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Structural models of corporate bond pricing with personal taxes

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ABSTRACT

The structural approach offers an integrated framework to deal with yield spreads and default probability simultaneously. However, structural models perform poorly in predicting corporate bond spreads. It is unclear whether this poor performance is caused by characteristics of individual models, missing factors, or different calibration procedures. This study evaluates the performance of four structural models by incorporating two important factors, personal taxes and the liquidity factor, and calibrating these models to data. To ensure our results are not contingent on the calibration method, we further apply the maximum likelihood estimation method to a large sample of individual bonds. Results consistently show that the ability of structural models to predict spreads improves considerably when personal taxes and liquidity are taken into account. Our findings suggest that the poor performance of standard structural models is more likely due to missing factors than the characteristics of individual models or the calibration procedure.

1. Introduction

The structural model pioneered by Black and Scholes (1973) and Merton (1974) has been used widely for contingent claims valuation. Notwithstanding its popularity, this model has shown serious limitations in predicting credit spreads. Jones et al. (1984) find that the Merton-type model predicts a credit spread well below the observed spread, a phenomenon widely referred to as the credit spread puzzle. The problem of spread underestimation is more severe for high-grade bonds.[1] Over the past several decades, many researchers have sought to extend the model in an attempt to improve its predictive ability for credit spreads. Among them, to name just a few, Longstaff and Schwartz (1995) consider stochastic interest rates. Leland and Toft (1996) introduce endogenous default boundaries. Anderson et al. (1996), and Mella-Barral and Perraudin (1997) consider strategic default by stockholders. Collin-Dufresne and Goldstein (2001) incorporate stationary leverage ratios.

Despite this enormous amount of efforts, empirical evidence has shown that these extensions do not materially improve the predictive ability of the model (see Huang and Huang, 2003; Eom et al., 2004). Using a new rating-based calibration approach to study a large class of structural models, Huang and Huang (2003) examine all important factors considered by previous studies, which include credit risk, liquidity, call and conversion features of corporate and Treasury bonds, time-varying asset premium, jump risk, leverage ratio, and strategic default. By calibrating these models to match historical default experience, they find that credit risk accounts for only a very small portion (20–30%) of the yield spread for high-grade bonds although it explains a larger portion of the spread for low-grade bonds. Collectively, these studies show that the issue of poor performance of structural models is still unresolved and missing factors are a potential cause. Recently, Bonfim (2009) shows that although the firm-specific financial situation is ultimately responsible for loan default at the microlevel, macroeconomic factors have additional contribution in explaining why firms default.

In this paper we generalize standard structural models to incorporate personal ordinary income and capital gains taxes in corporate bond pricing and evaluate their relative performance in predicting corporate bond spreads. The structural approach is particularly useful for analyzing the tax effect on spreads. First of all, the contingent claims approach offers an integrated framework to deal with the issues of spreads and default probability simultaneously. This contrasts with the reduced-form or time-series approach, which takes either spreads or default probability as exogenously given and deals with these two issues separately. A typical assumption of the reduced-form model is that the personal

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[1] See Elton et al. (2001).

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tax effect is independent of default risk (see Elton et al., 2001). However, in reality there is always a tax consequence when default occurs and so the independence assumption is clearly violated. More importantly, personal taxes increase cost of debt and affect firm's financing choices and default boundary, which in turn affects default probability. The structural approach allows us to directly incorporate the personal tax factor into the model to simultaneously deal with the issues related to leverage, default probability and spreads. An advantage of this approach is that the tax factor is permitted to interact with corporate financing decisions. As such, it provides deeper economic insight into debt financing mechanism and the role of personal taxes in the determination of corporate bond spreads.

Eom et al. (2004) provide a thorough evaluation of the performance of alternative structural models. They compare five structural models to determine which innovations in these models have improved the pricing of defaultable bonds. Like previous studies, they find that the Merton-type of models does not generate spreads as high as those observed in the bond market. Further, the performance of structural models is not stable. In particular, the newer models tend to severely overstate the credit risk of low-grade bonds and understate the credit risk of high-grade bonds. They attribute the dispersion in predicted credit spreads to the differences in model specifications such as the default boundary, recovery rates, coupons and interest rates. However, in a separate study Li and Wong (2008) find that structural models generate neither substantial underestimation nor extreme overestimation for yield spreads when these models are implemented properly with maximum likelihood estimation.

In this paper, we entertain missing factors as a potentially cause for the differential performances of structural models. Besides personal taxes, we consider the effect of liquidity in these models. In evaluating model performance, we control parameter values and employ the same calibration procedure for each model. This procedure provides consistent comparison across models while preserving the important features of each model. We then compare the predicted spreads associated with these structural models and examine the robustness of spread estimation by different models.

Our focus on the effects of personal taxes in this paper is related to a number of important studies. Elton et al. (2001) show that taxes can significantly affect bond prices. Liu et al. (2007) incorporate personal taxes into the standard reduced-form models. Liu et al. (2006) extend the Leland–Toft (1996) model to incorporate personal taxes on equity and debt in the structural model. Unlike these studies, we estimate the effective income tax rate implied by structural models, and compare the performance of major structural models with and without taxes to ascertain the main reason for their poor performance.

We find that on average the tax premium accounts for a significant portion of corporate bond spreads and incorporating personal taxes considerably improves the predictive power of all structural models. Personal taxes interact with default risk in a nonlinear fashion and as a consequence, the spreads due to personal taxes and default risk are not linearly separable. Previous studies (see, for example, Huang and Huang, 2003) document that standard structural models explain only 20–30% of yield spreads for investment-grade bonds. By contrast, we find that including the personal tax effect raises the proportion of spread explained by the structural models to about 40–97% for investment-grade bonds, a rather impressive improvement. Furthermore, including taxes in the model increases the predictive accuracy and reduces the dispersion of predicted spreads. Unlike Eom et al. (2004), we find that most structural models generate fairly stable spread estimates when effects of personal taxes are incorporated. Thus, the poor performance of structural models is partly due to missing factors.

To check the robustness of our results, we conduct maximum likelihood (ML) estimation using individual bond data to investigate whether our calibration method is unbiased and incorporating taxes indeed significantly reduces prediction errors of the structural models. Previous studies have shown that the ML estimation method significantly reduces the bias in calibration procedures (see, for example, Duan, 1994, 2000; Li and Wong, 2008). A major advantage of this method is that it estimates the structural model for individual bonds by inferring the unobservable asset volatility from equity data without *a priori* information for default probability. We employ the ML methodology suggested by Li and Wong (2008) in empirical investigations. We first evaluate the unbiasedness of different estimation methods using simulations and then perform ML estimation using individual firm data. The results of ML estimation confirm that incorporating taxes considerably improves the performance of the CG and LT models for a vast majority of the bonds in the sample. Furthermore, when backing out the marginal tax rates from the models using the ML method, we find these implied tax rates are quite close to previous estimates (see Graham, 1999). Thus, our findings for the importance of personal taxes appear to be robust to different estimation methods.

The remainder of this paper is organized as follows. Section 2 briefly describes the tax environment and assumptions for abstracting from complicated tax rules to focus on the key effects of personal taxes on bond pricing. Section 3 develops pricing models for defaultable bonds with personal taxes by generalizing the standard structural models with exogenous and endogenous leverage. Section 4 presents the calibration procedure based on data of rating groups to provide consistent comparison of predicted spreads across models. Section 5 shows the effects of personal taxes on corporate bond spreads and compares the performance of different structural models using the rating-based calibration method. Moreover, by calibrating the models to generate spreads consistent with observed spreads, we obtain implied marginal investors' income tax rates. Section 6 conducts simulations to compare the performance of the rating-based calibration and ML methods, and presents maximum likelihood estimates for individual bonds. Finally, Section 7 summarizes major findings and concludes the paper.

2. The tax environment

Before investigating the effects of personal taxes, it is essential to understand the current tax system and how it may affect bond pricing. In this section, we first outline the major elements of the US tax code relevant to taxation of fixed-income investments and recent changes in tax policy. Following this, we explain the model setting and introduce necessary assumptions for modeling defaultable bonds with personal taxes by emphasizing key aspects of the tax code that have a significant effect on bond price.

Under the current US tax laws, interest income and short-term capital gains are taxed at the ordinary income tax rate while long-term gains are taxed at a lower capital-gains rate.[2] The ordinary income tax rate for the highest income group is 35% for both individuals and corporations. Capital gains and losses are taxed in the year they are realized. Gain or loss from the sale or exchange of an asset is sale price less the basis. In most cases, the basis is purchase price plus cumulative amortization. Capital gains or losses are classified as either short-term or long-term depending on how long the asset was held. The required holding period for long-term gains or losses

[2] According to the tax bill passed May 23, 2003, the highest income tax rate for individuals is reduced to 35% retroactive to January 1, 2003. The next three rates are 33%, 28% and 25%. This tax act accelerates the tax reduction scheduled for 2004–2006 by the Economic Growth and Tax Relief Reconciliation Act of 2001.

is currently one year.[3] If a taxpayer has both long-term and short-term transactions in a tax year, each type is reported separately and gains and losses from each type are netted separately. Net short-term gains are taxed as ordinary income, while net long-term capital gains are subject to a maximum tax rate of 15% for individuals and 35% for corporations.[4] Short-term capital losses, including short-term loss carryovers from a prior year, are applied first to reduce short-term capital gains. A net short-term loss is then used to reduce net long-term capital gains.[5] Capital losses are deductible only to the extent of any capital gains plus, in the case of noncorporate taxpayers, ordinary income of up to $3000. Both net long-term and short-term capital losses may be used to offset up to $3000 of an individual's ordinary income. Any losses above $3000 can be carried forward or backward (for corporations only) but should be separated into short- and long-term categories. Individuals and noncorporate taxpayers may carry over a net capital loss indefinitely until the loss is exhausted. A firm can use losses for a tax year only to offset capital gains in that year but not against ordinary income. A firm is allowed to carry back unused capital losses to the three preceding tax years and to carry over losses to the five following tax years.

Amortization is an important feature of fixed-income investments. How one figures the amortization accrued each year depends on the date the bond was issued. The amortization rule may also depend on the type of transactions. For original issue bonds, discount or premium must be amortized each year. Original issue discount (OID) or premium may be amortized by the straight line method or the constant yield method, depending on the bond issuance date. Bonds purchased from the secondary market at a discount (premium), referred to as market discount (premium), are treated slightly differently. Again, the amortization method and tax treatment depends on the issuance date of the bond. Details of the amortization rules and the tax treatment associated with premium and discount amortization are described in Appendix B.

A distinct feature of the US tax system is that corporate bond income is subject to state taxes but Treasury bond income is not. Maximum state marginal income tax rates generally range from 5% to 10%. The effective tax rate of corporate bonds is equal to $\tau = \tau_F + \tau_S(1 - \tau_F)$, where τ_S is the state income tax rate and τ_F is the federal income tax rate. State taxes are deductible from income for the purpose of federal taxes and so the burden of state taxes is reduced by the federal tax rate.

Given the complex tax system, it is necessary to abstract from the details of tax rules and to focus on the most important aspects of the tax treatment that have material effects on bond pricing to keep results tractable. We assume that investors follow a buy-and-hold strategy.[6] The long-term capital gains tax rate is a constant fraction (α) of the regular income tax rate τ, which includes both federal and state taxes.[7] Tax rates applied to capital gains and losses, either long or short term, are equal (or symmetric) and there is no limit on loss deduction. In the event of default, a portion of the principal may be paid, and the remaining loss is treated as capital loss immediately deductible from an investor's taxable income. The tax rebate associated with default loss depends on the investor's tax status and the length of time the bond is held. Given this setting, we now turn to the derivation of structural models with personal taxes.

3. Pricing defaultable bonds with personal taxes

In this section, we generalize the structural models to account for the effects of personal taxes. Structural models are particularly suitable for investigating personal tax effects because the issues related to default probability, leverage and yield spreads can be addressed jointly. In fact, the so-called credit spread puzzle stems from an investigation within the contingent claims framework.[8] We consider two classes of structural models: exogenous and endogenous leverage. In the class of exogenous leverage, we consider the Longstaff and Schwartz (1995, hereafter LS) model and the Collin-Dufresne and Goldstein (2001, hereafter CG) model. In the class of endogenous leverage, we choose the Leland and Toft (1996, hereafter LT) model and the strategic default model of Anderson, Sundaresan and Tychon (1996, hereafter AST) and Mella-Barral and Perraudin (1997). In these models, leverage is optimally determined by maximizing firm value. Comparing models of endogenous and exogenous leverage allows us to assess the sensitivity of predicted tax spreads to different model specifications. Although we focus on the above models to assess personal tax effects, our approach can be easily applied to other term structure models.

Since bond default always has a tax consequence, the effects of personal taxes are inherently dependent on the probability of default.[9] Furthermore, personal taxes affect corporate financing decisions in the endogenous leverage models. Personal taxes affect the optimal leverage level and default boundary in these models, which in turn affect default risk, bond price and yield spread. In the following, we explore the personal tax effects first for the exogenous leverage models and then for the endogenous leverage models.

3.1. Structural models with exogenous leverage

Both LS and CG models assume the same process for the spot rate r_t and the geometric Brownian motion for the firm value V_t. A major difference between the two is that the LS model sets the default boundary as a constant K, while the CG model allows the default boundary to be a mean-reverting process. Default occurs when the firm value hits the threshold K, or $l_t = \ln(K/V_t) = 0$. The cumulative default probability $Q^T(r_0, l_0, T)$ at time zero under the T-forward measure is

$$Q^T(r_0, l_0, T) = \sum_{j=1}^{n_T} \sum_{i=1}^{n_r} q(r_i, l_0, t_j), \tag{1}$$

where $q(r_i, l_0, t_j)$ is the probability mass in a grid ($\Delta r \times \Delta t$) at the level of (r_i, t_j). The procedures of determining $q(r_i, l_0, t_j)$ for both the CG and LS models are summarized in Appendix A.

3.1.1. The pricing model of defaultable coupon bonds with personal taxes

We next extend the model to incorporate the effect of personal taxes. Consider a bond with coupon rate c and an initial principal value equal to 1. Following CG and LS, we assume that the expected default loss rate is a fraction L, or the risky bond pays a fraction

[3] The current law has a lower tax rate for assets held more than five years (effective now for taxpayers in the lowest regular tax bracket and as of January 1, 2006, otherwise).

[4] The top individual capital gains tax rate is reduced from 20% to 15% and for lower income taxpayers to 5% from 10%, effective May 6, 2003. For low-income taxpayers, the capital gains tax is phased out in 2007. The new law also reduces the top dividend tax rate from 38.5% to 15% retroactive to January 1, 2003. However, both capital gains and dividend tax cuts are "sunset" provisions because they would expire December 31, 2008 unless future Congresses extends them.

