Electronic Materials and Devices

2 Classical electrical and thermal conductance in solids

QQ Group:

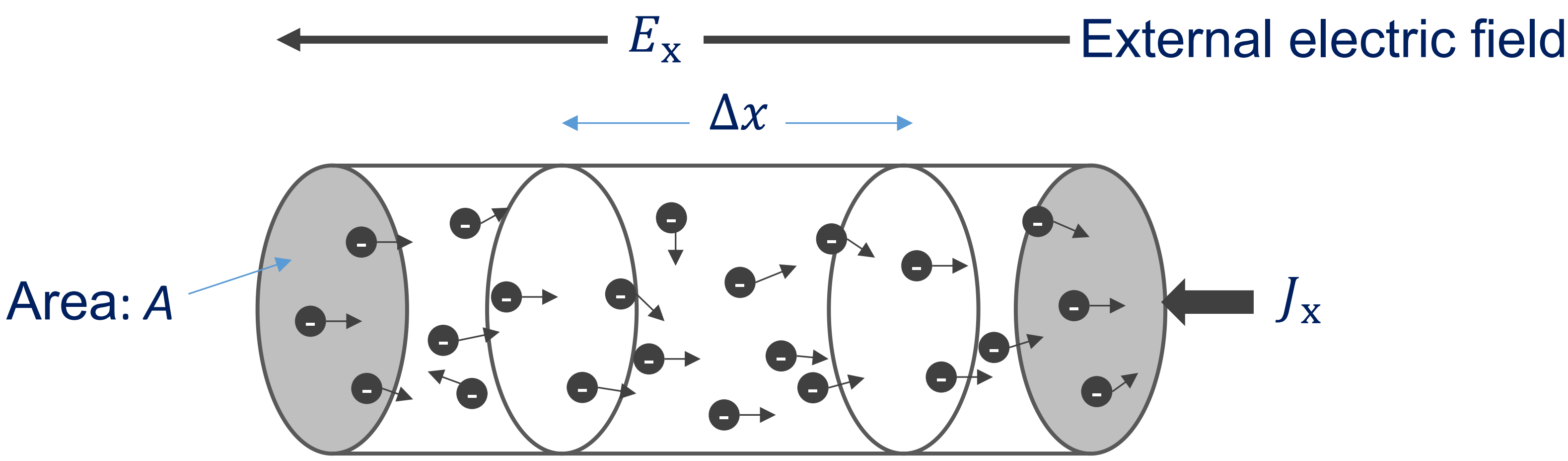
群名称:电子功能材料与器件

群　号:940368648

陈晓龙 Chen, Xiaolong

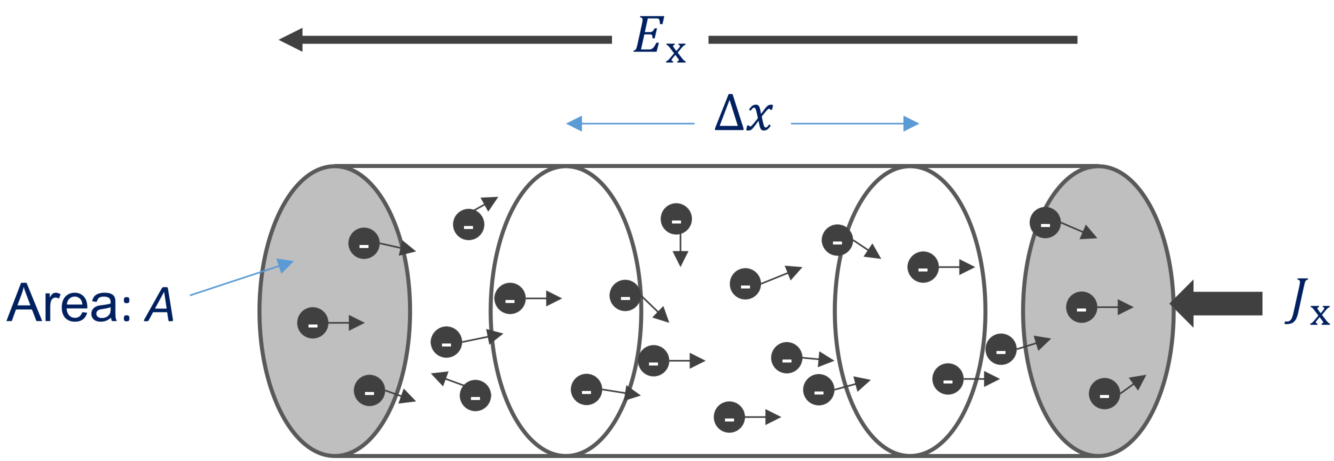
电子与电气工程系

2.1 Classic theory: the Drude model



The **electric current density J** is defined as the net amount of charge flowing across a unit area per unit time:

$$J_x = \frac{\Delta q}{A \Delta t}$$



Carrier density n: the number of conduction electrons per unit volume.

Drift velocity v_{dx}: the average velocity of the electrons in x-direction.

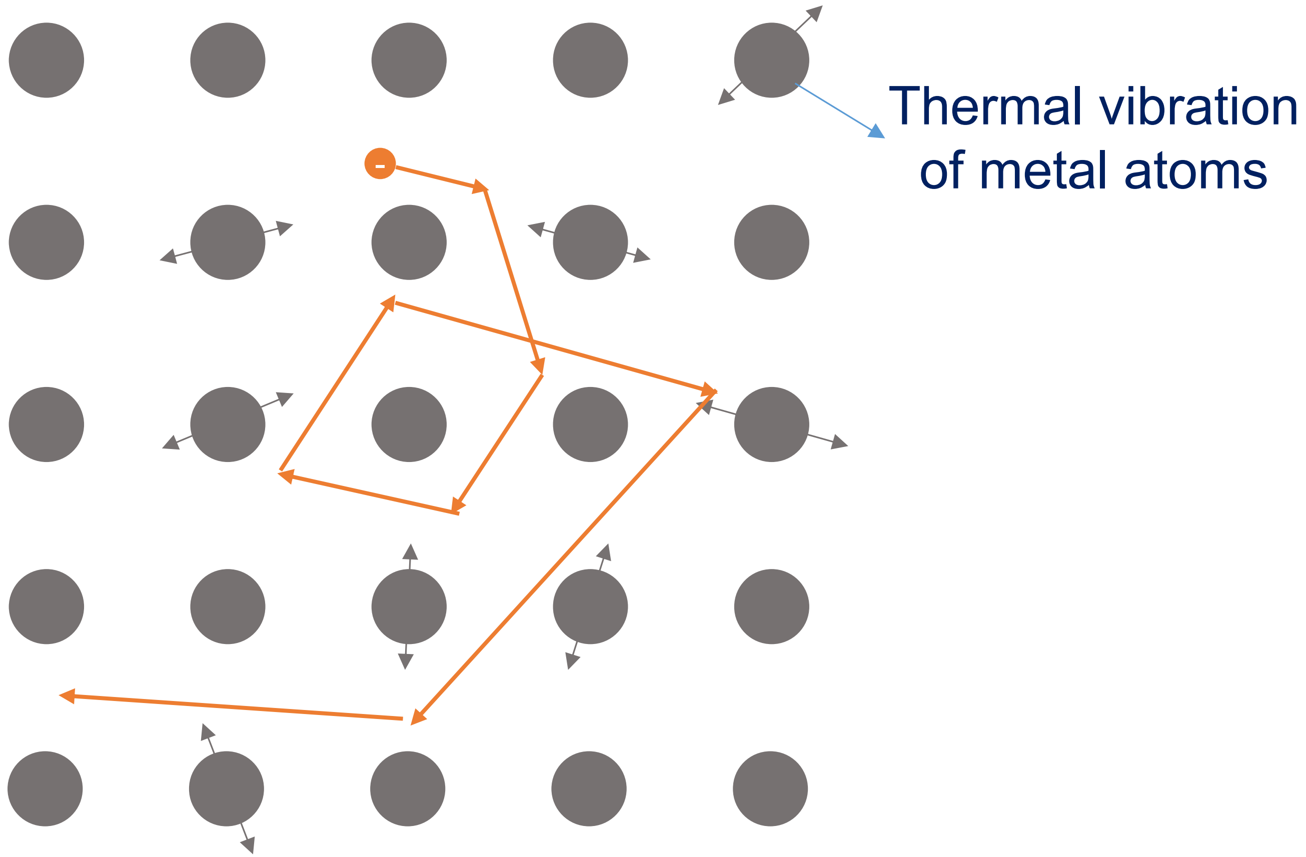
漂移速度

$$v_{dx} = \frac{1}{N}\sum_{i=1}^{N} v_{xi}$$

Electric current density : $$J_x = \frac{\Delta q}{A\Delta t} = \frac{enAv_{dx}\Delta t}{A\Delta t} = env_{dx}$$

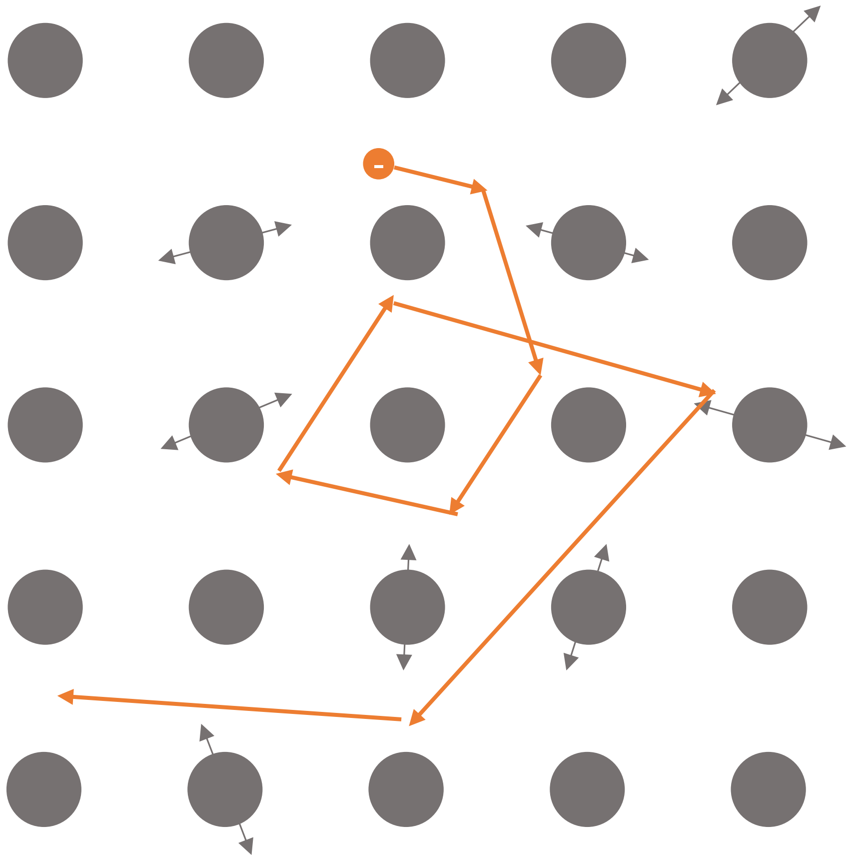
Question: Drift velocity v_{dx} *vs* mean speed of electron?

When there is no external electric field:



A conduction electron i^{th} in the electron gas moves about randomly in a metal with a mean speed u.

