Tianbo Qiu (tq213)

Problem 1

Assume the three doors are A, B, C. A is chosen, and the host opened door B, behind which there is no prize.

$$P(A_{prize} \mid B_{open}) = \frac{P(B_{open} \mid A_{prize}) P(A_{prize})}{P(B_{open} \mid A_{prize}) P(A_{prize}) + P(B_{open} \mid B_{prize}) P(B_{prize}) + P(B_{open} \mid C_{prize}) P(C_{prize})}$$

$$= \frac{\frac{1}{2} \times \frac{1}{3}}{\frac{1}{2} \times \frac{1}{3} + 0 \times \frac{1}{3} + 1 \times \frac{1}{3}} = \frac{1}{3}$$

~~P(A~~

$P(C_{prize} \mid B_{open}) = 1 - P(A_{prize} \mid B_{open}) = \frac{2}{3}$ Thus, she should switch doors.

Problem 2

The conjugate prior of multinomial is Dirichlet distribution

$X_i \sim \text{Multinomial}(\pi)$ $\quad P(X_i \mid \pi) = \frac{n!}{x_{i1}! \cdots x_{ik}!} \pi_1^{x_{i1}} \cdots \pi_K^{x_{ik}}$.

$\pi \sim \text{Dirichlet}(\alpha)$ $\quad P(\pi \mid \alpha) = \frac{(\alpha_1 + \cdots + \alpha_K - 1)}{(\alpha_1 - 1)! \cdots (\alpha_K - 1)!} \Pi_K^{\alpha_K - 1} = \frac{\Gamma(\alpha_1 + \cdots + \alpha_K)}{\Gamma(\alpha_1) \cdots \Gamma(\alpha_K)} \cancel{\Pi_K^{\alpha_K - 1}} \cdot \prod_{k=1} \pi_k^{\alpha_k - 1}$

$$P(\pi \mid X_1, \cdots X_N) = \frac{P(X_1, \cdots, X_N \mid \pi) p(\pi \mid \alpha)}{\int p(X_1, \cdots, X_N \mid \pi) p(\pi \mid \alpha) d\pi}$$

$$\propto \prod_{i=1}^{N} P(X_i \mid \pi) P(\pi \mid \alpha) \propto \prod_{i=1}^{N} \frac{n!}{x_{i1}! \cdots x_{ik}!} \pi_1^{x_{i1}} \cdots \pi_K^{x_{ik}} \cdot \frac{\Gamma(\alpha_1 + \cdots + \alpha_K)}{\Gamma(\alpha_1) \cdots \Gamma(\alpha_K)} \pi_K^{\alpha_K - 1}$$

$$\propto \pi_1^{\sum_{i=1}^{N} x_{i1} + \alpha_1 - 1} \cdot \pi_2^{\sum_{i=1}^{N} x_{i2} + \alpha_2 - 1} \cdot \cdots \cdot \pi_K^{\sum_{i=1}^{N} x_{iK} + \alpha_K - 1}$$

Thus, $p(\pi \mid X_1, \cdots X_N)$ is also Dirichlet (α'), with $\alpha_k' = \sum_{i=1}^{N} X_{ik} + \alpha_k$

The posterior distribution is the same with conjugate prior, with different parameters.

Problem 3

a) $X_i \sim \text{Poisson}(\lambda)$

$\lambda \sim \text{Gamma}(a, b)$

$$P(\lambda \mid x_1, \cdots, x_N) = \frac{P(x_1, \cdots, x_N \mid \lambda)\, p(\lambda \mid a, b)}{\int p(x_1, \cdots, x_N \mid \lambda)\, p(\lambda \mid a, b)\, d\lambda}$$

$$\propto \prod_{i=1}^{N} \frac{\lambda^{x_i} e^{-\lambda}}{x_i!} \cdot \frac{b^a}{\Gamma(a)} \lambda^{a-1} \cdot e^{-b\lambda}$$

$$\propto \lambda^{\sum_{i=1}^{N} x_i + a - 1} \cdot e^{-(N+b)\lambda}$$

Gamma distribution is conjugate prior of Poisson

Thus. $p(\lambda \mid x_1, \cdots, x_N) \sim \text{Gamma}\left(\sum_{i=1}^{N} x_i + a, N + b\right)$

$$P(\lambda \mid x_1, \cdots, x_N) = \frac{(N+b)^{\sum_{i=1}^{N} x_i + a}}{\Gamma\left(\sum_{i=1}^{N} x_i + a\right)} \cdot \lambda^{\sum_{i=1}^{N} x_i + a - 1} \cdot e^{-(N+b)\lambda}$$

b)

