Electronic Materials and Devices

5 Semiconductor

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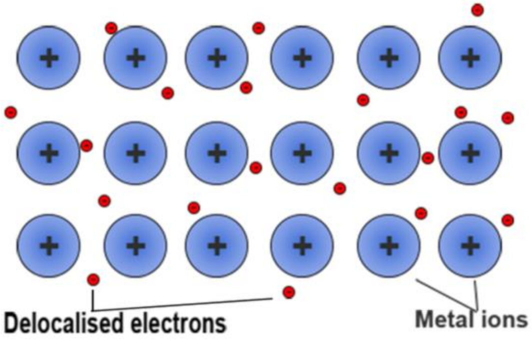
电子与电气工程系

4.1 Intrinsic semiconductor

Metal

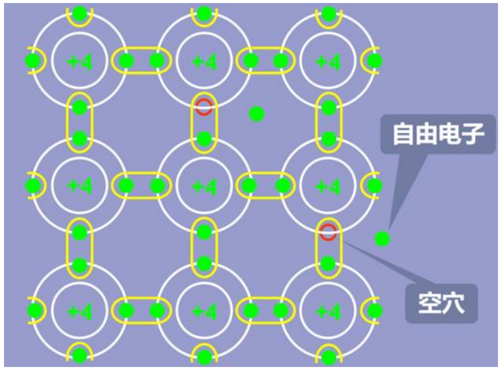
High conductivity



Free electron

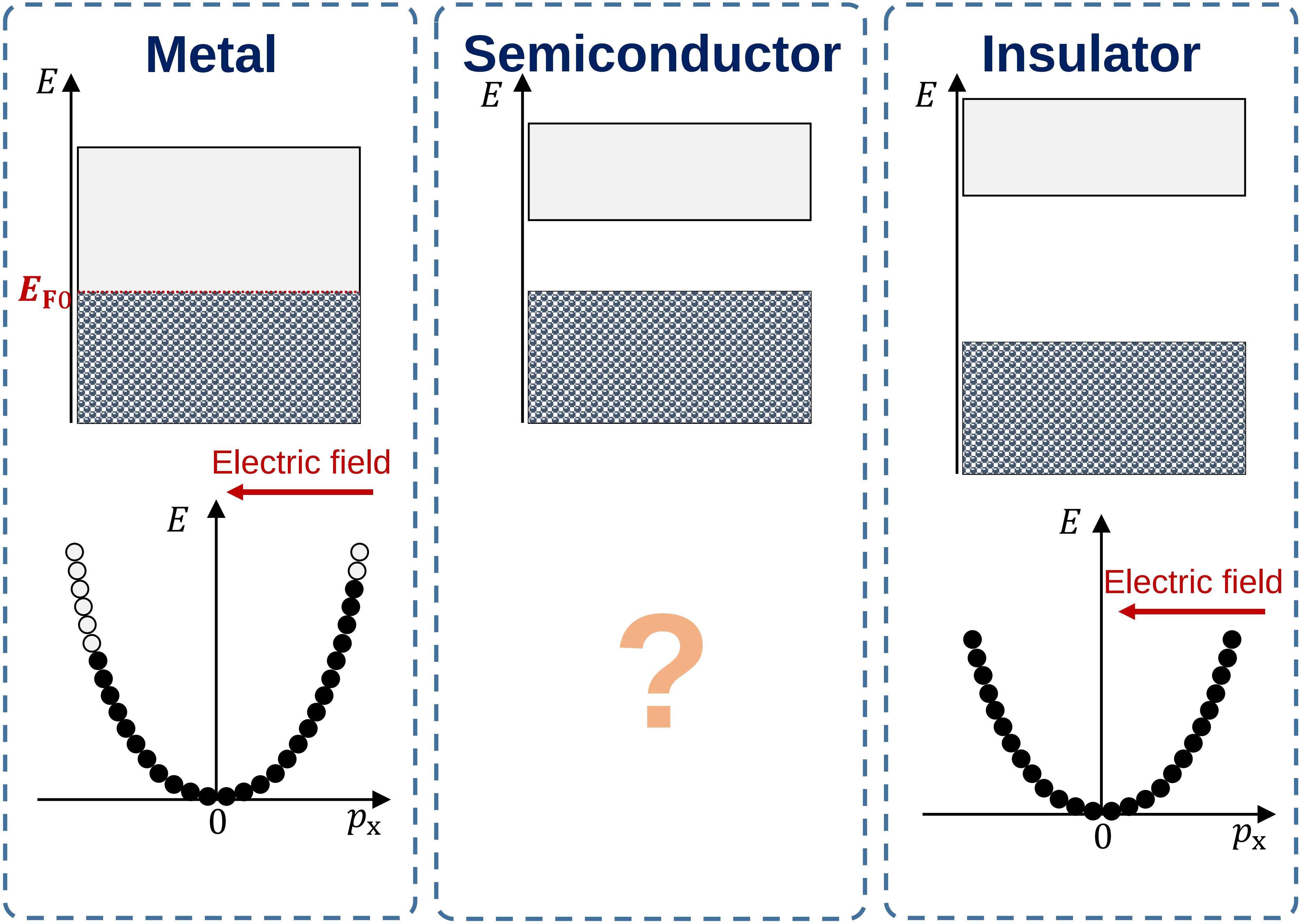
Semiconductor

Medium conductivity



Insulator

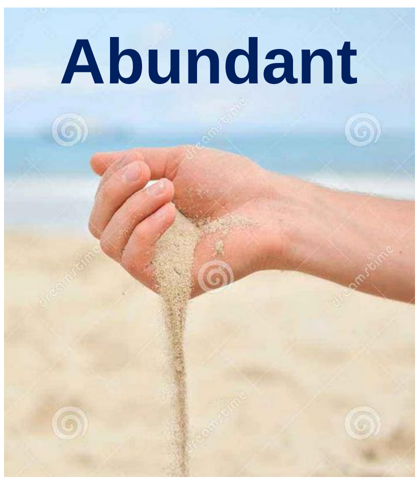
Extremely low conductivity

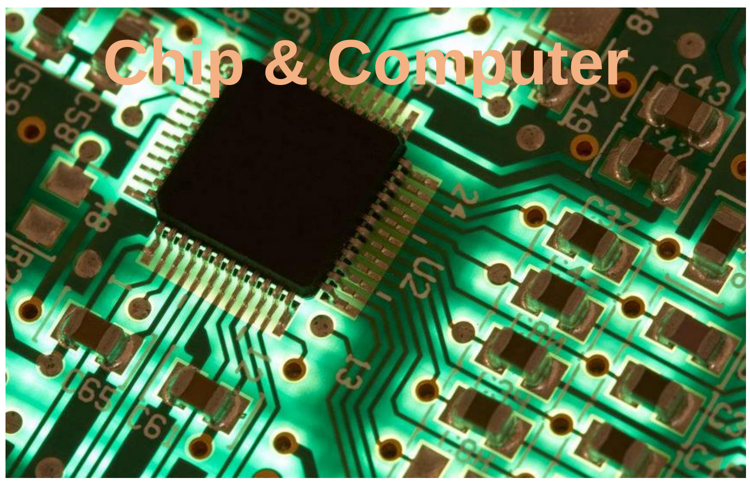


Metal
E
E F0
Electric field
E
0
p x
Semiconductor
E
?
Insulator
E
E
Electric field
0
p x

Most investigated and popular semiconductor is

Silicon 硅

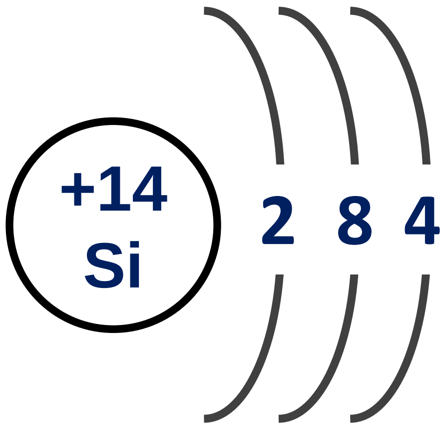


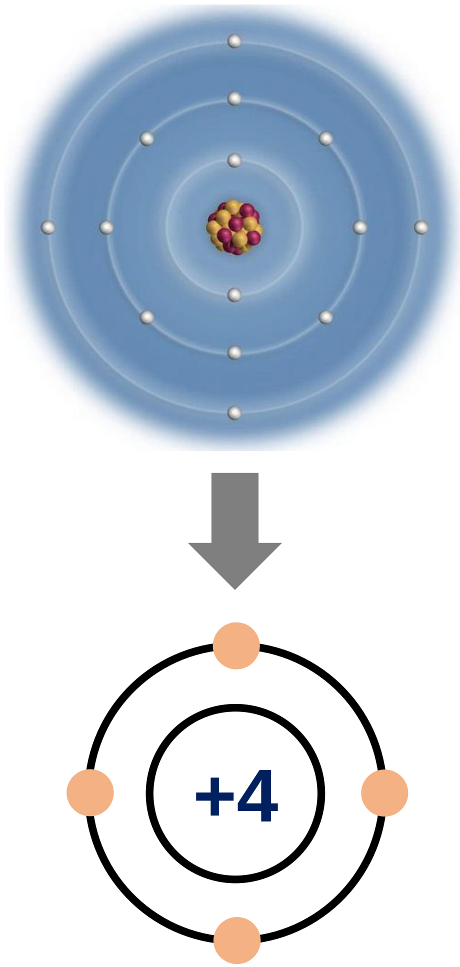


Abundant reserves

Widely used

Atom structure of Silicon

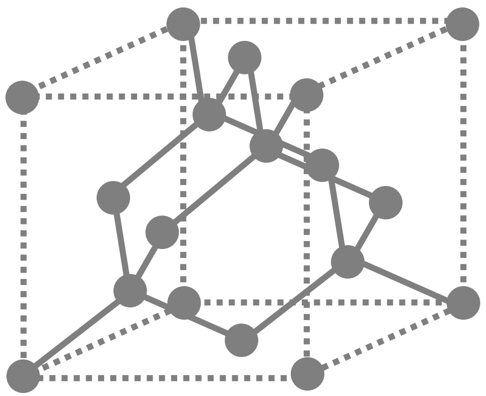




3s and 3p energy level are so close that the interactions result in 4 orbitals.

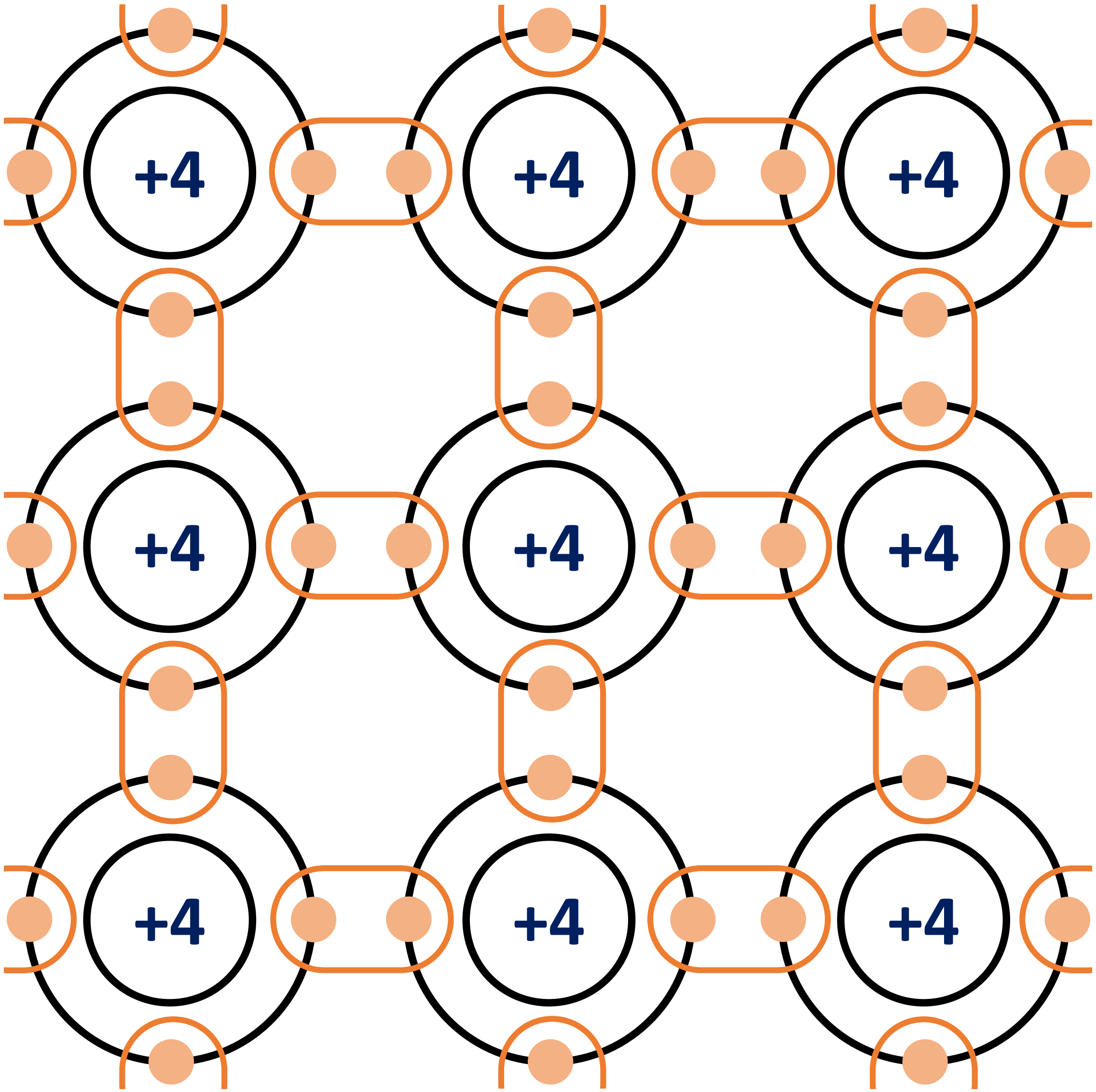
Crystal structure of silicon/germanium/diamond

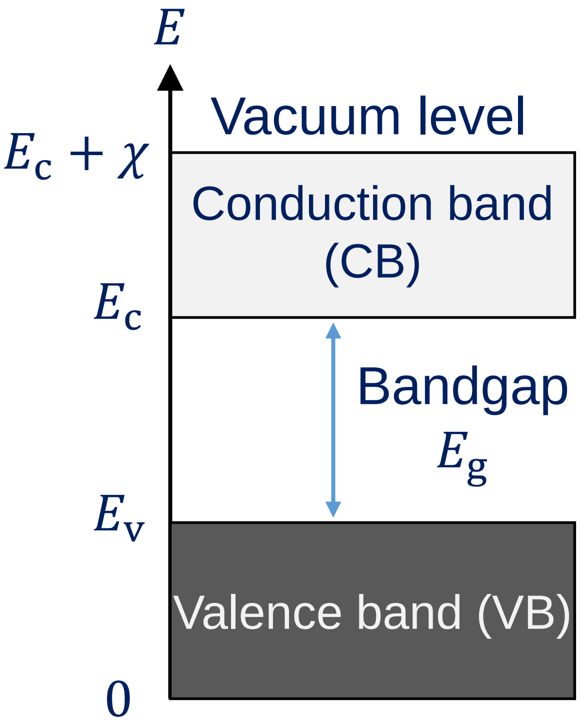
Unit cell



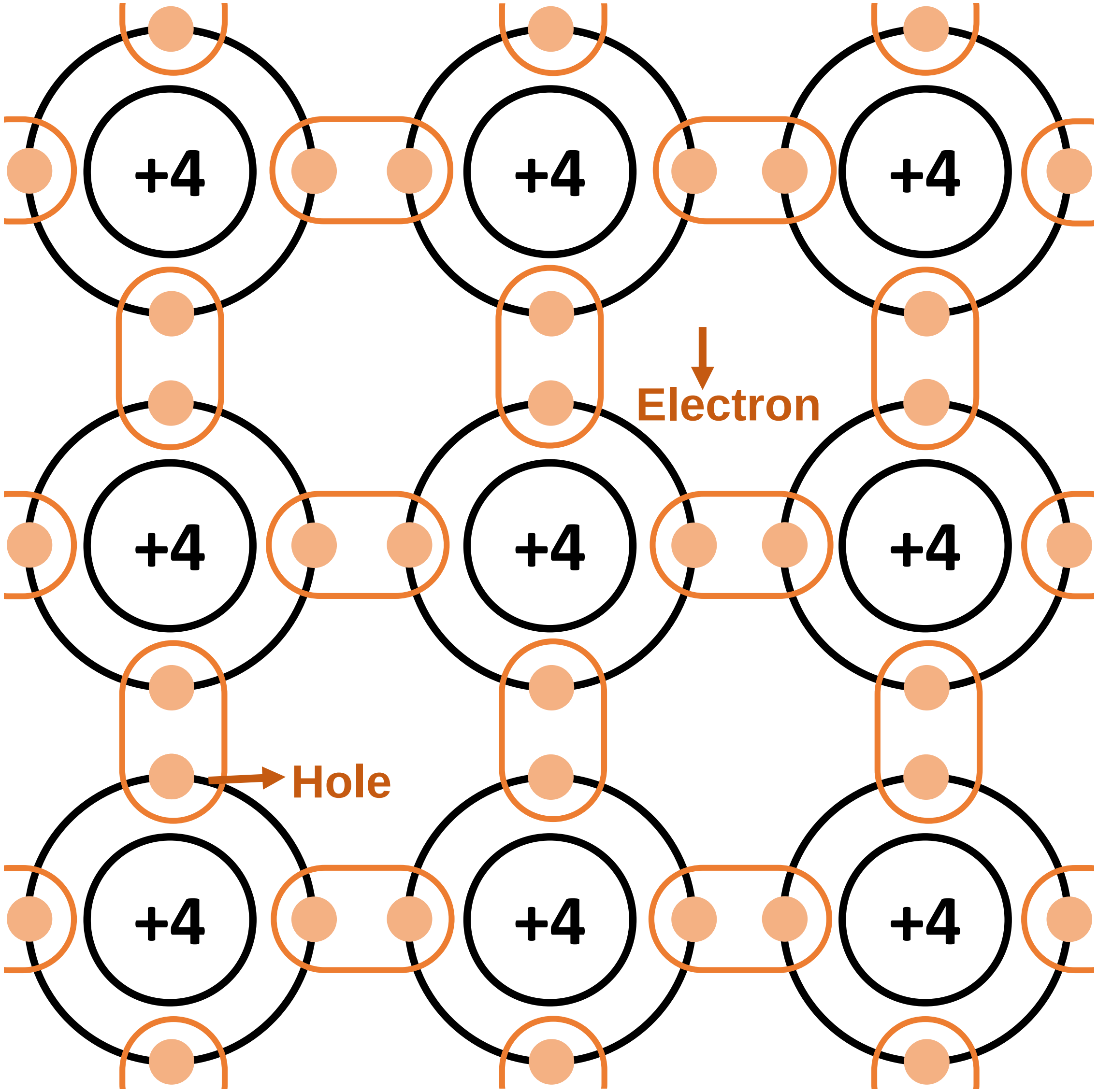
Covalent bond

2-dimensional schematic of silicon structure





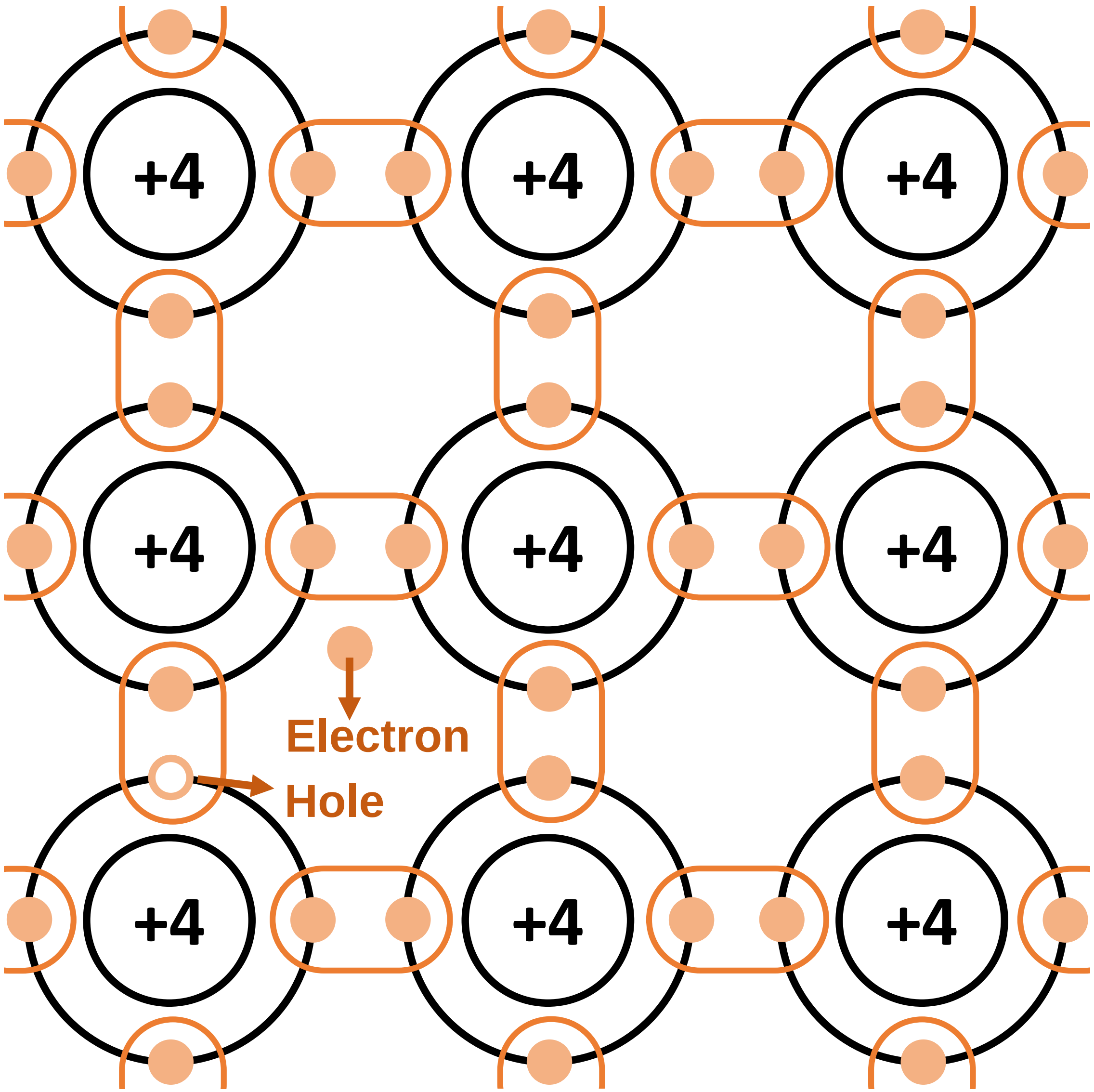
χ: electron affinity

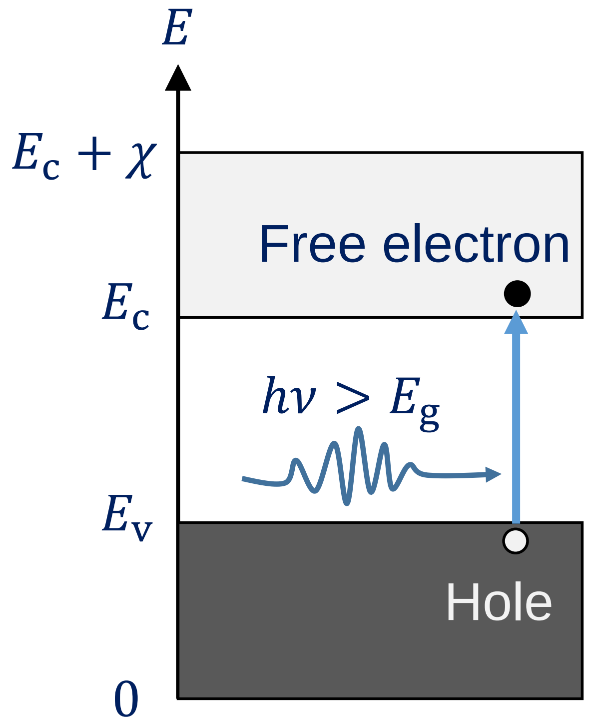


Thermal / light activation
热 / 光激发

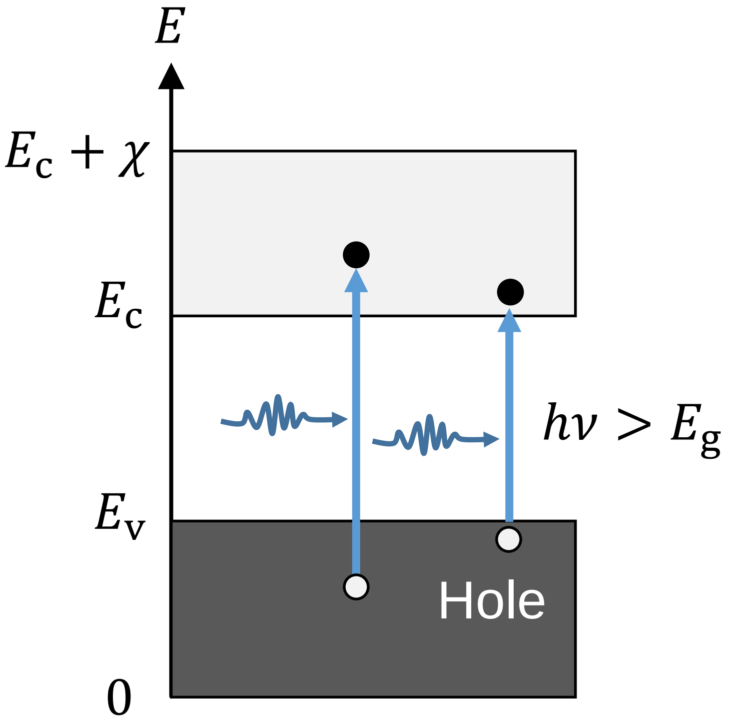
Bonded electrons are excited

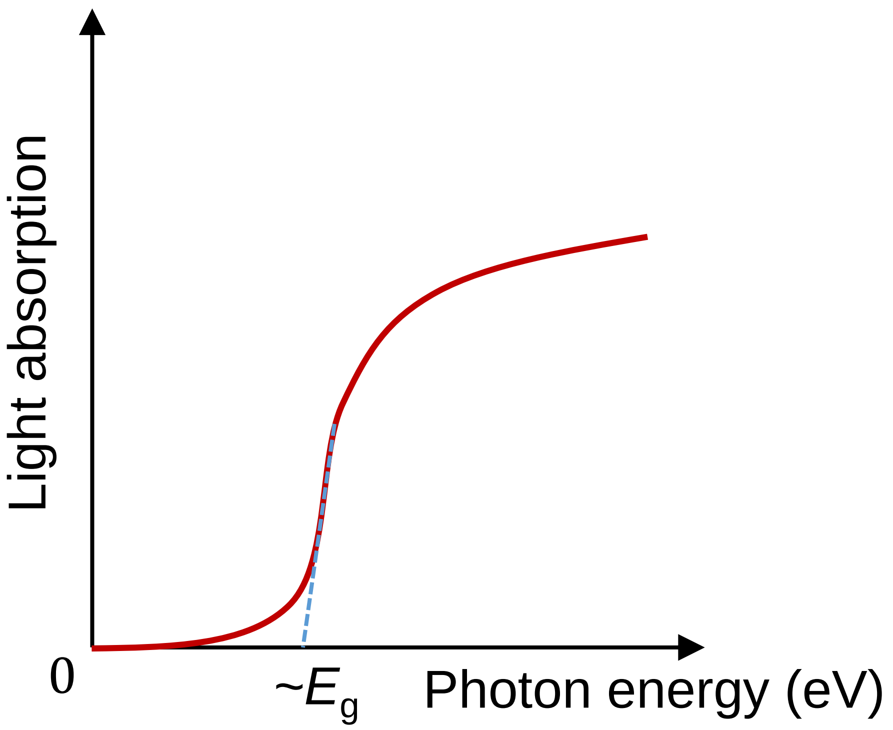
Electron and hole pair excitation
电子空穴对激发

