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SIHR rumor spreading model in social networks

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ABSTRACT

There are significant differences between rumor spreading and epidemic spreading in social networks, especially with consideration of the mutual effect of forgetting and remembering mechanisms. In this paper, a new rumor spreading model, Susceptible-Infected-Hibernator-Removed (SIHR) model, is developed. The model extends the classical Susceptible-Infected-Removed (SIR) rumor spreading model by adding a direct link from ignorants to stiflers and a new kind of people-Hibernators. We derive mean-field equations that describe the dynamics of the SIHR model in social networks. Then a steady-state analysis is conducted to investigate the final size of the rumor spreading under various spreading rate, stifling rate, forgetting rate, and average degree of the network. We discuss the spreading threshold and find the relationship between the final size of the rumor and two probabilities. Also Runge–Kutta method is used for numerical simulation which shows that the direct link from the ignorants to the stiflers advances the rumor terminal time and reduces the maximum rumor influence. Moreover, the forgetting and remembering mechanisms of hibernators postpone the rumor terminal time and reduce the maximum rumor influence.

1. Introduction

Rumors, as an important form of social communication and a typical social phenomenon run through the whole evolutionary history of mankind [1–5]. People disseminate rumors in order to achieve the purpose of increasing awareness, slandering others, manufacturing momentum, diverting attention, causing panic and so on [6–8]. For example, recently, the nuclear leakage in Japan caused a salt-buying frenzy in China. This salt-buying frenzy came from the rumors that the radiation leakage in Japan could pollute sea water and then sea salt, and additionally iodized salt could help to protect people from nuclear radiation. With the rumors spreading, this frenzy swept across China and caused social panic as well as instability in just a few days. In some cities, the salt price rose up more than ten times of the normal price and the amount of salt bought by some families is enough for several decades of normal consumption. Traditionally, rumors propagated by word of mouth. Nowadays, with the emergence of the internet, rumors spread by instant messengers, emails or publishing bloggings [9,10] that provide faster velocity of transmission.

The study of the rumor models began in the 1960s. As rumor spreading shows an interesting similarity to the epidemic spreading, most of the existing models of rumor spreading are based on the epidemic models [11,12]. Daley and Kendall [13] first proposed the basic DK model of rumor spreading. Maki and Thomson [14] focused on the analysis of the rumor spreading model based on mathematical theory and developed the MK model. The DK and MK models have been used extensively for quantitative studies of rumor spreading [15–21], but major shortcomings of these models are that they have not taken into account the topological characteristics of social networks and they were not suitable for describing rumor spreading mechanism on large-scale social networks. Sudbury [22] insisted that the dynamic behavior of rumor spreading matched

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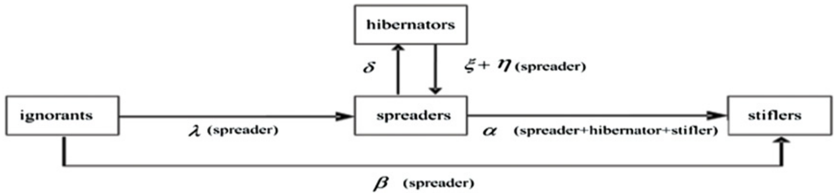


Fig. 1. Structure of SIHR rumor spreading process.

the SIR model. Zanette [23,24] established a rumor spreading model on small-world networks and provided a threshold of rumor spreading. Some scholars studied applications of the stochastic version of the MK model on scale-free networks and showed that the uniformity of the network has a major impact on the dynamic mechanism of rumor spreading [25–27]. Some scholars applied the MK model on complex networks and described a formulation of this model on networks in terms of interacting Markov chains [28,29]. Nekovee et al. [28] also derived the rumor spreading mean-field equations related to the arbitrary degree correlations networks and investigated the final size of rumors on general networks.

The dynamic behavior of rumor spreading assembles the SIR epidemic spreading model. The entire population is divided into three groups that are S, I, R. Here S, I, R stand for the people who are spreading rumor (Spreaders, similar to Infective), those who never heard rumor (Ignorants, similar to Susceptible), and the ones who heard rumor but do not spread it (Stiflers, similar to Removed), respectively. The rules of rumor spreading are as follows. ① When a spreader contacts with an ignorant, the ignorant becomes a spreader with a certain probability. ② When a spreader contacts with another spreader or a stifler, the initiating spreader becomes a stifler with a certain probability. The tendency of individuals to accept a rumor with a certain probability depends on the importance and credibility of the rumor. On the other hand, individuals no longer spread a rumor when they know the rumor is outdated or wrong. Besides, the spreaders may stop spreading a rumor spontaneously at a certain rate in consideration of forgetting mechanism [28]. The spreading process starts with one or more individuals being informed of a rumor and terminates when no spreaders are left in the population.