[5] In the case involving assets with different tax treatments, gains and losses are netted to arrive at a net gain or loss for each asset group. Any net short-term loss is then used first to reduce net long-term capital gain from the highest to lowest tax rate groups. Similarly, a net long-term loss from the highest tax rate group is used first to reduce gain from the next-highest tax rate group and then to reduce gain from a lower tax rate group.

[6] A similar assumption is made in Elton et al. (2001) and Liu et al. (2007).

[7] We assume $\alpha = 0.5$ later in empirical investigation.

[8] Empirical evidence shows that default probability implied by the yield spread has been much higher than historical default probability.

[9] For example, tax rebate is directly related to the timing of default and the value of recovery.

$(1-L)$ of face value if default occurs before maturity. By contrast, unpaid coupons have a 100% write-down (i.e., $L_{coupon} = 1$).[10] If default does not occur before maturity, the bondholder pays a regular income tax on each coupon, and a capital gains tax on the difference between the principal and purchase price at maturity. On the other hand, if default occurs before maturity, the bondholder receives a residual value, $(1-L)$ of the face value, plus a tax rebate from the government associated with default loss deduction. The tax rebate depends on the capital gains tax rate $\alpha\tau$ where τ is the ordinary income tax rate, and α is equal to one if the loss is short term or less than one if it is long term. The leverage ratio l dictates the likelihood of default before maturity. The price of the risky taxable coupon bond can then be written as

$$\begin{aligned} P^T(r_0,l_0) = &\sum_{i=1}^{M}(1-\tau)cD(r_0,t_i)[1-Q^T(r_0,l_0,t_i)] \\ &+ [1-\alpha\tau(1-P^T(r_0,l_0))]D(r_0,T)[1-Q^T(r_0,l_0,T)] \\ &+ [\delta-\alpha\tau(\delta-P^T(r_0,l_0))]D(r_0,T)Q^T(r_0,l_0,T) \end{aligned} \tag{2}$$

where $\delta = 1-L$ is the recovery rate and $D(r_0, t_i)$ is the price of the risk-free tax-free zero-coupon bond with maturity t_i. The first and second terms on the right side are the uncertain after-tax coupon and principal values, respectively, when there is no default. Coupon payments are subject to income taxes and so only the expected after-tax payments will affect bond valuation. If the bond is purchased at a discount, the investor must pay a capital gain tax $\alpha\tau$ on the difference between the face value and the discount price when the bond matures. The last term is the uncertain after-tax residual value when there is default. $\delta - P^T$ is the net loss to the investor and a tax rebate $\alpha\tau(\delta-P^T)$ is given by the government upon default. The size of this rebate partly depends on whether the default loss is short ($\alpha = 1$) or long term ($\alpha < 1$). Since the expected tax rebate also depends on the probability of default, the tax effect interacts with the default effect. Because this term is multiplicative, we can no long linearly separate the tax effect from the default effect. Solving (2) for $P^T(r_0, l_0)$ yields

$$P^T(r_0,l_0) = \frac{\sum_{i=1}^{M}(1-\tau)c[1-Q^T(r_0,l_0,t_i)]D(r_0,t_i) + (1-\alpha\tau)[1-LQ^T(r_0,l_0,T)]D(r_0,T)}{1-\alpha\tau D(r_0,T)}. \tag{3}$$

3.1.2. The pricing model of defaultable bonds with personal taxes and amortization

The analysis above ignores the effect of amortization on bond valuation. The pricing formula is correct only for corporate bonds purchased at par, the original discount bonds issued before May 28, 1969,[11] or the market discount bonds issued before July 19, 1984 (see Appendix B for the tax treatment of original and market discounts). In general, for bonds issued after May 27, 1969, discount or premium must be amortized using the straight line or constant yield method, depending on the date of issuance.[12] For those bonds subject to discount or premium amortization, their value will be affected by the tax treatment. In this section, we take into account the effect of amortization on corporate bond pricing. The derivations of pricing formulas using both straight line and constant yield amortization methods are shown in Appendix B. In the following, we illustrate the case for the straight line amortization.

With the straight line method, the amount of amortization in each period is $Amor = \frac{(P^T(r_0,l_0)-1)}{t_M-t_0}$, where t_0 denotes the time (= 0) when the bond is purchased, t_M is the time of maturity, and $P^T(r_0, l_0)$ is the purchase price. *Amor* is positive when the bond is purchased at a premium and negative at a discount. The basis of the bond is adjusted for the amount of amortization and the interest is accrued each period, if there is no default. The payoff adjusted for the amortization is $(1-\tau)c + \tau\left[\frac{P^T(r_0,l_0)-1}{t_M-t_0}\right]$ each period, where the second term reflects the adjustment due to amortization. Conversely, if default occurs at the time $t_m \leqslant t_M$, investors receive

$$B_m = \delta + \alpha\tau\left(P^T(r_0,l_0) - \left(P^T(r_0,l_0)-1\right)\frac{t_m-t_0}{t_M-t_0} - \delta\right), \tag{4}$$

where δ is the recovery rate, and the second term on the right side represents the tax rebate from default loss. As indicated, amortization affects the payoff at the time of default.

The pricing formula for the defaultable coupon bond under the straight line amortization is (see Appendix B)

$$P^T(r_0,l_0) = \frac{Z_M + \sum_{m=1}^{M}[((1-\alpha\tau)\delta + \frac{\alpha\tau\times t_m}{t_M})\Gamma_m] + \sum_{m=1}^{M}[(1-\tau)c - \frac{\tau}{t_M}]Z_m}{1-\frac{\tau}{t_M}\sum_{m=1}^{M}Z_m - \frac{\alpha\tau}{t_M}\sum_{m=1}^{M}[(t_M-t_m)\Gamma_m]}, \tag{5}$$

where

$$\Gamma_m = D(r_0,t_m)\Delta Q^T(r_0,l_0,t_m), \tag{6}$$

$$Z_m = D(r_0,t_m)[1-Q^T(r_0,l_0,t_m)], \tag{7}$$

and $\Delta Q^T(r_0, l_0, t_m)$ is the incremental default probability over $(t_{m-1}, t_m]$ under the T-forward measure using the price of riskless tax-free bond with maturity $t = t_m$, $D(r_0, t_m)$, as the numeraire.

3.2. Structural models with endogenous leverage

The endogenous leverage models offer an integrated framework that jointly determines capital structure, default probability and yield spreads. Leverage is optimally determined when firm value is maximized. This contrasts with the CG and LS models where the leverage ratio is exogenously given. In what follows, we first describe the generalized LT model and then the strategic default model with personal taxes.

3.2.1. The generalized Leland–Toft model with personal taxes

The evolution of the asset value of an unlevered firm, V, in the LT model follows the continuous diffusion process

$$\frac{dV}{V} = [\mu(V,t)-\xi]dt + \Sigma dW, \tag{8}$$

where $\mu(V, t)$ is the expected rate of return on the firm's asset, ξ is the total payout ratio, W is a standard Wiener process, and σ is the volatility parameter.

The firm continuously issues debt d at par per unit of time with maturity T, a constant coupon flow $c(T)$ and principal $p(T)$. Existing debts have time to maturity between 0 and T. For an outstanding debt with maturity t where $0 < t \leqslant T$, the associated coupon and principal are denoted as $c(t)$ and $p(t)$. The asset value process of the levered firm continues until it hits a default boundary V_B, at which the firm defaults on its debt, and bondholders receive a fixed portion ρ of the asset value V_B, or $(1-\rho)$ is the fraction of firm value lost due to default. Assuming there is a default-free asset with interest rate r, Liu et al. (2006) show that under the risk-neutral valuation with personal taxes, the debt value is

[10] See Collin-Dufresne and Goldstein (2001).

[11] This applies to bonds issued after 1954 (see IRS 2004 Publication 550).

[12] An exception is de minimis OID where the discount is treated as zero if it is less than one-fourth of 1% of the face value multiplied by the number of full years from the date of original issue to maturity.

$$d(T) = p(T) = \frac{\frac{(1-\tau)c}{r} - e^{-rT}\frac{(1-\tau)c}{r}[1-F(T)] + \left[(1-\alpha\tau)\rho(T)V_B - \frac{(1-\tau)c}{r}\right]G(T)}{1-\alpha\tau\{e^{-rT}[1-F(T)+G(T)]\} - e^{-rT}(1-\alpha\tau)[1-F(T)]}, \tag{9}$$

where $F(s, V, V_B)$ is the cumulative default probability up to time s, and $f(s, V, V_B)$ is the incremental default probability from time s to $s + \Delta s$ when the drift rate is $r - \xi$ and $G(t) = \int_{s=0}^{t} e^{-rs} f(s; V, V_B) ds$. The value of all outstanding debts D is determined by integrating $d(V, V_B, t)$ over the period of T, which depends on personal income tax rates as each individual component d is a function of the tax rate τ. $F(t)$ and $G(t)$ depends on the default boundary V_B, which is now a function of personal taxes. It can be shown that both the default boundary and debt value decreases as the tax rate increases.

3.2.2. The strategic default model with personal taxes

The financing decision in the LT model hinges on the tradeoff between tax savings of debt and the expected bankruptcy cost. In contrast, from the perspective of strategic competition between bondholders and equityholders, Anderson et al. (1996) and Mella-Barral and Perraudin (1997) argue that equityholders can default strategically to extract concessions from bondholders in the presence of bankruptcy cost. When equityholders possess the bargaining power, they can endogenously determine a default boundary. The payoff to bondholders is then equal to the firm value net of the bankruptcy cost bounded above by the face value of bond.

For a firm issuing *perpetual* bonds with the endogenous bankruptcy boundary V_B, the present value of expected bankruptcy cost is

$$BC(V, V_B) = (V_B - V_B\rho + K) \times \left(\frac{V_B}{V}\right)^{a+z}, \tag{10}$$

where $a = \frac{r-\xi-(\Sigma^2/2)}{\Sigma^2}$ and $z = \frac{[(a\Sigma^2)^2 + 2r\Sigma^2]^{1/2}}{\Sigma^2}$, K is the fixed component of bankruptcy cost, ρ is the recovery ratio, as a proportion of the asset value, or $(1-\rho)$ is the variable bankruptcy cost ratio. If K is set to zero, $(1-\rho)$ has the same meaning as β in the LT model. Following a similar procedure to derive the LT model with taxes, we can obtain the debt value

$$D(V, V_B) = \frac{\frac{(1-\tau)C}{r} \times \left(1 - \left(\frac{V_B}{V}\right)^{a+z}\right) + \mathrm{Max}[\rho V_B - K, 0] \times \left(\frac{V_B}{V}\right)^{a+z}(1-\alpha\tau)}{1 - \alpha\tau \times \left(\frac{V_B}{V}\right)^{a+z}}. \tag{11}$$

The debt value $D(V, V_B)$ is a function of the endogenous bankruptcy boundary V_B, and personal income tax rates (including both ordinary and capital gains taxes). The cumulative default probability up to time t is given by

$$\mathrm{Prob}_{def}(V, V_B, t) = N\left[\frac{\ln\left(\frac{V_B}{V}\right) - a\Sigma^2 t}{\Sigma\sqrt{t}}\right] + \left(\frac{V_B}{V}\right)^{2a} \times N\left[\frac{\ln\left(\frac{V_B}{V}\right) + a\Sigma^2 t}{\Sigma\sqrt{t}}\right], \tag{12}$$

where $N[\cdot]$ is the standard cumulative normal distribution. To transform the cumulative default probability under the physical probability measure to the risk-neutral measure, the interest rate r embedded in a and z should be replaced with $r + \vartheta$ where ϑ is the asset risk premium.

3.3. Coupon bond yield and spread

For a coupon bond with maturity $T = t_M$, price P and tax rate τ, yield-to-maturity $Y(T, \tau)$ at current time t_0 can be obtained by solving the following equation:

$$P^T = e^{-Y(T,\tau)T} + \sum_{i=1}^{M} c e^{-Y(T,\tau)t_i}, \tag{13}$$

where t_i is the time associated with the ith coupon payment and c is the coupon rate. Since corporate bonds are subject to default risk and state taxes while Treasury bonds are not, yield on corporate bonds must contain a premium to compensate investors for these disadvantages. The yield spread is the difference between the yield on a corporate bond $Y^C(T, \tau)$ and that on a Treasury $Y^T(T, \tau_F)$ with the same maturity T,[13]

$$YS(T) = Y^C(T, \tau) - Y^T(T, \tau_F), \tag{14}$$

We can compare the predictive ability of the models with and without personal taxes for corporate spreads. Unlike standard structural models, the generalized models incorporate the effects of personal taxes in each pricing formula of defaultable bonds. We first calculate the yield spread from the models with no taxes and then generate the spread from the generalized models with personal taxes. The difference between the two spreads is the tax premium.

4. Calibration of models

While structural models provide a rigorous framework for assessing personal tax effects on corporate bond spreads, in evaluating the performance of models, it is important to verify that the default probabilities implied by these models are consistent with historical default experience. In this section, we describe the procedure to calibrate the models with personal taxes such that they predict an expected level of default commensurate with historical data. The purpose of calibration is to ensure that the default probability and average loss rate used in the model to calculate spreads reflect actual default data. By calibrating models to generate a similar default rate, we provide consistent comparison of predicted spreads across models. As shown later, the calibrated models with taxes generate more reasonable credit spreads compared to previous studies, and do not produce excessively high spreads as in Eom et al. (2004). We choose three targets for calibration: default probability, equity risk premium, and recovery rate. Since calibrating models with exogenously given leverage is relatively straightforward (i.e., the initial leverage is a direct input of the model), in the interest of brevity we focus on the more complicated calibration procedure for the endogenous leverage models (LT and AST) below.

The calibration procedure involves three steps. First, we input the target recovery rate into the model and choose an initial firm value V_0 and asset volatility σ.[14] Based on these parameters, the model generates the optimal leverage l_0 and other variables including the values of debt D, equity E, coupon C, principal P, yield spread and firm value W. Second, we use the model-inferred optimal leverage l_0 and target equity risk premium to calculate asset risk premium. Based on the Modigliani–Miller (MM) model for equity return r_E, we employ the following formula to link the asset risk premium $r_A - r_f$ to the observed equity risk premium $r_E - r_f$[15]:

$$r_A - r_f = \frac{(r_E - r_f) + (1-\tau_C) \times \frac{l_0(r_D - r_f)}{1-l_0}}{1 + (1-\tau_C) \times \frac{l_0}{1-l_0}}, \tag{15}$$

where r_A is the expected asset return, r_f is the risk-free rate, τ_C is the corporate income tax rate, and r_D is the cost of debt capital. Third,

[13] In the LT model, the yield spread for the bond with maturity T is calculated as $YS = \frac{c}{d(V, V_B, T)} - r_f$, where r_f is the before-tax interest rate of riskless (Treasury) debt with the same maturity. For the CG and LS models, Treasury bond price is obtained from (3) and (5) by assuming default probability is zero and the yield is then calculated from price.

[14] We choose an initial value of asset volatility that implies a value of equity return volatility of about 30% annually.

[15] See Liu et al. (2006).

Table 1
Parameters for model calibration.

