Electronic Materials and Devices

3 Modern theory of solids

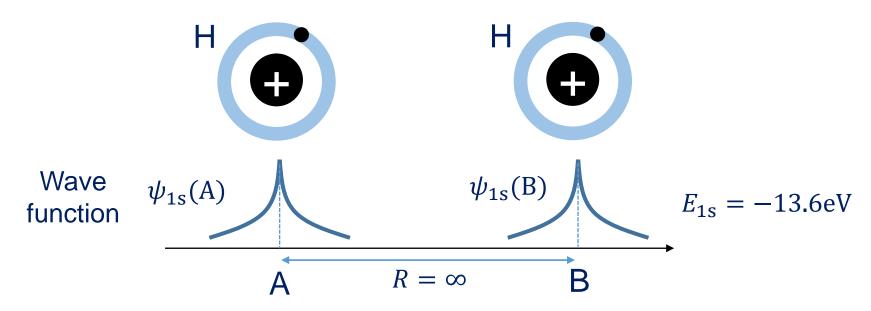
QQ Group:



陈晓龙 Chen, Xiaolong 电子与电气工程系

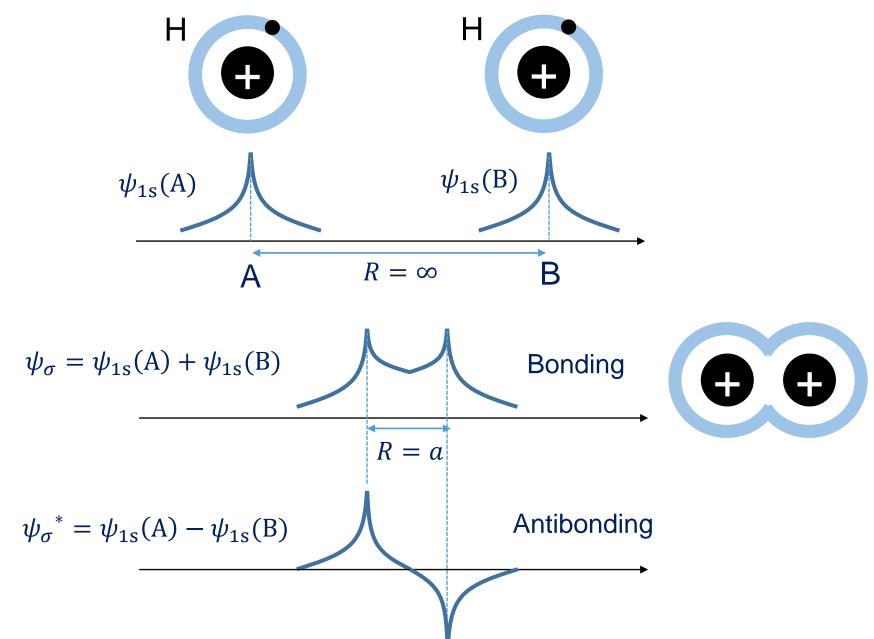
3.7 Hydrogen molecule: molecular orbital theory of bonding

分子轨道成键理论

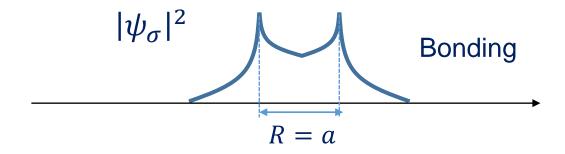


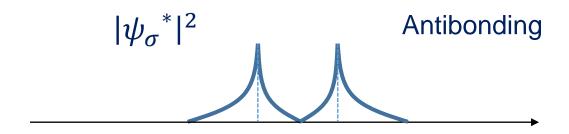
Q: What happens when two hydrogen atoms are brought together?

Linear combination of atomic orbitals 原子轨道线性组合

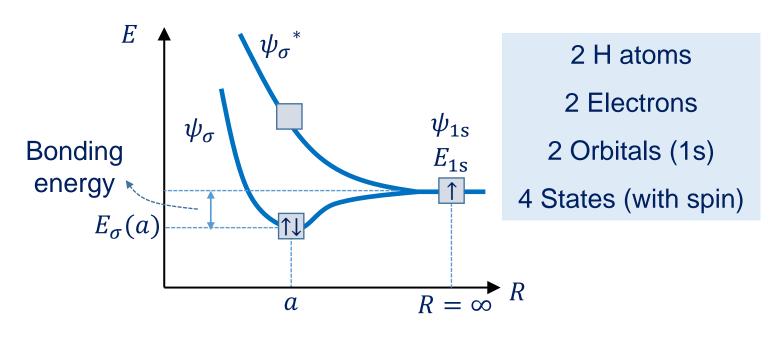


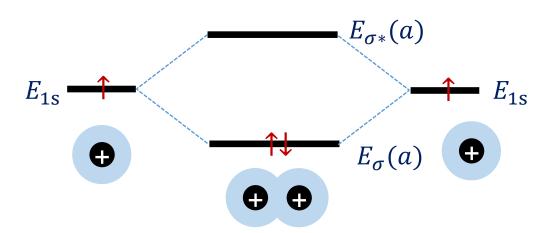
Electron probability distributions for bonding and antibonding orbitals



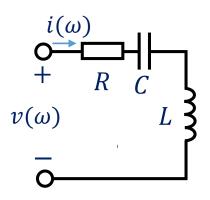


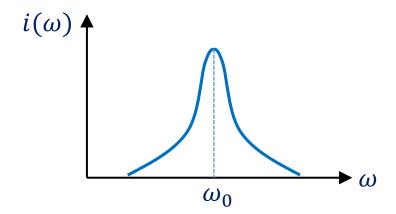
Energy for bonding and antibonding orbitals

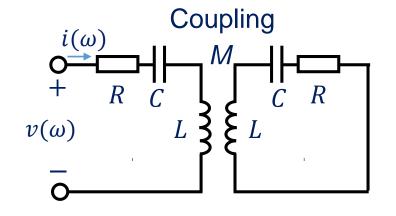


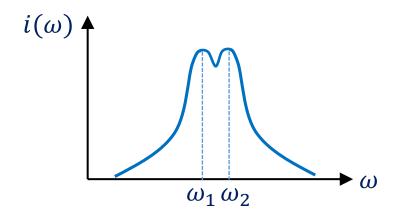


Analogy to RLC resonant circuit





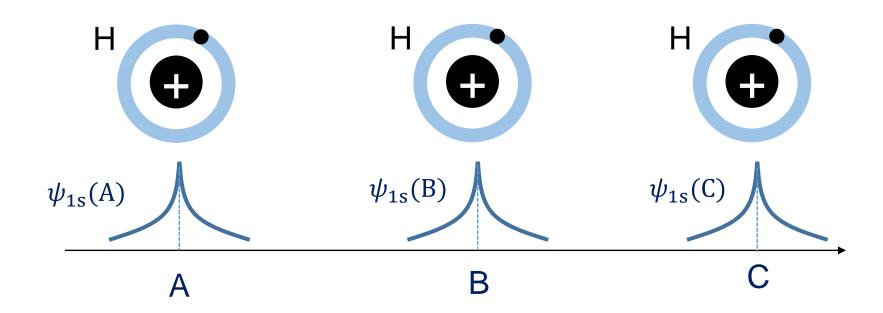




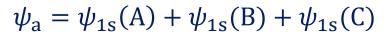
Q: What happens to 2s and 2p states?

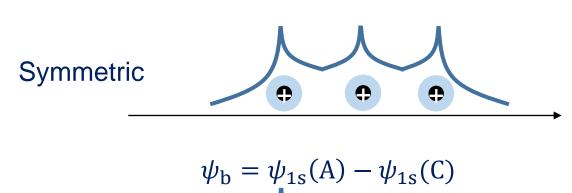
3.8 Band theory of solids 固体能带理论

Q: What happens when 3 H atoms are brought together?

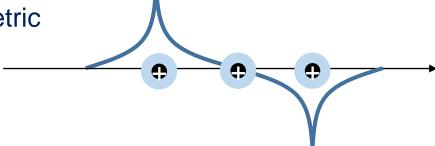


The wavefunction must be symmetric or antisymmetric!

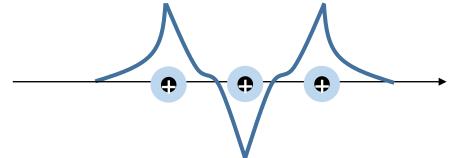




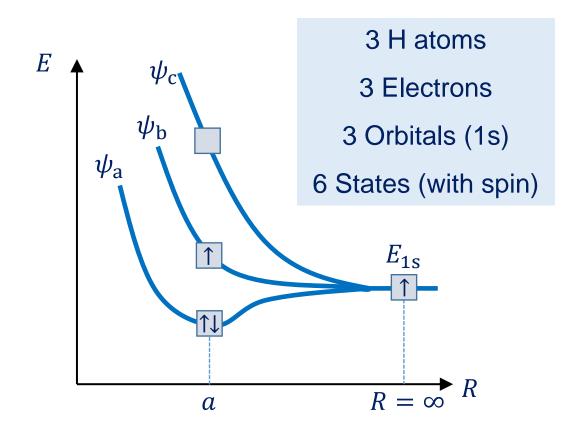
Antisymmetric



Symmetric $\psi_c = \psi_{1s}(A) - \psi_{1s}(B) + \psi_{1s}(C)$



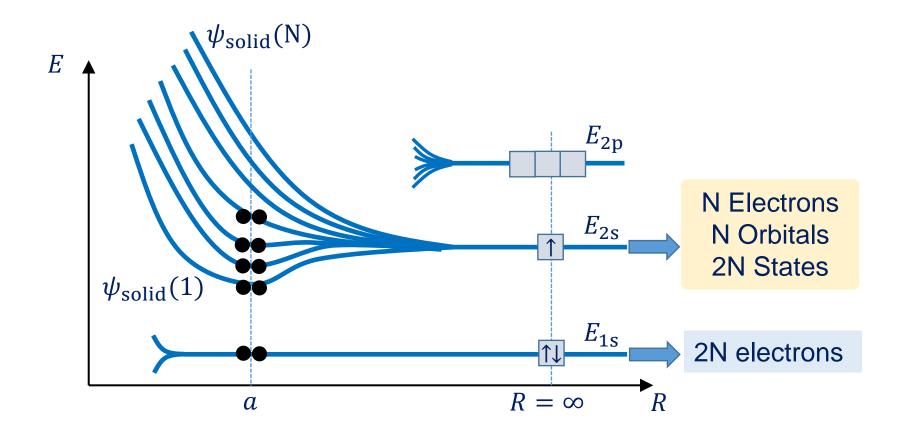
Energy for bonding and antibonding orbitals