When there is no external electric field:

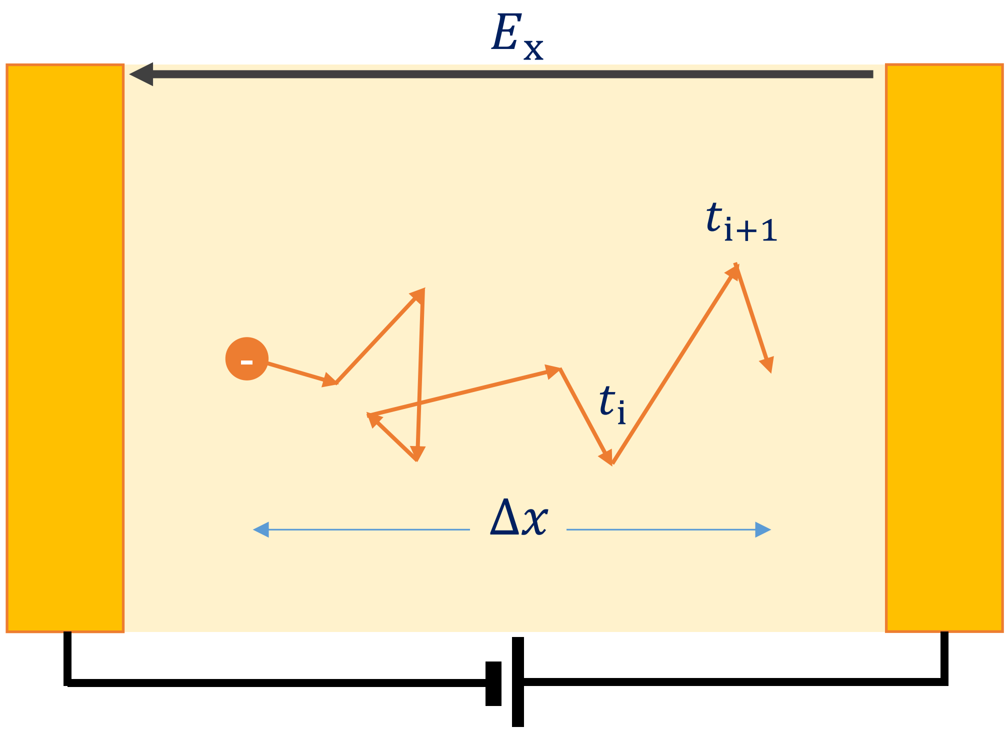


At time t, the average drift velocity in x-direction of all electrons $\frac{1}{N}\sum_{i=1}^{N} u_{\mathrm{xi}}$?

$$\frac{1}{N}\sum_{i=1}^{N} u_{\mathrm{xi}} = 0$$

No net current flow!

Apply an external electric field:



For the i^{th} electron, suppose its last collision was at time t_i, the velocity in x-direction at time t:

$$v_{xi} = u_{xi} + \frac{eE_x}{m_e}(t - t_i),\ t_i < t < t_{i+1}$$

The average drift velocity:

$$v_{\mathrm{dx}} = \frac{1}{N}\sum_{i=1}^{N} v_{\mathrm{xi}} = \frac{1}{N}\sum_{i=1}^{N} u_{\mathrm{xi}} + \frac{eE_{\mathrm{x}}}{m_{\mathrm{e}}}\frac{1}{N}\sum_{i=1}^{N}(t - t_{\mathrm{i}}) = \frac{eE_{\mathrm{x}}}{m_{\mathrm{e}}}\overline{(t - t_{\mathrm{i}})}$$

$\overline{(t - t_{\mathrm{i}})}$ is the average free time for N electrons between collisions.

$\boldsymbol{\tau} = \overline{(t - t_{\mathrm{i}})}$: **mean time between collisions/ mean scattering time 平均散射时间/ mean free time/ relaxation time弛豫时间.**

$$v_{\mathrm{dx}} = \frac{e\tau}{m_{\mathrm{e}}}E_{\mathrm{x}}$$

We introduce the electron **drift mobility 迁移率:** $\boldsymbol{\mu_{\mathrm{d}} = \frac{e\tau}{m_e}}$. m2 / (V s)

$$\boxed{v_{\mathrm{dx}} = \mu_{\mathrm{d}}E_{\mathrm{x}}}$$

The current density: $J_{\mathrm{x}} = env_{\mathrm{dx}} = en\mu_{\mathrm{d}}E_{\mathrm{x}}$

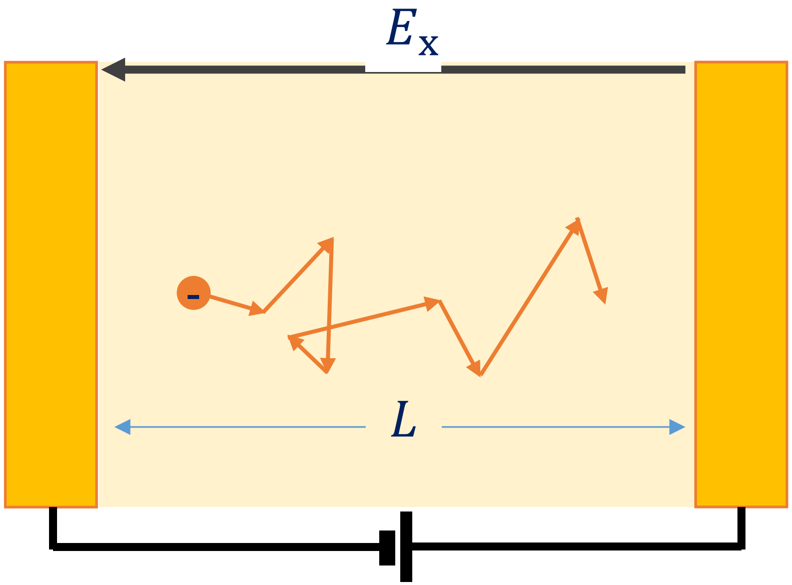
The **conductivity** is defined as $\boldsymbol{\sigma} = \frac{\boldsymbol{J_x}}{\boldsymbol{E_x}}$.

$$\boxed{\sigma = en\mu_{\mathrm{d}}}$$

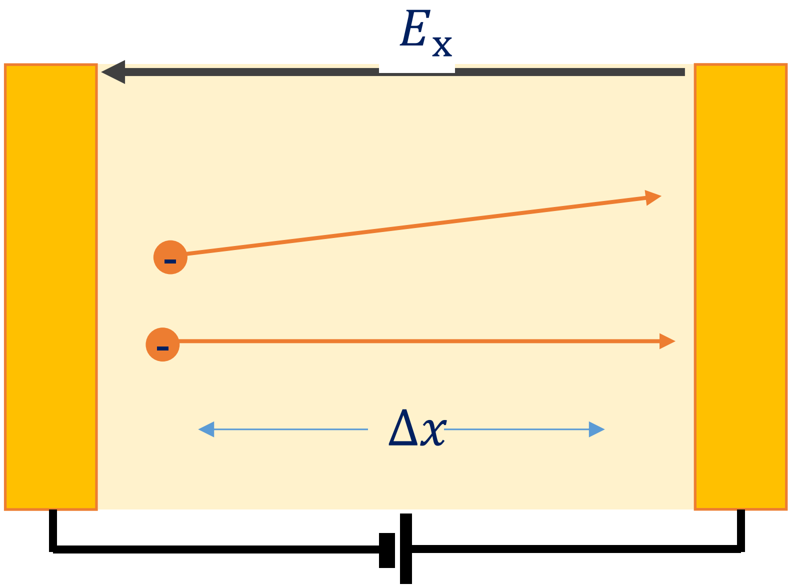
This is Drude model!

Q: What's the condition for this formula?

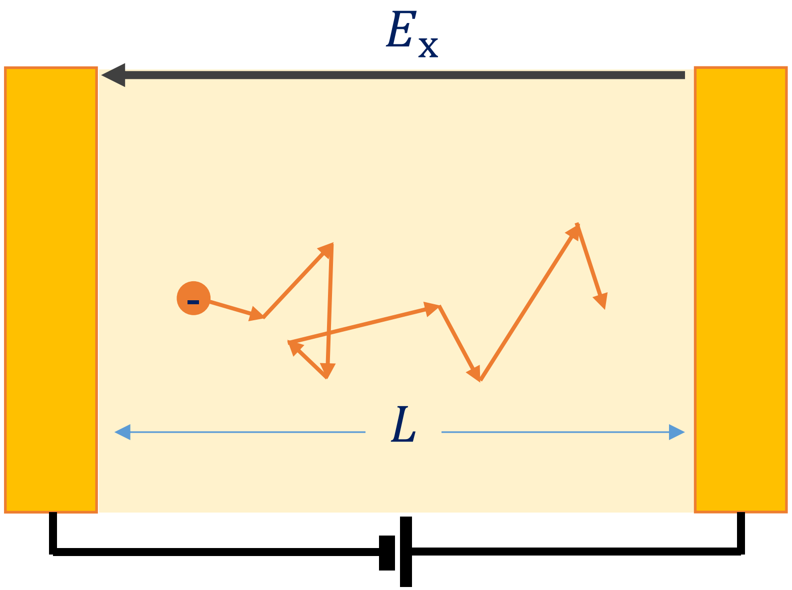
The condition for the formula $\sigma = en\mu_\mathrm{d}$



Diffusive transport ✓



Ballistic transport ✗



Diffusive transport

Mean free path 平均自由程:

$$l = u\tau$$

Mean velocity (not drift velocity)

Mean free time

$\sigma = en\mu_d$ is true when $l < L$

Paul Karl Ludwig Drude

July 12, 1863 - July 5, 1906

- Drude began his studies in mathematics at the University of Gottingen, but later changed his major to physics.
- Thus Drude began his professional career at the time Maxwell's theories were being introduced into Germany
- His first experiments were the determination of the optical constants of various solids. He then worked to derive relationships between the optical and electrical constants. In 1894 he was responsible for introducing the symbol "c" for the speed of light in a perfect vacuum.
- In 1905 he became the director of the physics institute of the University of Berlin. In 1906, at the height of his career, be became a member of the Prussian Academy of Sciences. A few days after his inauguration lecture, for inexplicable reasons, he committed suicide.

2.2 Temperature dependence of resistivity: ideal pure metals

Ideal pure metals: the conduction electrons are only scattered by thermal vibrations of the metal ions

Question: $\sigma \sim T$?