$$P(x^* \mid x_1, \cdots, x_n) = \int_0^\infty p(x^* \mid \lambda)\, p(\lambda \mid x_1, \cdots, x_n)\, d\lambda$$

$$= \int_0^\infty \left[\frac{\lambda^{x^*} e^{-\lambda}}{x^*!} \cdot \frac{(N+b)^{\sum_{i=1}^{N} x_i + a}}{\Gamma\left(\sum_{i=1}^{N} x_i + a\right)} \lambda^{\sum_{i=1}^{N} x_i + a - 1} \cdot e^{-(N+b)\lambda}\right] d\lambda$$

$$= \frac{(N+b)^{\sum_{i=1}^{N} x_i + a}}{\Gamma(x^*+1)\, \Gamma\left(\sum_{i=1}^{N} x_i + a\right)} \int_0^\infty \lambda^{x^* + \sum_{i=1}^{N} x_i + a - 1} \cdot e^{-(N+b+1)\lambda}\, d\lambda$$

$$= \frac{(N+b)^{\sum_{i=1}^{N} x_i + a}}{\Gamma(x^*+1)\, \Gamma\left(\sum_{i=1}^{N} x_i + a\right)} \cdot \frac{\Gamma\left(x^* + \sum_{i=1}^{N} x_i + a\right)}{(N+b+1)^{\left(x^* + \sum_{i=1}^{N} x_i + a\right)}}$$

$$= \frac{\Gamma\left(x^* + \sum_{i=1}^{N} x_i + a\right)}{\Gamma(x^*+1)\, \Gamma\left(\sum_{i=1}^{N} x_i + a\right)} \cdot \left(\frac{N+b}{N+b+1}\right)^{\sum_{i=1}^{N} x_i + a} \left(\frac{1}{N+b+1}\right)^{x^*}$$