Credit rating	Target parameters[a]			Average yield spread (bps)
	Leverage ratio[b] (%)	Equity premium (%)	Cumulative default probability (%)	
AAA	13.08	5.38	0.77	63
AA	21.18	5.60	0.99	91
A	31.98	5.99	1.55	123
BBB	43.28	6.55	4.39	194
BB	53.53	7.30	20.63	320
B	65.70	8.76	43.91	470

Notes: This table shows the target values of the parameters that the models conform to after calibration. The last column includes the average observed yield spreads. The data are for 10-year bonds in rating categories from B to AAA. Except for the recovery rates, the data presented here are directly taken from Huang and Huang (2003) for the period 1973–1993. Andrade and Kaplan (1998) and Eom et al. (2004) indicate that the cost of financial distress is in the range of 15–20% of the firm's going concern value. We choose 80% as the target recovery rate for models with endogenously determined leverage (i.e., the LT and strategic default models) where a bankruptcy boundary V_B can be determined. For models with exogenous leverage (i.e., the LS and CG model), we choose 44% of the bond face value as the recovered amount, which is close to that used by Huang and Huang (2003). Note that the recovery rate here is measured as the percentage of the firm value at default.

[a] A third target parameter is the bond recovery at bankruptcy. Its choices are explained above.

[b] These leverage ratio targets are used consistently for both the LS and the CG models. This contrasts with Huang and Huang (2003) who use two different sets of leverage ratio targets for these two models.

we use the asset premium and the initial asset volatility σ (given in the first step) to determine the cumulative default probability.[16] Finally, we check whether this model-inferred default probability is consistent with the historical default data. If not, we restart the first step with different initial values and repeat this process until the model-implied default probability matches the observed historical default probability.

We obtain the 10-year bond data for the period 1973–1993 from Huang and Huang (2003). The target values of the selected parameters and average historical spreads for each class of bonds with maturity of 10 years are displayed in Table 1. We focus on 10-year bonds since the ratings provided by the Moody's are based on 10-year default frequencies. When calibrating the model with endogenous leverage, we let the model endogenize the firm's leverage to generate an optimal capital structure consistent with historical default experience.

5. Results of calibration

5.1. General setting

In calibrating each model, we choose parameter values as close as possible to those used in the original model. For the LS and CG models, we chose $\beta = 0.1$, $\xi = 0.03$, $\sigma = 0.2$, $\eta = 0.015$, $\theta = 0.08$, $\lambda = 0.18$, $\varsigma = -0.2$, $\varphi = 2.8$, $\nu = 0.6$, $\delta = 0.44$ and the initial short rate $r_0 = 0.08$. These parameter values are selected from CG (2001). The recovery rate δ is in line with other studies (see Liu et al., 2007). We choose $r_L = 0.001$ and $r_U = 0.18$ as the lower and upper bounds for the range of the spot rate r_t and discretize this range into $n_r = 25$ equal intervals.[17] The coupon rate is set to 8%. For the endogenous leverage models, we employ the same interest rate ($r = 8\%$) and payout ratio ($\xi = 6\%$) as used by the LT and the strategic default models.[18] The bankruptcy cost rate is set to 20% of the firm value as suggested by Andrade and Kaplan (1998).[19] The interest coupon c is a decision variable, which is optimally generated by both the LT and strategic default models.

5.2. Calibration results

Table 2 reports the predicted spreads for the LS and CG models while Table 3 reports those for the LT and strategic default models. All spreads are generated from the calibrated models for debt with issuance maturity $T = 10$ years.[20] The results are obtained under five scenarios: no personal taxes, personal income tax rates of 10%, 20% and 30%, and a tax rate of 22.64% based on Graham's (1999) empirical estimate. Several issues should be clarified before we analyze the results. First, the main purpose of this exercise is to get a sense of the impact of alternative tax rates on the credit spread under different structural model specifications assuming that the tax rate is exogenously given. Initially, we made no assumption that the spread unexplained by the credit models (standard default-only models) must be due to taxes. Second, behind the calibration is the implicit assumption that there is a marginal tax rate associated with a marginal investor who is indifferent between all rated bonds, and this marginal tax rate does not change over time. Again, this marginal tax rate is exogenously assigned in the beginning. In the following, we first report the results without amortization and then with amortization.

5.2.1. Predicted spreads without amortization

Column 3 of Tables 2 and 3 shows yield spreads predicted by the models calibrated with no personal taxes ($\tau = 0$). Since personal taxes are assumed away, these model-inferred spreads largely reflect default premiums. The model-generated spreads are generally low for high-grade bonds but sometimes can be quite high for junk bonds (see B bonds in Panel A of Table 2 for the LS model), similar to the finding of Eom et al. (2004). As shown in Table 2, both CG and LS models (without taxes) perform poorly for high-grade bonds.[21]

The results in Table 3 show that the LT model generates higher predicted credit spreads than the CG model does when taxes are ignored. Default risk explains about 24–69% of the observed spreads for bonds of different ratings (see column 3 of Panel A). The proportion of spread explained by default risk is higher for junk bonds, consistent with previous findings.[22] The strategic default model predicts higher spreads than the LT model (see column 3 of Panel B). This is largely because the strategic default model includes an additional fixed bankruptcy cost K, which effectively increases the bankruptcy cost ratio relative to the LT model.[23] Similar to Huang and Huang (2003), we are unable to calibrate the strategic default model to obtain reasonable spreads for speculative-grade bond categories.

[16] To calibrate models with exogenously given leverage, we need to estimate the default probability numerically since there is no closed-form solution.

[17] Our numerical simulations show that 20 intervals are generally sufficient for the estimated cumulative default probability to achieve an accuracy of more than 99.9%.

[18] The interest rate of 8% represents very well the mean default-free rate over the sample period.

[19] Bruche and González-Aguado (2010) find that the recovery rate may exhibit some level of cyclicality. Nevertheless, the time-variation in recovery rate distribution does not amplify risk and the effect is much smaller than that of default probability on systematic risk. This lends support to the use of a constant recovery rate.

[20] For the LS and CG models, we set the coupon rate to 8% of face value, which is quite close to that (8.13%) used in Huang and Huang (2003) for the purpose of comparison.

[21] Note that the differences in the predicted default premiums (when $\tau = 0$) between the LS and CG models in Table 2 are larger than those in Huang and Huang (2003). This is partly because some parameters we choose are slightly different from theirs. For example, our recovery rate is 44% and coupon rate is 8%, while theirs are 51.31% and 8.13%, respectively. The more important reason for the difference between our predicted spreads and theirs is that Huang and Huang (2003) use very different leverage ratio targets for the two models (see their Tables 2 and 7).

[22] See, for example Huang and Huang (2003) and Eom et al. (2004).

[23] Another difference is that the strategic default model considers the perpetual bond.

Table 2
Exogenously determined leverage – yield spreads after calibration.

(1) Rating	(2) Observed spread (bps)	(3) Default only (bps) (%)	(4) τ = 10% (bps) (%)	(5) τ = 20% (bps) (%)	(6) τ = 30% (bps) (%)	(7) τ = 22.64% (bps) (%)
Panel A: The LS model						
AAA	63	13 (21%)	25 (40%)	40 (63%)	60 (95%)	45 (71%)
AA	91	20 (22%)	32 (35%)	47 (52%)	67 (74%)	52 (57%)
A	123	35 (28%)	48 (39%)	63 (51%)	82 (67%)	67 (54%)
BBB	194	84 (43%)	97 (50%)	112 (60%)	130 (67%)	116 (71%)
BB	320	266 (83%)	276 (86%)	287 (90%)	299 (93%)	289 (90%)
B	470	481 (102%)	492 (105%)	499 (106%)	508 (108%)	501 (107%)
Panel B: The CG model						
AAA	63	4 (6%)	15 (24%)	30 (48%)	50 (79%)	35 (56%)
AA	91	6 (7%)	18 (20%)	32 (35%)	52 (57%)	37 (41%)
A	123	9 (7%)	21 (17%)	35 (28%)	55 (45%)	40 (33%)
BBB	194	26 (13%)	38 (20%)	53 (27%)	72 (37%)	57 (29%)
BB	320	132 (41%)	143 (45%)	155 (48%)	170 (53%)	158 (49%)
B	470	322 (69%)	327 (70%)	331 (71%)	335 (71%)	332 (71%)
Panel C: The LS model with amortization						
AAA	63	13 (21%)	28 (44%)	49 (78%)	82 (130%)	57 (90%)
AA	91	20 (22%)	35 (38%)	57 (63%)	90 (99%)	65 (71%)
A	123	35 (28%)	52 (42%)	75 (61%)	88 (72%)	82 (67%)
BBB	194	84 (43%)	104 (54%)	130 (67%)	165 (85%)	138 (71%)
BB	320	266 (83%)	292 (91%)	321 (100%)	355 (111%)	329 (103%)
B	470	481 (102%)	535 (114%)	556 (118%)	573 (122%)	560 (119%)
Panel D: The CG model with amortization						
AAA	63	4 (6%)	17 (27%)	38 (60%)	69 (110%)	45 (71%)
AA	91	6 (7%)	19 (21%)	40 (44%)	72 (79%)	47 (52%)
A	123	9 (7%)	23 (19%)	44 (36%)	76 (62%)	51 (41%)
BBB	194	26 (13%)	41 (21%)	63 (32%)	95 (49%)	70 (36%)
BB	320	132 (41%)	152 (48%)	176 (55%)	208 (65%)	184 (58%)
B	470	322 (69%)	341 (73%)	364 (77%)	388 (83%)	370 (79%)

Notes: Column 2 shows the average observed spreads on 10-year bonds. Columns 3–7 report the spreads generated by the model under different personal tax scenarios. In column 3, the personal income tax rate is set equal to zero. In column 7, the income tax rate is chosen such that the equity return tax rate, estimated as a weighted average of personal income and capital gains tax rates, i.e., $\tau_E = (1-\xi)\alpha\tau + \xi\tau$, is equal to 12% estimated by Graham (1999). The numbers in parentheses show the model-generated spread as a percentage of the observed spread.

Table 3
Endogenously determined leverage – yield spreads after calibration.

(1) Rating	(2) Observed spread (bps)	(3) Default only (bps) (%)	(4) τ = 10% (bps) (%)	(5) τ = 20% (bps) (%)	(6) τ = 30% (bps) (%)	(7) τ = 22.64% (bps) (%)
Panel A: The LT model						
AAA	63	15 (24%)	39 (62%)	56 (89%)	75 (119%)	61 (97%)
AA	91	20 (22%)	43 (47%)	60 (66%)	78 (86%)	64 (70%)
A	123	32 (26%)	52 (42%)	68 (55%)	86 (70%)	72 (59%)
BBB	194	66 (34%)	79 (41%)	93 (48%)	110 (57%)	97 (50%)
BB	320	166 (52%)	173 (54%)	183 (57%)	198 (62%)	186 (58%)
B	470	324 (69%)	326 (69%)	336 (71%)	356 (76%)	340 (72%)
Panel B: The strategic default model						
AAA	63	18 (29%)	30 (48%)	43 (68%)	56 (89%)	47 (75%)
AA	91	23 (25%)	35 (38%)	48 (53%)	60 (66%)	51 (56%)
A	123	34 (28%)	46 (37%)	58 (47%)	68 (56%)	61 (50%)
BBB	194	76 (39%)	86 (44%)	95 (49%)	100 (52%)	97 (50%)
BB	–	–	–	–	–	–
B	–	–	–	–	–	–

Notes: The bankruptcy cost ratio is 20%, interest rate r = 8%, and payout ratio ξ = 6%. Column 2 shows the average observed spreads on 10-year bonds. Columns 3–7 report the spreads generated by the model under different personal tax scenarios. In column 3, the personal income tax rate (τ) is set equal to zero. In column 7, tax rate is chosen such that the equity tax rate matches an equity tax rate τ_E = 12% estimated by Graham (1999) through the formula $\tau_E = (1-\xi)\alpha\tau + \xi\tau$. The numbers in parentheses show the model-generated spread as a percentage of the observed spread.

Columns (4)–(6) report the results with personal taxes when income tax rates are 10%, 20% and 30%, respectively. Panel A of Table 2 shows that at τ = 10%, the spreads predicted by the LS model range from 25 to 492 basis points (bps). Panel B of Table 2 shows that predicted spreads for the CG model range from 15 to 327 bps. Results show that the proportion of yields explained by the models increases substantially for investment-grade bonds. For example, adding just a 10% income tax rate almost doubles the predicted spread of AAA bonds for the LS model, and quadruples for the CG model. The proportion of observed spreads predicted by the model increases substantially as income tax rate further increases from 10% to 30%. For example, at τ = 30%, the spread predicted by the model for AAA bonds increases from 13 bps when τ = 0–60 bps (or 21–95% of observed spreads) for the LS model, and 4 bps when τ = 0–50 bps (or 6–79% of observed spreads) for the CG model. The impact of tax rate on yield spreads is similar for both models for investment-grade bonds. This is because the tax effect for these bonds with low default risk depends more heavily on coupon, recovery rate and the income tax rate. Since these parameters are the same for both models, the tax effect on the yield spread of investment-grade bonds is comparable.

The spreads predicted by the LT and the strategic default models also increase substantially after incorporating the personal tax effect. Panel A of Table 3 shows that when τ is 10%, the spread predicted by the LT model ranges from 39 to 326 bps, or 41–69% of observed spreads. By contrast, Panel B of Table 3 shows that the predicted spread for the strategic default model ranges from 30 to 86 bps, or 37–48% of the observed spread for investment-grade bonds.[24] At τ = 30%, both models explain a much larger portion of corporate bond spreads, ranging from 75 to 356 bps for the LT model, and 56 to 100 bps (for investment-grade bonds only) for the strategic default model. On average, these models explain 66–83% of the investment-grade bond spread and 69% of the junk bond spread.

Results show that personal taxes explain a much larger proportion of spreads for investment-grade bonds. For high-grade bonds, the probability of default is small and so the tax premium accounts for a relatively high percentage of spreads. Conversely, for junk bonds, default risk is high and so the tax premium is relatively low in percentage terms compared to the default premium. Results also show that the tax spread in terms of basis points does not increase monotonically with default risk. This is because there is an interactive effect between default risk and taxes. For instance, the tax rebate upon default is contingent on default probability. The higher the default probability, the higher the tax rebate. The higher present value of tax rebate exerts a downward pressure on the yield spread of speculative bonds.

As the tax rate increases, the proportion of spreads predicted by the model rises. It would seem that if the selected tax rate were high enough, the model would be able to explain the entire corporate bond spread. However, this way of boosting predicted spreads is questionable because in reality the equilibrium marginal income tax rate is determined by the market. Also, tax rates are bounded by the statutory limit for the highest income bracket. Therefore, one cannot arbitrarily raise the level of personal income tax rate to generate higher spreads.

