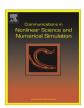
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Information spreading on dynamic social networks



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ABSTRACT

Nowadays, information spreading on social networks has triggered an explosive attention in various disciplines. Most of previous works in this area mainly focus on discussing the effects of spreading probability or immunization strategy on static networks. However, in real systems, the peer-to-peer network structure changes constantly according to frequently social activities of users. In order to capture this dynamical property and study its impact on information spreading, in this paper, a link rewiring strategy based on the Fermi function is introduced. In the present model, the informed individuals tend to break old links and reconnect to their second-order friends with more uninformed neighbors. Simulation results on the susceptible-infected-recovered (SIR) model with fixed recovery time T=1 indicate that the information would spread more faster and broader with the proposed rewiring strategy. Extensive analyses of the information cascade size distribution show that the spreading process of the initial steps plays a very important role, that is to say, the information will spread out if it is still survival at the beginning time. The proposed model may shed some light on the in-depth understanding of information spreading on dynamical social networks.

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1. Introduction

The epidemic spreading based on complex networks, where nodes represent individuals or organizations and links denote their interactions, has attracted an increasing attention in recent years [1–3]. Epidemic spreading is a dynamic process in which an item is transmitted from an infected individual to a susceptible individual through the link between them. Therefore, the network structure is a particularly important factor for the efficiency of epidemic spreading. Recently, many pioneering works about susceptible-infected-susceptible (SIS) and susceptible-infected-recovered (SIR) models indicate that a highly heterogeneous structure will lead to the absence of any outbreak threshold [4] while the epidemic spreading on small-world network exhibits critical behavior [5]. The voluntary vaccination strategy under game theory framework shows that the epidemic spreading on scale-free networks can be favorably and easily controlled [6,7]. However, all those interesting results are obtained based on the research of the static network, where interactions are always fixed. By contrast, in real online systems, people communicate with various individuals and might make new friends everyday. That is to say, the social communication network, also referred to as the peer-to-peer network, would change its topology dynamically. Consequently, it would be very suitable to study such dynamic networks with the *rewiring strategy* [8], where the network structure changes by breaking old links and forming new ones.

In the past few years, many researches have focused on the epidemic spreading problem in such dynamically contacting networks based on the link rewiring strategy [9–15]. The most important and widely used one is the *adaptive model* [10,11], in which the susceptible individuals try to avoid contacting the infected individuals [10,12,13]. Simulation results of *SIS*

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[10,13] and susceptible-infected-recovery-susceptible (SIRS) models [12] on adaptive networks show that the epidemic outbreak threshold would be larger than that on static networks. It indicates that the rewiring process typically tends to suppress epidemic spreading via isolating infected individuals. Recently, Yoo et al. [14] proposed a fitness-adaptive rewiring model where each individual's degree is preserved in the adaptive model [15]. They found that the speed of approaching the epidemic threshold is delayed and the prevalence is reduced comparing with adaptive models. Above all, those epidemic spreading researches on dynamic networks based on the adaptive model indicate that segregating infected individuals (or susceptible individuals) is an efficient strategy of reducing the fraction of susceptible-infected interactions, as well as preventing the outbreak of the whole spreading process.

However, information spreading is quite different from disease infections due to its specific features, such as time decaying effect [16], tie strength [17], information contents [18], memory effects [19], social reinforcement [20,21], non-redundancy of contacts [22], etc. In this paper, we propose a new rewiring model to study information spreading on dynamic networks where individuals will select the neighbors with larger payoff [23] following the Fermi function [24–26]. In conventional statistical physics. Fermi function is used to describe the probability of occupancy for an electron energy state at certain energy level by an electron. In the present model, we consider such energy level as payoff of rewiring strategy. That is to say, a given node will change its connection by comparing the alternatives' payoff. There is already a vast class of researches trying to apply Fermi function in modeling social dynamics. Fu et al. [26] used the Fermi function to evaluate the expected costs and benefits of vaccination via exploring the roles of individual imitation behaviour and population structure. Zhang et al. [27] considered that individual would adopt rewiring or migration reaction to adverse neighborhoods following the Fermi function and the mixture of different reactions led to much more favorable for the evolution of cooperation. Pacheco et al. [28] adopted the pair-wise comparison strategy for seeking new interactions of rational individuals based on Fermi function. Analogously, Santos et al. [29] proposed a computational model to allow individuals to be able to self-organize their social ties, based exclusively on their self-interest, in order to solve the evolutionary cooperative-competitive dilemma. Van Segbroeck et al. [30] alternatively introduced a model using the payoff-dependent Fermi function. They demonstrated that defectors were more rapid to break adverse links in order to achieve maximum fitness state, which finally led to a more heterogeneous network structure and improved cooperators' survivability.

