

The Impact of Heterogeneous Spreading Abilities of Network Ties on Information Spreading

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Abstract—Understanding the dynamics of information spreading over social networks has been of great interest for a long time. It has been convincingly demonstrated that the topology of a network can significantly affect information spreading over it. Information spreading is often compared on small-world networks and random networks. In this paper, we find that the heterogeneity of spreading abilities of network ties can lead to completely different spreading results over small-world and random networks. In more detail, the more heterogeneous the spreading abilities of network ties, the lower the efficiency of information spreading over small-world networks. Moreover, such heterogeneity not only slows down the spreading pace, but also lengthens the life span of the information spread over small-world networks. On the contrary, if the spreading abilities of network ties are similar or equal, small-world networks are much better than random networks for information spreading.

I. INTRODUCTION

Understanding the dynamics of information spreading in social networks is an interesting and challenging problem that has been studied by researchers from different fields for many years. At the early stage, researchers employed mathematical models, such as, the Susceptible-Infected (SI) model, the Susceptible-Infected-Susceptible (SIS) model [1], and the Susceptible-Infected-Removed (SIR) model [2], to characterize information spreading in social networks. Recently, with the emergence of online social networks, physicists and computer scientists became interested in adopting various types of social network data, such as, phone communication log files [3], friend relationships of Facebook [4], and retweets of Twitters [5], to analyze how the dynamics of information spreading is affected by network topology and intended to identify essential factors in the mechanism of information spreading.

In the real world, lots of networks (e.g., online social networks and telecommunication networks) have been identified as small-world networks [3], [4]. Therefore, to understand how epidemics, advertising slogans and computer viruses spread on this kind of networks is of much significance. Random networks, unlike small-world networks whose average shortest path is short, has long pathes between nodes and are also observed in the real world, such as, P2P networks [6]. Because of their obvious similarities and differences in topologies, these two types of networks are usually evaluated and even compared when studying information spreading.

Centola conducted an online experiment to verify whether small-world networks or random networks more favor information spreading, where volunteers were randomly chosen as a node in a small-world network or in a random network as created by the author [7]. These volunteers were recommended to adopt a healthy behaviour once their neighbours had done so. Results turned out that users received multiple recommendations could adopt the healthy behaviour with a higher probability than those who received fewer recommendations. Lots of redundant ties within a local community in small-world networks provided an ideal topology for multiple recommendations. Consequently, the adoption behaviour spread faster and father within small-world networks. Similarly, a more mathematical model was proposed in [8], where the author analyzed the impact of the social reinforcement on information spreading [9]. The above study concludes that small-world networks are more suitable for information spreading than random networks.

However, some other studies which focused on small-world networks obtain a finding that information is often trapped by local communities in small-world networks and it spreads slower than what is expected [10], [11], [12], [13], [14]. To explain this phenomenon, Karsai et al. adopted the SI model and null models to probe into possible factors [10]. They found that two factors leading to the inefficiency of information spreading in small-world networks. The first one are that the community structures and their correlation with the weights of network ties, while the second one are the inhomogeneous and bursty activity patterns on the ties. This reveals that local structures have an important impact on information spreading, which is overlooked in [7], [8].

Driven by the above two different behaviours of information spreading, we analyze the key differences between these studies [7], [8], [14]. One important difference is that the roles of network ties are treated differently. In [7], [8], ties are equal, that is, the neighbours of a node are of the same importance to this node, while in [14] network ties are supposed to have different weights according to the relationships of two connected nodes. Therefore, we take into consideration the different spreading abilities of network ties in our spreading model in order to see whether small-world or random networks more favor information spreading. In this study, the spreading abilities of network ties, which are also called weights or strengths in other studies, are referred as the probabilities of

TABLE I. THE NOTATION USED IN THE PROPOSED MODEL.

