questions which are beyond the scope of this article. In particular, this assumption rules out additional liquidation of assets (raising δ) to meet ongoing debt service payments. Bond covenants often rule out such liquidation.

⁵ Default can lead to restructuring in or outside of bankruptcy proceedings, or to liquidation—we do not examine what happens after default. See also Franks and Torous (1989). While default need not lead to bankruptcy, we shall refer to V_B as the “bankruptcy-triggering asset value,” and costs of restructuring or liquidation as “bankruptcy costs.”

fraction of the assets which will go to debt with maturity t , if bankruptcy occurs. Integrating the first term by parts yields

$$d(V; V_B, t) = \frac{c(t)}{r} + e^{-rt} \left[p(t) - \frac{c(t)}{r} \right] [1 - F(t)] + \left[\rho(t)V_B - \frac{c(t)}{r} \right] G(t) \quad (3)$$

where

$$G(t) = \int_{s=0}^t e^{-rs} f(s; V, V_B) ds.$$

We find expressions for $F(t)$ from Harrison (1990) and for $G(t)$ from Rubinstein and Reiner (1991):

$$F(t) = N[h_1(t)] + \left(\frac{V}{V_B} \right)^{-2a} N[h_2(t)] \quad (4)$$

$$G(t) = \left(\frac{V}{V_B} \right)^{-a+z} N[q_1(t)] + \left(\frac{V}{V_B} \right)^{-a-z} N[q_2(t)] \quad (5)$$

where

$$\begin{aligned} q_1(t) &= \frac{(-b - z\sigma^2 t)}{\sigma\sqrt{t}}; & q_2(t) &= \frac{(-b + z\sigma^2 t)}{\sigma\sqrt{t}}; \\ h_1(t) &= \frac{(-b - a\sigma^2 t)}{\sigma\sqrt{t}}; & h_2(t) &= \frac{(-b + a\sigma^2 t)}{\sigma\sqrt{t}} \\ a &= \frac{(r - \delta - (\sigma^2/2))}{\sigma^2}; & b &= \ln\left(\frac{V}{V_B}\right); & z &= \frac{[(a\sigma^2)^2 + 2r\sigma^2]^{1/2}}{\sigma^2} \end{aligned}$$

and $N(\cdot)$ is the cumulative standard normal distribution. Substituting these expressions into equation (3) gives a closed form solution for risky debt value, when V_B is constant. Defining $x = a + z$, note that as $t \rightarrow \infty$,

$$d(V; V_B, t) \rightarrow \frac{c(\infty)}{r} \left[1 - \left(\frac{V}{V_B} \right)^{-x} \right] + \rho(\infty)V_B \left(\frac{V}{V_B} \right)^{-x} \quad (6)$$

which is the same equation as Leland (1994a) derived for infinite-horizon risky debt when $\rho(\infty) = (1 - \alpha)$, where α is the fraction of asset value lost in bankruptcy, and $c(\infty)$ is the coupon paid by the infinite-maturity bond.

As in Longstaff and Schwartz (1995), the analysis above has assumed that V_B is an exogenous constant. For arbitrary capital structures, however, this is unlikely to be optimal. If the firm has only a single issue of debt outstanding, debt service requirements are time-dependent. Prior to maturity, asset value may be low (less than P) but still sufficient to justify paying the coupon to avoid

default. Only at maturity, when the required payment to debt holders “balloons,” will the firm need substantial asset value to avoid bankruptcy.⁶

In the next section we present a debt structure consistent with a constant endogenously-determined bankruptcy asset level V_B . This capital structure has *time-independent debt service payments*.

II. Endogenous Bankruptcy with a Stationary Debt Structure

A. A Stationary Debt Structure

Consider an environment where the firm continuously sells a constant (principal) amount of new debt with maturity of T years from issuance, which (if solvent) it will redeem at par upon maturity. New bond principal is issued at a rate $p = (P/T)$ per year, where P is the total principal value of all outstanding bonds. The same amount of principal will be retired when the previously-issued bonds mature. As long as the firm remains solvent, at any time s the total outstanding debt principal will be P , and have a uniform distribution of principal over maturities in the interval $(s, s + T)$. Without loss of generality, we define the current time $s = 0$.

Bonds with principal p pay a constant coupon rate $c = (C/T)$ per year, implying the total coupon paid by all outstanding bonds is C per year. Total debt service payments are therefore time-independent and equal to $(C + P/T)$ per year.

Later we shall show that this environment is consistent with a constant V_B . Now we assume this to be the case. Let $D(V; V_B, T)$ denote the total value of debt, when debt of maturity T is issued. The fraction of firm asset value lost in bankruptcy is α . The remaining value $(1 - \alpha)V_B$ is distributed to bond holders so that the sum of all fractional claims $\rho(t)$ for debt of all maturities outstanding equals $(1 - \alpha)$. For simplicity we will assume that $\rho(t) = \rho/T$ per year for all t .⁷ This in turn implies $\rho = (1 - \alpha)$.

We can now determine the value of all outstanding bonds:

$$\begin{aligned} D(V; V_B, T) &= \int_{t=0}^T d(V; V_B, t) dt \\ &= \frac{C}{r} + \left(P - \frac{C}{r} \right) \left(\frac{1 - e^{-rT}}{rT} - I(T) \right) + \left((1 - \alpha)V_B - \frac{C}{r} \right) J(T) \end{aligned} \quad (7)$$

⁶ For example, in Merton's (1974) analysis, bankruptcy occurs *only* at maturity (when $V < P$), since with zero coupon bonds there is never an endogenous reason to default prior to maturity.

⁷ This assumption, which implies equal seniority of all outstanding debt, is not required. For example, a formulation where $\rho(t)$ is exponentially declining (or increasing) can be accommodated in our framework. Also, following Leland [1994a; Section VI(C)], it is possible to relax the assumption made here of absolute priority—that equity holders receive no residual value in bankruptcy.

where

$$I(T) = \frac{1}{T} \int_0^T e^{-rt} F(t) dt$$

$$J(T) = \frac{1}{T} \int_0^T G(t) dt.$$

In Appendix A we show that

$$I(T) = \frac{1}{rT} (G(T) - e^{-rT} F(T))$$

$$J(T) = \frac{1}{z\sigma\sqrt{T}} \left(- \left(\frac{V}{V_B} \right)^{-a+z} N[q_1(T)] q_1(T) + \left(\frac{V}{V_B} \right)^{-a-z} N[q_2(T)] q_2(T) \right).$$

The total market value of the firm, v , equals the asset value plus the value of tax benefits, less the value of bankruptcy costs, over the infinite horizon. Tax benefits accrue at rate τC per year as long as $V > V_B$, where τ is the corporate tax rate. Following Leland (1994a, equation (12)) total firm value will be given by

$$v(V; V_B) = V + \frac{\tau C}{r} \left[1 - \left(\frac{V}{V_B} \right)^{-x} \right] - \alpha V_B \left(\frac{V}{V_B} \right)^{-x} \quad (8)$$

where we recall

$$x = a + z.^8$$

The value of equity is given by

$$E(V; V_B, T) = v(V; V_B) - D(V; V_B, T) \quad (9)$$

B. Bankruptcy: Determining the Bankruptcy-Triggering Asset Level V_B .

To determine the equilibrium bankruptcy-triggering asset value V_B endogenously, we invoke the smooth-pasting condition.⁹ V_B solves the equation

$$\left. \frac{\partial E(V; V_B, T)}{\partial V} \right|_{V=V_B} = 0. \quad (10)$$

⁸ In Appendix B, we consider the case where tax benefits are lost before bankruptcy, at some $V_T > V_B$. This affects not only the value of the firm, but also the endogenously determined V_B .

⁹ The smooth-pasting condition has the property of maximizing (with respect to V_B) both the value of equity, and the value of the firm, subject to the limited liability of equity ($E(V) \geq 0$ for all $V \geq V_B$), which also implies $E_{VV}(V_B) = \partial^2 E(V_B)/\partial V^2 \geq 0$. We have verified numerically that this inequality holds strictly for all examples considered.

The solution to equation (10) is independent of time, indicating that the earlier assumption of a constant V_B is warranted: it represents a rational expectations equilibrium. Using equations (7), (8), and (9), we can solve equation (10) for V_B :

$$V_B = \frac{(C/r)(A/(rT) - B) - AP/(rT) - \tau Cx/r}{1 + \alpha x - (1 - \alpha)B}, \quad (11)$$

where

$$\begin{aligned} A &= 2ae^{-rT}N(a\sigma\sqrt{T}) - 2zN(z\sigma\sqrt{T}) \\ &\quad - \frac{2}{\sigma\sqrt{T}}n(z\sigma\sqrt{T}) + \frac{2e^{-rT}}{\sigma\sqrt{T}}n(a\sigma\sqrt{T}) + (z - a) \\ B &= -\left(2z + \frac{2}{z\sigma^2T}\right)N(z\sigma\sqrt{T}) - \frac{2}{\sigma\sqrt{T}}n(z\sigma\sqrt{T}) + (z - a) + \frac{1}{z\sigma^2T} \end{aligned}$$

and $n(\cdot)$ denotes the standard normal density function. When $T \rightarrow \infty$, it can be shown that $V_B \rightarrow (1 - \tau)(Cx/r)/(1 + x)$, as in Leland (1994a).

Observe that V_B will depend upon the maturity of debt chosen, for any given values of total bond principal P and coupon rate C . In contrast, models consistent with flow-based bankruptcy (e.g., Kim, Ramaswamy, and Sundaresan (1993) and Ross (1994)) or with a positive net worth covenant (e.g., Longstaff and Schwartz (1995)) typically imply that V_B is *independent* of debt maturity. This leads to important differences in the effect of maturity on debt value between their models and ours.

Our bankruptcy condition can be further analyzed by computing the expected appreciation of equity around the endogenous bankruptcy trigger. Express the value of equity as a function of V and apply Ito's lemma. Since $E_V = 0$ when $V = V_B$, the appreciation of equity simplifies to

$$dE \Big|_{V=V_B} = \frac{1}{2} \sigma^2 V^2 E_{VV} \Big|_{V=V_B} dt \geq 0$$

Evaluating and further simplifying yields¹⁰

$$\begin{aligned} dE|_{V=V_B} &= \left((1 - \tau)C + \frac{P}{T} - \frac{(1 - \alpha)V_B}{T} - \delta V_B \right) dt \\ &= ((1 - \tau)C + p)dt - (d(V_B; V_B, T) + \delta V_B)dt \end{aligned} \quad (12)$$

where we recall that $p = P/T$ is the rate of principal redemption per year, and $d(V_B; V_B, t) = (1 - \alpha)V_B/T$ is the annual rate of newly issued debt market

¹⁰ We have proven equation (12) analytically for the case where $T = \infty$, and numerically verified that it holds in all cases considered when $T < \infty$.

value when $V = V_B$. The l.h.s. of equation (12) is the change in the equity value at $V = V_B$.¹¹ The r.h.s. is the additional cash flow required from equity holders for current debt service. It consists of the cost of debt service—the after-tax coupon expense plus the principal expense (redeeming the maturing bonds at par)—less the sum of the revenues from selling an equal principal amount of bonds at their market price, and the cash flow available for payout generated by the firm's activities.