Problem 4

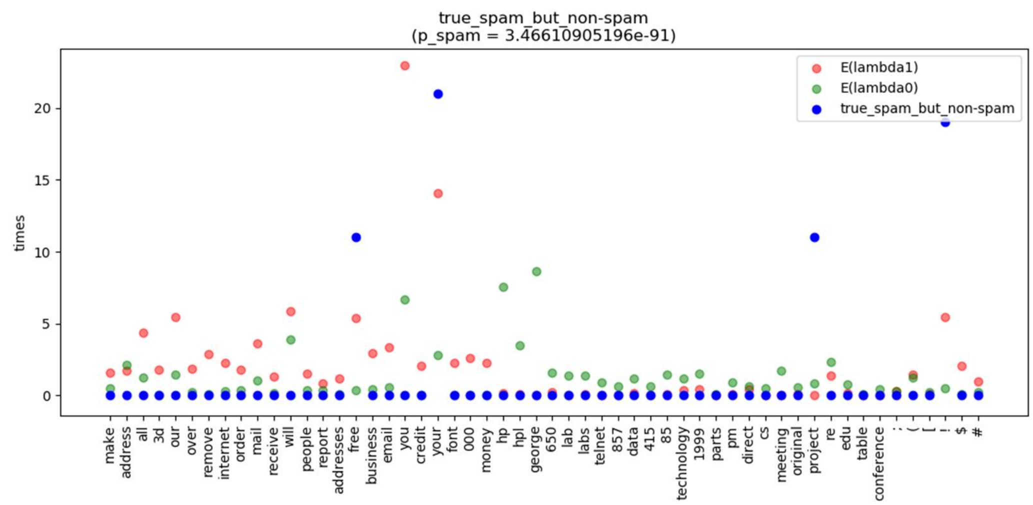
a. naiveBayesClassifier.py

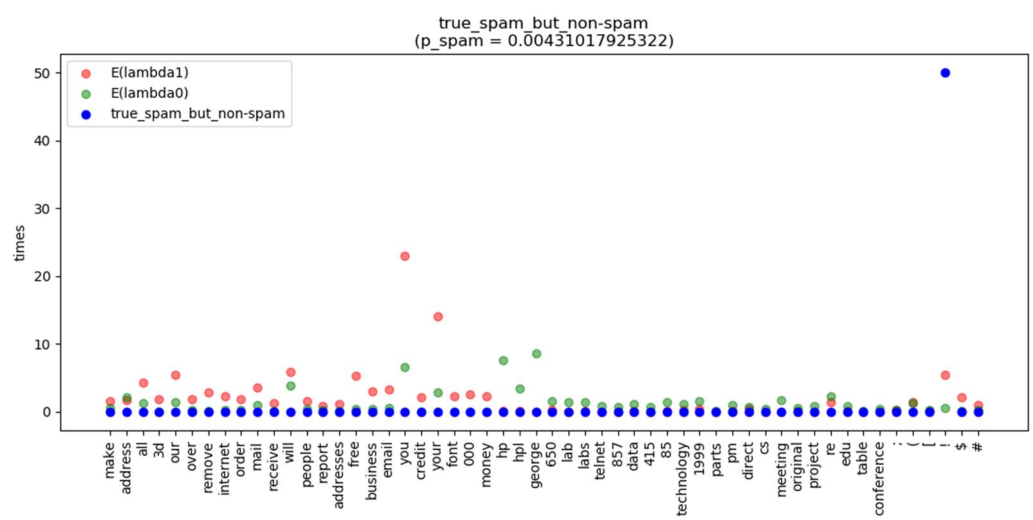
b. naiveBayesClassifier.py

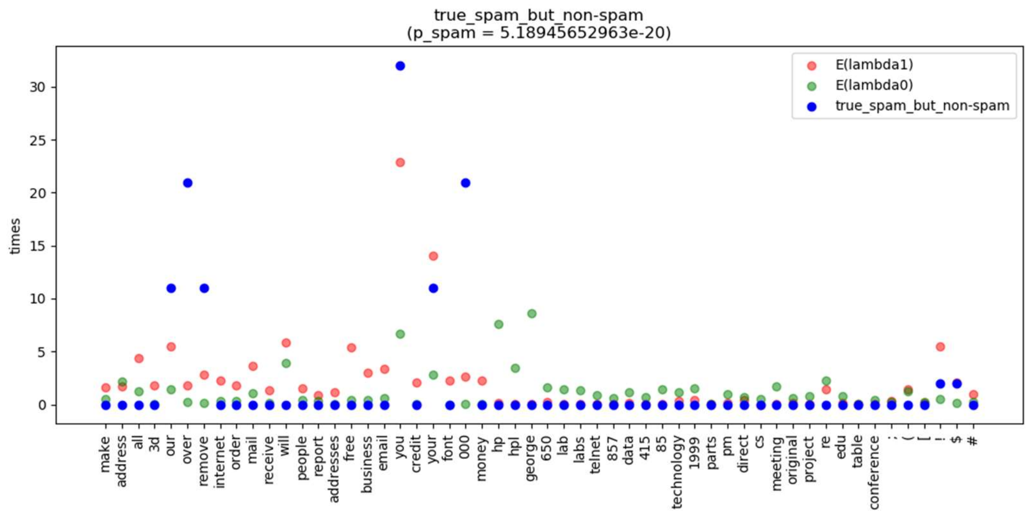
-----Confusion Matrix-----

actual / predict	spam	non-spam
spam	169	51
non-spam	13	228

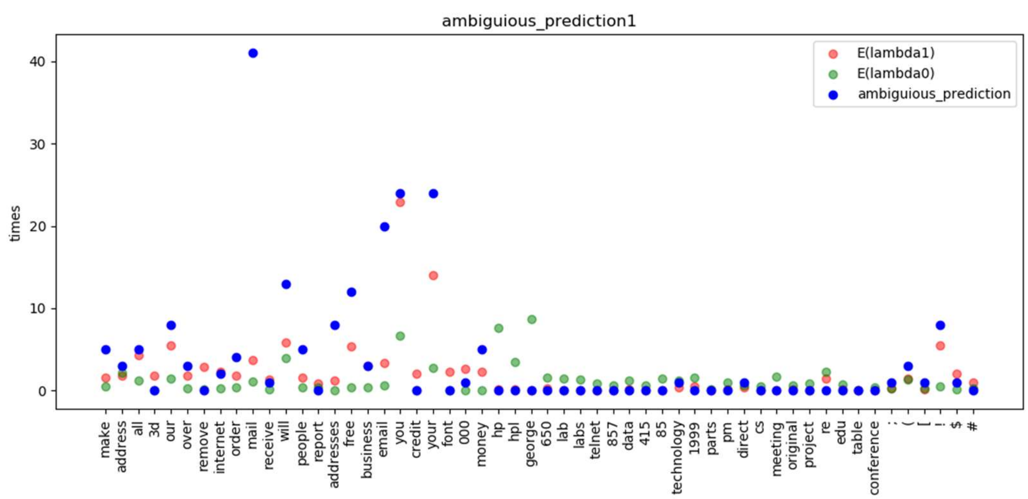
c. plot_p4c.py

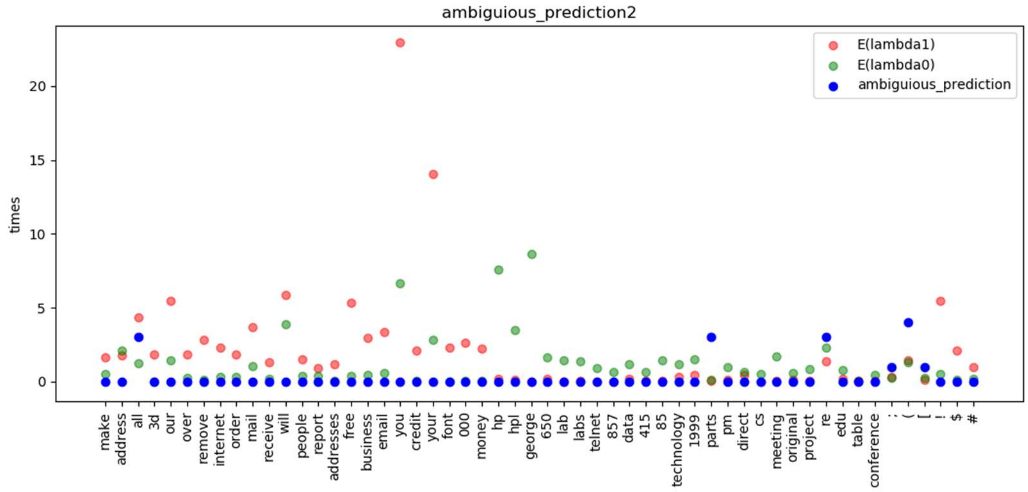




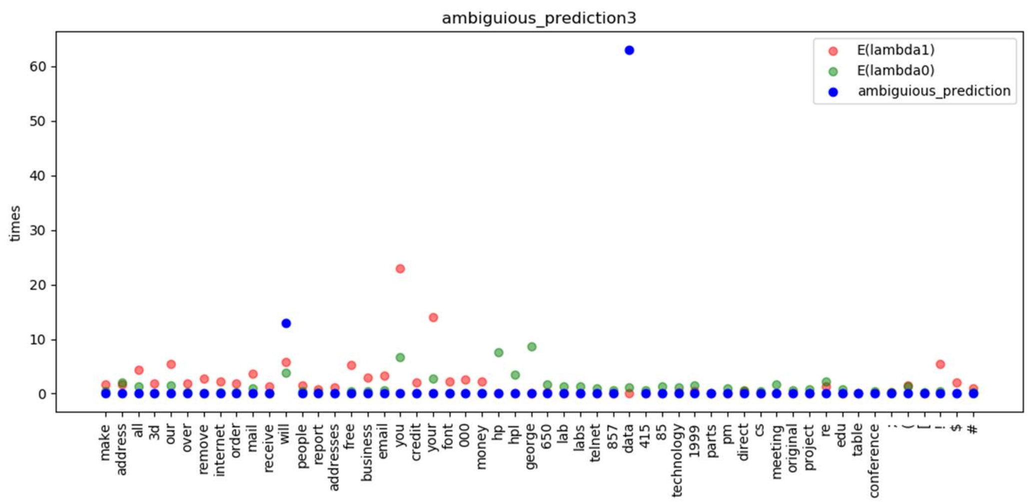

true_spam_but_non-spam
(p_spam = 5.18945652963e-20)
E(lambda1)
E(lambda0)
true_spam_but_non-spam
times
make address all 3d our over remove internet order mail receive will people report addresses free business email you credit your font 000 money hp hpl george 650 lab labs telnet 857 data 415 85 technology 1999 parts pm direct cs meeting original project re edu table conference ; ([! $ #

d.


ambiguious_prediction1
E(lambda1)
E(lambda0)
ambiguious_prediction
times
make address all 3d our over remove internet order mail receive will people report addresses free business email you credit your font 000 money hp hpl george 650 lab labs telnet 857 data 415 85 technology 1999 parts pm direct cs meeting original project re edu table conference ; ([! $ #



ambiguious_prediction2
E(lambda1)
E(lambda0)
ambiguious_prediction
times
0
5
10
15
20
make
address
all
3d
our
over
remove
internet
order
mail
receive
will
people
report
addresses
free
business
email
you
credit
your
font
000
money
hp
hpl
george
650
lab
labs
telnet
857
data
415
85
technology
1999
parts
pm
direct
cs
meeting
original
project
re
edu
table
conference
;
(
[
!
$
#



ambiguious_prediction3
E(lambda1)
E(lambda0)
ambiguious_prediction
times
0
10
20
30
40
50
60
make
address
all
3d
our
over
remove
internet
order
mail
receive
will
people
report
addresses
free
business
email
you
credit
your
font
000
money
hp
hpl
george
650
lab
labs
telnet
857
data
415
85
technology
1999
parts
pm
direct
cs
meeting
original
project
re
edu
table
conference
;
(
[
!
$
#