553.633/433

Homework #8

Due Monday 10/29/18

Three problems:

4.13. (textbook) (compute 95% symmetric confidence interval)

A. Suppose that we are testing a null hypothesis of $\boldsymbol{\theta} = \bar{\boldsymbol{\theta}}$ against the alternative hypothesis $\boldsymbol{\theta} \neq \bar{\boldsymbol{\theta}}$ for some nominal value $\bar{\boldsymbol{\theta}}$. In testing such a hypothesis, we treat $\bar{\boldsymbol{\theta}}$ as the true value of $\boldsymbol{\theta}$, and test whether the observed $\hat{\boldsymbol{\theta}}_n$ (an estimate of based on n i.i.d. data points) is consistent with this hypothesis. Because of the difficulties in interpreting simultaneous intervals for the multiple components of $\boldsymbol{\theta}$, it is common to map the difference $\hat{\boldsymbol{\theta}}_n - \bar{\boldsymbol{\theta}}$ into a scalar via the following inner product form of test statistic:

$$(\hat{\boldsymbol{\theta}}_n - \bar{\boldsymbol{\theta}})^T \boldsymbol{F}_n(\bar{\boldsymbol{\theta}})(\hat{\boldsymbol{\theta}}_n - \bar{\boldsymbol{\theta}}).$$

Using results in Sect. 1.15.3 of the textbook and slide 7 of `Chap4_bootstrap_handout.pdf`, show that this test statistic represents an approximate sum of p squared $N(0, 1)$ random variables where $\bar{\boldsymbol{\theta}}$ is some value being tested under the null hypothesis (i.e., the test statistic is approximately chi-squared distributed with p degrees of freedom).

B. Reproduce and expand on the results in slide 17 of the online class handout, `Chap4_bootstrap_handout.pdf` (use current version at the website). In particular do parts (a) and (b):

(a) Generate an original sample of 5000 replicates of sample means (each the average of 25 points) and give a histogram (like the left plot on slide 17). Then resample from the original sample (i.e., from the 5000 sample means) according to the EDF and produce a histogram of the bootstrap sample (like the right plot on slide 17). Note that the bootstrap samples do *not* need to be built from the individual points going into the sample means; rather, the bootstrap samples should come directly from the sample means themselves.

(b) Compare the two-sided 95% uncertainty bounds (i.e., 2.5% in each tail) centered around 0 of: (i) original sample, (ii) bootstrap sample, (iii) central limit theorem. Provide a brief discussion on how the three bounds differ and why they are like they are. Use the true variance, not the estimated variance, for part (iii). Note: The problem asks you to center around 0, which is convenient when considering the central limit theorem. However, it is trivial to modify the bounds to shift to the true mean 0.5. Please submit your answers centered around 0.