An appropriate approach for assessing the size of personal tax premium is to use a tax rate reasonably expected to be the marginal investor's income tax rate as an input to the model. In an extensive study, Graham (1999) documents a mean marginal income tax rate for equity returns (τ_E) of about 12%. This equity tax rate can be translated into an ordinary income tax rate (τ) of 22.64% given the payout ratio (ξ = 6%) and the equity tax formula $\tau_E = (1-\xi)\alpha\tau + \xi\tau$. Column 7 reports the model-implied spreads after incorporating this empirical income tax rate into the structural models. As shown, the proportion of corporate bond spreads predicted by the tax model is substantial. Taking AAA bonds as an example, the proportions of the observed spreads explained by the LS and CG models are 71% and 56%, respectively (see the last column in Panels A and B of Table 2) compared to only 21% and 6% when personal taxes are excluded. The predicted spread for AAA bonds is even higher for the LT and strategic default models, which reach 97% and 75% of the observed spread, respectively, when the empirical marginal tax rate is employed (see the last column in Panels A and B of Table 3).

One should be cautioned not to jump into the conclusion that the LS model performs better than the CG model merely based on the results in Table 2. Differences in predicted yield spreads are partly due to different structures of the two models. The calibration actually forces the model-generated cumulative default probability for both models to be the same as the observed default probability of 10-year bonds (at T = 10). Since the CG model assumes a stationary leverage ratio, a CG firm experiences essentially the same marginal default probability over time and the cumulative default probability increases linearly approximately. In contrast, the LS model assumes a constant default boundary and growing assets. As the value of assets grow, the firm's leverage ratio declines. This results in a decreasing marginal conditional default probability and a concave cumulative default probability. Although the calibration procedure determines the parameters for each model such that the cumulative default probability is the same, the marginal default probability curves of the two models differ.[25] The LS model has a concave cumulative default probability, which explains why the yield spread predicted by the LS model tends to be higher.[26] Which model is better ultimately depends on its ability to describe the leverage policy of the firm empirically.

It is also worth noting that after personal taxes are incorporated, the LT model tends to generate higher spreads than the strategic default model (see Panels A and B of Table 3). The optimal leverage ratios produced by the two models differ because the expected bankruptcy costs considered in each model are different. The strategic default model tends to be more responsive to tax changes by lowering the bankruptcy threshold V_B, resulting in a bigger drop in estimated default risk, and hence a lower predicted spread. Notwithstanding this difference, the general structures of the LT and strategic default models are similar, and the predictive ability of both models improves considerably after incorporating taxes, especially for investment-grade bonds.[27]

5.2.2. Predicted spreads with amortization

The results above assume away the effect of amortization. The pricing model of corporate bonds should take into account the effect of amortization on bond price when the purchase price deviates from the par value because amortization affects the after-tax cash flow of discount or premium bonds. In this section, we examine the effect of amortization on corporate bond spreads. In the LT and the strategic default models, bonds are always sold at par and so amortization will not affect the results of these two models. The analysis to follow thus focuses on the LS and CG models.

Panels C and D of Table 2 report the results, which include the effect of amortization. Considering the amortization effect generally increases the predicted spread given the same set of model parameters. In the present case, both interest rate and coupon rate are equal to 8% but because there is default risk, the bond is sold at a discount. The impact of amortization on the predicted spreads in percentage terms is higher for higher-grade bonds. For example, the last column of Panel C in Table 2 shows that when the personal tax rate equals 22.64%, the proportion of observed spreads for AAA bonds explained by the LS model increases to 90%, compared to an amount of 71% when the amortization effect is ignored (see the last column of Panel A). For the CG model, the predicted spreads increase from 56% to 71% for AAA bonds and from 71% to 79% for B bonds. Results show that the CG model performs much better

[24] Again, we are unable to obtain reasonable income tax rates from the calibrated strategic default model for BB and B bonds.

[25] This is due to the assumption underlying the model. For example, an LS firm issues debt only once and so initially the firm experiences much higher default risk. But as time goes on, the distance-to-default decreases exponentially because the firm value process follows a geometric Brownian motion with a positive drift.

[26] The concavity of cumulative default probability leads to higher level of default probability before maturity date T, and results in a higher yield spread than that for the CG model.

[27] However, we find that if we force the two models to produce the same leverage ratio, the LT model will consistently generate lower spreads due to (1) the perpetual bond pricing used in the strategic default model and the finite maturity bond pricing used in the LT model, and (2) the additional fixed bankruptcy cost included in the strategic default model.

Table 4
Exogenously determined leverage – implied tax rates.

(1) Credit rating	(2) Observed spread (bps)	(3) Liquidity component (bps)	(4) Non-liquidity component (bps)	(5) Implied personal tax rate	(6)	(7)	(8)
				The LS model		The CG model	
				Income (%)	Equity (%)	Income (%)	Equity (%)
Panel A: Estimates of implied tax rates without amortization							
AAA	63	16	47	23.8	12.6	28.8	15.3
AA	91	34	57	25.3	13.4	32.3	17.1
A	123	56	67	22.6	12.0	30.8	16.3
BBB	194	96	98	11.3	6.0	39.7	21.0
BB	320	100	220	–	–	48.3	25.6
B	470	100	370	–	–	53.8	28.5
Average implied tax rates			All ratings			38.9	20.6
			Investment grade	20.8	11.0	32.9	17.4
Panel B: Estimates of implied tax rates with amortization							
AAA	63	16	47	19.1	10.1	23.5	12.5
AA	91	34	57	19.9	10.5	25.8	13.7
A	123	56	67	17.2	9.1	27.7	14.7
BBB	194	96	98	8.2	4.3	30.6	16.2
BB	320	100	220	–	–	32.9	17.4
B	470	100	370	–	–	22.8	12.1
Average implied tax rates			All ratings			26.7	14.3
			Investment grade	16.1	8.5	27.2	14.4

Notes: This table reports the implied income tax rates obtained by calibrating the model to match the non-liquidity spread component. The liquidity component of spread in column 3 is taken from De Jong and Driessen (2007). The non-liquidity component in column 4 is obtained by subtracting the liquidity spread in column 3 from the observed spread in column 2. Estimated ordinary income tax rates are reported in columns (5) and (7). The equity tax rate is set to $\tau_E = (1-\xi)\alpha\tau + \xi\tau$ and reported in columns (6) and (8).

when the amortization effect is properly accounted for. However, the model still cannot fully explain the entire corporate bond spread, implying that there are still missing factors (e.g., liquidity) unaccounted by the model.

In summary, results show that amortization can significantly affect the predicted bond value and yield spread. Thus, the amortization effect should be taken into account when bonds are sold at a price significantly different from par in order to provide a more accurate estimate of the tax spread.

5.2.3. Estimates of implied personal income tax rates

In the analysis above, we examine how structural models perform after including personal taxes. This exercise helps us understand the relative importance of the personal tax effect and how it translates into the yield spread for corporate bonds. We next approach the issue from a different perspective by asking the question: given the observed spread, what is the marginal investor's income tax rate implied by the model? Using this approach, we can obtain the model-implied income tax rate to see if it is reasonably close to the empirical estimate documented by previous studies (e.g., Graham, 1999). However, to obtain a meaningful estimate of the implied tax rate, we must account for the liquidity risk effect as well, which is not explicitly considered by all structural models. Otherwise, the estimated implied tax rate would be biased because the liquidity risk effect could be misconstrued as the tax effect. One way to resolve this problem is to subtract the liquidity premium from the observed yield spread, and then recalibrate the model to generate spread estimates consistent with these liquidity-adjusted spreads by allowing tax rates to be endogenously determined. This procedure contrasts with that in Tables 2 and 3 where tax rates are exogenously assigned.

Tables 4 and 5 report the estimates of the implied marginal income tax rate for the models with exogenous and endogenous leverage, respectively. To control for the liquidity effect, we adjust the observed spread by the liquidity premium estimated by De Jong and Driessen (2007) by rating category to come up with the non-liquidity component of spread. Their estimates of the liquidity premium are shown in column 3, and the non-liquidity component or the adjusted spreads are reported in column 4 of each table.[28] We calibrate each model against these adjusted spreads to obtain the implied marginal income tax rates for the cases with and without amortization. In obtaining the implied tax rate, we assume that the remaining yield spread unexplained by default and liquidity is attributable to taxes. Although this appears to be a fairly strong assumption, the results should nevertheless provide useful information, given that the literature seems to suggest that default, liquidity and taxes are the dominating factors for corporate bond term structure.

Panel A of Table 4 reports the results without amortization for both LS and CG models. For the CG model, the implied income tax rates average about 33% for investment-grade bonds and 39% for all bonds combined. The estimates of the implied tax rate seem high, especially for junk bonds. The LS model predicts a lower average implied income tax rate of 21% for investment-grade bonds. However, we cannot obtain a reasonable marginal tax rate for junk bonds through calibration of the LS model. The equity tax rates implied by the LS model range from 6% to 13% with an average of 11%, which is close to Graham's (1999) estimate of 12%. By contrast, the equity tax rates implied by the CG model are higher (17% for investment-grade bonds). Overall, the CG model produces higher implied income tax rates. This is not surprising because given the selected parameter values, the CG model generates lower spreads than the LS model does (see Table 2). Since the leverage ratio is exogenously given in both models, the implied tax rate estimates would tend to be higher for the CG model to catch up with the observed spreads.

Panel B of Table 4 reports the estimates of implied marginal income tax rates for the LS and CG models when the amortization effect is taken into account. Considering the amortization effect lowers the implied tax rate estimates for both models. For the LS model, the average implied ordinary income tax rate is 16.1% whereas the equity return tax rate is only 8.5% for investment-grade firms. For the CG model, the average implied ordinary in-

[28] For AAA and AA bonds, due to a large discrepancy of liquidity premium between intermediate-term (5 years) and long-term bonds (10–22 years) reported by De Jong and Driessen (2007), we take the average of them.

Table 5
Endogenously determined leverage – implied tax rates.

(1) Credit rating	(2) Observed spread (bps)	(3) Liquidity component (bps)	(4) Non-liquidity component (bps)	(5) Implied personal tax rate	(6)	(7)	(8)
				The LT model		The strategic default model	
				Income (%)	Equity (%)	Income (%)	Equity (%)
AAA	63	16	47	14.6	7.7	22.4	11.9
AA	91	34	57	18.5	9.8	26.9	14.3
A	123	56	67	19.8	10.5	28.5	15.1
BBB	194	96	98	23.4	12.4	25.2	13.3
BB	320	100	220	39.2	20.8	–	–
B	470	100	370	34.4	18.2	–	–
Average implied tax rates ratings			All	25.0	13.2		
			Investment grade	19.1	10.1	25.8	13.6

Notes: This table reports the implied income tax rates obtained by calibrating the model to match the non-liquidity spread component. Bankruptcy cost ratio is 20%, interest rate r = 8% and payout ratio ξ = 6%. The liquidity component of spread in column 3 is taken from De Jong and Driessen (2007). The non-liquidity component in column 4 is obtained by subtracting the liquidity spread in column 3 from the observed spread in column 2. Estimated ordinary income tax rates are reported in columns (5) and (7). The equity tax rate is set to $\tau_E = (1-\xi)\alpha\tau + \xi\tau$ and reported in columns (6) and (8).

come tax rate is 27.2% for investment-grade bonds and 26.7% for all bonds combined. The corresponding equity return tax rates are about 14% for both investment-grade bonds and all bonds combined. Thus, after including the amortization effect, the marginal income tax rates implied by the CG model become more reasonable compared to previous empirical estimates.

Turning to the results for the endogenous leverage models in Table 5, we find that both the LT and strategic default models yield very reasonable income tax rate estimates. The average ordinary income tax rate implied by the LT model is 19% for investment-grade bonds and 25% for all bonds combined. The estimated income tax rates are a little higher for junk bonds but are still within the range estimated by Graham (1999, p. 161). The implied equity return tax rates for the LT model are reported in column 6 of Table 5. The marginal equity return tax rate averages about 10% for investment-grade bonds and 13% for all bonds combined, which are quite close to Graham's estimate. Similar to the LS model, we are only able to calibrate the strategic default model for the investment-grade bonds. The average ordinary income tax rate implied by the strategic default model is 25.8% whereas the average implied equity tax rate is 13.6%. Although the implied income tax rates are higher for the strategic default model, these tax rates are within the range of Graham's estimates.

Overall, our calibration results show that including personal tax rates considerably improves the performance of structural models, and the marginal income tax rates implied by the structural models are reasonable compared to previous estimates. These findings are quite encouraging and show promise for explaining the credit spread puzzle. To check the robustness of calibration results, we further conduct the maximum likelihood estimation to verify the importance of personal taxes in corporate bond pricing in the next section.

6. Model implementation with maximum likelihood estimation

In the preceding analysis, we show the importance of personal taxes in the determination of yield spreads using the calibration method, which matches the model-generated default probability to the observed default rate for each rating class. There are however several potential concerns for this rating-based calibration method. First, credit ratings may contain noise.[29] Second, the non-statistical calibration method may be biased (see Duan 1994, 2000). Third, since the default rate is only available by rating category (e.g., the Moody's default frequency data), this method is difficult to implement for individual bonds whose default probability is unobserved.

Duan (1994) indicates that the non-statistical calibration method does not have a sound theoretical foundation. Li and Wong (2008) examine alternative implementation methods for structural models using individual bond data and find that the ML estimation method is the soundest. To check whether our calibration results are spurious, we further investigate the importance of taxes using the maximum likelihood estimation method. This method estimates the parameters that determine the default probability directly from observed individual firm data.

In this section, we implement the ML estimation for two most representative structural models – the CG and the LT models.[30] A key strength of the ML method lies in its ability to infer the unobservable asset volatility σ from the observed equity data to estimate default probability and yield spreads based on the structural model. We employ the ML method suggested by Li and Wong (2008) to perform empirical estimation for individual bonds. In this exercise, we attempt to answer two questions: (1) does incorporating personal taxes reduce the prediction error of the structural models? and (2) after liquidity premium is considered, is the model-implied marginal tax rate reasonable?

6.1. The likelihood function

Li and Wong (2008) provide comprehensive analysis of the performance of the structural models using the ML method. Details about this estimation method can be found in their paper. We briefly outline the estimation procedure below.

Consider the density function of the log asset value

$$g(v_i|v_{i-1};\mu,\Sigma) = \varphi(v_i, v_{i-1}) - \varphi(v_i + v_{i-1} - 2\log H) \times \exp\left\{\left(\frac{2\mu}{\Sigma^2} - 1\right)(\log H - v_{i-1})\right\}, \tag{16}$$

where H denotes the default boundary, v_i is the log asset value at time t_i, σ is the asset volatility, and μ is asset drift rate under the physical probability measure, and

$$\varphi(x) = \frac{1}{\Sigma\sqrt{2\pi(t_i - t_{i-1})}} \exp\left\{-\frac{[x - (\mu - \Sigma^2/2) \times (t_i - t_{i-1})]^2}{2\Sigma^2(t_i - t_{i-1})}\right\}. \tag{17}$$

[29] For example, Güttler and Wahrenburg (2007) find evidence that the S&P and Moody's do not adjust their ratings to increasing default risk in the same fashion in a variety of aspects.

