



电子与电气工程系  
Department of Electrical and  
Electronic Engineering

# Diffusion

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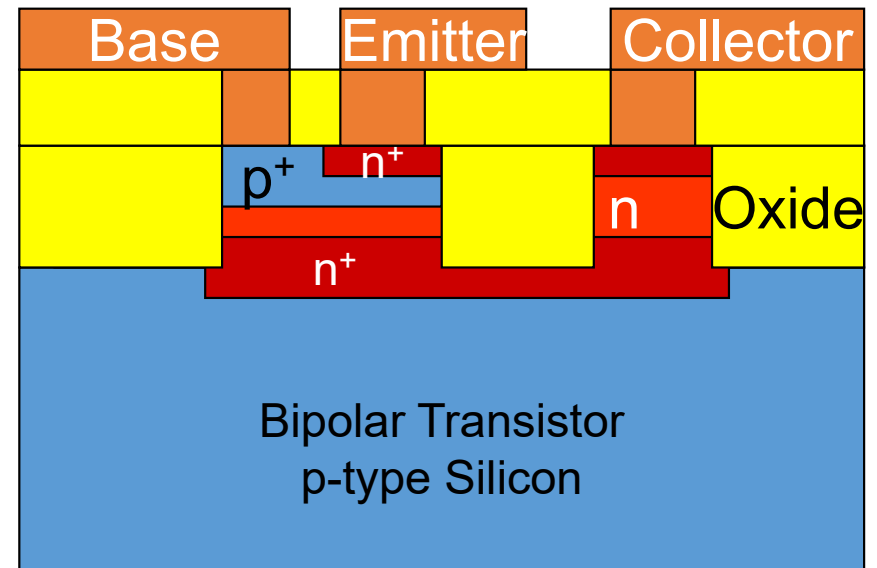
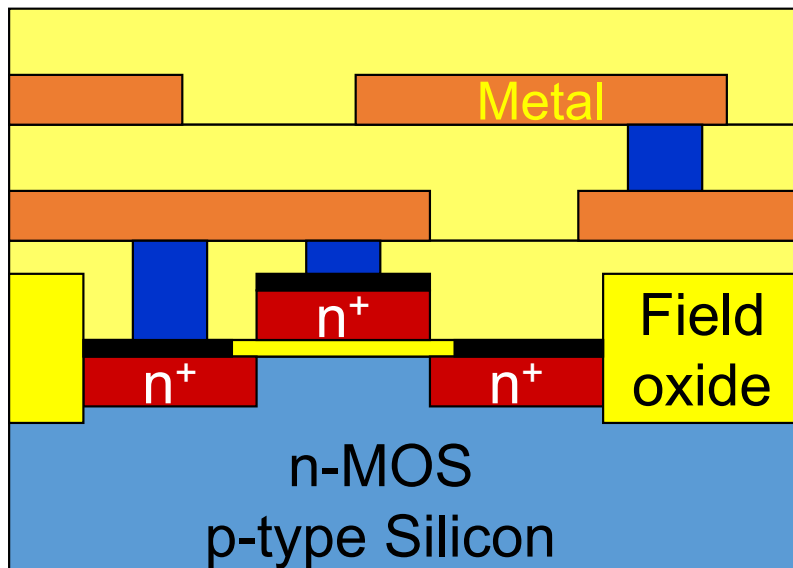
# Outline

- **Applications**
- **Methods & Equipment**
  - Predeposition
  - Drive-in
- **Diffusion mathematics**
  - The transport equation
  - The continuity equation
  - Field enhanced diffusion
- **Diffusion in solids**
- **Linear Diffusion**
  - Predeposition
  - Drive-in
- **Masking**
- **Segregation**
- **Diffusion & point defects**
  - High doping effects
  - Enhanced/retarded diffusion
- **Non-linear diffusion**
  - Predeposition
  - Drive-in
- **Evaluation**
  - Chemical & electrical conc.
  - Sheet resistance
  - Junction depth