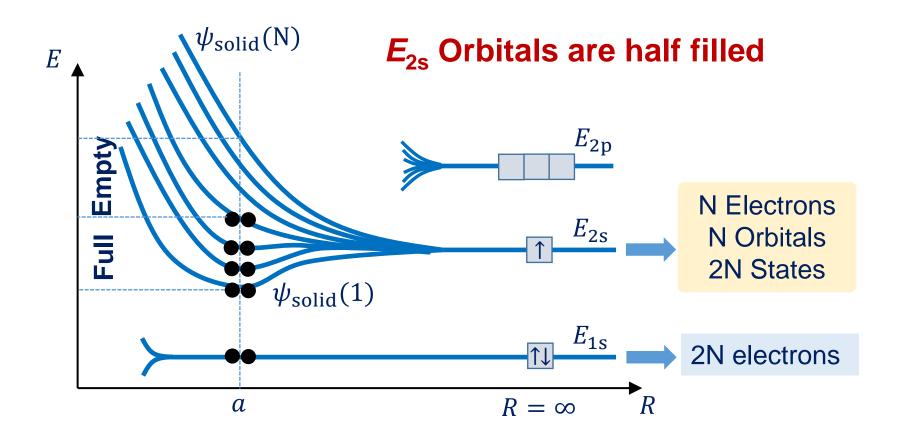
N atoms: Li solid systems

Electronic configuration of Li atoms: 1s²2s¹

The K shell (1s) are fully filled and the splitting of E_{1s} can be neglected.

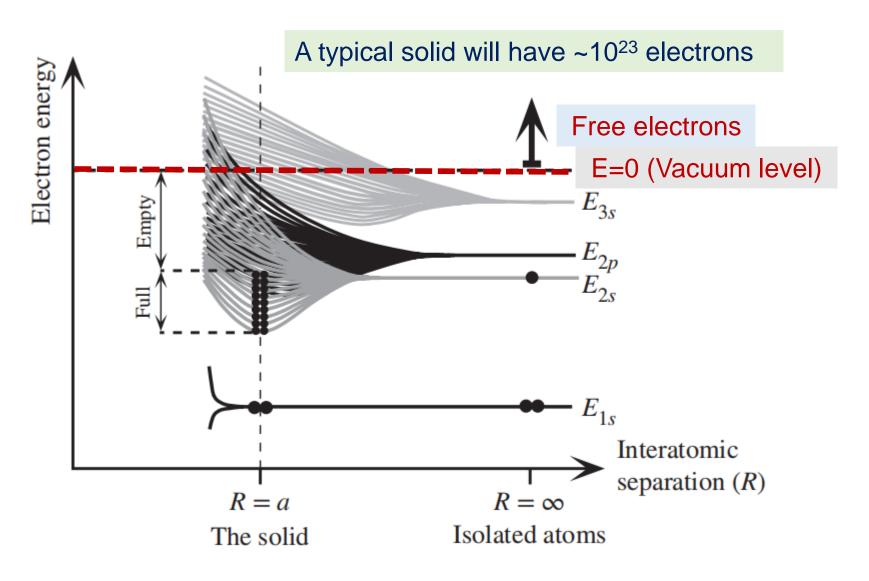


N atoms Li solid systems

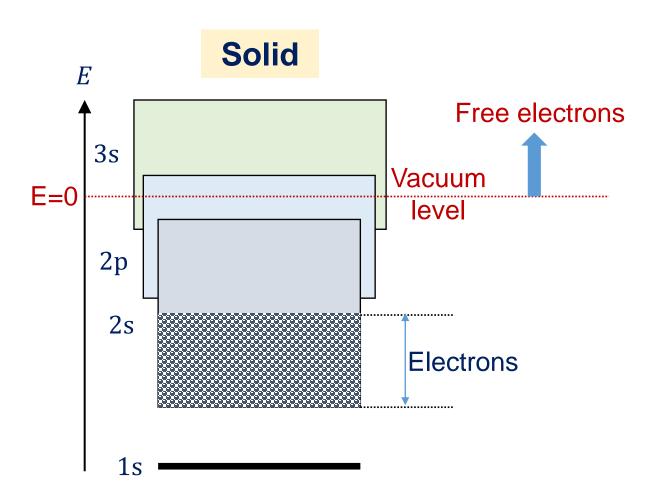


The single 2s energy level splits into N finely separated energy levels, forming an **energy band 能带**.

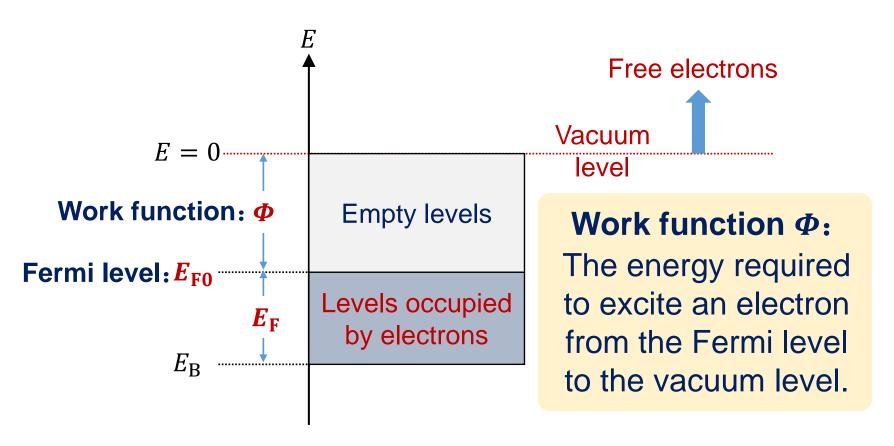
The overlap of energy bands give a single energy band that is only partially full of electrons.



Overlapping energy bands



The overlap of energy bands give a single energy band that is only partially full of electrons.



This is called energy band diagram 能带示意图

The work function of metals

Metal Metal								
	Ag	Al	Au	Cs	Cu	Li	Mg	Na
$\boldsymbol{\Phi}(\mathrm{eV})$	4.26	4.28	5.1	2.14	4.65	2.9	3.66	2.75
$E_{\rm F0}({\rm eV})$	5.5	11.7	5.5	1.58	7.0	4.7	7.1	3.2



Enrico Fermi 恩利克·费米 (1901/9/29-1954/11/28)

费米悖论、费米黄金定则、费米子、费米面、费米液体、费米伽马射线 探测器、费米实验室、费米常数、费米—狄拉克方程,托马斯—费米模 型

"被称为能与爱因斯坦比肩的20世纪最后的全才物理学家"

不但因为发现慢中子效应获得了1938年的诺贝尔物理学奖

建成了世界上第一座核反应堆,使人类正式踏入原子能的时代

https://www.sohu.com/a/129259329_224832

第一位华人诺贝尔得主杨振宁和李政道,还曾为"谁才是费米的学生"这个问题大动干戈。

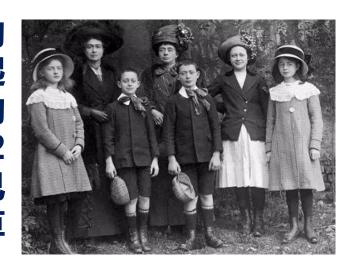
在《李政道传》中多次暗示李政道才是费米的博士生,而杨振宁不是。

而杨振宁之后对此回应说,我确实不是费米的博士研究生,但我才是费米的得意学生,因为在费米教过的学生中,只有我和他联名发表过理论物理论文。

Enrico Fermi 恩利克·费米

1901年,费米出生于意大利的一个铁路工人家中,他是家中最小的孩子。童年的他沉默寡言,看上去总是一副无精打采的样子,显得有些"木讷",被老师认为智力低下。

费米年纪稍大点就一反这种智商低下的偏见,表现其天才的的一面。从10岁起他就喜欢物理和科学,在没有老师教的情况下他就能理解类似x^2+y^2=r^2这种圆的方程式。此外,他还经常自己设计各种发动机模型和绘制飞机引擎草图。



他上完中学进入比萨高等师范学校就读,当时为了申请奖学金,他向学校提交了一篇题为《声音的特殊性能》的论文。论文第一页就写了关于振动弹簧的偏微分方程,之后更是用洋洋洒洒的20页篇幅来阐明了用本征函数所求出的解和特征频率等问题。一个刚离开中学的学生,竟然能研究得如此深入,这使当时的教授都大吃一惊,还特地找到他对他说:"你前途无量啊"。

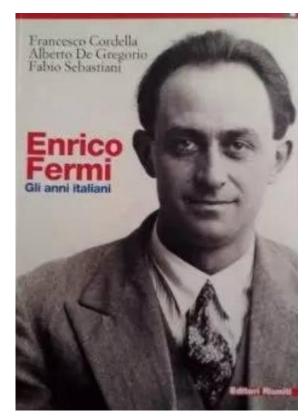
在进入高年级后,他积累的物理知识就比当时很多的教师要多,甚至对那时爱因斯坦的相对论也有独特的见解。那时很多教授不但不用给费米讲课,他们还特地邀请费米来给他们讲爱因斯坦的相对论。



