Electronic Materials and Devices

2 Classical electrical and thermal conductance in solids

QQ Group:

群名称:电子功能材料与器件
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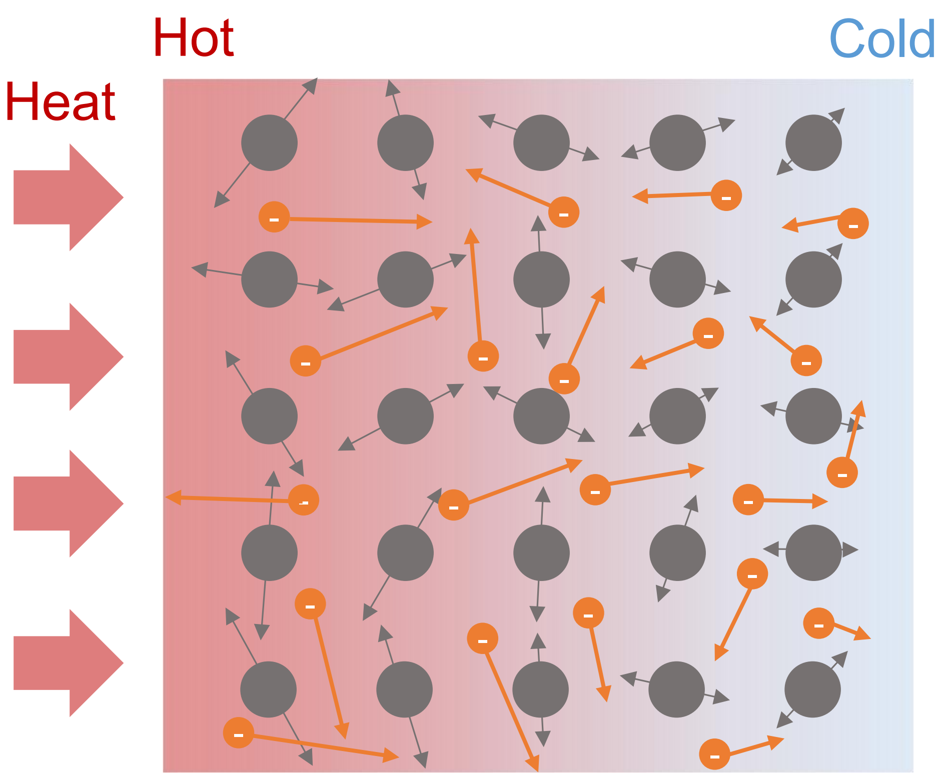
2.6 Thermal conductivity

Metals are both good electrical and good thermal conductors.



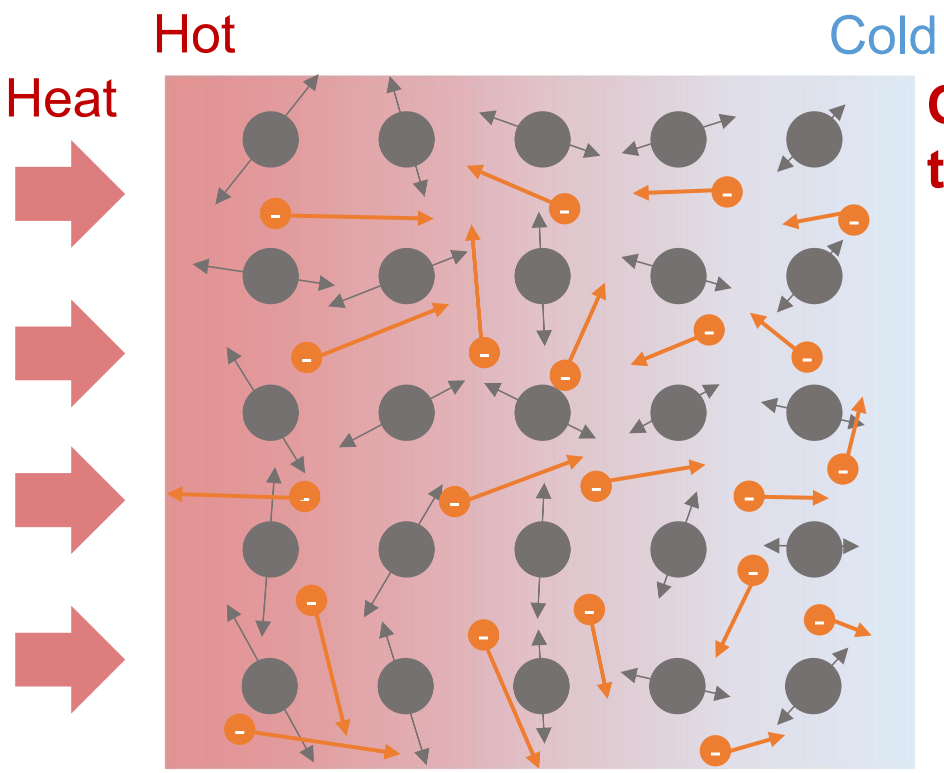
Q1: How thermal energy transfer from hot to cold end?