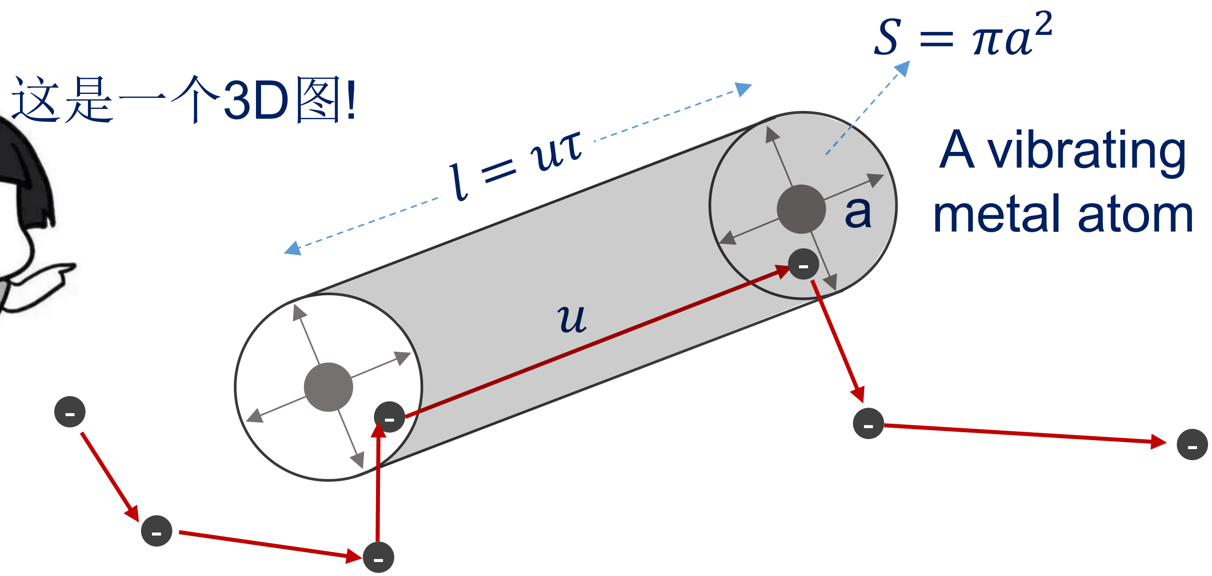
◆ **Let's start from the Drude model**

$$\sigma = en\mu_{\mathrm{d}} \qquad \mu_{\mathrm{d}} = \frac{e\tau}{m_{\mathrm{e}}}$$

τ: the mean free time due to thermal vibration scattering

◆ **Only τ strongly dependens on temperature**

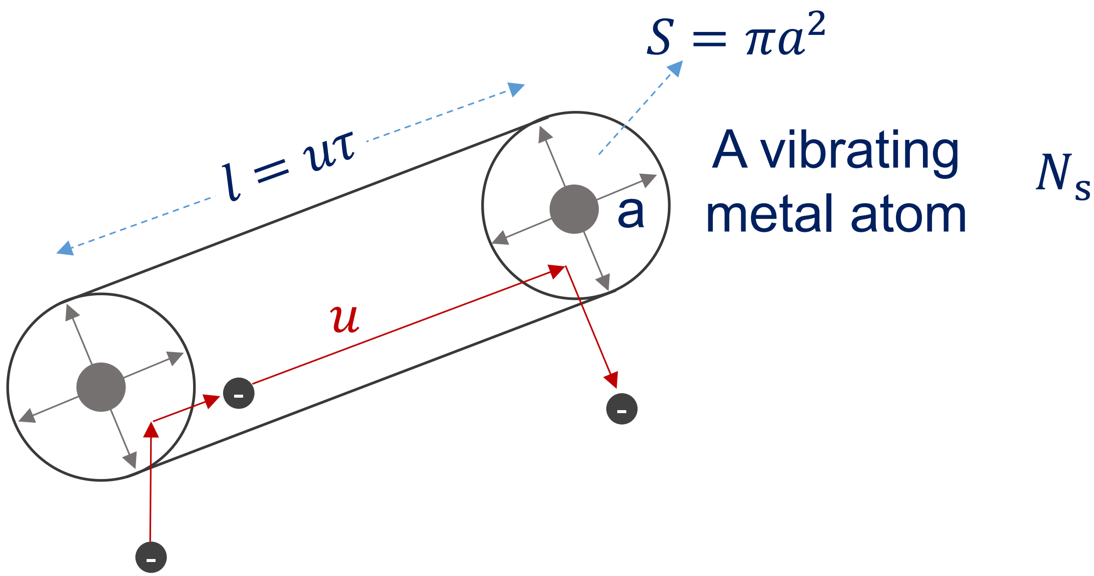
A simple classic model



- Scattering cross-sectional area: $S = \pi a^2$
- Mean free path: $l = u\tau$
- The concentration of scattering centers: N_s

Q: What's the relation between S, τ, and N_s?

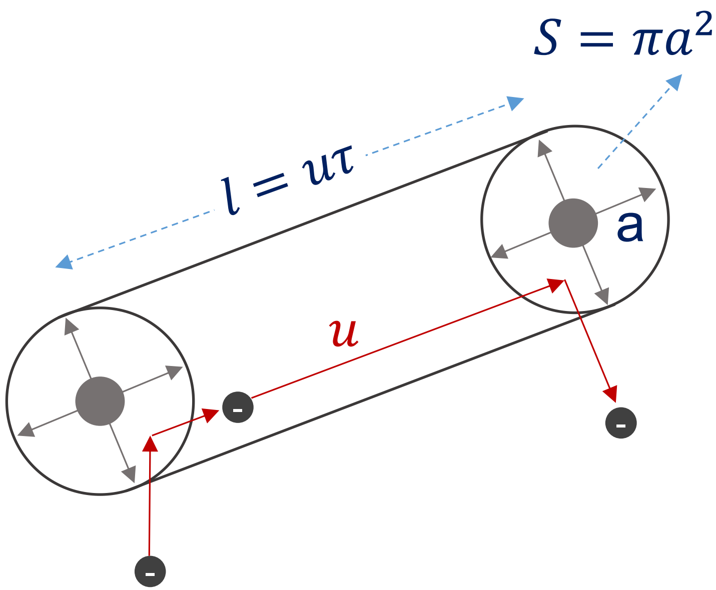
A simple classic model



There is 1 scattering event in the volume $SlN_s = 1$

$$\Rightarrow \quad \tau = \frac{1}{SuN_s} = \frac{1}{\pi a^2 u N_s}$$

Assume the mean speed u is constant.



$$\tau = \frac{1}{\pi a^2 u N_s}$$

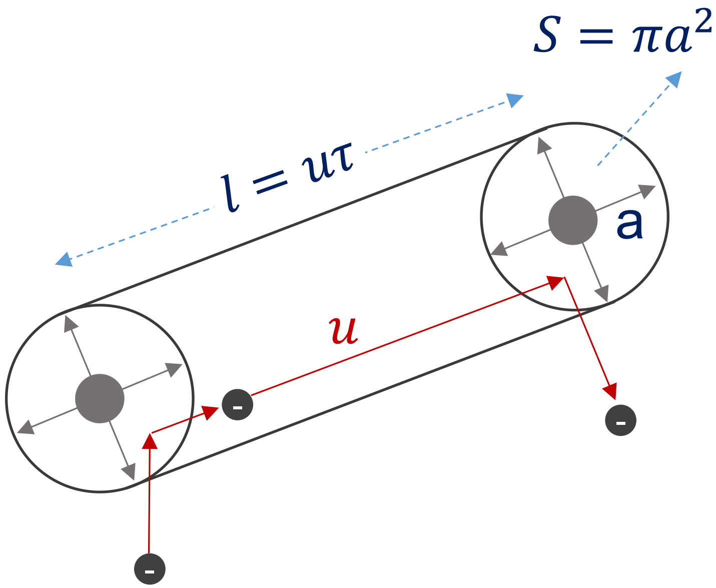
Q: The vibration amplitude a?

The vibration energy ≈ thermal energy

$$\frac{1}{4} M a^2 \omega^2 \approx \frac{1}{2} kT$$

$$\tau \propto \frac{1}{\pi a^2} \propto \frac{1}{T} \quad \text{or} \quad \tau = \frac{C}{T}$$

C is a temperature-independent constant.



Mobility: $\mu_{\mathrm{d}} = \dfrac{eC}{m_{\mathrm{e}}T}$

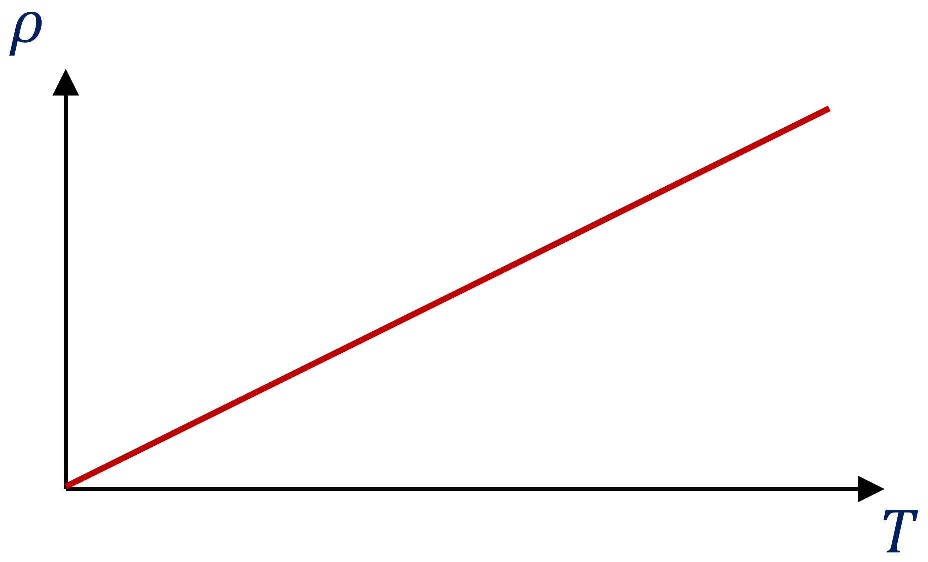
Lattice-scattering-limited conductivity and resistivity:

$$\rho_{\mathrm{T}} = \frac{1}{\sigma_{\mathrm{T}}} = \frac{1}{en\mu_{\mathrm{d}}}$$

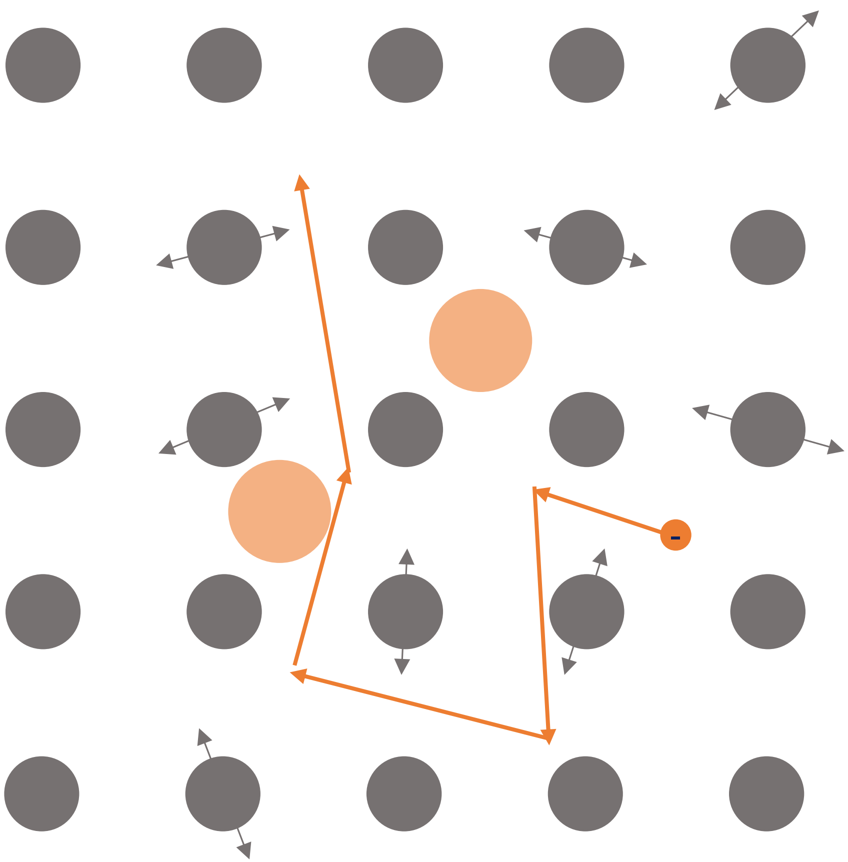
$$= \frac{m_{\mathrm{e}}T}{e^2 nC}$$

Lattice-scattering-limited resistivity:

$$\rho_{\mathrm{T}} = \frac{m_{\mathrm{e}} T}{e^2 n C}$$



2.3 Temperature dependence of resistivity: including impurity scattering



In real case, there must be impurities in metals.