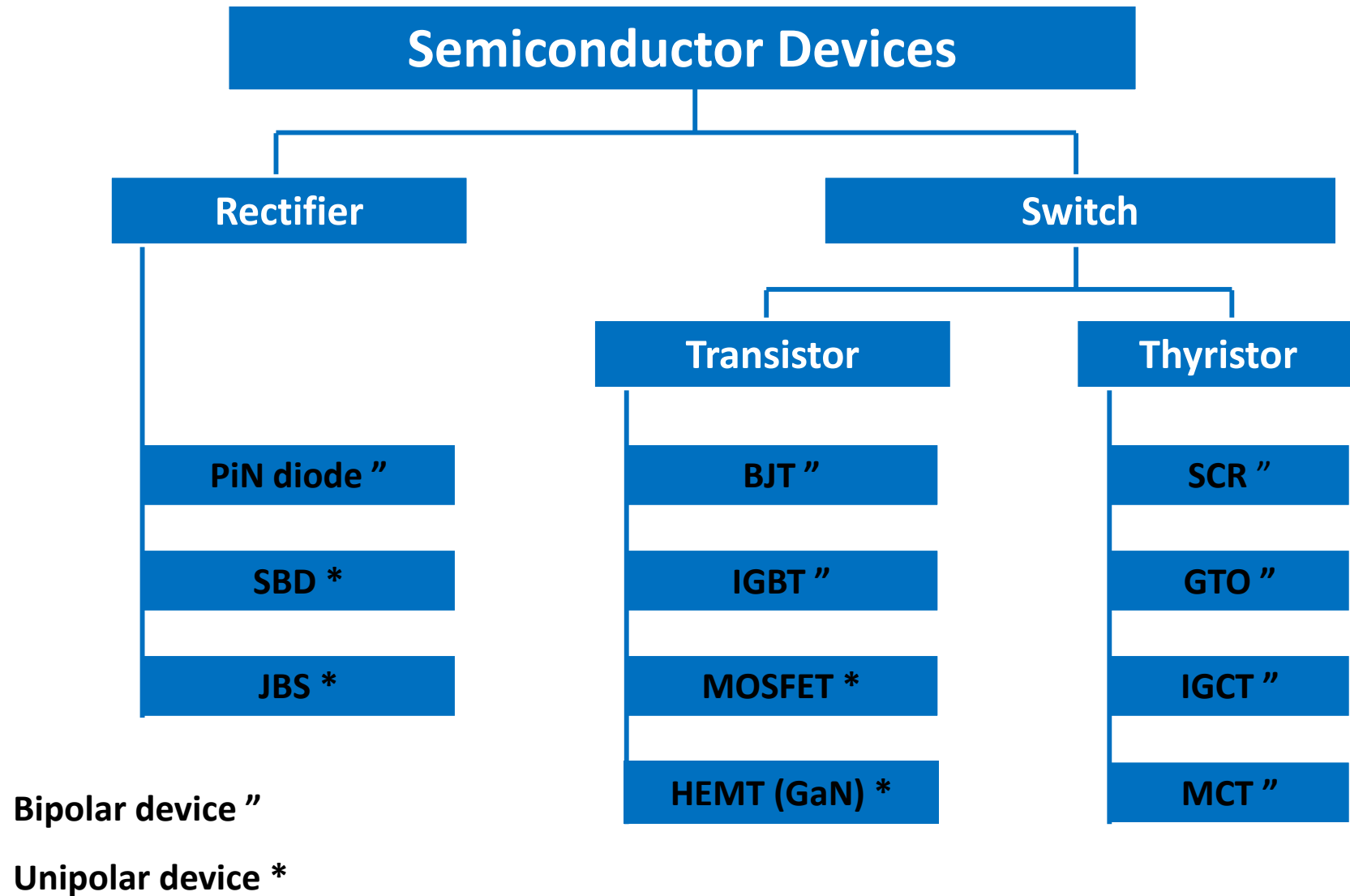


# Applications of Diffusion

- Diffusion of dopants  
pn-junctions; MOST; BJT; Resistors, piezoresistors
- Diffusion of contaminants  
Gettering