1926年,25岁的费米被任命为罗马大学的物理系教授,并开始了他研究的黄金时代。那时的他已经成为了公认的意大利物理学领袖,罗马大学还为他专门设立了意大利第一个理论物理学讲座。围绕在他身边的更是一群年轻的物理学家,物理学中的"罗马学派"就是在那时建立起来的。

有人说,费米不是理论物理学家中最天才的一个,也不是实验物理学家最厉害的一个。但是,在理论物理学和实验物理学都精通的全才上,他认第二,绝对没有人敢认第一。

无论在理论还是实验方面都如此踏实的费米,用杨振宁的话来形容就是"He has both feet on the ground"。



1925年著名理论物理学家泡利 (他是一位只会理论物理学的物理学家,对实验室的破坏技能完全点满)提出"不相容原理",即原子中不能容纳运动状态完全相同的电子。

在1926年,费米与狄拉克同时提出了服从泡利不相容原理的粒子所遵循的量子统计方程。后来人们把这种粒子服从的规律叫做费米——狄拉克统计,而服从这个规律的粒子也被称为费米子。

1934年,物理学界发生了一件大事件,小居里夫妇用α粒子轰击铝时发现了人工放射现象(小居里夫妇获得1935年的诺贝尔化学奖)。然而这种人工放射现象的"产率"却非常低,平均要用上百万个α粒子轰击铝才得到理想的人工放射现象。

对此,费米有一个绝妙的设想。他想铝原子核与α粒子均带电性,可能就是带电性使其产生了排斥作用,使α粒子对原子核的命中率大大降低。于是他决定试着用不带电性的中子进行轰击实验,看看会不会使"产率"大大提高。

然而那时候罗马大学的实验条件非常有限,根本无法给费米提供实验所需的仪器和设备。这时,费米在实验物理学这方面的才能就派上用场了。他自己制造了盖革计数器等好几种实验仪器,还到各个大学去搜集实验所需的元素材料。他用镭在衰变时发射出的α粒子源来轰击铍元素,获得了轰击原子核的理想"炮弹"中子。得到中子源之后,费米便开始用中子把已知的92种元素从头到尾轰击了个遍。

在这个过程中,他不但发现了许多新的同位素,还发现了著名的慢中子效应。当中子通过大量含氢物质时,与氢原子核碰撞之后,速度会变慢。用越慢的中子轰击原子核,所激发的放射性也就越强,这无疑就是给之后用中子轰击原子核的实验提供了一颗完美的"炮弹"——慢中子。



费米不但运用了薛定谔波动力学方程对此作出了理论解释,他还 给出了一种非常形象生动的形容,他说,中子进入原子核的情形 就像高尔夫球进洞一样,速度慢的高尔夫球会滚进洞内,而速度 快的高尔夫球会直接从洞顶飞过。

因为在中子轰击原子核和发现了慢中子效应的成就,他在1938年被授予了诺贝尔物理奖,同时被誉为"中子物理之父"。而这次的诺贝尔奖给他带来的不单是荣耀和奖金,还给他带来了逃离纳粹掌心的机会。

在费米接到诺贝尔委员会的电话之前,纳粹就在全国范围内发起"水晶之夜"事件,而意大利的统治者法西斯头目墨索里尼也开始到处逮捕犹太人。为了保护身份为犹太人的妻子劳拉,费米在借着到斯德哥尔摩领奖的机会,举家逃亡到了美国纽约。



初到美国,科学界就传来"中子被吸收后有时会引起铀原子裂变"的消息。费米马上意识到一个裂变的铀原子可以释放出更多的中子来引起一系列的链式反应,从而产生无法估量的巨大能量。

那时,科学界都在讨论这种链式反应用于军事的潜在性,但美国政府似乎对此不太感兴趣。然而此时德国却开始禁止捷克铀矿区的出口。因为铀是原子弹最重要的原材料,这使反法西斯同盟国开始意识到,德国可能已经在密谋着原子弹计划了。为此费米曾多次向海军反应,希望他们能针对德国为原子武器制定计划。

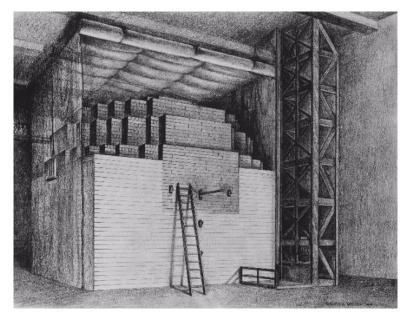
1941年12月,直到日本法西斯偷袭美国珍珠港,美国政府才决定启动"曼哈顿计划"。目的是在德国之前制造出原子弹,尽早结束第二次世界大战。



然而想要制成原子武器,首要的就是验证可控自持链式反应能否实现。因为费米是世界上研究中子的权威,又是集理论与实验于一身的全才,所以他被任命为世界上第一个核反应堆的攻关小组组长。1942年,费米来到芝加哥大学,主持对外称为"芝加哥冶金实验"的核反应堆工程。

1942年12月2日,在费米的指导设计下核反应堆"芝加哥Pile-1"首次运转成功。

随着这项实验的成功,曼哈顿工程全面启动,而费米也继续在这项工程中担任主要的科学顾问,继续发挥着自己的重要作用。在曼哈顿计划中,也发生了一件有趣的事情,后来被称为"费米纸片"。



1945年7月16日,第一颗原子弹在美国新墨西哥南部沙漠引爆。这颗原子弹爆炸后大约30分钟,强烈的冲击波到达费米所在的掩蔽处。只见费米把事先准备好的纸片一撒,随着纸片落地费米就说:这颗原子的威力相当于两万吨TNT炸药的能量。之后根据各种复杂测量仪器的记录数据来看,计算结果和费米的几乎无差,这使大家都为之叹服。

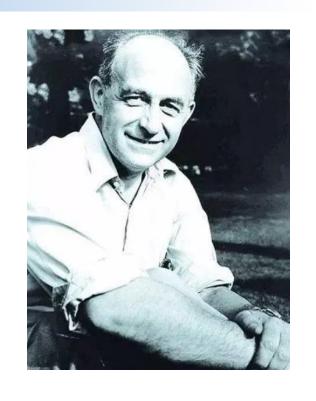
在战后,他并没有继续研制核武器,而是极力反对继续研制比原子弹威力更大的氢弹。从1945年以来,核武器便没有再用在战争上,而大量的反应堆也用于生产能源,造福着人类。

1949年,揭示宇宙线中原粒子的加速机制,研究了几介子、µ子和核子的相互作用,提出宇宙射线起源理论。

1952年,发现了第一个强子共振──同位旋四重态。

但是不幸的是,因长期接触放射性物质, 正直事业巅峰期的费米得了食道癌和胃癌。 1954年11月29日,费米与世长辞,享年 53岁。

为了纪念他,科学界将刚发现的100号元素命名为"镄Fm"。美国原子能委员会也建立了"费米奖",用于奖励在核能开发方面的人才。

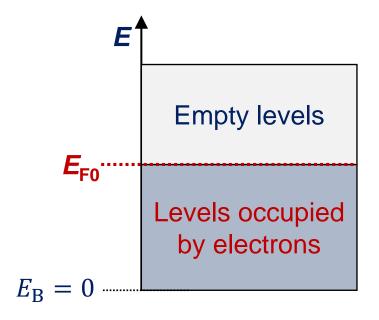


费米研究的领域非常广阔,他的成就和地位都很难衡量。但无论在哪一个领域,费米都是一个极其出色的存在。而更难能可贵的是国别、战争、诺贝尔奖、核反应堆、原子弹都没有成为限制他的标签。可以毫无疑问地说,他是20世纪最后一个全才。

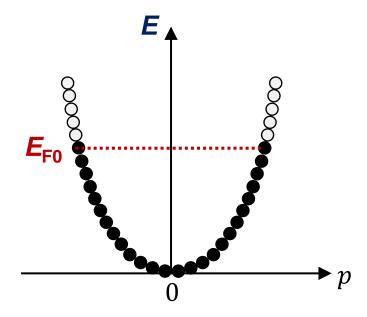
3.9 Properties of electrons in a band (for metals)

Electrons in metals are considered to be "free".

Energy-momentum:
$$E = \frac{p^2}{2m_e}$$

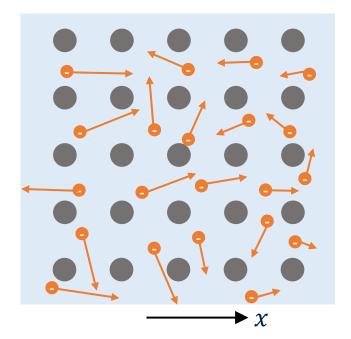


Energy band diagram



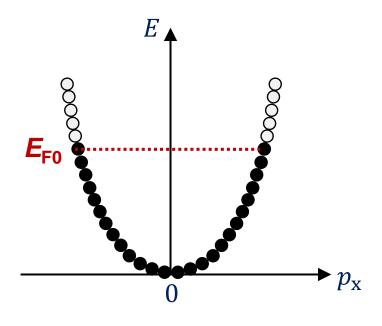
Energy-momentum space

Real space



Chaos!

Energy-momentum space



Ordered!

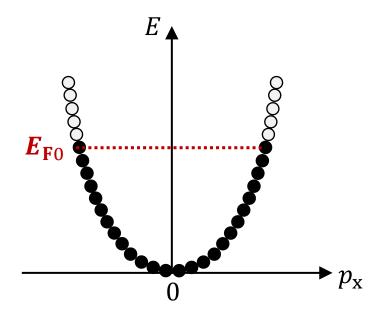
Real space



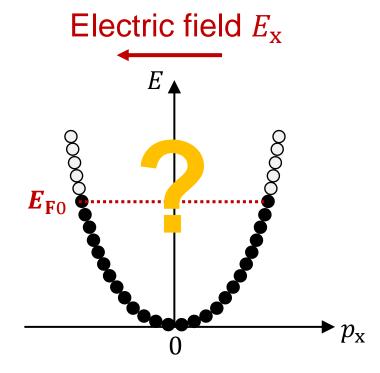
Energy space



No electric field $E_{\rm x}=0$

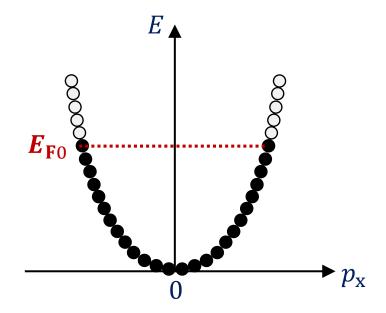


Average momentum in x-direction is zero!