From lattice vibrations

From collisions between electrons and lattice

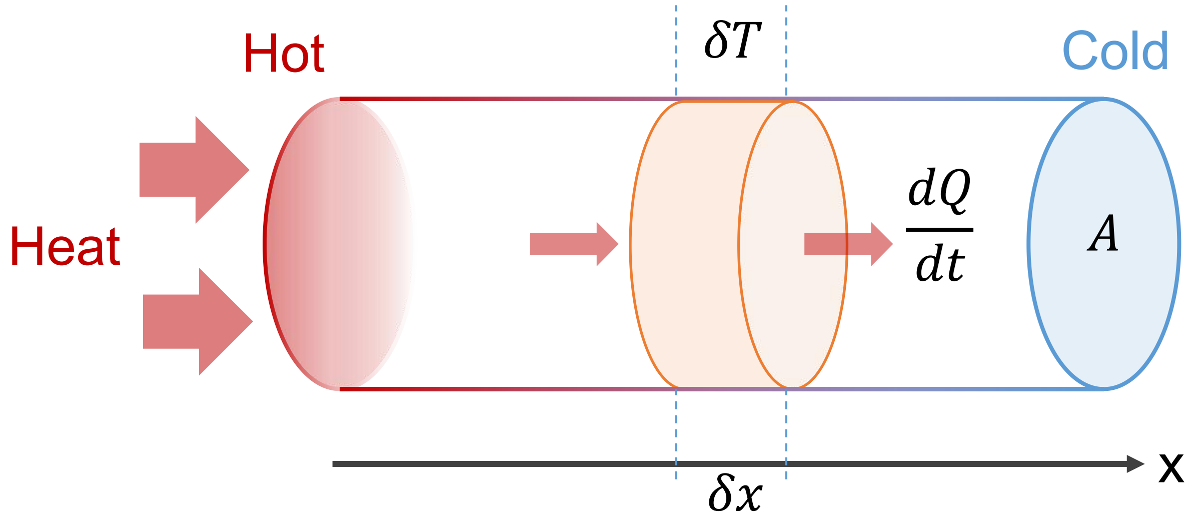
Electrons act as energy carriers.

In metals, energy mainly transfer through collisions between electrons and lattice.



Q2: The temperature of lattice = the temperature of electrons?

Q3: The temperature of an object is the temperature of lattice or electrons?

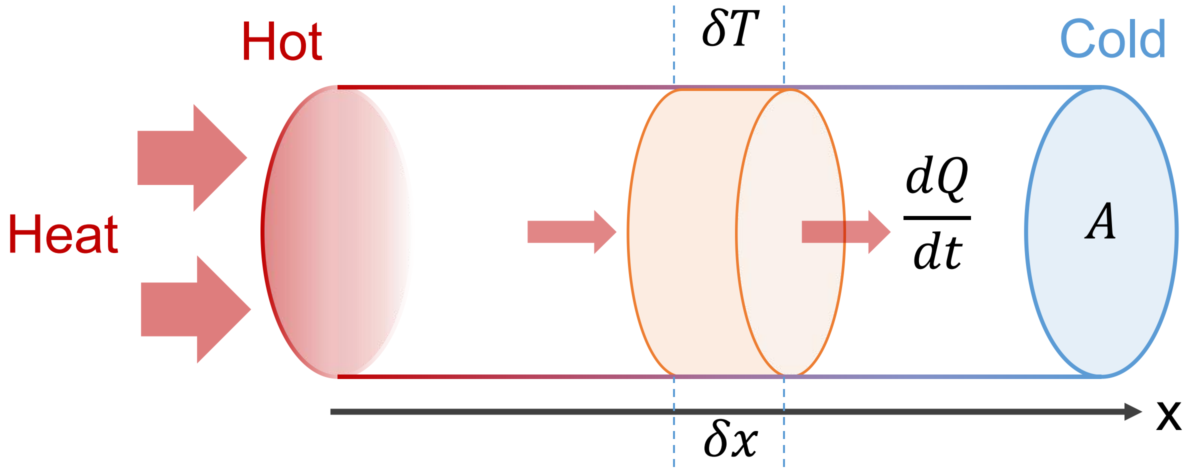


Fourier's law: $Q' = -A\kappa\frac{\delta T}{\delta x}$

Q: heat flow (Joule) $\quad Q' = \frac{dQ}{dt}$: the rate of heat flow (Watt)