Smooth pasting therefore implies a *flow* condition: at the bankruptcy point $V = V_B$, the change in value of equity just equals the additional cash flow that must be provided by equity holders to keep the firm solvent. When $V > V_B$, equity has positive value, and the risk-neutral expected equity appreciation exceeds the cash flows (if any) that must be contributed. Current shareholders will contribute (or be willing to suffer dilution from selling new equity), since the alternative is bankruptcy and a zero value of equity. We will never observe $V < V_B$, since equity appreciation would be less than the contribution required from equity holders to keep the firm solvent. Equity capital cannot be raised to meet the debt service, and the firm will default.

For long term debt structures, the endogenous bankruptcy-triggering asset value will typically be less than the principal value of debt: $V_B < P$. Hence the firm may continue to operate despite having negative net worth.¹² However, as $T \rightarrow 0$, $V_B \rightarrow P/(1 - \alpha)$, which exceeds P when $\alpha > 0$.¹³ Thus when debt is short term and $\alpha > 0$, bankruptcy will occur despite net worth being positive! This may seem puzzling: if the firm has assets $V = V_B$ that exceed the bond holders' principal P , why will bankruptcy be declared? The answer is that bankruptcy is triggered not because V falls beneath P , but rather because the anticipated equity appreciation does not warrant the additional contribution required from equity holders to avoid default on bond service payments.

Furthermore, note that the firm generally will *not* declare bankruptcy whenever its cash flow available for payout fails to equal its required net debt service payments.¹⁴ This observation follows immediately from equation (12). Recall that the r.h.s. of equation (12) is the cash flow required for net debt service, less the cash flow available (δV) at $V = V_B$. But at V_B , the r.h.s. of equation (12) equals dE , which is typically positive as discussed in Footnote 11. Therefore, bankruptcy will generally occur when net debt service requirements exceed the available cash flow. The firm survives to this point by raising

¹¹ Note that this term is nonnegative (and always strictly positive in the examples we have considered), since $E_{VV}(V_B) \geq 0$. It also is nonstochastic, since $E_V(V_B) = 0$ and $(dV/V)^2 = \sigma^2 dt$ with probability 1.

¹² We define "net worth" as the difference between the market value of (unleveraged) assets, V , and the principal value of debt, P . Only under special conditions will this definition coincide with accounting net worth.

¹³ This is unlike Leland (1994a), who postulates that (very) short run debt could be associated with $V_B = P$, not $P/(1 - \alpha)$. That reasoning is ad hoc, and not consistent with the endogenous bankruptcy condition except when $\alpha = 0$.

¹⁴ This point is observed by Cornell, Longstaff, and Schwartz (1996), and is in contrast to the assumption of Kim, Ramaswamy, and Sundaresan (1993) and Ross (1994).

additional cash from current equity holders, or, more realistically, by further equity issuance.

Appendix B extends the analysis to consider V_B in the case where tax deductibility is lost whenever V falls below some value V_T , where $V_T > V_B$. It is shown there that

$$V_B = \frac{(C/r)(A/(rT) - B) - AP/(rT)}{1 + x(\tau C/(rV_T) + \alpha) - (1 - \alpha)B} \quad (13)$$

When $V_T > V_B$, V_B in equation (13) will exceed V_B in equation (11). Thus the loss of tax deductibility raises the asset value at which bankruptcy will be declared.¹⁵ This lowers the value of debt and firm value. Although equation (7) will continue to be the correct expression for debt value (with the altered V_B), equation (8) for firm value must be modified as in Appendix B.

III. Applications

We may use equation (11) to substitute for V_B in equations (7), (8), and (9) to find *closed form expressions for the value of debt, the firm, and equity*, when bankruptcy is endogenously determined.¹⁶ In this section we consider some applications of the model. To facilitate comparisons, we consider a “base case” environment with the following parameters: the default-free interest rate $r = 0.075$, the corporate tax rate $\tau = 0.35$, the bankruptcy cost fraction $\alpha = 0.50$, and the asset risk $\sigma = 0.20$. The observable parameters were chosen to be consistent with the U.S. environment. The bankruptcy cost fraction generates credit spreads, bankruptcy rates, and bond value writedowns that are consistent with empirical estimates by Altman (1991) and others. The asset risk parameter is chosen such that an optimally leveraged firm’s equity will have a standard deviation of return of about 30 percent annually, which is consistent with the observed riskiness of equity.

We further assume that the firm pays out an amount to security holders equal to δV , with $\delta = 0.07$.¹⁷ Finally, we presume that the tax deductibility of

¹⁵ When the tax deduction of the coupon is lost at some value $V_T > V_B$, the flow condition (12) becomes $dE = [C + p]dt - [d(V_B; V_B, T) + \delta V_B]dt$ at $V = V_B$.

¹⁶ Alternatively, if $V_T > V_B$, we may substitute V_B from equation (13) into equation (7) to find debt value, and use the modified expression for firm value v found in Appendix B. Equity value will equal the difference between firm value and debt value.

¹⁷ This seems reasonable by historical standards for large and relatively mature firms. For example, General Motors paid interest, preferred dividends, and common dividends averaging about 6.6 percent of market value, over the 5-year period 1988–1993. Inclusive of reduction in debt over the period, total payouts to security holders averaged 8.7 percent per year. Later we show that, in our base case, a firm issuing 20-year debt at par when $V = 100$ will have optimal debt amount of 51.5 (paying interest at 8.6 percent) and an equity value of 60.5. If dividend payouts are 4.3 percent of equity value, then total payouts are 7.0 percent of asset value. Ibbotson Associates (1994) report the average dividend yield on large stocks is 4.7 percent for the period 1926–1993, and 4.3 percent over the postwar period. While today’s dividend rates are typically lower, payouts must include stock buybacks and dividends on preferred shares as well as common share dividends.

debt is lost when all available cash flow is needed to pay interest to bond holders. This will occur at an asset value $V = V_T$ such that $\delta V_T = C$.¹⁸ When $V_B < V < V_T$, current equity holders willingly contribute to the firm to avoid default—e.g., through a dilution of their holdings resulting from further stock issuance.

At the initial asset value ($V = 100$), we assume the coupon is set so that newly-issued debt sells at par value ($d = p$). This constraint requires that $c = C/T$ be the smallest solution to the equation

$$d(V; c, p)|_{V=100} = p \quad (14)$$

where we recall $p = P/T$. It is straightforward to find solutions to equation (14) numerically.

A. Optimal Leverage

We now examine the leverage ratio that maximizes the value of the firm v for alternative choices of debt maturity. Figure 1 plots this relationship for firms issuing debt with maturities 6 months, 5 years, 20 years, and infinity, respectively, given the parameters of the base case. Observe that the leverage ratio which maximizes firm value is larger for debt with longer maturity. The maximal firm value is also greater. Table I reports optimal leverage ratios and the values of key endogenous variables at optimal leverage, including the volatility of equity and debt when $V = 100$.

Optimal leverage increases from 19 percent for 6-month debt issuance to 46 percent for 20-year debt issuance, given the parameters of the base case. This is consistent with the empirical evidence of Barclay and Smith (1995a), who find a positive correlation between leverage and debt maturity. For any maturity, the optimal leverage ratio will fall when firm risk σ and/or bankruptcy costs α increase. A fall in the default-free interest rate r (keeping asset value V constant) will also decrease optimal leverage.

Credit spreads at the optimal leverage are negligible for issuance maturities of 2 years or less.¹⁹ Credit spread at the optimal leverage rises to 110 basis points for debt with 20 years to maturity. Longstaff and Schwartz (1995) report that, over the period 1977–1992, average credit spreads on Moody's Industrial bonds ranged from 48 basis points for Aaa-rated bonds to 184 basis points for

¹⁸ This analysis does not allow tax loss carryforwards, as they would introduce a form of path dependence. Thus, we may overstate the loss of tax shields. Kim, Ramaswamy, and Sundaresan (1993) assume that bankruptcy occurs at this point, since if $V < V_T$, available cash flow is insufficient to meet interest payments.

¹⁹ Even very high quality short term debt (e.g., commercial paper rated A1+ or Prime – 1), with virtually no past defaults, still pays a significant spread over Treasury bills. This spread has been attributed to several factors, including the greater liquidity of T-bills, and the exemption of their interest from state income taxes. We believe it makes sense (at least at the short end of the maturity spectrum) to consider our credit spreads as relative to commercial paper rather than T-bills.

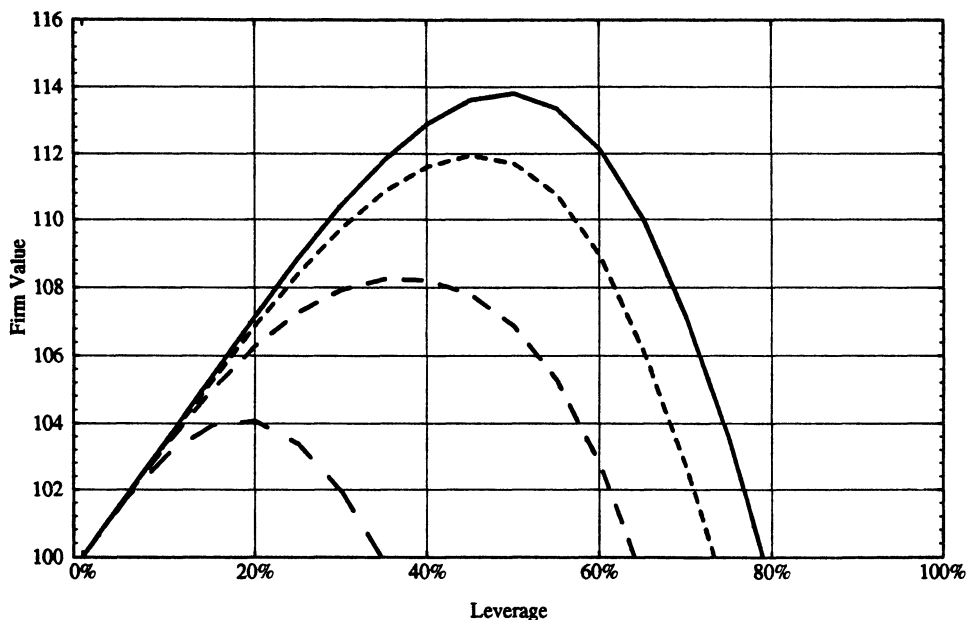


Figure 1. Firm value as a function of leverage. The lines plot the value of the firm as a function of leverage for firms issuing debt with maturity T' equal to 6 months (long dashed line), 5 years (medium dashed line), 20 years (short dashed line), and infinity (solid line), respectively. It is assumed that the riskfree interest rate $r = 7.5$ percent, the firm's payout rate $\delta = 7$ percent, the volatility of the firm's assets $\sigma = 20$ percent, the bankruptcy costs $\alpha = 50$ percent, and the corporate tax rate $\tau = 35$ percent. The value of the firm's underlying assets $V = 100$, and the bankruptcy trigger V_B is determined endogenously.