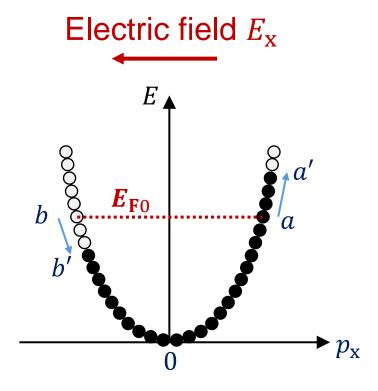


Average momentum in x-direction > zero!

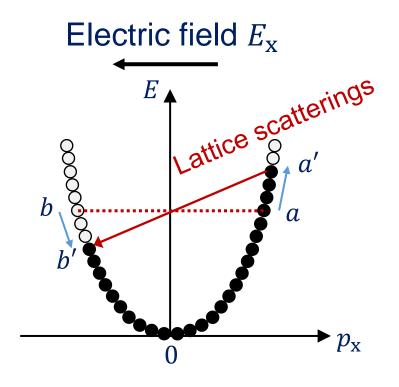
No electric field $E_{\rm x}=0$



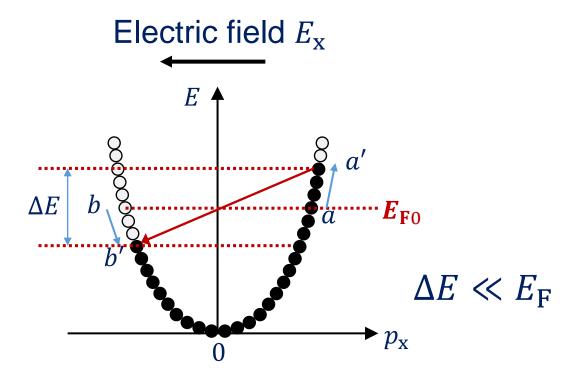
Average momentum in x-direction is zero!



Average momentum in x-direction > zero!



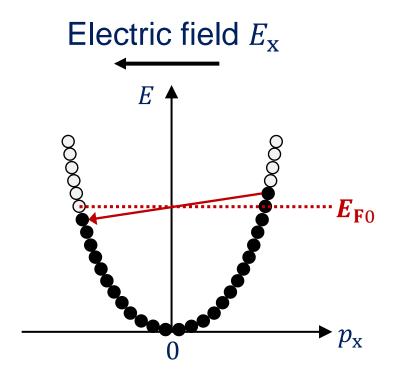
As we know, electrons cannot be accelerated to infinite speed, due to scattering with lattices and impurities.



Below b' level, the average momentum is zero.

Above b' level, the average momentum \neq zero.

We can summarize that conduction occurs by the drift of electrons at the Fermi level.

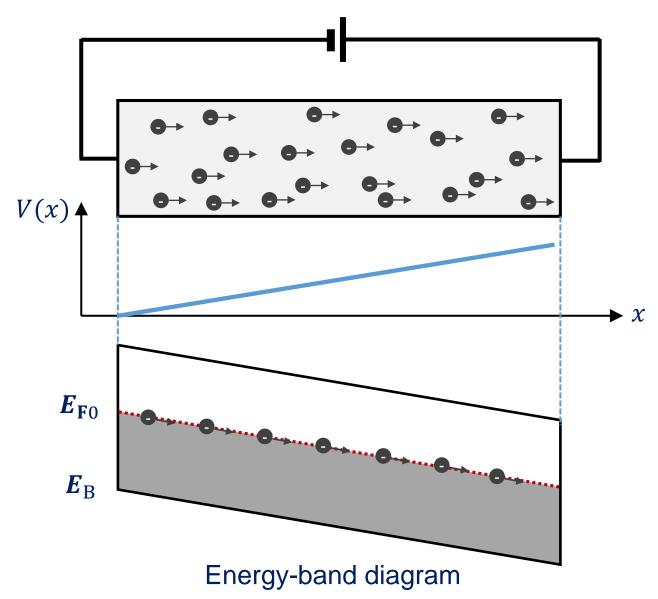


Electron velocity at Fermi level is called the Fermi velocity $v_{
m F}$

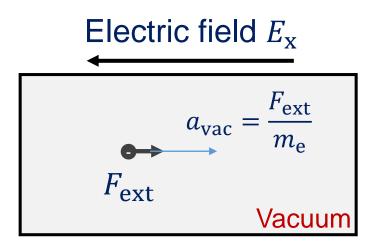
$$E_{\rm F0} = \frac{1}{2} m_{\rm e} v_{\rm F}^2$$

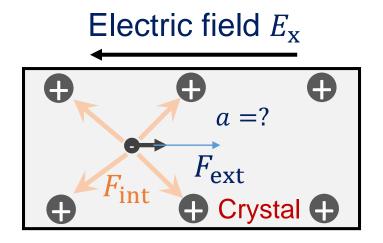
Q: What's the difference between Fermi velocity and drift velocity?

Explain the electrical conduction using energy-band diagram



Effective mass of electrons (for metals and semiconductors)





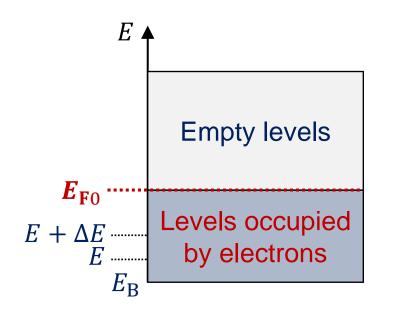
In crystal:
$$a_{\text{cryst}} = \frac{F_{\text{ext}} + F_{\text{int}}}{m_{\text{e}}}$$

The is F_{int} periodic, and can be solved by Schrodinger equation:

$$a_{\text{cryst}} = \frac{F_{\text{ext}}}{m_{\text{e}}^*}$$
 Effective mass

	Effe	ctive	mass o	f elec	trons	$m_{ m e}^*$ in	som	e met	als	
Metal	Ag	Au	Bi	Cu	Fe	K	Li	Mg	Na	Zn
$\frac{m_e^*}{m_e}$	1.0	1.1	0.008	1.3	12	1.2	2.2	1.3	1.2	0.85

3.10 Density of states 态密度 in an energy band



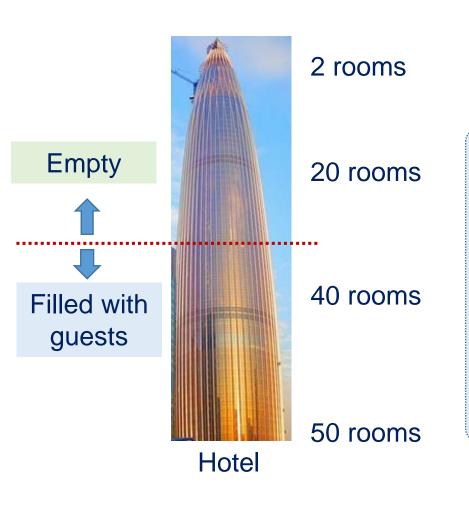
How many electrons can be filled between E to $E + \Delta E$?



Density of states: the number of states per energy per volume

$$g(E)$$
: m⁻³eV⁻¹

Q: For empty levels, will there be density of states?



Hotel: Energy band

Guests: Electrons

Rooms: Density of states

Assume we have N atoms $(\psi_A, \psi_B, \psi_C...)$ and N electron states $(\psi_1, \psi_2, ..., \psi_N)$

Density of states is highest in the central region of energy band.

Q: What's the value of density of states?

The value of density of states g(E): m⁻³eV⁻¹

Number of states per volume from 0 to E': $S_v(E')$?

$$S_{\mathbf{v}}(E') = \int_{0}^{E'} g(E) dE$$

Consider solid crystal as a 3-dimensional quantum well (size: $L \times L \times L$):

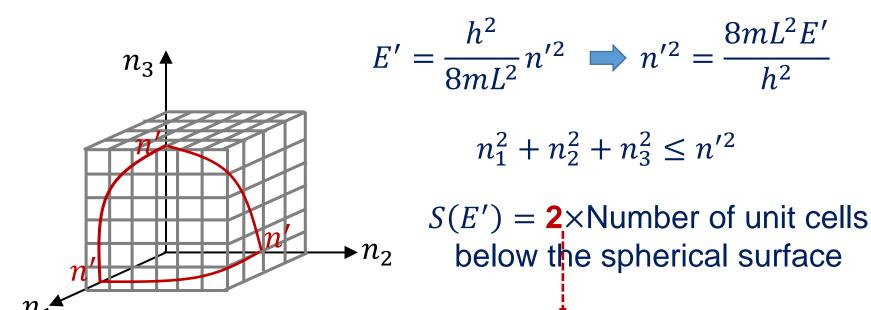
$$E = \frac{h^2}{8mL^2}(n_1^2 + n_2^2 + n_3^2)$$

$$n_1, n_2, n_3 = 1,2,3 \dots$$

Consider solid crystal as a 3-dimensional quantum well (size: $L \times L \times L$):

$$E = \frac{h^2}{8mL^2}(n_1^2 + n_2^2 + n_3^2) \qquad n_1, n_2, n_3 = 1,2,3 \dots$$

Q: What's the total number of states from 0 to E'?