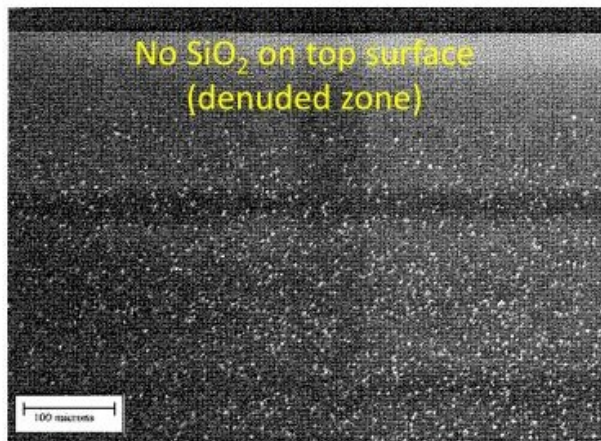
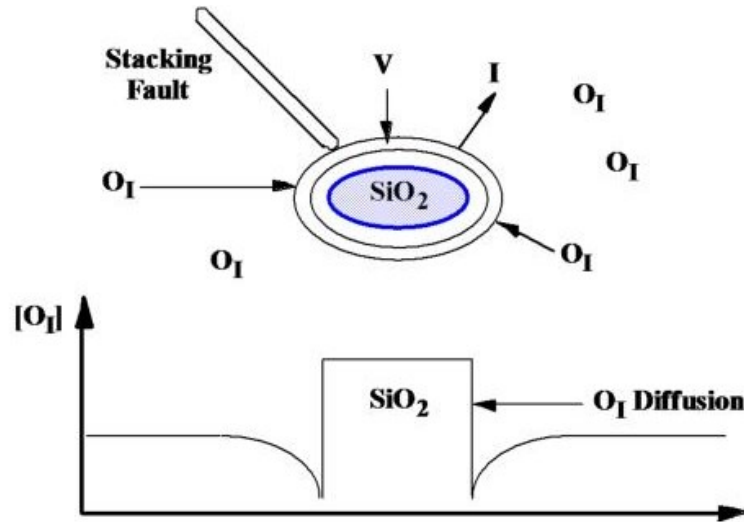