Baa-rated bonds, with an average credit spread of 109 basis points for investment-grade industrial bonds.

Writedown for debt is the fraction of the principal value of debt lost in bankruptcy, which in our model is $(1 - (1 - \alpha)V_B/P)$. For 20-year debt with a credit spread of 110 basis points (consistent with a Standard and Poor's rating between A and BBB, or a Moody's rating between A and Baa), the model implies a writedown of 65.7 percent. Altman (1991) calculates average writedowns for a sample of defaulted bonds over the period 1971–1991. Average writedowns are 52.8 percent for debt rated A when issued, 61.5 percent for BBB-rated debt, and 69.7 percent for BB-rated debt. It is difficult to verify that writedowns on short term debt are smaller, since the majority of short term debt is privately rather than publicly traded.

For firms with optimal leverage at $V = 100$, bankruptcy asset levels range from $V_B = 27.7$ for 6-month issuance maturity, to $V_B = 36.6$ for 10-year maturity. (Probabilities of bankruptcy are examined in subsection III(E) below). For short term debt, bankruptcy occurs at a higher asset level than debt principal ($V_B = 27.7 > 19.8 = P$), although after bankruptcy costs there are still substantial debt writedowns. In contrast, bankruptcy occurs at asset values considerably below principal value for long term debt. In the case of

Table I
Characteristics of Optimally Levered Firms

This table shows the characteristics of optimally levered firms issuing debt with maturities T ranging from 6 months to infinity. The average maturity of outstanding debt is $T/2$. It is assumed that the riskfree interest rate $r = 7.5$ percent, the volatility of the firm's assets $\sigma = 20$ percent, the bankruptcy costs $\alpha = 50$ percent, the corporate tax rate $\tau = 35$ percent, and the firm's payout rate $\delta = 7$ percent. The value of the firm's underlying assets $V = 100$ and the bankruptcy trigger V_B is determined endogenously.

Maturity (Years)	Coupon (Dollars)	Firm Value (Dollars)	Bankruptcy Trigger (Dollars)	Optimal Leverage (Percent)	Credit Spread, Total Debt (Basis Points)	Credit Spread, Newly Issued Debt (Basis Points)	Equity Volatility (Percent)	Total Debt Volatility (Percent)	Newly Issued Debt Volatility (Percent)
0.5	1.45	104.10	27.70	19	0	0	24.6	0.0	0.0
1.0	1.70	104.85	28.80	22	0	0	25.5	0.0	0.0
2.0	2.10	106.00	30.55	26	0	0	27.2	0.0	0.0
5.0	3.15	108.25	35.75	37	13	31	32.3	0.4	1.5
10.0	3.95	110.45	36.60	43	55	89	34.6	1.8	3.9
20.0	4.35	111.95	35.30	46	86	110	34.9	3.0	4.7
Infinity	4.80	113.80	32.80	49	107	107	34.9	4.6	4.6

Note: Dollar amounts in Table I are rounded to the nearest 5 cents.

20-year debt, $V_B = 35.3 < 51.5 = P$. Consistent with our discussion in Section II(B) above, our example illustrates a negative net worth when bankruptcy occurs with long term debt, and a positive net worth when bankruptcy occurs with short term debt.

Maximum firm value increases monotonically with debt maturity, rising from a value of 104.10 for firms using short term debt to 113.80 for firms using infinite maturity debt. Given these results, why would firms ever issue short term debt? The answer may well lie in the lower agency costs typically associated with shorter debt maturities.

In Section IV, we focus on one type of agency cost: the asset substitution problem. If equity holders benefit from increasing the risk of the firm's activities, bond holders will demand a higher coupon. We show that debt with shorter maturity reduces the incentives for equity holders to increase firm risk. Agency costs of asset substitution will therefore be lower when shorter term debt is used. Other agency costs may also be reduced by using shorter term debt—see for example Barclay and Smith (1995a).

B. Debt Value and Debt Capacity

Figure 2 plots the value of all outstanding debt D as a function of leverage (D/v) for different issuance maturities T , given that newly-issued debt sells at par. We use the parameters of the base case. In Figure 2, debt capacity is the maximal value of total debt. Note that debt capacity is smaller for shorter maturities. Debt capacity falls as volatility σ and/or bankruptcy costs α rise. Maximal debt value tends to occur at approximately the same leverage (about 80–85 percent) for debt of different maturities.

For any given maturity, the value of previously-issued debt falls as σ or the riskfree interest rate r increases, when leverage is low. However, debt value *increases* with σ and with r when leverage is (very) high and the bond is “junk.”²⁰ Leland (1994a) derives similar results for “junk” bonds, but only for the case of long term debt; short term debt (which in his model has an exogenously specified V_B) exhibits no such anomalies. Here, anomalies can occur even with short term debt. They occur principally because of endogenous bankruptcy. As r or σ increases, the endogenous bankruptcy level V_B is pushed down, providing additional “breathing room” for debt and hence increasing its value.²¹

Figure 3 shows bond values (standardized to a principal value of 100) and yields to maturity for each of a firm's outstanding bonds, whose maturities are

²⁰ Our numerical studies indicate that a firm would never optimally choose a leverage extreme enough for bonds to exhibit this behavior at issuance. However, if subsequent to issue the asset value V falls to a level close to V_B , the bond (now a “fallen angel”) could exhibit such perverse comparative statics.

²¹ Longstaff and Schwartz (1995) show that an increase in r (but not in σ) may increase risky debt value. Their explanation is different: an increase in r increases the risk neutral drift of V , and therefore reduces the (risk neutral) likelihood of hitting their exogenously-determined bankruptcy asset level V_B . Our sensitivity to changes in r will be much larger, since in addition to the drift effect, V_B will also shift downward when r increases.

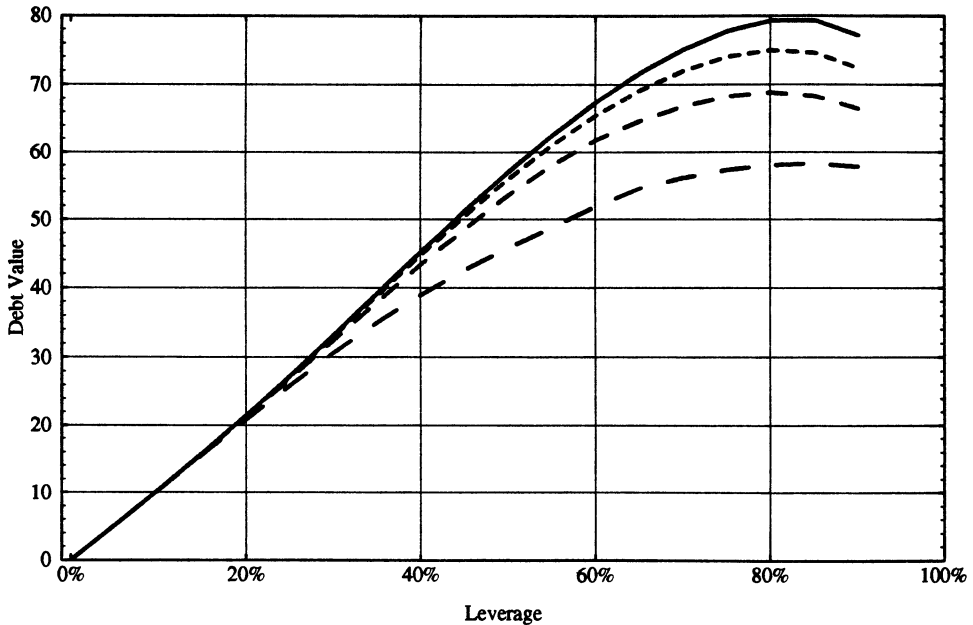


Figure 2. Debt value as a function of leverage. The lines plot the value of all outstanding bonds $D(V; V_B, T)$ as a function of leverage for firms issuing debt with maturities T equal to 6 months (long dashed line), 5 years (medium dashed line), 20 years (short dashed line), and infinity (solid line). It is assumed that the riskfree interest rate $r = 7.5$ percent, the firm's payout rate $\delta = 7$ percent, the volatility of the firm's assets $\sigma = 20$ percent, the bankruptcy costs $\alpha = 50$ percent, and the corporate tax rate $\tau = 35$ percent. The value of the firm's underlying assets $V = 100$, and the bankruptcy trigger V_B is determined endogenously.

uniformly distributed between zero (those about to be redeemed) and T (those that have just been issued). Three levels of leverage are considered: 30, 50, and 70 percent. Panels A and C refer to bonds with issuance maturity $T = 5$; Panels B and D to bonds with issuance maturity $T = 20$ years. Low and intermediate leverages show “humped” market values: newly-issued bonds (by construction) and about-to-be-redeemed bonds sell at par; bonds with remaining maturity between 0 and T sell above par. The hump is more pronounced for greater leverage levels and longer maturities. Yields to maturity of these bonds increase with their time to maturity.

Very risky bonds show a different price pattern. While newly-issued bonds and about-to-be-redeemed bonds sell at par, bonds with short remaining maturity sell substantially above par, while bonds of longer maturity (but less than T) sell below par. This behavior is more pronounced for longer term debt, and reflects the interplay between high coupon rates, the likelihood of bankruptcy, and time remaining to maturity. Yields to maturity of bonds in high-leverage firms increase rapidly with time to maturity initially, but then level off and may eventually fall as time to maturity approaches T .