Spin up and spin down for each (n_1, n_2, n_3)

$$n_3$$
 n_2
 n_1

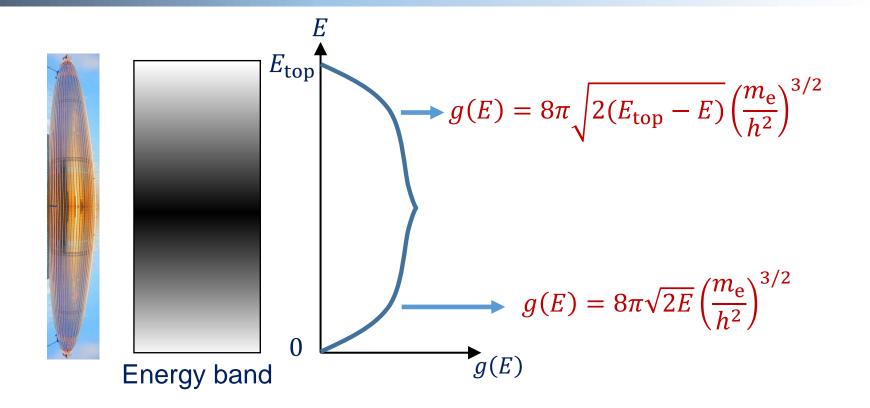
$$S(E') = 2 \times \frac{1}{8} \times (\frac{4}{3}\pi n'^3)$$
$$= \frac{1}{3}\pi n'^3$$
$$= \frac{\pi L^3 (8m_e E')^{3/2}}{3h^3}$$

Number of states per volume from 0 to E': $S_v(E')$

$$S_{\rm v}(E') = \frac{\pi (8m_{\rm e}E')^{3/2}}{3h^3}$$

Density of states: $g(E) = \frac{dS_v(E)}{dE}$

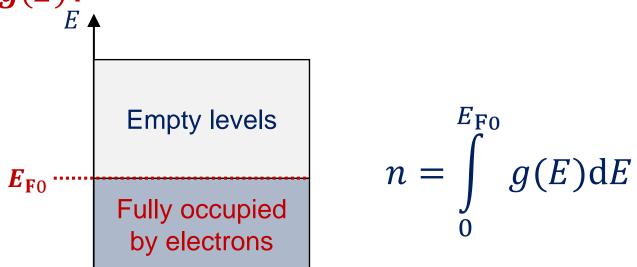
$$g(E) = 8\pi\sqrt{2E} \left(\frac{m_{\rm e}}{h^2}\right)^{3/2}$$



$$g(E) = 8\pi\sqrt{2E} \left(\frac{m_{\rm e}}{h^2}\right)^{3/2}$$

 $g(E) = 8\pi\sqrt{2E}\left(\frac{m_{\rm e}}{h^2}\right)^{3/2} \ \ \, \begin{cases} \ \, \text{is accurate for free electrons.} \\ \\ \ \, \text{is good approximation for metals and} \\ \\ \ \, \text{semiconductors near band edge.} \end{cases}$

Q: The relation between carrier density n and density of states g(E)?

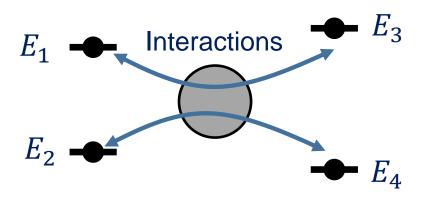


Q: The relation between carrier density n and density of states g(E), if the probability that an energy level is occupied is f(E)?

$$n = \int f(E)g(E)dE$$

3.11 Fermi-Dirac Statistics 费米-狄拉克统计

Classic model: Given a collection of classic particles in random motion and colliding with each other (**ignore Pauli exclusion principle**), the probability of an electron with energy *E* is *P*(E).



In thermal equilibrium (ignore Pauli exclusion principle):

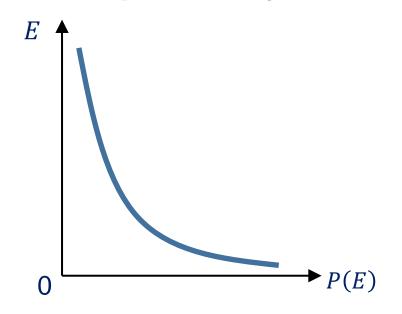
$$P(E_1)P(E_2) = P(E_3)P(E_4)$$

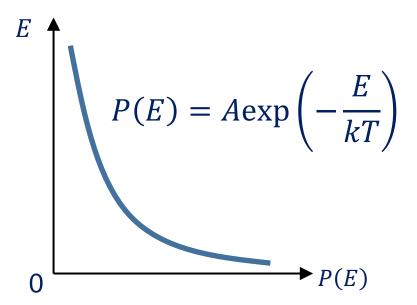
 $E_1 + E_2 = E_3 + E_4$

$$\begin{cases} P(E_1)P(E_2) = P(E_3)P(E_4) \\ E_1 + E_2 = E_3 + E_4 \end{cases}$$

$$P(E) = A \exp\left(-\frac{E}{kT}\right)$$

Boltzmann probability function





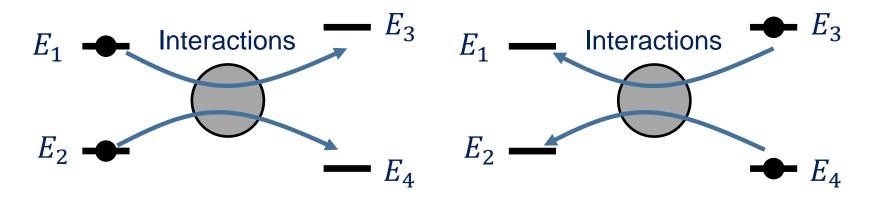
When temperature $T\rightarrow 0$, all particles are in the lowest energy level E=0.

This is called: Bose-Einstein condensation.



Particles that does not follow Pauli exclusion principles are called Boson 玻色子: photon, phonon...

Fermi-Dirac model: Given a collection of particles in random motion and colliding with each other (**follow Pauli exclusion principle**), the probability of an electron with energy *E* is *f*(E).



In thermal equilibrium (follow Pauli exclusion principle):

$$E_1 + E_2 = E_3 + E_4$$

$$f(E_1)f(E_2)[1 - f(E_3)][1 - f(E_4)]$$

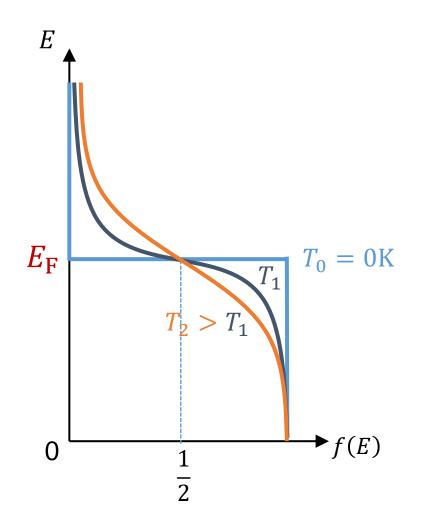
$$= f(E_3)f(E_4)[1 - f(E_1)][1 - f(E_2)]$$

By an "intelligent guess", the solution is:

$$f(E) = \frac{1}{1 + A \exp\left(\frac{E}{kT}\right)}$$

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

Fermi-Dirac function



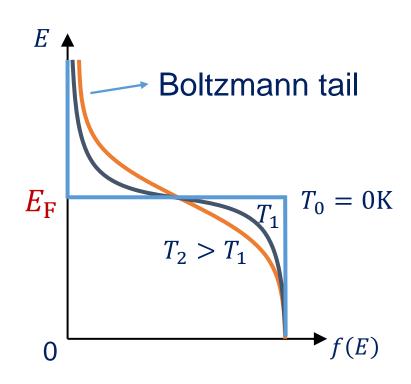
Particles follow Fermi-Dirac function are called Fermion费米子: electron.

When
$$E - E_{\rm F} \gg kT$$
,

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_{\rm F}}{kT}\right)} \implies f(E) = \exp\left(-\frac{E - E_{\rm F}}{kT}\right)$$

Fermi-Dirac function

Boltzmann function

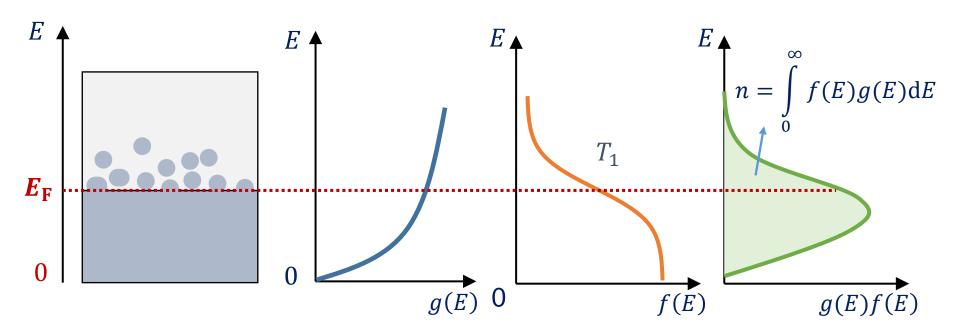


The carrier density n in metals

$$n = \int_{0}^{\infty} f(E)g(E)dE$$

$$g(E) = 8\pi\sqrt{2E} \left(\frac{m_{\rm e}}{h^2}\right)^{3/2}$$

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_{\rm F}}{kT}\right)}$$



Relationship between n and E_F

When T=0K:

$$E_{\rm F0} = \left(\frac{h^2}{8m_{\rm e}}\right) \left(\frac{3n}{\pi}\right)^{2/3}$$

When T>0K:

$$E_{\rm F} = E_{\rm F0} \left[1 - \frac{\pi^2}{12} \left(\frac{kT}{E_{\rm F0}} \right)^2 \right]$$

Fermi energy slightly depends on temperature.