Electrons also suffer the scatterings from impurities.

We define:

τ_T: mean free time from thermal scatterings/lattice vibrations

τ_I: mean free time from impurity scatterings

$$\left\{\begin{array}{l} \dfrac{1}{\tau_T}\text{: probability of scattering from lattice vibration} \\[2ex] \dfrac{1}{\tau_I}\text{: probability of scattering from impurities} \\[2ex] \dfrac{1}{\tau}\text{: the net probability of scattering} \end{array}\right.$$

$$\frac{1}{\tau} = \frac{1}{\tau_T} + \frac{1}{\tau_I}$$

Mobility: $\boldsymbol{\mu_{\mathrm{d}} = \frac{e\tau}{m_{\mathrm{e}}}}$

$$\left\{\begin{array}{ll} \mu_{\mathrm{d}} = \dfrac{e\tau}{m_{\mathrm{e}}} & \text{Drift velocity of the system} \\[2ex] \mu_{\mathrm{T}} = \dfrac{e\tau_{\mathrm{T}}}{m_{\mathrm{e}}} & \text{Lattice-scattering-limited drift velocity} \\[2ex] \mu_{\mathrm{I}} = \dfrac{e\tau_{\mathrm{I}}}{m_{\mathrm{e}}} & \text{Impurity-scattering-limited drift velocity} \end{array}\right.$$

$$\frac{1}{\mu_{\mathrm{d}}} = \frac{1}{\mu_{\mathrm{T}}} + \frac{1}{\mu_{\mathrm{I}}}$$

Resistivity: $\boldsymbol{\rho = \frac{1}{en\mu_\mathrm{d}}}$

$$\begin{cases} \rho = \dfrac{1}{en\mu_\mathrm{d}} & \text{Resistivity of the system} \\ \rho_\mathrm{T} = \dfrac{1}{en\mu_\mathrm{L}} & \text{Resistivity due to lattice vibration scatterings} \\ \rho_\mathrm{I} = \dfrac{1}{en\mu_\mathrm{I}} & \text{Resistivity due to impurity scatterings} \end{cases}$$

Matthiessen's rule

$$\rho = \rho_\mathrm{T} + \rho_\mathrm{I}$$

Matthiessen's rule: $\rho = \rho_{\mathrm{T}} + \underline{\rho_{\mathrm{R}}}$

Can be electrons scattering from dislocations and other crystal defects, as well as from grain boundaries. Hence ρ_{R} includes ρ_{I}.

ρ_{R}: very small temperature dependence

$$\rho = AT + B$$

A and B are temperature-independent constants.

The temperature coefficient of resistivity (TCR) α_0

$$\alpha_0 = \frac{1}{\rho_0}\left[\frac{\delta\rho}{\delta T}\right]_{T=T_0}$$

ρ_0: the resistivity at the reference temperature T_0, usually 273 K or 293 K.

Q: When resistivity follows $\rho = AT + B$, what's the relation between ρ and α_0?

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$$\begin{cases} \alpha_0 = \dfrac{1}{\rho_0}\left[\dfrac{\delta\rho}{\delta T}\right]_{T=T_0} = \dfrac{A}{\rho_0} \\ \rho_0 = AT_0 + B \end{cases}$$

$$\Downarrow$$

$$\begin{cases} A = \alpha_0\rho_0 \\ B = \rho_0 - \alpha_0\rho_0 T_0 \end{cases}$$

$$\Downarrow$$

$$\rho = \rho_0[1 + \alpha_0(T - T_0)]$$

Q: For pure metals, what's the relation between ρ and α_0?

When $T = 0$, $\rho = 0$.

$$\Downarrow$$

$$0 = \rho_0[1 + \alpha_0(0 - T_0)]$$

$$\Downarrow$$

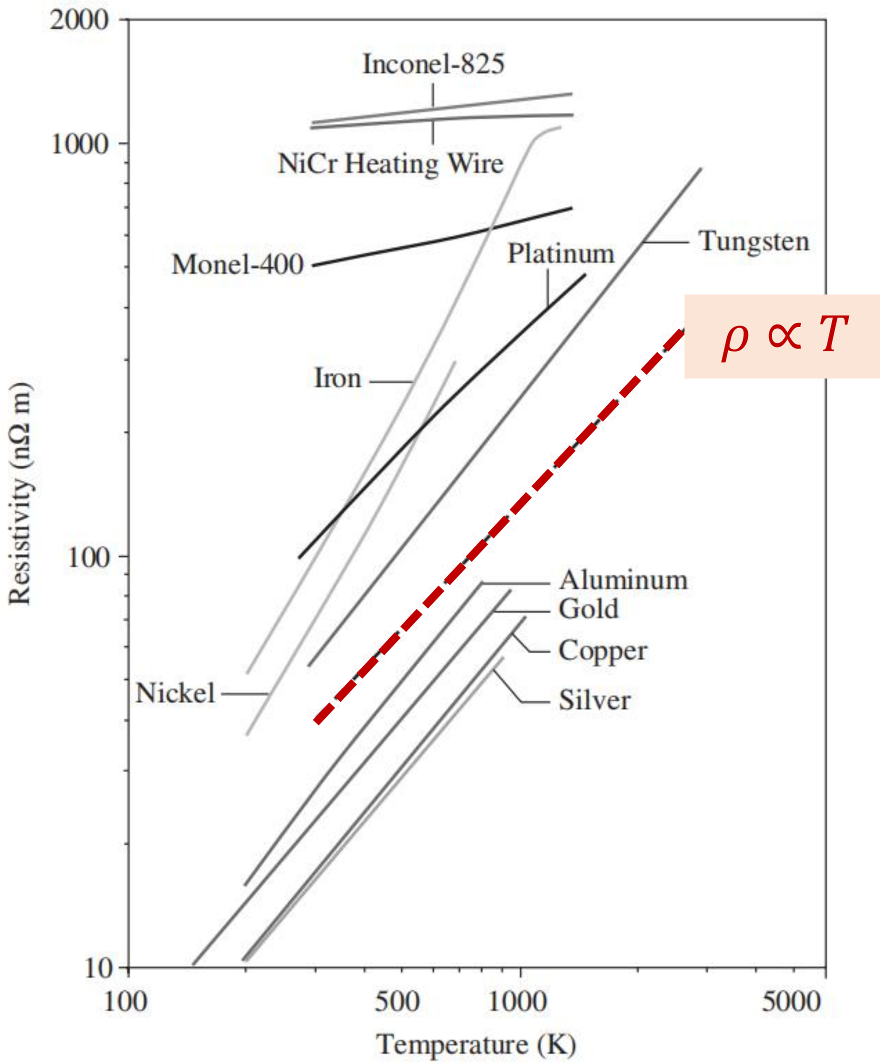
$$\alpha_0 = 1/T_0$$

$$\Downarrow$$

$$\boxed{\rho = \rho_0 \frac{T}{T_0}}$$

If the reference temperature $T_0 = 273\ \mathrm{K}$, $\alpha_0 = \frac{1}{273}\ \mathrm{K}^{-1}$.

In real conditions, only **some metals** follow this relation in **certain temperature range**!



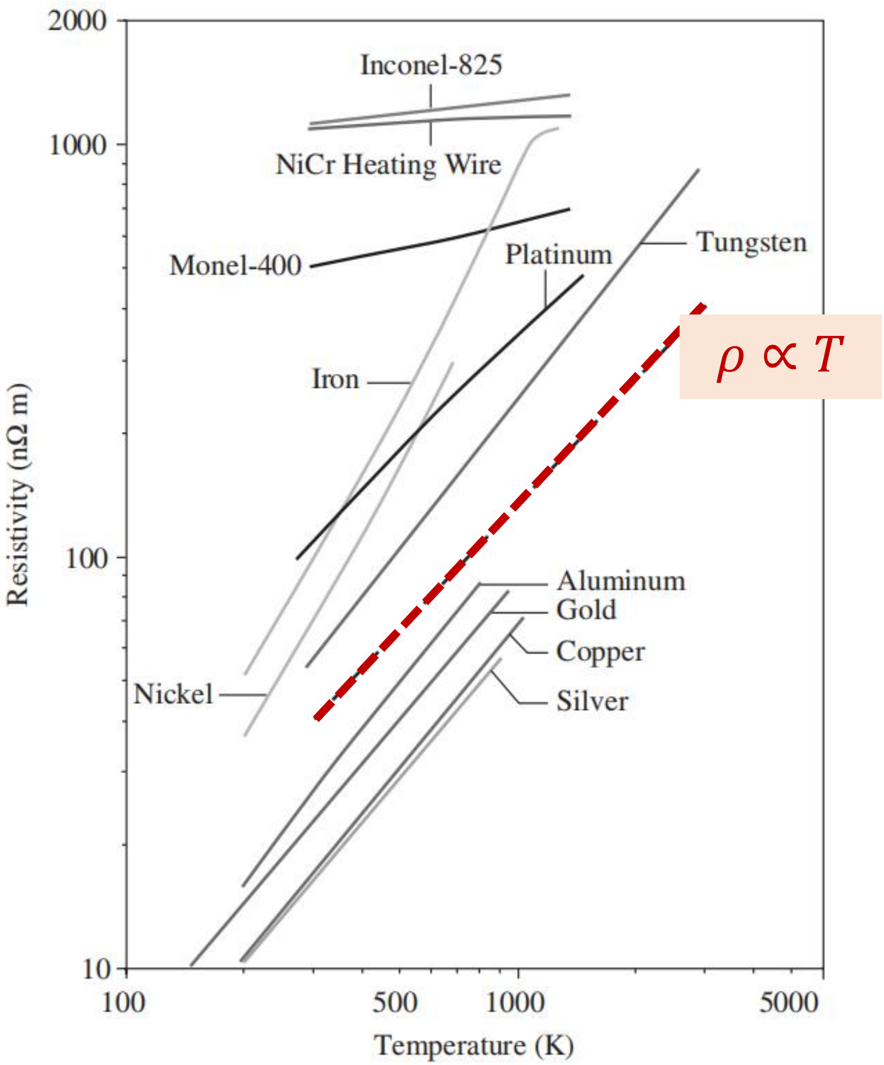
Aluminum, gold, copper, silver follows the $\rho = \rho_0 \frac{T}{T_0}$ quite well.

Magnetic metals such as iron, nickel deviate from $\rho = \rho_0 \frac{T}{T_0}$

The $\rho \sim T$ behavior can be described by a power law of the form:

$$\rho = \rho_0 \left[\frac{T}{T_0}\right]^n$$

In real conditions, only **some metals** follow this relation in **certain temperature range**!



