CSE5010 Wireless Network and Mobile Computing Fall 2023

Lab2

Signal Processing Basis

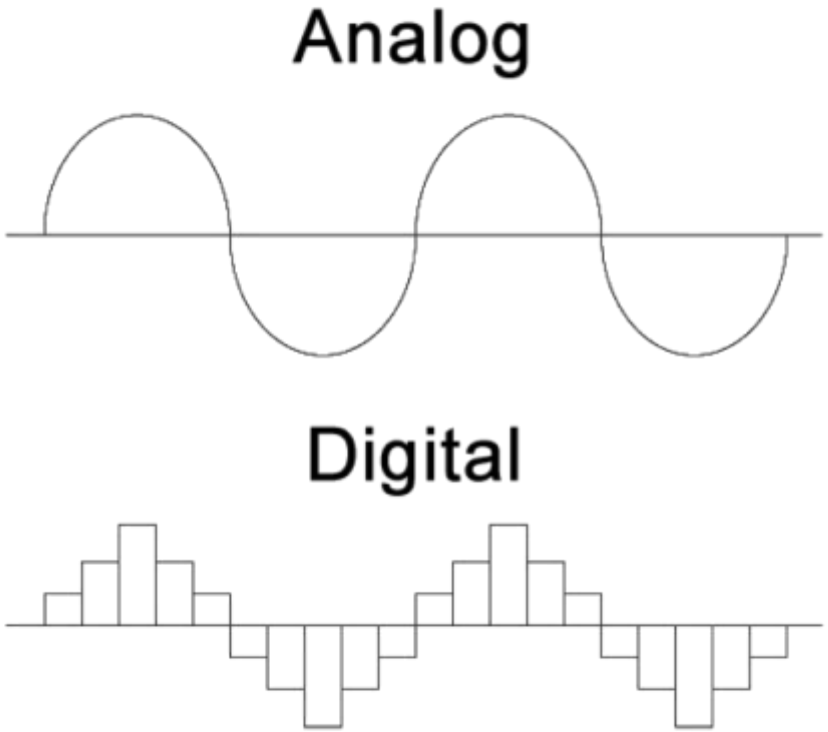
Schedule

Week 5	Paper Review
Week 8	Paper Presentation (Part 1)
Week 10	Project Proposal
Week 14	Paper Presentation (Part 2)

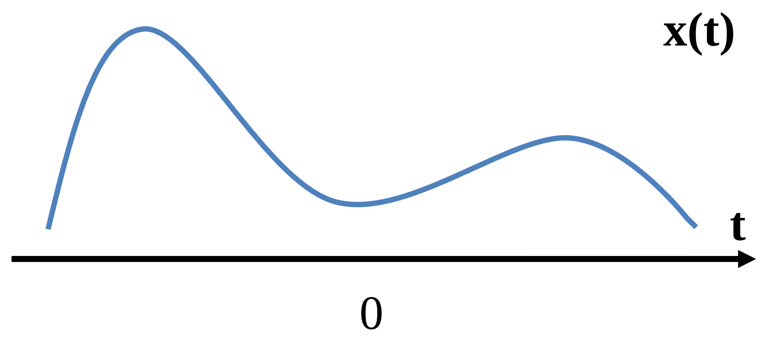
NYQUIST–SHANNON SAMPLING THEOREM

What is sampling

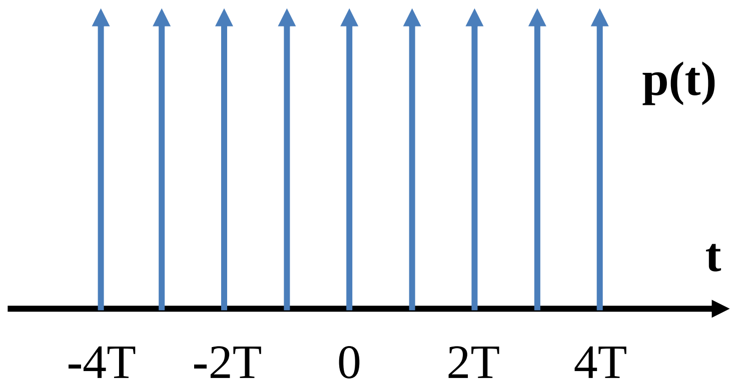
- From continuous signal to discrete signal
- From analog signal to digital signal (**substep**)
- From single curve to multiple points