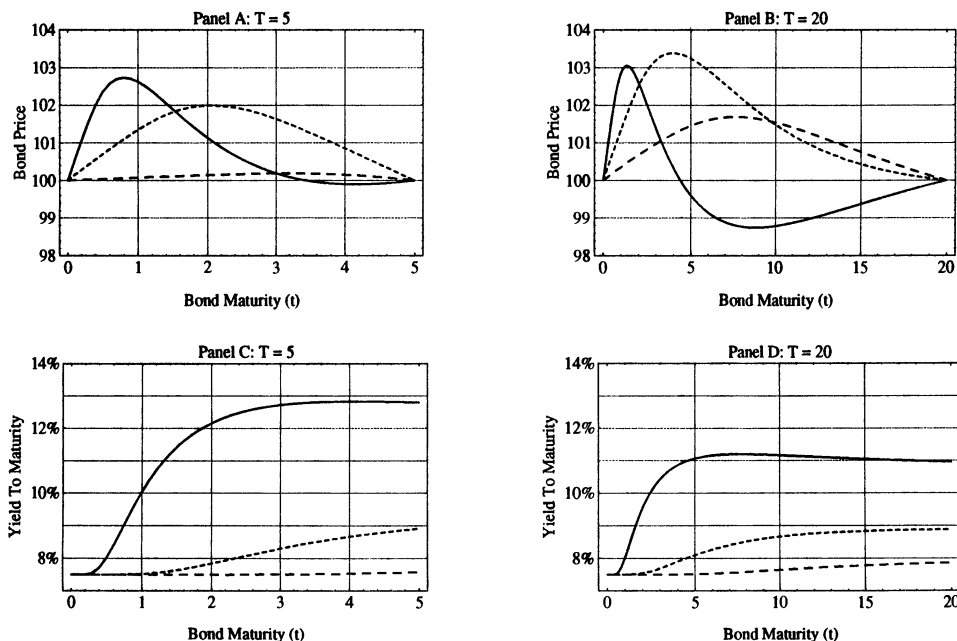


Figure 3. Bond price per \$100 face value and yield to maturity as functions of time to maturity. The plots examine bond prices per \$100 face value ($100d(t)/p(t)$) and corresponding yields to maturity as functions of bond maturity t . The debt issues illustrated in panels A and C are from a capital structure where new debt is issued with a maturity $T = 5$ years, while panels B and D plot bond prices and yields to maturity for debt in a firm which issues 20-year debt. Each panel plots bond prices or yields for 30 percent (medium dashed line), 50 percent (short dashed line), and 70 percent levered firms (solid line). It is assumed that the riskfree interest rate $r = 7.5$ percent, the firm's payout rate $\delta = 7$ percent, the volatility of the firm's assets $\sigma = 20$ percent, the bankruptcy costs $\alpha = 50$ percent, and the corporate tax rate $\tau = 35$ percent. The value of the firm's underlying assets $V = 100$, and the bankruptcy trigger V_B is determined endogenously.

C. The Term Structure of Credit Spreads

Figure 4 examines credit spreads ($c/d - r$) of newly-issued debt as a function of issuance maturity T , for alternative leverage ratios. For high leverage levels, spreads are high, but decrease as issuance maturity T increases beyond 1 year. For moderate-to-high leverage levels, spreads are distinctly humped: intermediate term debt offers higher yields than either very short or very long term debt. Finally, for firms that have low leverage, credit spreads are low but increase with issuance maturity T . These patterns are also observed in Merton's (1974) model of zero coupon debt and in Longstaff and Schwartz (1995). Such behavior has been verified empirically by Sarig and Warga (1989).²²

²² With high leverage, the term structure of yield spreads decreases from a peak at very short maturity, but still exhibits a "humped" shape. This is because as $T \rightarrow 0$ debt becomes riskless, since $V > V_B \rightarrow P/(1 - \alpha)$, and the firm remains solvent, even in the limit. In Merton (1974), the

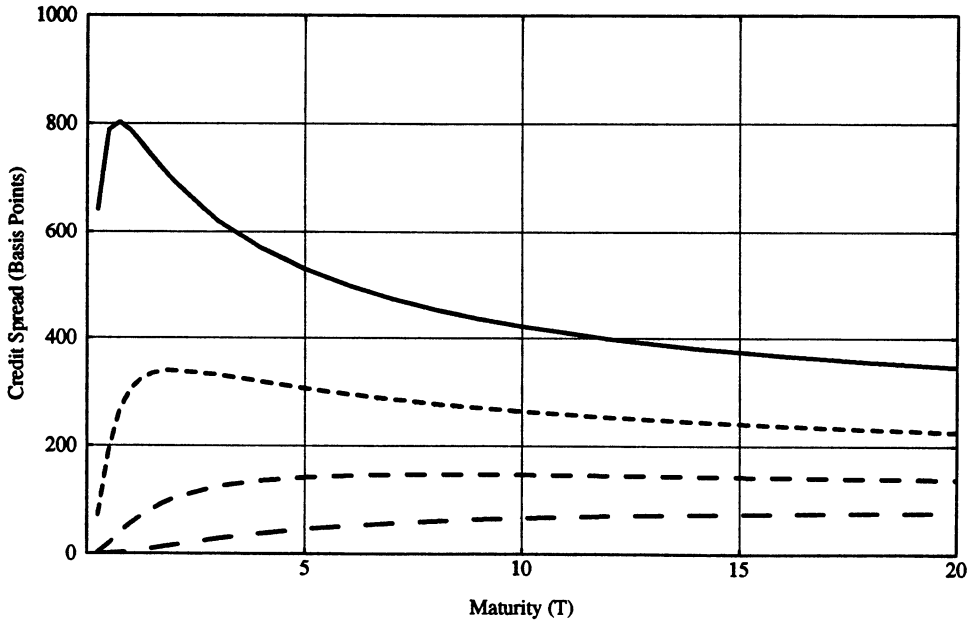


Figure 4. Credit spread as a function of debt maturity. The lines plot the credit spread (in basis points) of newly issued bonds (issued at par) as a function of maturity T for firms with leverage ratios of 40 percent (long dashed line), 50 percent (medium dashed line), 60 percent (short dashed line), and 70 percent (solid line). It is assumed that the riskfree interest rate $r = 7.5$ percent, the firm's payout rate $\delta = 7$ percent, the volatility of the firm's assets $\sigma = 20$ percent, the bankruptcy costs $\alpha = 50$ percent, and the corporate tax rate $\tau = 35$ percent. The value of the firm's underlying assets $V = 100$, and the bankruptcy trigger V_B is determined endogenously.

We now turn to the behavior of the term structure of credit spreads as asset risk, default-free interest rates, and bankruptcy costs change. Table II considers credit spreads for firms initially at optimal leverage levels. Panel A examines the effect of parametric changes on spreads for three different issuance maturities. It assumes that the coupon and principal of the outstanding bonds remain fixed at their optimal levels for the base case, but that V_B changes endogenously to reflect the altered parameters. (Note that bonds will no longer sell at par after the parameter shifts). Panel B considers the same parametric changes as Panel A, but assumes that V_B remains fixed.²³ Comparing the results in Panel A with Panel B therefore shows the effects of endogenous versus exogenous bankruptcy on sensitivities to parameter changes.

Optimal debt is essentially riskless for very short maturities, and the comparative statics of credit spreads are not of great interest in this case. Inter-

yield spread approaches infinity as maturity approaches zero in the case where $P/V > 1$. But this case implies that the firm is insolvent in the limit as $T \rightarrow 0$, since $V_B \rightarrow P > V$.

²³ We omit reporting situations where the fixed V_B is less than the endogenously determined V_B after the parameter change. Recall that the endogenous V_B is the lowest possible while preserving the limited liability of equity. A lower V_B would imply negative equity values when V is close to V_B .

mediate and long term debt, however, always pay lower credit spreads in Panel A than in Panel B. This is because the endogenous V_B is the lowest possible level subject to the limited liability of equity.²⁴ A higher V_B generates lower values for *both* debt and equity.

Credit spreads fall when the default-free interest rate rises—a somewhat unexpected result, although previously observed in the models of Kim, Ramaswamy, and Sundaresan (1993), Leland (1994a), and Longstaff and Schwartz (1995).²⁵ The latter provide some empirical evidence supporting this behavior. In cases of extreme leverage, our model predicts that bond prices can *increase* when default-free interest rates rise, and when asset risk rises. The latter result follows from the endogeneity of V_B .

Panel C of Table II considers a different comparative statics experiment: exogenous parameter changes *before* debt is issued. Thus P and C as well as V_B are adjusted in response to the parameter change. The comparative statics are somewhat different. For newly-issued debt, a rise in riskless rates will “tilt” credit spreads: a rise in the default-free rate will increase credit spreads of shorter term debt, but decrease spreads of long term debt. Lower bankruptcy costs lead to the use of particularly greater leverage for intermediate term debt, thereby increasing yield spreads for optimal debt with this maturity.

D. The Duration and Convexity of Risky Debt

Macaulay (1938) duration measures percent change of a bond price in response to a uniform change in default-free interest rates—for bonds with no default risk. A critical question follows: How sensitive are correct measures of duration to the presence of default risk? By “correct” measure of duration, we simply mean an expression that correctly predicts the percentage change in the (risky) bond value in response to a change in the default-free interest rate.

If the bond with maturity T sells at par, and R denotes the yield ($R = c/p$), the Macaulay duration Dur of the bond is

$$Dur = \frac{\int_{s=0}^T s e^{-Rs} c \, ds + T e^{-RT} p}{\int_{s=0}^T e^{-Rs} c \, ds + e^{-RT} p} = \frac{1 - e^{-RT}}{R}$$

Using the base case parameters, we compute the percent change in the value of a newly issued bond with maturity T , for a 1 percent change in the default-free interest rate r . This is the true or “effective” duration of the risky bond. Figure 5 plots this effective duration against the Macaulay duration, for bonds with different maturities and credit spreads. When the credit spread is 10 basis points (bps) or less, effective duration almost exactly equals Macaulay

²⁴ Notice that, in comparing exogenous and endogenous bankruptcy, we have only examined parametric changes that decrease V_B . Parametric changes that raise V_B imply the initial V_B could not be maintained without violating the limited liability of equity.

²⁵ Some caution in interpreting these results is warranted. While we have performed the standard ceteris paribus comparative statics, it should be observed that V may itself change with changes in the default-free interest rate.

Table II

Credit Spread and Endogenous Bankruptcy Trigger V_B for Various Parameter Values—Optimal Leverage
This table shows credit spreads (in basis points) and endogenous bankruptcy triggers for newly issued debt with 6 months, 5 years, and 20 years to maturity. In Panel A, the amount of outstanding debt is determined optimally for the base parameter case ($V = 100$, $r = 7.5$ percent, $\delta = 7$ percent, $\sigma = 20$ percent, $\alpha = 50$ percent, and $\tau = 35$ percent) and bankruptcy is determined endogenously subsequent to the parameter change. In Panel B, V_B is kept constant at the endogenous value for the base case parameters. In Panel C, both leverage and V_B are determined endogenously subsequent to the parameter change.