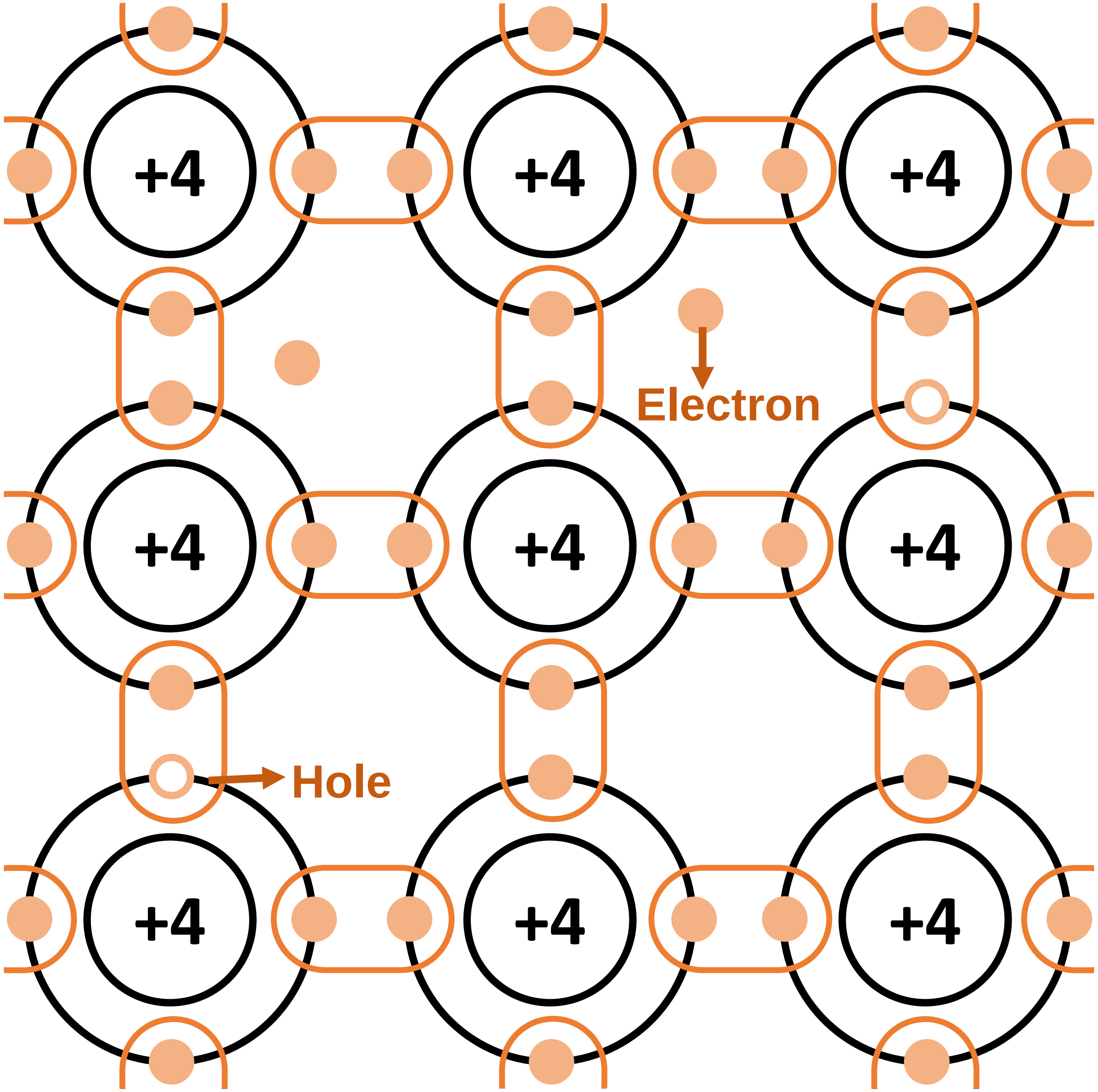

+4
Electron
Hole


E
$E_c + \chi$
Free electron
E_c
$h\nu > E_g$
E_v
Hole
0

Light absorption



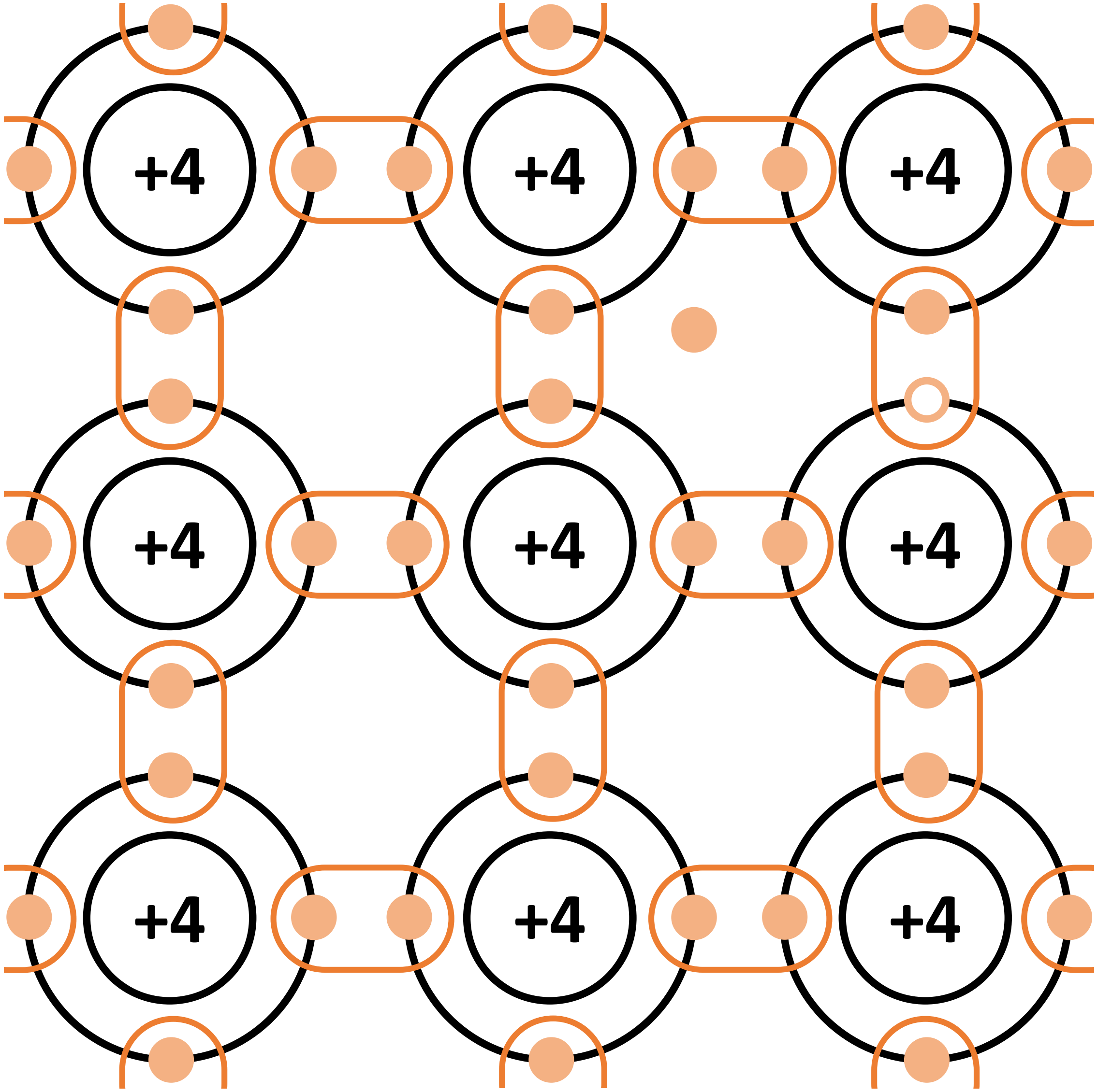




Higher temperature.

Electrons get more energy.

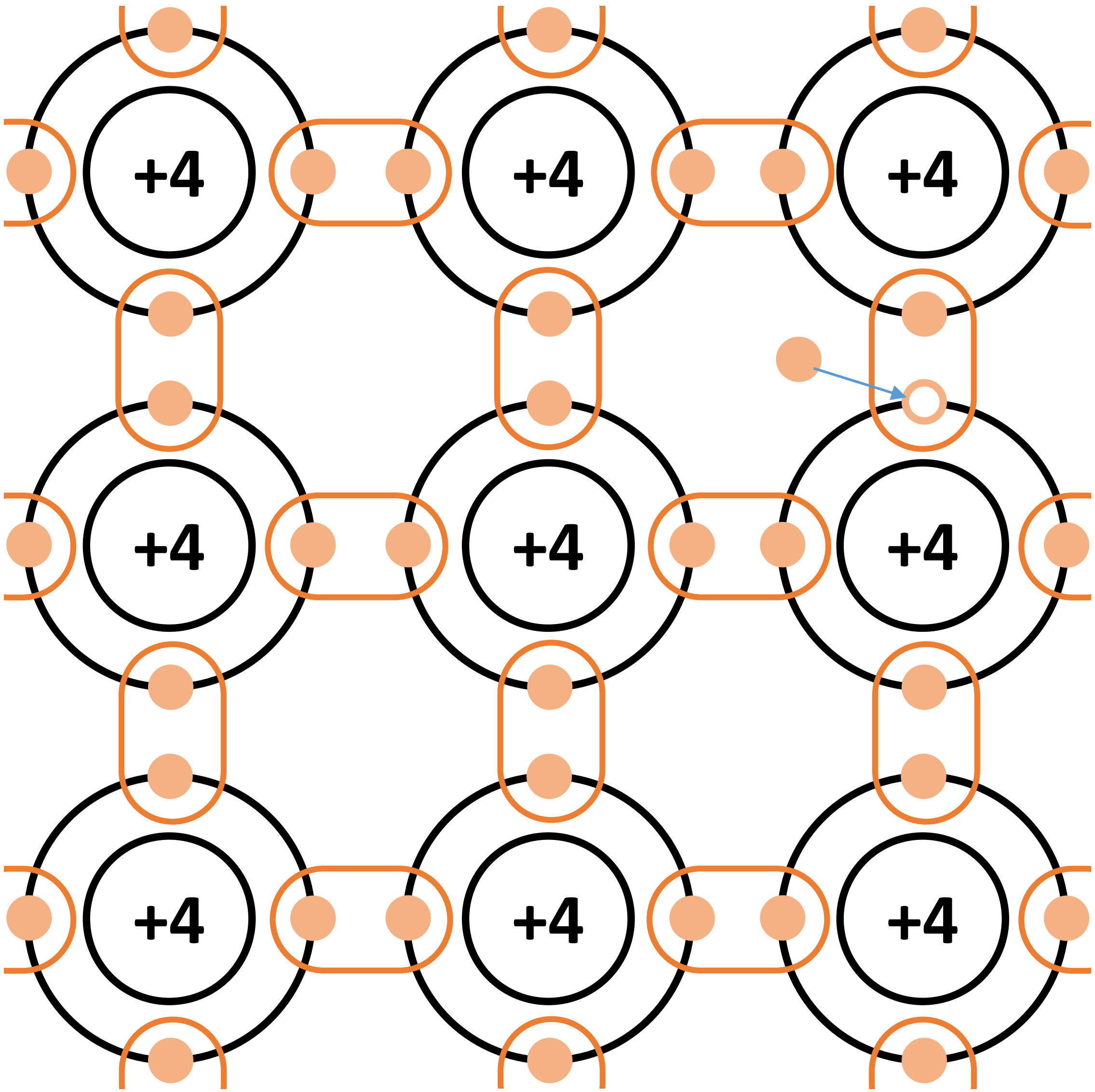
More electron and hole pairs.

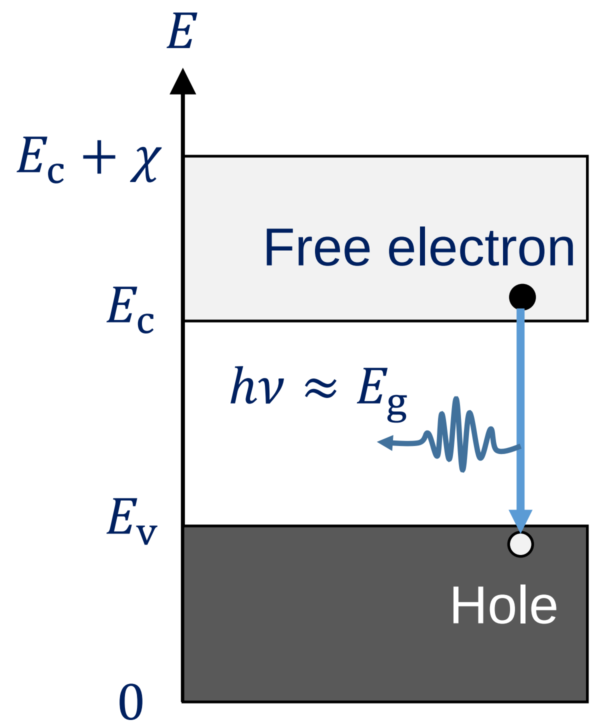


Electron and hole pair recombination
电子空穴对复合

At one time, there are both electron and hole pair excitation and recombination. They reach a dynamic equilibrium.

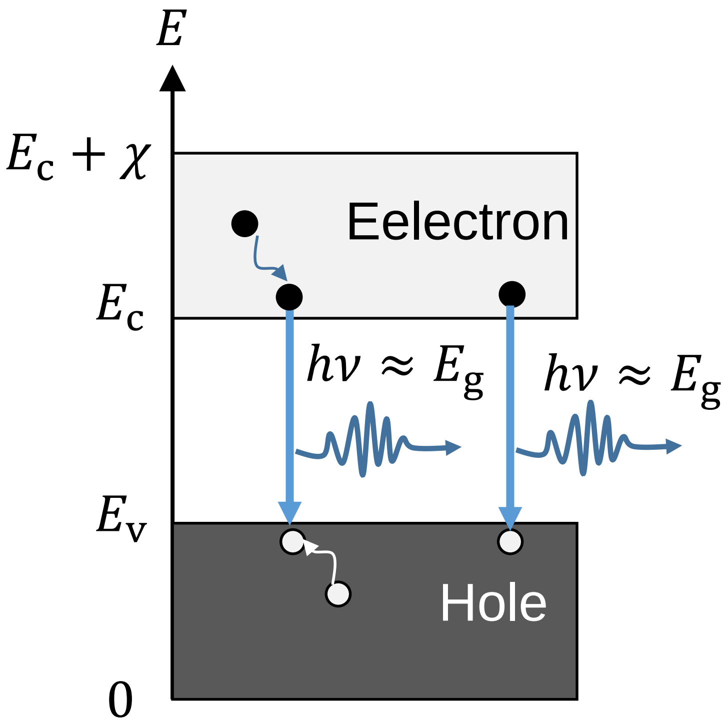
Electron and hole pair recombination

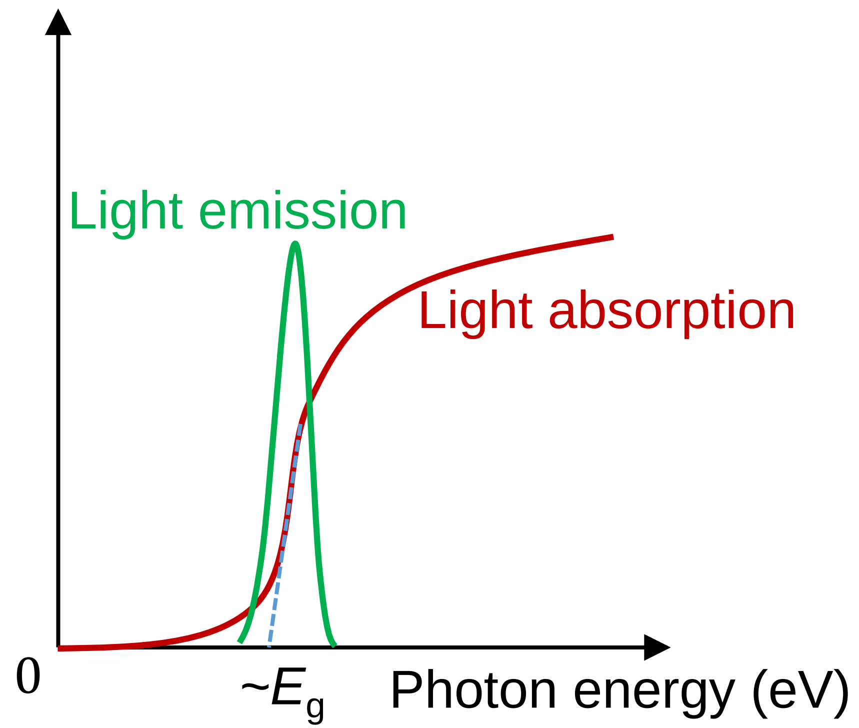


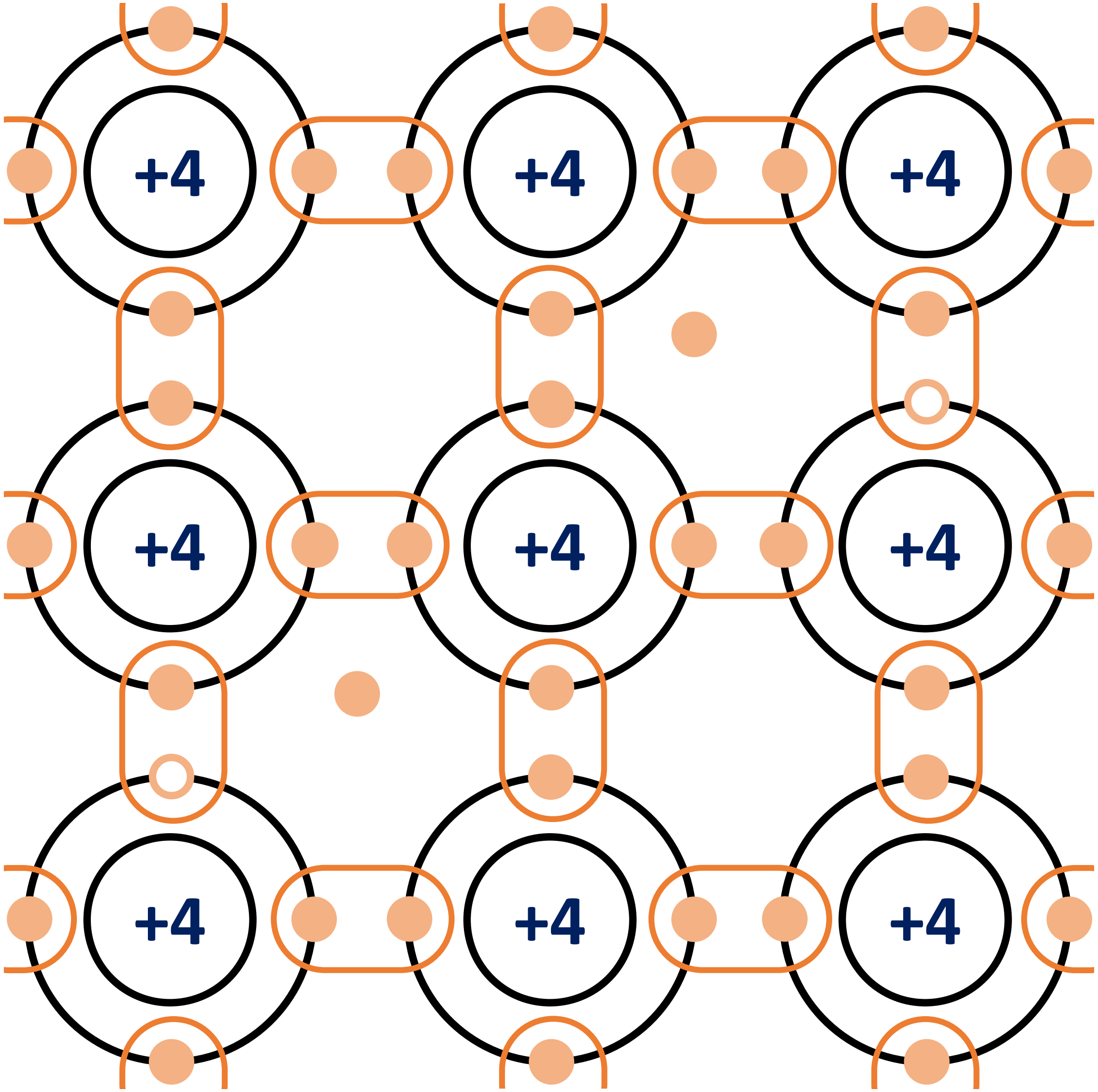


Emit photon or energy lose to lattice vibration.

Light emission





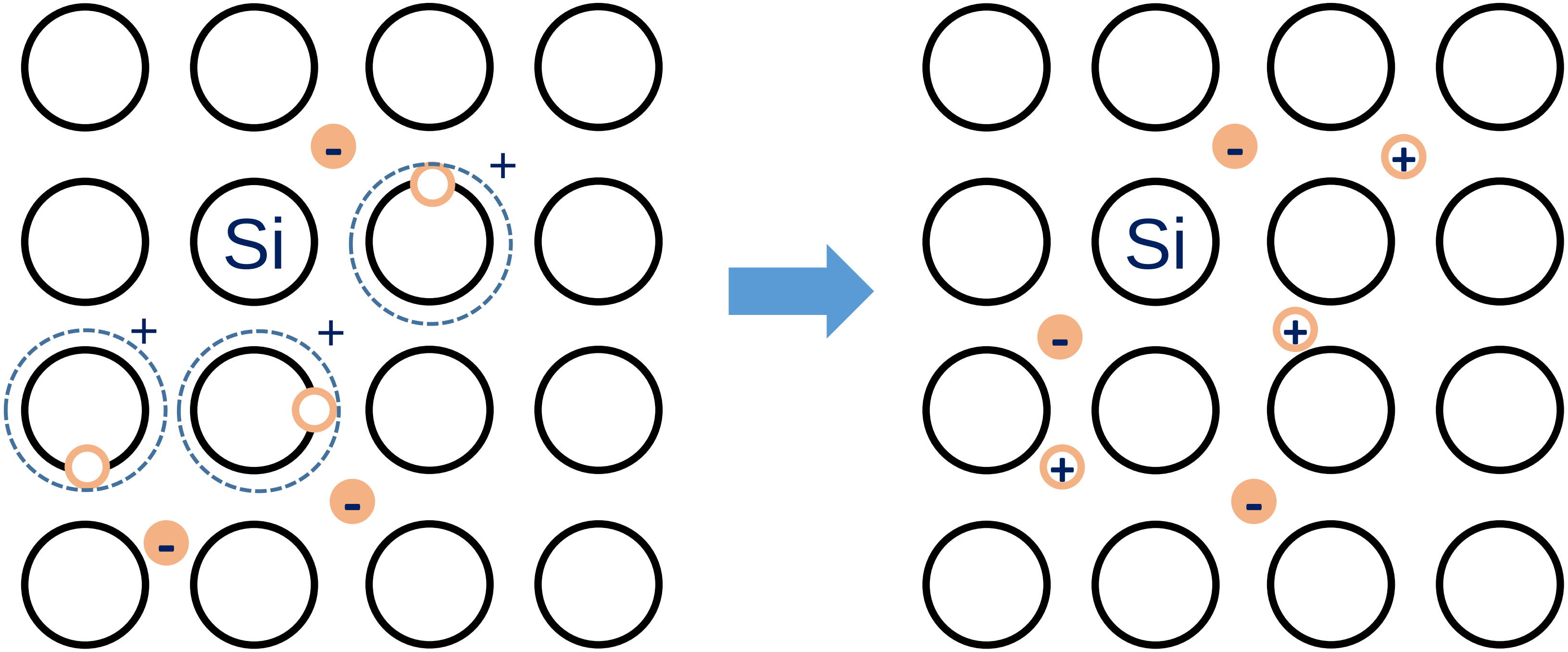


Free electron!

Free hole?

Free hole!

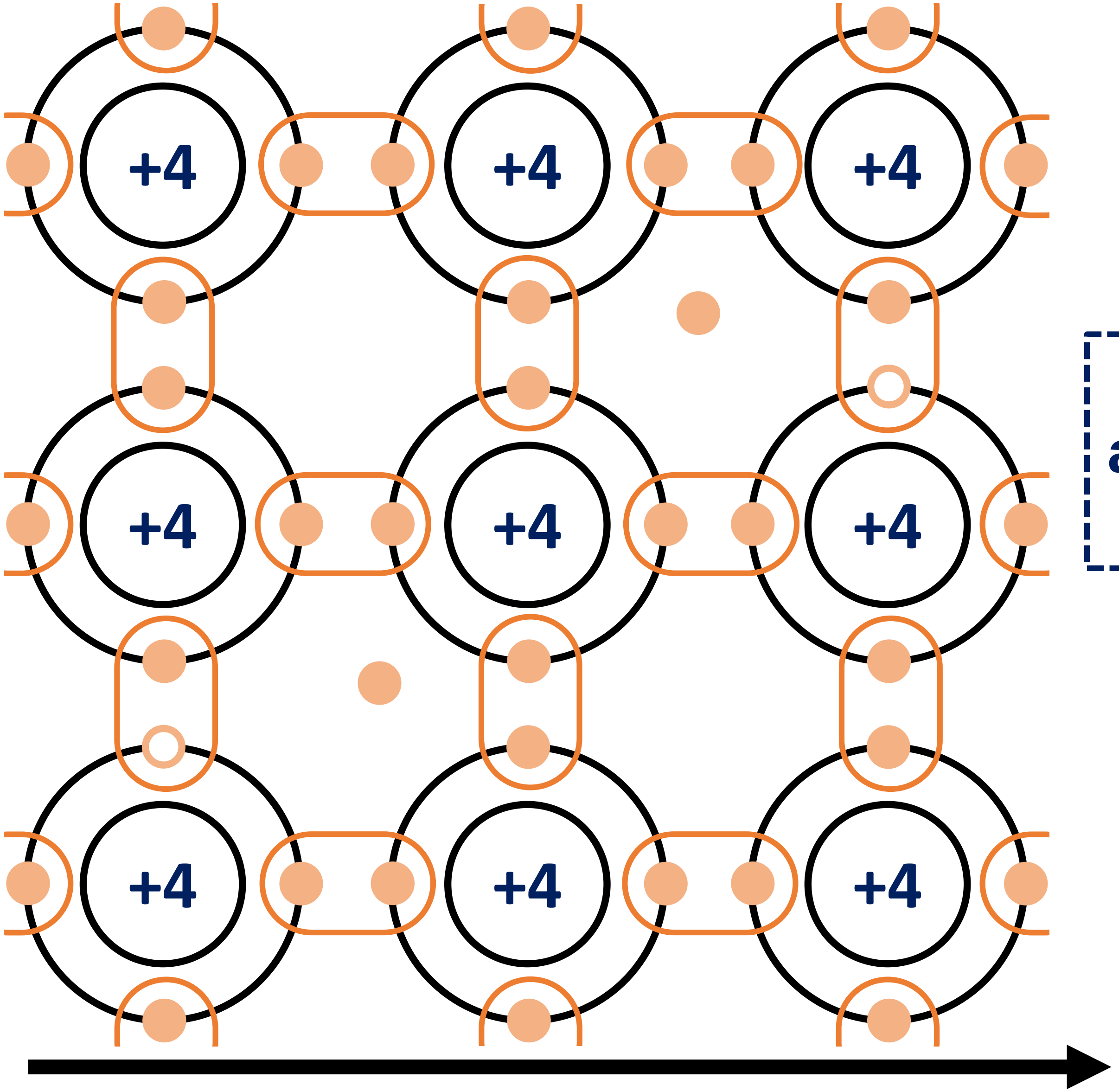
Equivalent to say holes as moving
电子的激发和复合可以等效成空穴在运动



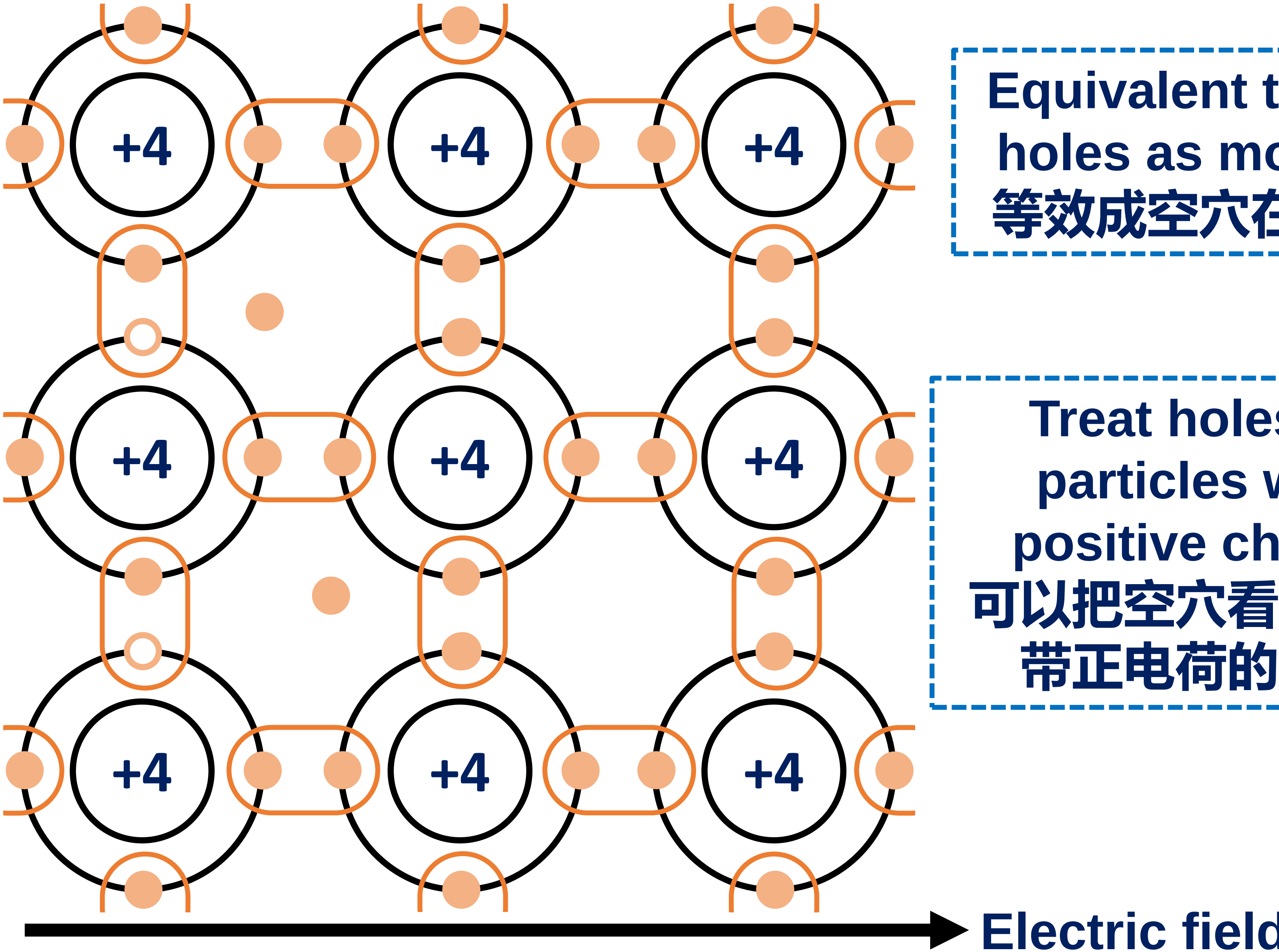
Treat holes as particles with positive charge.

The number of free electrons and holes in intrinsic semiconductor are equal.