Symbol	Description
Q_{max}	The maximal attention of a node
Q_i	The current activated attention of a node i
$node_i$	The i -th node in the network
q_{ij}	The spreading abilities from $node_j$ to $node_i$
f_{ij}	The number of common friends of $node_i$ and $node_j$
n_i	The number of friends of $node_i$

information spreading from one node to another. Therefore, the proposed model focuses on the heterogeneity of spreading abilities of ties and analyzes how such heterogeneity affects information spreading. A parameter β is employed to control the heterogeneity. When β is large, the heterogeneity is strong, indicating that the ties in the network have obvious different spreading abilities. When β is small, the heterogeneity is weak. Under such scenarios, different ties are treated similarly or even equally.

In general, we propose a new information spreading model in this paper, in which the heterogeneity of spreading abilities of network ties is elegantly characterized. Through simulations performed on artificial networks, we find that whether small-world or random networks are better for information spreading largely depends on the heterogeneity of spreading abilities of network ties. Specifically, if the heterogeneity is negligible, small-world networks are the best one for information spreading. On the contrary, if the heterogeneity is not negligible, information spreading over small-world networks is largely inhibited, while that over random networks is only slightly affected by the heterogeneity. It is also observed that the effect of this heterogeneity on small-world networks is twofold: While it certainly slows down the spreading process of information, it also unexpectedly lengthens its life span.

The remainder of this paper is organized as follows. In Section II, we propose a mathematical model that characterizes the heterogeneity of spreading abilities of network ties and we are able to control it by tuning the parameter β . In Section III, we briefly describe the design of our experiments for validating the proposed model. Section IV presents the results and findings of our experiments conducted on computer-generated networks.

II. THE PROPOSED MODEL

In this section, we will formally introduce our model. We first briefly describe the idea behind the model and then address the modelling issues of user attention, the spreading ability of network ties, and the heterogeneity of spreading abilities. Finally, we present a probability function for determining how a node spreads a message, as done in other models [8], [15], [16], [17].

The notation used in the proposed model is listed in Table I.

As aforementioned, this study is inspired and motivated by two previous works [8], [15]. In this study, we assume that every node in the network has the same capacity and attention for information/messages. Each neighbor of the node has the different ability to evoke a certain amount of its attention. Every time a node receives a message from its neighbours, it will try to spread it by a probability function. Past exposures to the same message are of positive effects on the same node. And

in our model, these exposures are reflected in the cumulative attention of a node.

A. Modelling users' attention

Weng et al. [15] has done a statistic research on real Twitter data, and finds that regardless other factors, all people happen to have equal attention to messages. In his study, the author allowed different messages from neighbours to compete for this limited attention, in other words, attracting users to spread it by a probability. Much like the method in his study, all nodes are assumed to have the same maximum attention $Q_{max} = 1$, and its all neighbours are able to evoke a certain amount of this attention. For a $node_i$, if its neighbours spread a message, much of $node_i$'s attention is caught and activated. For example, if $node_j$ and $node_i$ are friends and $node_j$ has spread a message, q_{ij} of $node_i$'s attention is caught and activated. Obviously, q_{ij} represents the spreading ability of this tie. How q_{ij} is computed will be discussed in next subsection.

B. Modelling heterogeneous spreading abilities of ties

In this subsection, we will mainly focus on a single node $node_i$ and its n_i neighbourhood, denoted as $(node_{ij_1}, node_{ij_2}, \dots, node_{ij_{n_i}})$. We will model how $node_i$ will spread a message if some of its neighbours have done so. Given a network or a graph, different ties have different spreading abilities, referred as the heterogeneity of spreading abilities and embodied in the parameter q_{ij} . It is difficult to measure these various abilities, but it is the common sense that people who have close relationships often form a local group. Conversations are likely to happen within a group instead of across groups [18]. It is suggested that the number of overlapping friends can serve as an indicator to this close relationships [14]. And also, it is pointed that ties connecting close neighbours have strong weights and large spreading abilities [14]. Therefore, it is very natural to allow q_{ij_k} to have linear relationship with number of common friends between $node_i$ and $node_{j_k}$. That is