The above rumor spreading mechanism has some pities. For example, when an ignorant contacts with a spreader, the ignorant may maintain the state of ignorance or accept the rumor to become a spreader. This assumption ignores the possibility that the ignorant become a stifler directly. This possibility lies in that some ignorants have a strong background knowledge, some ignorants have logical reasoning ability and some have little interest in the rumor. Consider the Chinese salt-buying frenzy case. Some people knew that iodized salt is extracted from mineral not sea. They would not believe the rumor and would not spread it. Another pity of the above rumor spreading mechanism is that although some researchers considered forgetting mechanism, they did not consider remembering mechanism. Individuals can spontaneously cease spreading the rumor by forgetting, also can spontaneously or stimulated restart spreading the rumor by remembering.

As a result, we extend the classical SIR rumor spreading model by ① considering the situation when an ignorant contacts a spreader, the ignorant becomes a stifler directly with a certain probability; ② adding an additional group called hibernators (H) coming from the spreaders due to forgetting mechanism and later becoming spreaders again due to remembering mechanism. The features of rumor propagation are community, repeatability, and variability. This new group, the hibernators, reflects the repeatability of rumor spreading. Based on the above rumor spreading mechanism, we build a new model called the SIHR model and derive the mean-field equations that describe the dynamics of the SIHR model in Section 2. In Section 3, we present analytical results for the steady-state of the SIHR model. In Section 4, numerical simulation on the dynamics results of the SIHR model is investigated to analyze the impact factors under different parameters. Conclusions and discussions are given in Section 5.

2. SIHR rumor spreading model

We consider a closed and homogeneously mixed population consisting of N individuals as a social network where individuals are vertices and contacts between people are edges. Then, an undirected graph $G = (V, E)$ can be obtained, where V is the set of vertices and E is the set of edges. We assume that the rumor is disseminated by direct contacts of spreaders with others, and the process of SIHR rumor spreading is shown in Fig. 1.

As shown in Fig. 1, the population is divided into four groups: ignorants, spreaders, hibernators, and stiflers. The SIHR rumor spreading rules can be summarized as follows.

(1) When an ignorant contacts a spreader, the ignorant becomes a spreader with probability λ, namely spreading rate.
(2) When an ignorant contacts a spreader, the ignorant becomes a stifler with probability β, namely refusing rate.
(3) We add the hibernators who come from the spreaders at a rate δ (forgetting rate) to reflect the forgetting mechanism. The remembering mechanism also exists for hibernators. Hibernators spontaneously become spreaders at a rate ξ (spontaneous remembering rate). When a hibernator contacts a spreader, the hibernator becomes a spreader with probability η, namely wakened remembering rate.
(4) When a spreader contacts with another spreader or a hibernator or a stifler, only the initiating spreader becomes a stifler at a probability α, namely stifling rate.

It should be noted that, in reality, the number of people that each person directly contacts is almost the same and this number approximately obeys Poisson distribution. Under this assumption, the following contents are discussed in the homogeneous network. $I(t)$, $S(t)$, $H(t)$, and $R(t)$ denote the density of population that are ignorants, spreaders, hibernators, and stiflers at time t, respectively. They satisfy the normalization condition:

$$I(t) + S(t) + H(t) + R(t) = 1.$$

Considering the SIHR rumor spreading mechanism, the mean-field equations can be described as follows:

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = -(\lambda + \beta)\bar{k}I(t)S(t), \tag{1}$$

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} = \lambda\bar{k}I(t)S(t) - \alpha\bar{k}S(t)(S(t) + H(t) + R(t)) - \delta S(t) + \xi H(t) + \eta\bar{k}H(t)S(t), \tag{2}$$

$$\frac{\mathrm{d}H(t)}{\mathrm{d}t} = \delta S(t) - \xi H(t) - \eta\bar{k}H(t)S(t), \tag{3}$$

$$\frac{\mathrm{d}R(t)}{\mathrm{d}t} = \beta\bar{k}I(t)S(t) + \alpha\bar{k}S(t)(S(t) + H(t) + R(t)). \tag{4}$$

Here $\bar{k}$ denotes the average degree of the network. We assume that there is only one spreader at the beginning of the rumor spreading. The initial condition for rumor spreading is given as follows:

$$I(0) = \frac{N-1}{N}, \qquad S(0) = \frac{1}{N}, \qquad H(0) = 0, \qquad R(0) = 0.$$

The classical SIR rumor spreading model and the classical SIR rumor spreading model with forgetting mechanism are special cases of the SIHR model. Moreover, if we only consider the forgetting mechanism, the SIHR model will be similar to the SIR epidemic model [27], as illustrated below.