4.2 Conduction in intrinsic semiconductors



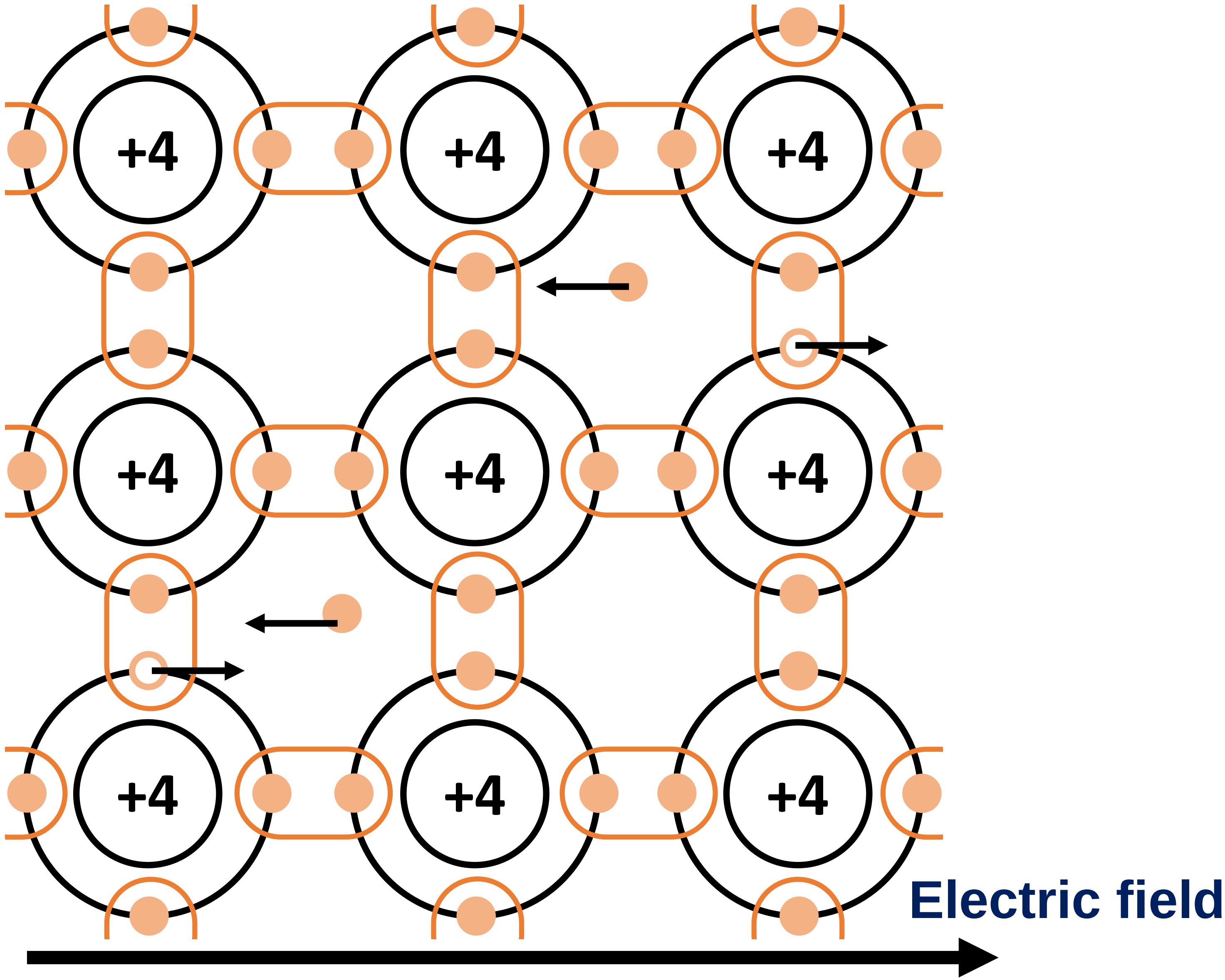
Conduction of activated electrons
本征激发电子导电

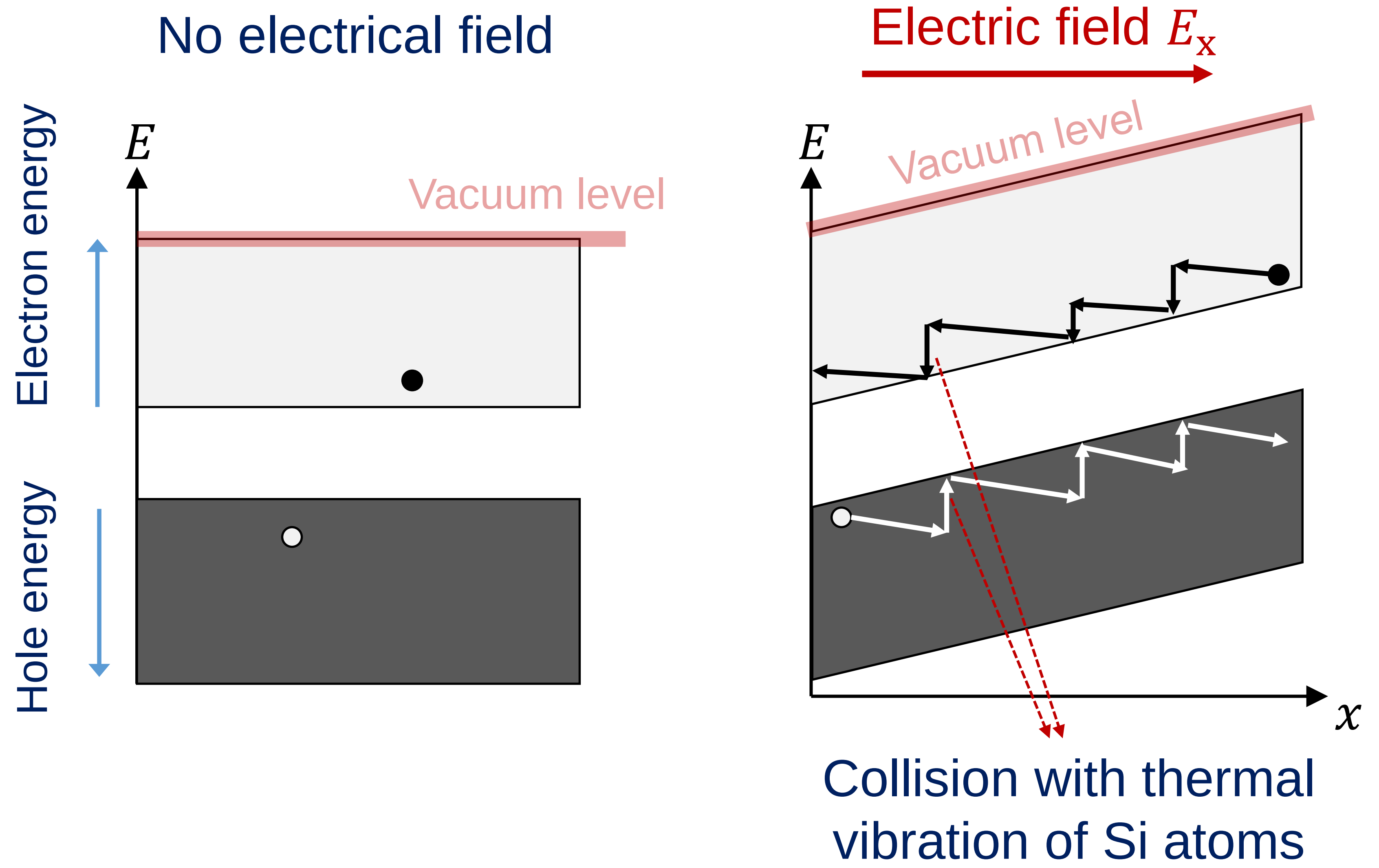


Equivalent to say holes as moving
等效成空穴在运动

Treat holes as particles with positive charge
可以把空穴看作一种带正电荷的粒子

The conduction of semiconductor is contributed from **both electrons and holes**





No electrical field
Electron energy
Hole energy
E
Vacuum level
Electric field E_x
E
Vacuum level
x
Collision with thermal vibration of Si atoms

Current density in semiconductors:

$$J = env_{\mathrm{de}} + epv_{\mathrm{dh}}$$

Concentrations of **electrons** and **holes**.

Drift velocity of electrons and holes:

$$v_{\mathrm{de}} = \mu_{\mathrm{e}} E_{\mathrm{x}} \qquad v_{\mathrm{dh}} = \mu_{\mathrm{h}} E_{\mathrm{x}}$$

Drift mobilities of **electrons** and **holes**.

$$\mu_{\mathrm{e}} = \frac{e\tau_{\mathrm{e}}}{m_{\mathrm{e}}^*} \qquad \mu_{\mathrm{h}} = \frac{e\tau_{\mathrm{h}}}{m_{\mathrm{h}}^*}$$

Effective mass of **electrons** and **holes**.

Conductivity in semiconductors:

$$\sigma = en\mu_{\mathrm{e}} + ep\mu_{\mathrm{h}}$$

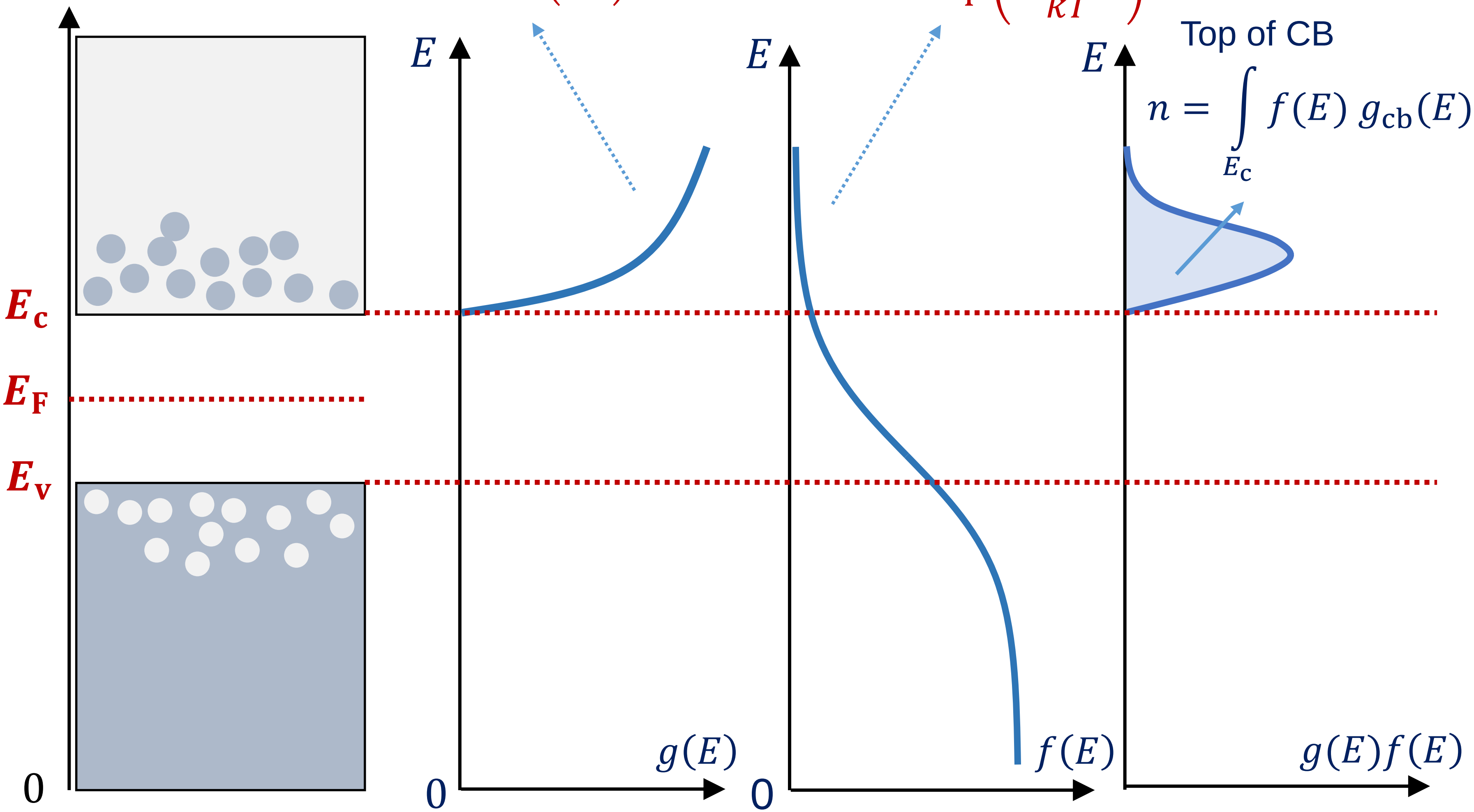
For intrinsic semiconductors:

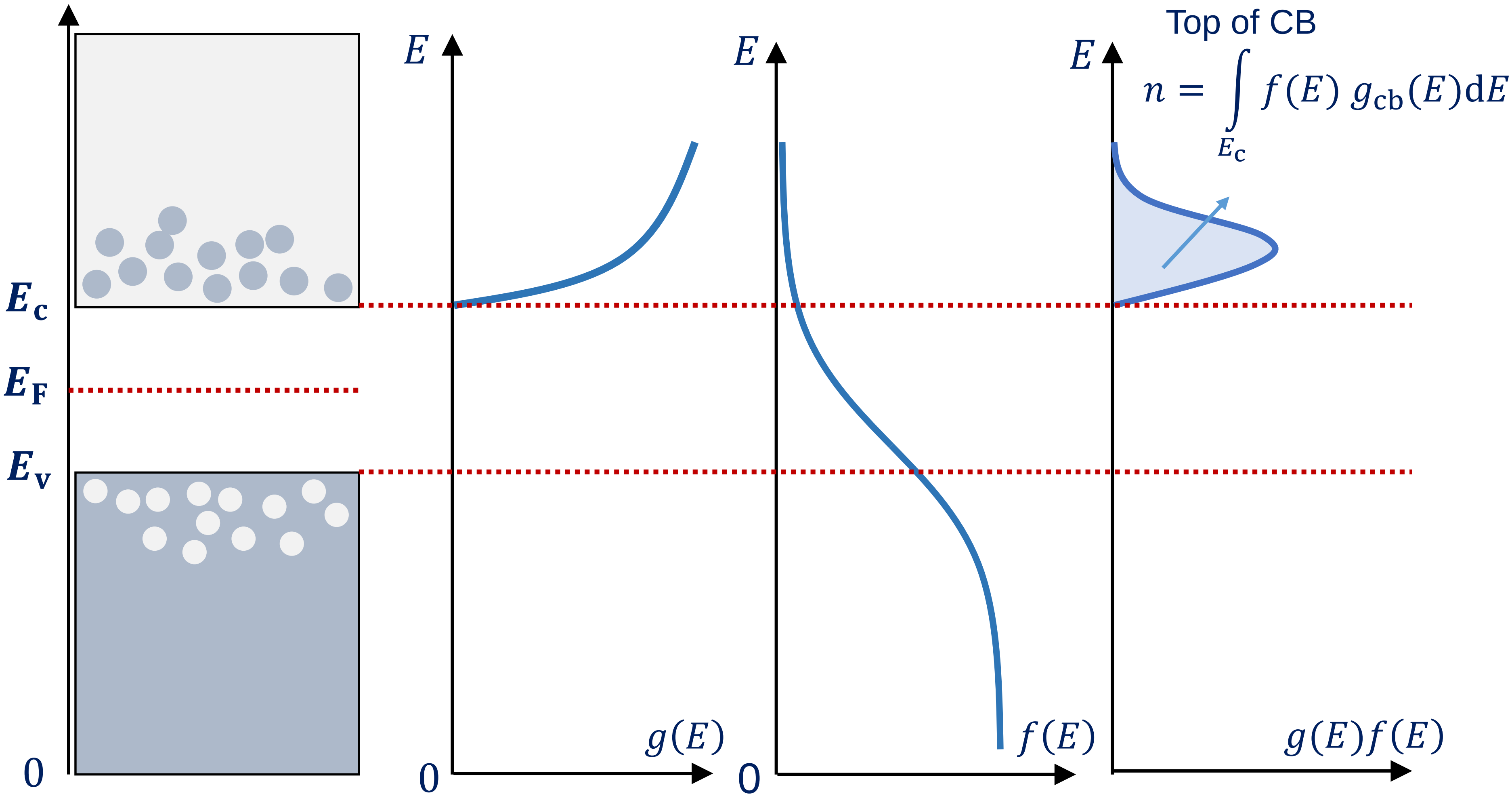
$$n = p$$

4.3 Electron and hole concentrations in intrinsic semiconductor

$$g_{\mathrm{cb}}(E) = 8\pi\sqrt{2(E - E_{\mathrm{c}})}\left(\frac{m_{\mathrm{e}}^{*}}{h^{2}}\right)^{3/2} \qquad f(E) = \frac{1}{1 + \exp\left(\frac{E - E_{\mathrm{F}}}{kT}\right)}$$

$$n = \int_{E_{\mathrm{c}}}^{\text{Top of CB}} f(E)\, g_{\mathrm{cb}}(E)\mathrm{d}E$$

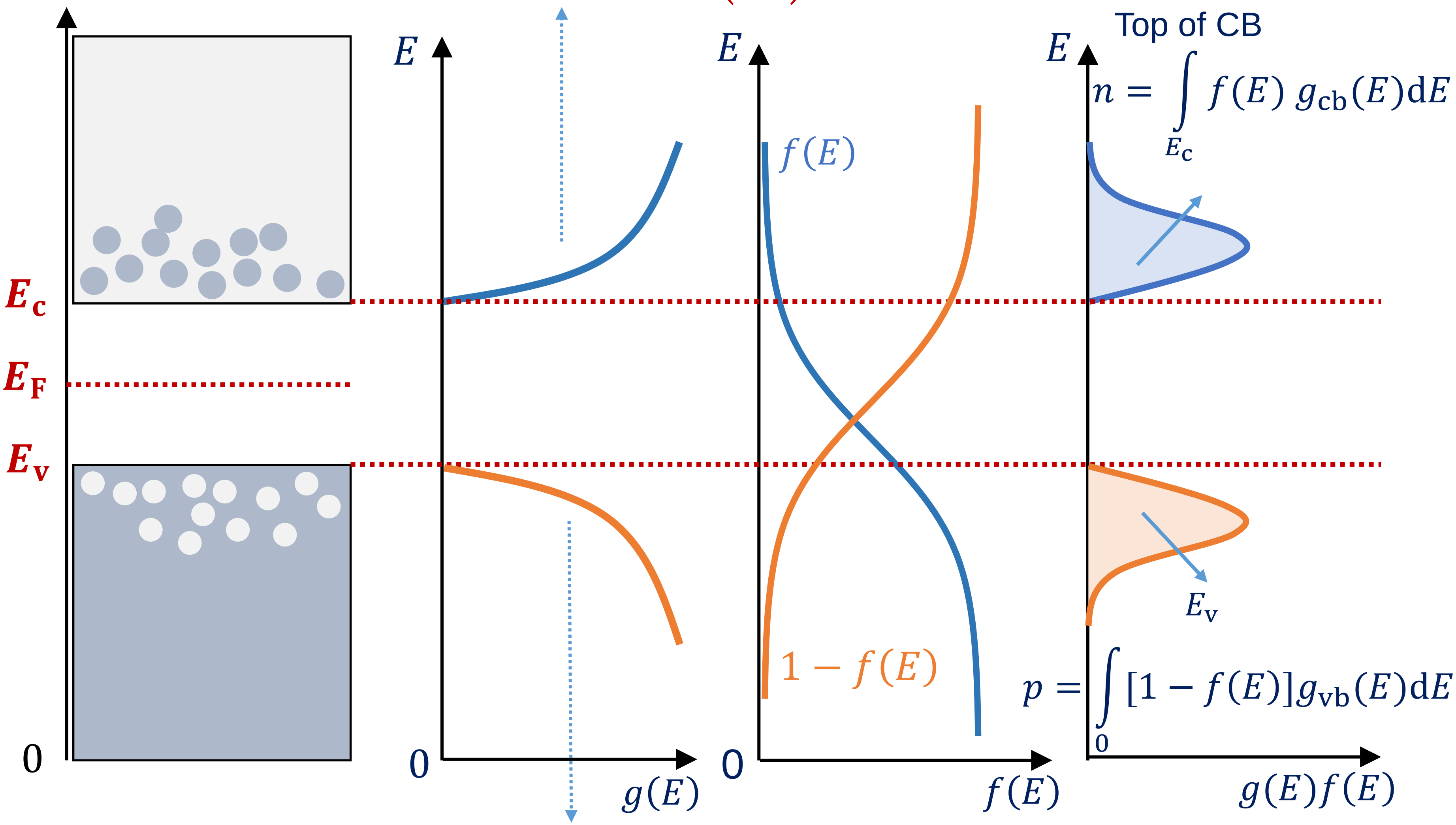




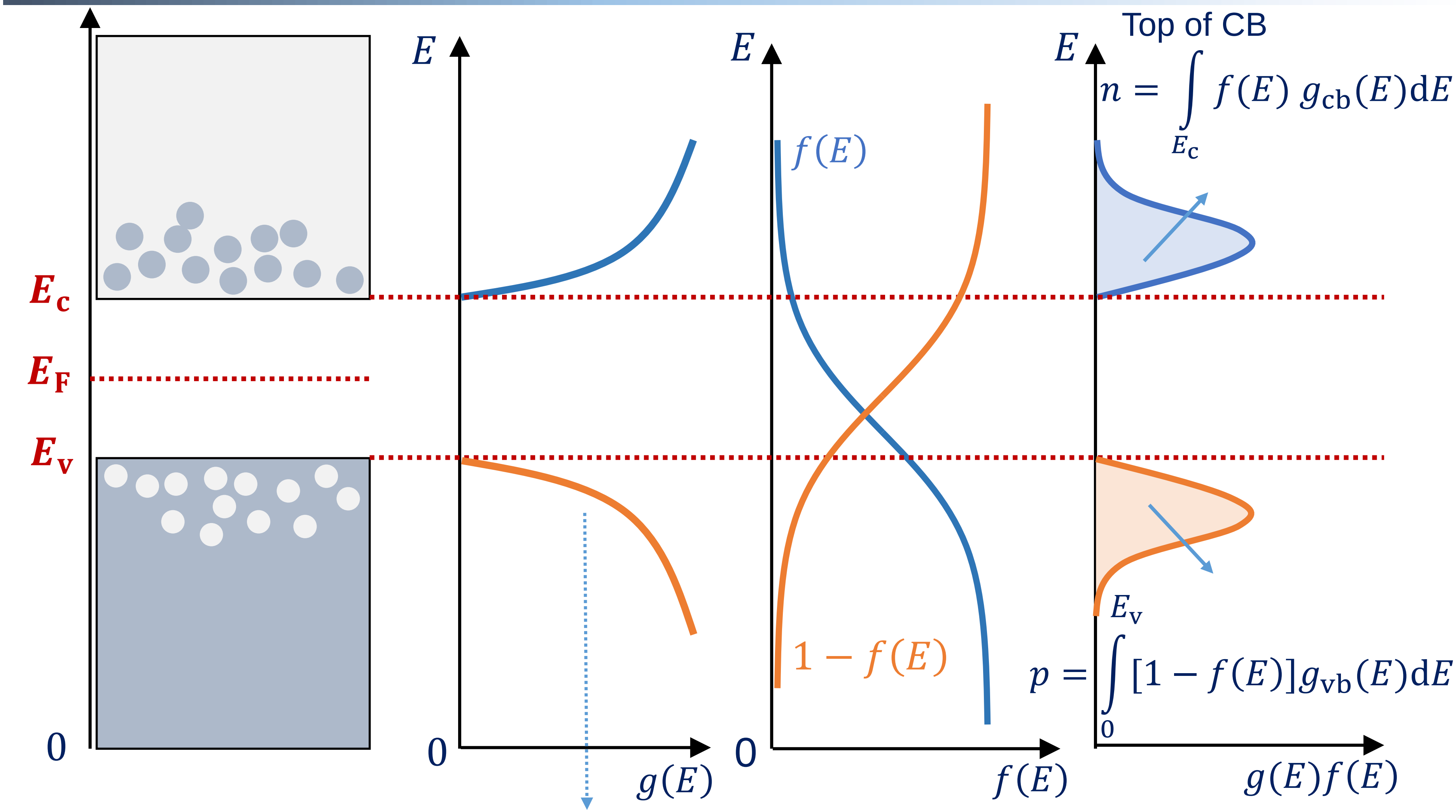
$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \approx \exp\left(-\frac{E - E_F}{kT}\right), E_c - E_F \gg kT$$

$$n = \int_{E_c}^{\text{Top of CB}} f(E) g_{cb}(E)\mathrm{d}E \approx N_c \exp\left(-\frac{E_c - E_F}{kT}\right), \quad N_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$$

$$g_{\mathrm{cb}}(E) = 8\pi\sqrt{2(E - E_{\mathrm{c}})}\left(\frac{m_{\mathrm{e}}^*}{h^2}\right)^{3/2}$$



$$g_{\mathrm{vb}}(E) = 8\pi\sqrt{2(E_{\mathrm{v}} - E)}\left(\frac{m_{\mathrm{h}}^*}{h^2}\right)^{3/2}$$



$$p = \int_0^{E_v} [1 - f(E)] g_{vb}(E) \mathrm{d}E \approx N_v \exp\left(-\frac{E_F - E_v}{kT}\right), \quad N_v = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$$

Electron concentration: $n = N_{\mathrm{c}} \exp\left(-\frac{E_{\mathrm{c}} - E_{\mathrm{F}}}{kT}\right)$, $N_{\mathrm{c}} = 2\left(\frac{2\pi m_{\mathrm{e}}^{*} kT}{h^2}\right)^{3/2}$

Hole concentration: $p = N_{\mathrm{v}} \exp\left(-\frac{E_{\mathrm{F}} - E_{\mathrm{v}}}{kT}\right)$, $N_{\mathrm{v}} = 2\left(\frac{2\pi m_{\mathrm{h}}^{*} kT}{h^2}\right)^{3/2}$

$$np = N_{\mathrm{c}} N_{\mathrm{v}} \exp\left(-\frac{E_{\mathrm{c}} - E_{\mathrm{v}}}{kT}\right) = N_{\mathrm{c}} N_{\mathrm{v}} \exp\left(-\frac{E_{\mathrm{g}}}{kT}\right)$$

$$n_{\mathrm{i}} = \sqrt{np} = \sqrt{N_{\mathrm{c}} N_{\mathrm{v}}} \exp\left(-\frac{E_{\mathrm{g}}}{2kT}\right)$$

For intrinsic semiconductors: $n = p = n_{\mathrm{i}}$

The Fermi energy of intrinsic semiconductor

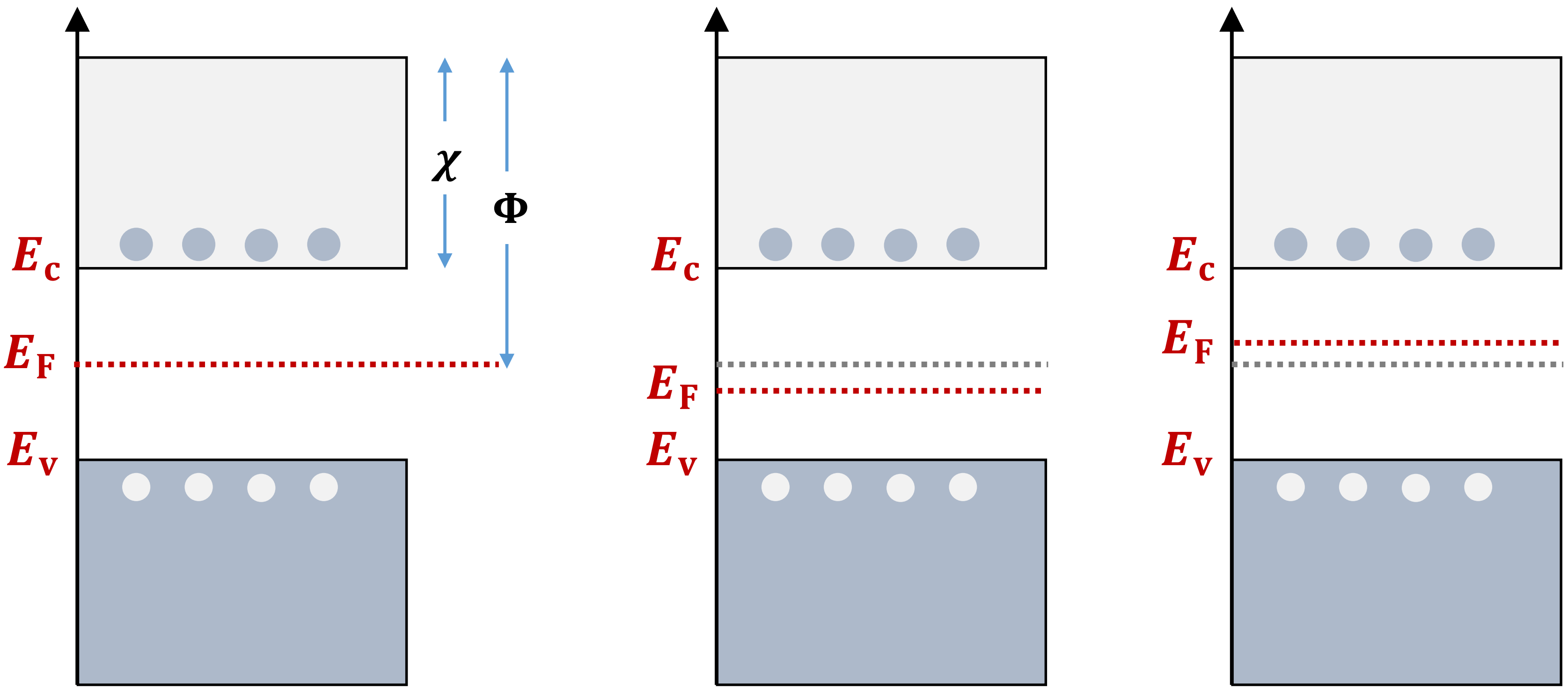
$$\begin{cases} p = n_\mathrm{i} = \sqrt{N_\mathrm{c}N_\mathrm{v}}\exp\left(-\dfrac{E_\mathrm{g}}{2kT}\right) \\ p = N_\mathrm{v}\exp\left(-\dfrac{E_\mathrm{F}-E_\mathrm{v}}{kT}\right) \end{cases}$$

$$E_\mathrm{F} = E_\mathrm{v} + \frac{1}{2}E_\mathrm{g} - \frac{1}{2}kT\ln\left(\frac{N_\mathrm{c}}{N_\mathrm{v}}\right)$$

$$E_\mathrm{F} = E_\mathrm{v} + \frac{1}{2}E_\mathrm{g} - \frac{3}{4}kT\ln\left(\frac{m_\mathrm{e}^*}{m_\mathrm{h}^*}\right)$$

The Fermi energy of intrinsic semiconductor

$m_e^* = m_h^*$ $\qquad$ $m_e^* > m_h^*$ $\qquad$ $m_e^* < m_h^*$



$$E_F = E_v + \frac{1}{2}E_g$$

$$E_F = E_v + \frac{1}{2}E_g - \frac{3}{4}kT\ln\left(\frac{m_e^*}{m_h^*}\right)$$

Q: The average energy of electrons in conduction band?

$$\begin{cases} \overline{E_{\mathrm{CB}}} = \dfrac{1}{n} \displaystyle\int_{\mathrm{CB\ band}} E f(E)\, g_{\mathrm{cb}}(E) \mathrm{d}E \\ n = \displaystyle\int_{\mathrm{CB\ band}} f(E)\, g_{\mathrm{cb}}(E) \mathrm{d}E \end{cases}$$

$$\Downarrow$$

$$\overline{E_{\mathrm{CB}}} = E_{\mathrm{c}} + \frac{3}{2} kT$$

KE: kinetic energy

Summary of intrinsic semiconductor

Electrical conduction is contributed from both:

electrons with negative charges, -

holes with positive charges. +

Electrical conduction of intrinsic semiconductor

Low conductivity

Strong temperature-dependence

Higher temperature, more electron-hole pairs, and higher conductivity

Suitable for thermistor and photoresistor.