[30] In reality, capital structures of most firms fall somewhere in between those in the CG and LT models.

Li and Wong (2008) derive the following log-likelihood function for barrier-dependent models,

$$L(\mu,\Sigma)=\sum_{i=2}^{n}\left\{\ln g(v_i|v_{i-1})-\ln[V_i\times\Delta(V_i)|_{V=V_i}]\right\}, \tag{18}$$

where the option delta $\Delta(V)$ is obtained by differentiating the pricing formula with respect to asset value V. The parameters can be estimated by solving the following optimization problem:

$$\max_{\mu,\Sigma} L(\mu,\Sigma) \quad \text{s.t.}\widetilde{E}(t_i)=E(V(t_i),H,\Sigma), \tag{19}$$

where $\widetilde{E}(t_i)$ and $E(V(t_i),H,\sigma)$ are observed and model-predicted equity values at time t_i, respectively. The CG model does not directly link asset volatility σ to equity value E since leverage is exogenously given. Therefore, we use the DOC (down and out call) formula for the CG model. This treatment is same as that for the LS model in Li and Wong (2008). Both the DOC pricing formula and the LT pricing formula are given in Appendix C.

6.2. Simulating the calibration methods – rating-based calibration and ML estimation

Before conducting the ML estimation using real data, we examine the reliability of the estimation methods by comparing their performance. This is done through simulations for both rating-based calibration and ML estimation methods.

6.2.1. Rating-based calibration using the simulated default rate

The rating-based calibration used earlier is implemented by matching the model-predicted default probability with the observed cumulative default rate for each rating category. In our simulation analysis for the calibration method, we first assume that the underlying process follows the CG and the LT mechanisms, respectively, with a pre-specified asset volatility σ to generate the value process V_t with an exogenous log-default threshold k_t in the CG case[31] or the endogenous default boundary V_B predicted by the LT model.[32] Then we generate a time series of asset values based on the geometrical Brownian motion in (A.1),[33] and keep track of the asset-over-threshold ratio, i.e., $f_t=V_t/e^{k_t}$ in the CG case and $f_t=V_t/V_B$ in the LT case. When $f_t\leq 1,\ \forall t\leqslant T$, where T is the given time horizon (e.g., 5 years), we stop the random value generating process and record one default event by performing $N_D=N_D+1$ with $N_D=0$ at $t=0$; otherwise we stop the process when $t=T$ without changing the value of N_D. We repeat this procedure M (= 500) times[34] and generate the cumulative default rate for T-year horizon as $\pi_D^T=N_D/M\times 100\%$. Each time interval Δt can represent a week or a month, depending on our choice.[35] After obtaining the default rate π_D^T, we apply both the CG and the LT models to match the model-predicted default probability to this simulated default rate. This is done by trying different values of σ until the match is close enough. This procedure backs out the unobserved asset volatility and generates bond price (yield) and yield spread. We simulate the rating-based calibration method with two different personal income tax rates, 10% and 30%, respectively. The simulation results are reported in Part I of Table 6.

We find that when the true default process is generated by the CG (or LT) mechanism, applying the correct model backs out the underlying asset volatility quite accurately and produces negligible prediction errors for prices and yields. This demonstrates that the structural models are sound in so far as the settings are identical to what they specify. Conversely, implementing a wrong model could result in sizable errors arising from misspecification of the model and the error is an increasing function of the tax rate. These findings are not surprising. First, different structural models impose different assumptions for the underlying value process. Therefore, if assumptions are not met, the implemented pricing model is expected to incur errors. Second, the error in the present context is mainly due to different treatments of capital structure by the two models and the tax shields therein. A higher tax rate will thus cause these models to behave more differently and result in a larger discrepancy in prediction accuracy.

6.2.2. ML estimation using the simulated price series

Similarly, to simulate the ML method we pre-specify the underlying data-generating processes of asset and equity and use the ML estimation method to infer the parameters. We choose values of parameters, such as tax rates, bankruptcy recovery rate and interest rate consistent with our earlier results.

We first generate equity values according to the CG assumptions (e.g., exogenously specified mean-reverting leverage ratio) and implement the ML estimation using the simulated series of equity values to back out the underlying asset volatility σ, which is then used in the structure model to predict the bond price, yield and spread. Since the value of asset volatility and the process to generate the equity process are known, we have the true yield spread and so can easily calculate the prediction error. The ML estimation is applied to both the CG and LT models using the same simulated equity values. We repeat this process 500 times for each model and the results are summarized in the left panel of Part II in Table 6. We simulate different leverages, coupon rates and maturities. Not surprisingly, the CG model is quite accurate in terms of both the percentage error and standard deviation of errors (in parentheses). By contrast, the LT model noticeably underperforms the CG model; not only does it have a larger average error but also exhibits much higher standard deviation, largely because the former model has different assumptions on capital structure and the value process from the latter.

We next repeat the above procedure with the equity value series simulated by the mechanism in accordance with the LT assumptions, i.e., firms endogenously optimize their leverage. The ML method is used to estimate unobserved asset volatility σ associated with each generated price series by maximizing L in (18). We then input this σ estimate in the LT model to obtain the predicted yields (prices) and spreads. Results of model predictions are reported in the right panel of Part II in Table 6. Since the coupon rate is not an independent variable within the LT setting, we choose four cases with different maturity and leverage only. As shown, the LT model performs much better than the CG model in terms of both percentage error and standard deviation of errors (in parentheses). Again, this discrepancy in prediction performance is mainly due to the fact that the latter imposes different assumptions on capital structure and the value process from the former.

6.2.3. Summary for the simulation results

Although the calibration and ML methods are technically different, simulation results show that both do a good job in assessing the performance of the structural models. The main message given from the simulation is that correct model specification is more important than the calibration method. If a correct model is used

[31] This threshold is mean-reverting and is a function of an exogenously given target v (see (A.3)).

[32] In our simulations, we set σ = 0.2, and 0.3 for the processes generated with CG and LT mechanisms, respectively. These σ values generate sufficient number of default events for our simulation analysis.

[33] Eq. (8) is also applicable in this regard; however, (A.1) is under the risk-neutral measure, which is simpler. In our simulation, we work under the risk-neutral measure for simplicity.

[34] This number of iterations is sufficient to guarantee convergence. We have also tried M = 1000 iterations and found the results are not sensitive.

[35] We find it more convenient to simulate on a monthly basis for T = 20 years, and weekly basis for T = 5 years.

Table 6
Simulation results of the CG and LT models.

Cases	Values generated by the CG model						Values generated by the LT model					
	% error in prices		% error in yields		% error in spreads		% error in prices		% error in yields		% error in spreads	
	CG	LT	CG	LT	CG	LT	CG	LT	CG	LT	CG	LT
Part I: The rating-based calibration method												
Panel A: Tax rate = 10%												
(1) Base case (T = 5 years, c = 5%, l = 30%)							(1) T = 5 years, l = 30%					
	0	−1.5	0	5.2	0	33.9	−1.8	0	7.0	0	307.1	0
(2) Different coupon rates (T = 5 years, c = 10%, l = 30%)							(2) T = 5 years, l = 60%					
	0	−0.6	0	1.9	0	10.6	1.9	0	−6.2	0	−31.3	0
(3) Different leverage ratios (T = 5 years, c = 5%, l = 60%)							(3) T = 20 years, l = 30%					
	0	−5.9	0	15.6	0	41.4	3.3	0	−3.9	0	−21.2	0
(4) Different maturities (T = 20 years, c = 5%, l = 30%)							(4) T = 20 years, l = 60%					
	0	−23.4	0	32.3	0	117.6	8.8	0	−9.4	0	−34.5	0
Panel B: Tax rate = 30%												
(1) Base case (T = 5 years, c = 5%, l = 30%)							(1) T = 5 years, l = 30%					
	0.0	4.4	0.0	−12.3	0.0	−42.1	−28.4	0.0	47.9	0.0	686.7	0.0
(2) Different coupon rates (T = 5 years, c = 10%, l = 30%)							(2) T = 5 years, l = 60%					
	0.0	7.3	0.0	−55.9	0.0	−19.6	−25.5	0.0	38.9	0.0	228.5	0.0
(3) Different leverage ratios (T = 5 years, c = 5%, l = 60%)							(3) T = 20 years, l = 30%					
	0.0	12.7	0.0	−26.3	0.0	−57.2	−11.4	0.0	14.5	0.0	59.6	0.0
(4) Different maturities (T = 20 years, c = 5%, l = 30%)							(4) T = 20 years, l = 60%					
	0.0	−24.6	0.0	83.5	0.0	31.8	−17.7	0.0	23.7	0.0	97.4	0.0
Part II. The maximum likelihood method[a]												
(1) Base case (T = 5 years, c = 5%, l = 30%)							(1) T = 5 years, l = 30%					
	−0.12	−1.81	0.39	6.15	2.16	34.52	5.03	0.08	−10.24	−0.37	−26.12	−2.08
	(0.62)	(2.79)	(2.06)	(9.77)	(11.56)	(54.89)	(9.82)	(0.12)	(24.44)	(3.43)	(62.35)	(3.39)
(2) Different coupon rates (T = 5 years, c = 10%, l = 30%)							(2) T = 5 years, l = 60%					
	−2.13	−19.81	0.37	0.94	1.61	4.02	5.10	0.26	−14.97	−0.95	−32.34	−2.61
	(0.59)	(2.28)	(2.01)	(9.12)	(8.62)	(39.16)	(12.04)	(1.23)	(27.16)	(4.21)	(79.15)	(6.22)
(3) Different leverage ratios (T = 5 years, c = 5%, l = 60%)							(3) T = 20 years, l = 30%					
	−0.26	−55.52	0.87	3.71	4.85	20.72	10.56	0.52	−17.24	−1.36	−39.06	−3.32
	(0.66)	(6.10)	(2.17)	(6.83)	(12.13)	(38.16)	(9.74)	(1.94)	(24.35)	(3.29)	(62.27)	(8.20)
(4) Different maturities (T = 20 years, c = 5%, l = 30%)							(4) T = 20 years, l = 60%					
	−0.14	−2.25	0.19	2.41	1.01	13.07	5.89	1.03	−22.54	−1.63	−44.11	−4.36
	(1.05)	(5.92)	(1.31)	(7.32)	(7.13)	(39.75)	(10.19)	(2.67)	(34.32)	(4.92)	(75.59)	(12.05)

Notes: This table summarizes the results of simulations to assess the unbiasedness of the calibration method and the ML estimation method. For the rating-based calibration method (Part I), we assume the underlying process follows the CG and the LT mechanisms, respectively, with a known asset volatility to generate the default probability. We then apply both the CG and the LT models to match this default probability and back out the unobserved asset volatility and finally bond prices and spreads. Since this is at the rating-based aggregate level (i.e., only one aggregate result for each rating category), if the true default process is generated by the CG (LT) mechanism, applying the correct model backs out the known asset volatility precisely. Conversely, implementing a wrong model would likely result in sizable errors. We carry out the simulation with two different income tax rates, 10% and 30%. Other parameters are the same as with the ML method in Part II. We compare results with different time horizon (T) and leverage (l). The percentage error is defined as the difference between actual and predicted values divided by the actual value, which can be prices, yields or yield spreads. For the ML method (Part II), we first generate a value process based on the settings in the CG and LT models, respectively. We then apply the ML method to the two models to estimate parameters and infer the yields spread. The ML results presented are the mean value and standard deviation (in parentheses). We assume that interest rate r = 6%, payout ratio ξ = 3%, corporate and personal income tax rates are 35% and 30%, respectively. Other parameters (e.g., bankruptcy cost ratio, etc.) are the same as used in the rest of this paper.

[a] Unlike the CG model which treats leverage and coupon rate independently, the LT model optimizes leverage ratio and coupon rate simultaneously and endogenously. We hence compare results with different time horizon (T) and leverage (l) in Part II.

to describe the data, estimation results are usually quite reliable regardless of whether we use the rating-based calibration or the ML method to estimate parameters and yield spreads. Conversely, if a wrong model is fitted to the data, the results of parameter estimation will be poor regardless of which method is used to estimate parameters.

However, there are pros and cons for the rating-based calibration method. On the one hand, the rating-based calibration method is more convenient to implement and imposes less computational burden. Unlike the ML method where the unobservable asset volatility σ is estimated (e.g., through (16)–(19) and (C5)), the rating-based method directly matches the model-generated default probability with the observed default data. This is why the rating-based calibration method is usually quite accurate when the underlying assumptions for the models are met. On the other hand, the rating-based calibration method cannot be applied to individual bond data because it requires the information for default probability, which is typically unobserved for individual bonds. Here the ML method has an edge because it can be used to estimate σ and other parameters from observed equity data without relying on the default probability information. Hence, it is ideal for individual bond estimation.

In summary, simulation results show that (1) models can behave quite differently depending on whether their underlying assumptions are consistent with data; and (2) both calibration and ML methods are quite appropriate for empirical investigations. With hindsight, these findings are not surprising because a model's prediction accuracy is expected to deteriorate if its underlying assumptions deviate substantially from the actual process. The CG and LT models adopt two distinct sets of assumptions about the capital structure. In reality, most firms are likely to fall in between these two polar cases. For example, a firm typically does not revert its capital structure to an exogenously set target by continuously redeeming or issuing debt as assumed by the CG model. Nor can it completely endogenously optimize its capital structure at one time point and expect its capital structure to remain optimal indefinitely as implied by the static capital structure assumption in the LT model. Therefore, we expect each model to have varying explanatory power when applied to real data. The calibration and ML methods complement each other and using both methods enhances the robustness of empirical results.

6.3. Maximum likelihood estimation

We next turn to estimation using the maximum likelihood method. Individual bond data are obtained from the Lehman Brothers Fixed Income Database distributed by Warga (1998). This database contains monthly price, accrued interest and return data on corporate bonds up to 1997. There are 5009 firms and 32,295 bonds in the database and a subset of corporate bonds is chosen for this study. We select our data sample using a screening procedure similar to Li and Wong (2008). We eliminate bonds that were

Table 7
Summary statistics of individual bonds in ML estimation.