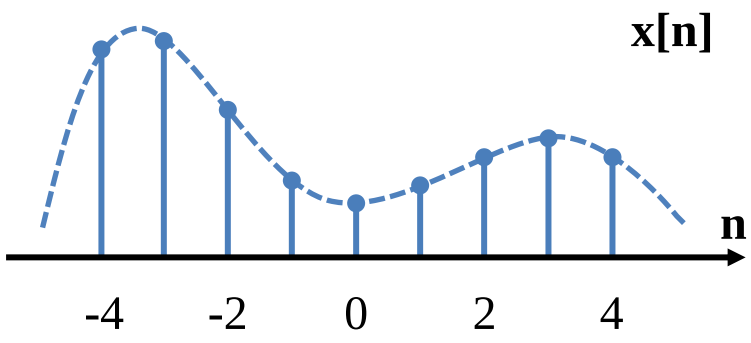
Impulse-train Sampling



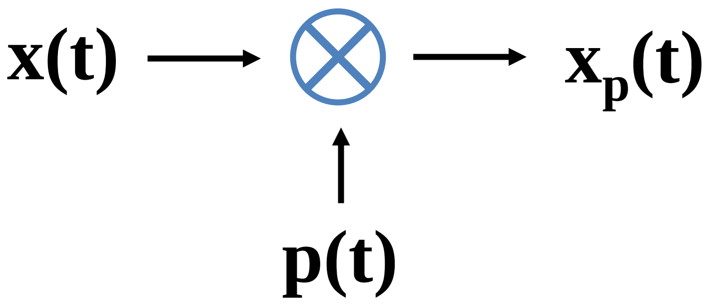
Continous Signal



Impluse Train

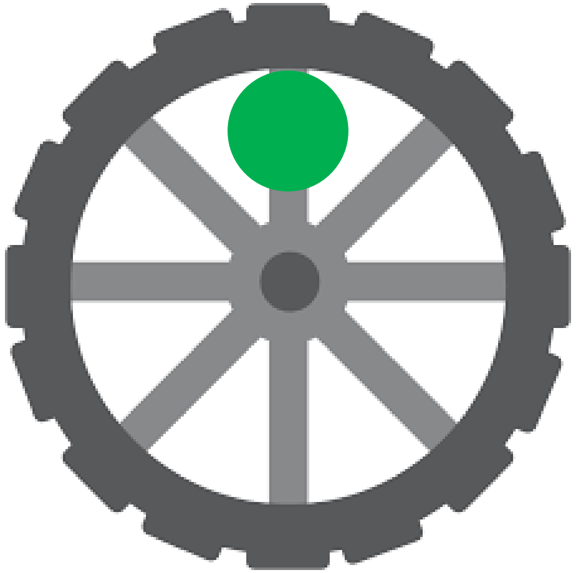


Discerete Signal



$$x[n] = x_p(nT) = x_p(t)|_{t=nT}$$

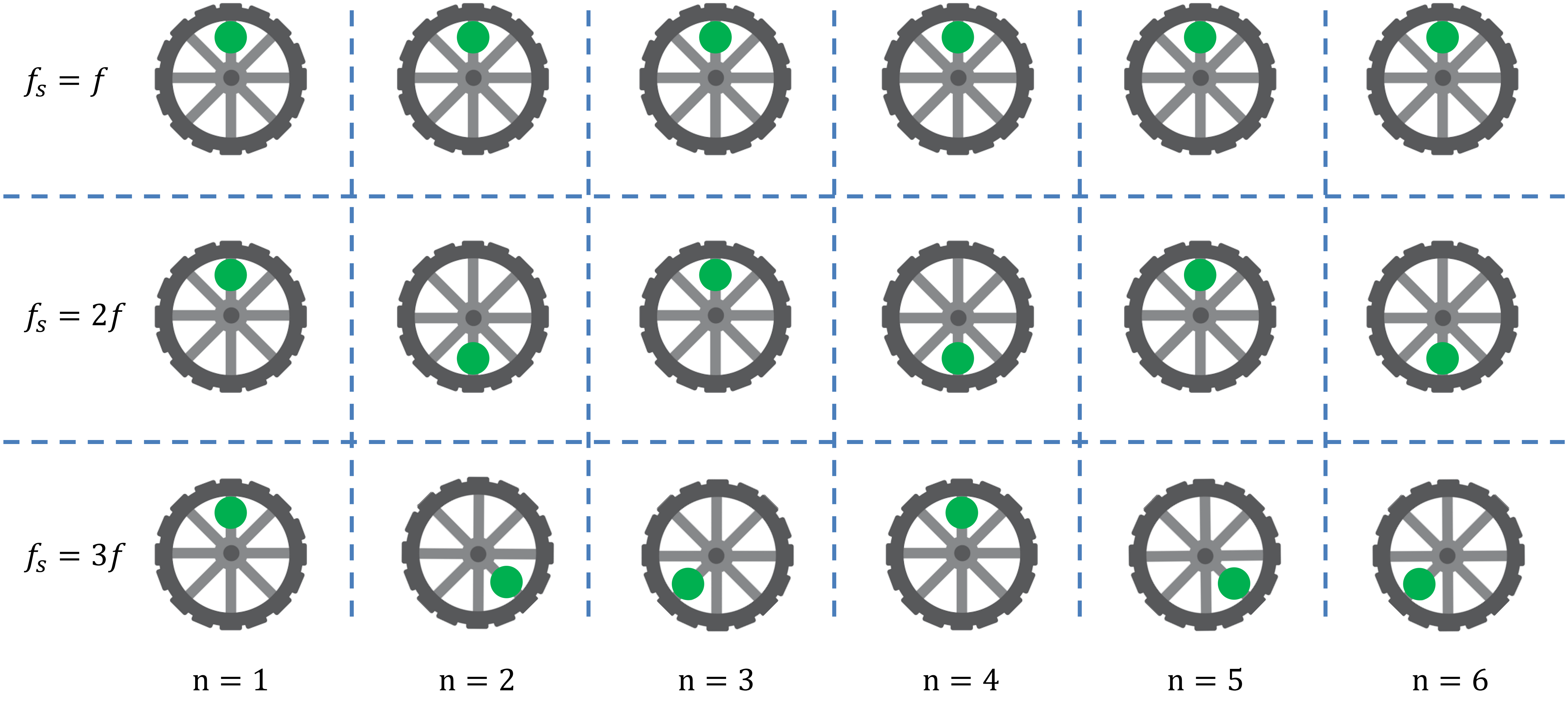
Sampling Frequency



A green sticker are sticked on the wheels to calibrate the rotation state.

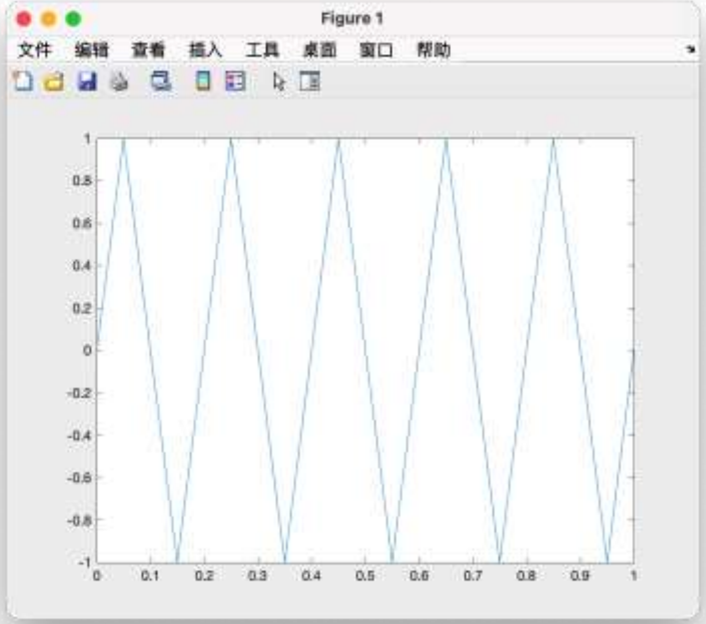
- cycle / period : T
- frequency : $f = 1/T$