	Panel A: Fixed P and C			Panel B: Fixed P , C , and V_B			Panel C: Optimal P , C , and V_B		
	$T = 0.5$	$T = 5$	$T = 20$	$T = 0.5$	$T = 5$	$T = 20$	$T = 0.5$	$T = 5$	$T = 20$
Base case parameters									
Credit spread	0.00	31.27	110.10	0.00	31.27	110.10	0.00	31.27	110.10
V_B	27.70	35.75	35.32	27.70	35.75	35.32	27.70	35.75	35.32
Higher volatility									
Credit spread	0.00	86.74	178.09	0.00	103.94	196.41	0.00	52.63	149.36
V_B	26.98	33.72	32.48	27.70	35.75	35.32	20.47	29.88	29.32
Higher riskfree rate									
Credit spread	0.00	10.23	39.56	0.00	16.03	48.93	0.00	42.54	66.22
V_B	26.15	33.03	32.14	27.70	35.75	35.32	39.64	42.10	38.67
Lower bankruptcy costs									
Credit spread	0.00	11.52	77.54	0.00	18.83	81.90	0.00	66.86	109.94
V_B	20.94	31.83	33.80	27.70	35.75	35.32	38.27	43.92	39.63

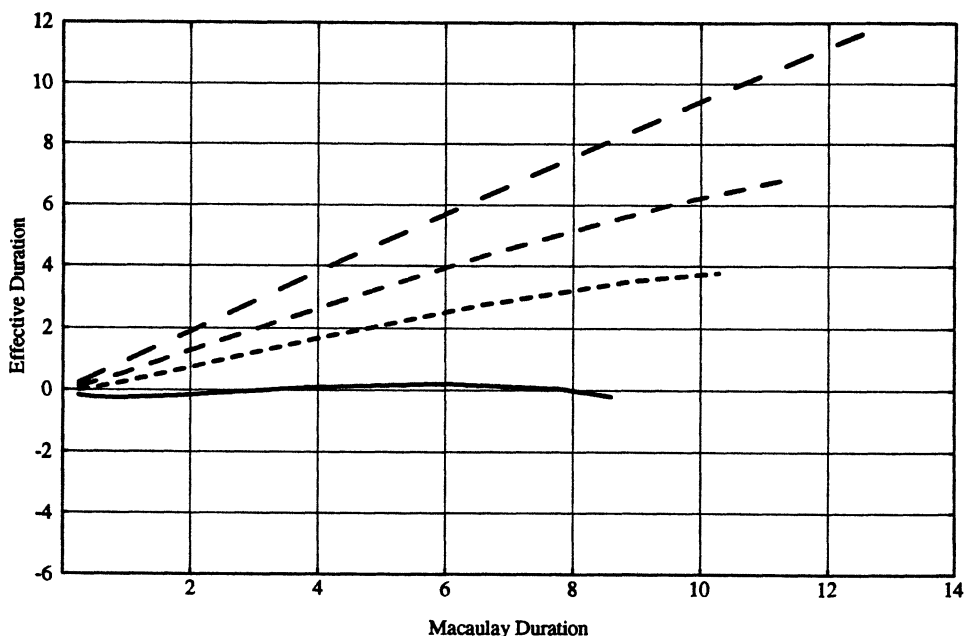


Figure 5. Effective duration as a function of Macaulay duration. The lines plot the effective duration of newly issued bonds (issued at par) as a function of these bond's Macaulay duration for bonds with credit spreads of 10 basis points (long dashed line), 100 basis points (medium dashed line), 200 basis points (short dashed line), and 400 basis points (solid line). It is assumed that the riskfree interest rate $r = 7.5$ percent, the firm's payout rate $\delta = 7$ percent, the volatility of the firm's assets $\sigma = 20$ percent, the bankruptcy costs $\alpha = 50$ percent, and the corporate tax rate $\tau = 35$ percent. The value of the firm's underlying assets $V = 100$, and the bankruptcy trigger V_B is determined endogenously.

duration. As leverage becomes larger, and credit spreads increase, effective duration becomes much shorter than Macaulay duration: with a credit spread of 200 bps, debt of 20-year maturity has Macaulay duration of about 6.5 years, but effective duration is only 2.7 years. When credit spreads exceed 400 bps, effective duration of shorter term debt becomes negative: as the riskfree rate rises, so do bond prices. Such dramatic differences in effective duration versus Macaulay duration suggest that immunization and related techniques using corporate bonds must explicitly reflect actual bond risk, and not rely on traditional duration-matching methods.

Riskless debt value is a convex function of the default-free interest rate r . Convexity is critical for managing a duration-matching strategy. A dynamic strategy must be followed, because duration increases as interest rates fall. However, if debt values were concave rather than convex in the riskfree interest rate, the *opposite* kind of dynamic hedging would be required. We find that the riskiness of debt, as well as its maturity, affects convexity. Figure 6 examines debt value as a function of the riskfree interest rate r , for credit

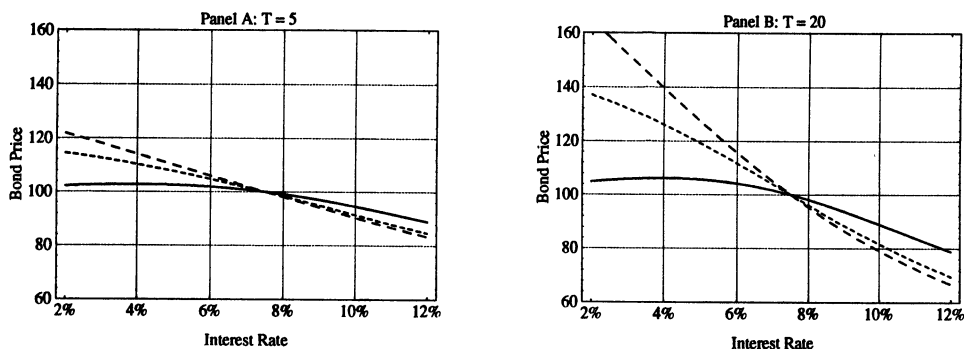


Figure 6. Bond price per \$100 face value as a function of interest rates. The plots examine bond prices per \$100 face value ($100d(T)/p(T)$) as functions of the riskfree rate of interest. The bonds shown in panel A are newly issued and have a maturity $T = 5$ years, while the bonds in panel B have maturity $T = 20$ years. The newly issued bonds are priced at par and they carry credit spreads of 10 basis points (medium dashed line), 50 basis points (short dashed line) and 200 basis points (solid line), when $r = 7.5$ percent. It is assumed that the firm's payout rate $\delta = 7$ percent, the volatility of the firm's assets $\sigma = 20$ percent, the bankruptcy costs $\alpha = 50$ percent, and the corporate tax rate $\tau = 35$ percent. The value of the firm's underlying assets $V = 100$, and the bankruptcy trigger V_B is determined endogenously.

spreads of 10, 50, and 200 basis points. Panels A and B examine debt with maturities of 5 years and 20 years, respectively.

The changes in convexity as credit spreads increase are pronounced, particularly for long term debt. As debt becomes increasingly risky, convexity is reduced and ultimately turns to concavity. The degree of concavity is most pronounced at lower interest rates. This again shows that hedging risky debt requires quite different strategies from those used to hedge riskfree debt.

E. Bankruptcy Rates and Bond Ratings

The prediction of default rates is important for bond ratings. Our model allows straightforward estimations of default rates.²⁶ The cumulative probability of the firm going bankrupt over the period $(0, s]$ is given by

$$N\left(\frac{-b - \lambda s}{\sigma \sqrt{s}}\right) + e^{-2\lambda b/\sigma^2} N\left(\frac{-b + \lambda s}{\sigma \sqrt{s}}\right) \quad (15)$$

where $\lambda = \mu - \delta - .5\sigma^2$.

Figure 7 plots the cumulative probability of bankruptcy over a 25 year period, for firms issuing an optimal amount of debt with maturities of 6 months, 5 years, and 20 years. Base case parameters are assumed, and μ (the

²⁶ Alternative models of the stochastic process leading to bankruptcy have been provided by Jarrow, Lando, and Turnbull (1993) and by Madan and Unal (1994). The former considers an exogenous finite state Markov process for the evolution of a firm's bond rating, with transition probabilities between ratings scaled to historical levels. The latter considers a jump process into bankruptcy, with a second process determining bankruptcy costs.

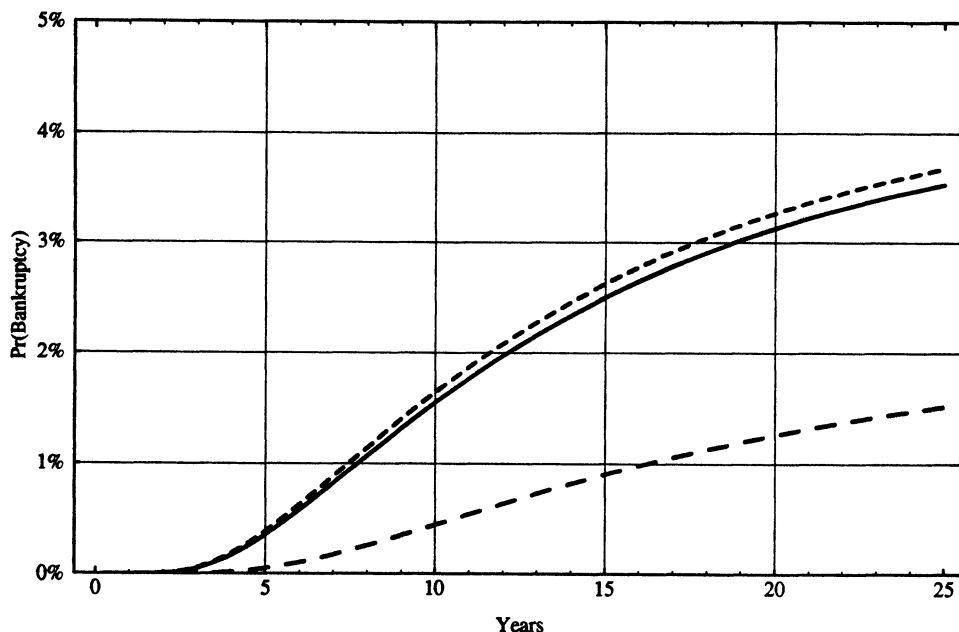


Figure 7. Cumulative bankruptcy probabilities. The lines plot optimally leveraged firms' cumulative bankruptcy probabilities as functions of time. Bankruptcy probabilities are shown for firms issuing debt with maturities equal to 6 months (medium dashed line), 5 years (short dashed line), and 20 years (solid line). The leverage ratios for these three capital structures are 19 percent, 37 percent, and 46 percent, respectively. It is assumed that the drift rate $\mu = 15$ percent, the riskfree interest rate $r = 7.5$ percent, the firm's payout rate $\delta = 7$ percent, the volatility of the firm's assets $\sigma = 20$ percent, the bankruptcy costs $\alpha = 50$ percent, and the corporate tax rate $\tau = 35$ percent. The value of the firm's underlying assets $V = 100$, and the bankruptcy trigger V_B is determined endogenously.

mean total rate of return of V) is assumed to be 15 percent per year, a risk premium of 7.5 percent over the default-free interest rate.

Optimal long term debt has a cumulative probability of bankruptcy which is negligible over the first 3 years, reaches about 1.5 percent after 10 years, and approximately 3.1 percent after 20 years. Thus, approximately 3.1 percent of debt which is issued with a 20-year maturity and pays 110 bps over the riskfree rate will default over a 20-year period.²⁷ Optimal intermediate term (5-year) debt, while offering a substantially lower credit spread than long term debt, has slightly higher cumulative bankruptcy probabilities. This is because V_B is marginally higher for the 5-year debt, as can be seen from Table I.²⁸

²⁷ The cumulative probability of default is quite dependent on the drift μ assumed for the asset process V . For example, if $\mu = 0.125$ (rather than 0.15), the probability of default of optimal 20-year debt over a 20-year horizon would be about 8.3 percent rather than 3.1 percent.

²⁸ Smaller credit spreads might appear to imply smaller cumulative probabilities of bankruptcy. For given asset risk and drift, the sole determinant of cumulative bankruptcy probabilities is V_B .

Optimal short term debt has virtually zero credit spread, and has lower bankruptcy probabilities even when rolled over for extended time periods.