Average energy of an electron in metal:

$$E_{\text{av}} = \frac{\int Eg(E)f(E)dE}{\int g(E)f(E)dE}$$

$$E_{\text{av}} \approx \frac{3}{5}E_{\text{F0}}\left[1 + \frac{5\pi^2}{12}\left(\frac{kT}{E_{\text{F0}}}\right)^2\right]$$

$$E_{\text{av}} \approx \frac{3}{5}E_{\text{F0}}$$

Average Kinetic energy (KE) of electron: $\frac{3}{5}E_{F0}$

Average speed of electron: $\frac{1}{2}m_{\rm e}v_{\rm e}^2 = \frac{3}{5}E_{\rm F0}$

Reexamine the conduction in metals using quantum theory (1-dimension model)

Electric field $E_{\rm x}$ $E_{\rm F0}$ $= \sum_{\rm attice} E_{\rm x}$ $= \sum_{\rm attice} E_{\rm x}$ $= \sum_{\rm attice} E_{\rm x}$ $= \sum_{\rm attice} E_{\rm x}$

Electrical conduction is contributed by electrons in a small range ΔE near E_F .

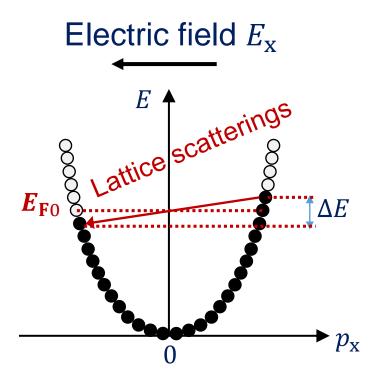
Electrons are accelerated by electric field Electrons are scattered by lattice.

In equilibrium, p_x gain= p_x loss

$$\Delta p_{\rm x} = e E_{\rm x} \tau$$

$$\Delta E = \frac{p_{\rm x}}{m_{\rm e}^*} \Delta p_{\rm x} = \frac{m_{\rm e}^* v_{\rm F}}{m_{\rm e}^*} (e E_{\rm x} \tau)$$
$$= e E_{\rm x} \tau v_{\rm F}$$

Electrical conduction is contributed by electrons in a small range ΔE near E_F .



$$J_{x} = en_{F}v_{F}$$

$$= e[g(E_{F})\Delta E]v_{F}$$

$$= e[g(E_{F})eE_{x}\tau v_{F}]v_{F}$$

$$= e^{2}v_{F}^{2}\tau g(E_{F})E_{x}$$

1-dimensional conductivity:

$$\sigma = e^2 v_{\rm F}^2 \tau g(E_{\rm F})$$

3-dimensional conductivity:

$$\sigma = \frac{1}{3}e^2v_{\rm F}^2\tau g(E_{\rm F})$$

Electrical conductivity in quantum model

$$\sigma = \frac{1}{3}e^2v_{\rm F}^2\tau g(E_{\rm F})$$

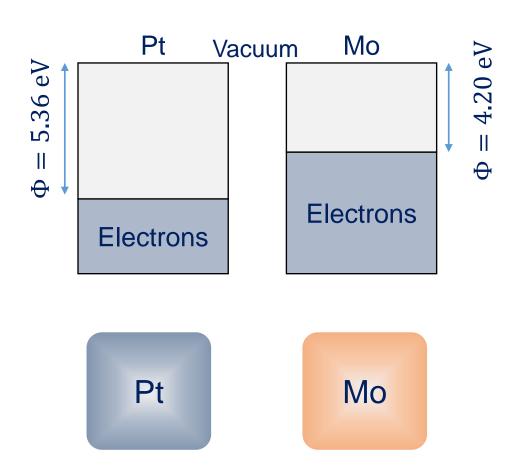
Electrical conductivity in classic Drude model

$$\sigma = \frac{e^2 n\tau}{m_{\rm e}^*}$$

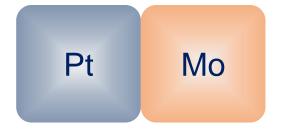
Homework 1-1: Prove that above two equations are identical

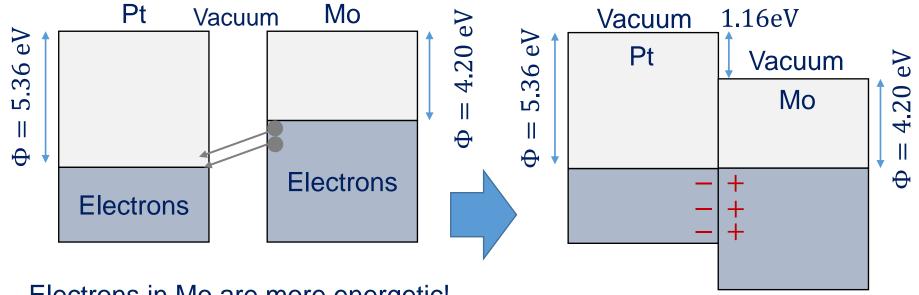
3.12 Fermi energy significance and device applications

Metal-metal contacts: contact potential 接触势



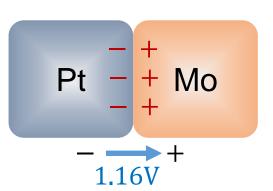
Q: Pt and Mo has different work function. What will happen when Pt and Mo contact?

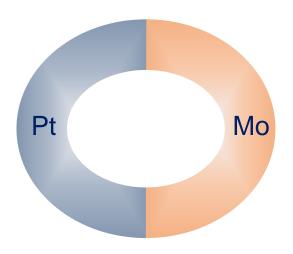




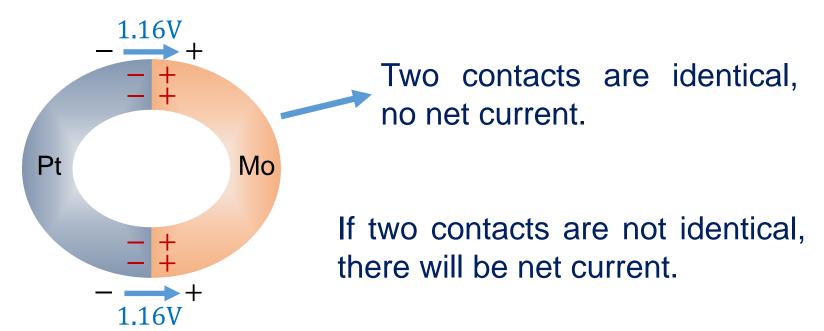
Electrons in Mo are more energetic!

Electrons in Mo will flow into Pt.

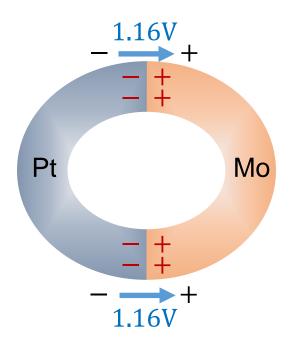




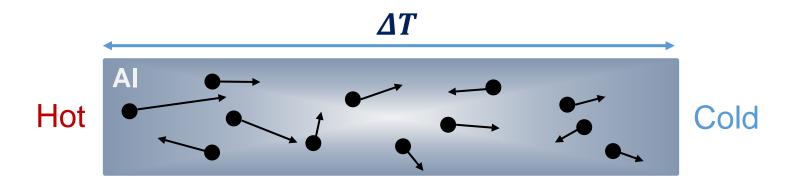
Q: In this configuration, will there be net current?



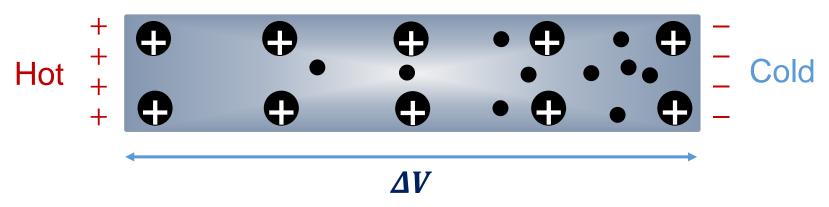
Q: What's the application of this structure?



Seebeck effect 塞贝克效应



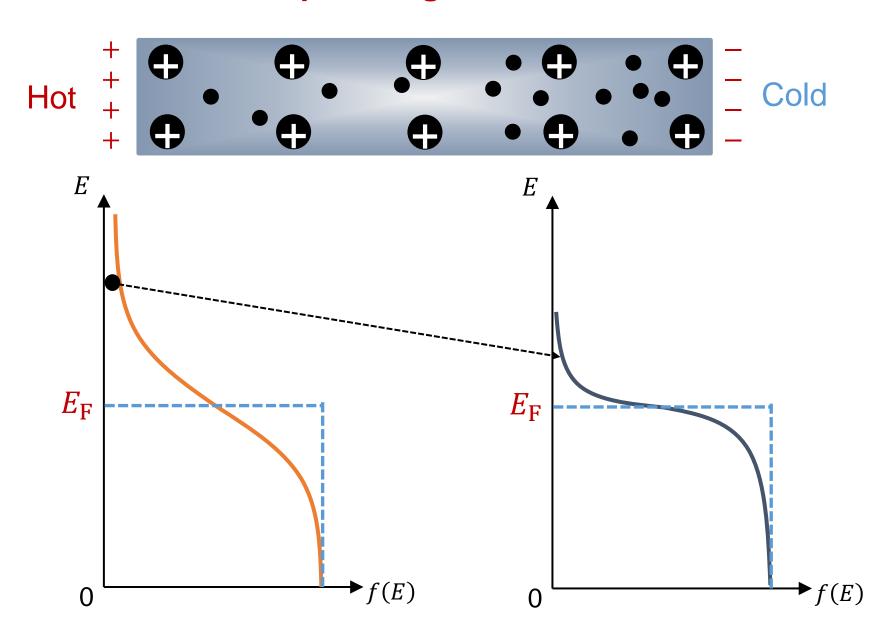
Electrons are more energetic in hot terminal.



Seebeck coefficient: $S = \frac{\mathrm{d}V}{\mathrm{d}T}$

The potential of cold side respect to the hot side.

Q: How to explain negative Seebeck effect?

