

553.633/433

Homework #10

Two problems due Wednesday 11/14/18

Paper review due Friday 11/16/18 (553.633 only)

Two problems + paper review (different due dates). Follow instructions carefully:

6.1. (textbook)

A. Suppose we wish to generate values from the $\text{beta}(4, 3)$ distribution (the target distribution); the density is

$$p_X(x) = \begin{cases} 60x^3(1-x)^2 & \text{if } 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Consider both the independent M-H sampler, where $q(w | x) = q(w)$, and the accept–reject (A-R) method of Chapter 2 in the textbook. For each of these methods, generate 500 replicates of 50 random samples per replicate (for the accept–reject method, the 50 samples are the values that are accepted). That is, run each of M-H and A-R 500 times, producing 50 random samples for each method for each replicate. The replicates should be independent across the 500 runs for both algorithms (i.e., different seed for each replicate), although M-H and A-R at the n th replicate may be statistically dependent (i.e., may use the same seed for the n th run of M-H and A-R). Let $q(w) = U(0, 1)$ for M-H and let $2.0736 I_{[0, 1]}$ be the majorizing function in A-R, where $I_{[0, 1]}$ denotes the indicator function over $[0, 1)$. Do the following:

(a) Compare the observed (empirical) acceptance rates for M-H and A-R across the full range of replicates and iterations. Assume no burn-in for the M-H implementation in calculating the acceptance rate. How does the observed A-R acceptance rate compare to its theoretical value?

(b) Determine the true mean for X . For each of the 500 replicates, compute $|\bar{X} - E(X)|$ for M-H and $|\bar{X} - E(X)|$ for A-R, where \bar{X} is either the sample (ergodic) mean of the 50 M-H iterates or the sample mean of the 50 A-R values. From the 500 independent values of $|\bar{X} - E(X)|$ for each algorithm, use a statistical two-sample test to determine if there is a significant difference between the accuracy of the sample means of the M-H values or the A-R values.

(c) Comment on whether the method with the higher acceptance rate is necessarily the “better” of the two methods, both in the specific problem above and in general.

NOT FOR STUDENTS IN 553.433

Paper review, Phase 2, completion of essay, due electronically to Turnitin by 5 PM, Friday, 11/16/18. In phase 1 (due 11/7/18), you submitted a proposed paper for review. You now do the review itself. Your essay should be brief and to the point—two or so typed paragraphs summarizing the essence of the paper and the *relationship to the material in the class*. Your essay should indicate that you actually read the paper and should *not* simply be a regurgitation of the author(s)’s abstract or introduction. You can comment on the problem formulation, potential shortcomings, connections to specific topics or examples in the course, comparative numerical results, etc., especially as relates to material covered in the class. Because of the length restrictions, you will probably not be able to discuss all of these aspects.

Your essay should use proper grammar and logic, but you are not expected to provide a formal mathematical proof of any claims that you make. You may pretend that you are writing to company manager with an M.S. degree in some technical area involving probability and stochastic processes, but who is *not* familiar with the details of the area of the paper. As usual, this essay *must be written in your own words* and not copied from other sources.

The length of your review is restricted to no more than one typed page, single-spaced (12 point font); shorter is fine (but not less than 3/4 of a page). (I will deduct points for reviews that are outside of the 3/4 to one page length and/or that use smaller font.) As a guideline, the review should be between 500 and 650 words (including heading information) when measured using the “Word Count” function in Word (or equivalent if using other package such as LaTeX). Submit the essay in electronic form *only* to Turnitin (no paper submission). The submission must be in either Word or PDF format; if the latter, the document must be created directly from the native Word or Latex files; do not submit a scanned file. The essay must include your name and the citation information for the paper (including link) at the top.