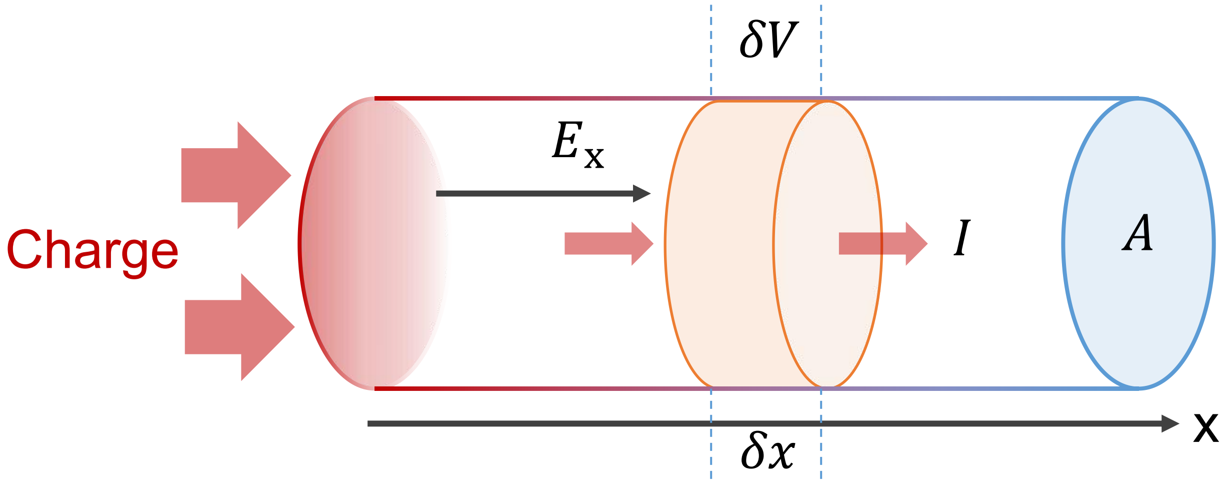
$\frac{\delta T}{\delta x}$: the temperature gradient (K/m)

κ: thermal conductivity of materials (W K^{-1} m^{-1})



Fourier's law: $Q' = -A\kappa \dfrac{\delta T}{\delta x}$

$\boldsymbol{\kappa}$:Thermal conductivity



$$I = AJ_{\mathrm{x}} = A\sigma E_{\mathrm{x}} = -A\sigma \frac{\delta V}{\delta x}$$