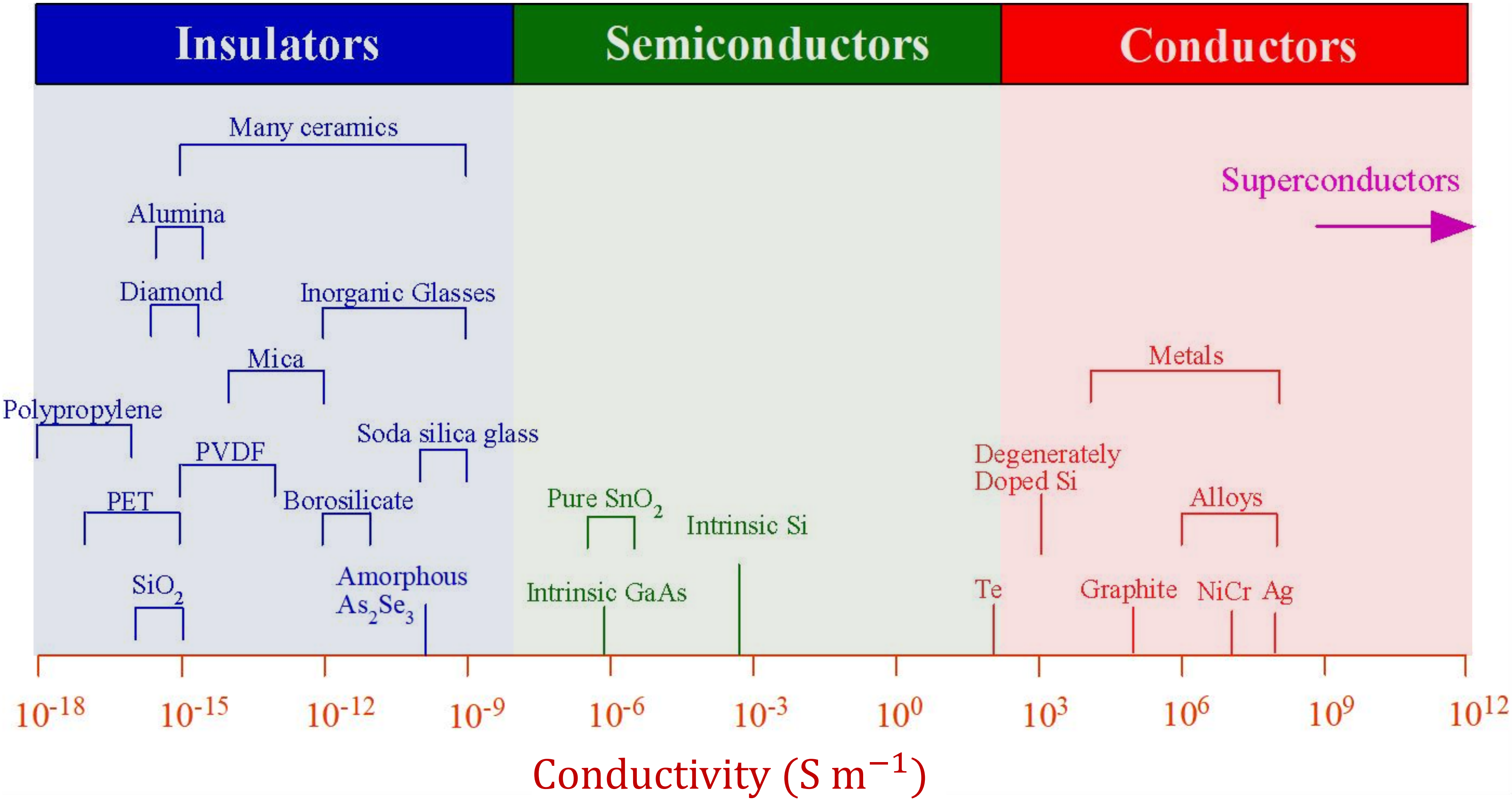
$$\rho = \rho_0 \left[\frac{T}{T_0}\right]^n$$

Q: the TCR α_0?

$$\alpha_0 = \frac{1}{\rho_0}\left[\frac{\delta\rho}{\delta T}\right]_{T=T_0}$$

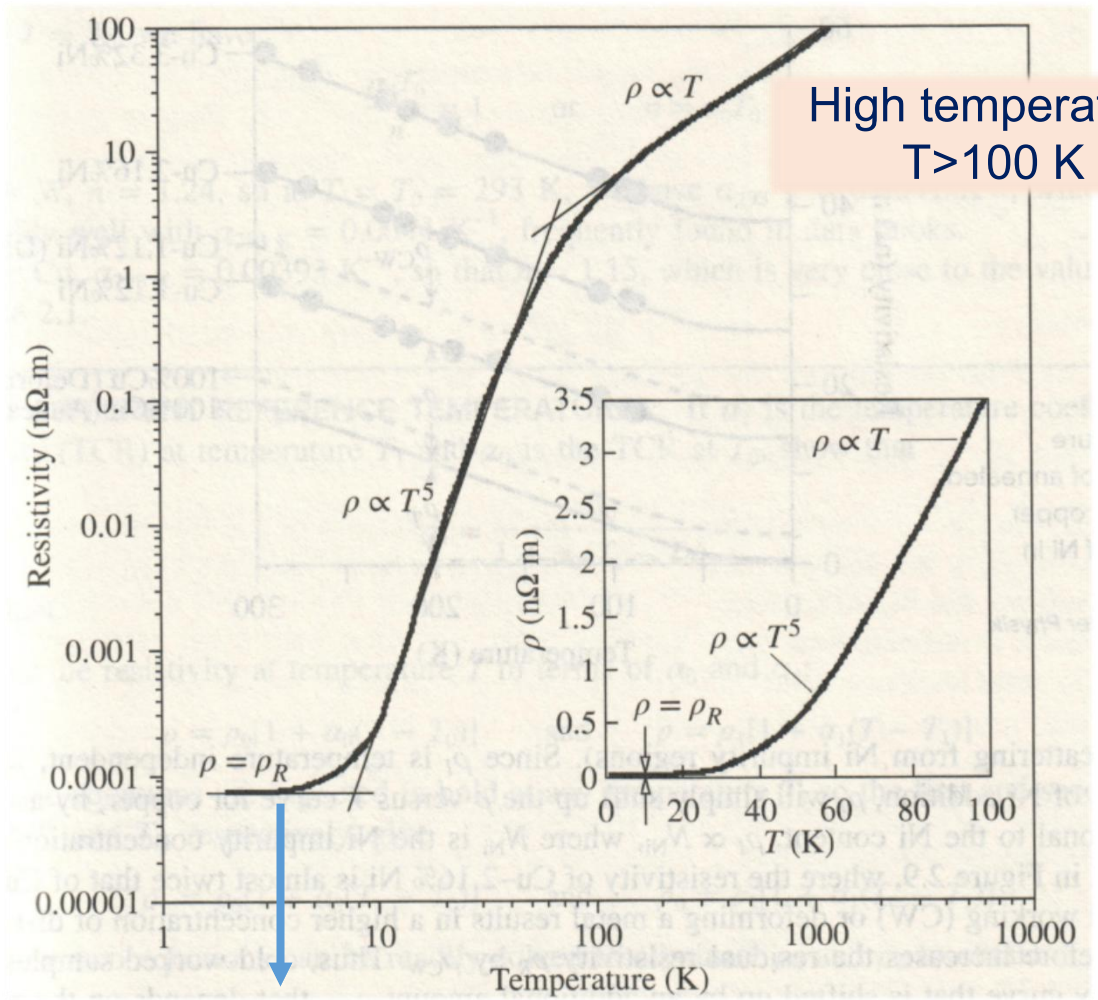
$$\alpha_0 = \frac{n}{T_0}$$

Metal 273 K	$\rho_0(\mathrm{n\Omega \cdot m})$	α_0 at $\mathbf{273\ K}(1/\mathrm{K})$	n	T range (K)
Aluminum, Al	24.2	1/227	1.20	200-800
Antimony, Sb	390	1/215	1.27	80-400
Copper, Cu	15.4	1/233	1.16	200-1100
Gold, Au	20.5	1/242	1.13	225-1000
Indium, In	80	1/208	1.31	200-400
Molybdenum, Mo	48.5	1/226	1.21	200-2400
Platinum, Pt	98.1	1/256	1.01	200-1273
Silver, Ag	14.7	1/242	1.13	200-1100
Strontium, Sr, 锶	123	1/276	0.99	273-800
Tin, Sn	115	1/248	1.10	200-490
Tungsten, W	48.2	1/210	1.24	200-3000
Iron, Fe (Magnetic)	85.7	1/159	1.73	200-900
Nickel, Ni (Magnetic)	61.6	1/155	1.76	200-700



Insulators
Semiconductors
Conductors
Many ceramics
Alumina
Diamond
Inorganic Glasses
Mica
Polypropylene
Soda silica glass
PVDF
PET
Borosilicate
SiO_2
Amorphous As_2Se_3
Pure SnO_2
Intrinsic Si
Intrinsic GaAs
Superconductors
Metals
Degenerately Doped Si
Alloys
Te
Graphite
NiCr
Ag
10^{-18}
10^{-15}
10^{-12}
10^{-9}
10^{-6}
10^{-3}
10^{0}
10^{3}
10^{6}
10^{9}
10^{12}
Conductivity (S m^{-1})

Resistivity at low temperature (Copper)



High temperature
T>100 K

Impurity-scattering-limited resistivity

2.4 Solid solution 固溶体 and Nordheim's rule

Solid solution: an isomorphous alloy of metals, that is, a alloy.

Q: The TCR and resistivity in a binary alloy is higher or lower compared with pure metal?

Matthiessen's rule: $\rho = \rho_{\mathrm{T}} + \rho_{\mathrm{R}}$

The impurity concentration increases with the concentration of solute atoms.

↓

The temperature-independent term ρ_{R} increases.

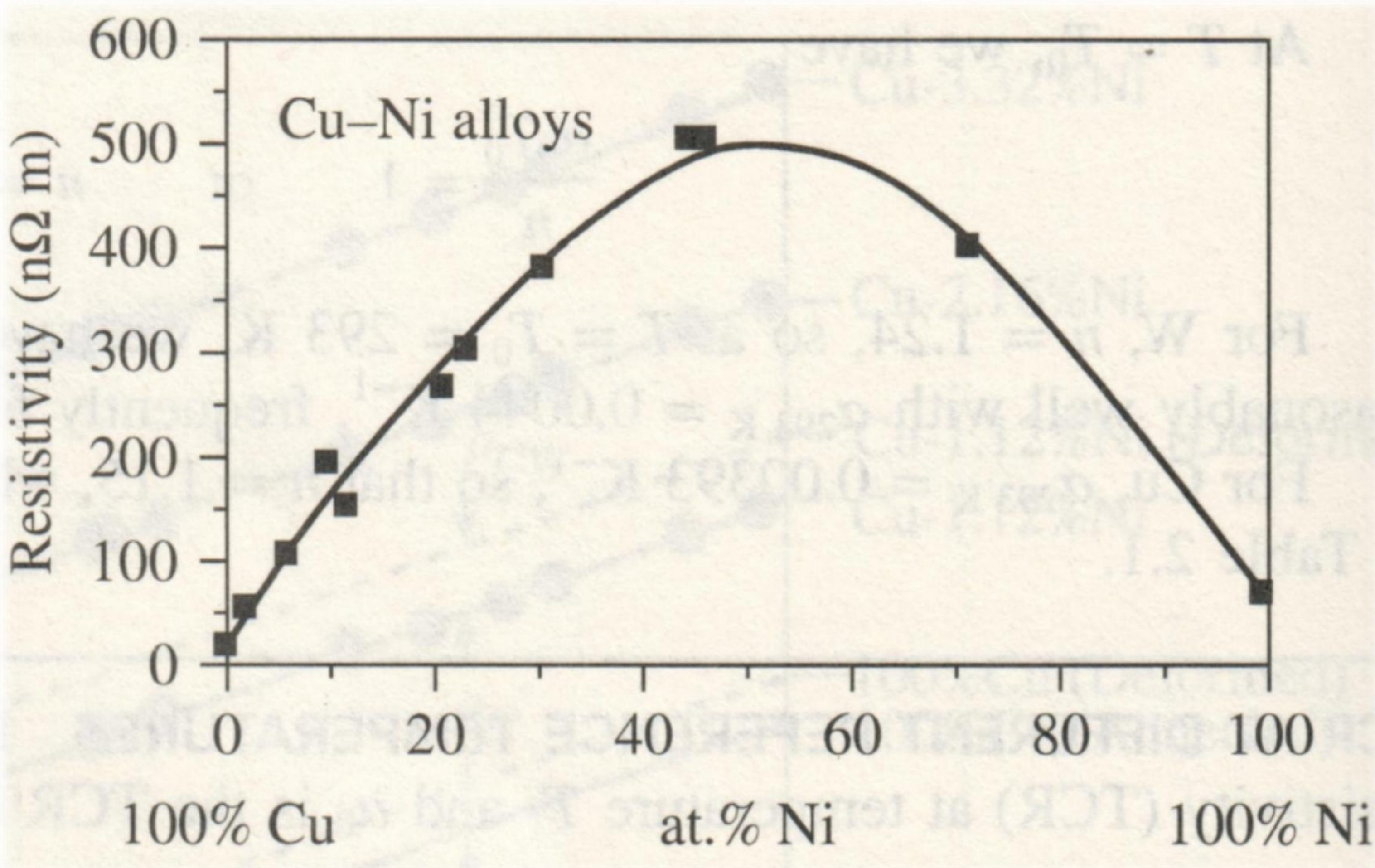
↓

The total resistivity ρ increases and is less sensitive to temperature.