Different from adaptive models, in this model, the informed individual will break the susceptible-infected link if the susceptible individual's payoff (the number of susceptible neighbors of the considering individual) is less than one randomly selected among its second-order neighbors, and rewire the link to the selected susceptible individual (see Fig. 1). Simulation results on various networks show that the spreading on dynamic networks is more efficient than that on static networks. Especially for the scale-free network, the information spreading prevalence forms two regimes, indicating that the information diffusion either dies out quickly or spreads into a finite fraction of the total population.

2. Model

In this paper, we use the SIR model with fixed recovery time T=1 to illustrate the proposed information spreading process. All individuals in the system must be one of the three discrete states: the uninformed individuals (defined as S-state), the active informed individuals (defined as I-state) which would transmit information to their S neighbors and the inactive informed individuals (defined as R-state) which know the information but would not transmit it any more. Initially, a node is randomly selected as the I-state, which is considered as the seed for the information spreading, and all other individuals are set as the S-state. At each time step, the I individuals transmit the information to the S nodes through S-I links with the spreading rate λ . In social network, the individuals usually won't transfer an information item more than once to the same neighbor, namely the non-redundancy property [22]. Therefore, each S-I link can just be used once in our model, no matter the transmission through this link is successful or not. That is to say, when one I individual transmits the information to all its S neighbors at one time step, the I individual will change to R-state and would not be able to transmit information any more. The information spreading process stops until there is no I-state individual in the system.

Generally, information decays very fast [16], that is to say, some information would lose attraction within a very short period. Hence, how to spread the information quickly is a critically important problem in the social system [31]. The link

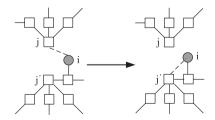


Fig. 1. Illustration of the one-step rewiring process for a given node *i*. The left panel is the original network, and the gray circle *i* represents the informed individual (*I*-state), while squares represent the uninformed individuals (*S*-state). In the original network, individual *j* is *i*'s neighbor and *j'* is *i*'s second-order neighbor. The payoffs of *j* and *j'* are $\pi_j = 3$ and $\pi_{j'} = 4$ respectively. Node *i* will break the link to *j* (left panel) and reconnect to *j'* (right panel) with probability $p_f = \frac{1}{1+e^{-\frac{\pi}{2}(\pi_{j'}-\pi_{j'})}}$.

rewiring strategy is one of the possible methods to enhance the information spreading efficiency through changing the network structure. Consequently, we consider the link rewiring strategy as the following way (see Fig. 1). For each S node (such as j in Fig. 1) that connects to the I-state nodes (i), I node will rewire the link to a randomly chosen S node (j') among the i's second-order neighbors with probability p_f , which is determined according to the Fermi function from statistical physics [24–26,32]

$$p_f = \frac{1}{1 + e^{-\beta(\pi_f - \pi_j)}},$$
 (1)

where π_j , $\pi_{j'}$ are respectively the number of the S neighbors of two target nodes S_j , $S_{j'}$. And π_j can be considered as the payoff [25] of strategy that connecting with the individual j. The individual with more connections (corresponding to the large payoff) is regarded as *information hungry*, for they would be more likely to communicate with others, hence the information is more likely to spread out through such individuals. In general, individuals may know only the payoff with a small range instead of all social systems, so that the rewiring individual (node j' in Fig. 1) is randomly chosen in the second-order neighbors of the corresponding I individual in this model.