$$q_{ij_k} = \frac{f_{ij_k}}{\sum_{r=1}^{n_i} f_{ij_r}} \quad (1)$$

However, there is a risk directly using this equation. Consider if two nodes have no common friends ($f_{ij} = 0$), which is usually the case in random networks, then the spreading ability of this tie will decrease to 0, causing the message to stop spreading. As the common sense, if there is a tie existing between any pair of two nodes, then it means these two nodes are known with each other, and the spreading ability of this edge shouldn't be zero, even if there are no common friends between these two nodes. Therefore, a parameter β ($\beta > 0$) is included to smooth the above equation.

$$q_{ij_k} = \frac{\frac{1}{\beta} + f_{ij_k}}{\frac{1}{\beta} \times n_i + \sum_{r=1}^{n_i} f_{ij_r}} \quad (2)$$

Note that n_i denotes the number of friends of $node_i$. The merits of the above equation are obvious. Firstly, the heterogeneity is represented by q_{ij} , and parameter β is the key parameter which can amplify or lessen this heterogeneity. If β is large, q_{ij} is sensitive to f_{ij} , meaning that a small perturbation in

f_{ij} can lead to a big change of q_{ij} . On the other hand, if β is small, q_{ij} is insensitive to f_{ij} and in a small region around $\frac{1}{n_i}$ even for very large f_{ij} . Therefore, large β results in strong heterogeneity and small β results in weak heterogeneity. Second, no matter what value is assigned to β , for a node i , $\sum_{r=1}^{n_i} q_{ij_r} = 1$ is always the case for every node in the networks. Therefore, all nodes play fairly. The key problem we analyse in this study is how this heterogeneity affect the information spreading. So, we will assign different value to β , to see what behaviours emerge.

There is one special case that we have to point out and discuss it. Consider if a node $node_j$ has only one neighbour $node_i$, which means it is at the edge of the network, then spreading ability along this tie from $node_i$ to $node_j$ is always 1. This ability will lead to a large spreading probability along this tie, which is some what unexpected. However, we argue that, since node $node_i$ is the only information source for $node_j$, this tie is important to $node_j$ and $node_j$ is much likely to adopt messages from this neighbour. For example, in a real network, a person has followed only one friend, then the message from this only friend is much likely adopted by this person.

C. Modelling spreading probability

In this subsection, that how attention is transformed to spreading probabilities is discussed. The number of exposures to the same message has an impact on the behaviour whether people choose to transmit it or ignore it [19]. It is also been found that the social reinforcement has a positive effect on people adopting a message [7], [8], [20]. Thereby, in our model, these multiple-exposure effects are embodied in Q_i , that is to say, past exposures to the same message all have effect on the spreading behaviour. Again, let's focus on a single node i . Q_i is denoted to represent its current activated attention. At first, its activated attention is set to 0. If a neighbour $node_j$ has spread the message, q_{ij} attention of $node_i$ is activated. To embody these multiple-exposure effects, we set

$$Q_i = \sum_k q_{ij_k} \quad (3)$$

for all neighbours $node_{j_k}$ that have already spread the message. Equation (2) ensures that Q_i has the upper bond 1 for all nodes. Then, the following probability equation is proposed to determine whether a node will adopt and spread the message that passed to it:

$$P_i = \tanh(\alpha \times Q_i^2) \quad (4)$$

In this equation, Q_i and P_i have a square relationship rather than a linear relationship, which allows that a small Q_i has a little spreading probability, while a large Q_i may lead to large spreading chances. This is an empirical way to deal with the spreading probabilities. Parameter α suggests the intrinsic value of a message that how popular and interesting it is. If a message is popular, then it will be transmitted by users at a higher chance [21]. The results that emphasized in this paper are independent of parameter α unless it is too small or too large under which circumstances the message will reach nobody or anybody in the networks. The tanh function ensures that this probability is always within 0 and 1 ($P_i \in (0, 1)$).