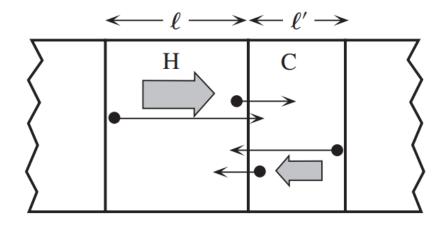


Metal	S at 27 °C (μV/K)	E _F (eV)
Al	-1.7	11.7
Au	+2.08	5.53
Cu	+1.94	7.00
K	-13.7	2.12
Li	+11.4	4.74
Na	-6.3	3.24
Mg	-1.46	7.08
Ni	-19.5	~7.4
Pt	-4.92	~6.0

S < 0: cold side is negative.

S > 0: hot side is negative, electrons diffusive from cold to hot end.

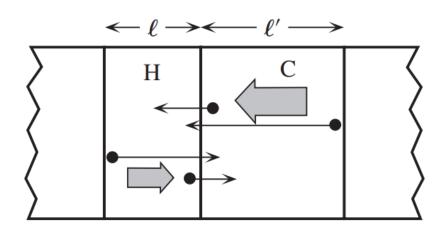
(a) S negative



Energy (hot) > Energy (cold)

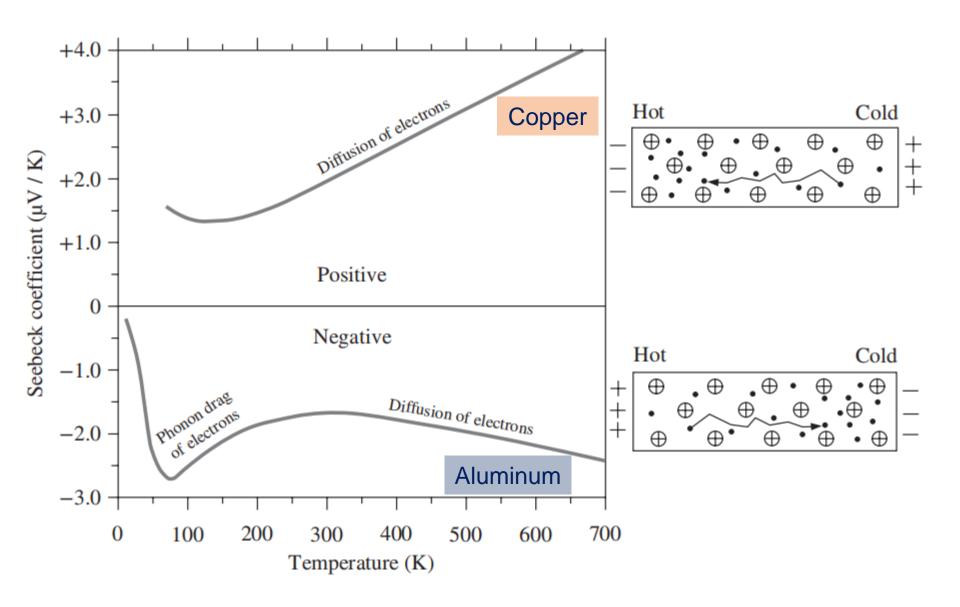
Mean free path (hot) > Mean free path (cold)

(b) S positive



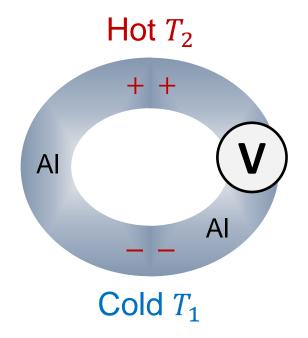
Energy (hot) > Energy (cold)

Mean free path (hot) < Mean free path (cold)



Thermocouple 热电偶

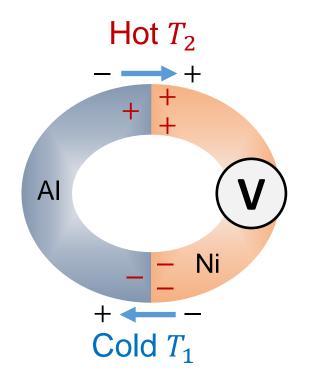
Metal	S at 27 °C (μV/K)
Al	-1.7
Ni	-19.5



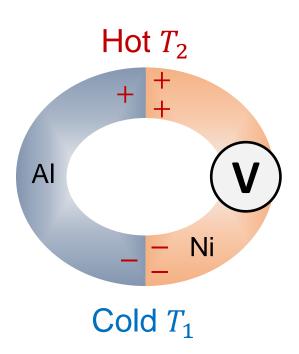
Voltage meter: 0 V

Thermocouple 热电偶

Metal	S at 27 °C (μV/K)
Al	-1.7
Ni	-19.5



Thermocouple 热电偶



Metal	S at 27 °C (μV/K)
Al	-1.7
Ni	-19.5

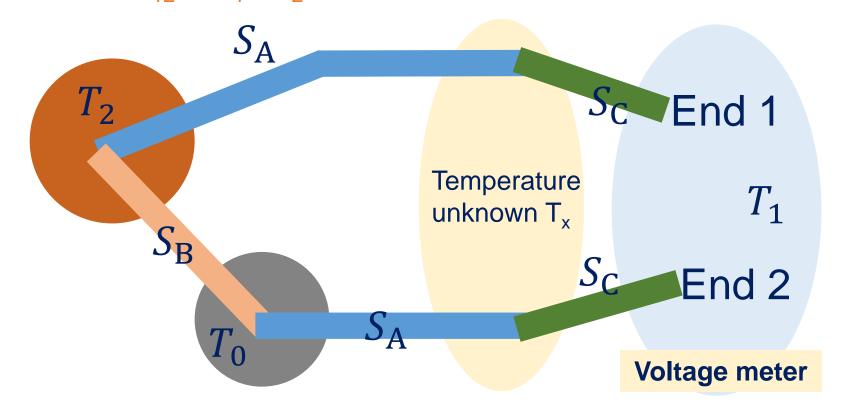
The electromotive force 电动势 between two metal wires A and B:

$$V_{AB} = \int_{T_1}^{T_2} (S_A - S_B) dT$$

Thermoelectric power 热电功 for the thermocouple pair:

$$S_{AB} = S_A - S_B$$

Homework 1-2: The electromotive force/voltage between End 1 and End 2: $V_{12} = V_1 - V_2$.

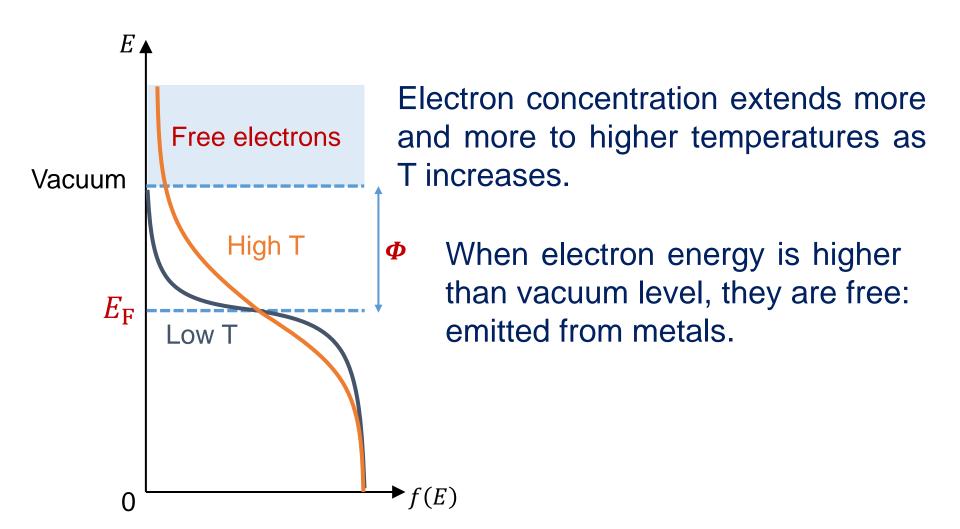


3 types of metals: A, B, C

Assume Seebeck coefficient is independent of temperature.

3.13 Thermionic emission and vacuum tube devices

What happens when temperature of a metal is too high?



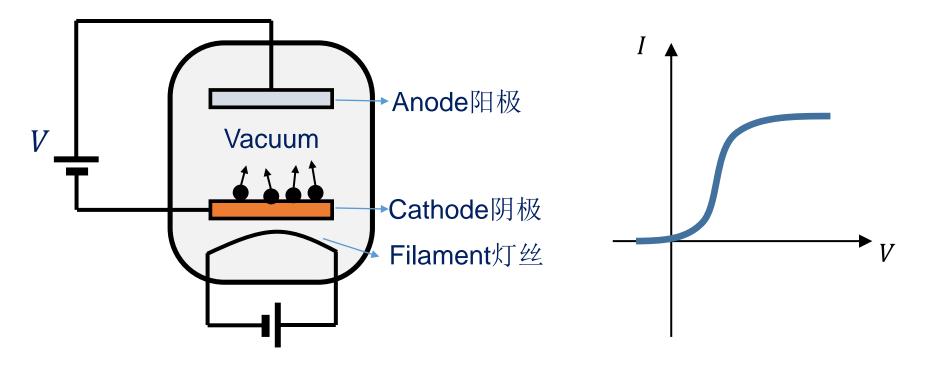
Vacuum tube

1st generation electronic device 第一代电子器件

Electronic (vacuum) tube 电子管 (真空管)



- a. Vacuum shield
- b. Cathode 阴极
- c. Heated cathode will emit electrons阴极可由灯丝加热,使温度升高,发射出电子
- d. Current: Electrons motion under the electric field and magnetic field电子受外加电场和磁场的作用下,在真空中运动就形成了电子管中的电流



Thermionic emission current density:

$$J = B_0 T^2 \exp\left(-\frac{\Phi}{kT}\right)$$
, $B_0 = \frac{4\pi e m_e k^2}{h^3}$