We assume that individuals are of finite rationality following the common practice. That is to say, individuals prefer to choose strategies with higher payoff, while they are also possible to select those with lower payoff. Fermi function is a widely used way to achieve this purpose with incorporating stochastic element into the model. For small β , individuals are less responsive to the payoff differences, while large β would weaken this stochastic effect, and individuals may switch to the nodes with higher payoff, even if the payoff difference is small [25,26]. In our model, β is generalized to $[-\infty, +\infty]$. The individual with larger payoff will be chosen with larger probability when $\beta > 0$, and vice verse. The rewiring probability becomes neutral when $\beta = 0$, which corresponds to the case of random rewiring.

3. Results & analysis

The proposed model is performed on four representative networks with the same total population N and average degree $\langle k \rangle$. (1) Regular network: a ring lattice with N nodes and k edges per node [33]; (2) Small-world network: rewiring each edge at random with probability p_s based on the regular network [33]; (3) Random network: randomly rewiring probability $p_s = 1$ [33]; and (4) Scale-free network: m = k/2 in the BA model where m is the number of edges for the new node [34], and the network exhibits a power-law degree distribution $p(k) \sim k^{-\gamma}$ with $\gamma = 3$. To alleviate the effect of randomly selecting the spreading seed, all the simulation results are obtained by averaging over 10,000 independent realizations. In all simulations, we set N = 10,000 and $\langle k \rangle = 6$.

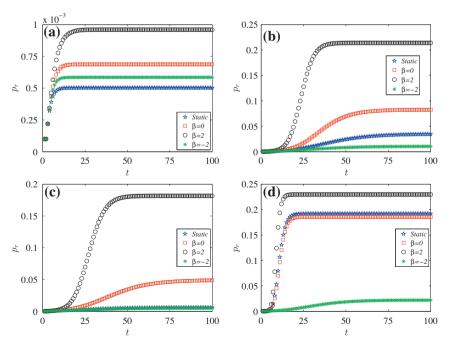


Fig. 2. Dynamics of p_r with different methods: static network (blue pentagram), $\beta = 0$ (red square), $\beta = 2$ (black circle) and $\beta = -2$ (green star). All simulations are run on four representative networks: (a) regular network; (b) random network; (c) small-world network; (d) scale-free network. The spreading rate is set to $\lambda = 0.2$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

To illustrate the spreading process, we firstly focus on the time evolution of the proportion of R-state individuals (p_r) in the systems. p_r indicates the information diffusion range, and larger p_r value in the stationary state shows the broader spreading. Fig. 2 illustrates the dynamics of p_r with the spreading rate $\lambda = 0.2$. It can be seen that the information spreading on scale-free network is both broader and faster than that on the other networks. This is caused by the heterogeneous degree distribution of the scale-free network where the hub nodes hold the connective in the spreading process (see Fig. 8). The number of R-state nodes follows the relation "random > small-world > regular" which is consistent with the traditional understanding of the epidemic spreading [35]. This is different from the information spreading model considering the social reinforcement [22], in which the spreading on regular network is faster and broader than that on random network. For each kind of network, we show the results of $\beta = 2, 0, -2$ and the static network as the baseline to illustrate the influence of the link rewiring strategy on the information spreading process. Fig. 2 shows that in all observed networks, the spreading results obtained for $\beta = 2$ is broader than others, and the enhancement is significant compared with the static network. $\beta > 0$ indicates the case that the I-state nodes are more likely to rewire the links to nodes with more S neighbors, and the informed chance of the corresponding S nodes will increase through the rewired links. Once the information is transmitted to the large payoff S individuals, it will spread out quickly for more S neighbors will be informed through the corresponding individuals at the next step. In contrast, the spreading process will die quickly for β < 0. Furthermore, it is interesting to find out that the influence of the link rewiring strategy on the scale-free network is also quite different from others. Neutrally rewiring strategy (β = 0) enhances the spreading on regular, small-world and random networks, while slows the diffusion on scale-free networks, since it breaks the already present preferential attachment. According to Fig. 2(a), it is obtained that the enhancement of the rewiring strategy on the regular network is very weak. With considering the limited attention [36] of the individuals in social systems, the present rewiring range is confined in the second-order neighbors of the corresponding *I*-state individual. But the second-order neighbors are also just the neighbors with large probability in the regular network for the large clustering coefficient. As a result, the information spreading enhancement is very limited with the proposed rewiring strategy for the regular network.