Sampling Frequency

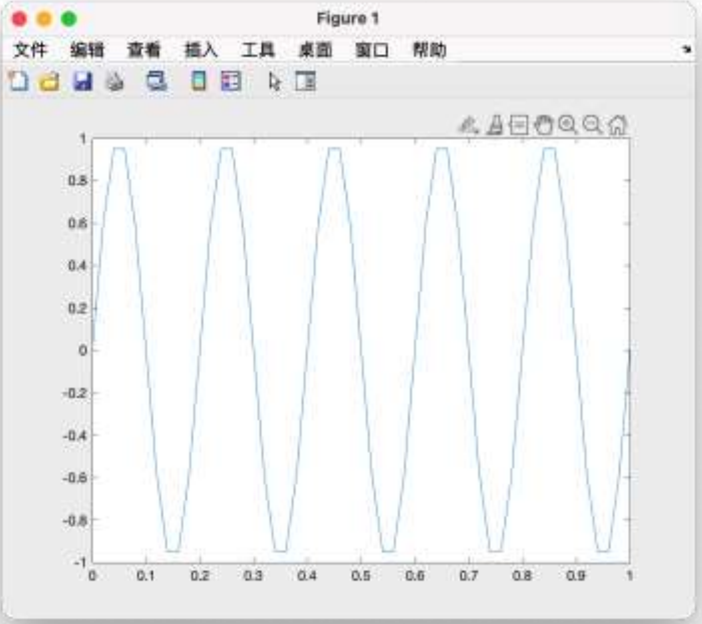


Sampling Frequency

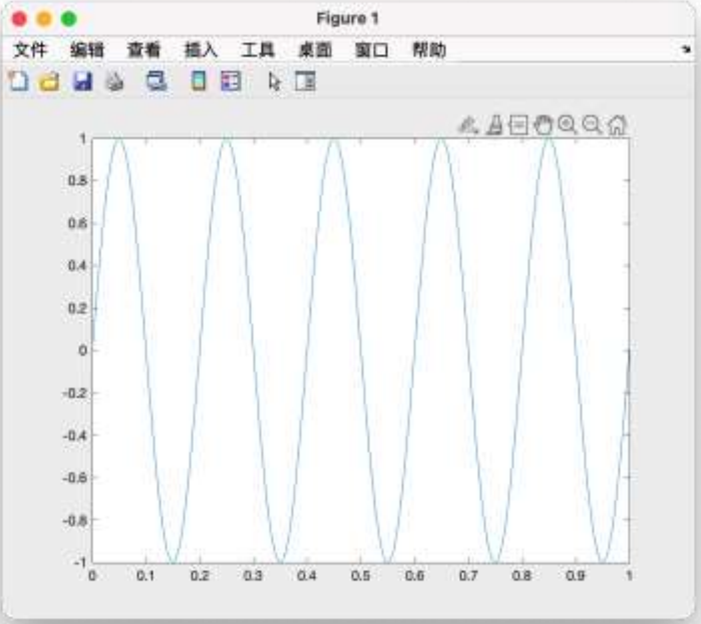
```
clear
clc

fs = 200;   % Sampling Frequency (Unit: Hz)
f = 5;      % Signal Frequency   (Unit: Hz)
T = 1;      % Time Duration      (Unit: s )

t = 0 : 1/fs : T;
y = sin(2*pi*f*t);  % Generate a Sine Signal
plot(t, y);
```



$f_s = 20Hz$



$f_s = 50Hz$



$f_s = 200Hz$

How many sampling points are enough?

Nyquist–Shannon Sampling Theorem:

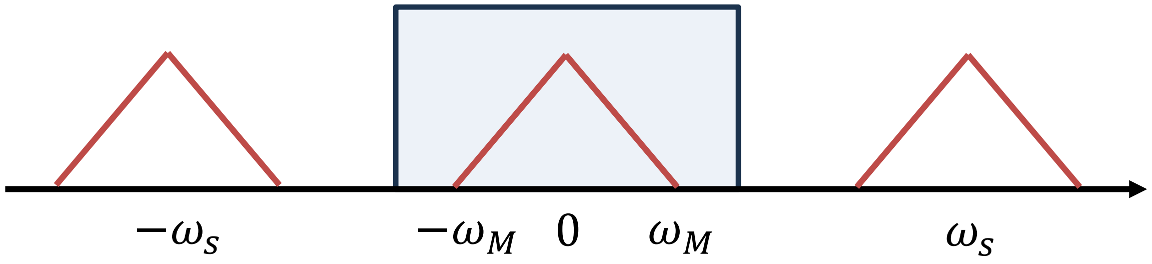
A signal must be sampled at a rate, f_S, which is at least twice the highest frequency component, f_M, present in the signal to avoid the loss of information.

为了不失真地恢复模拟信号，采样频率应该不小于模拟信号频谱中最高频率的2倍。

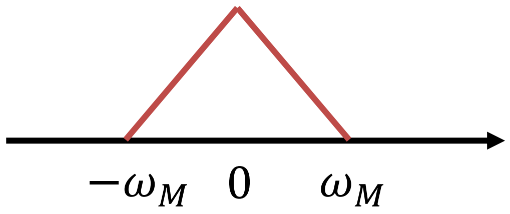
Nyquist–Shannon Sampling Theorem

- $\omega_s = 2\pi f_s = \frac{2\pi}{T}$
- $\omega_M = 2\pi f_M$

$\omega_s \geq 2\omega_M$

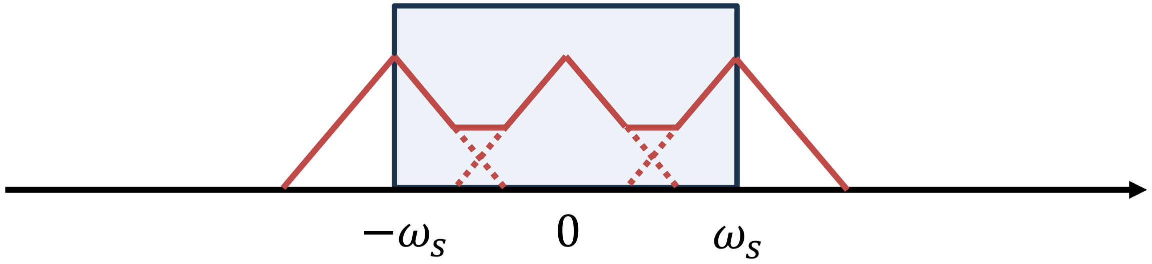


Sampled Signal (High Sampling Frequency)