Altman (1991) examines long term bond defaults over a ten-year period subsequent to issuance. A-rated bonds have a cumulative default probability of 0.67 percent after five years and 1.20 percent after 10 years. BBB-rated bonds have a cumulative default probability of 1.70 percent after five years and 3.98 percent after 10 years. These default rates span the default rates seen for long term debt in Figure 7.

In principle, our techniques could be used to produce bond ratings themselves. An important question is “what are we trying to measure?” with a bond rating. Is it probability of default during the debt’s life, or credit spread? Credit spread seems the more important variable to predict when market prices are unavailable. While these two are related, the relation is complex. Predicted credit spreads reflect exogenous variables such as current asset value, risk, debt maturity, bankruptcy costs, payout rate, and the default-free interest rate, as well as the (total) bond coupon and principal. Bankruptcy probabilities depend upon the drift of the asset value V as well.

Current bond rating methodologies focus on flow measures, such as interest coverage ratios. The way in which these ratios interact with exogenous parameters such as asset risk to determine ratings is somewhat murky. At first glance, our approach, which focuses on values, seems quite different from commercial bond-rating approaches, which focus on ratios of flows. However, recall that our equations for the value of debt and for V_B are homogeneous of first degree in V , C , and P . This allows V_B/V and D/V (as well as credit spreads) to be expressed in terms of the ratios C/V and P/V . If we consider δV as a proxy for cash flow (or some constant fraction thereof), credit spreads and bankruptcy probabilities can now be rewritten as functions of $C/\delta V$ and $(P/T)/\delta V$. $C/\delta V$ is simply the inverse of “times interest covered.” $(P/T)/\delta V$ is the ratio of long-term debt becoming a current liability, to cash flow.

Credit spreads and bankruptcy probabilities therefore can be expressed as functions of flow ratios. Our analysis is not as different from the ratio analysis used by rating agencies as it first appeared. In fact, our analysis shows precisely *how* these ratios should interact with one another and with the exogenous parameters to determine credit spreads and bankruptcy probabilities. The results may be viewed as formalizations of bond-rating methodologies.

IV. Agency Effects: Debt Maturity and Asset Substitution

Since Black and Scholes (1973) and Jensen and Meckling (1976), it has been a tenet of financial economics that, after debt is issued, equity holders will wish to increase the riskiness of the firm’s activities. This is presumed to transfer value from debt to equity, creating the “asset substitution” problem. This

Nonetheless, shorter term debt can have smaller credit spreads than long term debt, despite having a higher V_B , because the chances of reaching V_B *before maturity* is less.

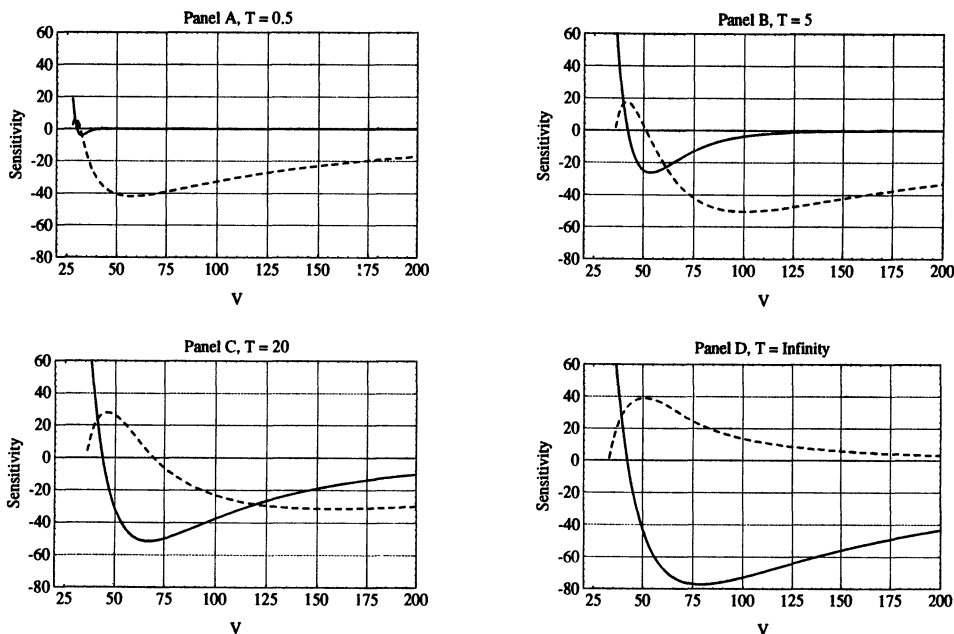


Figure 8. Effects of an increase in risk σ on bond and equity values. The panels plot the partial derivative of the equity value (dashed line) and the partial derivative of the value of all outstanding debt (solid line) with respect to risk, σ , as a function of underlying asset value. The maturity T of newly issued debt equals 6 months (panel A), 5 years (panel B), 20 years (panel C), and infinity (panel D). The total annual coupon payment is determined such that the capital structure is optimal when the firm value $V = 100$. These optimal annual coupon payments are 1.45, 3.15, 4.35, and 4.80 for panels A, B, C, and D, respectively. It is assumed that the riskfree interest rate $r = 7.5$ percent, the firm's payout rate $\delta = 7$ percent, the volatility of the firm's assets $\sigma = 20$ percent, the bankruptcy costs $\alpha = 50$ percent, and the corporate tax rate $\tau = 35$ percent. The bankruptcy trigger V_B is determined endogenously.

presumption follows from regarding equity as a call option on the firm's assets, as indeed is the case when debt has no coupon, and taxes and bankruptcy costs are ignored—the case studied by Merton (1974). Barnea, Haugen, and Senbet (1980) further explore this analogy, and suggest that shorter term debt may reduce shareholder incentives to increase risk.

Equity in our model, however, is not precisely analogous to an ordinary call option. First, default may occur at any time—not only at debt maturity. The bankruptcy value V_B , roughly analogous to the option's strike price, varies with the risk of the firm's activities. Most importantly, the existence of tax benefits (and their potential loss in bankruptcy) implies that debt and equity holders do not split a claim whose value depends only on the underlying asset value.²⁹

Figure 8 plots the partial derivatives of equity value and debt value with

²⁹ It has been brought to our attention that many of these reasons were anticipated by Long (1974).

respect to changes in asset risk σ , as the underlying asset value V changes. Panels 8A–8D plot these partial derivatives for optimal debt levels (when $V = 100$) with issuance maturities of 6 months, 5 years, 20 years, and infinity. The dashed line maps $\partial E/\partial \sigma$; the solid line maps $\partial D/\partial \sigma$. As $V \rightarrow \infty$, debt becomes risk free, and the partial derivative of debt with respect to risk σ approaches zero from below. Observe that

- (i) For either short term or intermediate term debt, increasing risk will benefit neither bond holders *nor* shareholders, except when bankruptcy is imminent.
- (ii) The incentives for increasing risk are much more pronounced for longer term debt. For very long term debt, $\partial E/\partial \sigma > 0$ for all asset values.
- (iii) At all maturities, the incentives for increasing risk become positive for both equity holders and bond holders, as bankruptcy V_B is approached. However, incentives to increase risk become positive for equity holders before they become positive for bond holders.

The incentive compatibility problem exists only for the range of V for which $\partial E/\partial \sigma > 0$, and $\partial D/\partial \sigma < 0$. For the optimal amount of short term debt ($T = 6$ months), this range is minuscule. For intermediate term debt ($T = 5.0$ years), the range is approximately $42 < V < 51$. The range extends to $44 < V < 69$ with 20-year maturity debt; for very long term debt ($T = \infty$) the asset substitution problem exists whenever $V > 43$.³⁰ Thus, while the option analogy is not exact, the conclusion that adverse incentives are greater with longer term debt remains correct.

The existence of potential agency costs implies that firms with higher asset risk σ will shorten their optimal debt *maturity* as well as decrease their optimal amount of debt. This is because, for higher risk σ , the increase in value from optimally using long term versus short term debt falls, while the agency costs of using long term debt are greater. Both effects suggest that optimal debt maturity as well as amount should be reduced when risk increases.

Firms with higher bankruptcy costs α will also choose lower optimal amounts of debt, for any given maturity. But they will *lengthen* the maturity of debt in the presence of asset substitution agency costs. Here the effects are the opposite of those when firm risk rises: The incremental firm value from using long term debt versus short term debt increases with α , while agency costs associated with asset substitution fall. Both effects indicate that optimal debt maturity should be raised when bankruptcy costs increase. This is not entirely surprising, since longer maturity tends to reduce V_B and thereby serves to postpone bankruptcy and its associated costs.

Firms with a lower payout rate δ should have a lower optimal amount of long term debt, but a higher optimal amount of short term debt. The incremental

³⁰ A possible relative measure of the asset-substitution agency costs is the Lebesgue measure of the set of values V over which equity holders will wish to raise risk at bond holders' expense. A different measure, yielding similar conclusions, would be the value of a claim paying a unit cash flow per year in the region where debt holders and equity holders have conflicting interests.

firm value using long versus short term debt is significantly reduced when δ falls. If other agency costs remain the same or increase, as Barclay and Smith (1995a) suggest, the optimal maturity of debt will be shortened. However, the incremental agency costs associated with asset substitution alone are lower for firms with lower payout rates. In this environment, our theory cannot predict whether optimal maturity should be shortened when δ falls.

Firms with greater growth prospects typically have lower cash flows (as a fraction of asset value) available for payout to security holders, greater risk, and higher bankruptcy costs. Our model predicts that such firms should have less debt in their optimal capital structure. Our prediction for optimal maturity is less certain, when asset substitution is the only agency cost considered. Higher risk suggests shorter debt maturity; higher bankruptcy costs suggest longer debt maturity. Lower cash flows available for payout have an indeterminate effect.

Barclay and Smith's (1995a) empirical results show that firms with greater growth prospects do use shorter term debt. This is consistent with our results if the most important characteristic of high growth firms is their greater volatility—or if, as seems likely, there are additional agency costs to long term debt besides those of asset substitution.³¹

Finally, a lower default-free interest rate r will reduce both the amount and maturity of optimal debt. (Recall that the comparative statics experiment presumes asset value V remains unchanged.) Lower required coupons will reduce the tax deduction of debt and the optimal degree of leverage.³² They will also diminish the value advantage of long term versus short term debt, while raising agency costs. This will lead to shorter term debt being substituted for longer term debt.

The extent of conflict between equity holders and bond holders increases when tax rates τ and bankruptcy costs α decline. This is because outside parties have less claim on firm values, and the “game” between bond holders and equity holders approaches a zero sum game. Figure 9 illustrates the effect of increasing risk on stock and bond values when both $\alpha = 0$ and $\tau = 0$, and the firm is 50 percent leveraged. Panel A considers firms issuing 6-month debt; Panel B examines a firm issuing 20-year debt. In both cases there is a direct conflict between bond holders and equity holders, although the magnitude of the problem is significantly reduced when short term debt is used.

