Study on the Growth and Decline of SNSs by Using the Infectious Recovery SIR Model

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***Abstract*—The dynamics of network services' growth and decline is similar to infectious disease. So, it is expected that we can explain the growth and decline of a network service by using epidemiological models. The irSIR model is proposed as one of promising model for describing epidemiological transition of network services. To study the validity of applying the irSIR model, this paper focuses on SNSs transition occurred in Japan, and investigates the transition between SNSs by analyzing the sequence of the number of word searches of SNS. By comparing the well-known SIR model and irSIR model, we show that irSIR model gives better fitting to the real data. This result indicates that the transition of SNS is accelerated by invitation of new service users.**

I. Introduction

In the last decade, we have been able to exchange information and message easily by using social networking services (SNSs). In these period, some SNSs have emerged and declined. Only few of them grew to the major SNS that has a lot of users. After a new major SNS emerges, the existing major SNS became to be declined. In Japan, Mixi is the first major SNS. After Facebook that is the second major SNS was emerged, many users of Mixi moved to Facebook.

The dynamics of SNS growth and decline is similar to infectious disease. So, it is expected that we can explain the growth and decline by using epidemiological models. SIR model is a most popular and well-known model to describe the epidemiological growth of SNSs. As an extended model of SIR model, the irSIR model is proposed as one of promising model for describing epidemiological transition of network service. To study the validity of applying the irSIR model, this paper focuses on SNSs transition of users from Mixi to Facebook, and investigates the transition between SNSs by analyzing the time sequence of the number of word searches of SNS. By comparing SIR model and irSIR model, we show that irSIR model gives better fitting to the real data. This result indicates that the transition of SNS is accelerated by invitation of new service users.

II. Models

A. SIR model

First, we define some notations: t is time, $S(t)$ is the number of susceptible people, $I(t)$ is the number of infected people, and $R(t)$ is the number of recovered people. The SIR model for the spread of infectious disease is shown as follows [1].

$$\frac{\mathrm{d}}{\mathrm{d}t} S(t) = -\beta\, S(t)\, \frac{I(t)}{N(t)}, \tag{1}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} I(t) = \beta\, S(t)\, \frac{I(t)}{N(t)} - \gamma\, I(t), \tag{2}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} R(t) = \gamma\, I(t), \tag{3}$$

where β is the infection rate, and γ is the recovery rate. In addition, N is the total number of population, therefore $S(t)+I(t)+R(t)=N$, and it is independent of time.

Infectious disease spreads by contact with people, and infected people recover by acquiring immunity naturally. Similarly, the main motivation of a person newly joining a SNS might be occurred by contact with the person's friends who are the persons already knows. In short, growth of SNS is similar to infectious disease because they are driven by contacting with people. Therefore, we can expect that the SIR model is a model that can show the growth and decline of SNS. Becoming SNS user corresponds to infection, and quitting the SNS user corresponds to recovering from infection.

B. irSIR model

The irSIR model is shown as follows [2].

$$\frac{\mathrm{d}}{\mathrm{d}t} S(t) = -\beta\, S(t)\, \frac{I(t)}{N(t)}, \tag{4}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} I(t) = \beta\, S(t)\, \frac{I(t)}{N(t)} - \nu\, I(t)\, \frac{R(t)}{N(t)}, \tag{5}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} R(t) = \nu\, I(t)\, \frac{R(t)}{N(t)}, \tag{6}$$

where the most of the parameters are the same as SIR model, and ν is the new recovery rate.

This model is a modified model of the SIR model. Significant difference from the SIR model is in the fact that the recovery rate of the infected people is accelerated by contact with the recovered people who have immunity already. In actual infectious disease, since the recovered people can hardly give immunity to the infected people, the recovery rate is independent of the number of the recovered people. However in the SNS, people who have moved from the current major SNS to the new emerging SNS influences the current major

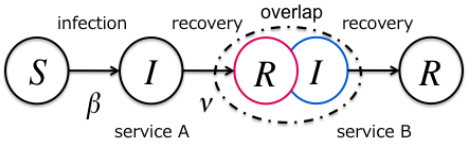

Fig. 1. irSIR model

SNS users. The contact between them accelerates the transition of users from the current major SNS to the new emerging SNS. Considering that, the current major SNS users correspond to the infected people and people who have moved from the current major SNS to the new emerging SNS correspond to the recovered people. Then we can also use the modified infectious recovery model to show the growth and decline of SNS. We show the situation in Fig. 1 which service A is the former and service B is the latter. The transition from service A to service B can be interpreted in two ways. One is the recovery process from service A (quitting service A), and the other is the infection process to service B (joining service B). This means the temporal sequence of the growth and decline of SNSs is relative.