# Category of Main Semiconductor Devices

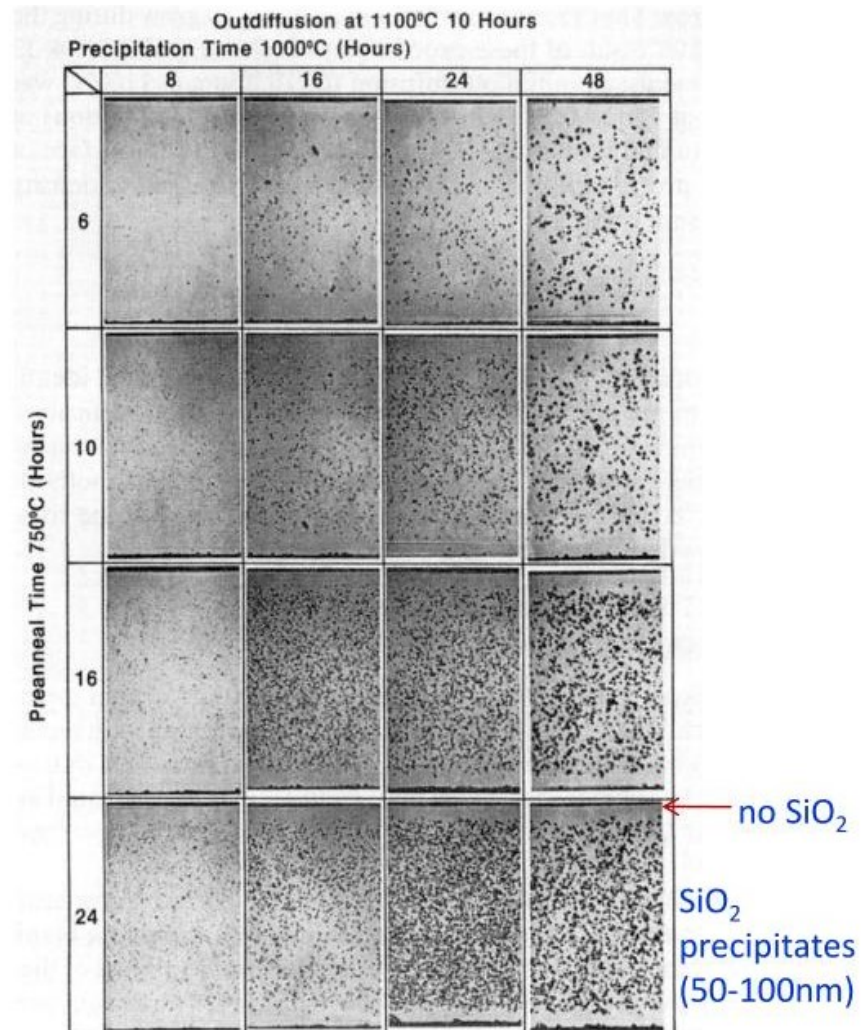


# Applications of Diffusion

- Diffusion of contaminants → Gettering



$\text{SiO}_2$  precipitates (white dots) in bulk Si



# Doping of Semiconductors

## Dopants in Silicon

- Donors (V): P, As, Sb
- Acceptors (III): B, Al, Ga, In

## Doping a two step process:

### 1. Predeposition -

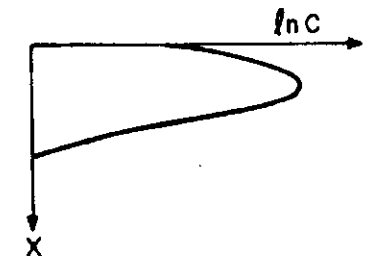
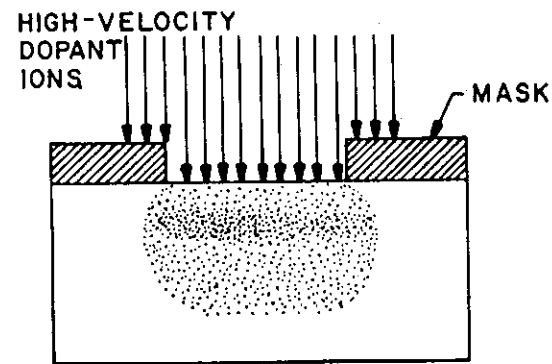
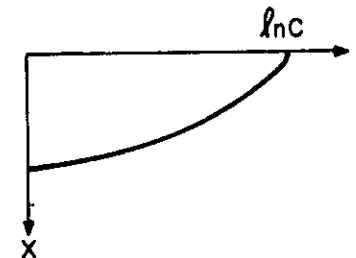
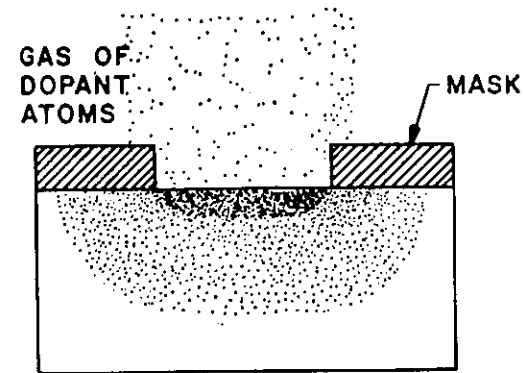
Introduce dopants

- Diffusion from gas-phase
- Diffusion from thin films
- Ion implantation
- Grown-in dopants

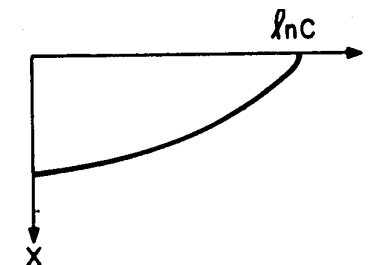
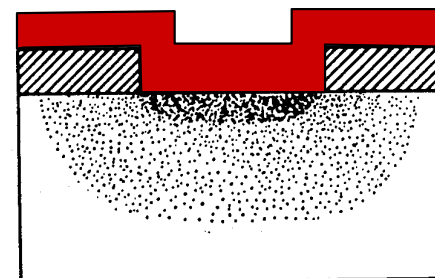
### 2. Drive-in -

Redistribute dopants

- Furnace Anneal
- Rapid Thermal Annealing



### Doped film



# Predeposition: Gas Phase or Ion Implantation

## Gas Phase / Doped Film:

### Advantages

- Batch process
- No damage
- Low cost

### Disadvantages

- Only SiO<sub>2</sub> masks
- Only moderate/high doses
- Only high surface conc.
  - $C < C_{sol}$

## Ion Implantation:

### Advantages

- All materials mask
- Precise Dose Control
- $10^{11}$ - $10^{16}$ /cm<sup>2</sup> Doses
- Buried profiles