Conductivity of intrinsic semiconductor is too low for practical applications.

Q: How to improve the conductivity of semiconductors?

4.4 Doped/Extrinsic semiconductor 掺杂半导体

Substitute intrinsic Si/Ge atoms with foreign atoms.

Depends on dopant (掺杂物), doped semiconductors can be categorized as

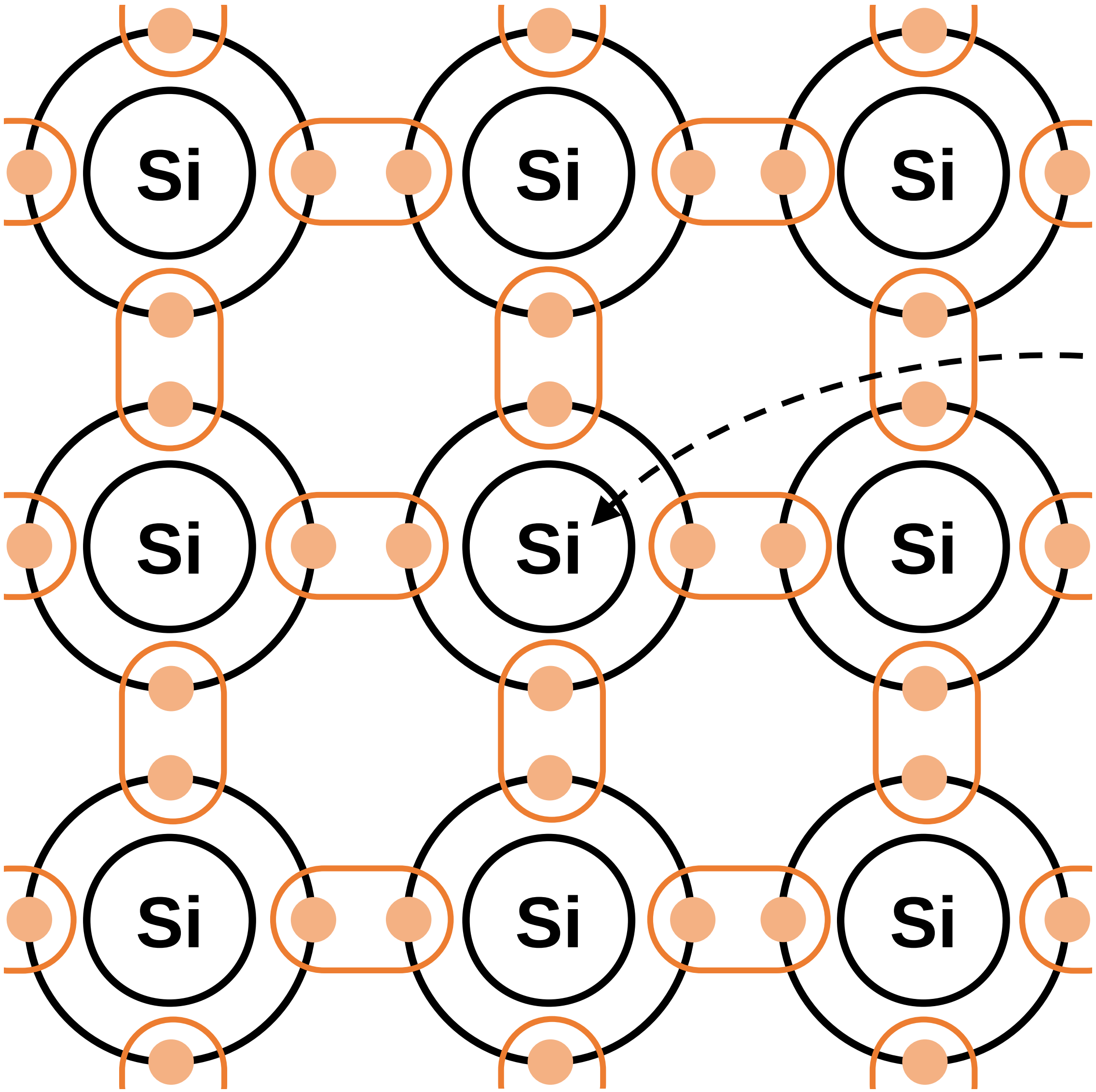
N-type semiconductor N型半导体

N represents “Negative”. Electrons are major conducting carriers.

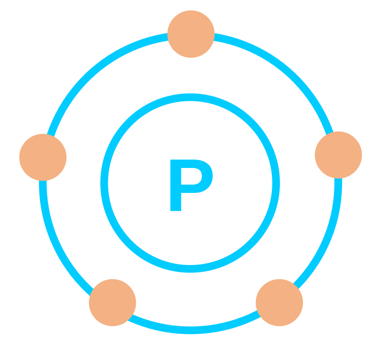
P-type semiconductor P型半导体

P represents “Positive”. Holes are major conducting carriers.

N-type semiconductor

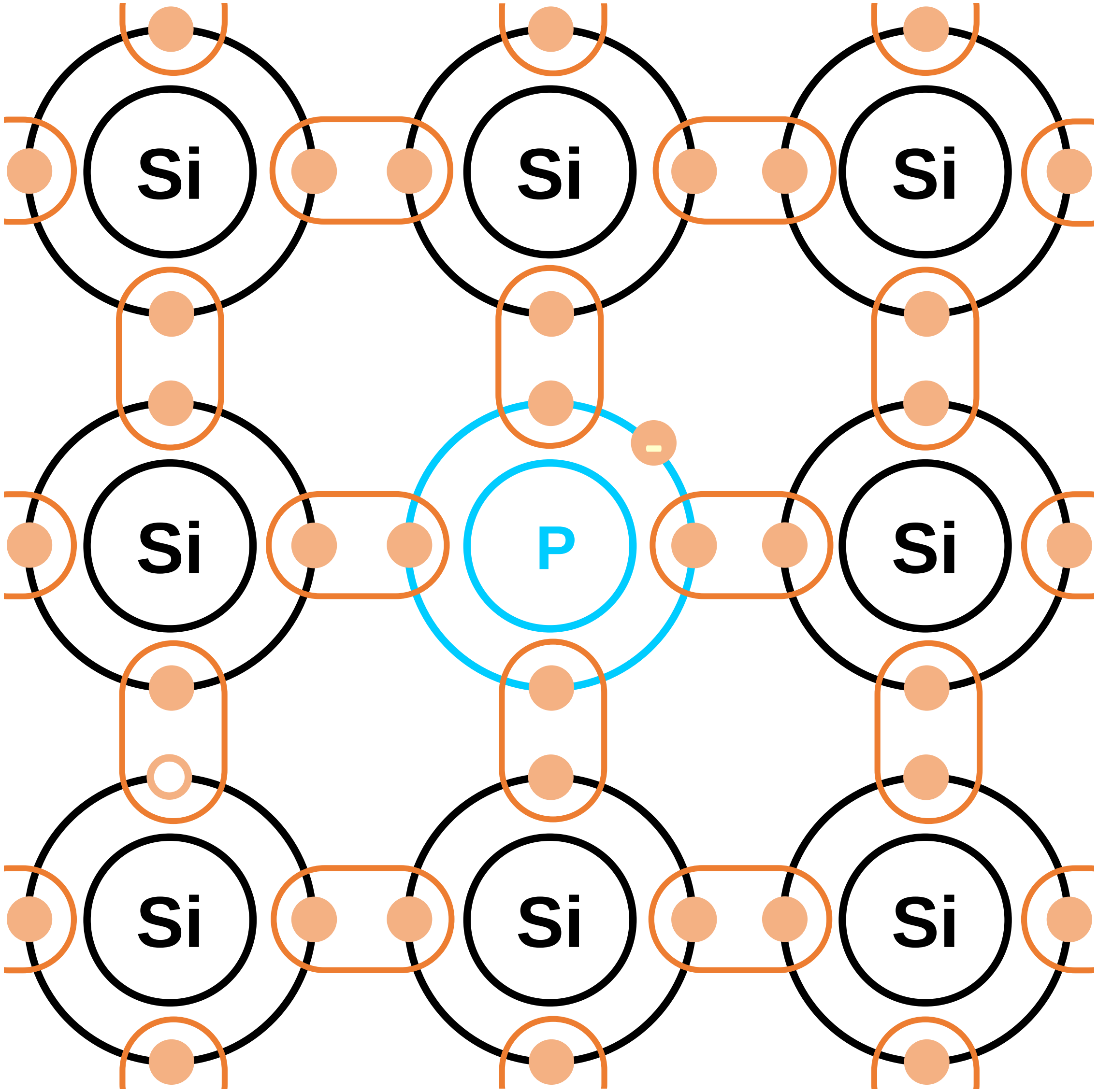


Dopant：
Phosphorus (磷)
and Arsenide (砷)



5 electrons in
outmost shell

Dopant：Phosphorus and Arsenide



Q: Is this electron free?

Binding energy of electron in a hydrogen atom model:

$$E_b = -E_1 = \frac{m_e e^4}{8\varepsilon_0{}^2 h^2} = 13.6\ \mathrm{eV}$$

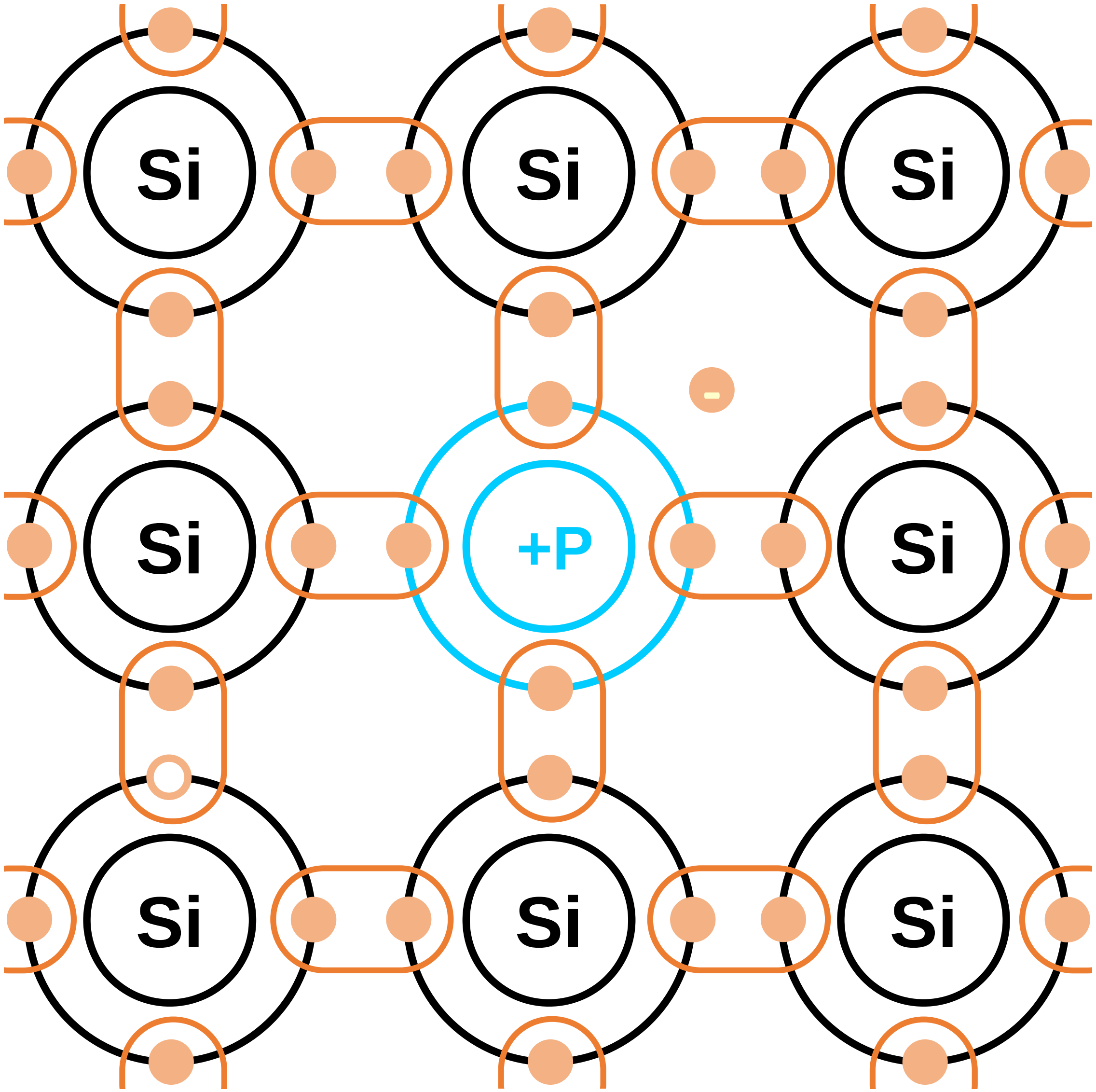
In real situation:

$$E_b = \frac{m_e^* e^4}{8(\varepsilon_0 \varepsilon_r)^2 h^2} = 0.032\ \mathrm{eV}$$

$$\varepsilon_r = 11.9, m_e^* \approx \frac{m_e}{3}$$

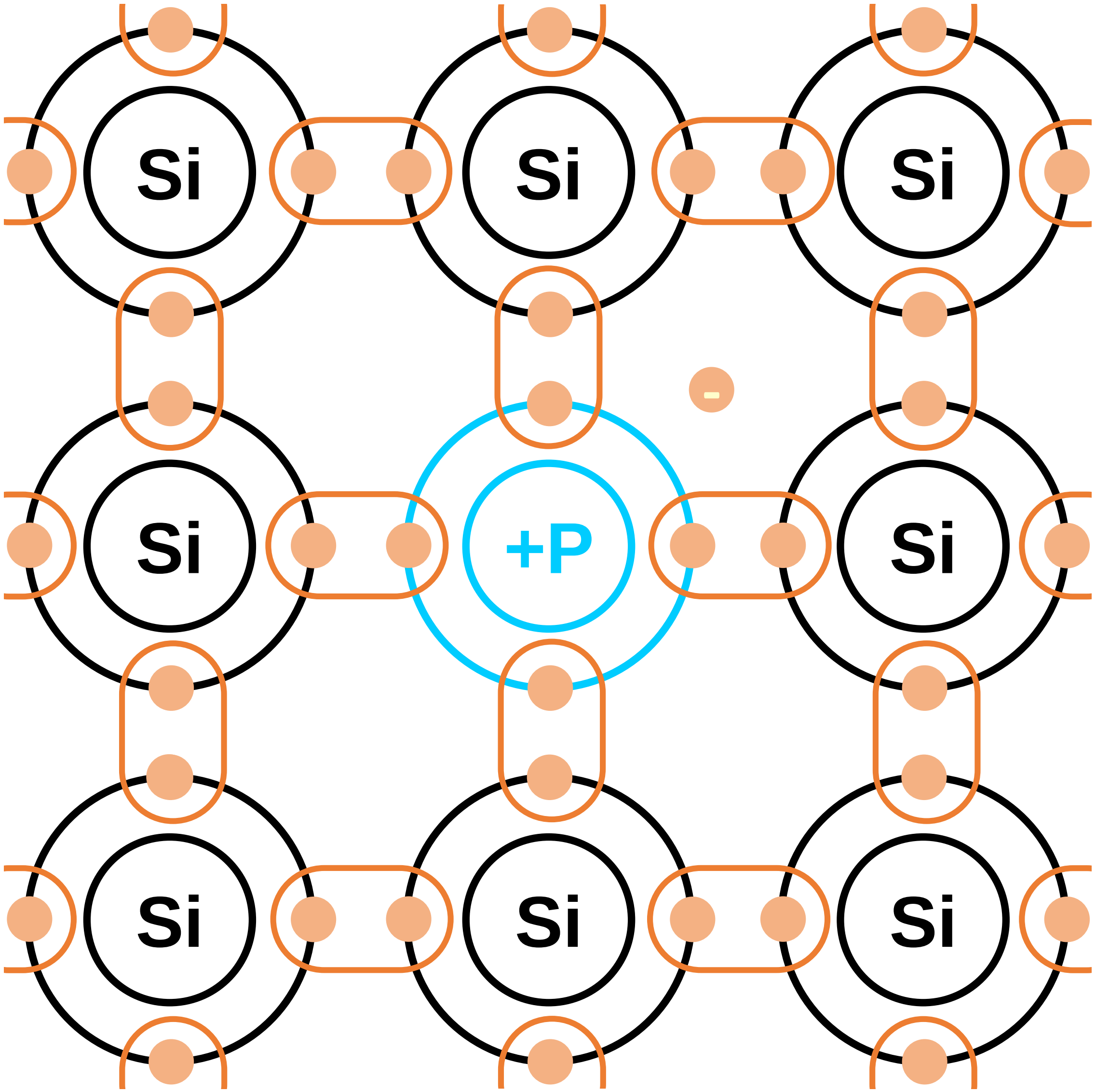
Electron can be treated as "free electron".

Dopant：Phosphorus and Arsenide



Electron can be treated as "free electron".

Dopant：Phosphorus and Arsenide



One more free electron and one positive ion

Electron number > hole number

Electron: major carriers

电子:多数载流子

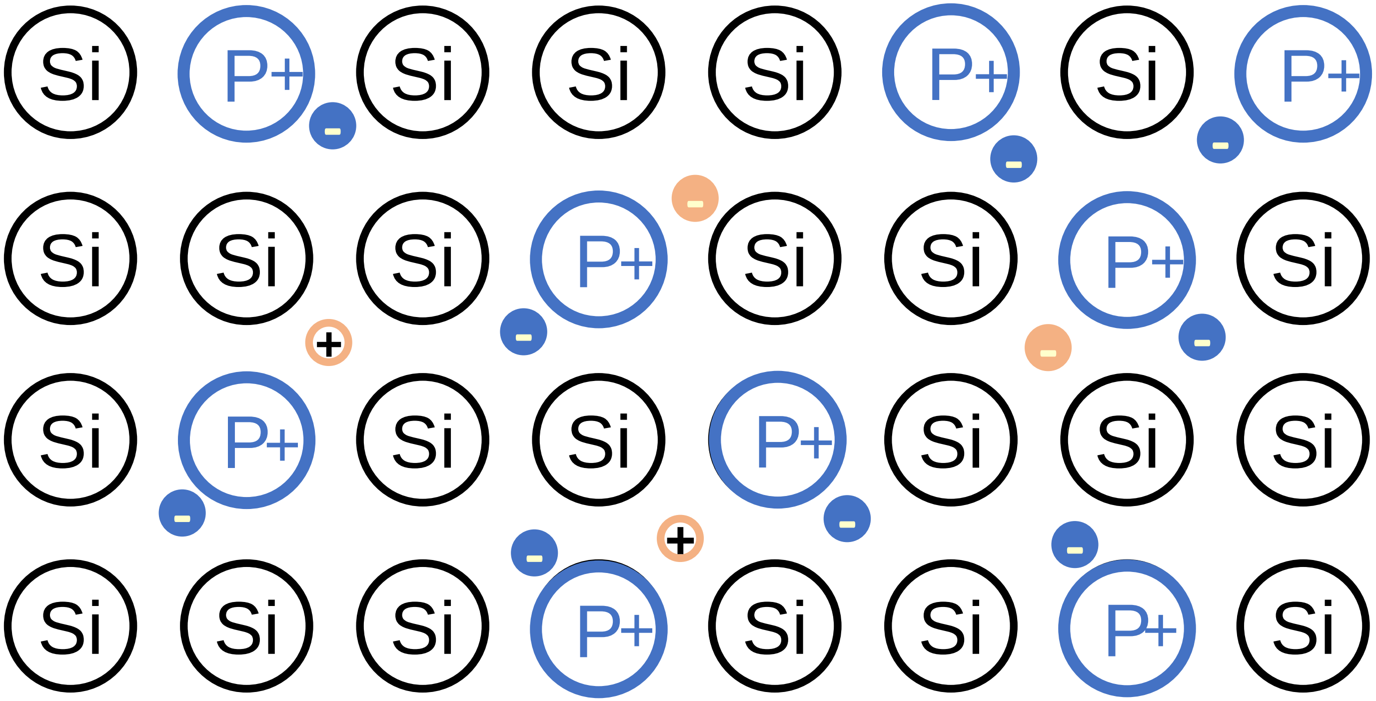
Hole: minor carriers

空穴:少数载流子

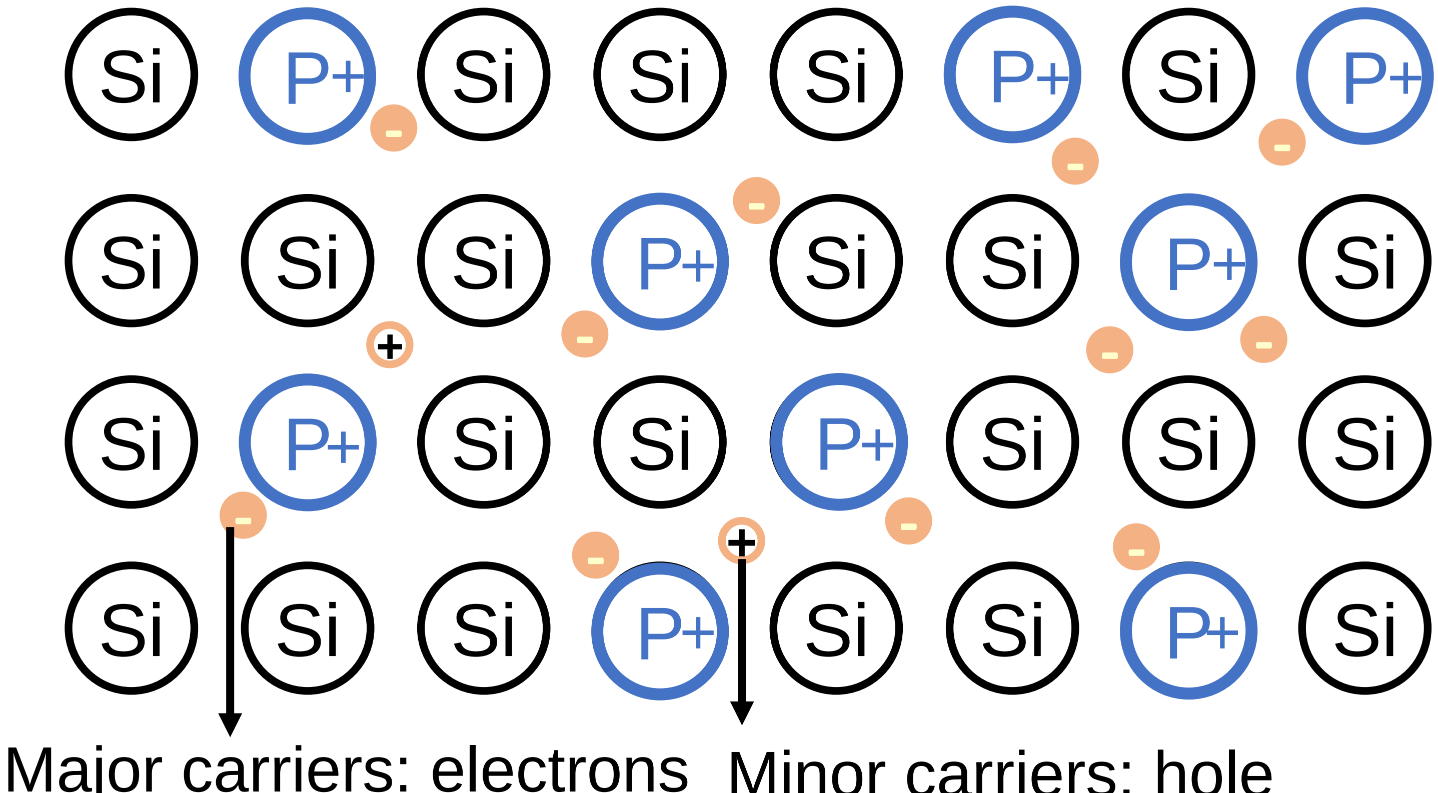
P and As: donor

施主杂质

N-type semiconductor

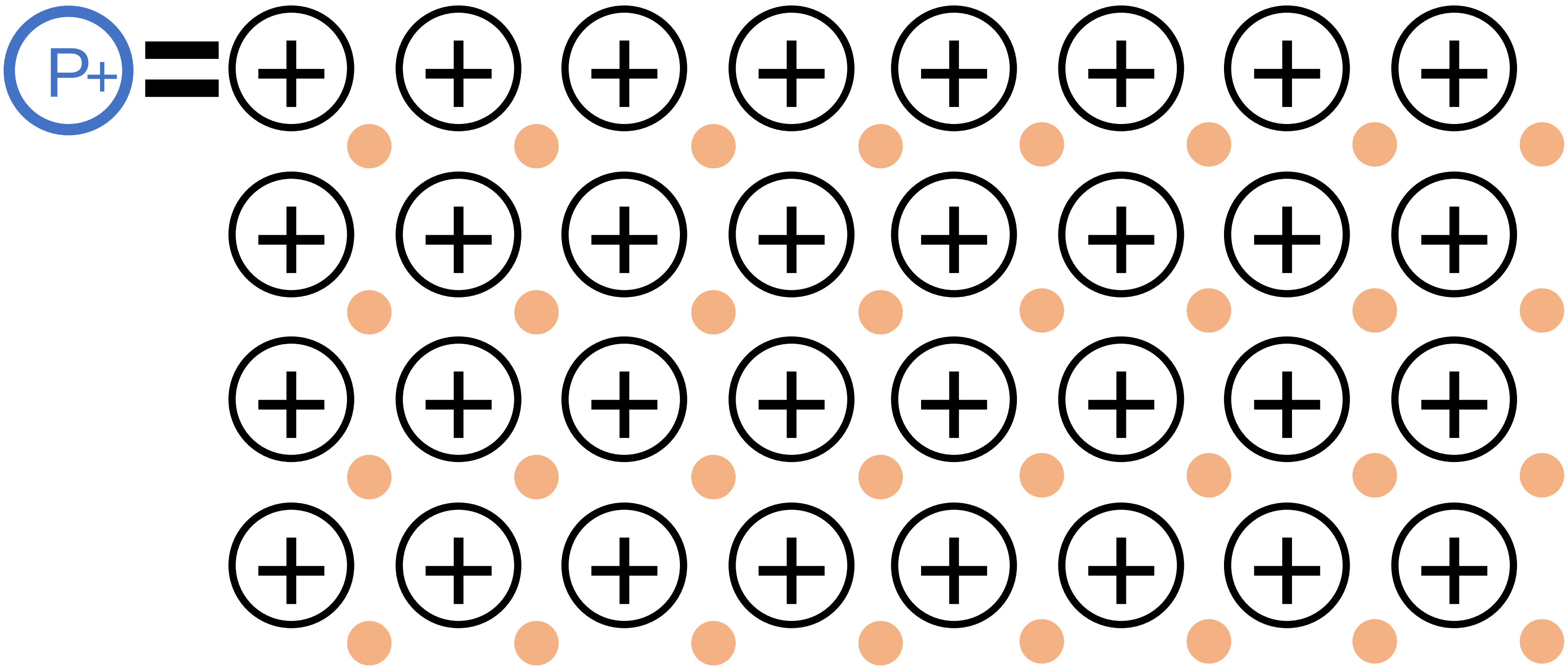


N-type semiconductor



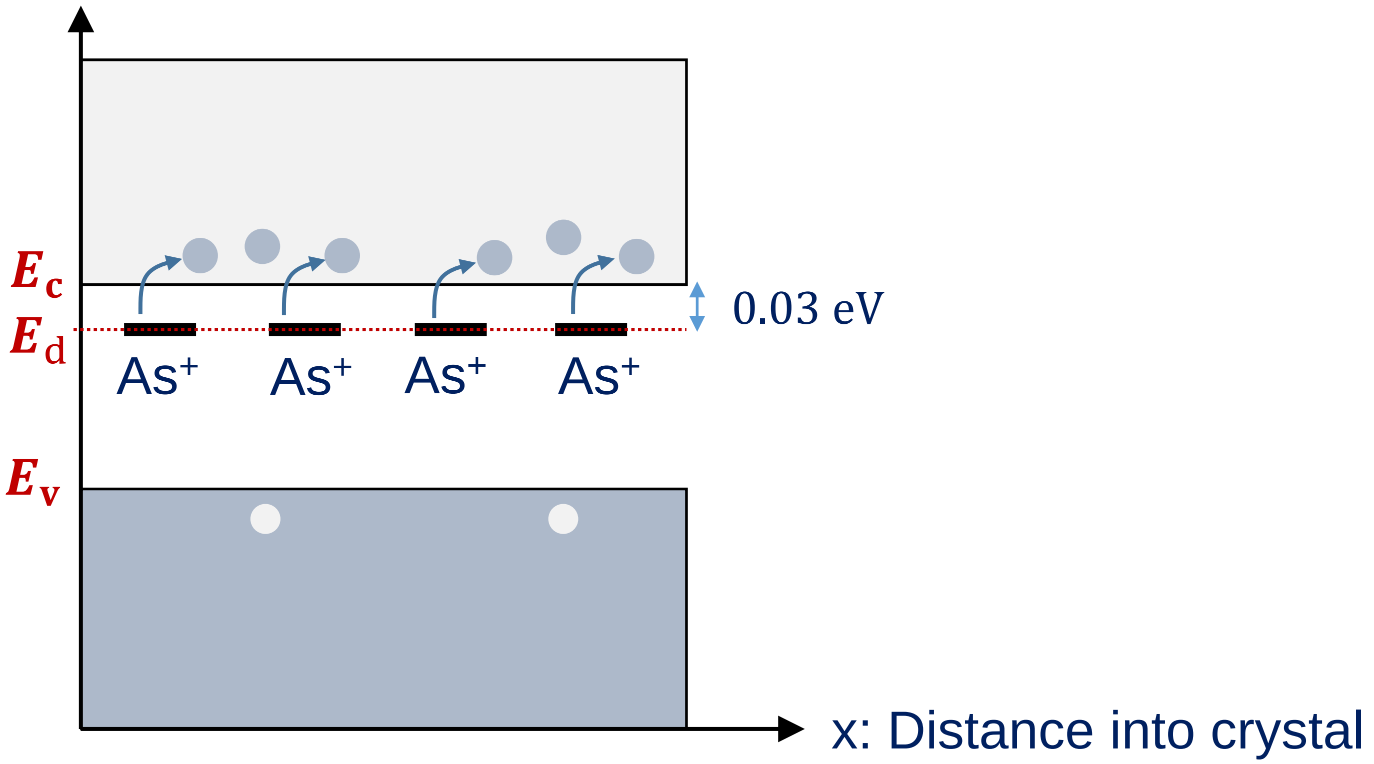
Schematic 示意简图 for N-type semiconductor

Donor ions 施主杂志正离子

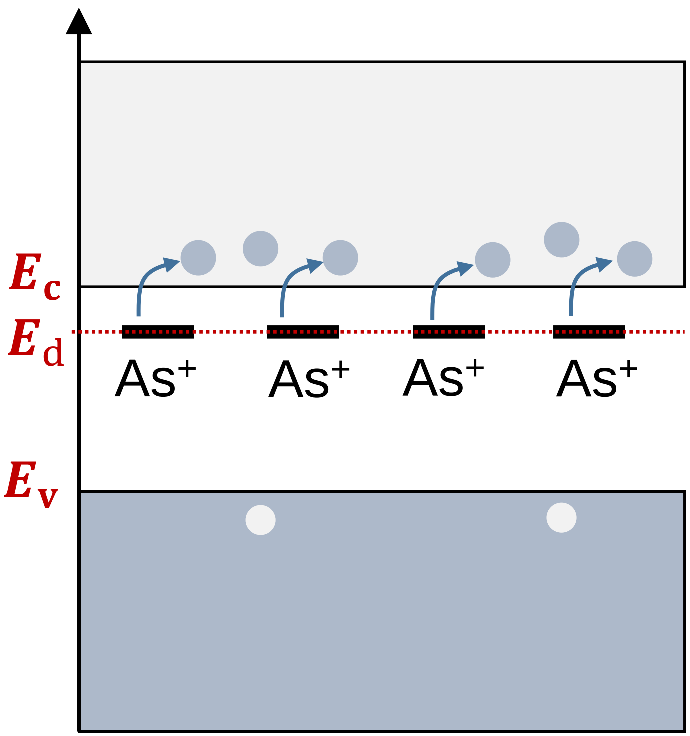


The schematic neglects Si atoms and holes
简图中硅原子和少量空穴没有标识出来

Band diagram of N-type semiconductor



Electron and hole concentrations in N-type semiconductor



Hole concentration in VB :

$$p = N_v \exp\left(-\frac{E_F - E_v}{kT}\right), \quad N_v = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$$