Original Signal

$\omega_s < 2\omega_M$

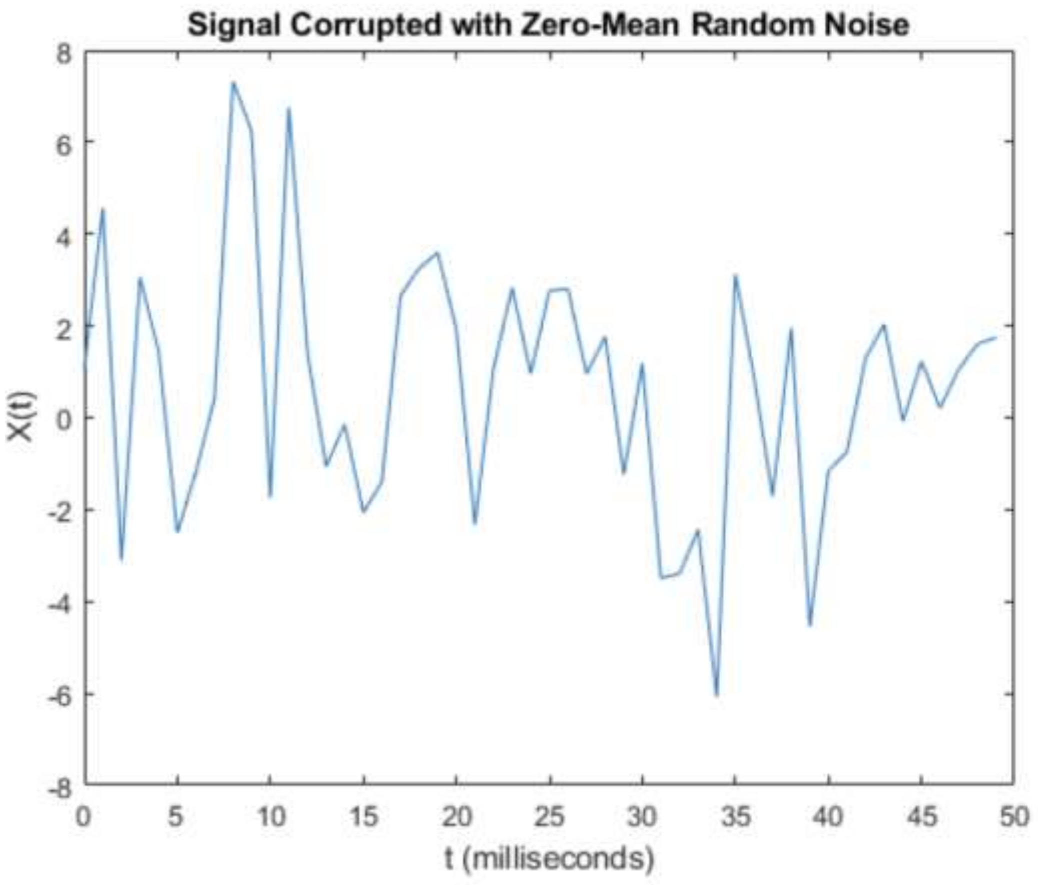


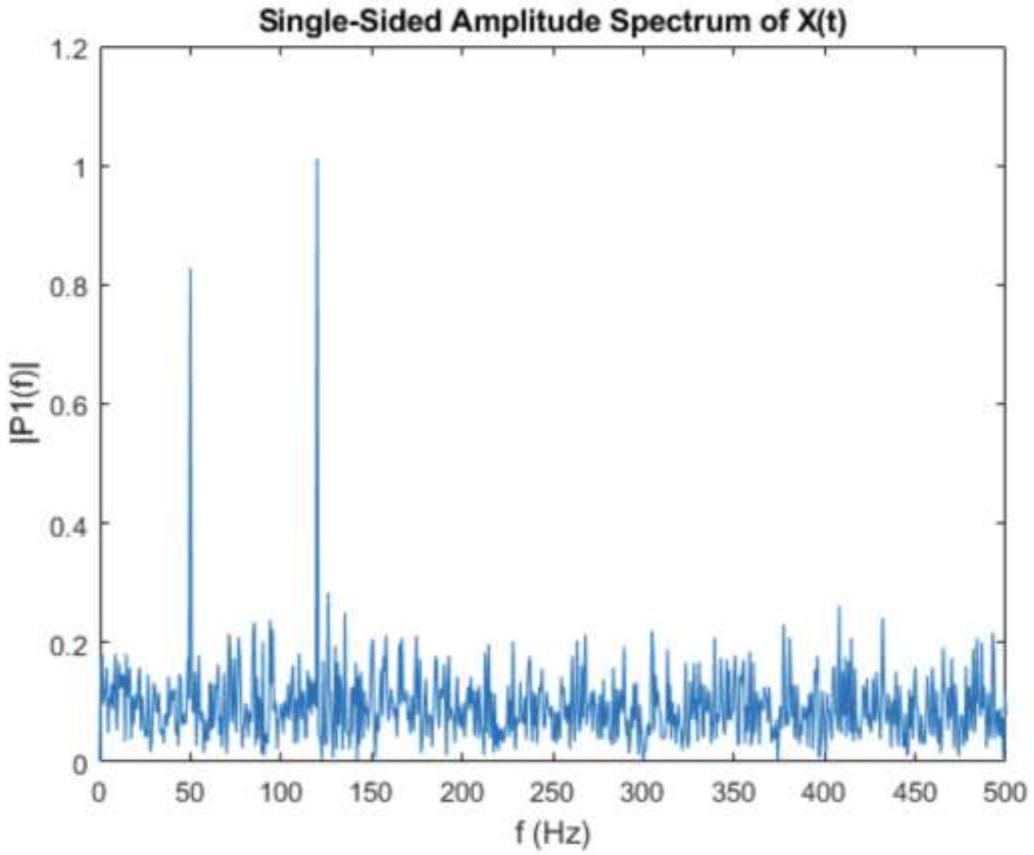
Sampled Signal (Low Sampling Frequency)

FAST FOURIER TRANSFORM (FFT)

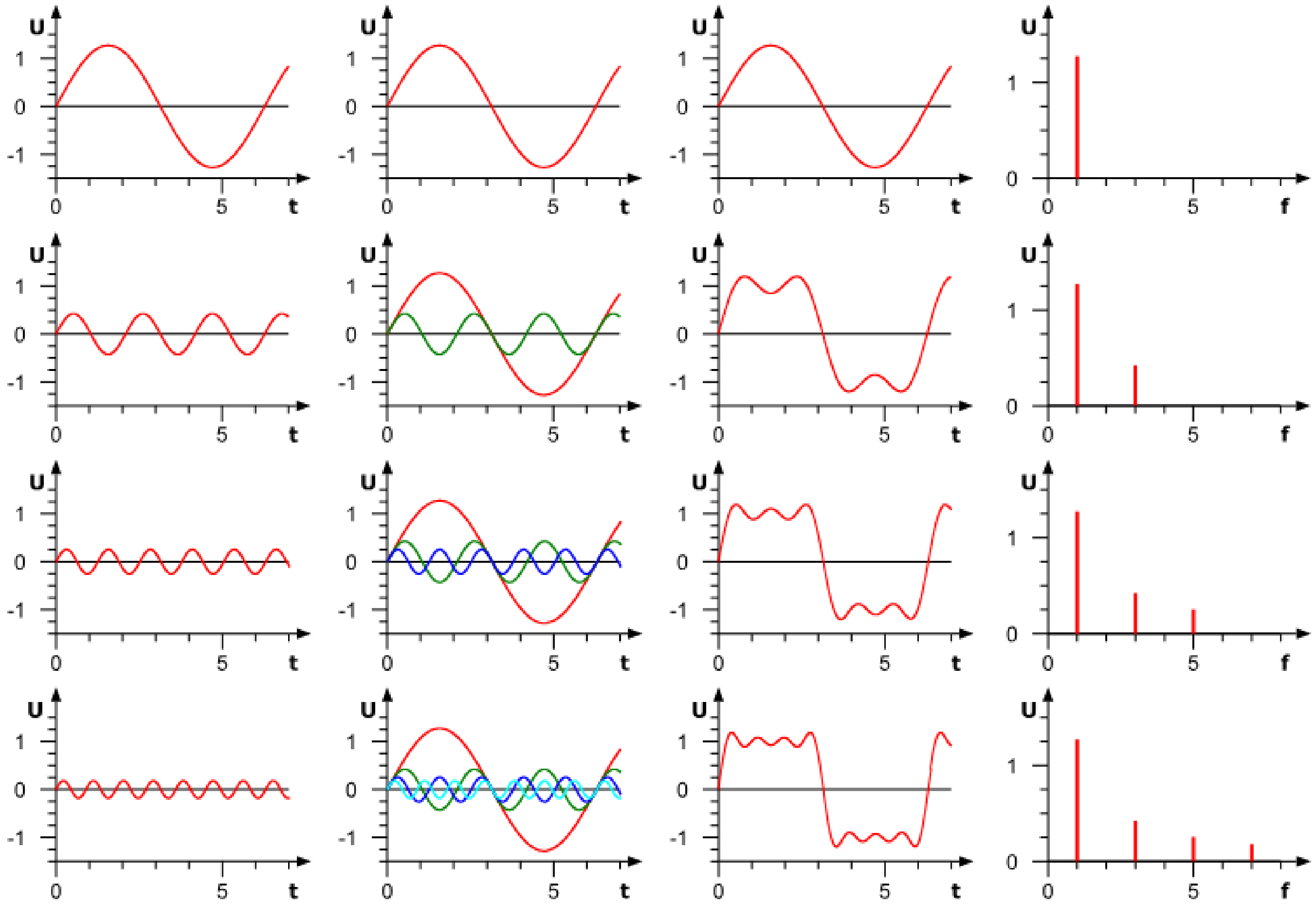
Time Domain & Frequency Domain

A signal containing a 50 Hz sinusoid of amplitude 0.7 and a 120 Hz sinusoid of amplitude 1