Richardson-Dushman equation

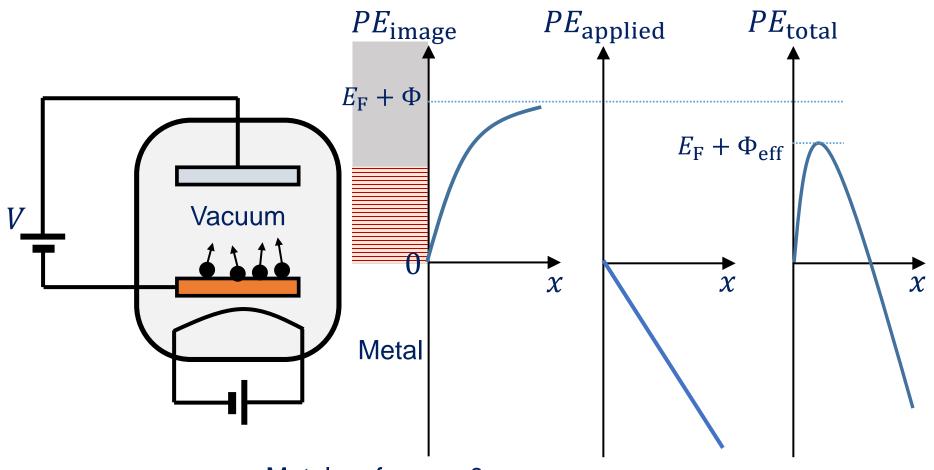
Richardson-Dushman equation

$$J = B_0 T^2 \exp\left(-\frac{\Phi}{kT}\right), B_0 = \frac{4\pi e m_e k^2}{h^3} \approx 1.2 \times 10^6 \text{Am}^{-2} \text{K}^{-2}$$

In real case, electrons will have chance to be reflected back.

$$J = B_{\rm e}T^2 \exp\left(-\frac{\Phi}{kT}\right)$$
, $B_{\rm e} = (1 - R)B_0$

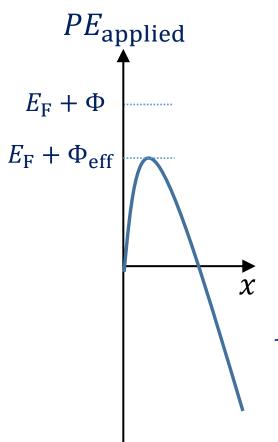
Effect of applied voltage



Metal surface: x=0

$$PE_{\text{image}} = E_{\text{F}} + \Phi - \frac{e^2}{16\pi\varepsilon_0 x}$$
 $PE_{\text{applied}} = -exE$

Schottky effect 肖特基效应



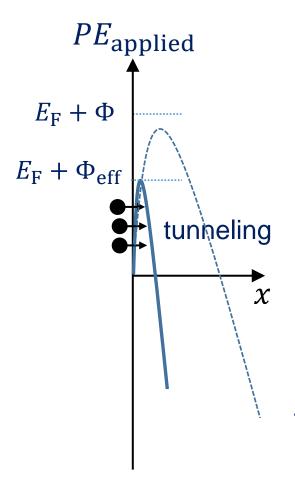
Schottky effect: using electric field to lower the potential barrier (PE).

$$\Phi_{\rm eff} = \Phi - \left(\frac{e^3 E}{4\pi\varepsilon_0}\right)^{\frac{1}{2}} = \Phi - \beta_{\rm S}\sqrt{E}$$

$$\int_{X} J = B_{\rm e} T^2 \exp\left(-\frac{\Phi - \beta_{\rm s} \sqrt{E}}{kT}\right), B_{\rm e} = (1 - R)B_0$$

 β_s : Schottky coefficient

Field emission 场发射



When electric field is very large: $E > 10^7 \text{V/cm}$

Barrier is very narrow.

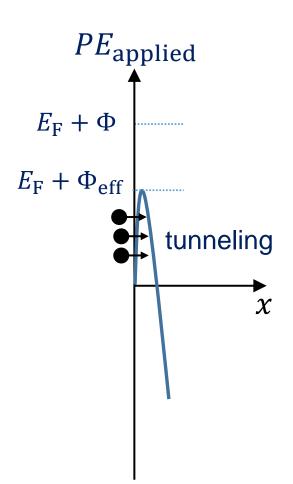


Electrons can directly tunnel into vacuum.



Since tunneling is temperature independent, the emission process is called field emission.

Field emission 场发射

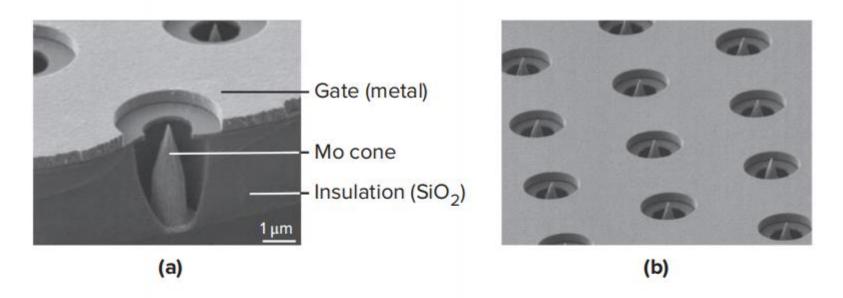


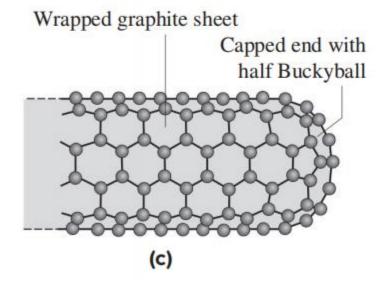
$$J_{\text{field}} = CE^2 \exp\left(-\frac{E_{\text{c}}}{E}\right),\,$$

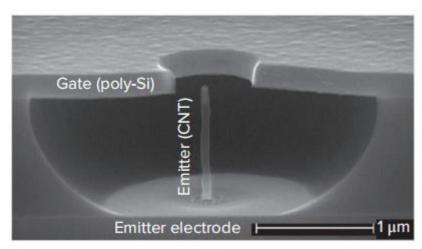
$$C = \frac{e^3}{8\pi h \Phi}$$
, $E_{\rm c} = \frac{8\pi (2m_{\rm e}\Phi^3)^{1/2}}{3eh}$

Q: The advantages of field emission compared with thermionic emission?

Applications of field emission effect



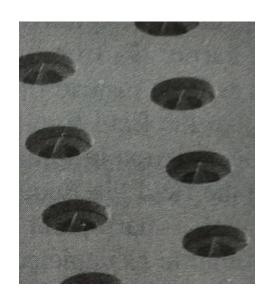


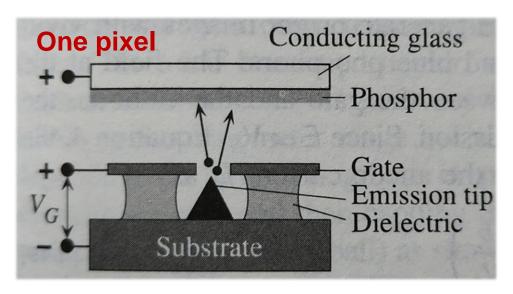


(d)

Applications of field emission effect

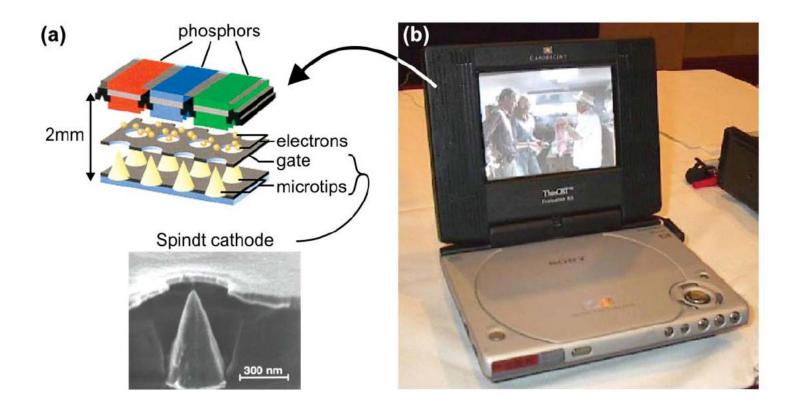
Field emission displays (FED)



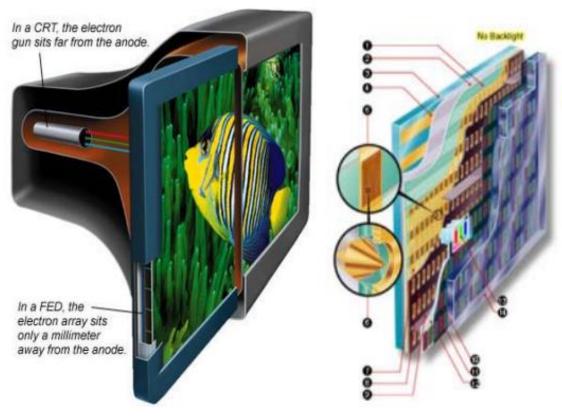


FED can be very thin <6 mm, low weight, low power consumption.

Sony was leading the investigation of FED since 2000, but it was not successful as LCD (liquid crystal displays).



- (a) Cross-section of a field emission display showing a Spindt tip cathode;
- (b) Sony portable DVD player using a field emission display.



Cross-Section of a FED

- 1. Dielectric
- 2. Patterned Resister Layer
- 3. Cathode Glass
- 4. Row Metal
- 5. Emitter Array
- 6. Single Emitter Cone & Gate Hole
- 7. Column Metal
- 8. Focusing Grid
- 9. Wall
- 10. Phosphor
- 11. Black Matrix
- 12. Aluminum Layer
- 13. Pixel On
- Faceplate Glass

Hot cathode

Cold cathode

Advantages: Thinner, lighter, vivid color, fast response, wide viewing angle etc.

Applications of field emission effect

Electronic gan 电子枪

Scanning electron microscope (SEM)



TEM

