553.633/433

Homework #9

Due Wednesday 11/7/18

Two problems for those enrolled in 553.433. Three problems for those enrolled in 553.633, including phase 1 of paper review:

5.3. (textbook)

A. Consider importance sampling (IS) for the purpose of estimating tail probabilities. We will compare the standard method against the IS approach. Suppose we wish to estimate $\ell \equiv P(|X| \geq c)$ for $X \sim N(0, 1)$ and $c > 0$ based on N simulation runs. Do the following:

(a) Define the function $H(X)$ for use with ℓ. What is $\text{var}(\hat{\ell})$ for $\hat{\ell}$ determined in the standard Monte Carlo way (as in Sect. 4.3 of the textbook)? The answer will be in terms of ℓ and N.

(b) Consider an IS estimate, say $\hat{\ell}^{\text{IS}}$, with a sampling (proposal) density $N(\mu, 1)$ for $\mu > 0$. What is $\text{var}(\hat{\ell}^{\text{IS}})$? The answer should be in terms of ℓ, N, μ, and $\Phi(\cdot)$ with Φ being the cdf for the standard normal distribution (i.e., for $N(0, 1)$).

(c) Let $c = 3.5$. What is the true value of ℓ? (As needed, you may use numerical integral values or probability values that are determined via table look-ups or online.) Let $N = 20{,}000$ and $\mu = 3.5$ for the proposal distribution for IS. Calculate the relative error RE in (4.6) (textbook) for both the standard method and the IS method and offer a few comments on how the accuracy of the two methods compare.

(d) Using the values of N, c, and μ in part (c), calculate 10 estimates $\hat{\ell}$ in the standard way (as in Sect. 4.3 of the textbook) and 10 estimates $\hat{\ell}^{\text{IS}}$. Are the observed accuracies of the two methods consistent with the accuracies predicted in part (c)?

NOT FOR STUDENTS IN 553.433

Paper review, Phase 1. The purpose of this assignment is to help familiarize you with the vast literature on simulation and Monte Carlo, and to allow you to study an application of *your* choice. In this assignment, you will write a short essay on a previously published paper that is relevant to 553.633. This assignment has two phases; phase 1 (below) is due with HW9; phase 2 (the review itself) will be due at a later date:

Phase 1, selection of paper. You are to identify one technical paper in the literature related to an application of ideas of Chapters 2 – 5 of the textbook or Higham (2001). In particular, the paper that you review must be related to material covered through week 10

of the class (as defined in the current online class schedule at https://sites.google.com/site/jhu553633/syllabus). The eligible topics are computer-based (pseudo) random number generation (LCGs, MRGs, inverse transform method, accept-reject method), stochastic differential equations (SDEs) and simulation of Brownian motion, discrete event models, reliability models, queuing models, statistical analysis of discrete event and general simulation models (confidence intervals; batch means; regenerative processes), bootstrap methods for uncertainty analysis, and the variance reduction methods of common and antithetic random numbers or importance sampling. (Note that this list does *not* include topics to be covered in weeks 11 – 14 of the class, such as MCMC, particle filters, etc.) The paper *must* satisfy the following:

- It must deal with a real system of some type, and can optionally include real data.
- The simulation-related aspect of the paper (i.e., the aspect related to 553.633) must be a significant fraction of the paper (not a minor part), with the associated discussion of the implementation of the simulation method and the results occupying *at least* 1/4 of the length of the paper.
- The article must be from a refereed journal or refereed conference proceedings or other "quality" publication; it should not be a textbook excerpt or other type of article. Contact one of the TAs or me if you have questions about the appropriateness of an article.

The JHU Library and its librarians are an excellent resource for assistance if needed (beyond Google, etc.) if you need help choosing a paper; you are entitled and encouraged to use that resource as a student of JHU.

For phase 1, you are to submit three pieces of information: (i) the complete citation information for the paper you plan to review (including a link to the paper), (ii) the real-world application area of the paper, and (iii) the explicit connection to the class material in Chapters 2 – 5 of the textbook or Higham (2001). *You do not have to submit the essay on the paper* (that is for phase 2). See the example below* for information to be submitted in phase 1.

Note 1: No more than three students may review a given paper. All written submissions for phase 2 must by your own (no group submissions). If more than three students choose the same paper in phase 1, at least some of the students involved will be required to pick a different paper.

Note 2: If you are doing a paper related to SDEs, then the paper must *explicitly* address at least one of the issues in Section 5 of the Higham reading (strong and/or weak convergence of the EM method) or Section 7 of Higham (linear stability). It is not acceptable to review a paper that simply uses SDEs, but does not directly address at least one of the areas in Sections 5 or 7 (as we discussed in class). The student must comment on these connections in the essay.

Note 3: The choice of paper (in terms of meeting criteria above) will be graded, with the grade included as part of HW 9.

*Here is an example of the type of format you should use in submitting the information for phase 1:

Paper to be reviewed: Spall, J. C. and Chin, D. C. (1997), "Traffic-Responsive Signal Timing for System-Wide Traffic Control," *Transportation Research, Part C*, vol. 5, pp. 153–163.
http://dx.doi.org/10.1016/S0968-090X(97)00012-0

Real-world application area: Traffic control for urban traffic networks.

Connection to class: Statistical analysis of simulation output representing alternative strategies for a discrete-event system. The paper uses methods of Chapter 3 and Sections 4.1 – 4.4 of the textbook.