Time-Frequency Domain Transform



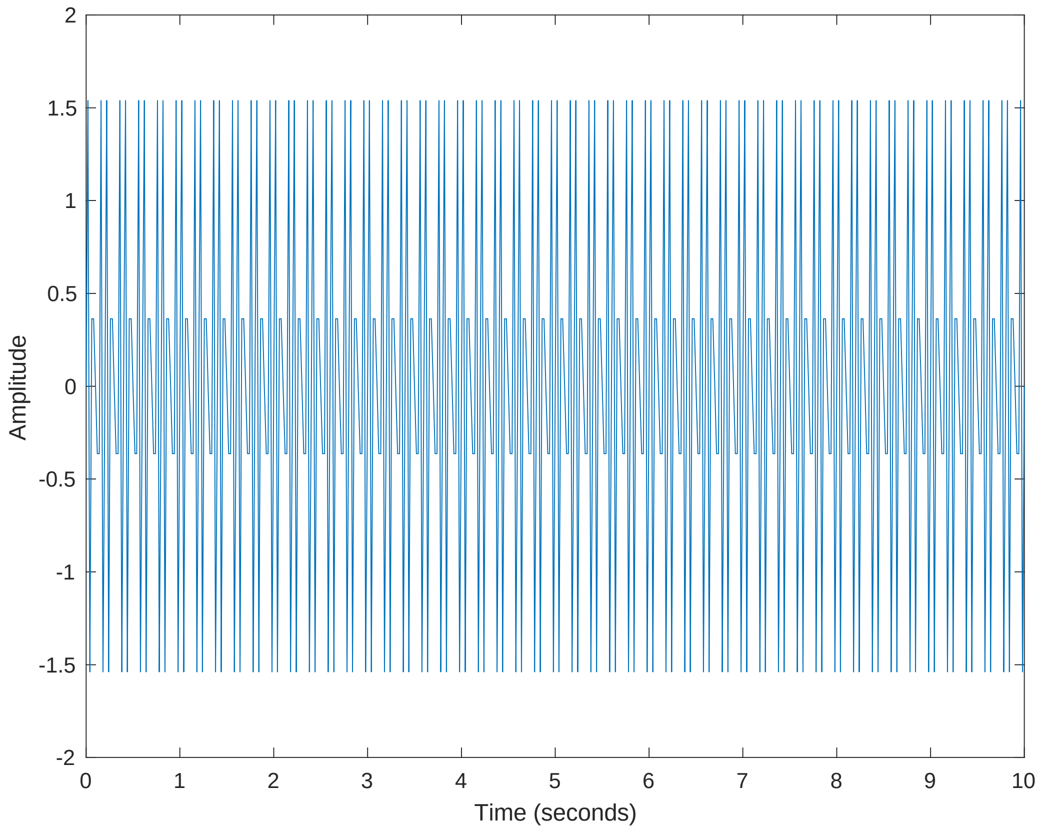
Fourier Transforms

Fourier transform is a mathematical formula that transforms a signal sampled in time or space to the same signal sampled in temporal or spatial frequency. In signal processing, the Fourier transform can reveal important characteristics of a signal, namely, its frequency components.

傅里叶变换是将按时间或空间采样的信号与按频率采样的相同信号进行关联的数学公式。在信号处理中，傅里叶变换可以揭示信号的重要特征（即其频率分量）。

FFT in MATLAB

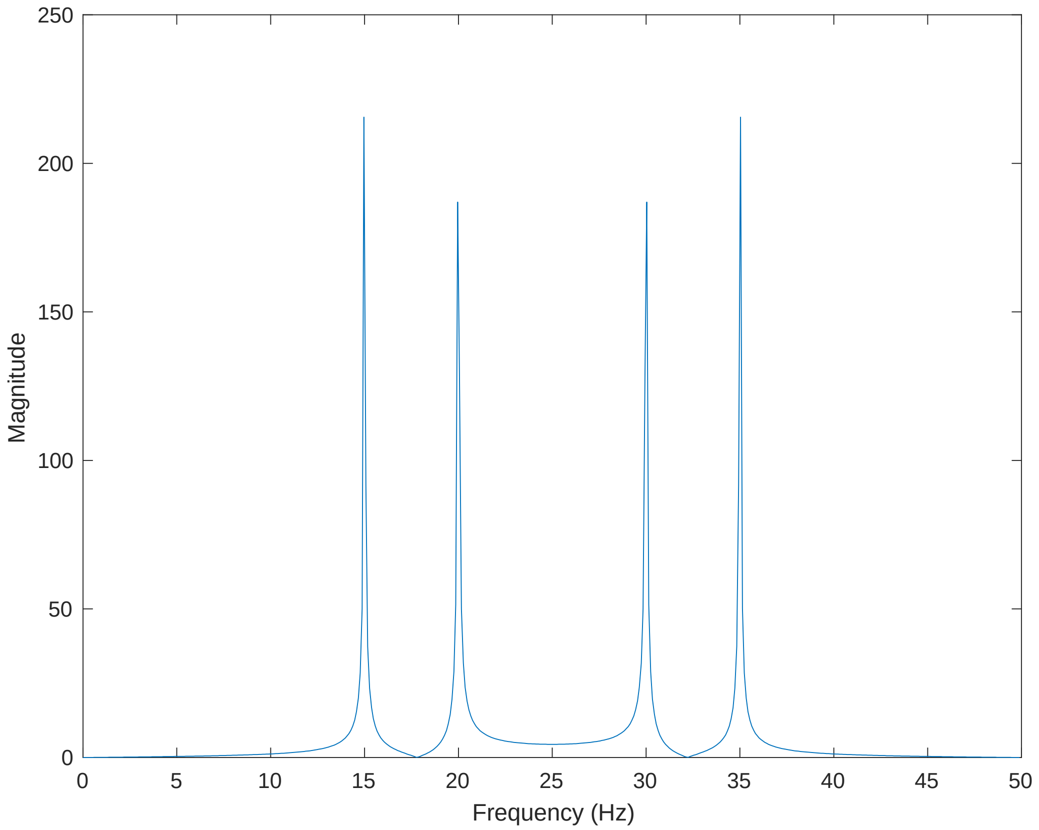
```
T = 10;
fs = 50;

t = 0 : 1/fs : T;
x = sin(2*pi*15*t) + sin(2*pi*20*t);

plot(t, x);
xlabel('Time (seconds)');
ylabel('Amplitude');
```



FFT in MATLAB

When you plot the magnitude of the signal as a function of frequency, the spikes in magnitude should correspond to the signal's frequency components of 15 Hz and 20 Hz.