Case i: When $\beta = 0, \delta = 0, \xi = 0, \eta = 0$, the SIHR model becomes the classical SIR rumor spreading model.

Case ii: When $\beta = 0, \delta \neq 0, \xi = 0, \eta = 0$, and there is no consideration of hibernators. The SIHR model becomes Nekovee's rumor spreading model with forgetting mechanism [28].

Case iii: When $\beta = 0, \delta \neq 0, \xi = 0, \eta = 0, \alpha = 0$, in other words, the termination of the rumor only depends on the forgetting mechanism.

3. Steady-state analysis

In the whole process of rumor spreading, the number of spreaders first increases, then decreases and reach zero when the rumor dies out. At that time, the system reaches an equilibrium state and has only ignorants and stiflers. We analyze the final size of a rumor R, here $R = \text{final}\{R(t)\} = \lim_{t\to\infty} R(t) = R(\infty)$, which can be used to measure the level of rumor influence. Taking $R = 0.8$ as an example, it means that 80% of individuals have heard of the rumor in the end. Dividing Eq. (1) by Eq. (4), we can get

$$\begin{aligned}\frac{\mathrm{d}R(t)}{\mathrm{d}I(t)} &= \frac{\beta\bar{k}I(t)S(t) + \alpha\bar{k}S(t)(S(t) + H(t) + R(t))}{-(\lambda + \beta)\bar{k}I(t)S(t)} \\ &= \frac{\beta\bar{k}I(t)S(t) + \alpha\bar{k}S(t)(1 - I(t))}{-(\lambda + \beta)\bar{k}I(t)S(t)} \\ &= \frac{(\alpha - \beta)I(t) - \alpha}{(\lambda + \beta)I(t)},\end{aligned}$$

that is,

$$\mathrm{d}R(t) = \frac{\alpha - \beta}{\lambda + \beta}\mathrm{d}I(t) + \frac{-\alpha}{(\lambda + \beta)I(t)}\mathrm{d}I(t). \tag{5}$$

Eq. (5)'s both sides can be integrated for $R(t)$ and $I(t)$ from the initial state to the stable state. Notice that $I(0) = \frac{N-1}{N} \approx 1 (N \to \infty)$, $R(0) = 0$, $I(\infty) = 1 - R(\infty) = 1 - R$, so we can get

$$-\frac{\lambda + \alpha}{\alpha}R = \ln(1 - R). \tag{6}$$

Taking the logarithm of e for Eq. (6), we obtain the following transcendental equation:

$$R = 1 - \mathrm{e}^{-\varepsilon R}, \tag{7}$$

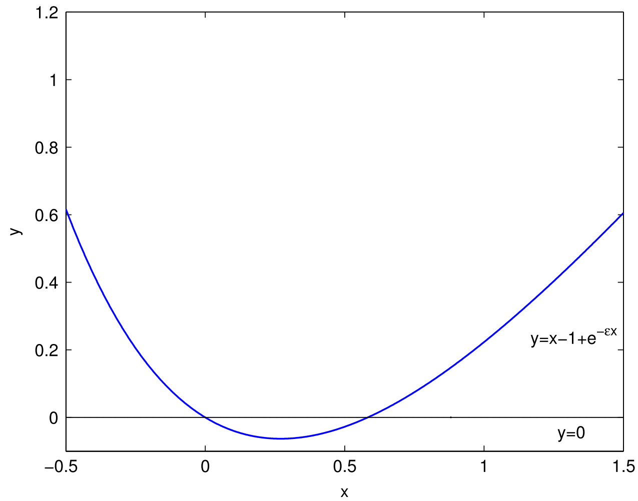


Fig. 2. Shape of function $y = x - 1 + e^{-\epsilon x}$.

where

$$\varepsilon = \frac{\lambda + \alpha}{\alpha}. \tag{8}$$

Obviously, Eqs. (7)–(8) are the same as the transcendental equation of the SIR rumor spreading model [26].

Theorem 1. *If* $\epsilon > 1$, $x = 1 - e^{-\epsilon x}$ *has two solutions: zero and a nontrivial solution R, where* $0 < R < 1$.