Electron concentration in CB:

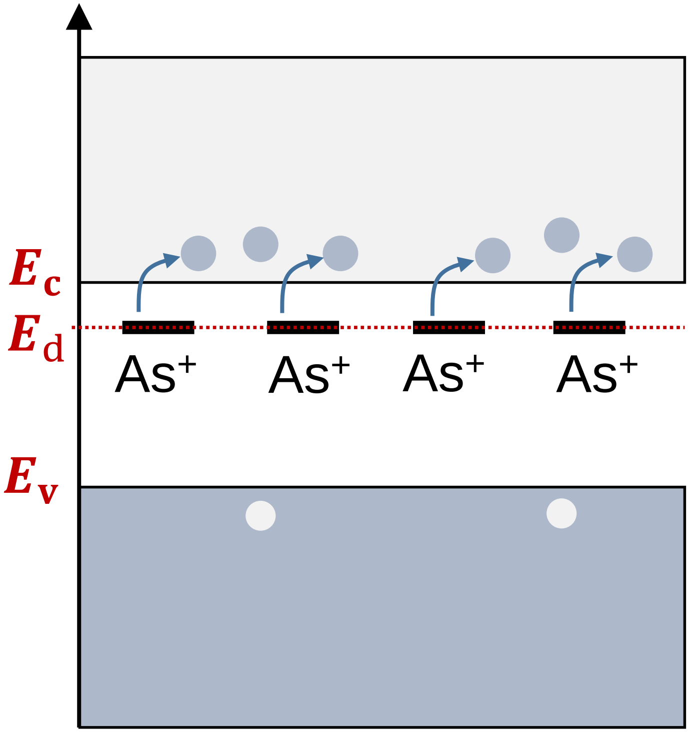
$$n = N_c \exp\left(-\frac{E_c - E_F}{kT}\right), \quad N_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$$

$$np = {n_i}^2, \; n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT}\right)$$

$$\boldsymbol{n > p}$$

Fermi energy is closer to CB.

Electron and hole concentrations in N-type semiconductor



$$n > p$$

Total electron concentrations=

hole concentrations

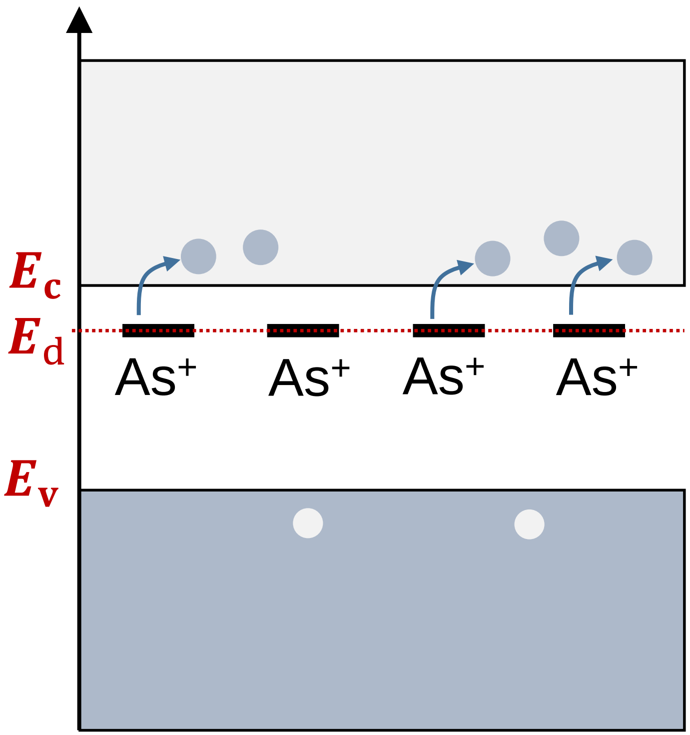
+

electron concentration from donors n_d

$$n = p + n_d$$

Q: How to get n_d?

Electron concentration from donors $n_d \neq$ Donor concentrations N_d



Probability of finding an electron at E_d:

$$f_d(E) = \frac{1}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)}$$

At E_d: either spin up or spin down, but not both.

Q: What's the relation between n_d and N_d?

$$n_d = N_d[1 - f_d(E)]$$

Total electron concentrations=hole concentrations + electron concentration from donors n_d

$$n = p + n_\mathrm{d}$$

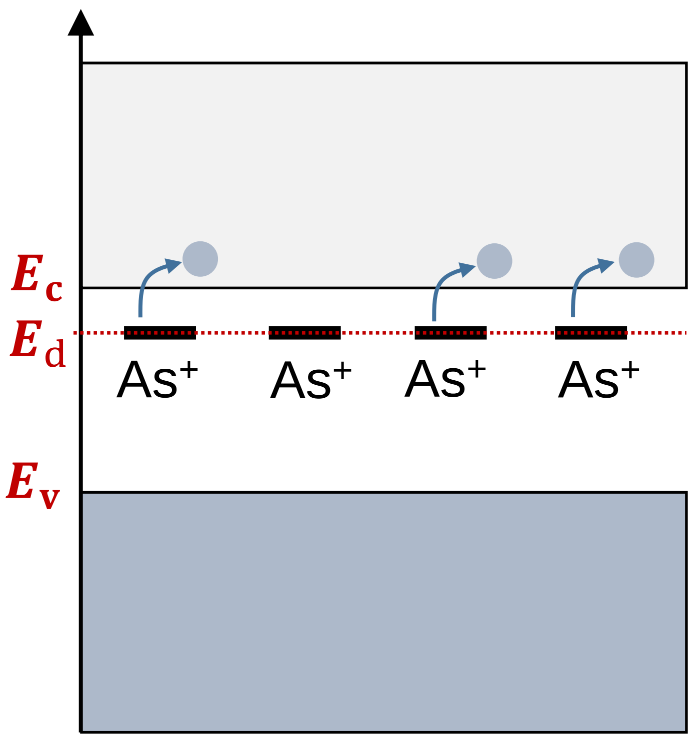
$$N_\mathrm{c}\exp\left(-\frac{E_\mathrm{c}-E_\mathrm{F}}{kT}\right) = N_\mathrm{v}\exp\left(-\frac{E_\mathrm{F}-E_\mathrm{v}}{kT}\right) + N_\mathrm{d}\frac{1}{1+2\exp\left(\frac{E_\mathrm{F}-E_\mathrm{d}}{kT}\right)}$$

Question: For N-type semiconductor (*n*>>*p*):

(1) The electron concentration and the Fermi energy at low temperature T.

(2) When T → 0K, the electron concentration and the Fermi energy.

(1) The electron concentration and the Fermi energy at low temperature T.



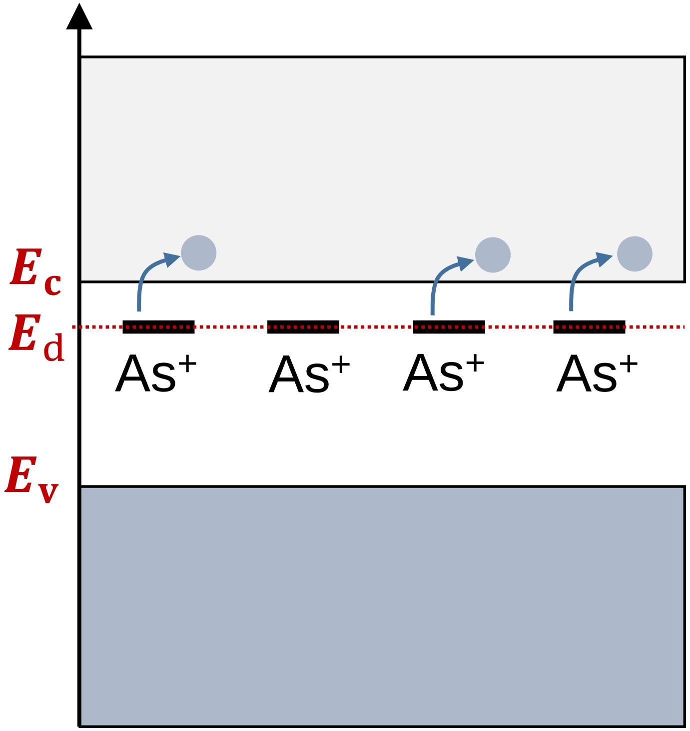
$$\begin{cases} n = p + n_d \\ np = {n_i}^2 = N_c N_v \exp\left(-\dfrac{E_g}{kT}\right) \end{cases}$$

$$\Downarrow$$

$$n = \frac{{n_i}^2}{n} + n_d$$

$$\Downarrow$$

$$n = \frac{1}{2} n_d + \left[\frac{1}{4} {n_d}^2 + {n_i}^2\right]^{1/2}$$



At low temperature:

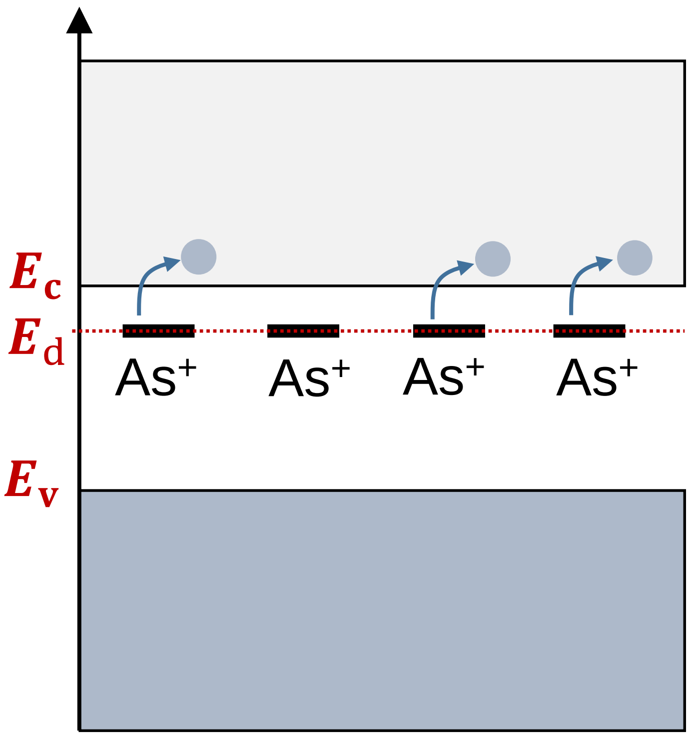
$$n_i \ll n_d$$

$$n = \frac{1}{2} n_d + \left[\frac{1}{4} {n_d}^2 + {n_i}^2\right]^{1/2}$$

$$\approx n_d$$

$$= N_d \frac{1}{1 + 2\exp\left(\frac{E_F - E_d}{kT}\right)}$$

$$\approx \frac{N_d}{2} \exp\left(-\frac{E_F - E_d}{kT}\right)$$



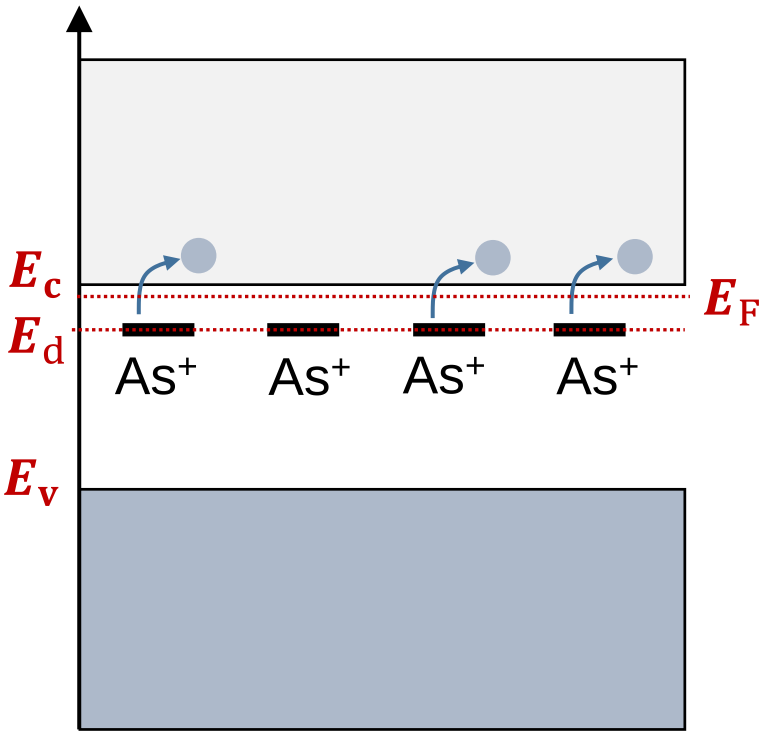
$$\begin{cases} n = \dfrac{N_\mathrm{d}}{2}\exp\left(-\dfrac{E_\mathrm{F}-E_\mathrm{d}}{kT}\right) \\ n = N_\mathrm{c}\exp\left(-\dfrac{E_\mathrm{c}-E_\mathrm{F}}{kT}\right) \end{cases}$$

$$n^2 = \frac{1}{2}N_\mathrm{c}N_\mathrm{d}\exp\left(-\frac{E_\mathrm{c}-E_\mathrm{d}}{kT}\right)$$

$$n = \sqrt{\frac{1}{2}N_\mathrm{c}N_\mathrm{d}}\exp\left(-\frac{E_\mathrm{c}-E_\mathrm{d}}{2kT}\right)$$

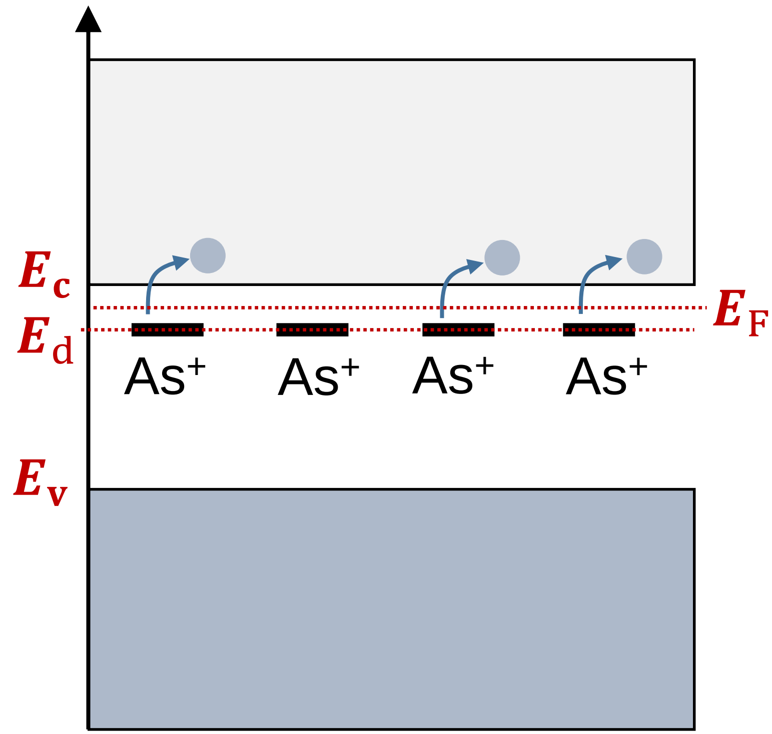
$$E_\mathrm{F} = \frac{E_\mathrm{c}+E_\mathrm{d}}{2} + \frac{1}{2}kT\ln\left(\frac{N_\mathrm{d}}{2N_\mathrm{c}}\right)$$

$T > 0$



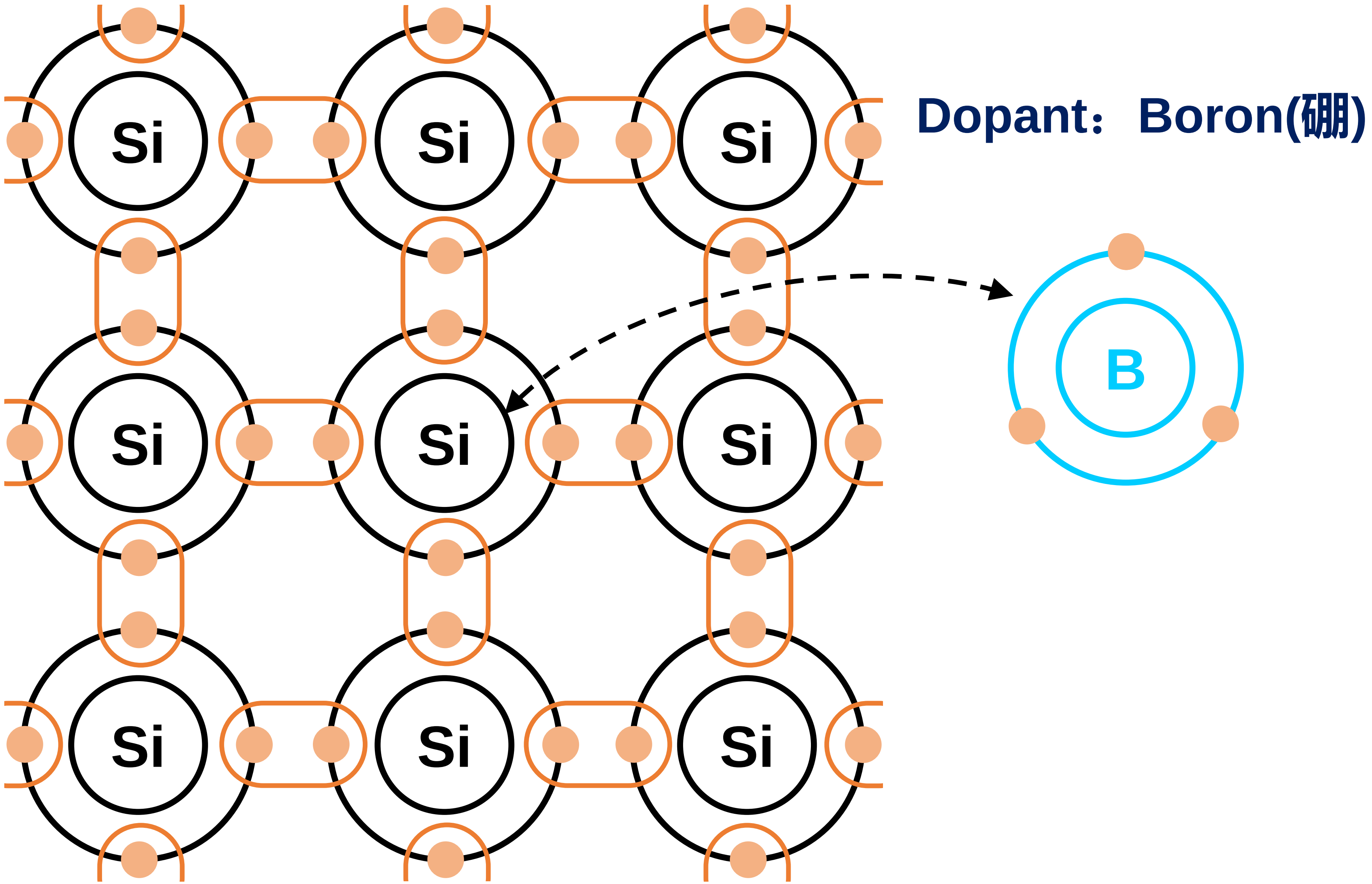
$$E_F = \frac{E_c + E_d}{2} + \frac{1}{2} kT \ln\left(\frac{N_d}{2N_c}\right)$$