The informed individuals in the network can be considered as the cascade [37], which is a set of the informed individuals generated by the information spreading process. Fig. 3 shows the cascade size distribution (CSD) over 10,000 independent realizations. The CSD can be considered as two regimes: a power-law for small sizes (the blue area in Fig. 3), and the rest part for large sizes which is very different over the network structure and parameter β . According to Fig. 3, it can be found that the stronger heterogeneity of the network is, the more sharply the power-law range decays and the more higher peaks the large size part exhibits. The CSD for large size (n > 10) on the scale-free network (the bottom four subgraphs which are marked with d) can be described as a log-normal function, which is very similar to the empirical results on Digg¹ where the information spreading on the fans' network with the power-law degree distribution structure [38]. For the same network structure, there are more cascades with large size for $\beta > 0$, resulting in more informed individuals in the stationary state which coincides with Fig. 2. It is interesting to find out that the two regimes of the CSD are separated absolutely in the scale-free network for $\beta = 2$ (Fig. 3(d3)) and there is nearly no cascade with the size ranging from 10 to 4000, which indicates that the information will either spread to a high level or die quickly. Therefore, the spreading at the initial steps are very important. If the information is still survival after the beginning several steps, it will spread into a fraction of the total population. In order to illustrate the evolution of the information cascade, we investigate the dynamics of I-state individuals proportion p_i in Fig. 4. In this model, the Istate individuals are just the newly informed individuals in the last step. For all the networks, the p_i increases sharply at several initial steps, then decrease until the spread process stops. The p_i with positive β is much larger than the corresponding static networks, which means that the information spreads much faster with the link rewiring strategy. In the scale-free networks, the spreading is quite quick that more than 8% of the total population are informed in a single step.

Fig. 5 displays the final spreading level for each network structure as a function of the spreading rate λ . It is easy to realize that p_r increases with λ for all the rewiring strategies and network structures. When λ is small, the information spreading enhancement with the positive β could be very significant. In addition, the curve with positive β shows that the p_r peaks suddenly with small λ , which means that positive β makes the critical λ value diminish. For large λ , nearly all the individuals will be changed to the R-state. Furthermore, it is very obvious that there is the inhibitory effect on information spreading using the rewiring strategy with negative β on the scale-free network according to Fig. 5(d).

The aforementioned results show that spreading process on regular network and random network are quite different. The network structure could be parameterized by the randomly rewiring probability p_s (namely 'small-world' parameter), where $p_s = 0$ and $p_s = 1$ correspond to regular and random network, respectively. We plot p_r at the stationary state as a function of p_s in Fig. 6. Similar to the previous studies [5,39], we also find out that p_r increases with the small-world parameter p_s . In the small-world network, as the existence of the long range connection, the average distance of the network will decrease [33]. The information spreading should be faster and broader, as the distance between I and S nodes becomes shorter. The influence of the link rewiring with Fermi function is robust with the change of p_s , and the information spreading is much broader for p_s 0. Experiments show that, with positive p_s even very small p_s can bring a remarkable improvement in promoting p_r .