### Disadvantages

- High cost
- Damage causing:
  - Enhanced diffusion
  - Dislocations

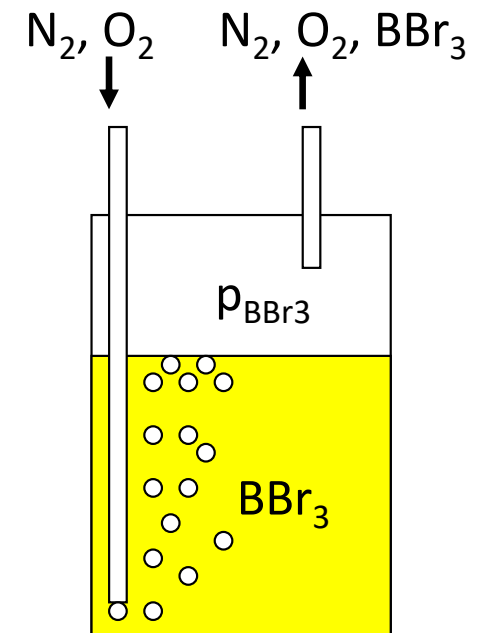
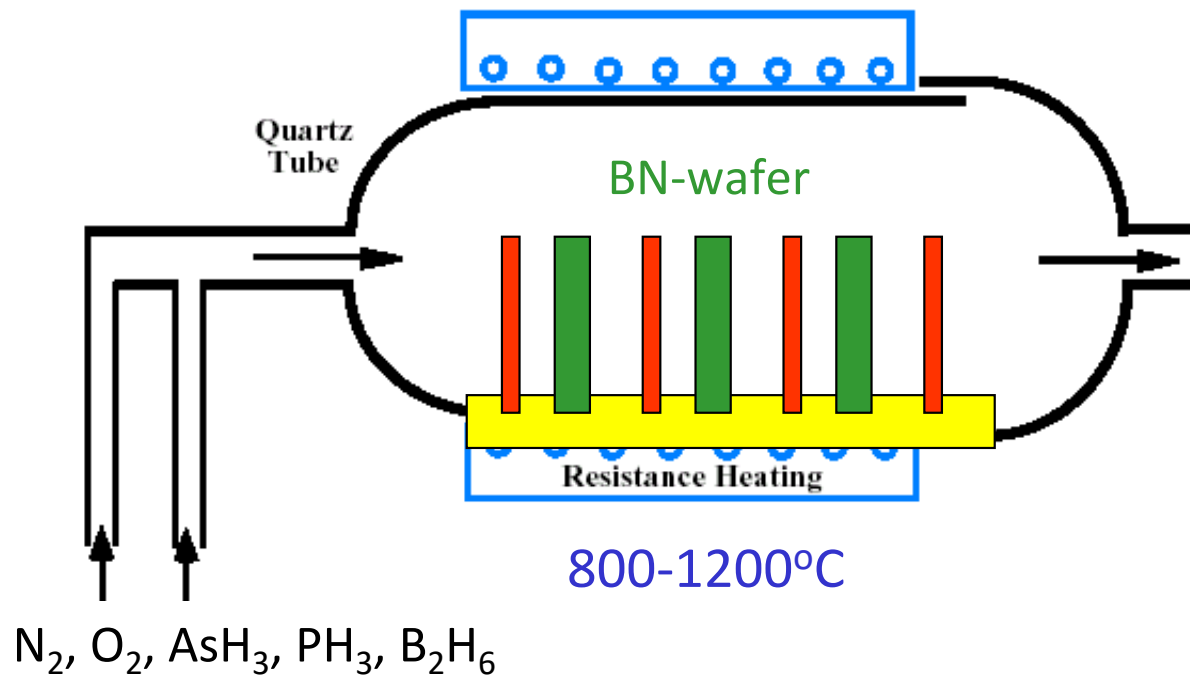


# Gas Phase Predeposition

A furnace process similar to thermal oxidation.