The merits of the above model are obvious. A single parameter β is able to control to what extent this heterogeneity can be. The transmitting behaviour is a monotonous consecutive increase function of activated attention Q_i , meaning that larger Q_i has larger transmitting chances P_i and this probability is always within the range $(0, 1)$, which is controlled by the tanh function. Specifically, when $Q_i = 0$, $P_i = 0$ and when $Q_i = 1$, $P_i = \tanh(\alpha)$, which is close to 1.

III. EXPERIMENT DESIGN

In order to validate the proposed spreading model and particularly investigate the impact of the heterogeneity of spreading abilities of network ties, we carry out experiments using the proposed model on computer-generated networks. In this section, we present the design of the experiments, including parameter settings, the generation of artificial social networks, and simulation manners.

A. Parameter settings

In our experiments, α varies from 6 to 9, while β is set between 0.1 and 1. These ranges of parameters are carefully designed, within which different of spreading behaviours can be captured. We briefly justify the rationality of these values by epidemic threshold study, which is another heated topic for years. Although our model and scenarios differs from those in epidemic threshold studies, we can still make some approximations. In epidemic threshold studies [22], [23], different methods are proposed to analyze the boundaries of epidemic outbreaks and deaths. In one aspect, the threshold depends on the average connectivity (denoted as K) of the network. In our computer-generated networks, K equals to 6 as the same in work [8], [24]. Roughly, if the spreading probability is smaller than $\frac{1}{K}$, the epidemic dies out quickly and only reaches a small scope of nodes in the network. However, if this probability is larger than $\frac{1}{K}$, the epidemic is likely to reach majority of nodes in the network. So, the parameter α is set within 6 and 9. Consider, if β is close to 0, then, if a node receives a message at first time, its spreading probability is close to $p_a = \tanh(\frac{\alpha}{K^2})$. If $\alpha = 6$, then $p_a = \tanh(\frac{1}{6}) \approx 0.1651$ ($K = 6$), which is very close to $\frac{1}{K}$. That's the reason why the message reaches only a small scope of nodes as shown below when $\alpha = 6$. Therefore, for computer-generated networks, the lower bond of α is 6, and the upper bond is 9. If α is larger than 9, the simulation process will cover almost all nodes in the network, in which circumstance, no obvious behaviours can be captured regardless how β is tuned. The average connectivity of our computer-generated networks is 6, so parameter β is set between 0.1 and 1, and $\frac{1}{\beta}$ is between 10 and 1. We believe this range of value is wide enough to control this heterogeneity compared with K .

B. How computer-generated networks are generated

Three kinds of networks are generated by algorithm introduced in [24]. First, a network with N nodes is constructed, and each node is connected to its K nearest neighbours (in the experiment, $K = 6$, $N = 10000$). This is a fully ordered network and regular network. Small-world networks and random networks can be generated by reshuffling links on this regular network. At each reshuffling step, a link $a-b$ is chosen, and with probability p this link is broken and

rewired to a-c where node c is randomly chosen. Perform this step on all links in this regular network. Parameter p indicates the randomness of the networks. The larger the value of p , the more random of the network. Normally, $p \in [0, 0.01]$ represents regular networks and $p \in (0.1, 1]$ represents random networks. When $p \in [0.01, 0.1]$, it best shows significant small-world properties [24], [25]. Note that the exact boundaries between small-world and random networks or regular networks are not well established.