In order to illustrate how the parameter β influences the spreading process, we plot p_r as a function of parameter β on the scale-free network in Fig. 7. The simulation result shows that a transition occurs between a regime where the spreading process dies out within a small neighborhood of the *seed*, and a regime where it spreads over a finite fraction of the whole

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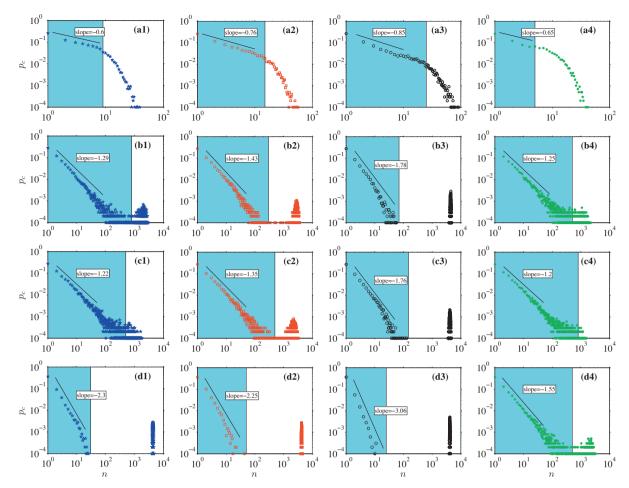


Fig. 3. The cascade size distribution p_c with randomly selecting the spreading *seed* for different networks. The figures from left to right are simulated with (1) static network, (2) β = 0, (3) β = 2 and (4) β = -2 respectively; from top to bottom are (a) regular network, (b) random network, (c) small-world network and (d) scale-free network respectively. The spreading rate is set to λ = 0.2. The results are obtained by averaging over 10,000 independent realizations.

population. The spreading is very limited (p_r is round 1%) for β < 0, where the number of R-state individuals raises very slightly with increasement of β . It means that rewiring active links to the nodes with less S neighbors will prevent information spreading. And this could be used as a strategy to control the spreading for virus and rumors. The spreading broadens very sharply when β increases around 0, which indicates that β = 0 should be the saltation point. For β > 0, the active link will be more likely to rewire to the nodes with more S neighbors. In addition, as long as β is positive, the information will spread into a very broad range (p_r around 23%). The behavior of the informed numbers around β = 0 indicates that, the symbol of β rather than the accurate numerical value is the most significant factor to affect the information spreading with the proposed rewiring strategy.

4. Discussion

The simulation results show that the positive β induces a broader and faster information spreading process. To interpret the reason why the positive β enhances the spreading, we investigate the dynamics of the average number of S nodes connecting with the newly informed nodes in Fig. 8. The results of the four curves at the beginning of the spreading process coincide with Fig. 2, which suggests that the information item will spread broader and faster if the nodes with more S neighbors are informed as the spreading initializes. In addition, the result on the scale-free network is consistent with Barthélemy's work [40], where the dynamics of the spreading is characterized by a hierarchical structure, that the information is transmitted to large degree nodes firstly, then pervades the network via smaller degree nodes rapidly. And the hierarchical spreading patten would be more obvious when $\beta > 0$ (see Fig. 8(d)). However, the large-degree nodes don't always speed up the spreading process, such as the game-theoretic models of the innovation spread [41].

For the positive β on the generalized Fermi function, we will obtain large rewiring probability by $\frac{1}{1+e^{-\beta(n_j-n_j)}}$ if the payoff of S-state node j' is larger than j, and vice verse. That is to say, the I nodes will be more likely to rewire the link to the S nodes

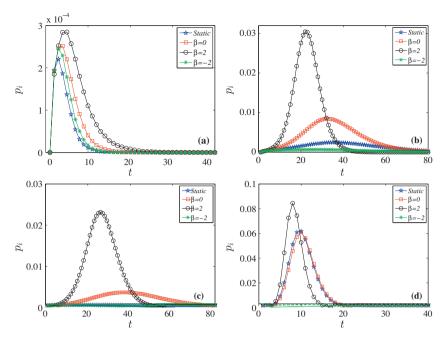


Fig. 4. Dynamics of p_i with different methods: static network (blue pentagram), $\beta = 0$ (red square), $\beta = 2$ (black circle) and $\beta = -2$ (green star). All simulations are run on four representative networks; (a) regular network; (b) random network; (c) small-world network; (d) scale-free network. The spreading rate is set to $\lambda = 0.2$. The results are obtained by averaging over 10^4 independent realizations. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