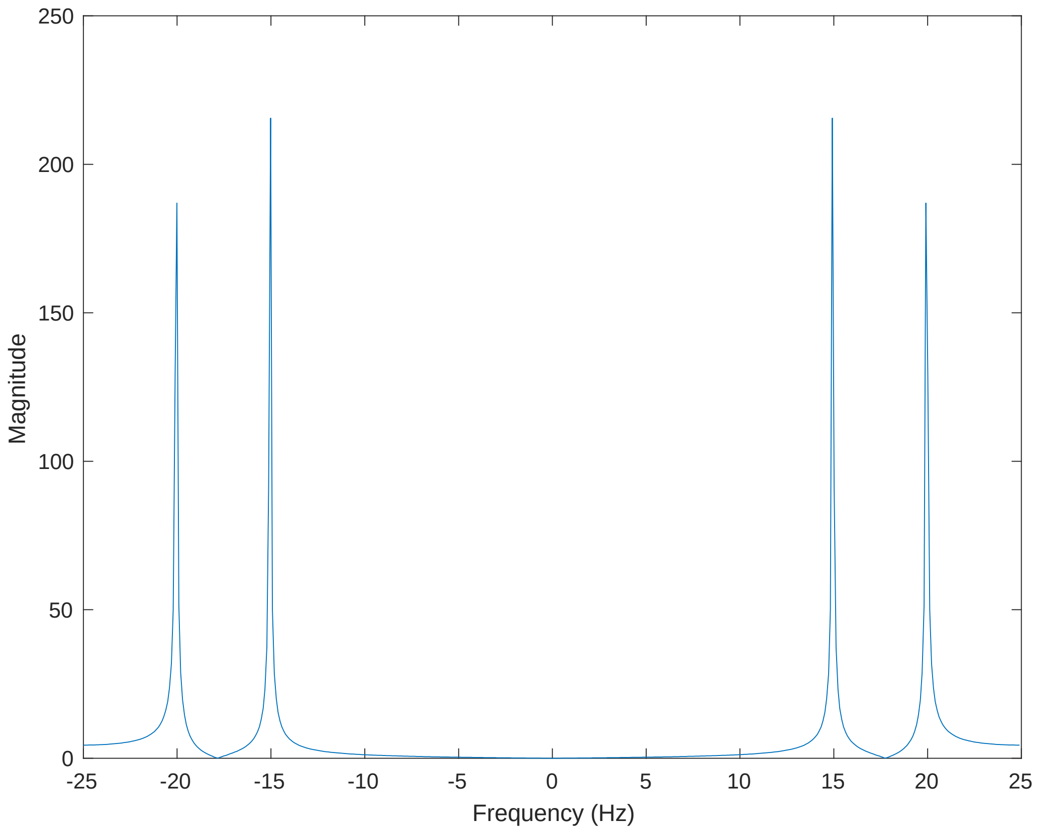
```
N = length(x);

y = fft(x);
f = (0:N-1)*fs/N;

plot(f, abs(y));
xlabel('Frequency (Hz)');
ylabel('Magnitude');
```



FFT in MATLAB

The transform also produces a mirror copy of the spikes, which correspond to the signal's negative frequencies. To better visualize this periodicity, you can use the **fftshift** function, which performs a zero-centered, circular shift on the transform.

```
yshift = fftshift(fft(x));
fshift = (-N/2:N/2-1)*fs/N;

plot(fshift, abs(yshift));
xlabel('Frequency (Hz)');
ylabel('Magnitude');
```



Fourier Analysis

Continuous Periodic Signal	Fourier Series
Continuous Aperiodic Signal	Fourier Transform
Discrete Periodic Signal	Discrete Fourier Series & Discrete Fourier Transform (FFT in MATLAB)
Discrete Aperiodic Signal	Discrete Time Fourier Transform

- Only when the original signal is a periodic signal and the slicing time is an integer multiple of the signal period, can DFT accurately reveal the spectrum.
- The spectrum calculated by DFT may differ from the one based on the original, and the phenomenon is commonly known as **"spectrum leakage"**.

Homework

Generate three square wave signals with the same pattern but different sampling rates. Transform you generated time-domain signals into frequency-domain.

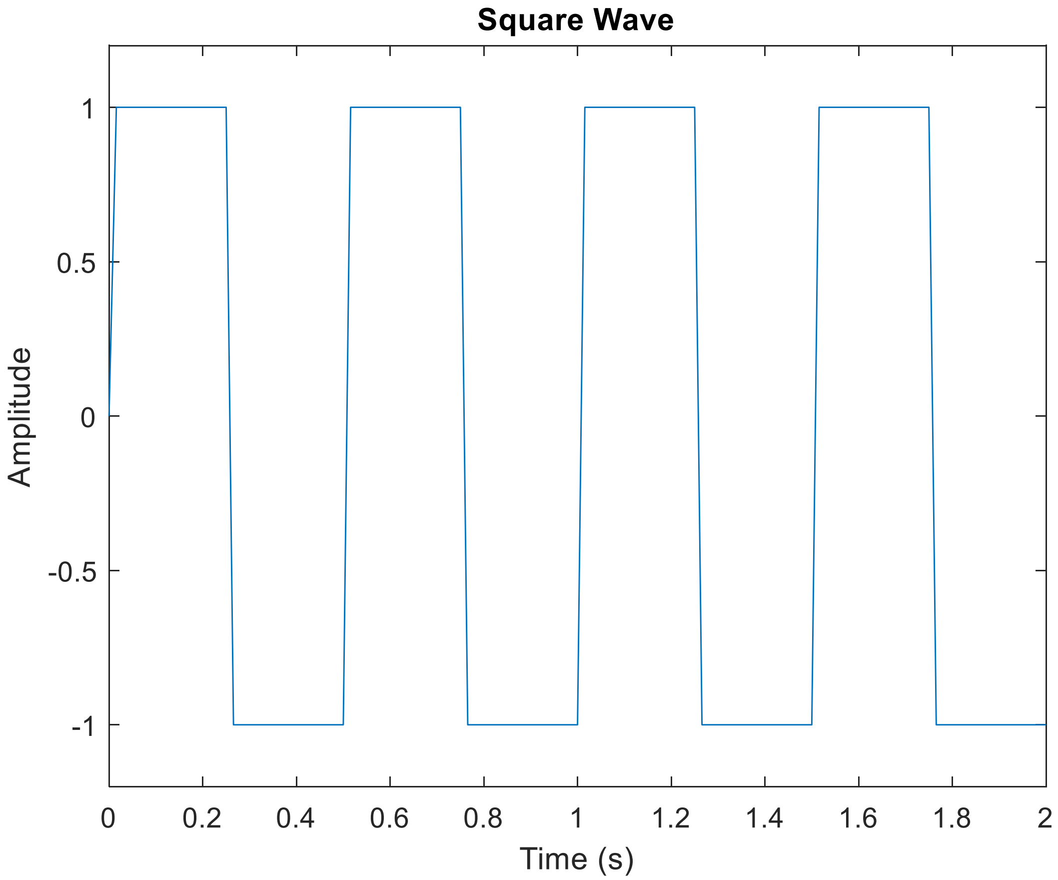
Plot the generated signals and corresponding FFT results (totally 6 plots), in one figure. (***subplot*** function is recommend to be used.)