C. Simulation

In terms of the simulation method, when we say a node adopting or transmitting a message, it means that a node posts the message coming from one of its neighbours, and then all of its neighbours can notice this message. Firstly, all Q_i are set to be 0 and a node is randomly chosen as the seed which spreads the message to all of its neighbours. And all of its neighbours will update their current activated attention Q_i according to Equations (2) and (3), and consider whether to adopt the message or not according to Equations (4). In next step, only those who received the message in the last step will make the same decision whether adopting it or not. If a node has already adopted (transmitted) the message, it will lose its interest in this process at once and will not join in this process. When no new node receives the message from its neighbours, the simulation comes to the end. We denote how many nodes that have adopted and transmitted the message to indicate what kind of networks is more suitable for information spreading.

IV. EXPERIMENTAL RESULTS

In this section, we present the experimental results on computer-generated networks, and discuss the behaviours we observe. For every combination of α and β , 1000 independent runs are performed on three kinds of networks with different p , and the numeric results are averaged over these simulations.

A. What kinds of networks more favor information spreading

In general, we want to explore which kind of networks is more efficient for information spreading under different strengths of the heterogeneity. To achieve this, we present our results with a single value of β in each subplot and then we can observe different behaviours with different β as Figure 1 shows. In this plot, we measure the number of adopted nodes as a function of randomness. Obviously, when the heterogeneity is strong as shown in Figure 1(a-b) ($\beta=0.9, 0.7$), no matter what value of α is, the peaks of the curves all appear in random networks region ($p \in [0.1, 1]$). It indicates that under the strong effect of the heterogeneity, random networks provide the best topology for information spreading, supporting the results in [26]. However, it is difficult to compare regular networks ($p \in [0, 0.01]$) with small-world networks ($p \in [0.01, 0.1]$). Roughly, for the large value of β , when α is large, for example, $\alpha = 9$, small-world is better than regular networks for information spreading, while when α is small, for example, $\alpha = 6$, it is on the contrary.

But as the heterogeneity becomes weaker (Figure 1(c-d) with smaller value of β), it can be observed that small-world networks (randomness p is around 0.1, some peaks are at randomness slightly larger than 0.1) eventually outperform

random networks, which is consistent with the results in Refs. [7], [8].

Our parameter α can control the overall spreading results by assigning a large or a small value to it. The number of adopted nodes can be sharply increased or decreased by tuning the value of α . As discussed above in Section II, the first spreading can not be designed too large (compared to average degree K), or the message will eventually reach a significant scope of the network no matter on what types of networks. For instance, illustrated in Figure 1(d), when α equals to 9, even on regular networks, it can cover over 40% nodes on the network.

B. How heterogeneity affects the spreading

Next, we investigate how the heterogeneity affects the spreading by tuning the value of β , given that the value of α is fixed.

It is clearly manifested in Figure 2 that, once α is fixed for a network, as heterogeneity becomes stronger (from 0.1 to 0.9), the spreading is more inhibited in small-world networks and regular networks. This means that if the heterogeneity is more amplified, the spreading is more inhibited, except for the case of regular networks with $\alpha = 6$, under which circumstances the spreading is limited to an extremely small scope regardless how parameter β changes.

But when p is close to 1, the random networks are affected little. This suggests that the heterogeneity has no significant impact on random networks.

C. The Impact of heterogeneity on spreading pace and life span

We next investigate the impact of heterogeneity in spreading abilities of ties on information spreading from another two views, the spreading pace and the life span.

Recall in the last paragraph of Section III, where simulation method is defined. So, we define the life span as the total number of steps of the simulation process and spreading pace as the average number of nodes that adopt the message at each step.

It can be observed that when the heterogeneity is overlooked, the message travels at a very fast pace while it travels slowly when considering this heterogeneity on small-world networks. This is easy to understand in common sense.

However, as shown in Figure 3(a), it is amazing to find this strange behaviour that while the existence of heterogeneity does slow down the pace that a message travels but it lengthens the life span of the message unexpectedly. The relationship between this heterogeneity and pace and life span is obvious. The stronger the heterogeneity, the slower the pace and the longer the life span. In the real circumstances, this behaviour can be explained as people needing more sources (multi-copies of the message from their neighbors) to approve and adopt the message from neighbours who are not familiar with, thus lengthening the time and reducing the speed.