Sample characteristics	Mean	S.D.	Minimum	Median	Maximum
Panel A: Overall summary statistics of bonds for the whole sample period					
Time to maturity T (years)	8.45	5.98	2.00	7.00	30.00
Coupon rate c (%)	9.56	2.28	0.00	9.40	16.38
Spread s (bps)	207	118	2	198	750
Moody's ratings	5.02	2.88	1.00	5.00	15.00
S&P ratings	4.95	3.09	1.00	5.00	24.00
Firm value FV (millions $)	47,997	44,248	291	34,323	164,001
Leverage (%)	61.25	25.22	12.20	68.93	99.00

Observation year	Number of bonds	T (years)	c (%)	s (bps)	Moody's rating	S&P rating	FV (millions $)	Liability (millions $)
Panel B: Summary statistics of bonds by year								
1986	46	7.76	9.37	238	5.46	5.33	22,622	6754
1987	50	10.86	8.62	208	4.38	4.28	7750	3570
1988	72	12.97	9.57	251	5.26	5.17	26,660	15,544
1989	44	8.95	10.23	223	5.82	5.93	57,191	38,688
1990	62	8.66	10.20	178	5.60	5.40	71,225	52,162
1991	105	8.17	10.03	175	4.95	4.68	78,003	61,689
1992	64	8.72	9.78	187	4.84	4.59	61,654	52,865
1993	65	7.75	9.40	221	5.28	5.00	83,743	78,437
1994	109	7.57	9.18	211	4.61	4.96	34,046	19,447
1995	22	6.68	9.03	188	4.32	4.18	36,600	8,958
1996	79	6.97	9.86	206	4.90	4.90	43,967	20,648
1997	41	4.98	8.60	221	4.83	4.83	14,162	2,560

Rating score	Moody's	S&P and Fitch.
Panel C: Bond rating numerical conversions		
1–2	Aaa	AAA
3–5	Aa	AA
6–8	A	A
9–11	Baa	BBB
12–14	Ba	BB
15–17	B	B
19–20	Caa	CCC
21	Ca	CC
22	C	C
23	D	D
24	NR	NR

Notes: This table presents summary statistics of 759 bonds issued by 307 firms in the ML estimation. Panel A shows mean, standard deviation (S.D.), minimum, median and maximum of time to maturity (T), coupon rate (c), spreads (s), the Moody's and S&P's ratings, firm value (FV) and leverage. Panel B shows average values of bond characteristics and rating scores by year. The bond rating numerical conversions are summarized in Panel C.

Table 8
Prediction errors for models with and without taxes.

	CG		LT	
	% error in yield prediction	Error in spread prediction	% error in yield prediction	Error in spread prediction
No tax	−1.72%	−1.88%	5.61%	6.09%
	(1.36%)	(1.50%)	(7.69%)	(8.34%)
With 2.07%	taxes	−0.05%	−0.06%	1.91%
	(1.16%)	(1.27%)	(3.19%)	(3.47%)

Notes: This table reports the model prediction errors for yields and spreads before and after taxes are incorporated in the structural models. Personal ordinary income tax rate is chosen such that the equity tax rate matches an equity tax rate $\tau_E = 12\%$ estimated by Graham (1999) through the formula $\tau_E = (1-\xi)\alpha\tau + \xi\tau$, where ξ is the payout ratio, τ is the income tax rate and $\alpha\tau$ is the capital gains tax rate where α equals 0.5. The prediction error is the difference between the predicted and observed values. The percentage prediction error is the prediction error divided by the observed value. Standard deviation is reported in parentheses. The percentage error in yields is calculated as the model-predicted yield minus the observed yield and then divided by the observed yield. The error in spread prediction is the model-predicted spread minus the observed spread.

matrix-priced because matrix prices are problematic. We also exclude bonds with options (callable, puttable, or sinking funds), floating rate bonds, flower bonds, inflation-indexed bonds, and bonds with odd frequencies of coupon payments. We then match the data to bond characteristics and firm information. Equity and firm data are from Compustat, which include monthly price, monthly return, share number, firm value, and leverage ratios. We exclude firms in finance, real estate finance, public utility, insurance, and banking industries. Three-month Treasury rates are obtained from the Federal Reserve Board and used as the short-term interest rate. The final sample consists of 759 bonds issued by 307 firms over the period of 1986–1997.

Table 7 provides a summary of the data sample. As shown, the sample covers a wide range of bonds, in terms of maturity, leverage, coupons, and ratings, issued by firms of different size. It includes a sufficient number of premium and discount bonds for examining the tax effects due to different tax treatments. Bond maturity ranges from 2 to 30 years with a median of 7 years. This contrasts with the rating-based sample in Table 1, which focuses on 10-year bonds. The sample also covers a whole spectrum of bond rating categories from D to AAA and non-rated (NR) bonds. Thus, our individual bond sample is well balanced. From the empirical viewpoint, a well-balanced sample is important for obtaining estimation results representative of the entire corporate bond universe.

We assume that in the event of default, the firm is liquidated and the residual value to equity holders is zero. For the DOC pricing formula, we assume that the book value of debt X is equal to the barrier H. CG (2001) assume a recovery rate of 44% of the face va-

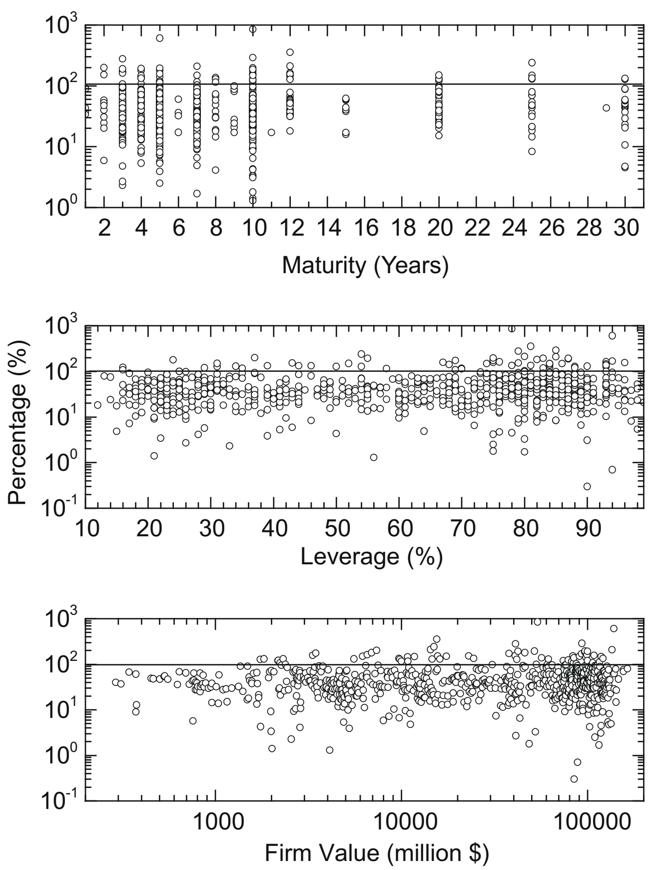

Fig. 1. Relative prediction errors for the CG model with and without taxes using the maximum likelihood estimation method. The relative prediction error is in percentage of the prediction error by the model without taxes (shown as the 100% baseline). The vertical axis is logarithmically scaled and relative prediction errors are plotted by maturity, leverage and firm value. Out of 759 bonds, the model performance improves for 698 bonds after incorporating personal taxes. The tax rate is chosen such that the equity tax rate τ_E matches the empirical estimate (12%) by Graham (1999).

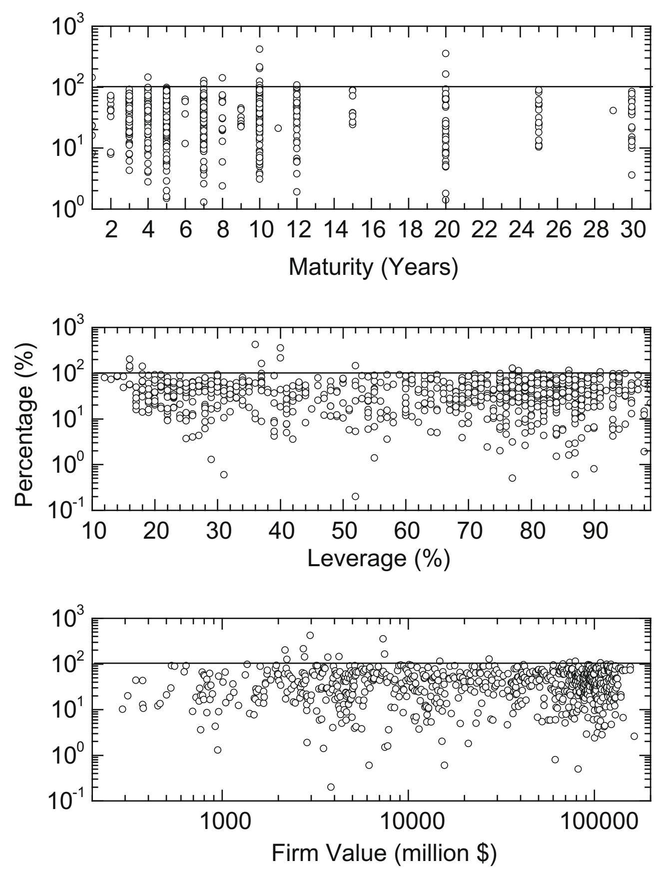

Fig. 2. Relative prediction errors for the LT model with and without taxes using the maximum likelihood estimation method. The relative prediction error is in percentage of the prediction error by the model without taxes (shown as the 100% baseline). The vertical axis is logarithmically scaled and relative prediction errors are plotted by maturity, leverage and firm value. Out of 759 bonds, the model performance improves for 745 bonds after incorporating personal taxes. The tax rate is chosen such that the equity tax rate τ_E matches the empirical estimate (12%) by Graham (1999). Compared to the CG model, the performance of the LT model improves more significantly by including personal taxes.

Table 9
Implied income tax rates.

Overall average for the entire sample (%)	Minimum (%)	Maximum (%)	Median (%)	Standard deviation (%)
Panel A: The CG model				
26.2 (13.9)	0.0 (0.0)	35.3 (18.7)	27.0 (14.3)	6.3 (3.3)
Average implied income tax rate (%) based on asset and leverage quartiles[a]				
	1st Quartile	2nd Quartile	3rd Quartile	4th Quartile
Asset-based average	27.1 (14.4)	26.6 (14.1)	25.1 (13.3)	25.8 (13.7)
Asset-based std.	6.1 (3.2)	5.6 (3.0)	6.5 (3.4)	6.5 (3.4)
Leverage-based average	26.9 (14.3)	26.7 (14.2)	26.0 (13.8)	25.1 (13.3)
Leverage-based std.	5.7 (3.0)	6.1 (3.2)	6.4 (3.4)	6.7 (3.6)
Implied income tax rate (%)				
Panel B: The LT model				
30.2 (16.0)	0.0 (0.0)	35.3 (18.7)	32.2 (17.1)	8.3 (4.4)
Average implied income tax rate (%) based on asset and leverage quartiles[a]				
	1st Quartile	2nd Quartile	3rd Quartile	4th Quartile
Asset-based average	30.4 (16.1)	31.1 (16.5)	29.4 (15.6)	29.9 (15.8)
Asset-based std.	8.2 (4.3)	7.1 (3.8)	9.4 (5.0)	8.1 (4.3)
Leverage-based average	31.4 (16.6)	29.8 (15.8)	29.5 (15.6)	33.1 (17.5)
Leverage-based std.	7.6 (4.0)	8.9 (4.7)	8.4 (4.5)	8.0 (4.2)

Notes: This table reports mean and standard deviation (std.) of implied income tax rates obtained by considering both default and liquidity risk premia in the structural models. The liquidity premium is assumed to be 20% of the total yield spread (see Jarrow et al., 2010). The personal income tax rates are obtained by matching the model-inferred spreads to the observed spreads, subject to the lower bound $\tau \geqslant 0$ and the upper bound $\tau \leqslant 35.25\%$. Given the statutory federal tax rate $\tau_F = 30\%$ and the average statutory state tax rate $\tau_S = 7.5\%$ (see Elton et al., 2001), the overall maximum income tax rate is $\tau = \tau_F + \tau_S(1-\tau_F) = 35.25\%$. Panels A and B report results for the CG and the LT models, respectively. The values outside the parentheses are estimates of regular income tax rates and the values in parentheses are estimates of equity return tax rates in percentage. The equity return tax rate is given by $\tau_E = (1-\xi)\alpha\tau + \xi\tau$, where ξ is the payout ratio.
[a] The 1st quartile (4th quartile) contains one-fourth of the bonds ranked the lowest (highest) by firms' asset value or leverage ratio.

lue. To be consistent with our analysis using the rating group data earlier, we assume the same recovery rate for corporate bonds when implementing the CG model.[36] On the other hand, if measured against the going concern value, bankruptcy cost is found to be around 20% (see Andrade and Kaplan, 1998). Following Huang and Huang (2003), we choose the maximum of the two bankruptcy cost measures when implementing the LT model.

6.4. Results of the ML estimation

To see whether incorporating personal taxes helps improve model performance, we compare model prediction errors before and after taxes are considered. The prediction error is defined as the difference between the model-implied and observed values and the percentage prediction error is the prediction error divided by the observed value. Table 8 summarizes the mean and standard deviation of prediction errors.

As shown, incorporating taxes significantly improves the performance of both models in terms of both mean prediction errors and standard deviation. For the CG model, considering personal taxes reduces the mean percentage prediction error in yields from −1.72% to −0.05% and standard deviation of percentage errors from 1.36% to 1.16%. The next column shows the difference between predicted and observed yield spreads. Again, there is a substantial improvement after incorporating personal taxes. The mean prediction error in spreads reduces from −1.88% to −0.06%, and the standard deviation of prediction errors drops from 1.5% to 1.27%. Results show that incorporating taxes not only produces smaller prediction errors but also generates estimates with a greater precision and a higher degree of confidence.

The results for the LT model are reported in the third and fourth columns of Table 8. The improvement due to incorporating taxes is even more striking. The mean percentage prediction error of yields decreases sharply from 5.61% to 1.91% and the standard deviation decreases from 7.69% to 3.19%. Similarly, the mean prediction error for the yield spread decreases dramatically from 6.09% to 2.07% and the standard deviation of prediction errors decreases from 8.34% to 3.47%.

To assess the tax effect more easily, we further calculate the relative prediction error for the models with and without taxes. For example, based on the ML estimation, if the model predicts a spread of 100 bps without taxes and 140 bps with taxes while the observed spread is 200 bps, then the relative prediction error of the model with taxes to that without taxes is $(200-140)/(200-100)*100\% = 60\%$. Thus, if incorporating taxes does improve model performance, one will expect a relative prediction error below the 100% baseline. Figs. 1 and 2 plot the relative prediction errors for the CG and the LT models, respectively.

For the CG model, the average relative prediction error for the entire sample is 51.2%, suggesting that on average incorporating taxes reduces the error by a half. Incorporating taxes leads to an improvement in spread predictions for 698 (out of 759) bonds as exhibited in Fig. 1 where the relative error is shown in logarithmic scale against maturity, leverage and firm value, respectively. The 100-percent gridline provides a convenient reference line to visualize the extent of improvement by incorporating taxes.

For the LT model, the improvement in model performance is even more impressive. As shown in Fig. 2, the relative error is reduced for 745 (out of 759) bonds when taxes are incorporated in the model. The average relative error is 43%. Compared to the CG model, the LT model's improvement appears to be more uniform and much stronger in terms of average errors and prediction precision (standard deviation).