$\boldsymbol{\sigma}$:electrical conductivity

$$Q' = -A\kappa\frac{\delta T}{\delta x}$$

κ:Thermal conductivity

$$I = -A\sigma\frac{\delta V}{\delta x}$$

σ:electrical conductivity

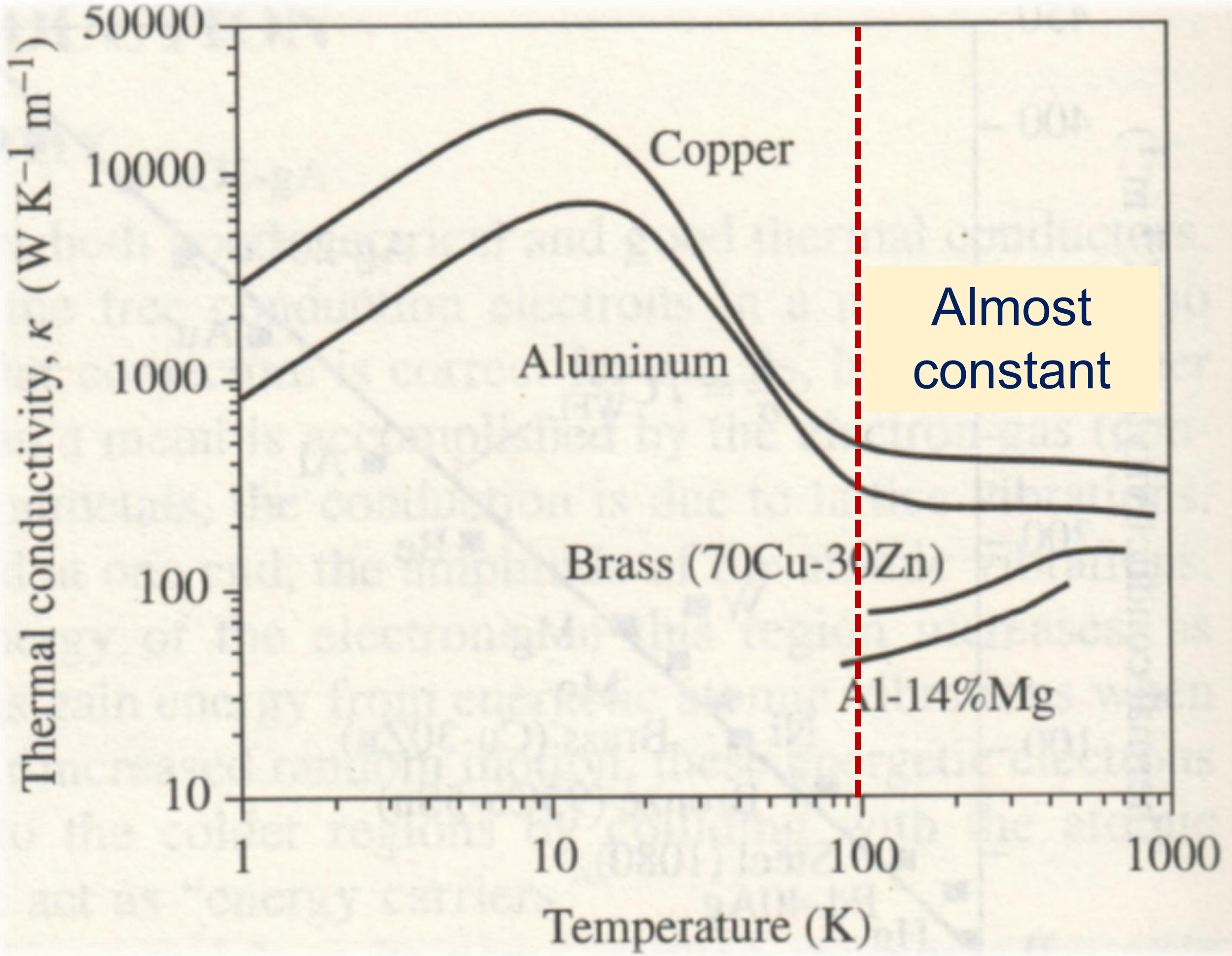
Only for metals

Wiedemann-Franz-Lorentz law: $\frac{\kappa}{\sigma T} = C_{\mathrm{WFL}}$

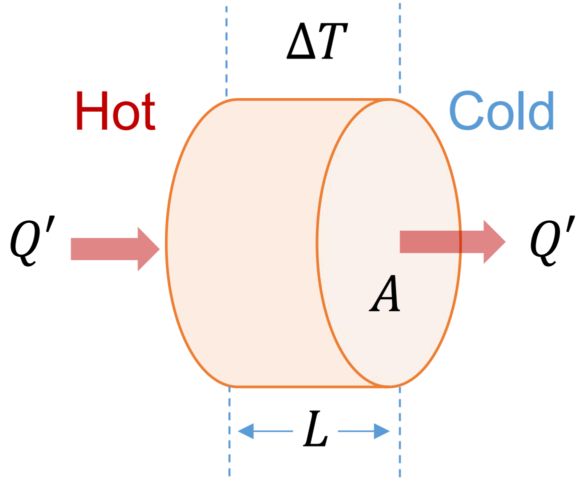
Lorentz number/WFL coefficient: $C_{\mathrm{WFL}} = \frac{\pi^2 k^2}{3e^2} = 2.44 \times 10^{-8}\ \mathrm{W\Omega K^{-2}}$

Q4: What's the temperature-dependence of κ in ideal pure metal?

$$\kappa = C_{\mathrm{WFL}}\sigma T = \frac{C_{\mathrm{WFL}}T}{\rho}$$



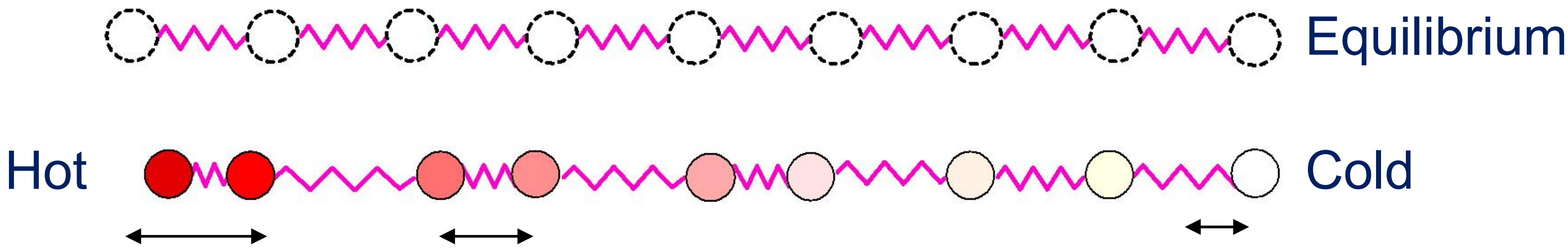
Thermal resistance



$$Q' = A\kappa\frac{\Delta T}{L} = \frac{\Delta T}{L/A\kappa} = \frac{\Delta T}{\theta}$$

Thermal resistance: $\theta = \dfrac{L}{\kappa A}$

Thermal conductivity in nonmetals



In nonmetals, energy mainly transfer through lattice vibrations.

Hence, the efficiency of heat transfer depends on the coupling strength of atoms. The stronger the coupling, the greater will be the thermal conductivity.