Proof. Obviously, $x = 0$ is a solution of $x = 1 - e^{-\epsilon x}$. Supposing $y = x - 1 + e^{-\epsilon x}$, and taking the derivative of y with respect to x, we have $y' = 1 - \epsilon e^{-\epsilon x}$ and $y'' = \epsilon^2 e^{-\epsilon x} > 0$. So y is a convex function, shown in Fig. 2.
And notice $y'(0) = 1 - \epsilon < 0$, $y(1) = e^{-\epsilon x} > 0$, which completes the proof. □

According to Theorem 1, Eq. (7) always admits the zero solution $R = 0$. And at the same time, since the condition:

$$\left.\frac{d}{dR}(1 - e^{-\varepsilon R})\right|_{R=0} > 1, \tag{9}$$

can be easily appreciated for all nontrivial values of the parameters λ and α, Eq. (7) also has another solution. That is, there is no rumor threshold in the SIHR rumor spreading model.

The above analysis indicates that the new added factors have no effect on the final size of a rumor R. R is a function of spreading rate λ and stifling rate α. Fig. 3 shows variations of the final size R as changes of the parameters λ and α. As can be seen from Fig. 3, the rumor influence R has great changes, and almost any value from 0 to 1 can be taken. Visually, given a fixed α, R increases as λ increases, but it looks like that given a fixed λ, R first decreases, then increases as α increases. Actually, since $\epsilon = \lambda/\alpha + 1$, parameters λ and α should have opposite impact on R. The reason of having this wrong vision is because the surface of Fig. 3 is distorted.

We now give a formal proof of the monotonic result between R and these two parameters, α and λ, in the following theorem.

Theorem 2. *Given a fixed* α, *R increases as* λ *increases. Similarly, given a fixed* λ, *R decreases as* α *increases.*

Proof. Since λ/α can be written as $\frac{1/\alpha}{1/\lambda}$, we only need to show the first half of the theorem. As shown above, the equation

$$R = 1 - e^{-\frac{\lambda+\alpha}{\alpha}R} \tag{10}$$

has two solutions, zero and a nontrivial solution $0 < R < 1$. Given a fixed α, the nontrivial solution R of the above equation is a function of λ. Taking the derivative of R with respect to λ, we have

$$R' = \left[\frac{R}{\alpha} + \frac{\lambda + \alpha}{\alpha}R'\right] e^{-\frac{\lambda+\alpha}{\alpha}R},$$

which means

$$[1 - \epsilon e^{-\epsilon R}]R' = \frac{R}{\alpha} e^{-\epsilon R} > 0,$$

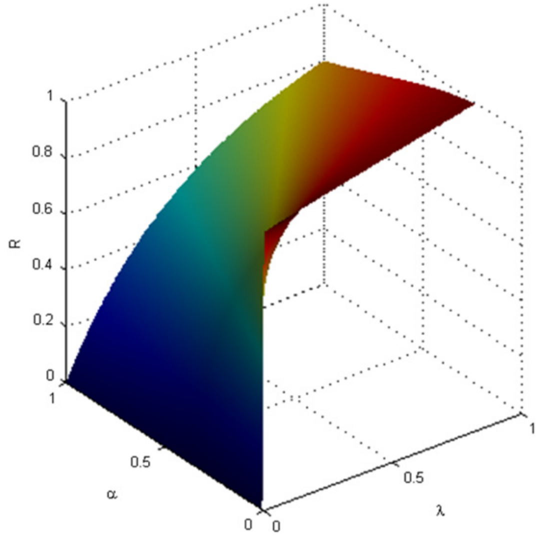


Fig. 3. The final size of the rumor R is shown as a function of λ and α.

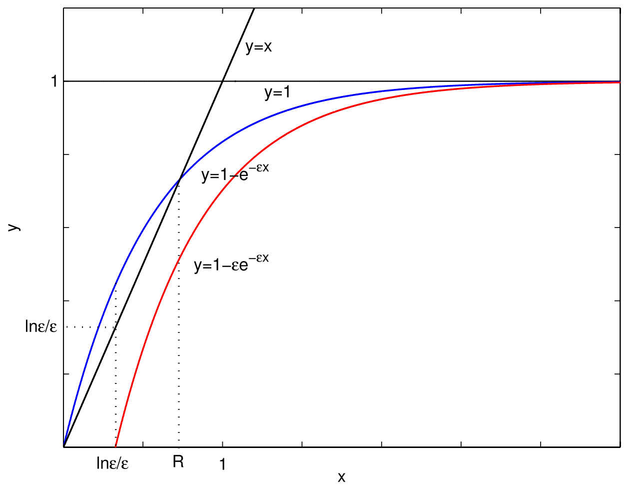


Fig. 4. Illustration of the proof of Theorem 2.