Overall, results show that incorporating personal taxes improves model performance substantially. Consistent with the rating-based calibration analysis, results of the ML estimation based on individual bonds show that considering personal taxes produces much improvement in the predictive power of the models. Thus, our findings are robust to different capital structure assumptions and estimation methods.

Apart from default risk and tax effects, the liquidity premium has been shown to be another important component of yield spreads. As mentioned earlier, it is important to account for the

[36] This is in line with empirical studies such as Li and Wong (2008).

liquidity premium when inferring the personal tax rate.[37] We next obtain the implied tax rate based on the ML estimation. Using a similar dataset, Jarrow et al. (2010) employ a reduced-form model to estimate yield spreads for individual bonds and find that on average liquidity premium accounts for about 20% of the yield spread. We use their estimate of the liquidity premium here to infer the implied tax rate for individual bonds because their data sample is close to ours. Assuming that our sample bonds have a similar liquidity premium, we apply the ML estimation method to back out the implied income and equity tax rates. Panels A and B of Table 9 show the estimates of tax rates for the CG and LT models, respectively, where the equity return tax rates are reported in parentheses.

As shown in Panel A, the CG model implies an average income tax rate of 26.2% and an equity tax rate of 13.9% using the entire sample. These values are quite close to our earlier estimates based on the rating-based data. For example, in Panel B of Table 4, the CG model implies an income tax rate of 26.7% and an equity tax rate of 14.3%. These values are also close to Graham's (1999) estimate of 12% for the equity tax rate. By contrast, as shown in Panel B of Table 9, the LT model implies an average of 30.2% income tax rate and 16% equity tax rate, compared to the rating-based calibration estimates of 25% and 13.2%, respectively.

Overall, results show that the implied tax rates are substantially above zero. This confirms that personal taxes are important for corporate bond pricing. More detailed results by asset and leverage groups for both models reported in the lower parts of Panels A and B of Table 9 show the cross-sectional distributional pattern and provide a similar conclusion.

7. Conclusions

In this paper, we examine the effects of personal taxes on yield spreads of defaultable bonds using both rating group and individual bond data. We first generalize four structural models of defaultable bonds to account for personal tax effects. To provide consistent comparison of the predicted yield spreads, these models are calibrated to generate default probabilities commensurate with historical default experience of bonds in different rating classes. Furthermore, we apply the maximum likelihood (ML) estimation to individual bond data. Our empirical results consistently show that including personal taxes significantly improves the performance of the structural models by increasing their explanatory power for corporate bond spreads. Furthermore, the implied income tax rates estimated from the models are quite close to those estimated by Graham (1999). These findings are robust to different estimation methods and data samples. Results strongly suggest that personal taxes are an important factor for corporate bond pricing.

The poor performance of standard term structure models can be due to their inability to predict the probability of default, or omission of important factors which are relevant to corporate bond pricing but not necessarily related to credit risk. Our results suggest that the failure of structural models to explain corporate bond spreads is partly due to missing factors and that structural models should include personal taxes in order to explain corporate bond spreads more satisfactorily.

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Appendix A

In this appendix, we summarize the key features of the CG and LS models and illustrate the procedures for estimating the default probability used in this paper. Detailed derivations can be found in CG (2001) and LS (1995). Both models assume that firm value V_t under the risk-neutral measure follows a geometric Brownian process:

$$\frac{dV_t}{V_t} = (r_t - \xi)dt + \Sigma dz_1^{\Pi}(t), \tag{A.1}$$

where ξ is the payout ratio, and σ is the volatility parameter. The risk-free tax-free spot rate r_t obeys the following process:

$$dr_t = \beta(\theta - r_t)dt + \eta dz_2^{\Pi}(t), \tag{A.2}$$

where β, θ and η are constants, and $dz_1^{\Pi} dz_2^{\Pi} = \varsigma dt$.

Unlike the LS model, CG employs a stationary leverage ratio process and postulates that the log-default threshold k_t has the following mean-reverting process:

$$dk_t = \lambda[y_t - \nu - \phi(r_t - \theta) - k_t]dt, \tag{A.3}$$

where $(y_t - \nu)$ sets the target threshold with ν being a constant and $y_t = \log V_t$, λ adjusts the mean-reverting speed, and $\varphi \geqslant 0$. If there is no default before maturity, the bond investor receives the principal (= 1) at maturity. If default occurs prior to maturity, a fractional loss in principal, L, is incurred. The price of a risky tax-free discount bond is thus given by

$$P^T(r_0, l_0) = D(r_0, T)[1 - LQ^T(r_0, l_0, T)], \tag{A.4}$$

where $D(r_0, T)$ is the price of the risk-free zero-coupon bond and $Q^T(r_0, l_0, T)$ is the cumulative probability default before maturity T under the T-forward measure.

Using the method of the first passage time probability density and discretizing the interest rate space (with a lower bound r_L and an upper bound r_U) into n_r equal intervals by $r_i = r_L + i\Delta r = r_L + i \times (r_U - r_L)/n_r$ for $i \in (1, 2, \ldots, n_r)$, and time into n_T equal intervals with $t_j = jT/n_T = j\Delta t$ for $j \in (1, n_T)$, where $\Delta t = T/n_T$, one can obtain the cumulative default probability $Q^T(r_0, l_0, T) = \sum_{j=1}^{n_T}\sum_{i=1}^{n_r} q(r_i, l_0, t_j)$ in (1). Under the T-forward measure and given the initial leverage ratio l_0, $q(r_i, l_0, t_j)$ is the default probability mass in a grid $(\Delta r \times \Delta t)$ at the level of (r_i, t_j), and is given by

$$q(r_i, l_0, t_1) = \Delta r \Psi(r_i, t_1, l_0), \quad \forall i \in (1, 2, \ldots, n_r), \tag{A.5}$$

$$q(r_i, l_0, t_j) = \Delta r\left[\Psi(r_i, t_j, l_0) - \sum_{v=1}^{j-1}\sum_{u=1}^{n_r} q(r_u, t_v, l_0)\Phi(r_i, t_j | r_u, t_v)\right], \quad \forall i \in (1, 2, \ldots, n_r) \quad \text{and} \quad \forall j \in (2, \ldots, n_T), \tag{A.6}$$

$$\Psi(r_t, t, l_0) = \pi(r_t, t | r_0, 0) N\left(\frac{\mu(r_t, t | l_0, r_0, 0)}{\Sigma(r_t, t | l_0, r_0, 0)}\right), \tag{A.7}$$

$$\Phi(r_t, t | r_s, s) \equiv \pi(r_t, t | r_s, s) N\left(\frac{\mu(r_t, t | l_s = 0, r_s, s)}{\Sigma(r_t, t | l_s = 0, r_s, s)}\right), \quad \forall (t > s). \tag{A.8}$$

Here $\mu(r_t, t | l_s, r_s, s)$ and $\Sigma^2(r_t, t | l_s, r_s, s)$ are the expected value and variance of log-leverage conditional on (l_s, r_s, s) at date t, and $\pi(r_t, t | r_s, s)$ is the transition probability density of the interest rate process given by

[37] We note that liquidity risk is more difficult to capture. For example, Van Landschoot (2008) shows that even bonds denominated in different currencies (US dollar and Euro) can have different liquidity risk. In this study we ignore these additional complications.

$$\pi(r,t,r_s,s) \equiv G\left(r_t - r_s e^{-\beta(t-s)} - \theta(1-e^{-\kappa(t-s)}), \frac{\eta^2}{2\kappa}(1-e^{-2\kappa(t-s)})\right), \tag{A.9}$$

where $G(\mu_{gk}, t-s)$ is the Gaussian kernel,

$$G(\mu_{gk}, t-s) = \frac{1}{\sqrt{2\pi(t-s)}} e^{-\frac{\mu_{gk}^2}{2(t-s)}}, \tag{A.10}$$

and μ_{gk} is the kernel mean.

For the Gaussian process (r, l), the conditional moments of a joint bivariate normal system can be expressed as a function of unconditional moments by the projection theorem:

$$\mu(r_t, l_s, r_s) \equiv E_s^T[l_t, r_t] = E_s^T[l_t] + \frac{Cov_s^T[l_t, r_t]}{Var_s^T[r_t]}(r_t - E_s^T[r_t]), \tag{A.11}$$

$$\Sigma^2(r_t, l_s, r_s) \equiv Var_s^T[l_t, r_t] = Var_s^T[l_t] + \frac{Cov_s^T[l_t, r_t]^2}{Var_s^T[r_t]}. \tag{A.12}$$

A.1. The Collin–Dufresne and Goldstein model

Under the T-forward measure, the conditional moments of the bivariate normal system are

$$\begin{aligned} E_u^T[l_t] &= l_u e^{-\lambda(t-u)} - (1+\lambda\varphi)\left(r_u + \frac{\eta^2}{\beta^2} - \theta\right) e^{-\beta(t-u)} B_{(\lambda-\beta)}^{(t-u)} \\ &\quad - \left(\frac{\eta\varsigma\Sigma}{\beta} + (1+\lambda\varphi)\frac{\eta^2}{2\beta^2}\right) e^{\beta(T-t)} B_{(\lambda+\beta)}^{(t-u)} \\ &\quad + (1+\lambda\phi)\frac{\eta^2}{2\beta^2} e^{\beta(T-t)-2\beta(t-u)} B_{(\lambda-\beta)}^{(t-u)} + \left(\frac{\eta\varsigma\Sigma}{\beta} + \lambda\bar{l}^{Q} - (1+\lambda\phi)\left(\theta - \frac{\eta^2}{\beta^2}\right)\right) B_{\beta}^{(t-u)}, \end{aligned} \tag{A.13}$$

$$E_u^T[r_t] = r_u e^{-\beta(t-u)} + \left(\theta\beta - \frac{\eta^2}{\beta}\right) B_\beta^{(t-u)} + \frac{\eta^2}{\beta} e^{-\beta(T-t)} B_{2\beta}^{(t-u)}, \tag{A.14}$$

$$\begin{aligned} Var_u^T[l_t] &= \left(\frac{(1+\lambda\phi)\eta}{\lambda-\beta}\right)^2 B_{2\beta}^{(t-u)} + \left[\Sigma^2 + \left(\frac{(1+\lambda\phi)\eta}{\lambda-\beta}\right)^2 \right. \\ &\quad \left. - \left(2\frac{\varsigma\Sigma(1+\lambda\phi)\eta}{\lambda-\beta}\right)\right] B_{2\lambda}^{(t-u)} + 2\left(\frac{\varsigma\Sigma(1+\lambda\phi)\eta}{\lambda-\beta}\right) + \left(\frac{(1+\lambda\phi)\eta}{\lambda-\beta}\right)^2 B_{(\lambda-\beta)}^{(t-u)}, \end{aligned} \tag{A.15}$$

$$Var_u^T[r_t] = \eta^{2[r_t]} B_{2\beta}^{(t-u)}, \tag{A.16}$$

$$Cov_u^T[l_t, r_t] = -\frac{(1+\lambda\phi)\eta^2}{\lambda-\beta} B_{2\beta}^{(t-u)} - \left(\varsigma\Sigma\eta - \frac{(1+\lambda\phi)\eta^2}{\lambda-\beta}\right) B_{(\lambda-\beta)}^{(t-u)}, \tag{A.17}$$

where log-leverage ratio $l_t \equiv k_t - y_t$ with k_t being the mean-reverting stochastic log-default threshold and $y_t = \ln V_t$, $\bar{l}^Q(t) \equiv \frac{-r_t + \xi + \frac{\Sigma^2}{2}}{\lambda} - \upsilon$, and $B_\beta^{(u)} \equiv \frac{1}{\beta}(1 - e^{-\beta u})$.

A.2. The Longstaff and Schwartz model

A key feature of the LS model is that the default boundary is a constant. It may therefore be considered as a special case of the CG model where $dk_t \equiv 0$. Under the T-forward measure, the mean, variance and covariance of interest rate r_t and l_t are

$$E_u^T[l_t] = l_u - \left(\theta - \frac{\eta^2}{\kappa^2} - \xi - \frac{\Sigma^2}{2} - \frac{\varsigma\Sigma\eta}{\kappa}\right)(t-u) - \left(r_u - \theta + \frac{\eta^2}{\kappa^2} + \frac{\varsigma\Sigma\eta}{\kappa} e^{-\kappa(T-t)}\right) B_\kappa^{(t-u)} - \frac{\eta^2}{2\kappa} e^{-\kappa(T-t)} B_\kappa^{(t-u)^2}, \tag{A.18}$$

$$E_u^T[r_t] = r_u e^{-\kappa(t-u)} + \left(\theta\kappa - \frac{\eta^2}{\kappa}\right) B_\kappa^{(t-u)} + \frac{\eta^2}{\kappa} e^{-\kappa(T-t)} B_{2\kappa}^{(t-u)}, \tag{A.19}$$

$$Var_u^T[l_t] = \left(\Sigma^2 + 2\frac{\Sigma\varsigma\eta}{\kappa} + \frac{\eta^2}{\kappa^2}\right)(t-u) - \left(\frac{\Sigma\varsigma\eta}{\kappa} + \frac{\eta^2}{\kappa^2}\right) B_\kappa^{(t-u)} + \frac{\eta^2}{\kappa^2} B_{2\kappa}^{(t-u)}, \tag{A.20}$$

$$Var_u^T[r_t] = \eta^2 B_{2\kappa}^{(t-u)}, \tag{A.21}$$

$$Cov_u^T[l_t, r_t] = \frac{\eta^2}{\kappa} B_{2\kappa}^{(t-u)} - \left(\frac{\eta^2}{\kappa} + \varsigma\eta\Sigma\right) B_\kappa^{(t-u)}, \tag{A.22}$$

where $B_\kappa^{(u)} \equiv (1/\kappa)(1 - e^{-\kappa u})$.

Appendix B

In this appendix, we first outline the tax treatment for amortization of premium and discount. We then derive the pricing formulas for coupon bonds under alternative amortization rules.

For original issue bonds, premium or discount must be amortized each period.[38] For bonds issued on or before July 1, 1982, the straight line method is prescribed.[39] For bonds issued after July 1, 1982, the constant yield method is applied.

On the other hand, bonds purchased at premium or discount from the secondary market, referred to as market discount (or premium), are treated differently. For bonds issued on or before September 27, 1985, investors may amortize the premium using the straight line method. For bonds issued after September 27, 1985, investors have an option to amortize the premium using the constant yield method. In any cases, investors can opt to deduct the premium as a loss at maturity. Obviously, amortizing the premium is a better choice. For bonds purchased from the secondary market at a discount, if they are issued on or before July 18, 1984, there is no amortization and the discount is treated as a capital gain at maturity or sale. If the bond is issued after July 18, 1984, the discount is amortized using either the straight line or constant yield method, and the accumulated amortization (with a maximum equal to the appreciation) is taxed at maturity or sale at the ordinary income tax rate. However, it is interesting to note that recent tax laws have granted investors a choice to use either the straight line or constant yield method for amortizing market discount and premium (see IRS 2004 Publication 550, p. 15).