Metal	κ at 298K ($\mathrm{W\ m^{-1}\ K^{-1}}$)
Fe	80
Al	250
Cu	390
Ag	420

Metal alloys	κ at 298K ($\mathrm{W\ m^{-1}\ K^{-1}}$)
Stainless steel	12-16
1080 steel	50
Bronze (95% Cu-5% Sn)	80
Brass (63% Cu-37% Sn)	125

Ceramics and glasses	κ at 298K ($\mathrm{W\ m^{-1}\ K^{-1}}$)
Silica-fused (SiO_2) 石英	1.5
Alumina (Al_2O_3)	30
Sapphire (Al_2O_3)	37
Diamond	~1000

Polymers	κ at 298K ($\mathrm{W\ m^{-1}\ K^{-1}}$)
PVC	0.17
Polycarbonate	0.22
Nylon 6,6	0.24
Teflon	0.25

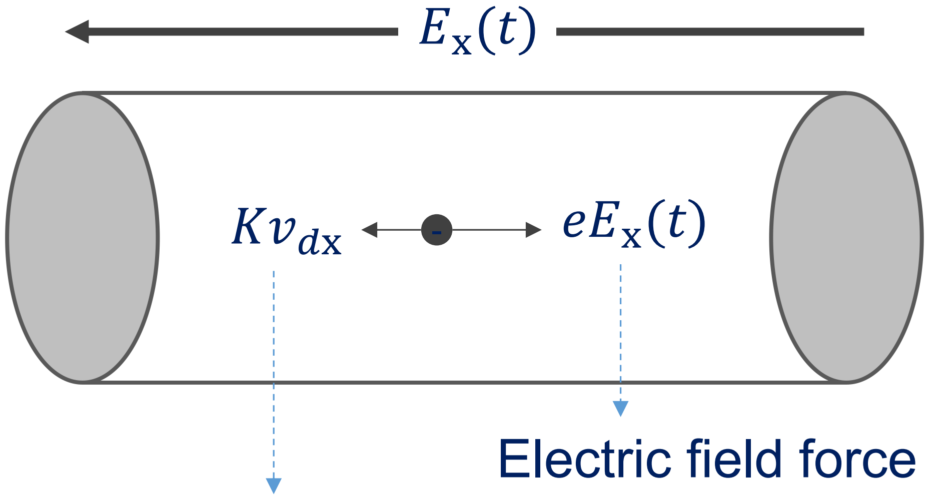
2.7 AC electrical conductivity

DC conductivity: steady state motion of electrons.

$$v_{\mathrm{dx}} = \frac{e\tau}{m_{\mathrm{e}}} E_{\mathrm{x}} = \mu_{\mathrm{d}} E_{\mathrm{x}} \qquad \sigma = en\mu_{\mathrm{d}}$$

AC conductivity: dynamic state motion of electrons.

A model of dynamic motion of electrons



An equivalent force: describing the collisions with and deflections from the metal ions

The general equation of electron motion:

$$eE_x - Kv_{dx} = m_e \frac{dv_{dx}}{dt}$$

The general equation of electron motion:

$$eE_{\mathrm{x}} - Kv_{d\mathrm{x}} = m_{\mathrm{e}}\frac{\mathrm{d}v_{\mathrm{dx}}}{\mathrm{d}t}$$

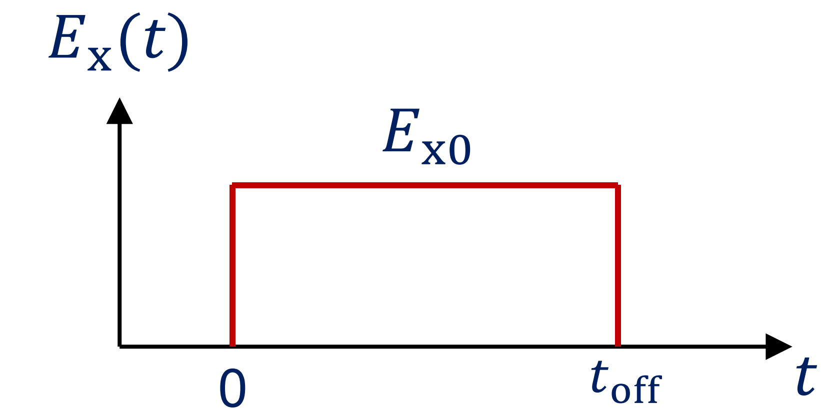
What's the value of K?

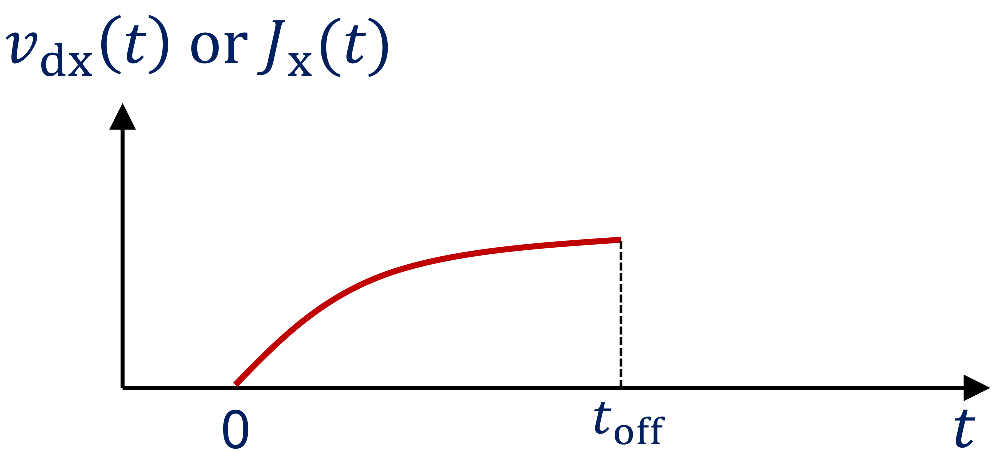
Steady case: $\frac{\mathrm{d}v_{\mathrm{dx}}}{\mathrm{d}t} = 0$ $\quad v_{\mathrm{dx}} = \frac{e\tau}{m_{\mathrm{e}}}E_{\mathrm{x}}$