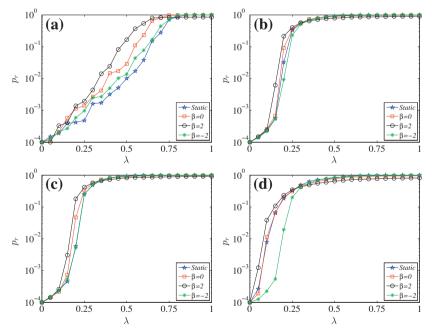


Fig. 5. The dependence of p_r at the final state on the spreading rate λ with different methods: static network (blue pentagram), β = 0 (red square), β = 2 (black circle) and β = -2 (green star). All simulations are run on four representative networks: (a) regular network, (b) random network, (c) small-world network and (d) scale-free network. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

with more *S* neighbors. Following this strategy, the nodes with more *S* neighbors have more chance to be informed, and the information could be more likely to spread out through such nodes. Therefore, we can observe more *S* neighbors of the newly

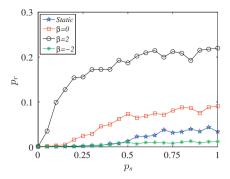


Fig. 6. p_r at the final state as a function of parameter p_s with different methods: static network (blue pentagram), $\beta = 0$ (red square), $\beta = 0$ (black circle) and $\beta = -2$ (green star). The spreading rate is set to $\lambda = 0.2$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

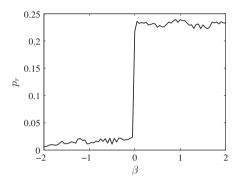


Fig. 7. p_r at the final state as a function of the parameter β on scale-free network. The spreading rate is set to $\lambda = 0.2$.

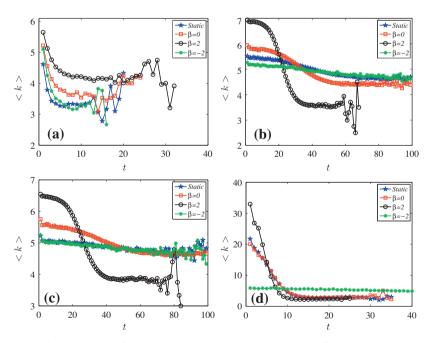


Fig. 8. The average number $(\langle k \rangle)$ of the *S* neighbors of the *I*-state individuals at each time step with different methods: static network (blue pentagram), $\beta = 0$ (red square), $\beta = 2$ (black circle) and $\beta = -2$ (green star). All simulations are run on four representative networks: (a) regular network; (b) random network; (c) small-world network; (d) scale-free network. The spreading rate is set to $\lambda = 0.2$. The results are obtained by averaging over 10^4 independent realizations. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

informed individuals with positive β (Fig. 8). For β < 0, the I individuals will more likely reconnect to the S individuals with less S neighbors, leading to quick annihilation of the spreading.

5. Conclusion

In this paper, we proposed a dynamic model for the information spreading process that considers the link rewiring based on the SIR model with fixed recovery time T=1. The rewiring probability is chosen following the generalized Fermi function based on the payoff between the two selected uninformed individuals. Simulation results on four representative networks show that the information will spread broader and faster when the parameter $\beta>0$, because that the uninformed individuals with more uninformed neighbors are more likely to be informed at the beginning spreading steps. Through those uninformed hubs, the information item can spread into a finite proportion of the population quickly. The cascade size distribution indicates that the initial steps are very important in the information spreading process, where the information can spread into a finite fraction if it can survive at the beginning several steps. In addition, the negative β can be used as a strategy to control the spreading of virus and rumors.

Recently, the research of the information spreading based on temporal networks has attracted more and more attention [42,43]. Simulation results in this paper demonstrated that with the large payoff trend, the rewiring strategy can significantly enhance the information spreading process. Moreover, it is found that the human communication pattern is of critical importance in the information diffusion [17], which provides a promising way to enhance the efficiency of *Information Filtering* [44–46] in the era of big data. For a more detailed evaluation, the temporal patterns of human communication such as the burst activity should be studied on the rewiring strategy in future work.

Acknowledgements

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