Metal	ρ_0 at $\mathbf{293}$ K $(\mathrm{n}\boldsymbol{\Omega}\cdot\mathrm{m})$	$\boldsymbol{\alpha}_0$ at $\mathbf{293}$ K $(\mathbf{1}/\mathrm{K})$
Nickel	69	1/156
Chrome	129	1/333
Nichrome (80% Ni-20% Cr)	1100	1/2500

Resistivity is less sensitive to temperature

Cu and Ni are both FCC crystals.



Nordheim's rule: an important semiempirical equation to predict the resistivity of an alloy.

The impurity resistivity to the atomic fraction X of solute atoms in a solid solution:

$$\rho_{\mathrm{I}} = CX(1 - X)$$

X: **atomic fraction of solute atoms.**

C: a constant termed the **Nordheim coefficient.**

C represents the effectiveness of the solute atom in increasing the resistivity.

The resistivity of an alloy of composition *X* is:

$$\rho = \rho_{\mathrm{matrix}} + CX(1 - X)$$

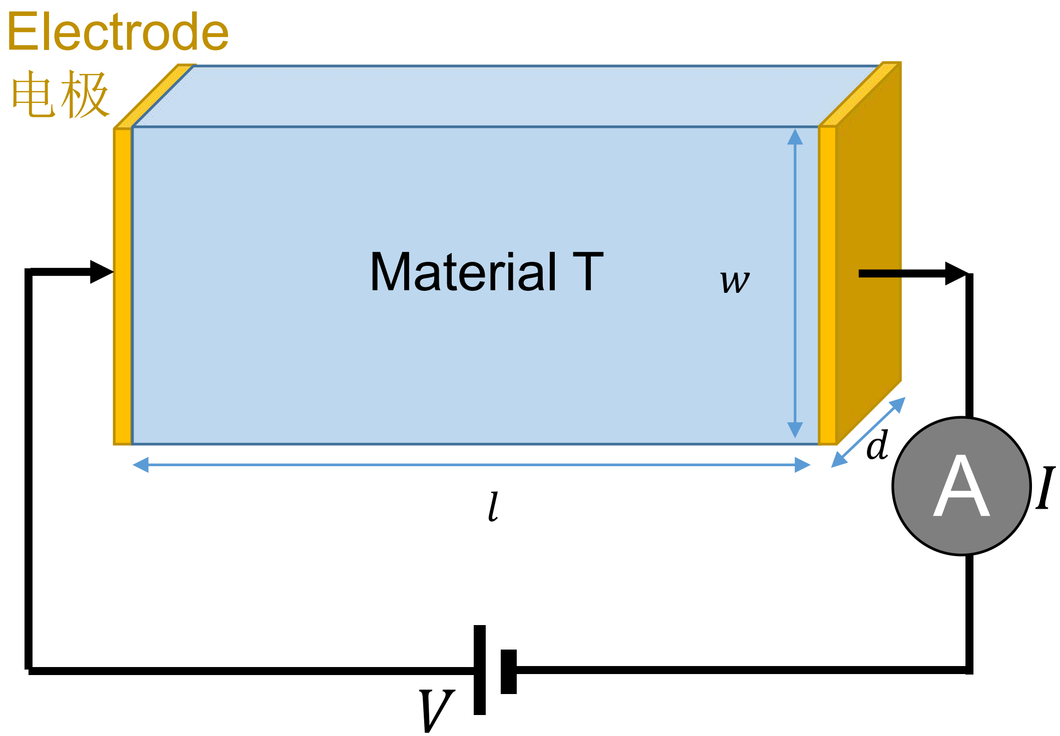
Where $\rho_{\mathrm{matrix}} = \rho_{\mathrm{T}} + \rho_{\mathrm{R}}$ is the resistivity of the matrix due to scattering from thermal vibrations and from other defects

Table: Nordheim coefficient C (at 20 ^{o}C) for **dilute alloys**

Solute in Solvent (element in matrix)	C (nΩ m)	Maximum Solubility at 25 °C (at.%)
Au in Cu matrix	5500	100
Mn in Cu matrix	2900	24
Ni in Cu matrix	1200	100
Sn in Cu matrix	2900	0.6
Zn in Cu matrix	300	30
Cu in Au matrix	450	100
Mn in Au matrix	2410	25
Ni in Au matrix	790	100
Sn in Au matrix	3360	5
Zn in Au matrix	950	15

2.5 How to measure resistivity?

Two-terminal measurement



Resistance: $R = \dfrac{V}{I}$

Resistivity: $\rho = R\dfrac{wd}{l}$

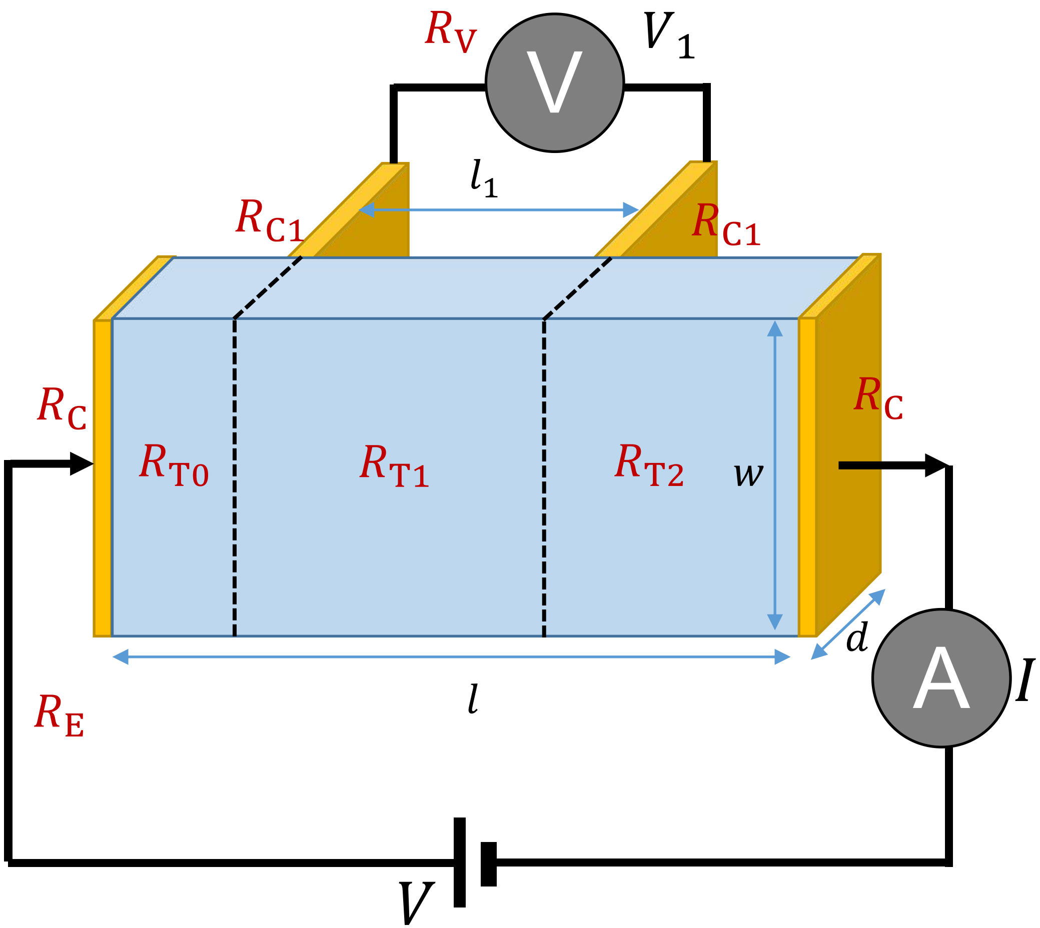
Q: If this measurement is accurate, what condition should be satisfied?

$$R = R_{\mathrm{T}} + 2R_{\mathrm{C}} + R_{\mathrm{E}}$$

R_{C}: Contact resistance between electrode and material

R_{E}: Resistance of electrode and wires

Four-terminal measurement



R_V: Resistance of voltage meter
电压表内阻

$R_V \gg R_E, R_T$ and R_{C1}

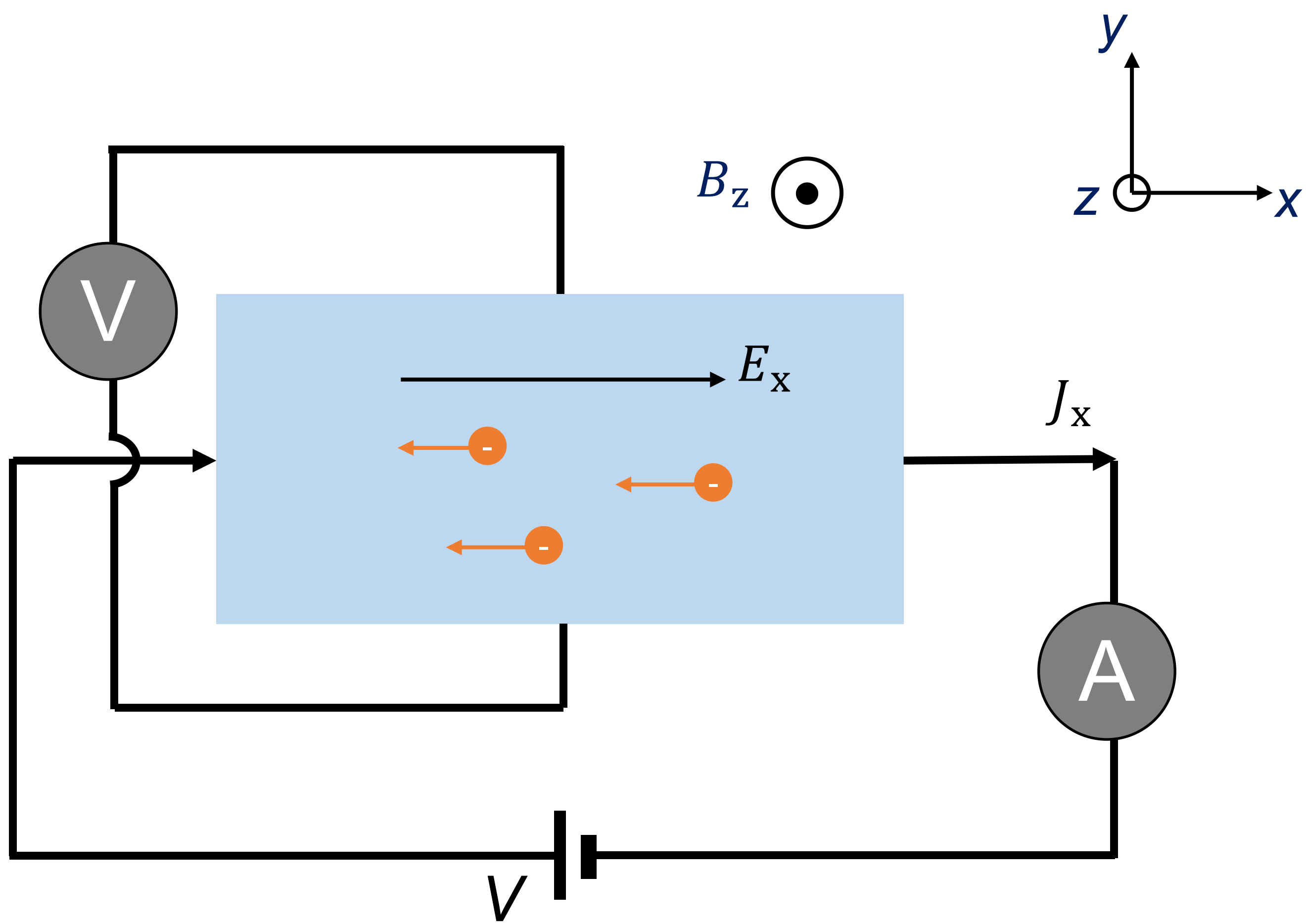
Q: What's the accurate value of R_T?