where $\epsilon = (\lambda + \alpha)/\alpha > 1$. If we can show that $1 - \epsilon e^{-\epsilon R} > 0$, we then prove the theorem. Note that R is a function of λ and also is the nontrivial solution of Eq. (10). Therefore, point R (more formally, point $(R, 0)$) is the point on the horizontal axis corresponding to the intersection (not the origin) of line $y = x$ and curve $y = 1 - e^{-\epsilon x}$. For any given $0 < \lambda < 1$, we can draw the following Fig. 4. On this figure, since $1 - e^{-\epsilon x} > 1 - \epsilon e^{-\epsilon x}$, curve $y = 1 - e^{-\epsilon x}$ is above curve $y = 1 - \epsilon e^{-\epsilon x}$.

Curve $y = 1 - \epsilon e^{-\epsilon x}$ crosses the horizontal axis at $x = \frac{\ln \epsilon}{\epsilon}$. Now if we can show that on the horizontal axis point $x = \frac{\ln \epsilon}{\epsilon}$ is on the left side of point $x = R$, we then conclude that $1 - \epsilon e^{-\epsilon R} > 0$. This is as same as to show that $1 - e^{-\epsilon(\ln \epsilon)/\epsilon} > \frac{\ln \epsilon}{\epsilon}$, i.e., $1 - \frac{1}{\epsilon} - \frac{\ln \epsilon}{\epsilon} > 0$.

Define function $f(\epsilon) = 1 - \frac{1}{\epsilon} - \frac{\ln \epsilon}{\epsilon}$. Since $f'(\epsilon) = \frac{\ln \epsilon}{\epsilon^2} > 0$ and $f(1) = 0$, we have $f(\epsilon) = 1 - \frac{1}{\epsilon} - \frac{\ln \epsilon}{\epsilon} > 0$ for any $\epsilon > 1$, which completes the proof. □

As mentioned earlier, in some conditions the SIHR model would reduce to some special cases. The transcendental equation of Case i is exactly the same as the SIHR model, which shows there is no rumor threshold in Case i, too. So we will not discuss further. Now, we consider the steady-state in Case ii, here Eqs. (1)–(4) turn into

$$\frac{dI(t)}{dt} = -\lambda \bar{k} I(t) S(t), \tag{11}$$

$$\frac{dS(t)}{dt} = \lambda \bar{k} I(t) S(t) - \alpha \bar{k} S(t)(S(t) + H(t) + R(t)) - \delta S(t), \tag{12}$$

$$\frac{dH(t)}{dt} = \delta S(t), \tag{13}$$

$$\frac{dR(t)}{dt} = \alpha \bar{k} S(t)(S(t) + H(t) + R(t)). \tag{14}$$

There are ignorants, hibernators and stiflers left in the end of rumor spreading. In the steady-state, the combination of hibernators and stiflers is recorded as U, that is, $U = H + R = \text{final}\{H(t) + R(t)\} = \lim_{t\to\infty}(H(t) + R(t))$. Dividing Eq. (11) by the sum of Eqs. (13) and (14), and using the former approach similarly, the transcendental equation can be obtained as follows:

$$U = 1 - e^{-\varepsilon_1 U}, \tag{15}$$

where

$$\varepsilon_1 = \frac{\lambda \bar{k} + \alpha \bar{k}}{\delta + \alpha \bar{k}}. \tag{16}$$

While $U = 0$ is always a solution of Eq. (15), in order to have a nontrivial solution the following condition must be fulfilled:

$$\frac{d}{dU}(1 - e^{-\varepsilon_1 U})\Big|_{U=0} > 1. \tag{17}$$

This condition is equivalent to the constraint $\lambda > \lambda_c$, where the threshold λ_c takes the value $\lambda_c = \delta \bar{k}^{-1}$ in this special Case ii.

In the steady state, dividing Eqs. (13) and (14) by Eq. (11) respectively, we obtain

$$H = \frac{\delta}{-\lambda \bar{k}} \ln I, \tag{18}$$

and

$$R = -\frac{\alpha}{\lambda} \ln I + \frac{\alpha}{\lambda} I - \frac{\alpha}{\lambda}. \tag{19}$$

From Eqs. (18), (19) and notice that $I = 1 - H - R$, we can get

$$\frac{H}{R} = \frac{\lambda\delta + \alpha\delta}{\lambda\alpha\bar{k} - \alpha\delta}. \tag{20}$$

Eq. (20) illustrates that in the steady state the proportion of the hibernators and the stiflers is a function of spreading rate λ, refusing rate α, forgetting rate δ and average degree $\bar{k}$ in Case ii.