Since our model assumes the buy-and-hold strategy, we primarily consider original issue discount (or premium) bonds. Our model applies equally well to the pricing of market discount (premium) bonds where once the bond is purchased, the investor is assumed to hold it to maturity. For original issues, discount and premium must be amortized each year and the basis adjusted accordingly.[40] A slight complication is that our data sample covers the period 1973–1993. In the earlier part of this period (before July 2, 1982), the straight line amortization rule is prescribed while in the later part, the constant yield amortization rule is mandated. In our simulation, we choose the straight line amortization method but it can be easily extended to the constant yield method. In the following, we derive the pricing model of corporate bonds first using the straight line and then the constant yield method.

B.1. Straight line amortization

Denote bond price at purchase time $t = t_0\,(= 0)$ as $P^T(r_0, l_0)$, where $T\,(= t_M)$ is the time to maturity, and r_0 and l_0 are the initial interest rate and leverage ratio. The coupon rate is c and the face value is normalized to 1. Under the straight line amortization rule, the amount of amortization in each period (year) is $Amor = \frac{(P^T(r_0, l_0) - 1)}{t_M - t_0}$.

If default occurs at the time t_m, the bond investor receives

$$B_m = \delta + \alpha\tau\left(P^T(r_0, l_0) - (P^T(r_0, l_0) - 1)\frac{t_m - t_0}{t_M - t_0} - \delta\right), \tag{B.1}$$

[38] See IRS publications 550 and 1212.

[39] Exceptions are Series E (issued before July 1980) and EE US government bonds where investors can elect to have the discount amount taxed at maturity. Treasury bills are taxed upon sale and small de minimis discounts are not taxed.

[40] The original issue discount (OID) is the difference between the issue price of a bond and the amount that is payable to the investor when the bond matures. If the bond is a taxable bond, the investor reports the discount as interest income per Form 1099-OID. On the other hand, if the bond is purchased in the secondary market and if there is any adjustment to the original issue discount reported on Form 1099-OID, the investor needs to list it separately on Schedule B as the OID adjustment. See IRS Publication 1212 for information about the original issue discount and how to report on Schedule B.

where α is less than one if the default loss is long term and is equal to one if it is short term. The sum of the first two terms in the parentheses is the basis of the bond, and the difference between the basis and the recovery value δ is the capital loss due to default. The investor receives a tax rebate upon default. The default probability is $\Delta Q(r_0, l_0, t_m) = [Q(r_0, l_0, t_m) - Q(r_0, l_0, t_{m-1})]$, where $Q(r_0, l_0, t_m)$ is the cumulative default probability under the risk-neutral measure. If default has not occurred up to time $t_m \leqslant t_M$, the investor receives

$$M_m = (1-\tau)c + \tau\left[\frac{P^T(r_0, l_0) - 1}{t_M - t_0}\right], \tag{B.2}$$

over the time period $(t_{m-1}, t_m]$ with a probability of $1 - Q(r_0, l_0, t_m)$. If the bond does not default before maturity t_M, the terminal value of the bond at maturity is

$$U_M = 1 \tag{B.3}$$

with a probability of $1 - Q(r_0, l_0, t_M)$. Therefore, bond price $P^T(r_0, l_0)$ can be expressed as the expected present value of all future cash flows:

$$\begin{aligned} P^T(r_0, l_0) = E^Q\Bigg[&\sum_{m=1}^{M} B_m e^{-\int_0^{t_m} r_t \mathrm{d}t} \times \Delta Q(r_0, l_0, t_m) \\ &+ \sum_{m=1}^{M} M_m e^{-\int_0^{t_m} r_t \mathrm{d}t}[1 - Q(r_0, l_0, t_m)] \\ &+ U_M e^{-\int_0^{t_M} r_t \mathrm{d}t}[1 - Q(r_0, l_0, t_M)]\Bigg], \end{aligned} \tag{B.4}$$

where $E^Q[\bullet]$ denotes the expectation under the risk-neutral measure. Both the interest rate r_t and firm value V_t are assumed to be stochastic (see (A.1) and (A.2)) and permitted to be correlated with a coefficient of correlation ς. Thus, the discount term $e^{-\int_0^{t_m} r_t \mathrm{d}t}$ and default process $Q(r_0, l_0, t_m)$ are not independent under the equivalent martingale measure (Q). However, this dependence can be removed under the T-forward measure, which allows us to rewrite price as

$$\begin{aligned} P^T(r_0, l_0) = &\sum_{m=1}^{M} B_m D(r_0, l_0, t_m) \times \Delta Q^T(r_0, l_0, t_m) \\ &+ \sum_{m=1}^{M} M_m D(r_0, l_0, t_m)[1 - Q^T(r_0, l_0, t_m)] \\ &+ U_M D(r_0, l_0, t_M)[1 - Q^T(r_0, l_0, t_M)], \end{aligned} \tag{B.5}$$

where $D(r_0, l_0, t_m) = E^Q[e^{-\int_0^{t_m} r_t \mathrm{d}t}]$ is the present value of a default-free bond (subject to interest rate risk only) with face value of 1 and time to maturity t_m, $Q^T(r_0, l_0, t_m)$ is the T-forward cumulative default probability up to $t = t_m$, and $\Delta Q^T(r_0, l_0, t_m)$ is the incremental default probability over the period $(t_{m-1}, t_m]$ under the T-forward measure using $D(r_0, t_m)$, the price of default-free bond with maturity $t = t_m$, as the numeraire. Due to the discrete nature in our numerical computation, we can also express $\Delta Q^T(r_0, l_0, t_m)$ under the T-forward measure using $D(r_0, t_{m-1})$ as the numeraire. Under the risk-neutral measure, we have the exact relationship, $\Delta Q(r_0, l_0, t_m) = [Q(r_0, l_0, t_m) - Q(r_0, l_0, t_{m-1})]$. However, under the T-forward measure, this relationship is only held approximately, $\Delta Q^T(r_0, l_0, t_m) \approx Q^T(r_0, l_0, t_m) - Q^T(r_0, l_0, t_{m-1})$. This is because $Q^T(r_0, l_0, t_i)$ is strictly associated with numeraire $D(r_0, t_i)$ and the approximation results from using two numeraires, $D(r_0, t_m)$ and $D(r_0, t_{m-1})$. To have the exact relationship under the T-forward measure, $\Delta Q^T(r_0, l_0, t_i)$ must be measured based on the same numeraire. In simulation, we can use either the approximate or exact form because the difference is negligible. Substituting (B.1), (B.2), and (B.3) into (B.5), and rearranging, we obtain the pricing formula in (5).

B.2. Constant yield amortization

Under the constant yield amortization, the amount of amortization in each period m is

$$Amor(m) = c - y \times Basis(m), \tag{B.6}$$

where y is the yield-to-maturity of the bond. We assume that the initial basis is the issuing price

$$Basis(1) = P^T(r_0, l_0). \tag{B.7}$$

The basis is adjusted each period by the amount of amortization. The basis changes according to

$$Basis(m) = Basis(m-1) - Amor(m-1). \tag{B.8}$$

When there is no default, the net after-tax cash flow to the bondholder is

$$\sum_{m=1}^{M}[c(1-\tau) + \tau \times Amor(m)] \times D(r_0, t_m) + D(r_0, t_M), \tag{B.9}$$

where $Amor(m)$ is the amortization in period m and the ordinary income tax rate times the amount of amortization is the benefit (or cost) of amortizing the premium (or discount) in each period. The risk-neutral probability of no default is $[1 - Q(r_0, l_0, t_M)]$.

When there is default over the period $(t_{m-1}, t_m]$, the investor receives

$$[\delta + \alpha\tau(Basis(m) - \delta)]D(r_0, t_m), \tag{B.10}$$

which represents the present value of the residual and tax rebate upon default. The risk-neutral probability in the period $(t_{m-1}, t_m]$ is $\Delta Q(r_0, l_0, t_m) = [Q(r_0, l_0, t_m) - Q(r_0, l_0, t_{m-1})]$. Combining (B.6)–(B.10), and switching to the T-forward measure (see the procedure above for the straight line method), we can obtain the bond pricing formula under the constant yield method as

$$P^T(r_0, l_0) = \frac{J_1 + J_2}{1 + \tau \times J_3 - \alpha\tau \times J_4}, \tag{B.11}$$

$$\begin{aligned} J_1 &= \sum_{m=1}^{M} c(1 + \tau y \sum_{j=0}^{m-2}(1+y)^j) \times D(r_0, t_m)[1 - Q^T(r_0, l_0, t_m)] \\ &\quad + D(r_0, t_M)[1 - Q^T(r_0, l_0, t_M)], \\ J_2 &= \sum_{m=1}^{M}\left\{[(1-\alpha\tau)\delta - \alpha\tau c \sum_{j=0}^{m-2}(1+y)^j]D(r_0, t_m)\Delta Q^T(r_0, l_0, t_m)\right\}, \\ J_3 &= y\sum_{m=1}^{M}(1+y)^{m-1}D(r_0, t_m) \times [1 - Q^T(r_0, l_0, t_m)], \\ J_4 &= \sum_{m=1}^{M}(1+y)^{m-1}D(r_0, l_0, t_m) \times \Delta Q^T(r_0, l_0, t_m). \end{aligned}$$

Appendix C

In this appendix, we outline the procedure for implementing the maximum likelihood estimation for the LT and the CG model.

C.1. The LT model with personal taxes

Under the endogenously determined default barrier, H, Liu et al. (2006) show that the firm's equilibrium equity value is

$$E(V, H, T) = \frac{V + \left(1 - \frac{(1-\tau_C)(1-\tau_E)}{1-\tau}\right)\frac{C}{r}\left[1 - \left(\frac{H}{V}\right)^{a+z}\right] - \beta H\left(\frac{H}{V}\right)^{a+z} - D(V, H, T)}{1 - \tau_{EC}\left(\frac{H}{V}\right)^{a+z}}, \tag{C.1}$$

where τ_E is the effective tax rate on equity returns (i.e., the weighted average of dividend and capital gains tax rates), τ_{EC} is the capital gains tax rate on equity returns, β is the bankruptcy ratio, C is the total annual coupon, D is the total debt value, V is the underlying asset value, and

$$F(t) = N[h_1(t)] + \left(\frac{H}{V}\right)^{2a} N[h_2(t)], \quad \text{and}$$

$$G(t) = \left(\frac{H}{V}\right)^{a-z} N[q_1(t)] + \left(\frac{H}{V}\right)^{a+z} N[q_2(t)]. \tag{C.2}$$

We use $N[^*]$ to denote the cumulative normal distribution. The default barrier H is determined by the smooth-pasting condition,

$$\left.\frac{\partial E}{\partial V}\right|_{V=H} = \frac{1 + \left(1 - \frac{(1-\tau_C)(1-\tau_E)}{1-\tau}\right)\frac{C(a+z)}{rH} + \beta(a+z) - \int_0^T \Omega \mathrm{d}t}{1 - \tau_{EC}} = 0, \tag{C.3}$$

where

$$\begin{cases} \Omega = \frac{B}{A^2}\left\{\left.\frac{\partial G(t)}{\partial V}\right|_{V=H} - e^{-rt}\left.\frac{\partial F(t)}{\partial V}\right|_{V=H}\right\} + \frac{1}{A}\left\{(a_2c - b_1p)\left.\frac{\partial F(t)}{\partial V}\right|_{V=H} + (b_3H - a_1c)\left.\frac{\partial G(t)}{\partial V}\right|_{V=H}\right\} \\ A = 1 + \alpha\tau[-e^{-rt} + e^{-rt}F(t) - G(t)];\ B = b_2\{a_1c + (b_1p - a_2c)[1 - F(t)] + (b_3H - a_1c)G(t)\} \end{cases}. \tag{C.4}$$

The parameters, such as $a_1, a_2, b_1, b_2, b_3, p, a, z, h_1, h_2, q_1, q_2$, total debt D, $\frac{\partial F(t)}{\partial V}|_{V=H}$ and $\frac{\partial G(t)}{\partial V}|_{V=H}$ are functions of H, time to maturity, interest rate, payout ratio, tax rate, and asset volatility (see Liu et al., 2006). Given the asset volatility σ and drift rate μ, we can numerically estimate the corresponding equity value $E(t)$ by adjusting $V(t)$ in (C.1) to the data at t, which allows us to calculate the option delta Δ. Finally, given these values, we are able to compute the likelihood $L(\mu, \sigma)$ through (18). The asset volatility σ that maximizes the likelihood function in (18) is then used in the LT model to generate yield spreads.

C.2. The pricing formula of the down and out call (DOC) option

Equity value E can be treated as a call option written on the underlying asset V and its value is given by

$$\begin{aligned} E(V,H) = {} & VN(a) - Xe^{-rT}N(a - \Sigma\sqrt{T}) - V(H/V)^{2\eta}N(b) \\ & + Xe^{-rT}(H/V)^{2\eta-2}N(b - \Sigma\sqrt{T}) + R(H/V)^{2\eta-1}N(c) \\ & + R(H/V)N(c - 2\eta\Sigma\sqrt{T}), \end{aligned} \tag{C.5}$$

where V is the asset value, X is the book value of liabilities, H is the barrier level, σ is the asset volatility, r is the risk-free interest rate, T is the time to maturity, R is the rebate paid to the equity holders upon default, and

$$a = \begin{cases} \frac{\ln(V/X) + (r + \Sigma^2/2)T}{\Sigma\sqrt{T}}, & \text{for } X \geqslant H, \\ \frac{\ln(V/H) + (r + \Sigma^2/2)T}{\Sigma\sqrt{T}}, & \text{for } X < H, \end{cases}$$

$$b = \begin{cases} \frac{\ln[H^2/(VX)] + (r + \Sigma^2/2)T}{\Sigma\sqrt{T}}, & \text{for } X \geqslant H, \\ \frac{\ln(H/V) + (r + \Sigma^2/2)T}{\Sigma\sqrt{T}}, & \text{for } X < H, \end{cases}$$

$$c = \frac{\ln(H/V) + (r + \Sigma^2/2)T}{\Sigma\sqrt{T}},$$

$$\eta = \frac{r}{\Sigma^2} + \frac{1}{2}.$$

Further assuming that the equity value at default is zero, we have

$$\begin{aligned} E(V,H) = {} & VN(a) - Xe^{-rT}N(a - \Sigma\sqrt{T}) - V(H/V)^{2\eta}N(b) \\ & + Xe^{-rT}(H/V)^{2\eta-2}N(b - \Sigma\sqrt{T}). \end{aligned} \tag{C.6}$$

Similar to the LT model, the observed equity $\widetilde{E}(t)$ can be matched by adjusting the unobservable asset value $V(t)$ at a given asset volatility σ and drift rate μ through (C.6), which permits us to calculate the maximum likelihood $L(\mu, \sigma)$ through (18). The asset volatility σ that maximizes the likelihood function in (18) is then used in the CG model to generate yield spreads.

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