However, no obvious effects of this heterogeneity on random networks emerge. As Figure 3(b) shows, the number of adopted nodes at each step is steady regardless how parameter

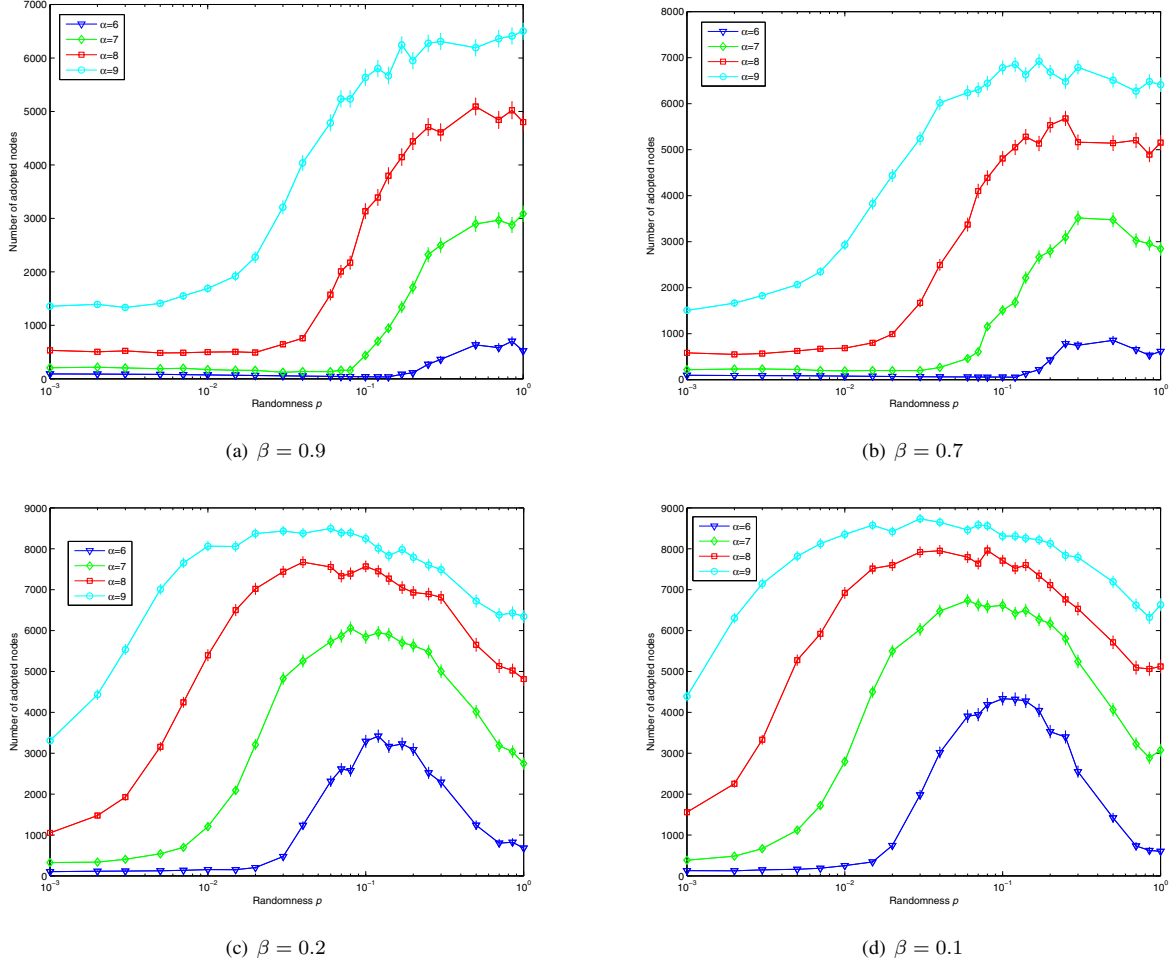


Fig. 1. The number of adopted nodes as a function of randomness p given β . Other parameters are set as $K = 6$, $N = 10000$. The results are averaged over 1000 independent simulations.