III. Actual Data and Analysis

We analyze the transition between SNSs by using the SIR and irSIR models. There are two typical SNSs in Japan. One is Mixi and the other is Facebook. Main function of Mixi is community entertainment to meet new people, and Facebook has almost the same function. The Mixi was released in February 2004 and obtained a lot of users. The Mixi was dominant SNS during a few years in Japan. Facebook was also released in 2004. However, because the Japanese interface of Facebook was released in 2008, the growth of Facebook in Japan occurred after the domination of Mixi. Although there were many Mixi users at the time of Facebook emerged, most of Mixi users have moved to Facebook at the present time.

To analyze these services, we take the google search query data from Google Trends [3]. It gives us the sequence of the frequency of search of various words. In Google Trends, the frequency is denoted in the normalized range between 0 to 100. The frequency of the most frequently searched period is denoted as 100. We assume that the activity of the SNS users is proportional to the word search frequency of the SNSs: Mixi or Facebook. By investigating time sequence data of the search frequency, we estimate the temporal transition of the number of users of the corresponding SNS.

We use weekly data of the word search frequency in Mixi or Facebook, and it is shown in Fig. 2. The period of the data is from the 4th of January 2004 to the 23th of August 2014. The Mixi is denoted as a blue line and the Facebook data is denoted as a red one. Note that the Facebook data has a large gap from 100 to 19 at the end of 2010. This unnatural gap might be caused by upgrade of the google system. Therefore we can remove the gap by changing the scale before the jump into small. According to the Facebook COO report, the number of

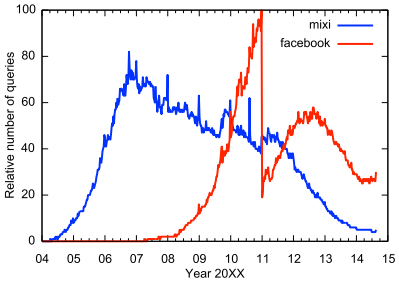

Fig. 2. Search frequency in Japan from Google Trends.

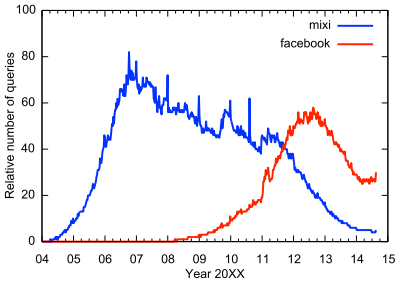

Fig. 3. The scaled sequence of search frequencies.

active users reached 15 million in September 2012 [4]. On the other hand, the number of monthly active users of Mixi were 14 million in that time [5]. The number of active Facebook users exceeded that of Mixi users around that time. The scaled data is shown in Fig. 3. This graph fortunately shows that the magnitude correlation of frequencies was reversed around that time. The number of Mixi users at time t is denoted by $I(t)$. We calculate the temporal evolution of $I(t)$ by using the SIR model or the irSIR model, and try to fit it to real data shown in Fig. 3. The goodness of fit of the models are measured by the sum of squared error (SSE) between the real data and the generated $I(t)$ calculated by SIR and irSIR models.

The best fit curve is achieved by choosing the initial input parameters $S(0)$, $I(0)$, $R(0)$, β, γ or ν and is defined as the $I(t)$ curve that minimizes the SSE.

To study on the transition of SNS, we apply the SIR model and irSIR model to Mixi data and fit the curve.

IV. Results: The Goodness-of-Fit of irSIR Model

To validate that the irSIR model can describe the transition of SNSs, we compare SIR model with irSIR model. If Facebook does not exist, the transition from Mixi user to Facebook user does not occur. So, before Facebook emerged, we apply the SIR model for describing the temporal evolution of the number of Mixi users $I(t)$. After Facebook emerged in 2008, we compare the SIR model with the irSIR model, by fitting them to the real data. Figure 4 shows the best fit curve calculated by the SIR model. In this figure, the best fit curve is denoted as the black line and Tables I shows the each parameter and the corresponding SSE.

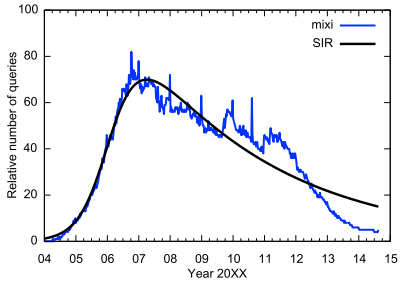

Fig. 4. The best fit curve calculated by SIR model.