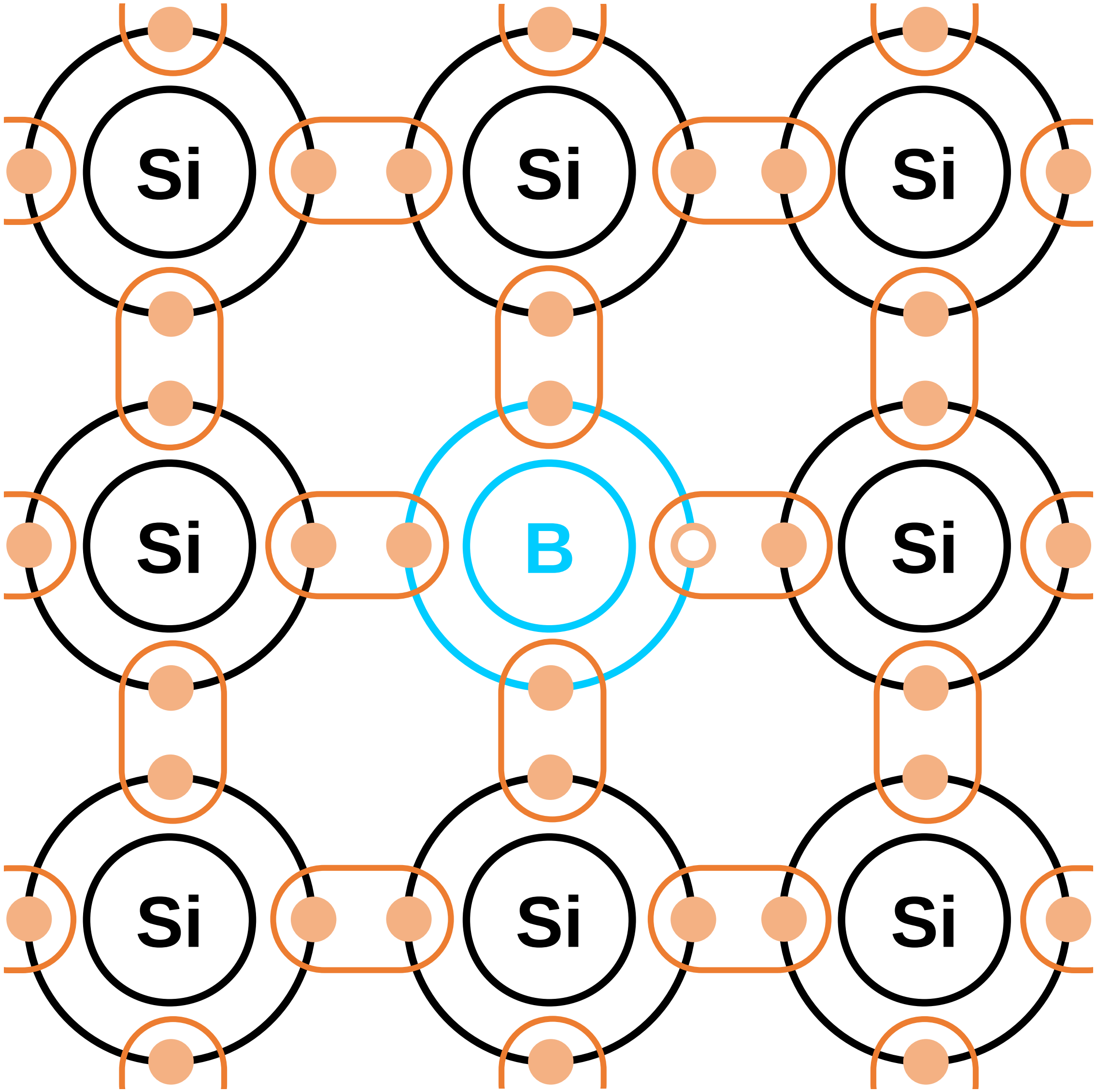
$T \to 0$

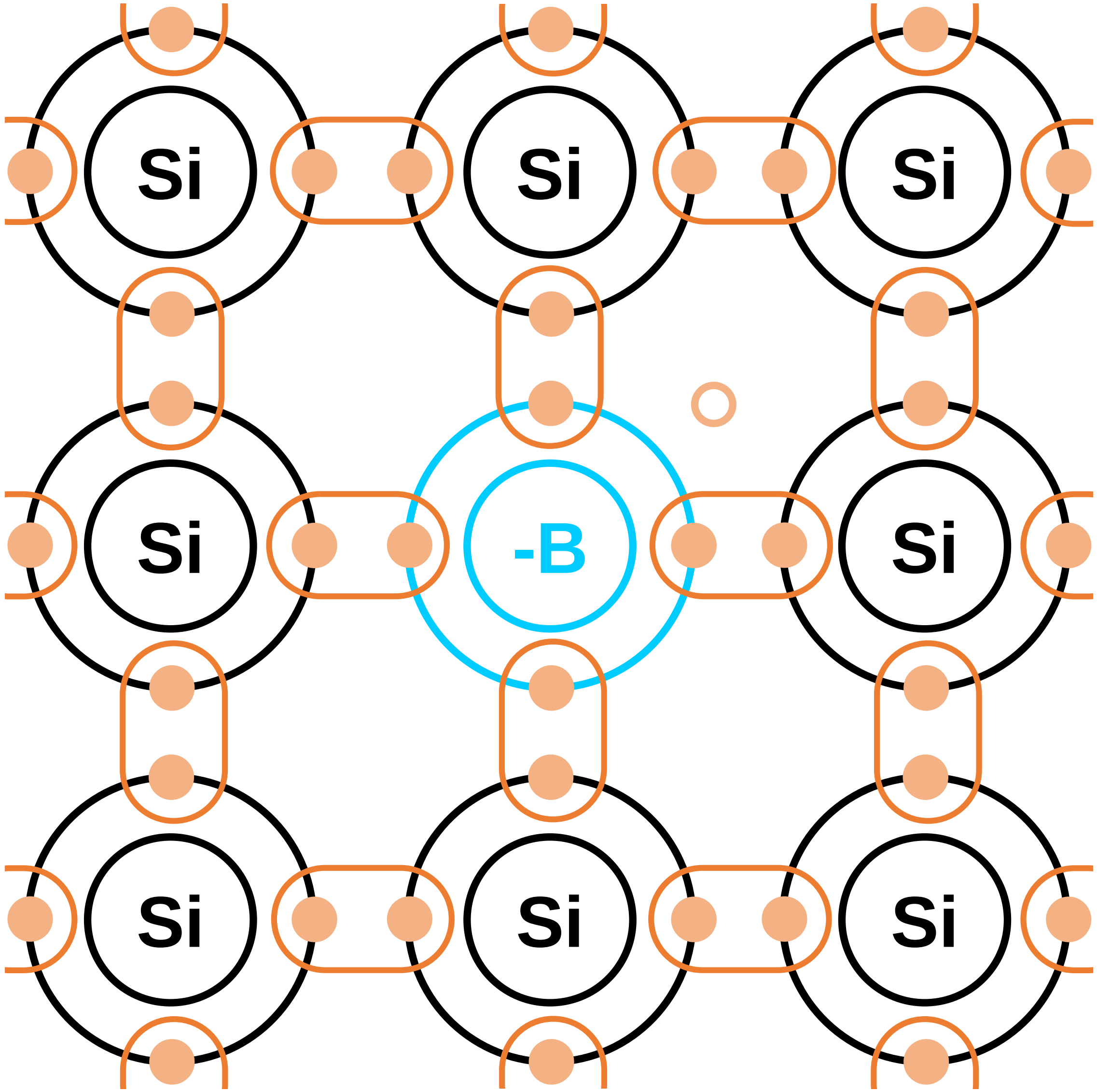


$$E_F = \frac{E_c + E_d}{2}, \quad n = 0$$

P-type semiconductor




Si
Si
Si
Si
B
Si
Si
Si
Si



One more free hole
and one negative ion

Hole number >
electron number

Hole: major carriers

空穴:多数载流子

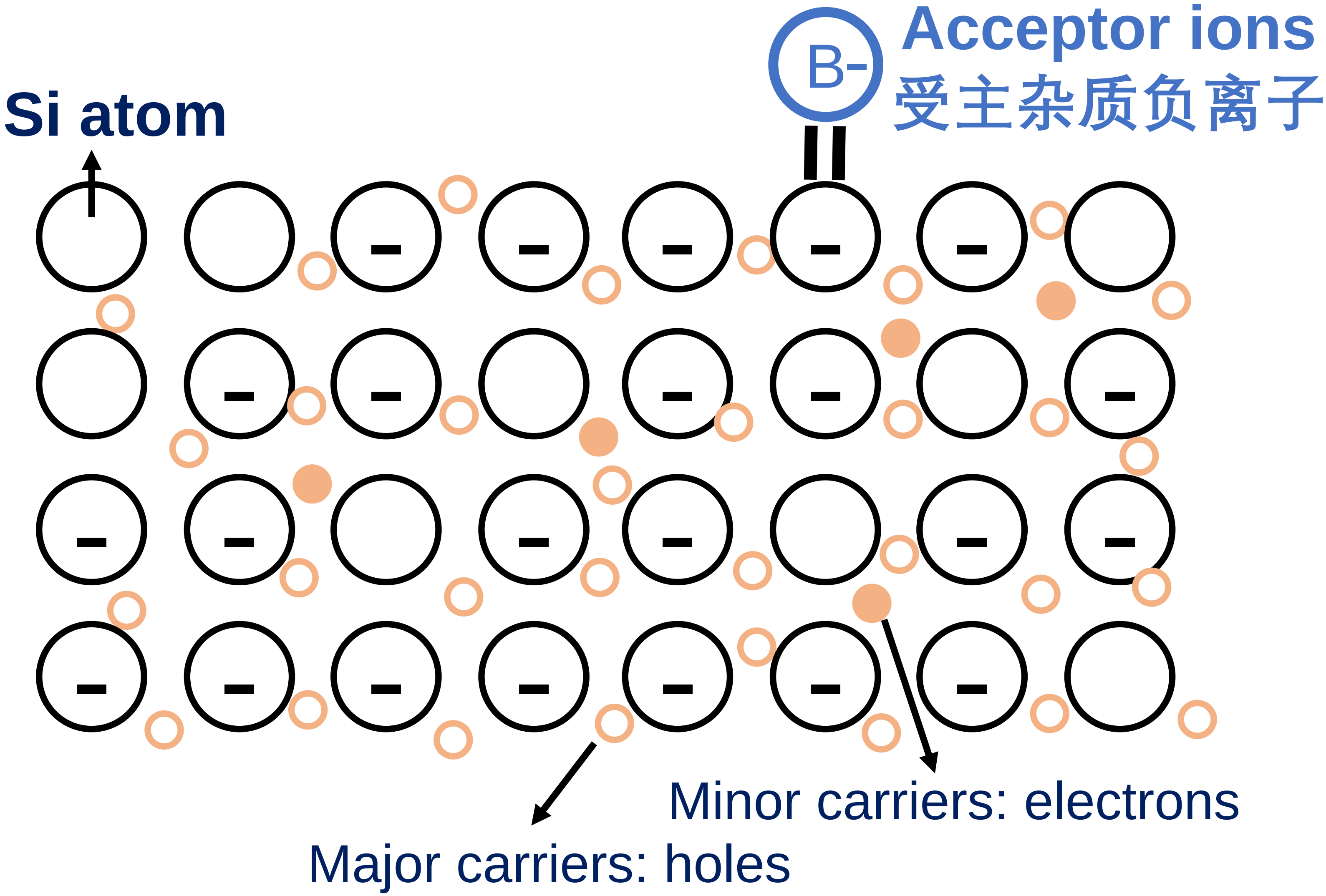
Electron: minor carriers

电子:少数载流子

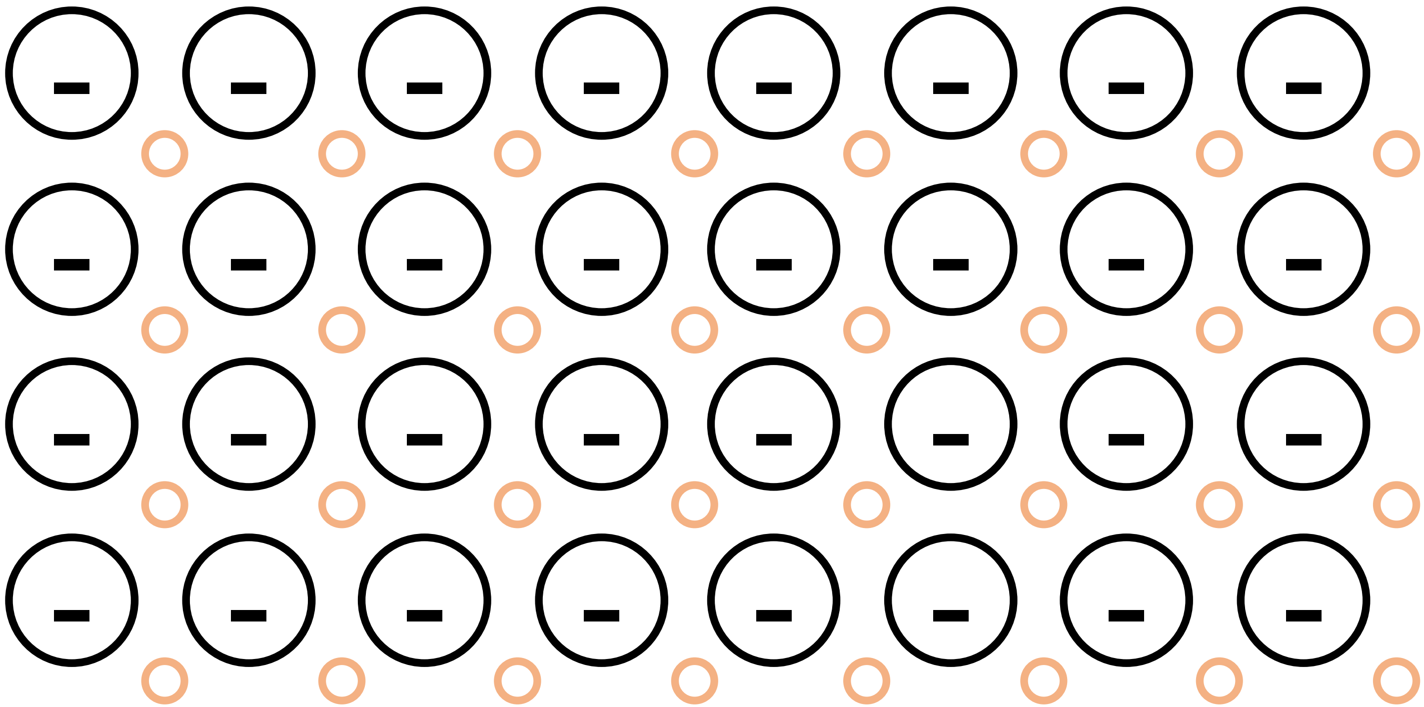
B: Acceptor

受主杂质

P-type semiconductor

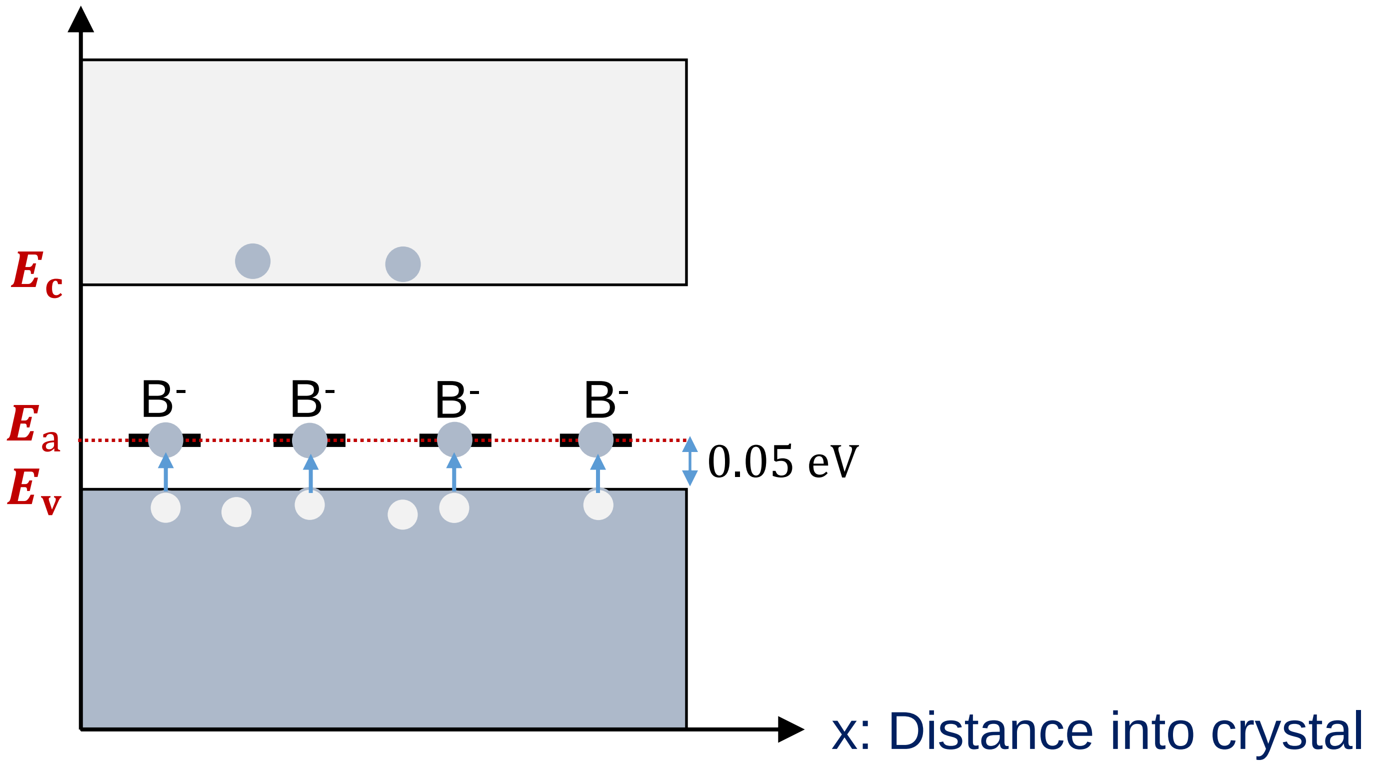


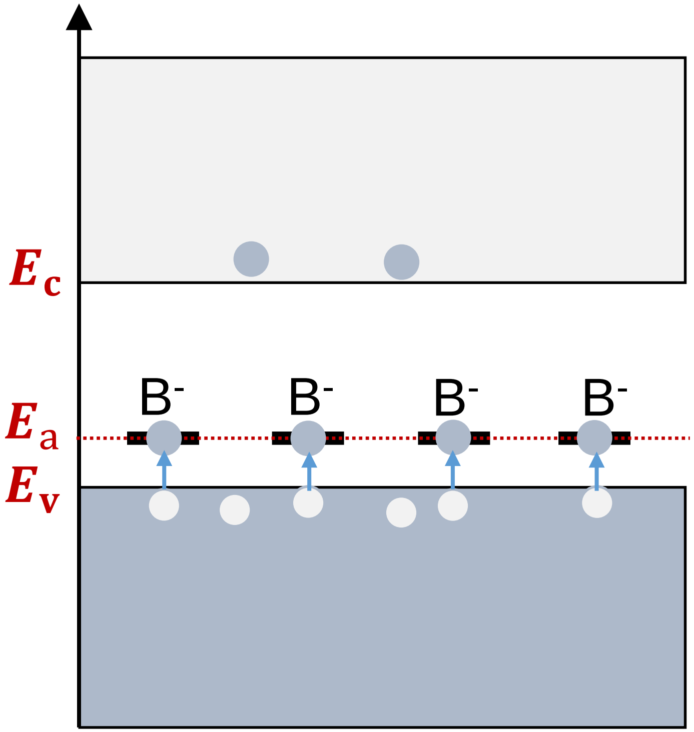
Schematic 示意简图 for P-type semiconductor



The schematic neglects Si atoms and electrons
简图中硅原子和少量电子没有标识出来

Band diagram of P-type semiconductor





Probability of finding a hole at E_a:

$$f_a(E) = \frac{1}{1 + \frac{1}{2}\exp\left(\frac{E_F - E_a}{kT}\right)}$$

Practice: For P-type semiconductor (p>>n), (1) the hole concentration and the Fermi energy at relatively low temperature T.

(2)When T→0K, the hole concentration and the Fermi energy.

4.5 Temperature dependence of conductivity in doped semiconductor

Conductivity in semiconductors:

$$\sigma = en\mu_{\mathrm{e}} + ep\mu_{\mathrm{h}}$$

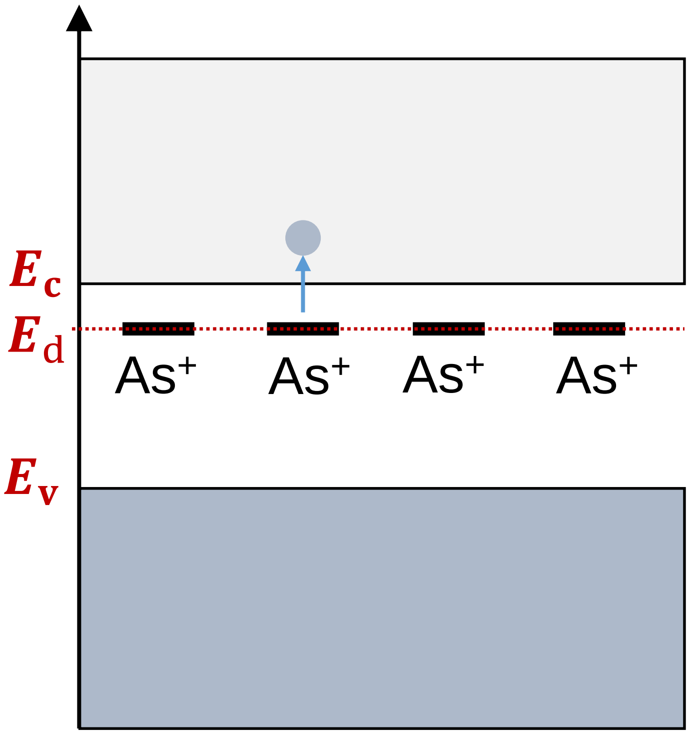
(1) Temperature dependence of *n* and *p*

(2) Temperature dependence of μ_{e} and μ_{h}

(3) Temperature dependence of σ

In following section, we use **n-type semiconductor** to illustrate the temperature-dependent properties.

Temperature dependence of *n* and *p*



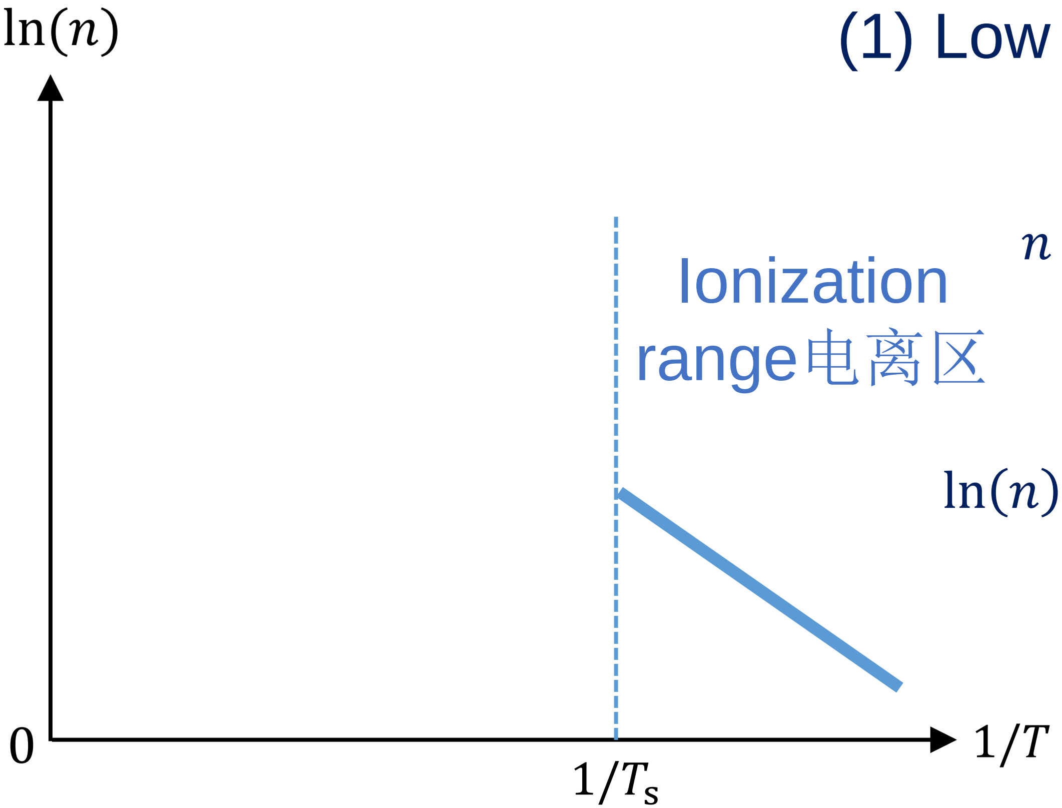
T_s is called **saturation temperature**: all donors are ionized, and the number of ionized donors are saturated.

(1) Low temperature range: $T < T_s$

A portion of donors are ionized.

$$n \approx n_d = \sqrt{\frac{1}{2} N_c N_d} \exp\left(-\frac{E_c - E_d}{2kT}\right)$$

(1) Low temperature range: $T < T_s$

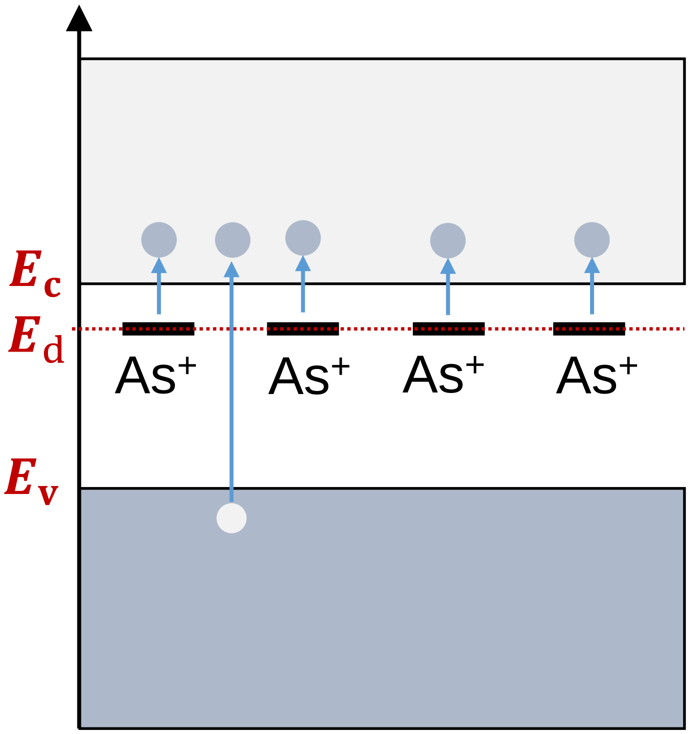


$$n = \sqrt{\frac{1}{2}N_c N_d}\exp\left(-\frac{E_c - E_d}{2kT}\right)$$

$$\ln(n) = -\frac{E_c - E_d}{2kT} + \ln\sqrt{\frac{1}{2}N_c N_d}$$

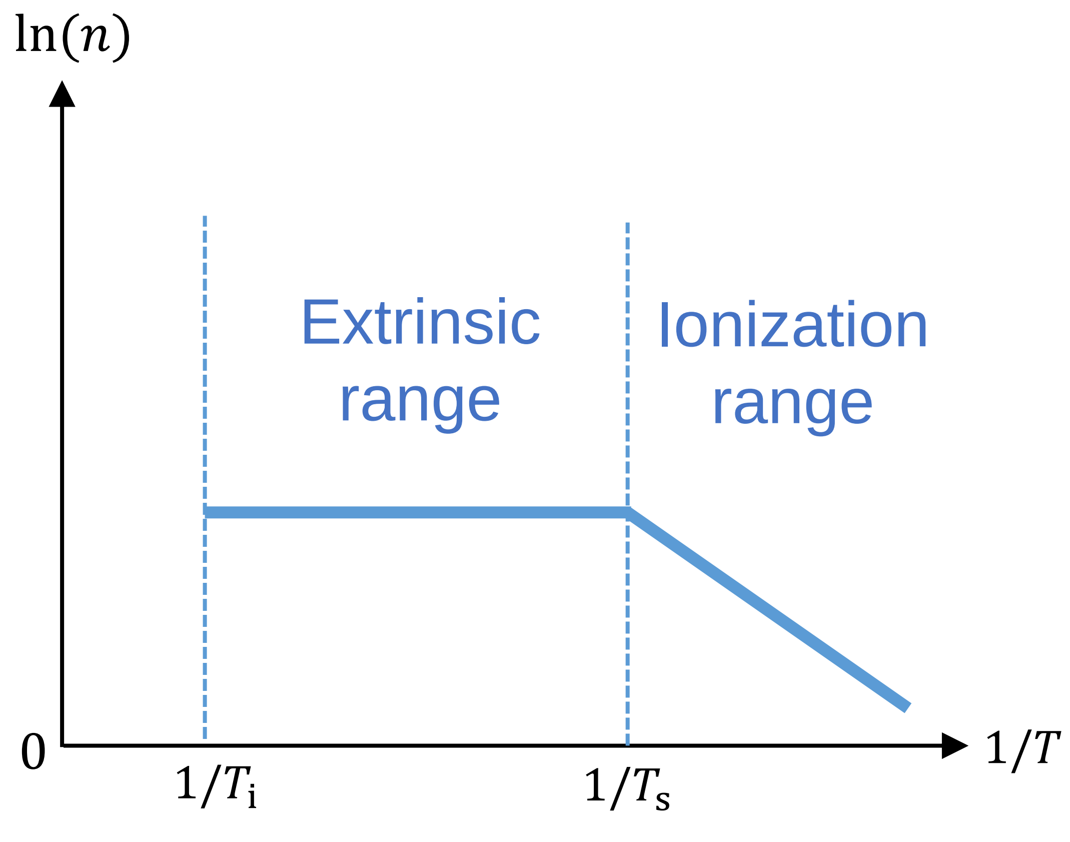
$$\approx -\frac{E_c - E_d}{2kT}$$

(2) Medium temperature range: $T_s < T < T_i$



All donors are ionized and $N_d \gg p$.

In this range, $n \approx N_d$.

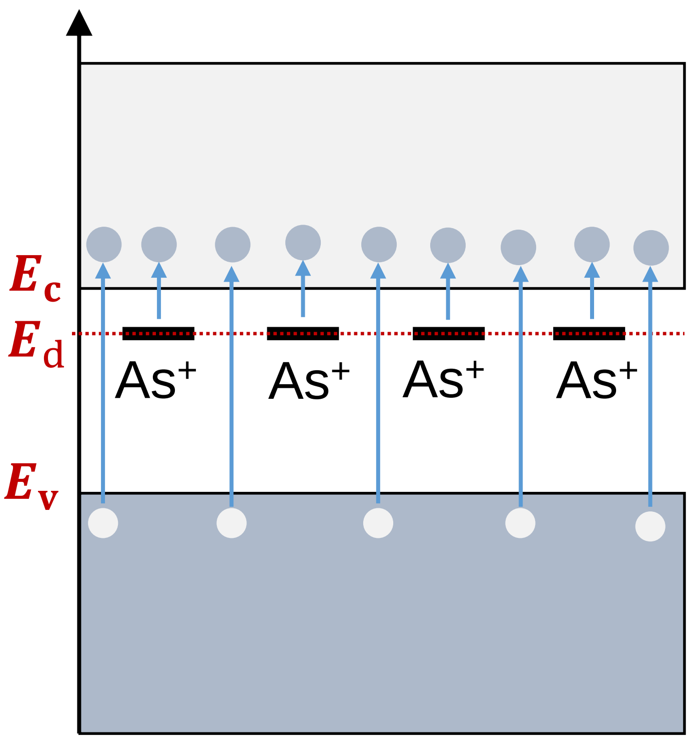


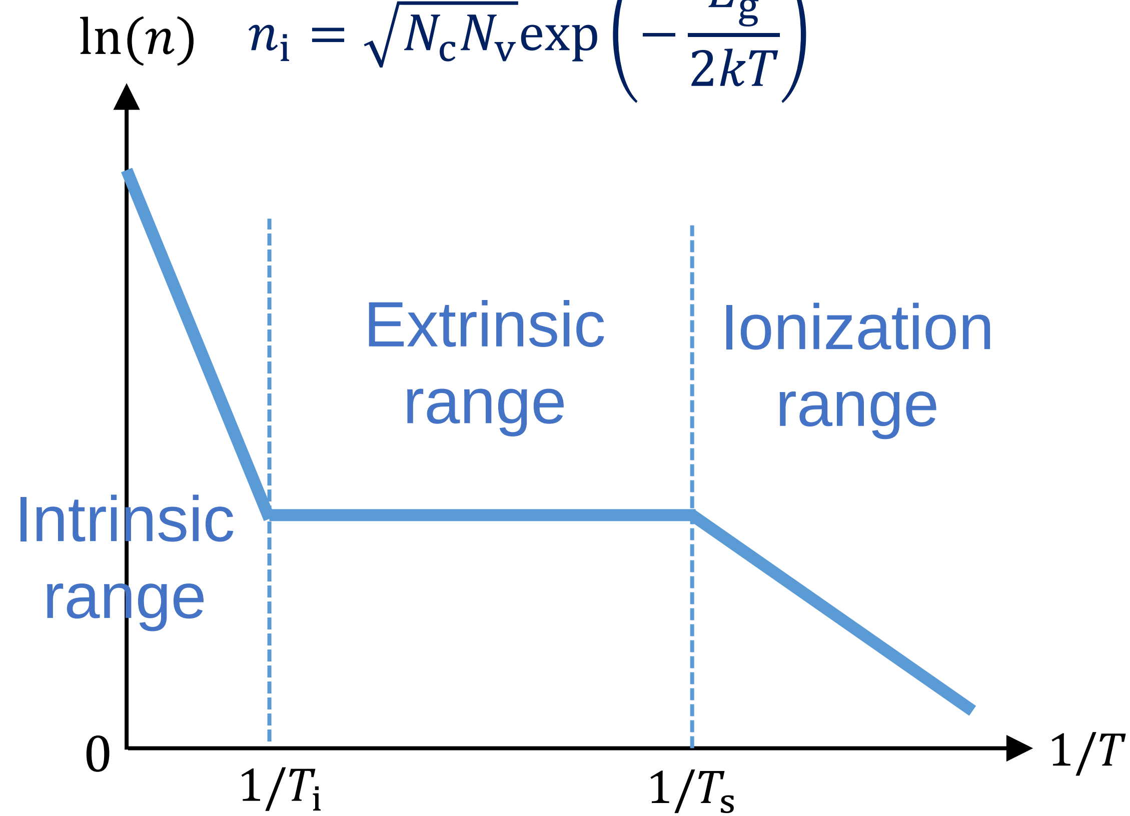
(3) High temperature range: $T_{\mathrm{i}} < T$

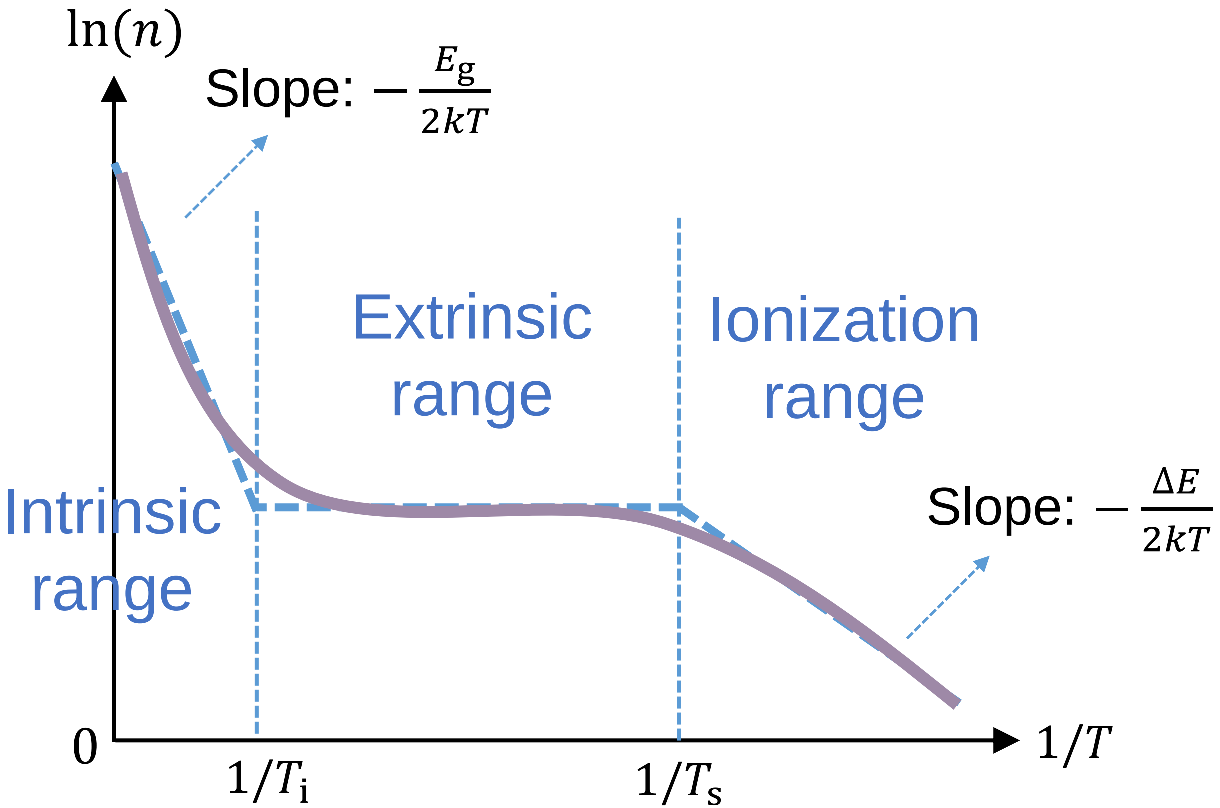
In this range, $n_{\mathrm{i}} \gg N_{\mathrm{d}}$.