β changes, meaning that random networks acquire autoimmunity for this heterogeneity.

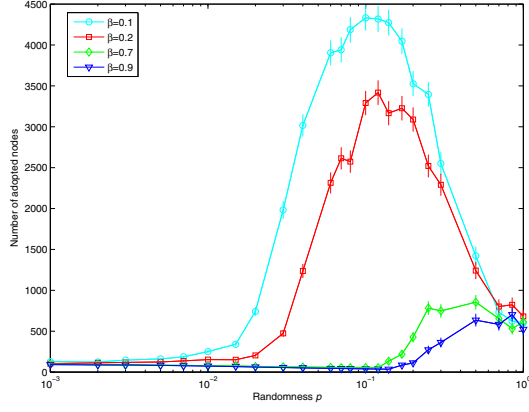
D. Remarks

Here we briefly conclude all the results we have got. First of all, the heterogeneity has very obvious and significant impacts on small-world networks. The strong heterogeneity can lead to less efficiency of information spreading, while the weak heterogeneity has no bad effects on small-world networks. However, we find no obvious impacts on random networks when p is close to 1. This phenomenon is due to a variety of reasons. First of all, the un-uniform distributed spreading abilities cause the message to stop spreading along a weak tie, that is, if node a and node b are bridge between two communities, the spreading from a to b is difficult under the strong heterogeneity, for the simple reason q_{ab} is quite small. However, when the heterogeneity is weak, q_{ab} is almost the same no matter a and b are within or across communities, and nodes in small-world have much more chances spreading this message, leading to the high efficiency of spreading. Second, the topology structures are very important. In small-world,

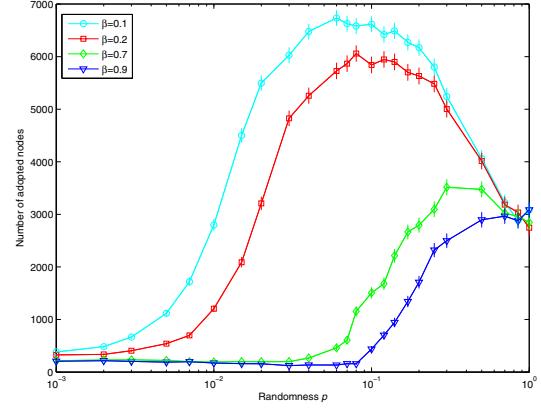
nodes are formed by groups, in which case, nodes are sensitive to parameter f_{ij} , this leading to very big difference in q_{ij} . However, nodes in random networks are randomly connected, in which case, f_{ij} is very similar for every node. Therefore, the heterogeneity has no impact on random networks.

V. CONCLUSIONS

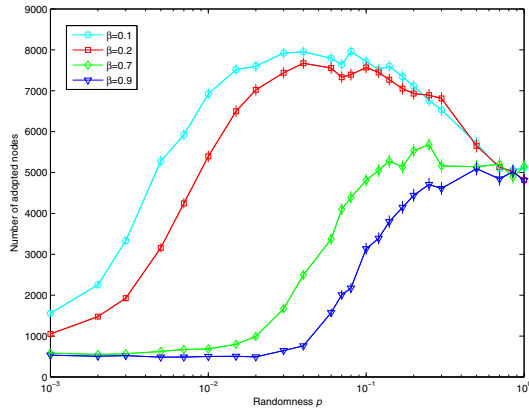
In this paper, we have investigated whether a message spreads faster and broader over small-world networks than over random networks, which has been debated for many years. For this purpose, we have proposed a unified model, based on which we have shown that different heterogeneity in spreading abilities of network ties can lead to completely different results. Experiments indicate that if the heterogeneity in spreading abilities of ties is strong, small-world networks are to large extent inhibited. To be specific, the spreading speed of a message is slowed down and its life span is lengthened under the impact of the strong heterogeneity. A possible explanation is that in social networks, if a message travels in an un-trusted path, people need more sources (more copies from neighbours) to judge it, which lengthens its life span. If the heterogeneity