Figure 10 details how values of all outstanding bonds react differentially to increases in firm risk, in the base case scenario with optimal leverage. Panel A considers a firm issuing optimal debt with 5 years maturity; Panel B

³¹ For example, Mello and Parsons (1992) consider the agency costs of debt with respect to the investment and operating decisions of a mining firm.

³² It may seem counterintuitive that lower interest rates should lead to less debt, since the cost of debt financing is less. But recall that in the “pure” Modigliani and Miller (1958) world with no taxes or bankruptcy costs, financial structure is irrelevant: lower costs of debt financing will not induce preference for debt. Thus the changing costs of debt financing are important only for their impact on tax deductions and bankruptcy costs. We have shown that lower interest rates make debt less desirable in these dimensions.

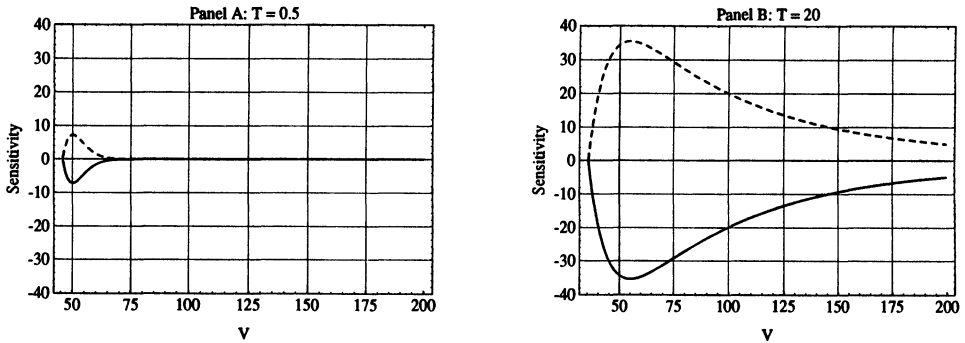


Figure 9. Effects of an increase in risk σ on bond and equity values—zero taxes and bankruptcy costs. Panels A and B plot the partial derivative of equity value (dashed line) and the partial derivative of the value of all outstanding debt (solid line) with respect to risk, σ , as a function of underlying asset value. The maturity T of newly issued debt equals 6 months and 20 years for panels A and B, respectively. The total annual coupon payments are determined such that the firms are 50 percent leveraged. It is assumed that the riskfree interest rate $r = 7.5$ percent, the firm's payout rate $\delta = 7$ percent, the volatility of the firm's assets $\sigma = 20$ percent, the bankruptcy costs $\alpha = 0$ percent, and the corporate tax rate $\tau = 0$ percent. The bankruptcy trigger V_B is determined endogenously.

examines a firm issuing optimal 20-year debt. In both cases, the sensitivity of outstanding bond values to changes in firm risk rises with debt maturity, up to the issuance maturity T . Furthermore, near bankruptcy, outstanding bonds with a longer time to maturity have a greater window of incentive incompatibility with equity—the interval V for which $\partial E / \partial \sigma > 0$ and $\partial d / \partial \sigma < 0$ —than outstanding bonds with a shorter time to maturity. Near bankruptcy it is possible that both share holders and bond holders with bonds about to mature wish to increase firm risk, while longer term bond holders do not want risk to increase.

In sum, these results suggest that the “general” asset substitution problem may have been overstated, except when debt is very long term, or when taxes and bankruptcy costs are minimal. The results illustrate that incentive incompatibilities arise as bankruptcy is approached. On the very brink of bankruptcy, incentive compatibility is again restored: both stock and bond holders want to raise risks to avoid bankruptcy costs and preserve the (potential) tax shelters for debt.

V. Multiple Classes and Seniorities of Debt

Multiple debt securities, with differing maturity and seniority, can be brought within the framework developed here. For example, we could consider a firm that simultaneously issues (and rolls over, as above) 5-year debt and 20-year debt, with the former being senior. Each different debt issue will be valued according to equation (3), with the amount received by each class in

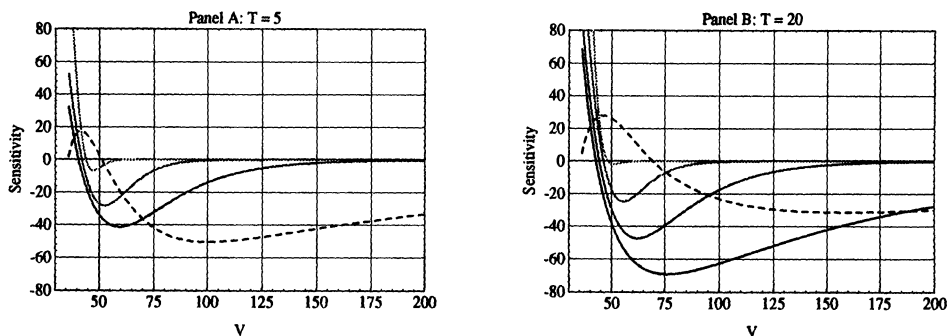


Figure 10. Effects of an increase in risk σ on equity values and bond prices of various maturities outstanding. The panels plot the partial derivative of equity (dotted line) and the partial derivatives of the values of the individual issues of debt (solid lines) with respect changes in risk, σ , as a function of underlying asset value (the bond prices are scaled to facilitate comparisons with Figures 8 and 9). Panel A illustrates the substitution effect for a firm with newly issued debt maturity $T = 5$ years, while the firm in panel B has newly issued debt with a maximum maturity $T = 20$ years. Panel A plots the sensitivities for 6-month (light gray line), 2-year (gray line), and 5-year debt (dark gray line) while panel B also shows the sensitivity for 20-year (black line) debt. The total annual coupon payments are determined such that the capital structures are optimal when the firm value $V = 100$. These optimal coupon payments are 3.15 per year for panel A and 4.35 per year for panel B, respectively. It is assumed that the riskfree interest rate $r = 7.5$ percent, the firm's payout rate $\delta = 7$ percent, the volatility of the firm's assets $\sigma = 20$ percent, the bankruptcy costs $\alpha = 50$ percent, and the corporate tax rate $\tau = 35$ percent. The bankruptcy trigger V_B is determined endogenously.

bankruptcy reflecting its seniority.³³ The asset value V_B that triggers bankruptcy must satisfy the smooth-pasting condition for equity, which will reflect the total value of the firm from equation (8) less the *sum* of debt values. The framework developed in this paper may therefore be used to analyze optimal capital structure and yield spreads of firms with much more complex capital structures.

In principle, sinking fund schedules could also be included (e.g., a constant amount of principal of all outstanding bonds is retired each moment), as long as new debt is issued at market value in order to keep total coupon payments and principal amounts constant. Leland (1994b) develops a simple model similar to Leland (1994a), in which infinite life debt is continuously issued, and subsequently retired at a proportional rate through a sinking fund. By choosing the proportional rate of retirement, the average maturity of debt can be made to match the average maturity of any given capital structure. The resulting formulas for leverage and bond prices are relatively simple, while preserving much of the richness of behavior observed with the model presented here.

³³ The net bankruptcy proceeds $((1 - \alpha)V_B)$ will be divided among debtholders according to their seniority. If all debt is of equal seniority, each class would receive a fraction of net bankruptcy proceeds—for example, a fraction equal to its share of total debt principal.

VI. Conclusions

This article develops a model of optimal leverage and risky corporate bond prices for arbitrary debt maturity. Bankruptcy is determined endogenously and will depend on the maturity of debt as well as its amount. Both value and flow conditions that characterize the bankruptcy point are presented. They show that bankruptcy can occur at asset values that may be either lower or higher than the principal value of debt. And a cash flow shortfall relative to required debt service payments need not result in default—it may be optimal for equity holders to raise further funds to avoid bankruptcy.

Optimal leverage depends upon debt maturity, and is markedly lower when the firm is financed by shorter term debt. Credit spreads increase with maturity up to 20 years at the optimal leverage ratio. This confirms empirical evidence that firms tend to issue higher-rated short term debt than long term debt.

The fact that longer term debt generates higher firm value poses the question of why firms issue short term debt. One answer to this question is that short term debt reduces agency costs. We explicitly consider the problem of asset substitution. In contrast with conventional wisdom, we find that equity holders of firms issuing short term debt generally will *not* have an incentive to raise firm risk. Consequently, short term debt holders do not have to protect themselves from misincentives by demanding higher coupon rates, and equity holders will benefit.

The choice of debt maturity therefore represents a tradeoff between tax advantages, bankruptcy costs, and agency costs. Our results suggest that, in the presence of agency costs, riskier firms should issue shorter term debt in addition to using less debt. A firm with higher bankruptcy costs will prefer longer term debt. The tax advantages of long term debt are significantly reduced when cash flows relative to asset value are small, as may be the case for firms with high growth opportunities. In the presence of agency costs beyond asset substitution, higher-growth firms will want to use shorter term debt, an important empirical finding of Barclay and Smith (1995a).

The model relates bond values to firm value, asset risk, leverage, bankruptcy costs, tax rates, total payout rate, and the default-free interest rate. We find that risky corporate debt behaves very differently from default-free debt. Effective duration may be far shorter than Macaulay duration—it may even become negative. Convexity can become concavity. Proper hedging techniques for risky fixed income portfolios must therefore explicitly consider potential default risks, even when default is not imminent.

Predicted term structures of credit spreads are similar to those which have been observed empirically by Sarig and Warga (1989). Surprisingly, credit spreads of debt decrease as riskless rates rise. Longstaff and Schwartz (1995) derive a similar result, and provide empirical evidence that the prediction is correct.

Our techniques allow computation of the default probabilities, given the actual (not risk-neutral) drift of the asset value process. When cash flows are

proportional to asset value, our predictions of credit spreads and default probabilities can be expressed as functions of traditional financial ratios, and thus can be viewed as a formalization of standard bond-rating approaches. Predictions of bankruptcy rates and asset writedowns in default are generally consistent with Altman's (1991) findings.

The model is simple, yet robust. The default-free interest rate is assumed to be constant, and the firm always replaces retired debt with the same amount of new debt—the same coupon, and the same principal. Despite these simplifications, our model provides predictions of optimal leverage ratios, equity risk, credit spreads, bankruptcy probabilities, writedowns in bankruptcy, and dividend yields that are consistent with long term historical levels.

Appendix A. Derivation of Expressions for $I(T)$ and $J(T)$.

First, consider the integral $I(T)$:

$$\begin{aligned} I(T) &= \frac{1}{T} \int_0^T \exp(-rt)F(t) dt = \frac{1}{rT} \int_0^T \exp(-rt)f(t) dt - \frac{1}{rT} \exp(-rT)F(T) \\ &= \frac{1}{rT} [G(T) - \exp(-rT)F(T)], \end{aligned}$$

where all definitions are given in the main body of the text. The second equality is derived by integration by parts while the last simplification is obtained from the definition of $G(T)$.