$$K = \frac{m_{\mathrm{e}}}{\tau}$$

$$eE_{\mathrm{x}} - \frac{m_{\mathrm{e}}}{\tau}v_{\mathrm{dx}} = m_{\mathrm{e}}\frac{\mathrm{d}v_{\mathrm{dx}}}{\mathrm{d}t}$$

Transient behavior 瞬时行为





Solve the equation:

$$eE_x - \frac{m_e}{\tau} v_{dx} = m_e \frac{dv_{dx}}{dt}$$

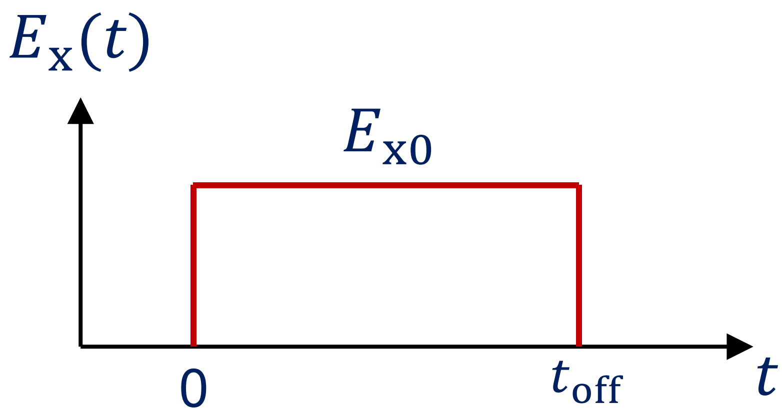
(1) $0 \leq t < t_{off}$

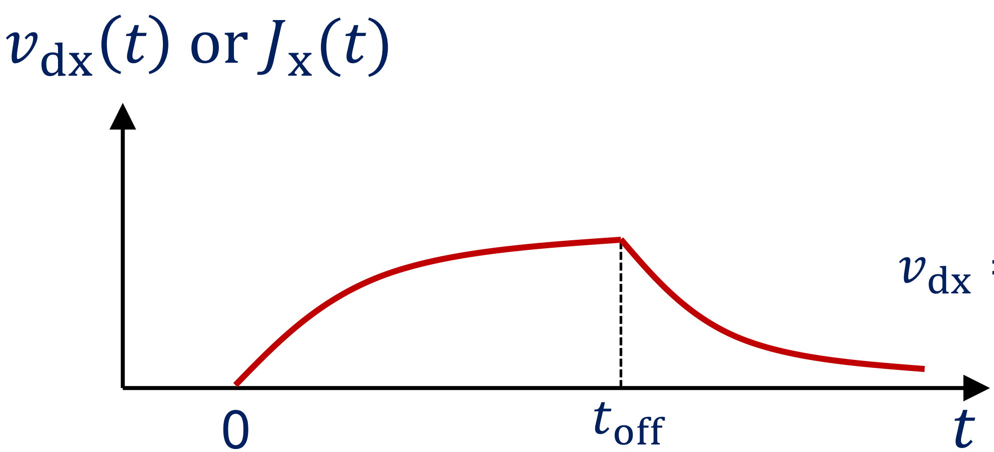
$$eE_{x0} - \frac{m_e}{\tau} v_{dx} = m_e \frac{dv_{dx}}{dt}$$

$$v_{dx} = \frac{e\tau E_{x0}}{m_e}[1 - \exp(-\frac{t}{\tau})]$$

$$v_{dx}(t_{off}) = \frac{e\tau E_{x0}}{m_e}[1 - \exp(-\frac{t_{off}}{\tau})]$$

Transient behavior 瞬时行为





(2) $t_{off} \leq t$

$$-\frac{m_e}{\tau} v_{dx} = m_e \frac{dv_{dx}}{dt}$$

$$\Downarrow$$

$$v_{dx} = \frac{e\tau E_{x0}}{m_e}\left[\exp\left(\frac{t_{off} - t}{\tau}\right) - \exp\left(-\frac{t}{\tau}\right)\right]$$

AC conductivity

$$E_\mathrm{x} = E_\mathrm{x0}\exp(\mathrm{j}\omega t)$$

Solve the equation: $eE_\mathrm{x0}\exp(\mathrm{j}\omega t) - \dfrac{m_\mathrm{e}}{\tau}v_\mathrm{dx} = m_\mathrm{e}\dfrac{\mathrm{d}v_\mathrm{dx}}{\mathrm{d}t}$