$$n \approx p \approx n_{\mathrm{i}} \gg N_{\mathrm{d}}$$

$$n_{\mathrm{i}} = \sqrt{N_{\mathrm{c}} N_{\mathrm{v}}} \exp\left(-\frac{E_{\mathrm{g}}}{2kT}\right)$$







$\ln(n)$
Slope: $-\frac{E_g}{2kT}$
Extrinsic range
Ionization range
Intrinsic range
Slope: $-\frac{\Delta E}{2kT}$
0
$1/T_i$
$1/T_s$
$1/T$

Temperature dependence of μ_e and μ_h

$$\mu = \frac{e\tau}{m_e{}^*}$$

$$\left\{\begin{array}{l}\text{Scatterings from lattice vibrations} \\ \\ \text{Scatterings from ionized donor atoms}\end{array}\right.$$

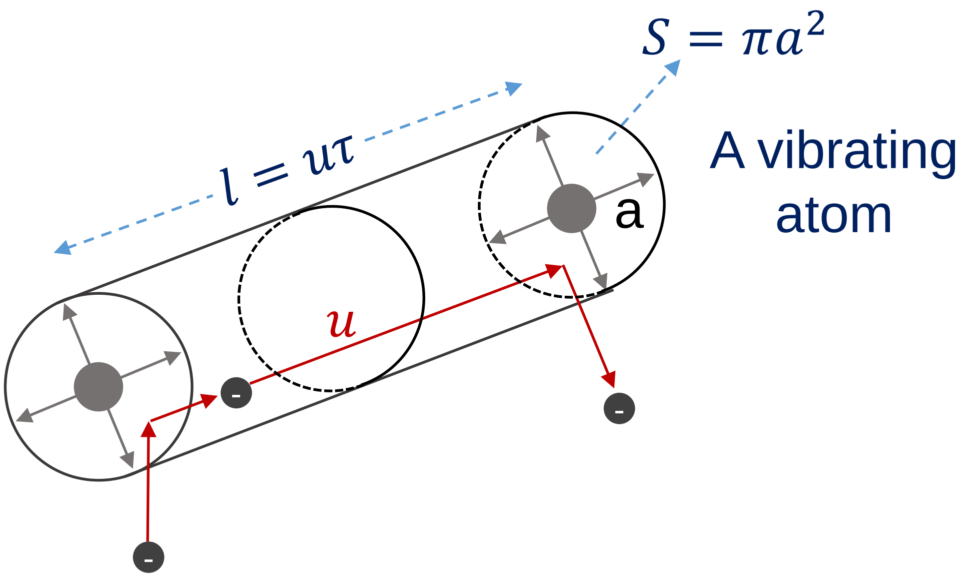
Mobility limited by lattice vibrations

$$\mu = \frac{e\tau}{m_e^*}$$

$$\tau_L = \frac{1}{S v_{th} N_s}$$

$$\frac{1}{2} m {v_{th}}^2 = \frac{3}{2} kT$$

$$v_{th} \propto \sqrt{T}$$



Scattering cross-sectional area: $S = \pi a^2$

Mean free path :$l = v_{th}\tau$

The concentration of scattering centers: N_s

Mobility limited by lattice vibrations

$$\begin{cases} \tau_{\mathrm{L}} = \dfrac{1}{S v_{\mathrm{th}} N_{\mathrm{s}}} \\ v_{\mathrm{th}} \propto \sqrt{T} \\ S \propto T \end{cases}$$

$$\Downarrow$$

$$\tau_{\mathrm{L}} \propto T^{-3/2}$$

$$\Downarrow$$

$$\mu_{\mathrm{L}} \propto T^{-3/2}$$

Mobility limited by ionized atoms

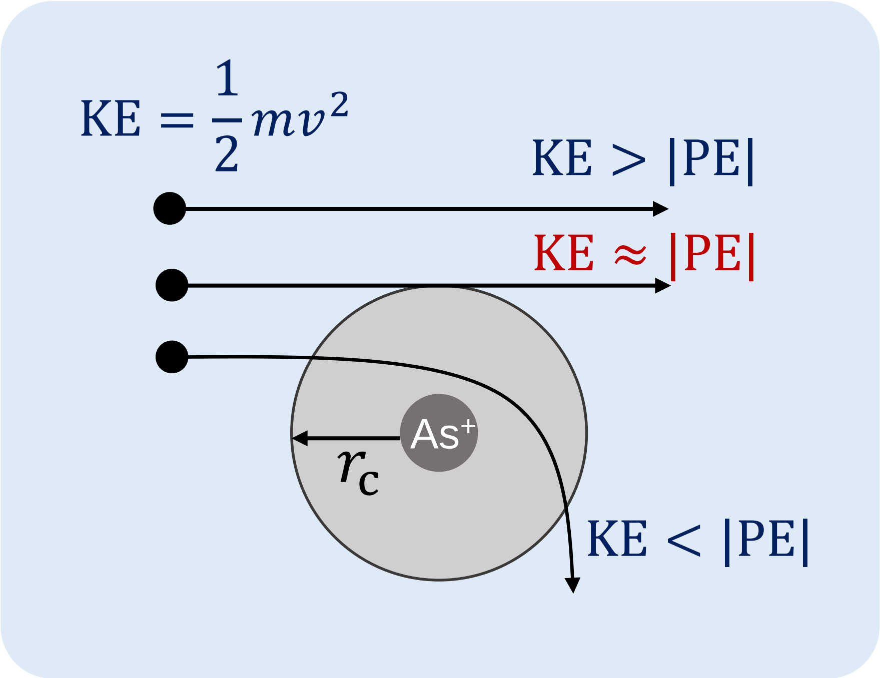
$$\tau_{\mathrm{I}} = \frac{1}{S v_{\mathrm{th}} N_{\mathrm{I}}}$$

$$\mathrm{KE} \approx |\mathrm{PE}|$$

$$\frac{3}{2}kT = \frac{e^2}{4\pi\varepsilon_0\varepsilon_{\mathrm{r}} r_{\mathrm{c}}}$$

$$r_{\mathrm{c}} \propto T^{-1}$$

$$S \propto T^{-2}$$



Mobility limited by ionized atoms

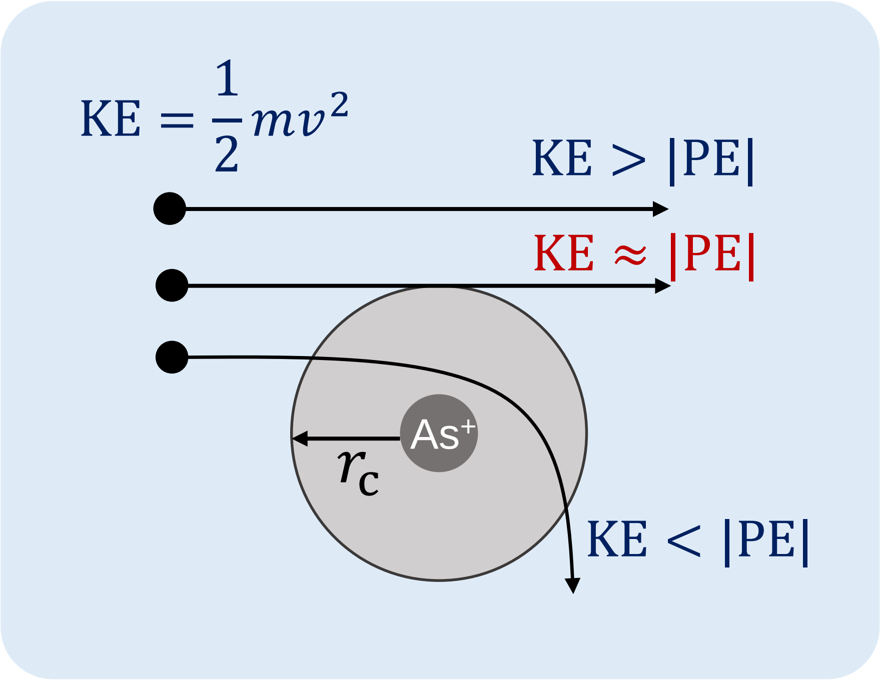
$$\begin{cases} \tau_{\mathrm{I}} = \dfrac{1}{S v_{\mathrm{th}} N_{\mathrm{I}}} \\ S \propto T^{-2} \\ v_{\mathrm{th}} \propto \sqrt{T} \end{cases}$$

$$\Downarrow$$

$$\tau_{\mathrm{I}} \propto \frac{T^{3/2}}{N_{\mathrm{I}}}$$

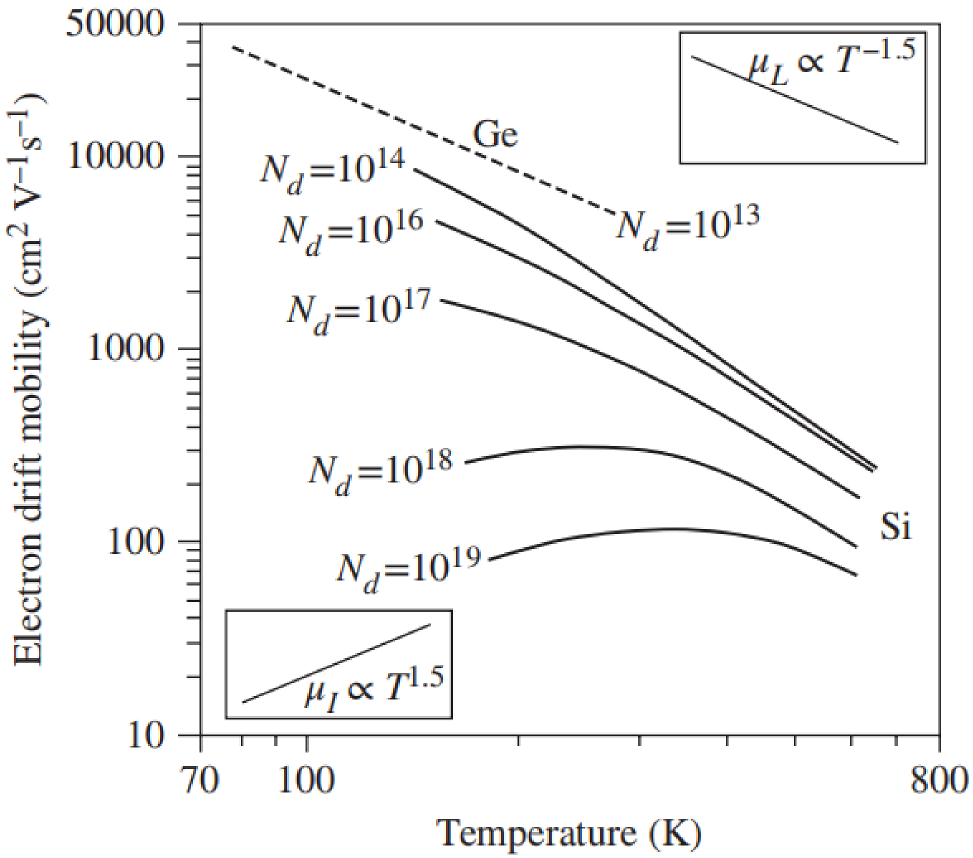
$$\Downarrow$$

$$\mu_{\mathrm{I}} \propto \frac{T^{3/2}}{N_{\mathrm{I}}}$$

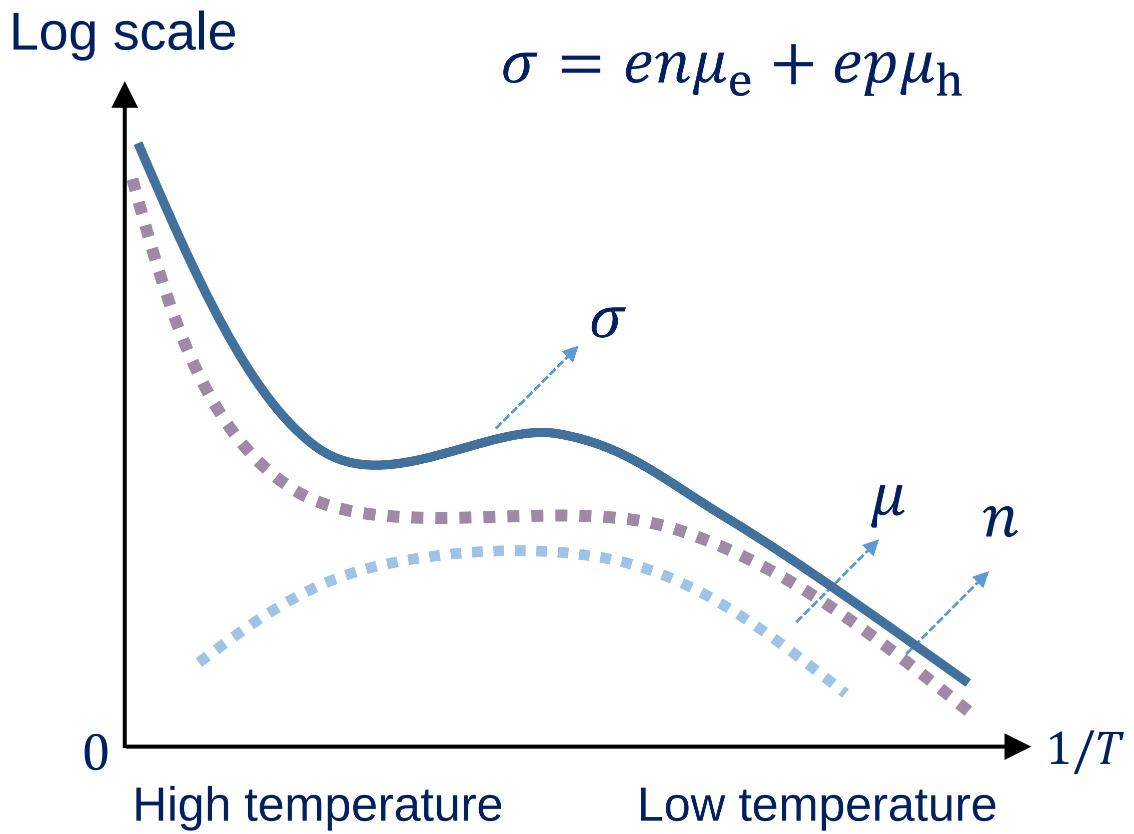


Total mobility: Matthiessen's rule

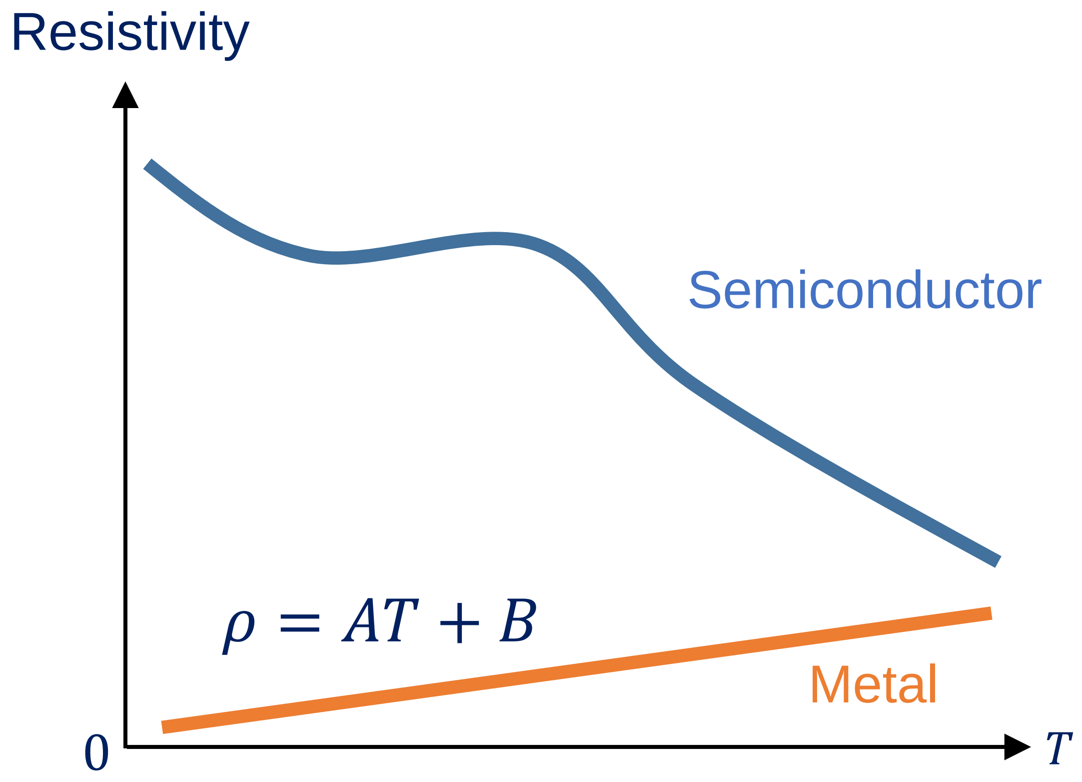
$$\frac{1}{\mu_{\mathrm{d}}} = \frac{1}{\mu_{\mathrm{L}}} + \frac{1}{\mu_{\mathrm{I}}} \qquad \begin{cases} \mu_{\mathrm{L}} \propto T^{-3/2} \\ \mu_{\mathrm{I}} \propto \dfrac{T^{3/2}}{N_{\mathrm{I}}} \end{cases}$$



Conductivity temperature dependence



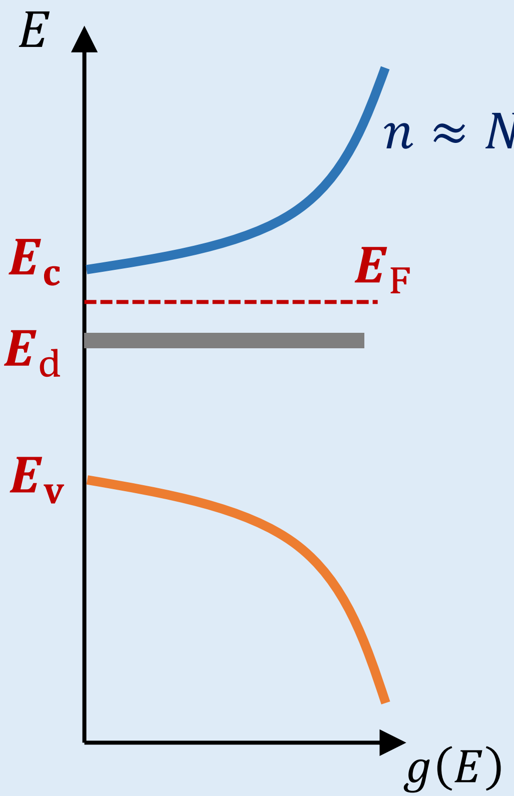
Comparison between semiconductor and metal



4.6 Nondegenerate and degenerate semiconductors

非简并与简并半导体

Nondegenerate semiconductors



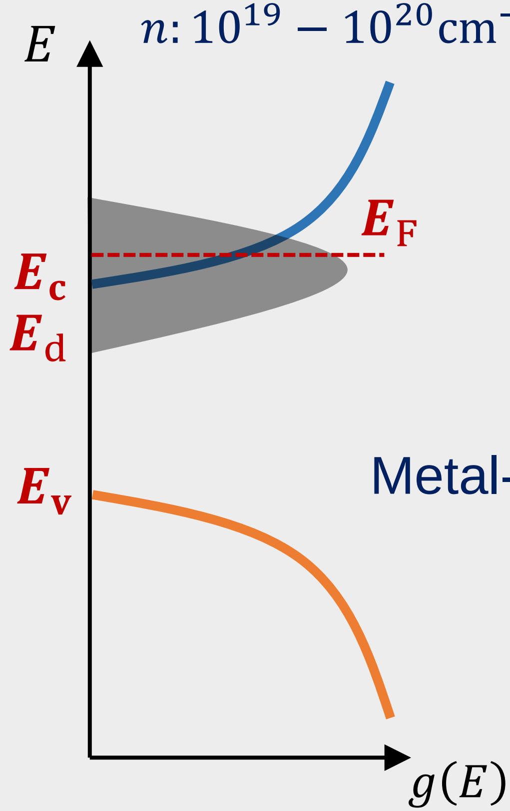
$$n \approx N_c \exp\left(-\frac{E_c - E_F}{kT}\right)$$

$$n \ll N_c$$

$$p \ll N_v$$

Degenerate semiconductors

$n: 10^{19} - 10^{20}\,\mathrm{cm}^{-3}$ is very large



$$n > N_c$$

$$p > N_v$$

$$np \neq {n_i}^2$$

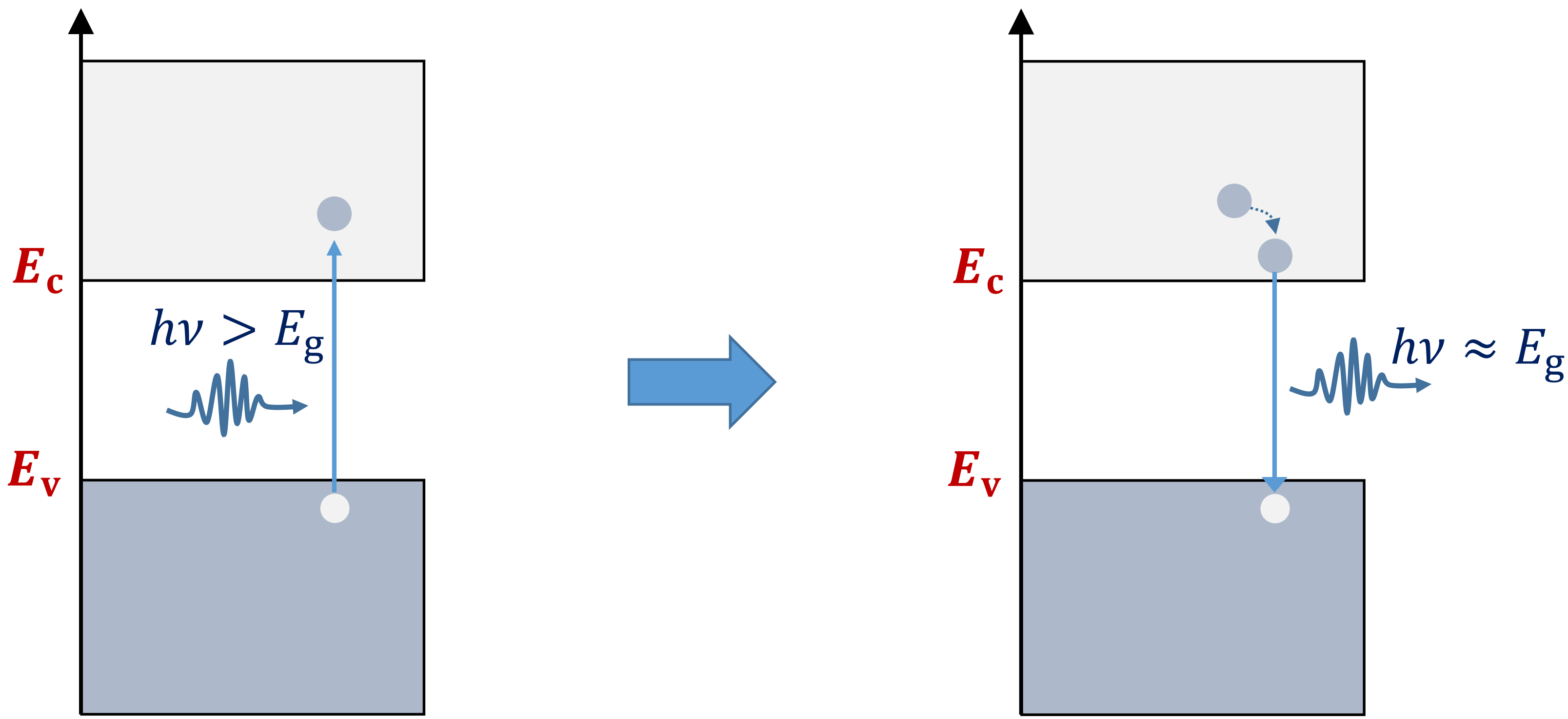
Metal-like properties:

$$\rho \propto T$$

4.7 Direct and indirect bandgap semiconductor

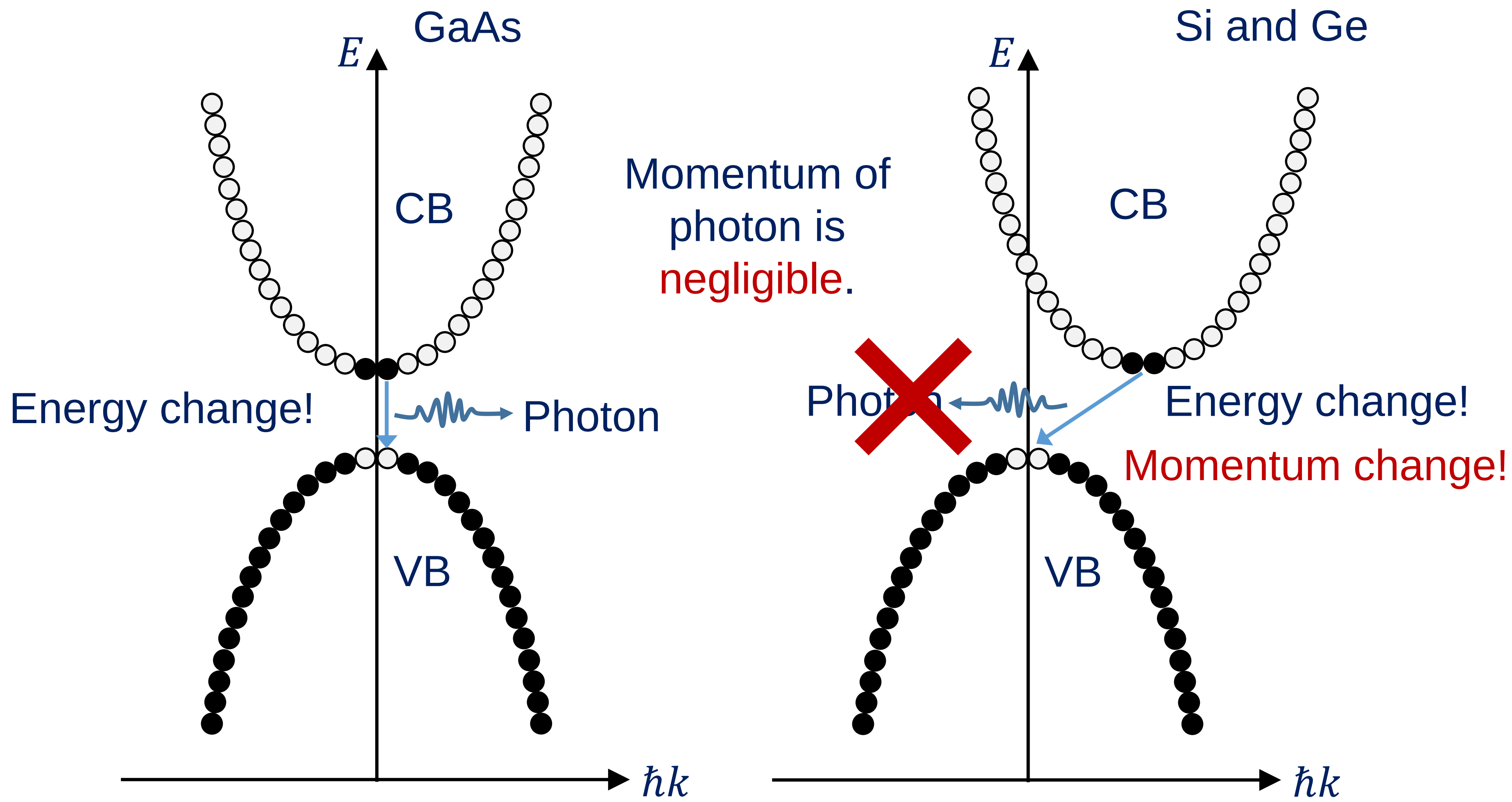
直接与间接带隙半导体

The concept of direct and indirect bandgap is very important for materials for LED and laser applications.



Excitation of an electron-hole pair Recombination of an electron-hole pair

Energy-momentum relation



Direct bandgap material

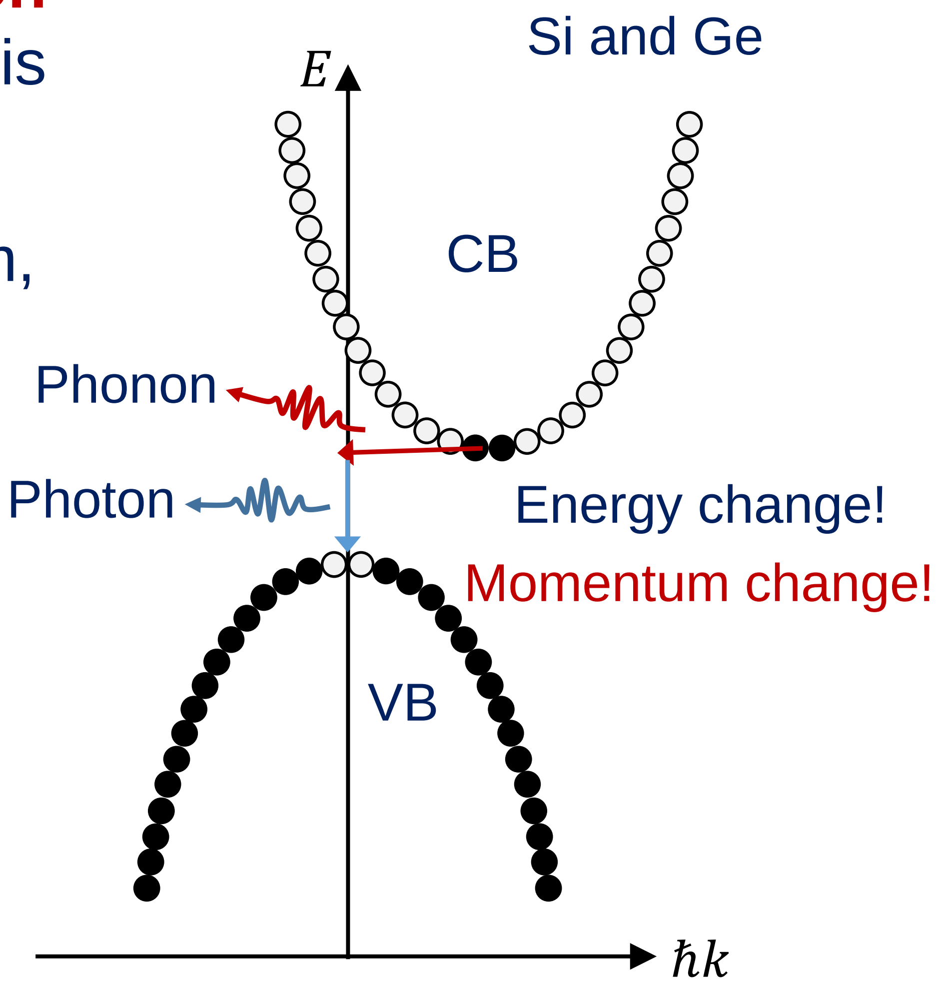
Indirect bandgap material

A third body, such as **phonon** (lattice vibration modes) is involved in this process.