Dopant sources: (The real source is always the oxide)

- Gas:  $\text{AsH}_3$ ,  $\text{PH}_3$ ,  $\text{B}_2\text{H}_6$
- Vaporised liquid:  $\text{POCl}_3$ ,  $\text{BBr}_3$ ,
- Vapours of a solid:  $\text{B}_2\text{O}_3$ ,  $\text{P}_2\text{O}_5$ ,  $\text{As}_2\text{O}_5$



Vaporiser:  
Controlled  
temperature



# Sheet Resistance

**Homogenous sample :**

$$R = \rho \frac{L}{Wh} = \frac{\rho}{h} \frac{L}{W} = \frac{L}{W} R_{sh}$$

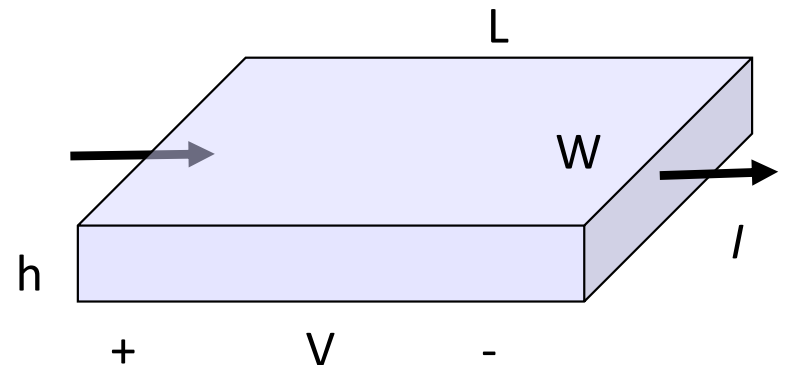
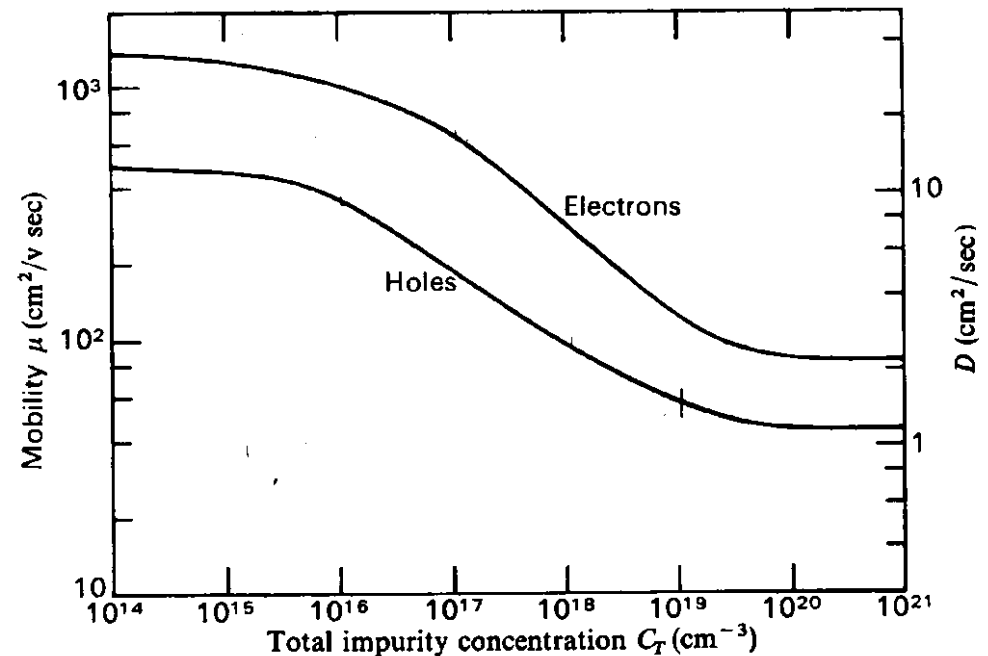
**Sheet resistance :**  $R_{sh} = \frac{\rho}{h}$

