Synchronous Sequential Logic I

CS207 Lecture 9

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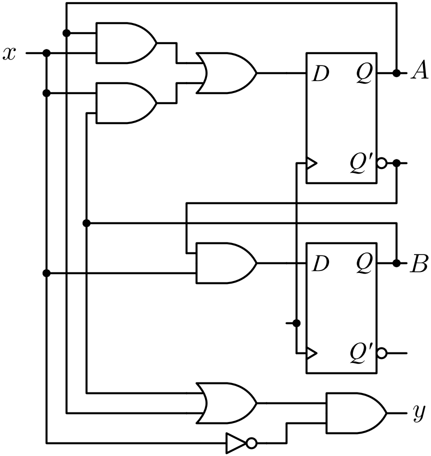
Analysis of clocked sequential circuits

- Describes what a given circuit will do under certain operating conditions.
  - Obtaining a table or a diagram for the time sequence of inputs, outputs, and internal states.
  - Or obtain the Boolean expressions that describes the behavior of the circuit.
- A diagram is a clock sequential circuit, if it includes flip-flops and clock inputs.

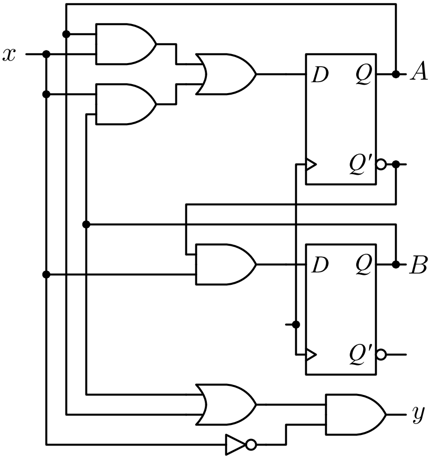
Analysis of clocked sequential circuits

- The behavior can be described with state equations, or transition equations.



Analysis of clocked sequential circuits

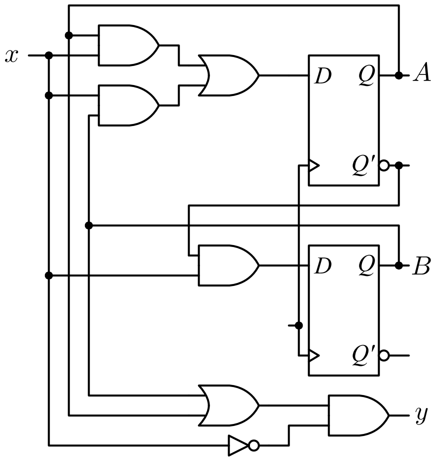
- Two D flip-flops:
  - $A(t+1) = A(t)x(t) + B(t)x(t)$,
  - $B(t+1) = A'(t)x(t)$.
- The output:
  - $y(t) = [A(t) + B(t)]x(t)'$.
  - Since all signals are labeled by t, we can also write $y = (A+B)x'$.



Analysis of clocked sequential circuits

- Time-sequence of inputs, outputs, FFs can be enumerated in a *state table*.

Present		Input	Next		Output
A	B	x	A	B	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0



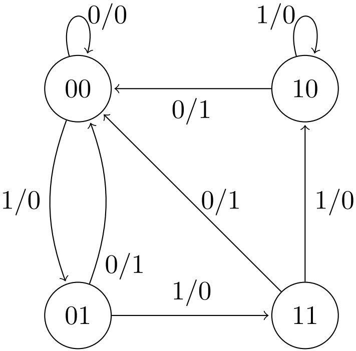
Analysis of clocked sequential circuits

- State table has 2^{m+n} rows for m flip-flops and n inputs, which is very long.
- Or with three sections, with input in the next state and output column.

Present		Next $x=0$		Next $x=1$		Output $x=0$	Output $x=1$
A	B	A	B	A	B	y	y
0	0	0	0	0	1	0	0
0	1	0	0	1	1	1	0
1	0	0	0	1	0	1	0
1	1	0	0	1	0	1	0

Analysis of clocked sequential circuits

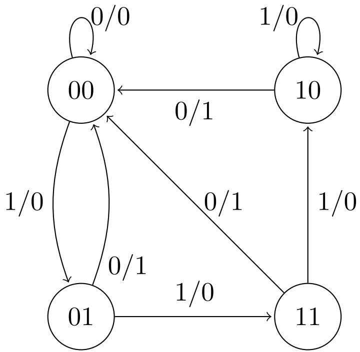
- *State diagram*:
  - Each state as a circle.
  - Transitions between states are directed lines connecting the circles.



Present		Next				Output	
		$x=0$		$x=1$		$x=0$	$x=1$
A	B	A	B	A	B	y	y
0	0	0	0	0	1	0	0
0	1	0	0	1	1	1	0
1	0	0	0	1	0	1	0
1	1	0	0	1	0	1	0

Analysis of clocked sequential circuits

- Circuit diagram → Equations → State table → State diagram.
  - State table is easier to derived from circuit diagram and state equations.
  - State diagram gives a pictorial view of state transitions.
- The shown diagram is a 1-detector.



Analysis of clocked sequential circuits

- The logic diagram of a sequential circuit consists of flip-flops and gates.
- The interconnections among the gates form a combinational circuit and may be specified algebraically with Boolean expressions.
- The part of the combinational circuit that generates external outputs is described algebraically by a set of Boolean functions called *output equations*.
- The part of the circuit that generates the inputs to flip-flops is described algebraically by a set of Boolean functions called *flip-flop input equations* (or, sometimes, *excitation equations*).