Phonon: large momentum, small energy ~10^{-2} eV

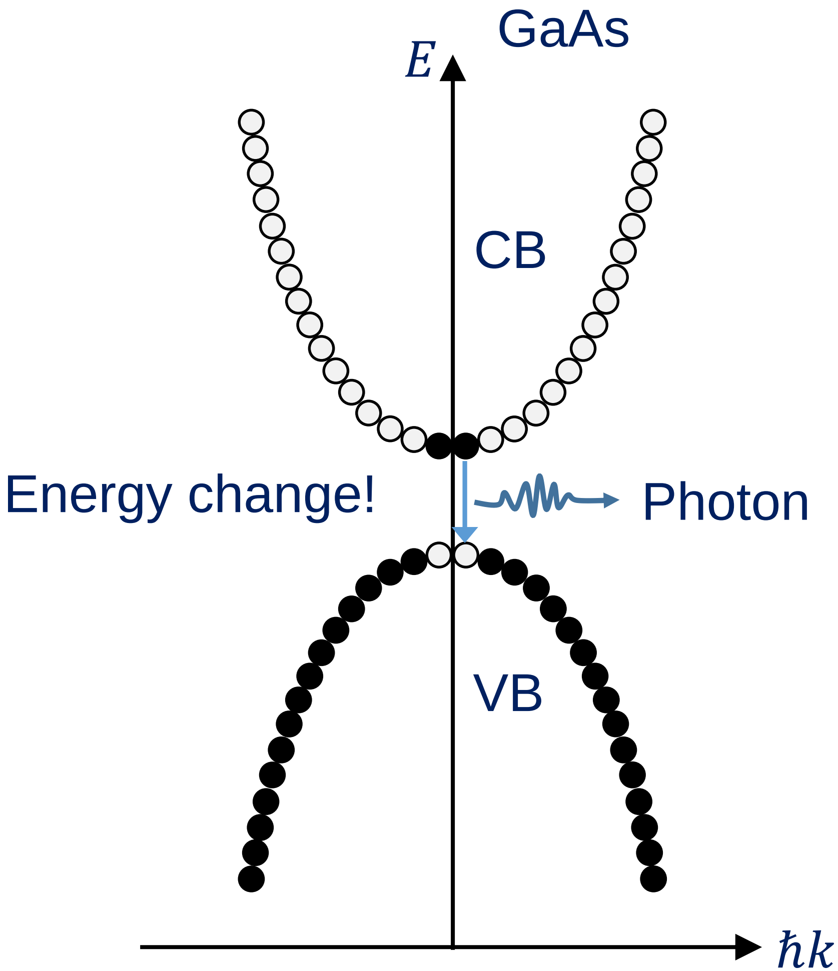
Electron can recombine with hole and emit photon with **the assistance of phonon**.

The efficiency to emit a photon is very low compared with direct bandgap materials.

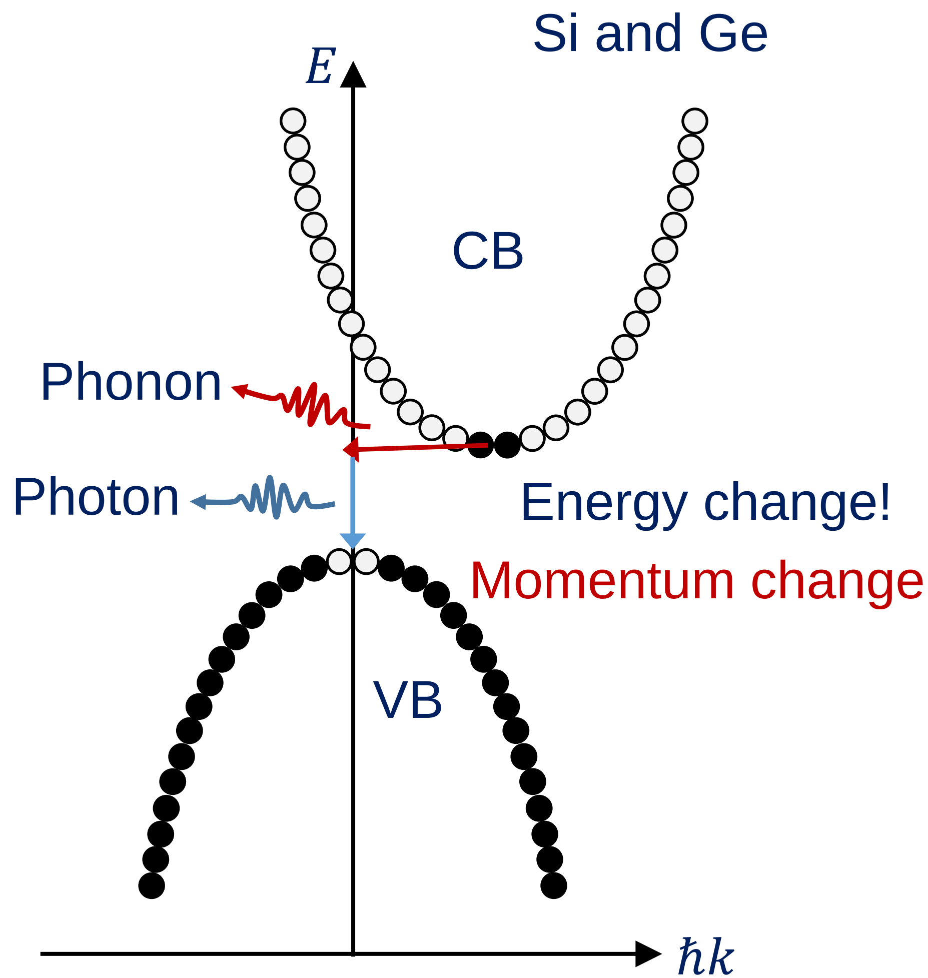


Indirect bandgap material

Direct bandgap material



Indirect bandgap material



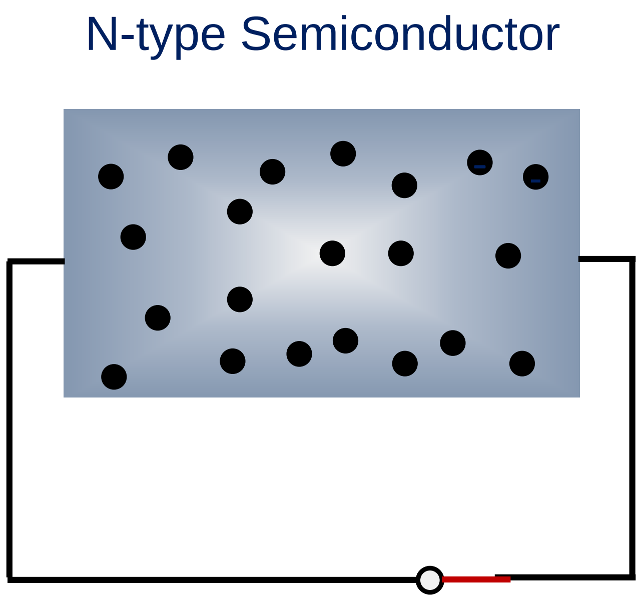
High efficiency!

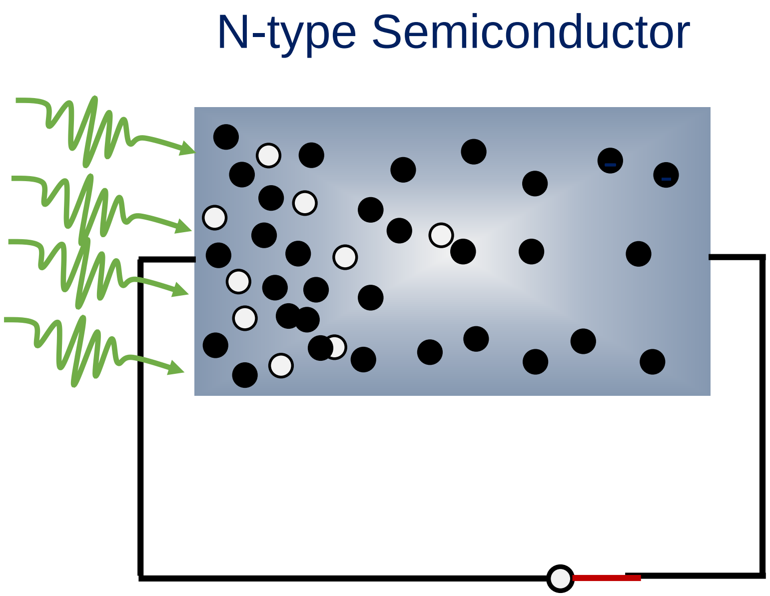
Good for LED applications!

Low efficiency!

Not good for LED applications!

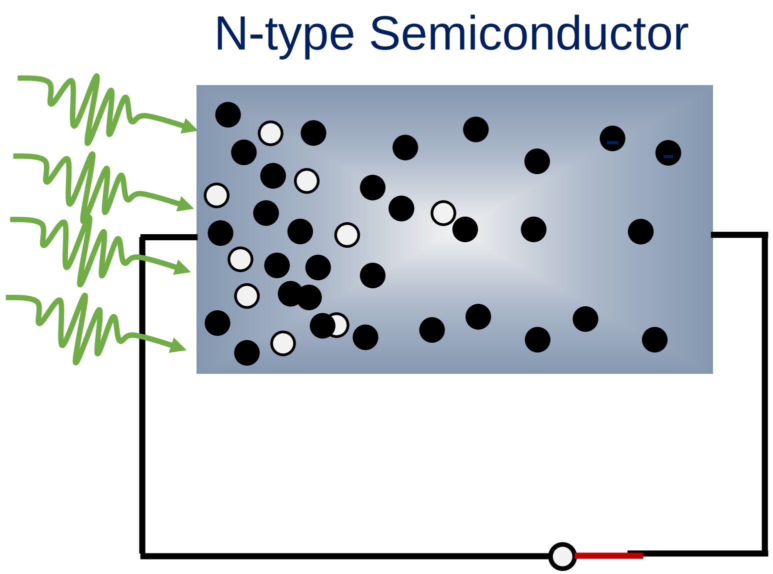
4.8 Diffusion and conduction equations



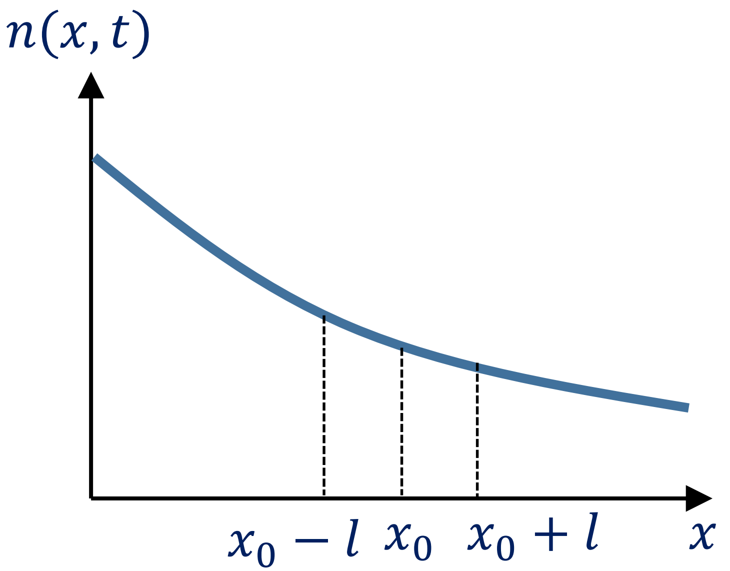


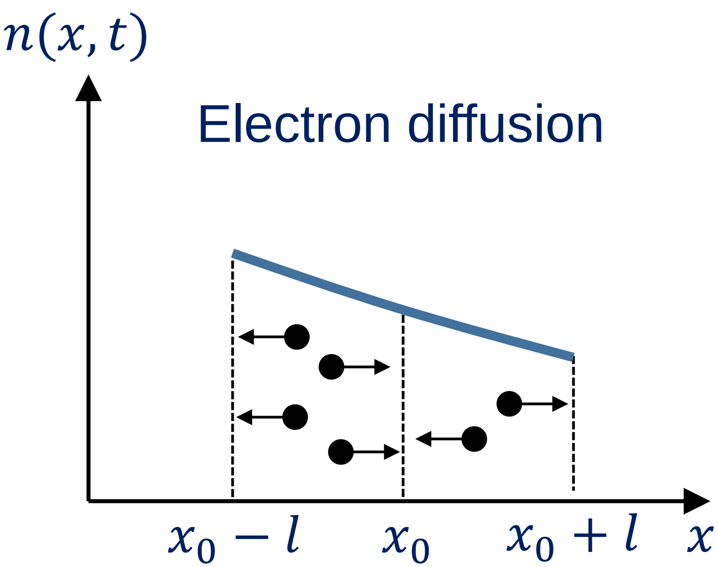
Electron and hole concentrations are not uniform everywhere.

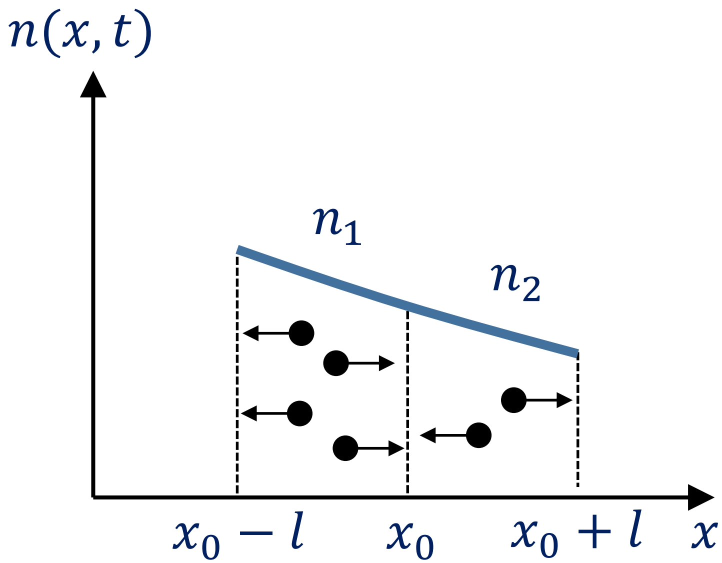
Electrons and holes will diffusive 扩散 from high to low concentration region.



Electron concentration along x-direction







Particle flux density 粒子通量:

$$\Gamma = \frac{\Delta N}{A\Delta t}$$

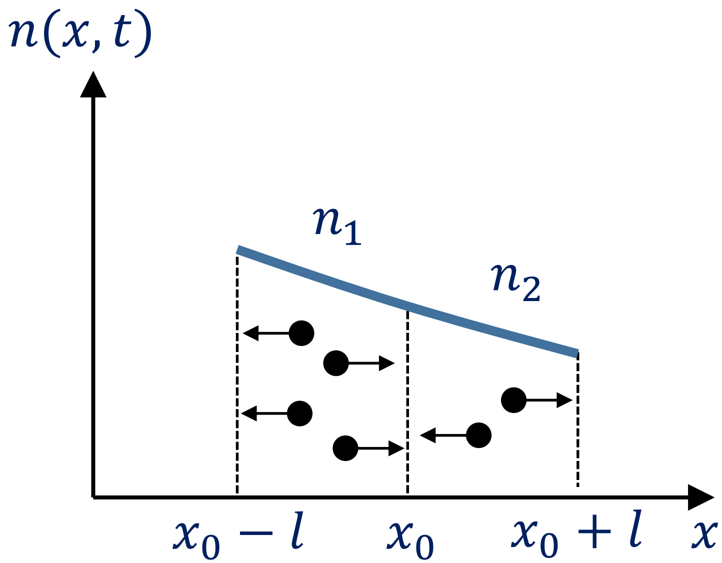
ΔN: total number of particles passing through the plane area A in time Δt

Current density for e$^-$ and h$^+$:

$$J = \mp e\Gamma$$

Since l is very small, n_1 and n_2 can be treated as constant in region $[x_0 - l, x_0]$ and $[x_0, x_0 + l]$, respectively.

$$\Gamma_\mathrm{e} = \frac{\frac{1}{2}n_1Al - \frac{1}{2}n_2Al}{A\tau} = -\frac{l}{2\tau}(n_2 - n_1)$$



Particle flux density at x_0:

$$\Gamma_\mathrm{e} = -\frac{l}{2\tau}(n_2 - n_1)$$

Because l is very small:

$$n_2 - n_1 = \frac{\mathrm{d}n}{\mathrm{d}x} l$$

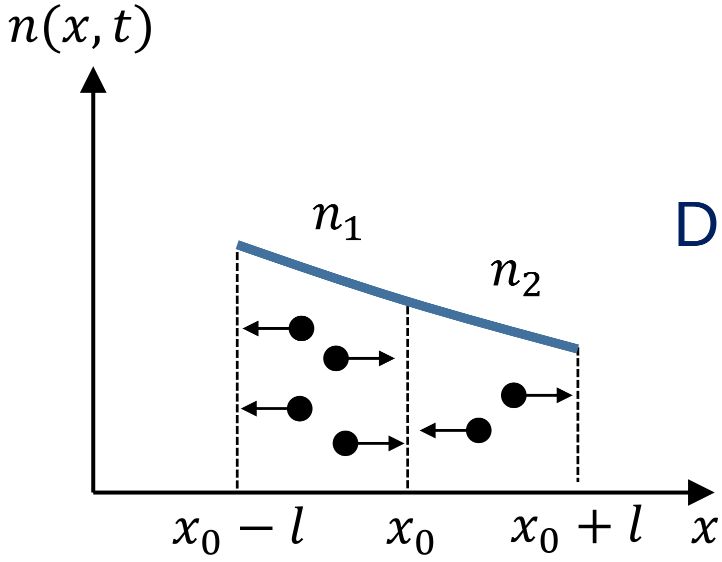
$$\Gamma_\mathrm{e} = -\frac{l^2}{2\tau}\frac{\mathrm{d}n}{\mathrm{d}x}$$

D_e: electron diffusion coefficient 电子扩散系数

$$\Gamma_\mathrm{e} = -D_\mathrm{e}\frac{\mathrm{d}n}{\mathrm{d}x}$$

Our oversimplified model: $D = \dfrac{l^2}{2\tau}$

More accurate model: $D = \dfrac{l^2}{\tau}$



$$\Gamma_\mathrm{e} = -D_\mathrm{e}\frac{\mathrm{d}n}{\mathrm{d}x}$$

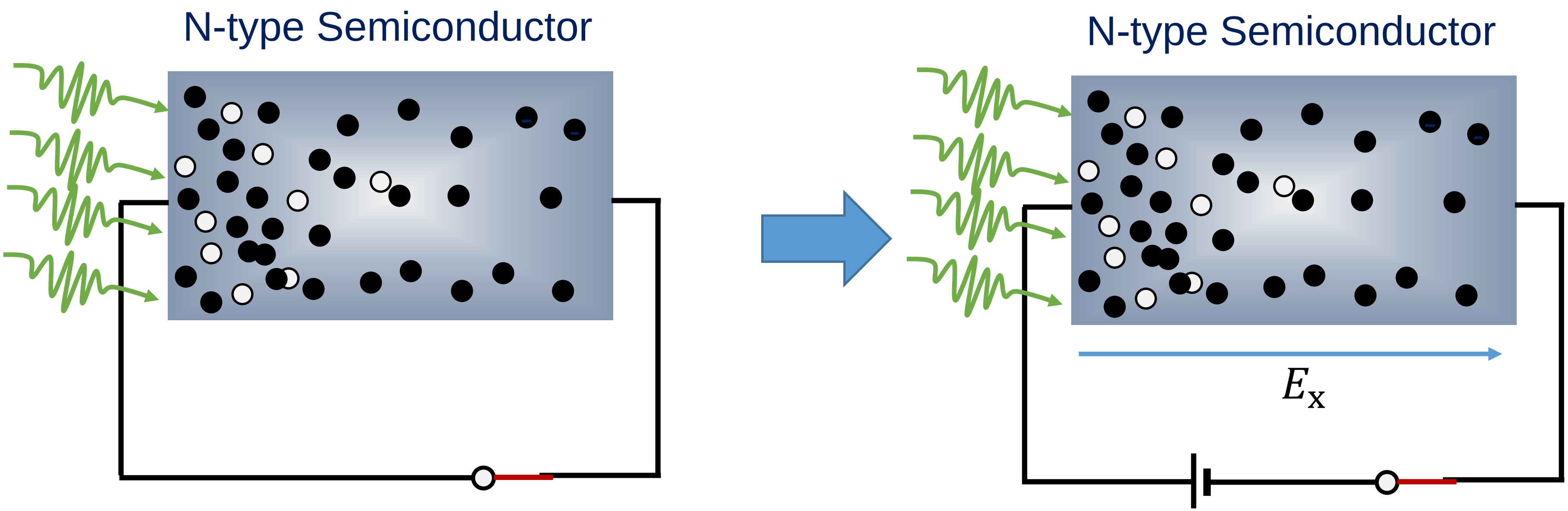
Diffusion current density of electrons:

$$J_\mathrm{e} = -e\Gamma_\mathrm{e} = eD_\mathrm{e}\frac{\mathrm{d}n}{\mathrm{d}x}$$

For hole carriers:

$$\Gamma_\mathrm{h} = -D_\mathrm{h}\frac{\mathrm{dp}}{\mathrm{d}x}$$

$$J_\mathrm{h} = +e\Gamma_\mathrm{h} = -eD_\mathrm{h}\frac{\mathrm{d}p}{\mathrm{d}x}$$



Q: What's the current density when there is external electric field?

Electron: $J_e = en\mu_e E_x + eD_e \frac{dn}{dx}$

Hole: $J_h = en\mu_h E_x - eD_h \frac{dp}{dx}$

Drift漂移 Diffusion扩散

Electron: $J_e = en\mu_e E_x + eD_e \frac{dn}{dx}$

Hole: $J_h = en\mu_h E_x - eD_h \frac{dp}{dx}$

Drift漂移 Diffusion扩散

D: is a measure of the ease with which the diffusing charge carriers move in the medium.

μ: is a measure of the ease with which the charge carriers move in the medium.

$$\frac{D_e}{\mu_e} = \frac{kT}{e} \quad \text{and} \quad \frac{D_h}{\mu_h} = \frac{kT}{e}$$

Einstein relation

Einstein relation

$$\frac{D_{\mathrm{e}}}{\mu_{\mathrm{e}}} = \frac{kT}{e} \quad \text{and} \quad \frac{D_{\mathrm{h}}}{\mu_{\mathrm{h}}} = \frac{kT}{e}$$

$$D_{\mathrm{e}} = \frac{l^2}{\tau} = \frac{(v_x \tau)^2}{\tau} = v_x^2 \tau$$

In CB band of semiconductor and in one-dimension, the mean *KE* of electrons is:

$$\frac{1}{2} m_{\mathrm{e}}^* v_x^2 = \frac{1}{2} kT \Rightarrow v_x^2 = \frac{kT}{m_{\mathrm{e}}^*}$$

$$D_{\mathrm{e}} = \frac{kT\tau}{m_{\mathrm{e}}^*} = \frac{kT}{e}\left(\frac{e\tau}{m_{\mathrm{e}}^*}\right)$$

$$\boxed{D_{\mathrm{e}} = \frac{kT}{e} \mu_{\mathrm{e}}}$$

Einstein relation

$$\frac{D_{\mathrm{e}}}{\mu_{\mathrm{e}}} = \frac{kT}{e} \quad \text{and} \quad \frac{D_{\mathrm{h}}}{\mu_{\mathrm{h}}} = \frac{kT}{e}$$

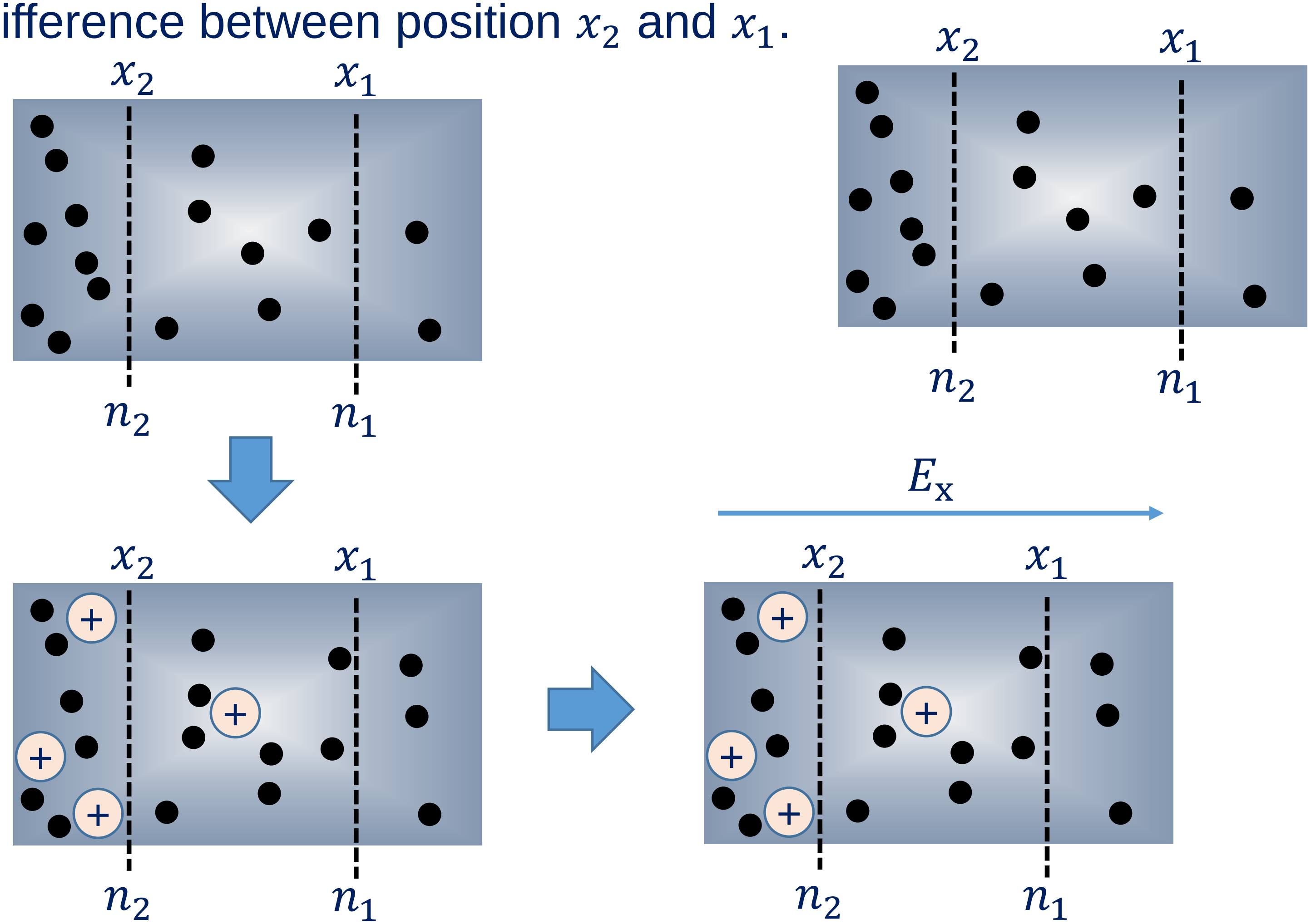
For semiconductors: ✓

For metals: ? ✗

The assumption we used is: $\frac{1}{2} m_{\mathrm{e}}^{*} v_{x}^{2} = \frac{1}{2} kT$

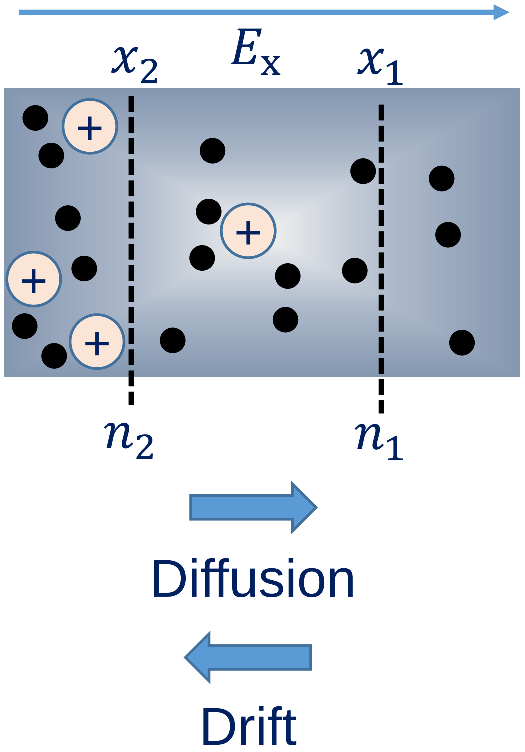
Einstein relation is only valid for electrons and holes in non-degenerated semiconductor.

[Example]: The doping concentration is non-uniform in a n-type semiconductor. The dopant concentration is higher near left surface. The potential difference between position x_2 and x_1.



Electron diffusion, charged donors are left.

Electric field is formed to prevent the diffusion of electrons.



When the system reaches equilibrium:

$$J_e = en\mu_e E_x + eD_e \frac{dn}{dx} = 0$$

$$-en\mu_e \frac{dV}{dx} + eD_e \frac{dn}{dx} = 0$$

$$-e\mu_e dV + eD_e \frac{dn}{n} = 0$$

$$V_2 - V_1 = \frac{kT}{e} \ln(\frac{n_2}{n_1})$$