Following we discuss the steady-state in Case iii, Eqs. (1)–(4) turn into

$$\frac{dI(t)}{dt} = -\lambda \bar{k} I(t) S(t), \tag{21}$$

$$\frac{dS(t)}{dt} = \lambda \bar{k} I(t) S(t) - \delta S(t), \tag{22}$$

$$\frac{dH(t)}{dt} = \delta S(t). \tag{23}$$

There are ignorants and hibernators left. In the steady-state, the hibernators are recorded as H, $H = \text{final}\{H(t)\} = \lim_{t\to\infty} H(t)$. The transcendental equation can be obtained as follows:

$$H = 1 - e^{-\varepsilon_2 H}, \tag{24}$$

where

$$\varepsilon_2 = \frac{\lambda \bar{k}}{\delta}. \tag{25}$$

While $H = 0$ is always a solution of Eq. (24), in order to have a nontrivial solution the following condition must be fulfilled:

$$\frac{d}{dH}(1 - e^{-\varepsilon_2 H})\Big|_{H=0} > 1. \tag{26}$$

This condition is equivalent to the constraint $\lambda > \lambda_c$, where the threshold λ_c takes the value $\lambda_c = \delta \bar{k}^{-1}$ in this special Case iii.

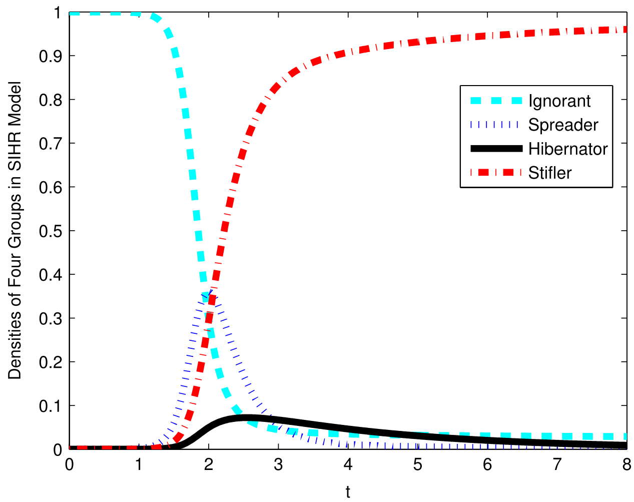


Fig. 5. Densities of ignorants, spreaders, hibernators, and stiflers over time with $\lambda = 0.8$, $\beta = 0.2$, $\alpha = 0.3$, $\delta = 0.6$, $\xi = \eta = 0.5$.

4. Numerical simulation

The Runge–Kutta method can be used to solve the differential equations (1)–(4) and analyze the effects on the rumor spreading process by the new factors. According to the above description in Section 2, here we construct a homogeneous network consisting of N individuals, each person as a node and the contact between two people as an edge. In the following simulation we assume $N = 10^6$, the average degree $\bar{k} = 10$, and in the initial condition there is only one spreader in the network, thus $S(0) = \frac{1}{10^6}$, $I(0) = \frac{10^6-1}{10^6}$, $H(0) = 0$, and $R(0) = 0$.

Fig. 5 shows the general trends of the four kinds of agents in the SIHR rumor spreading model. From the following simulation we can find there is a sharp increase in the number of spreaders as spreaders begin to propagate a rumor. With further spreading of the rumor, the number of spreaders reaches a peak and thereafter declines. Finally, the number of spreaders is zero and this leads to the termination of rumor spreading. In this whole process, the number of ignorants always reduces while the number of stiflers always increases until they reach the balance, respectively. The variation trend of the number of hibernators is similar to that of the spreaders, which increases at first and then decreases to zero. But, the trend of increasing and decreasing processes of the hibernators is much more moderate than the spreaders. The numerical changes of the hibernators is much smaller than those of the spreaders'.

Fig. 6 shows how the densities of spreaders change over time for different refusing rate. The red solid line represents the scenario that refusing rate is zero, i.e. refusing mechanism is not considered. The blue dashed line represents the scenario with refusing rate at 40%, and the black spotted line represents the scenario with refusing rate at 80%. The peak value of spreader density $\max\{S(t)\}$, the highest density of people are spreading the rumor, can be used to measure the maximum rumor influence. From Fig. 6, we can see that the higher the refusing rate β, the smaller the maximum rumor influence, and the higher the refusing rate β, the faster the rumor terminates. Therefore, the new added factors have affected the process of rumor spreading. If more people have the ability to see through the rumor or do not spread rumor (become stiflers immediately), the maximum rumor influence becomes smaller and the rumor termination time comes earlier.