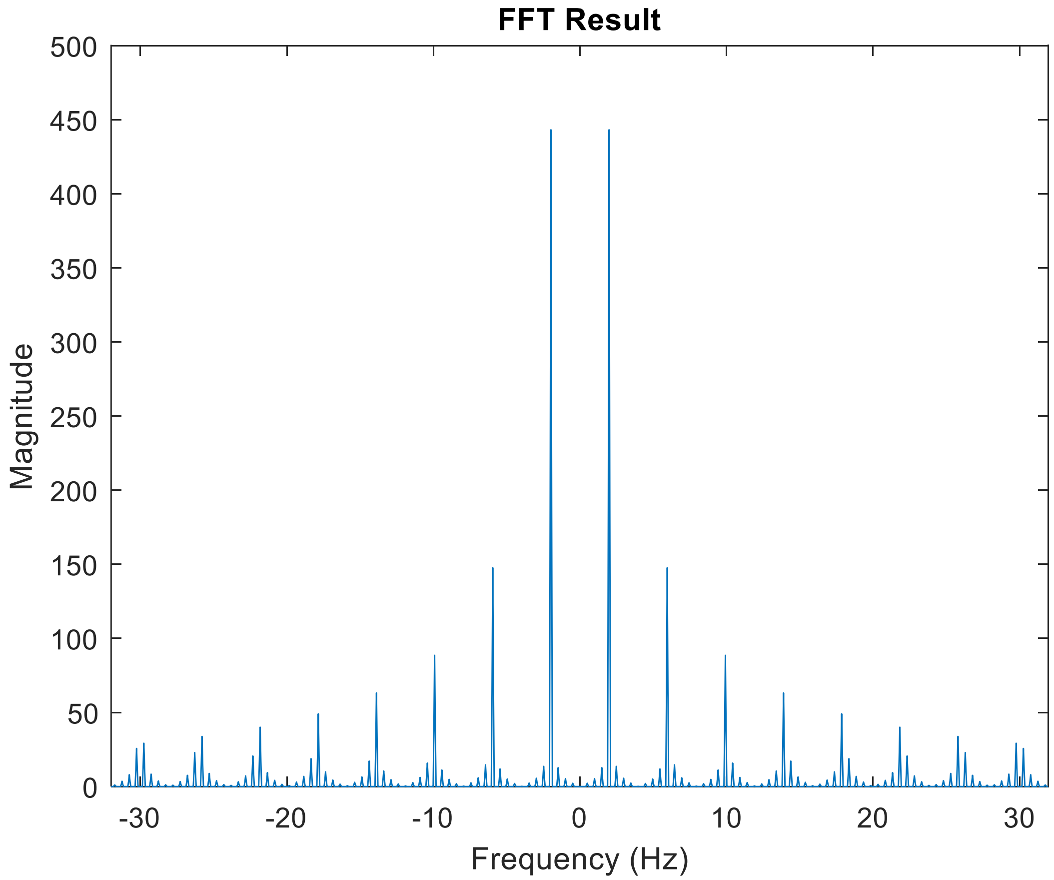
Reconstruct one of the generated square wave signal using the main frequency components based on FFT results.

Show how many components are used when human ear cannot distinguish the original and the reconstructed one. Save the original and the reconstructed square wave signal.

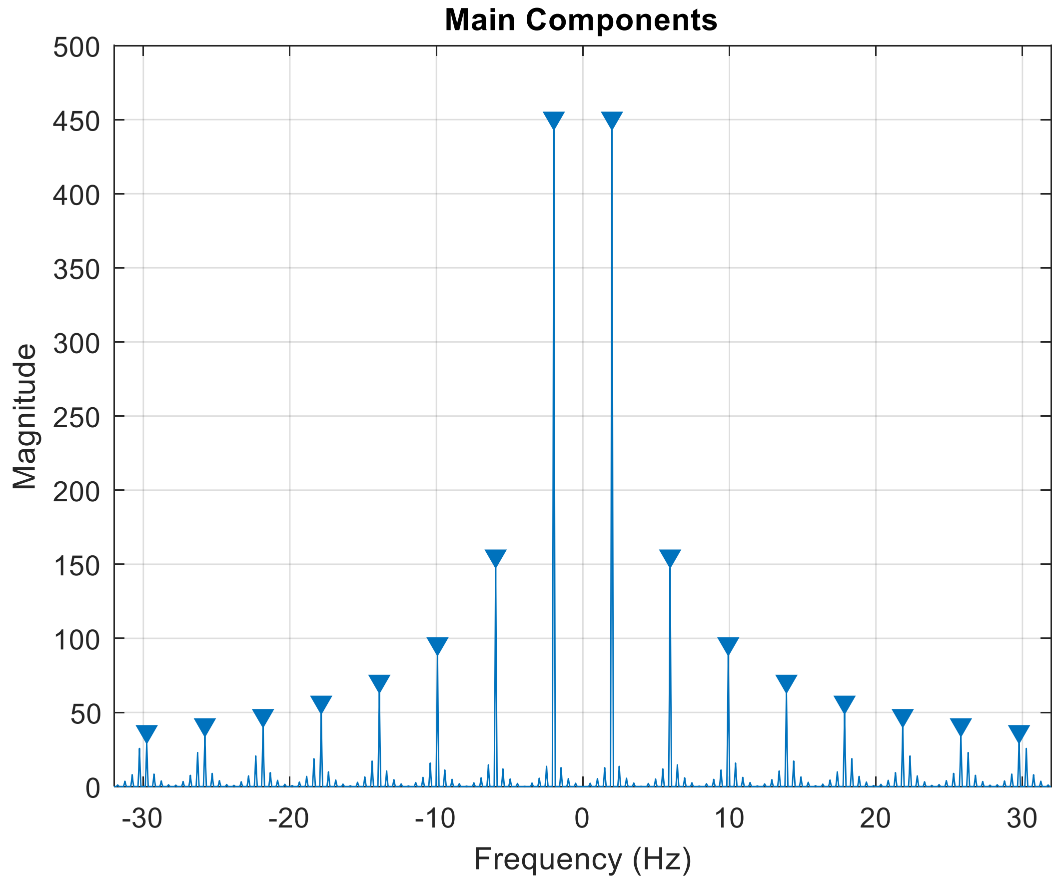
Pack your codes, figure and wav files into SID.zip. Hand in your SID.zip in bb system.