Let: $v_\mathrm{dx} = v_\mathrm{dx0}\exp(\mathrm{j}\omega t)$

$$\Downarrow$$

$$v_\mathrm{dx} = \frac{e\tau E_\mathrm{x0}}{m_\mathrm{e}(1+\mathrm{j}\omega\tau)}\exp(\mathrm{j\omega t})$$

$$\Downarrow$$

$$\sigma = \frac{J_\mathrm{x}}{E_\mathrm{x}} = \frac{env_\mathrm{dx}}{E_\mathrm{x}}$$

AC conductivity

$$\sigma_{\mathrm{ac}} = \frac{e^2 n\tau}{m_{\mathrm{e}}(1 + \mathrm{j}\omega\tau)}$$

$$\sigma_{\mathrm{ac}} = \frac{\sigma_{\mathrm{dc}}}{(1 + \mathrm{j}\omega\tau)}$$

$\sigma_{\mathrm{dc}} \rightarrow$ DC conductivity

$$\sigma_{\mathrm{ac}} = \sigma' - \mathrm{j}\sigma''$$

$$\sigma' = \frac{\sigma_{\mathrm{dc}}}{1 + \omega^2\tau^2}, \qquad \sigma'' = \frac{\sigma_{\mathrm{dc}}\omega\tau}{1 + \omega^2\tau^2}.$$

AC conductivity

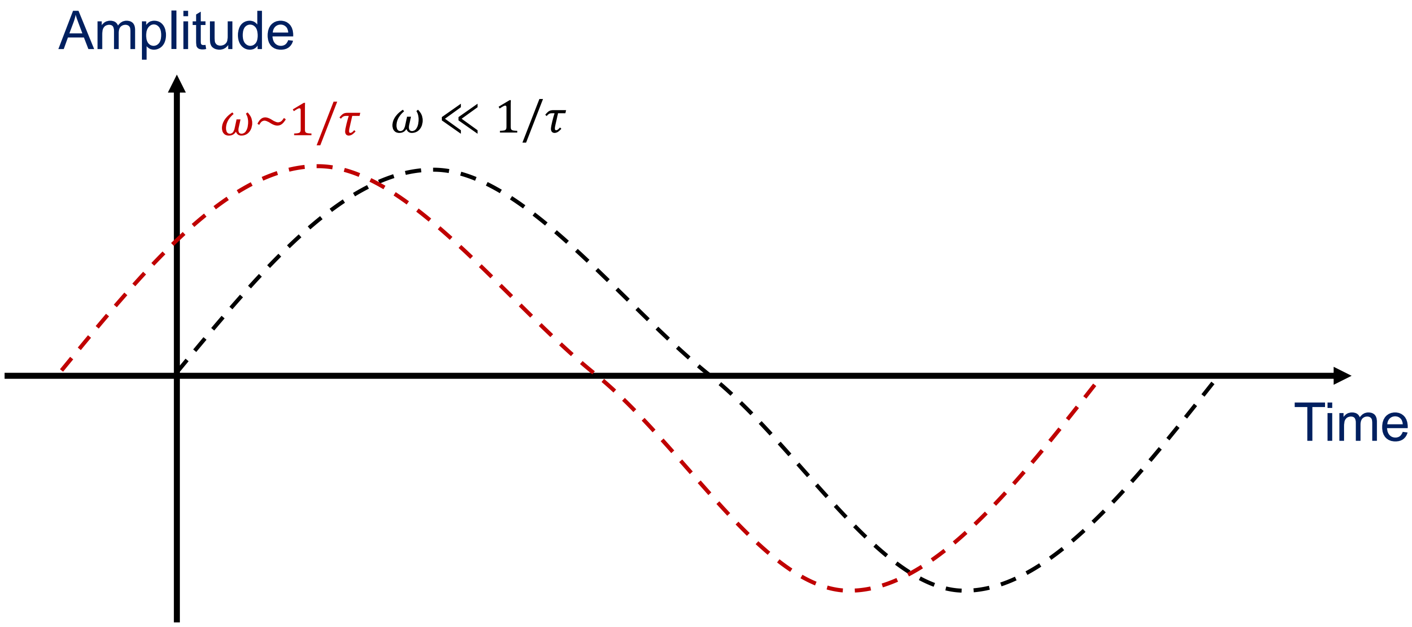
$$\sigma_{\mathrm{ac}} = \sigma' - \mathrm{j}\sigma''$$

$$\sigma' = \frac{\sigma_{\mathrm{dc}}}{1+\omega^2\tau^2}, \qquad \sigma'' = \frac{\sigma_{\mathrm{dc}}\omega\tau}{1+\omega^2\tau^2}.$$

Determine the Joule loss: energy dissipation per unit volume associated with I^2R $\quad \frac{1}{2}\sigma' {E_{\mathrm{x0}}}^2$