$$V = I \times (R_T + 2R_C + R_E)$$

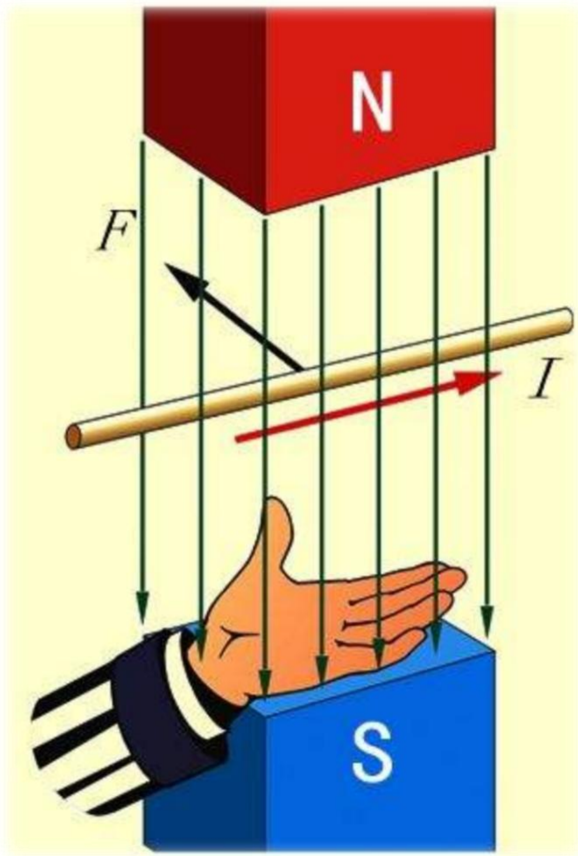
$$V_1 = I \times R_{T1}$$

$$\rho = R_{T1}\frac{wd}{l_1} = \frac{V_1}{I}\frac{wd}{l_1}$$

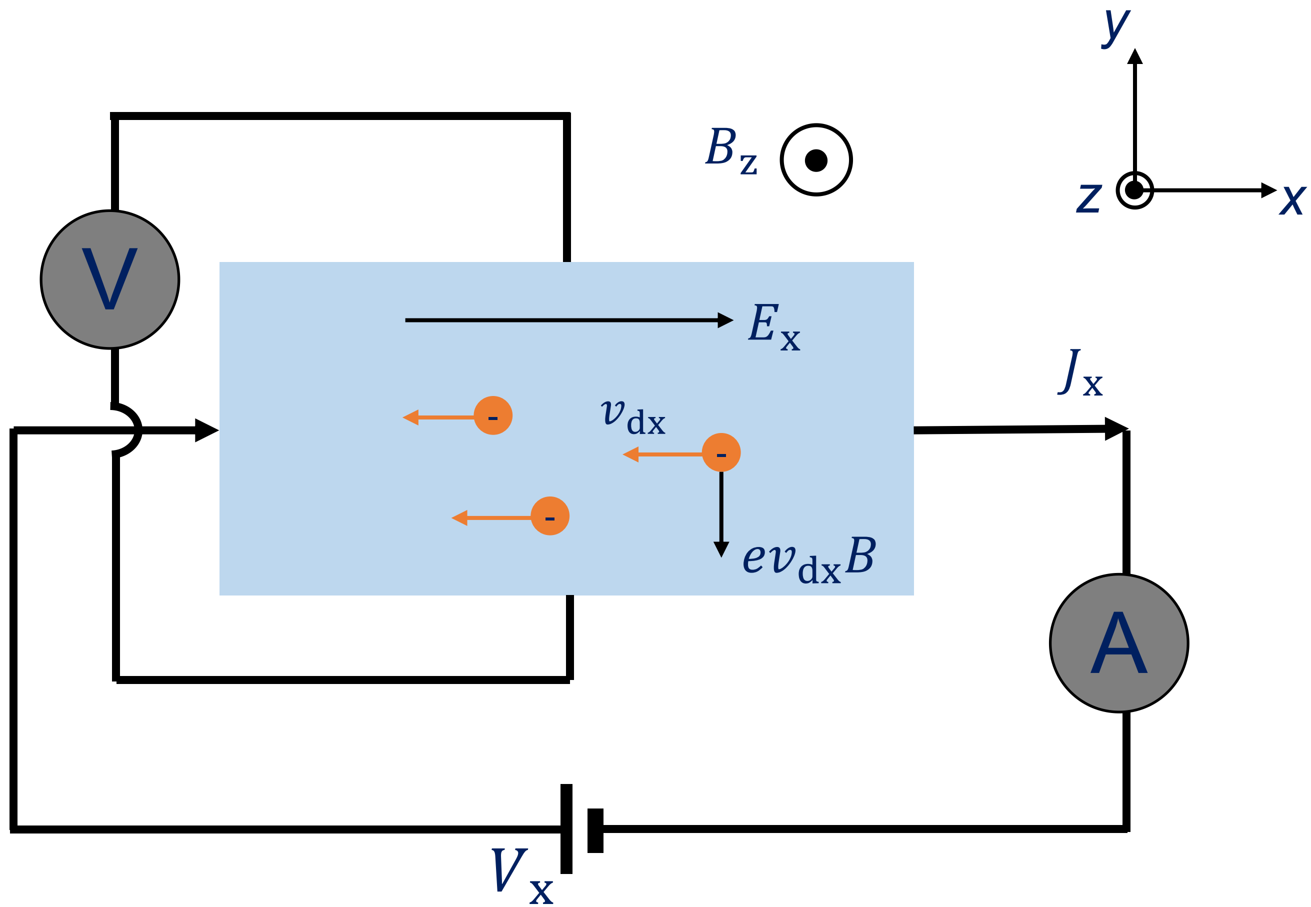
2.6 The Hall effect and Hall devices



Q: The direction of Lorentz force?