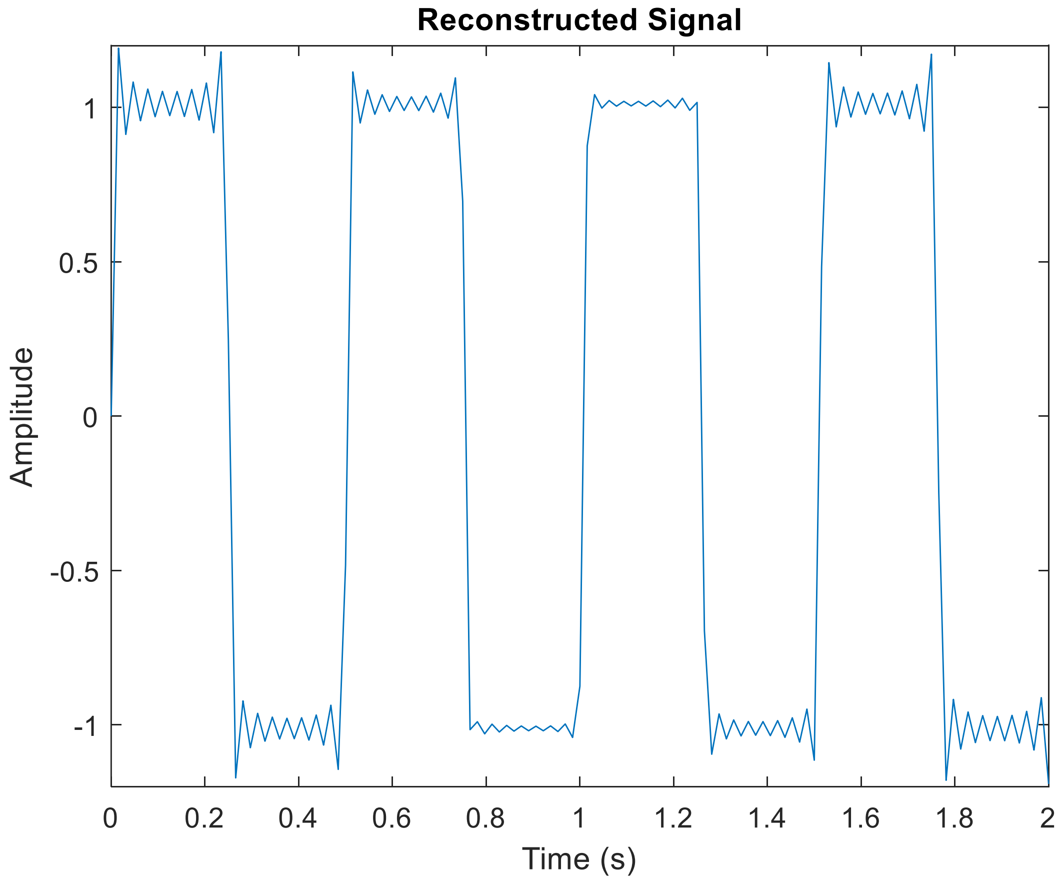
Homework


Square Wave
Amplitude
1
0.5
0
-0.5
-1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Time (s)

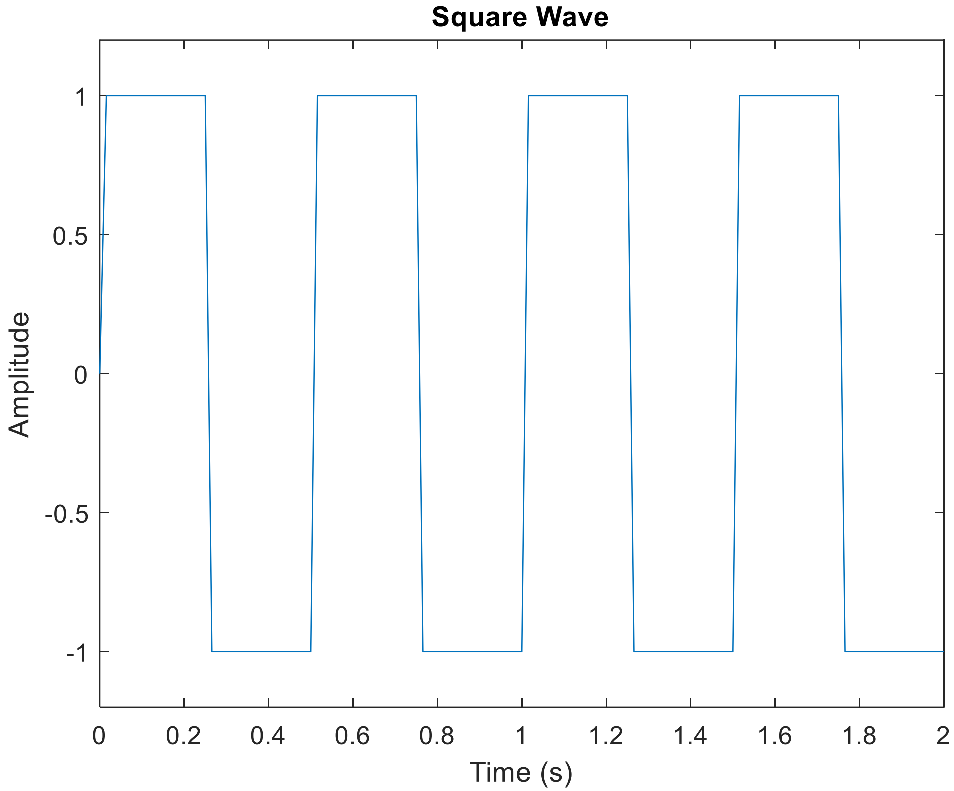

FFT Result
Magnitude
500
450
400
350
300
250
200
150
100
50
0
-30
-20
-10
0
10
20
30
Frequency (Hz)

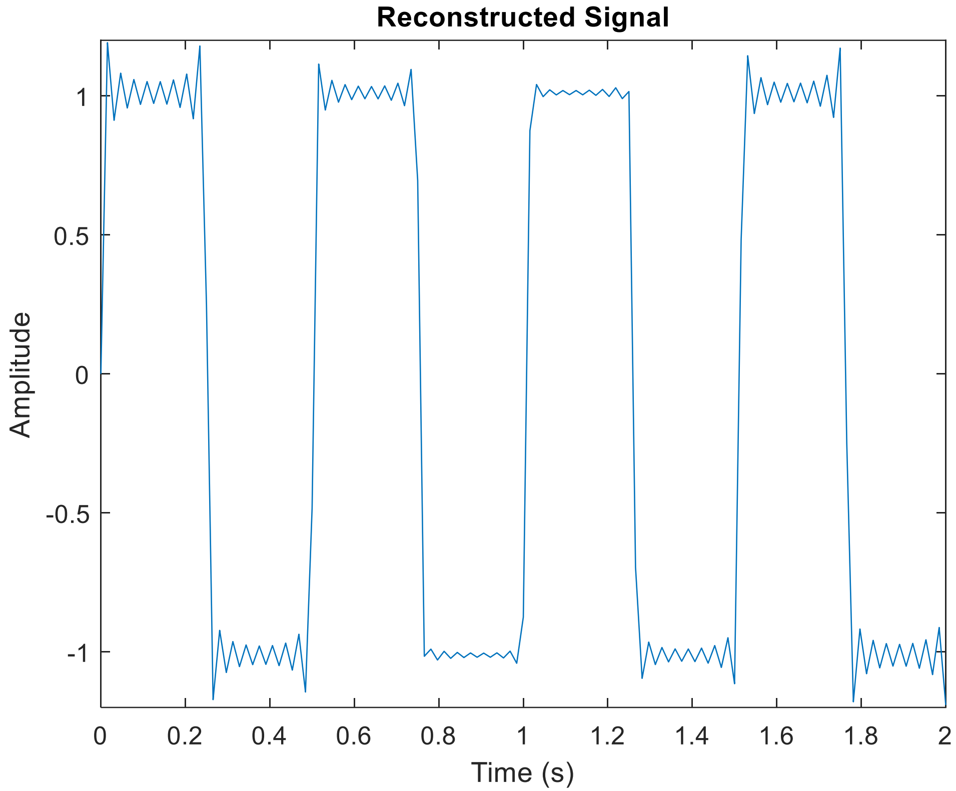
Homework


Main Components
Magnitude
Frequency (Hz)
500
450
400
350
300
250
200
150
100
50
0
-30
-20
-10
0
10
20
30


Reconstructed Signal
Amplitude
Time (s)
1
0.5
0
-0.5
-1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

Homework





Gibbs Phenomenon:

The partial sum in the vicinity of the discontinuity exhibits **ripples** whose amplitude does **NOT** seem to decrease with increasing *N*.

Reference

物联网前沿实践 - 清华大学 - 王继良 （https://iot-book.github.io）

- 第2章 信号处理基础
- 第3章 傅立叶分析