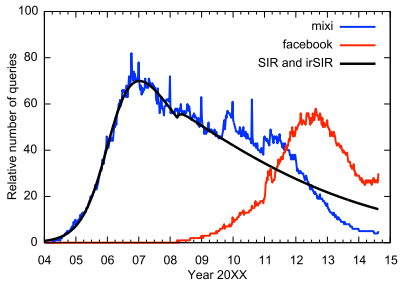

Fig. 5. The best fit curve calculated by irSIR model.

Next, let the time period be splitted into two parts at the beginning of 2008, these periods are before or after Facebook emerged. For after 2008, let compare irSIR and SIR models. In this comparison, we use the SIR model for the period during 2004 to 2007, and Table II shows the each parameter and the corresponding SSE for the best fit curve in this period. Figures 5 and 6 show the best fit curve calculated by irSIR and SIR models after 2008, respectively. In these figures, the best fit curves are denoted as the black line. Tables III (a) and (b) show the each parameter and the corresponding SSE for the best fit curves after 2008 calculated by irSIR and SIR models, respectively. From Tables II and III (a), the total SSE for the Fig. 5 is about 24,351. From Tables II and III (b), the total SSE for the Fig. 6 is about 29,794. The SSE for Fig. 6 is greater than that of irSIR model shown in Table I. This result implies that irSIR model gives better description of the SNS user transition. The goodness of irSIR model might be caused by splitting the period. However, this possibility is denied because the SSE for Fig. 6 is greater than that for Fig. 5.

V. Conclusion

In this paper, we studied on the infectious recovery model by using a case study, Mixi and Facebook. In our analysis, we used the google search query data from Google Trends. We assumed that the activity of the SNS users is proportional to the frequency of search the words “Mixi” or “Facebook”. By investigating time sequence data of the search frequency, we estimated the temporal transition of the number of users of the corresponding SNS. By comparing SIR model and irSIR model, we showed that irSIR model gives better fitting to the real data. This result indicates that the transition of SNS is accelerated by invitation of new service users.

TABLE I
Fitting parameters and SSE of the SIR model for the entire period

β	γ	$S(0)$	$I(0)$	$R(0)$	SSE
0.044721	0.00442	102.6504	1.202182	0(fixed)	26341.38

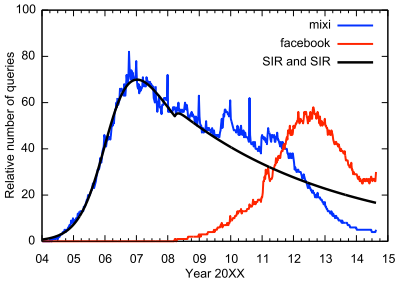

Fig. 6. The best fit curve calculated by SIR model.

TABLE II
Fitting parameters and SSE of the SIR model before 2007

β	γ	$S(0)$	$I(0)$	$R(0)$	SSE
0.052025	0.007692	121.80437	0.781982	0(fixed)	1480.52

TABLE III
Fitting parameters and SSE after 2008

(a) For irSIR model

β	ν	$S(221)$	$I(221)$	$R(221)$	SSE
0.469827	0.005648	3.34916	54.10110	65.13607	22871.20

(b) For SIR model

β	γ	$S(221)$	$I(221)$	$R(221)$	SSE
0.484591	0.0037116	3.34916	54.10110	65.13607	28314.76

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References

[1] W. O. Kermack, and A. G. McKendrick, “A contribution to the mathematical theory of epidemics,” in *Proc. Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, Vol. 115, No. 772, pp. 700–721, Aug. 1927.
[2] J. Cannarella and J. A. Spechler, “Epidemiological modeling of online social network dynamics,” arXiv:1401.4208 [cs.SI], Jan. 2014.
[3] Google trends. Available: `http://trends.google.com` Accessed: 2014-8-24.
[4] Interview of Sheryl Sandberg of The Nikkei, Sep. 2012. Available: `http://www.nikkei.com/article/DGXBZO46086670T10C12A9000000/`(In Japanese)
[5] Monthly Active Users of Mixi, pp. 14, Aug. 2012. Available: `http://v4.eir-parts.net/v4Contents/View.aspx?templateir_material_for_fiscal_ym&sid=12018&code=2121`(In Japanese)