Fig. 7 illustrates how the densities of spreaders change over time for different forgetting rate δ, the spontaneous remembering rate ξ, and the wakened remembering rate η. The red solid line represents the scenario that forgetting rate and remembering rates are zero, i.e. forgetting and remembering mechanisms are not considered. The blue dashed line represents the scenario with the three rates at 40%, and the black spotted line represents the scenario with the three rates at 80%. Fig. 7 shows that the stronger the effect of the forgetting and remembering mechanisms, the smaller the maximum rumor influence, but the longer time it takes for the spreaders to decline to zero. With a higher δ value, more spreaders become the hibernators, which leads to smaller number of the spreaders. But the hibernators can transform back to the spreaders due to the remembering mechanism. The combined effect of forgetting and remembering mechanisms postpone the time of the spreaders' final disappearance.

Fig. 8 describes how the densities of hibernators and stiflers change with the spontaneous remembering rate ξ over time. Fig. 8(a) divides hibernators' densities into two cases. When $\xi \neq 0$, the higher the spontaneous remembering rate ξ, the smaller the number of hibernators and the faster the hibernators reduce down to 0. When $\xi = 0$, the hibernator density increases over the spreading time and finally reaches a balanced value. Fig. 8(b) also divides stiflers' densities into two cases. When $\xi \neq 0$, the higher the spontaneous remembering rate ξ, the sooner the stiflers reach equilibrium. When $\xi = 0$, the final size of stifler is smaller than the final size of the case when $\xi \neq 0$, and the lost part becomes hibernators. In fact, in the steady-state, whether hibernators exist in the system or not is entirely decided by the parameter ξ, not the parameter η. When $\xi = 0$, the system in the stable state would have hibernators, conversely it is not true. This is because the way with

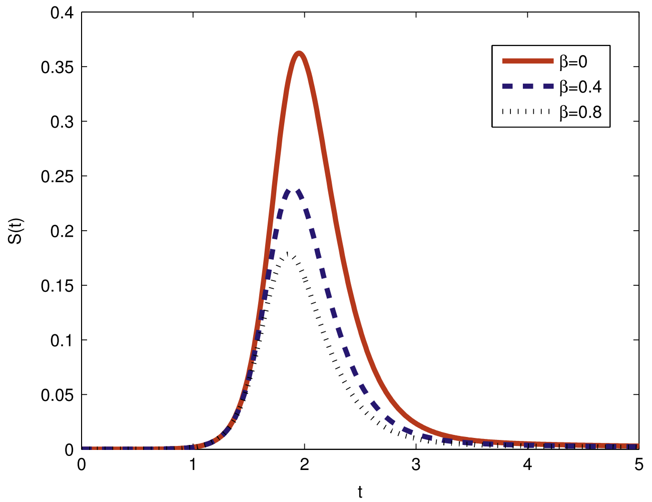


Fig. 6. Density of spreaders over time under different refusing rate β with $\lambda = 0.8$, $\alpha = \delta = \xi = \eta = 0.5$.

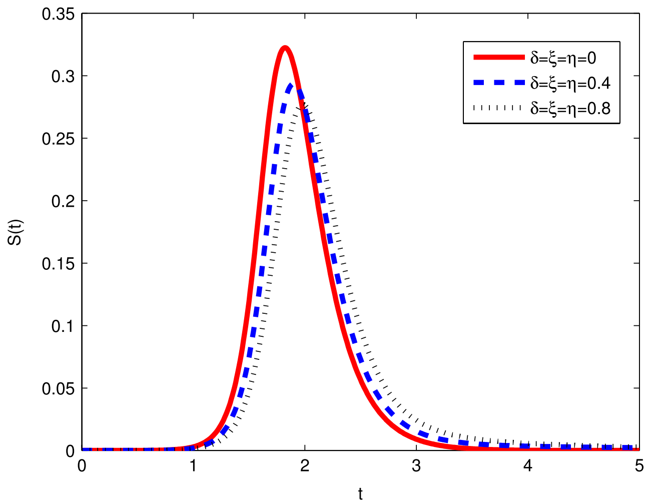


Fig. 7. Density of spreaders over time under different forgetting rate δ, remembering rates ξ and η with $\lambda = 0.8$, $\beta = 0.2$, $\alpha = 0.5$.