Second, consider the integral $J(T)$:

$$\begin{aligned} J(T) &= \frac{1}{T} \int_0^T G(t) dt = G(T) - \frac{1}{T} \int_0^T t \exp(-rt)f(t) dt \\ &= G(T) - \frac{1}{T} \int_0^T \frac{t \exp(-rt)b}{\sigma \sqrt{t} 2\pi} \exp\left(-\frac{1}{2}\left(\frac{b + \eta t}{\sigma \sqrt{t}}\right)^2\right) dt \\ &= G(T) - \frac{1}{T} \exp\left(\frac{b\theta - b\eta}{\sigma^2}\right) \int_0^T \frac{b}{\sigma \sqrt{t} 2\pi} \exp\left(-\frac{1}{2}\left(\frac{b + \theta t}{\sigma \sqrt{t}}\right)^2\right) dt, \end{aligned}$$

where we for convenience introduce the definitions

$$\eta = r - \delta - \frac{1}{2} \sigma^2, \quad \theta = \sqrt{\eta^2 + 2r\sigma^2}.$$

The second equation is derived by integration by parts, the third by the definition of $f(t)$, and the fourth equation is obtained by completing the square. To complete the derivation we must evaluate the time integral

$$K(T) = \int_0^T \frac{b}{\sigma \sqrt{t} 2\pi} \exp\left(-\frac{1}{2} \left(\frac{b + \theta t}{\sigma \sqrt{t}}\right)^2\right) dt.$$

First, define

$$\bar{\sigma} = \frac{2\theta}{\sigma}, \quad Y = V^{2\theta/\sigma^2}, \quad Y_B = V_B^{2\theta/\sigma^2}.$$

The integral $K(T)$ can then be rewritten as

$$\begin{aligned} K(T) &= \frac{2b\theta}{Y\sigma^2\bar{\sigma}^2} \int_0^T \frac{\bar{\sigma}Y}{\sqrt{t}2\pi} \exp\left(-\frac{1}{2} \left(\frac{\ln(Y/Y_B) + \frac{1}{2}\bar{\sigma}^2 t}{\bar{\sigma}\sqrt{t}}\right)^2\right) dt \\ &= \frac{4b\theta}{Y\sigma^2\bar{\sigma}^2} \int_0^{\bar{\sigma}} \frac{\sqrt{T}Y}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} \left(\frac{\ln(Y/Y_B) + \frac{1}{2}\epsilon^2 T}{\epsilon\sqrt{T}}\right)^2\right) d\epsilon. \end{aligned}$$

The last equation is obtained by substituting

$$\epsilon = \frac{\bar{\sigma}\sqrt{t}}{\sqrt{T}}.$$

$K(T)$ is now written as a constant multiplied by an integral of the partial derivative of a Black-Scholes call option price with respect to volatility for an option on an underlying asset with value Y , strike price Y_B , interest rate and dividend yield equal to zero, and time to maturity T . Since $Y > Y_B$, we can use the fundamental theorem of calculus to obtain

$$\begin{aligned} K(T) &= \frac{4b\theta}{Y\sigma^2\bar{\sigma}^2} \left[YN\left(\frac{\ln(Y/Y_B) + (1/2)\bar{\sigma}^2 T}{\bar{\sigma}\sqrt{T}}\right) \right. \\ &\quad \left. - Y_B N\left(\frac{\ln(Y/Y_B) - (1/2)\bar{\sigma}^2 T}{\bar{\sigma}\sqrt{T}}\right) - (Y - Y_B) \right] \\ &= \frac{b}{\theta} \left[-N\left(\frac{-b - \theta T}{\sigma\sqrt{T}}\right) + \exp\left(-\frac{2b\theta}{\sigma^2}\right) N\left(\frac{-b + \theta T}{\sigma\sqrt{T}}\right) \right]. \end{aligned}$$

Substituting this result into the expression for $J(T)$ yields the desired result

$$\begin{aligned} J(T) &= \left(1 + \frac{b}{\theta T}\right) \exp\left(\frac{b\theta - b\eta}{\sigma^2}\right) N\left(\frac{-b - \theta T}{\sigma\sqrt{T}}\right) \\ &\quad + \left(1 - \frac{b}{\theta T}\right) \exp\left(\frac{-b\theta - b\eta}{\sigma^2}\right) N\left(\frac{-b + \theta T}{\sigma\sqrt{T}}\right) \\ &= \frac{1}{z\sigma\sqrt{T}} \left[-\left(\frac{V}{V_B}\right)^{z-a} N(q_1(T))q_1(T) + \left(\frac{V}{V_B}\right)^{-z-a} N(q_2(T))q_2(T) \right], \end{aligned}$$

where the last equality follows from the definitions of $q_1(T)$, $q_2(T)$, z , a , and b given in the main body of the text.

Appendix B. Endogenous V_B When $V_T > V_B$.

In this appendix we derive the endogenous bankruptcy trigger V_B for the case where tax-deductibility is lost immediately when the value of the firm's assets falls below V_T . The firm value v in this situation is derived by Leland (1994a) as

$$\begin{aligned} v &= V + A_1V + A_2V^{-x} - \alpha V_B \left(\frac{V}{V_B}\right)^{-x} \quad V_B < V \leq V_T \\ v &= V + \frac{\tau C}{r} + B_2V^{-x} - \alpha V_B \left(\frac{V}{V_B}\right)^{-x} \quad V_T < V. \end{aligned} \tag{B1}$$

where

$$\begin{aligned} A_1 &= \frac{\tau C}{r} \frac{x}{x+1} \frac{1}{V_T}, \\ A_2 &= -\frac{\tau C}{r} \frac{x}{x+1} \frac{V_B^{x+1}}{V_T}, \\ B_2 &= -\frac{\tau C}{r} \frac{x}{x+1} \frac{1}{V_T} \left(V_B^{x+1} + \frac{1}{x} V_T^{x+1} \right). \end{aligned}$$

The value of equity is given by $v - D$, with v and D defined by equations (B1) and (7), respectively. We are now in a position to find the optimal bankruptcy trigger by applying the smooth pasting condition and solving

$$\left. \frac{\partial v(V)}{\partial V} \right|_{V=V_B} = \left. \frac{\partial D(V)}{\partial V} \right|_{V=V_B}$$

for V_B . This yields the optimal bankruptcy trigger

$$V_B = \frac{(C/r)(A/(rT) - B) - AP/(rT)}{1 + x(\tau C/(rV_T) + \alpha) - (1 - \alpha)B},$$

where A and B are defined in the main body of the text.

REFERENCES

- Altman, E., 1991, Defaults and returns on high-yield bonds through the first half of 1991, *Financial Analysts Journal* 47, 67–77.
- Anderson, R., and S. Sundaresan, 1996, Design and valuation of debt contracts, *Review of Financial Studies* 9, 37–68.
- Barclay, M., and C. Smith, 1995a, The maturity structure of corporate debt, *Journal of Finance* 50, 609–631.
- Barclay, M., and C. Smith, 1995b, The priority structure of corporate debt, *Journal of Finance* 50, 899–917.
- Barnea, A., R. Haugen, and L. Senbet, 1980, A rationale for debt maturity structure and call provisions in the agency theory framework, *Journal of Finance* 35, 1223–1234.
- Black, F., and J. Cox, 1976, Valuing corporate securities: Some effects of bond indenture provisions, *Journal of Finance* 31, 351–367.
- Black, F., and M. Scholes, 1973, The pricing of options and corporate liabilities, *Journal of Political Economy* 81, 637–654.
- Brennan, M., and E. Schwartz, 1978, Corporate income taxes, valuation, and the problem of optimal capital structure, *Journal of Business* 51, 103–114.
- Cornell, B., F. Longstaff, and E. Schwartz, 1996, Throwing good money after bad? Cash infusion and distressed real estate, *Real Estate Economics* 24. Forthcoming.
- Franks, J., and W. Torous, 1989, An empirical investigation of U.S. firms in reorganization, *Journal of Finance* 44, 747–769.
- Harrison, J. M., 1990, *Brownian motion and stochastic flow systems* (Krieger Publishing Company: Malabar, Florida).
- Ibbotson Associates, 1994, Stocks, bonds, bills, and inflation, *1994 Yearbook*.
- Jarrow, R., D. Lando, and S. Turnbull, 1993, A Markov model for the term structure of credit spreads, Working paper, Cornell University (revised March 1995).
- Jensen, M. and W. Meckling, 1976, Theory of the firm: Managerial behavior, agency costs, and ownership structure, *Journal of Financial Economics* 3, 305–360.
- Jones, E., S. Mason, and E. Rosenfeld, 1984, Contingent claims analysis of corporate capital structures: An empirical investigation, *Journal of Finance* 39, 611–627.
- Kim, I., K. Ramaswamy, and S. Sundaresan, 1993, Does default risk in coupons affect the valuation of corporate bonds?: A contingent claims model, *Financial Management*, 117–131.
- Leland, H., 1994a, Corporate debt value, bond covenants, and optimal capital structure, *Journal of Finance*, 49, 1213–1252.
- Leland, H., 1994b, Bond prices, yield spreads, and optimal capital structure with default risk, Working paper No. 240, IBER, University of California, Berkeley.
- Long, J., 1974, Discussion, *Journal of Finance* 29, 485–488.
- Longstaff, F., and E. Schwartz, 1995, A simple approach to valuing risky fixed and floating rate debt, *Journal of Finance* 50, 789–819.
- Macaulay, F., 1938, *Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields, and Stock Prices in the U.S. Since 1856* (National Bureau of Economic Research: New York).
- Madan, D., and Unal, H., 1994, Pricing the risks of default, Working paper, College of Business and Management, University of Maryland.
- Mella-Barral, P., 1995, The dynamics of corporate debt forgiveness and contract renegotiation, Working paper, London School of Economics.

- Mello, A., and Parsons, J., 1992, Measuring the agency costs of debt, *Journal of Finance* 47, 1887–1904.
- Merton, R., 1974, On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance* 29, 449–470.
- Miller, M., 1977, Debt and taxes, *Journal of Finance* 32, 261–275.
- Modigliani, F., and M. Miller, 1958, The cost of capital corporation finance, and the theory of investment, *American Economic Review* 48, 261–277.
- Moody's Special Report*, 1992, Corporate bond defaults and default rates (Moody's Investors Service, New York).
- Nielsen, L., J. Saá-Requejo, and P. Santa-Clara, 1993, Default risk and interest rate risk: The term structure of default spreads, Working paper, INSEAD.
- Ross, S., 1994, Capital structure and the cost of capital, Financial Economics Working paper No. 43, Series F, Yale University.
- Rubinstein, M., and E. Reiner, 1991, Breaking down the barriers, *Risk Magazine* 4, 28–35.
- Sarig, O., and A. Warga, 1989, Some empirical estimates of the risk structure of interest rates, *Journal of Finance* 44, 1351–1360.
- Toft, K., 1994, Options on leveraged equity with default risk, Working paper No. 238, IBER, Haas School of Business, University of California, Berkeley.
- Vasicek, O., 1977, An equilibrium characterization of the term structure, *Journal of Financial Economics* 5, 177–188.