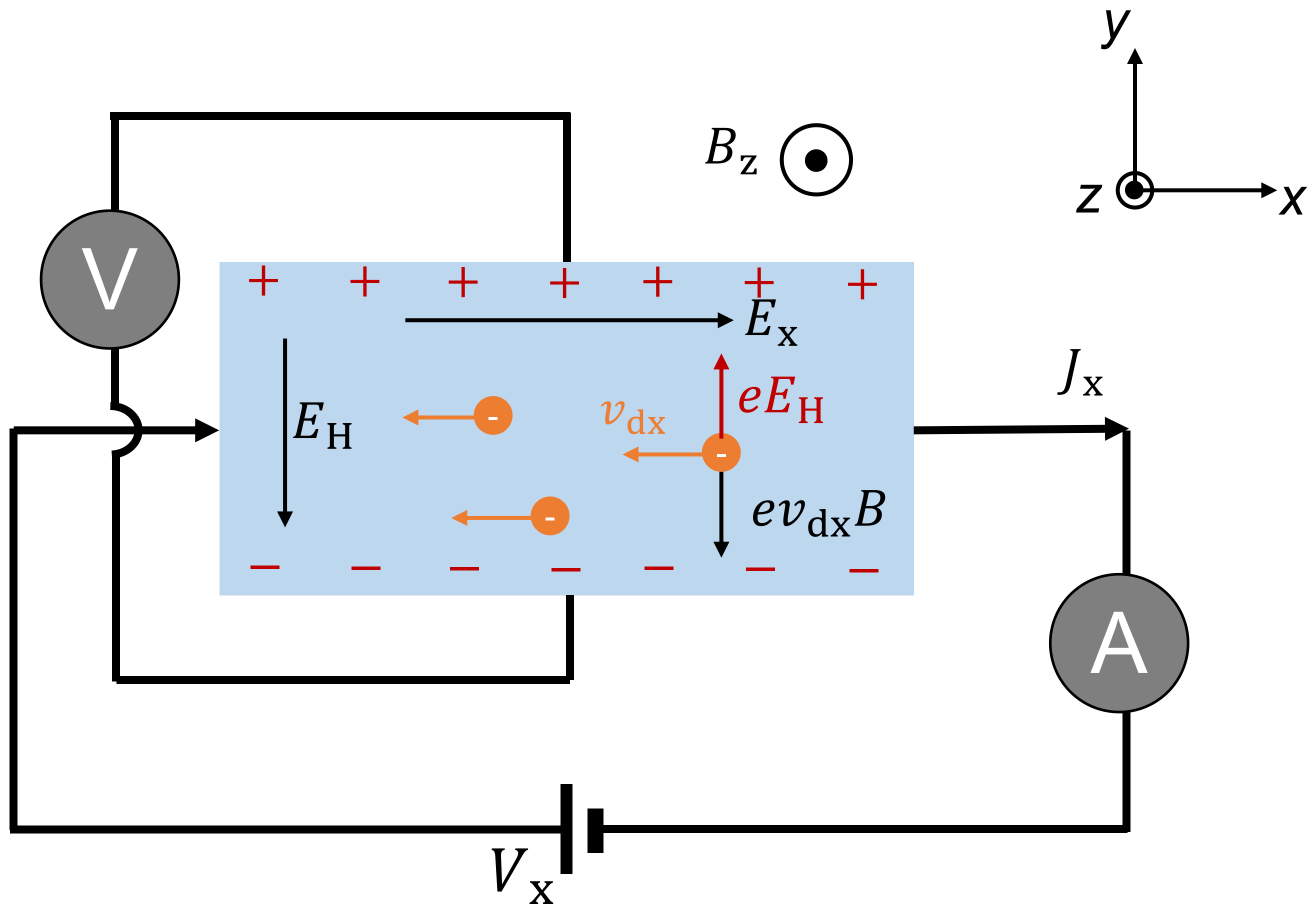


The direction of Lorentz force:
Left-hand rule 左手定则



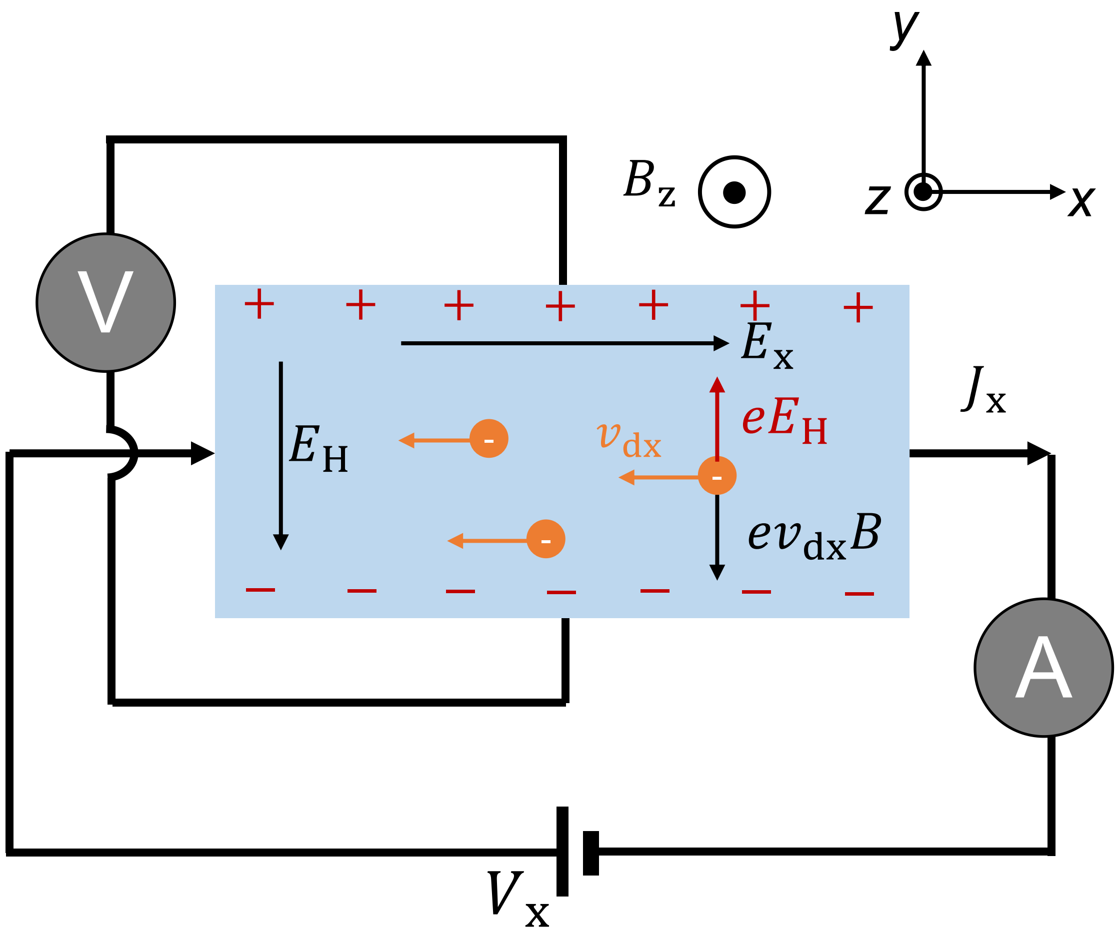
Lorentz force $\vec{F} = q\vec{v} \times \vec{B}$

$$F_y = -ev_{dx}B_z$$



In the steady state: $ev_{dx}B_z = eE_H$

E_H is called the **Hall field**.



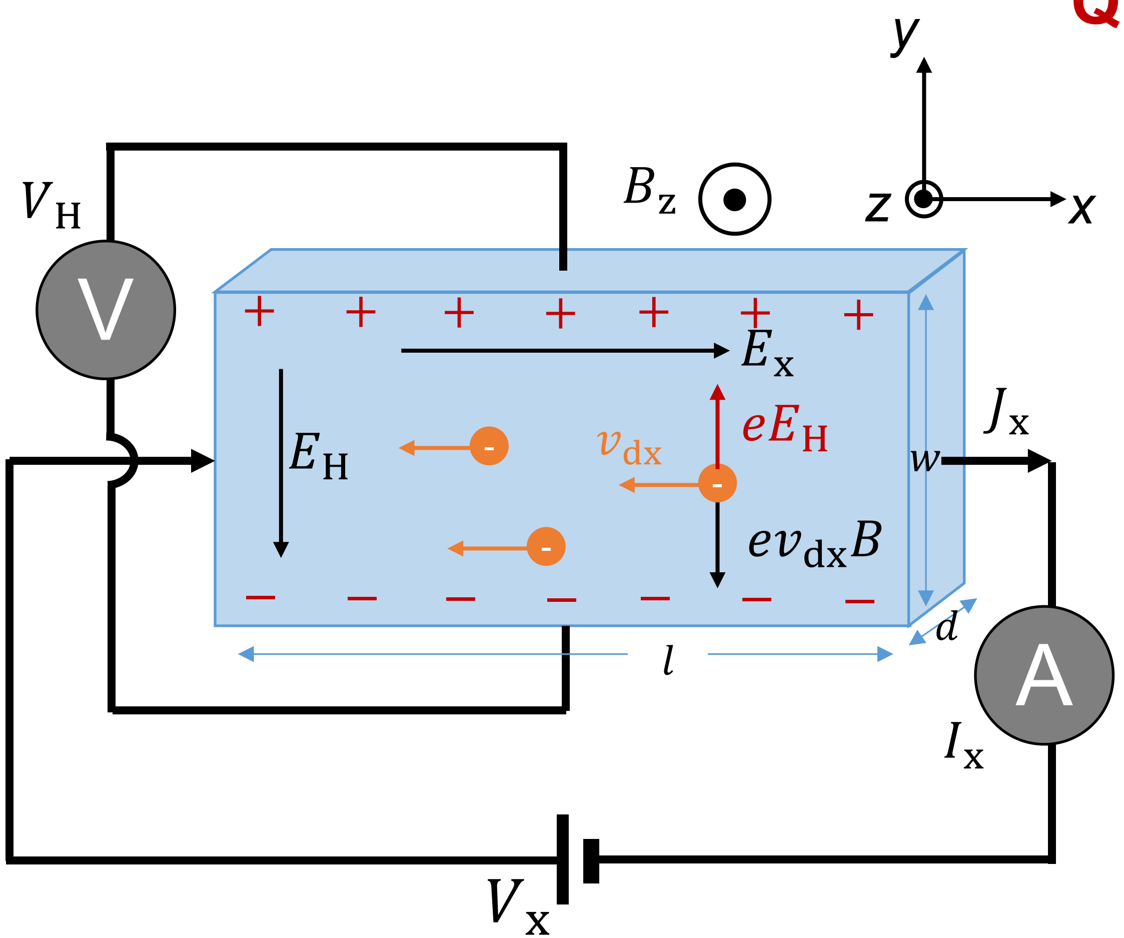
$$\begin{cases} E_H = v_{dx} B_z \\ J_x = env_{dx} \end{cases}$$

$$E_H = \frac{J_x}{en} B_z$$

Introduce **Hall coefficient: R_H**

$$R_H = -\frac{E_H}{J_x B_z} = -\frac{1}{ne}$$

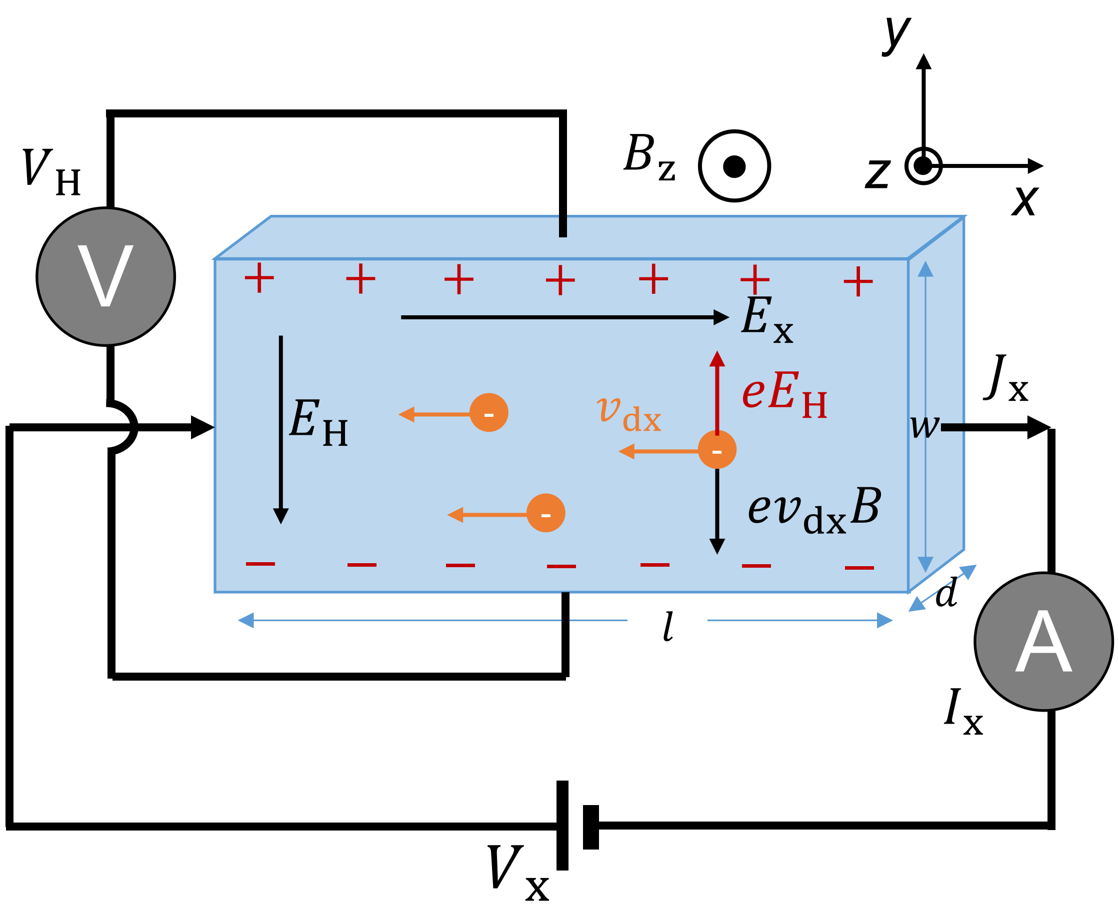
Q: What we can get from Hall measurement?



$$\begin{cases} E_H = \frac{J_x}{en} B_z \\ E_H = \frac{V_H}{w} \\ J_x = \frac{I_x}{wd} \end{cases}$$

$$n = \frac{I_x B_z}{V_H d e}$$

$$R_H = -\frac{V_H d}{I_x B_z}$$



$$\mu_d = \frac{\sigma}{ne} =- R_H\sigma$$

The product σ of and $-R_H$ is called the **Hall mobility**.

$$\sigma = \frac{J_x}{E_x} = \frac{I_x/wd}{V_x/l} = \frac{I_x l}{V_x wd}$$

$$\mu_d = \frac{V_H l}{V_x B_z w}$$

$$\tau = \frac{m_e \mu_d}{e}$$

Hall coefficient and Hall mobility of selected metals at room temperature

Metal	Valency	R_H ($m^3\ A^{-1}\ s^{-1}$) (Experiment) $\times 10^{-11}$	R_H ($m^3\ A^{-1}\ s^{-1}$) (Theory) $\times 10^{-11}$	$\mu_H = \lvert\sigma R_H\rvert$ ($cm^2\ V^{-1}\ s^{-1}$)
Na	1	−24.8	−24.6	50.8
K	1	−42.8	−47.0	57.9
Ag	1	−9.0	−10.7	53.9
Cu	1	−5.4	−7.4	31.6
Au	1	−7.2	−10.6	31.9
Mg	2	−8.3	−7.2	18.5
Al	3	−3.4	−3.5	12.6

Hall measurement is an effective method to obtain mobility, carrier density and mean scattering time