Analysis of clocked sequential circuits

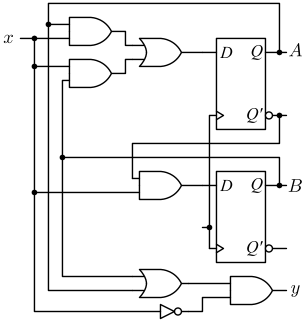
- We will adopt the convention of using the flip-flop input symbol to denote the input equation variable and a subscript to designate the name of the flip-flop output.
  - For example, the following input equation specifies an OR gate with inputs x and y connected to the D input of a flip-flop whose output is labeled with the symbol Q:

$$D_Q = x + y$$

Analysis of clocked sequential circuits

- The sequential circuit consists of two D flip-flops A and B, an input x, and an output y.
- The logic diagram of the circuit can be expressed algebraically with two flip-flop input equations and an output equation:

$$
\begin{aligned}
D_A &= Ax + Bx, \\
D_B &= A'x, \\
y &= (A+B)x'.
\end{aligned}
$$



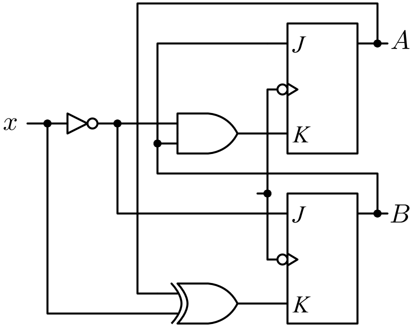
Analysis with JK/T flip-flops

- JK flip-flops and T flip-flops are fifferent from D flip-flops whose state equation is the same as the input equation.
  - Refer to the corresponding characteristic equation.
- Next-state values can be derived by
  - Determining input equations.
  - Listing binary values for each input equation.
  - Using the corresponding flip-flop characteristic table to determine next state values.

An example

- Determine input equations:

$$J_A = B,\ K_A = Bx',$$
$$J_B = x'\ K_B = A \oplus x.$$



An example

- List binary values for each input equation.
- Use the corresponding flip-flop characteristic table to determine next state values.
  - $Q(t+1) = JQ' + K'Q$.
- $A(t+1) = J_A A' + K'_A A = A'B + A(Bx')'$.
- $B(t+1) = J_B B' + K'_B B = x'B + B(A \oplus x)'$.

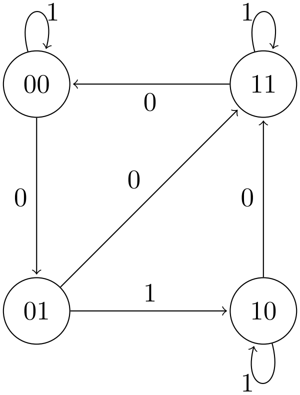
An example

- $A(t+1) = J_A A' + K'_A A = A'B + A(Bx')'$.
- $B(t+1) = J_B B' + K'_B B = x'B + B(A \oplus x)'$.

Present		Input	Next		FF Inputs			
A	B	x	A	B	J_A	K_A	J_B	K_B
0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0	1
0	1	0	1	1	1	1	1	0
0	1	1	1	0	1	0	0	1
1	0	0	1	1	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	1	1	1	1
1	1	1	1	1	1	0	0	0

An example

Present		Input	Next		FF Inputs			
A	B	x	A	B	J_A	K_A	J_B	K_B
0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0	1
0	1	0	1	1	1	1	1	0
0	1	1	1	0	1	0	0	1
1	0	0	1	1	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	1	1	1	1
1	1	1	1	1	1	0	0	0

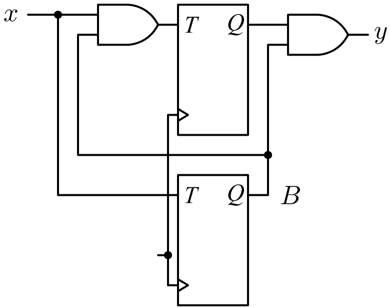


Another example

- Determine input equations:

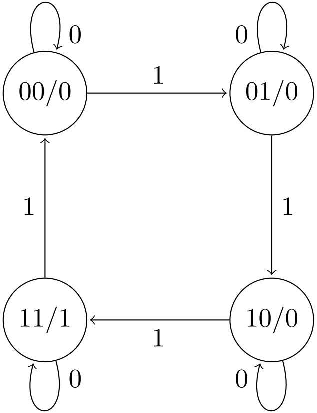
$$T_A = Bx, T_B = x$$
$$y = AB.$$

- List binary values for each input equation.
- Use the corresponding flip-flop characteristic table to determine next state values.
  - $Q(t+1) = T \oplus Q = T'Q + TQ'$.
- $A(t+1) = (Bx)'A + (Bx)A' = AB' + Ax' + A'Bx$.
- $B(t+1) = x \oplus B$.



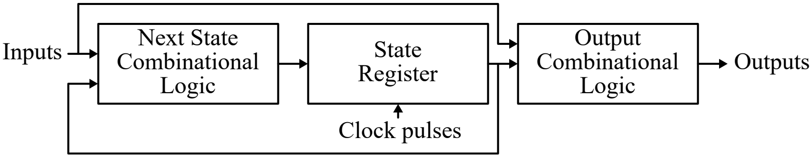
Another example

Present		Input	Next		Output
A	B	x	A	B	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	1	0
0	1	1	1	0	0
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	1



Finite state machine

- The most general model of a sequential circuit has inputs, outputs, and internal states.
- It is customary to distinguish between two models of sequential circuits:
  - *Mealy model*.



  - *Moore model*.