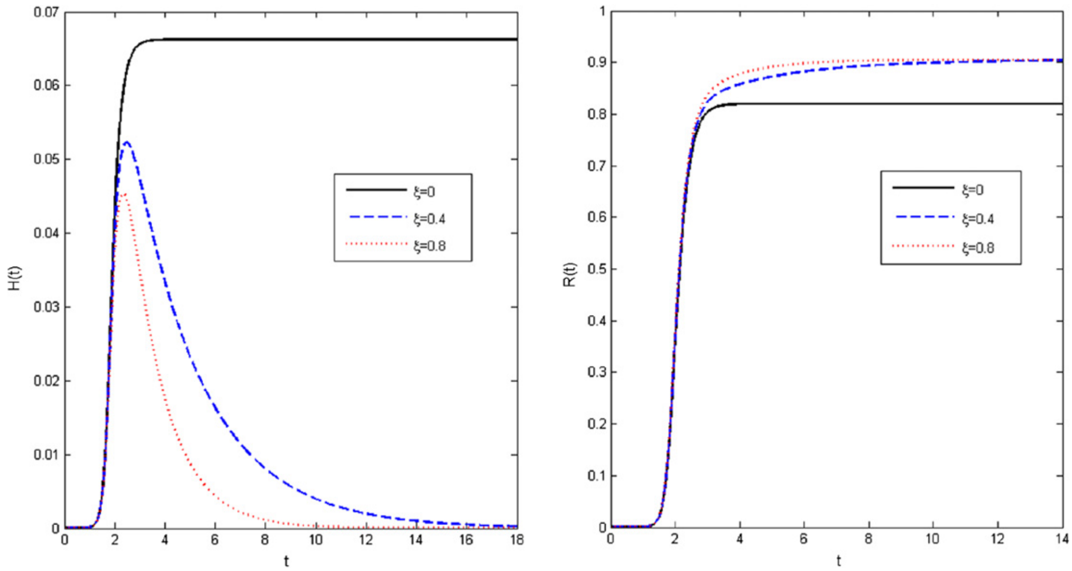
the transformation probability ξ requires no conditions. As long as the time is long enough, hibernators always completely transform into spreaders. But the way with the transformation probability η needs a condition that the hibernators need to meet the spreaders. Therefore, when there are no spreaders, the transformation of hibernators will be stopped and make themselves surplus. This shows categorizing people depends on whether ξ is zero or not in the steady-state.

5. Conclusions

In this paper, we considered some more realistic factors and extended the classical SIR rumor spreading model to make it closer to real life.

(1) We added a direct link from the ignorants to the stiflers and a new group hibernator which embodies forgetting and remembering mechanisms. We then established the SIHR rumor spreading model and derived mean-field equations. The classical SIR rumor spreading model and the Nekovee's rumor spreading model with forgetting mechanism are special cases of the SIHR model.

(2) In the steady-state, we obtained a transcendental equation of the SIHR model. The transcendental equation is as same as that of the classical SIR rumor spreading model. Theorem 1 and the analysis method of Moreno et al. [26] revealed that there is no threshold in the SIHR model. The figure of the final size R of a rumor changing with rates λ and α was provided. We give Theorem 2 to prove that given a fixed α, R increases as λ increases, and given a fixed λ, R decreases as α increases. Then we got the transcendental equations and discussed the thresholds of the three special cases. There is no rumor threshold in the special Case i, either. The special Case ii has a threshold $\lambda_c = \delta\bar{k}^{-1}$. Moreover, we illustrated



(a) Density of hibernators.

(b) Density of stiflers.

Fig. 8. Density of hibernators and stiflers over time under different spontaneous remembering rate ξ with $\lambda = 0.8$, $\beta = 0.2$, $\alpha = \delta = \eta = 0.5$.

the proportion of the hibernators and the stiflers in the steady-state in the special Case ii. The special Case iii has the same rumor threshold as that of Case ii.

(3) The Runge–Kutta method was used to conduct the numerical simulation for the mean-field equations of the SIHR model. New added factors have no effect on the final size of a rumor *R*, but they have effect on the process of rumor spreading. The most important influences are the maximum rumor influence(spreader peak) and the rumor terminal time. The direct link from the ignorants to the stiflers accelerates the terminal time and reduces the maximum rumor influence. The combined effect of forgetting and remembering mechanisms reduces the maximum rumor influence and postpones the rumor terminal time. Moreover, through simulation we found in the steady-state, whether the hibernators exist in the system or not is entirely decided by the spontaneous remembering rate ξ, not the wakened remembering rate η. When $\xi = 0$, the system would have the hibernators, conversely it is not true.

The role of forgetting and remembering mechanisms reflects the repeatability of the spreading characteristic of the rumor. How to reflect the variability of a rumor is an interesting topic for future research.

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