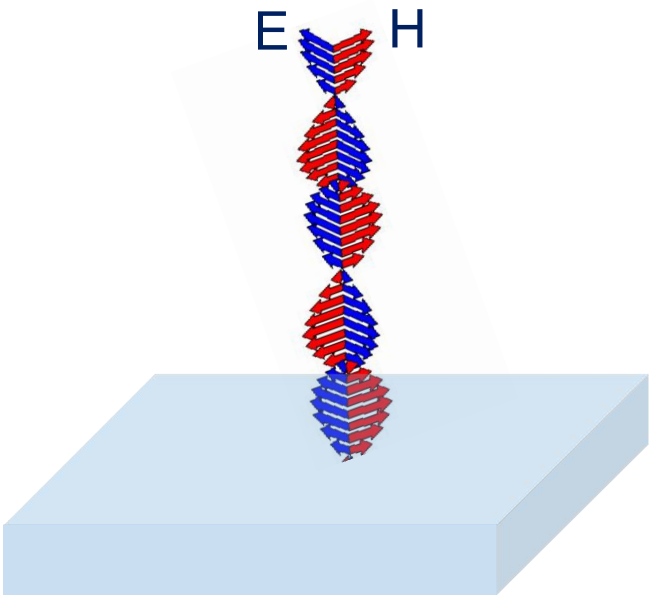
σ' is also related to light absorptions in some materials.

Phase lag



Electromagnetic wave propagate in conducting materials

An electromagnetic field (including light) is shining to a metal. Its angular frequency is ω



Maxwell's Equations ($\rho = 0$)

$$\begin{cases} \nabla \cdot \boldsymbol{E} & = 0 \\ \nabla \cdot \boldsymbol{H} & = 0 \\ \nabla \times \boldsymbol{E} & = -\frac{1}{c}\frac{\partial \boldsymbol{H}}{\partial t} \\ \nabla \times \boldsymbol{H} & = \frac{4\pi}{c}\boldsymbol{j} + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} \end{cases}$$

The wave-equation is obtained:

$$-\nabla^2 \boldsymbol{E} = \frac{\omega^2}{c^2}\varepsilon(\omega)\boldsymbol{E}$$

Complex dielectric constant:

$$\varepsilon(\omega) = 1 + \frac{4\pi i\sigma}{\omega}$$

When $\omega\tau \gg 1$:

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\omega_p^2 = \frac{4\pi n e^2}{m}$$

When $\omega > \omega_{\mathrm{p}}$:

- Wave propagation in metal will occur
- Metals become transparent

Below which the alkali metals become transparent

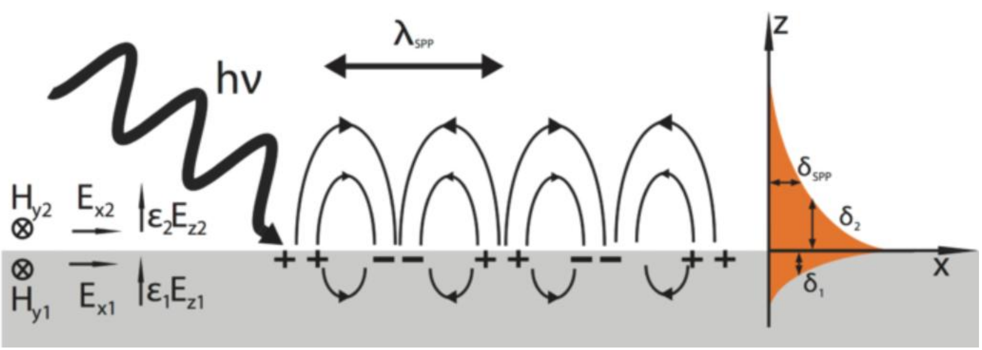
Alkali Metals	Theoretical λ (nm)	Experimental λ (nm)
Li	150	200
Na	200	210
K	280	310
Rb	310	360
Cs	350	440

Charge density oscillation (Plasmon)

- Solutions of the type $\rho(\boldsymbol{r},t)=\rho(\boldsymbol{r},\omega)e^{-i\omega t}$, from
  - charge conservation$(\frac{dq}{dt}+\int_S \boldsymbol{j}\cdot\hat{\boldsymbol{n}}dS=0)$
  - Gauss Theorem

$$\begin{cases}\nabla\cdot\boldsymbol{j} & =-\frac{\partial\rho}{\partial t}\\ \nabla\cdot\boldsymbol{E} & =4\pi\rho\end{cases}$$

- Non trivial solutions for $1+\frac{i4\pi\sigma(\omega)}{\omega}=0\Longrightarrow\omega=\omega_p$



Plasmon 等离子体激元

Occur at metal surface

Absorb significant amount of light

Practice

The mean free time in copper is $2.5 \times 10^{-14}\ s$ and the room temperature conductivity is $5.9 \times 10^{5}\ \Omega^{-1}\mathrm{cm}^{-1}$. What is the change in the conductivity of copper from dc to 10 GHz to 1 THz to 100 THz?