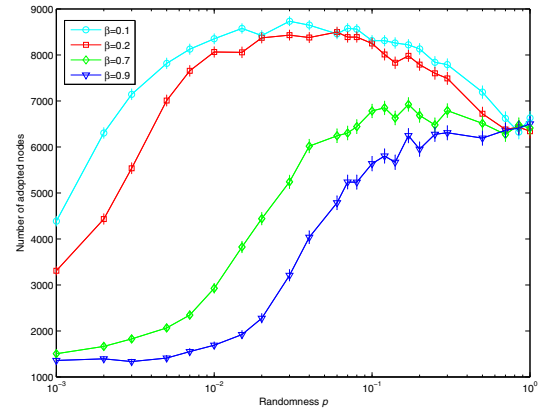
(a) $\alpha = 6$



(b) $\alpha = 7$

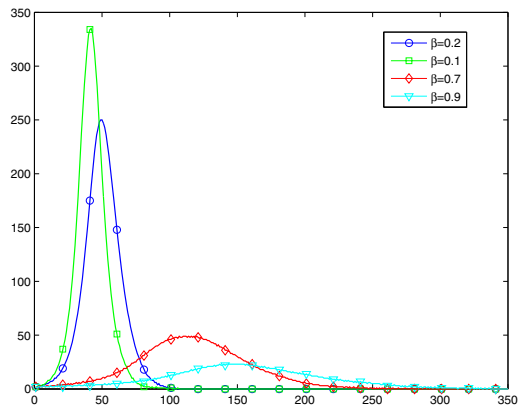


(c) $\alpha = 8$

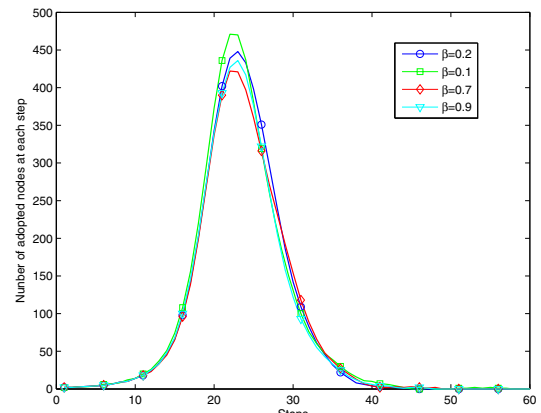


(d) $\alpha = 9$

Fig. 2. The number of adopted nodes as a function of randomness p given α . Other parameters are set as $K = 6$, $N = 10000$. The results are averaged over 1000 independent simulations.



(a) Small-world networks with $p = 0.1$



(b) Random networks with $p = 1.0$

Fig. 3. The number of adopted nodes at each step. Other parameters are set as $\alpha = 8$, $N = 10000$, $K = 6$. The results are averaged over 1000 independent simulations.

in spreading abilities of network ties is weak, small-world networks outperform random and regular networks, which supports the experiments of two recent studies [7], [8].

In fact, the heterogeneity in spreading abilities of network ties, modeled by the number of common friends of every two connected nodes, has little effect on random networks. This is due to the fact that in random networks, closed triangles are negligible, thus every two connected nodes have very few common friends and each tie has almost equal ability for spreading.

However, there are many other factors that can affect information spreading. It may be interesting to identify the most crucial factors that lead to transmitting a message on individual level which is rarely studied so far. People transmit a message for its content or just because of the influence of their neighbours, and so on. There are lots of to be done in this field and which will be our future work.

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