**Inhomogenous :**

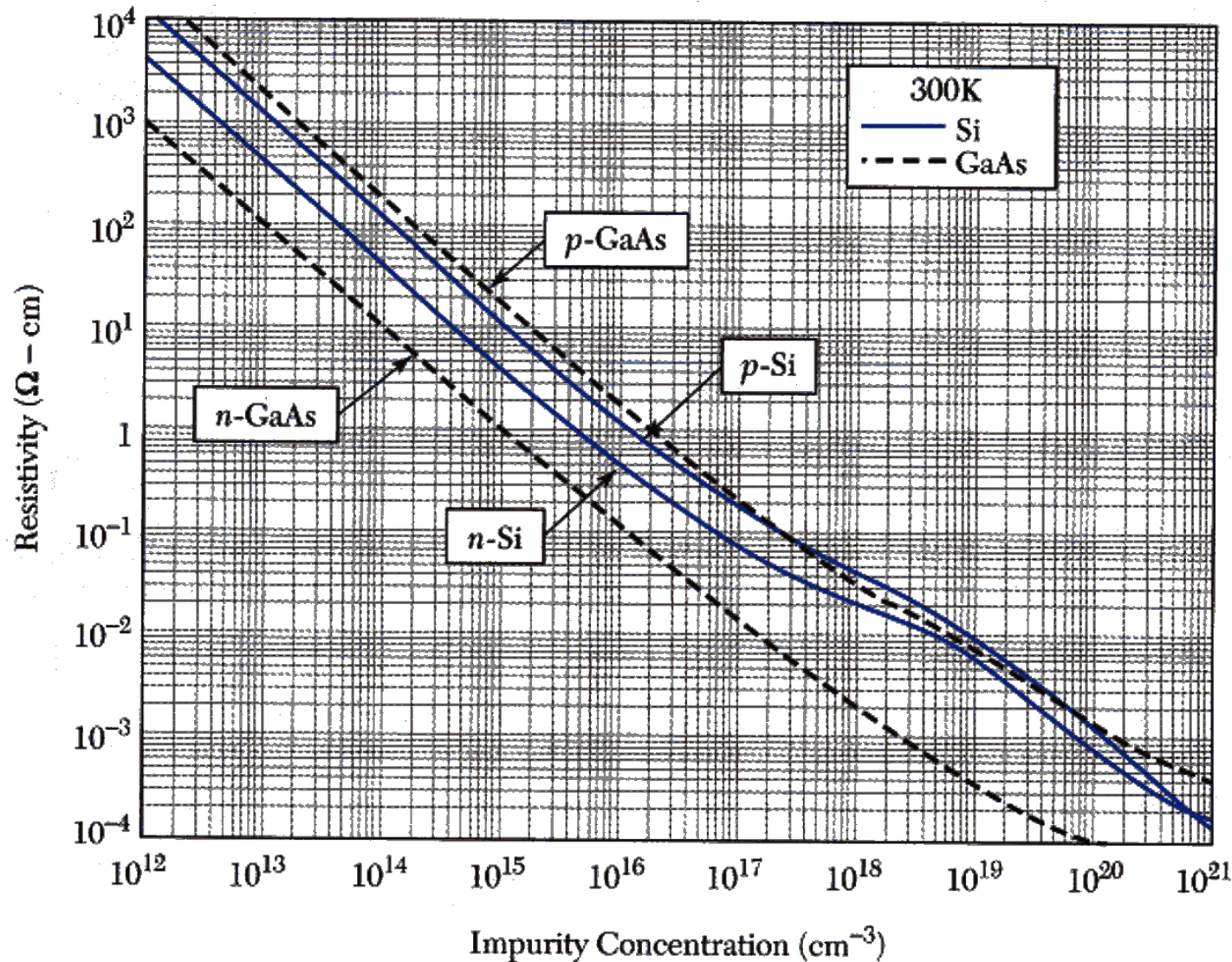
**Conductivity :**  $\sigma = q\mu_n n + q\mu_p p$

$$\frac{1}{R} \equiv \frac{W}{L} \int_0^h \sigma(x) dx \equiv \frac{W}{L} \frac{1}{R_{sh}} \Rightarrow$$

$$R_{sh} = \frac{1}{\int_0^h \sigma(x) dx} \cong \frac{1}{\int_0^h q\mu_n n dx} \approx \frac{1}{q\mu_n Q}$$



# Resistivity of Doped Silicon



## Dopants:

Donors: **P, As, Sb**

Acceptors: **B, Al, Ga**

$$\rho = \frac{1}{q\mu_n n + q\mu_p p}$$

$$\rho_n \approx \frac{1}{q\mu_n N_D}$$

$$\rho_p \approx \frac{1}{q\mu_p N_A}$$



# Solid Solubility

Dopants are soluble in bulk silicon up to a maximum value before they precipitate into another phase

## Purpose:

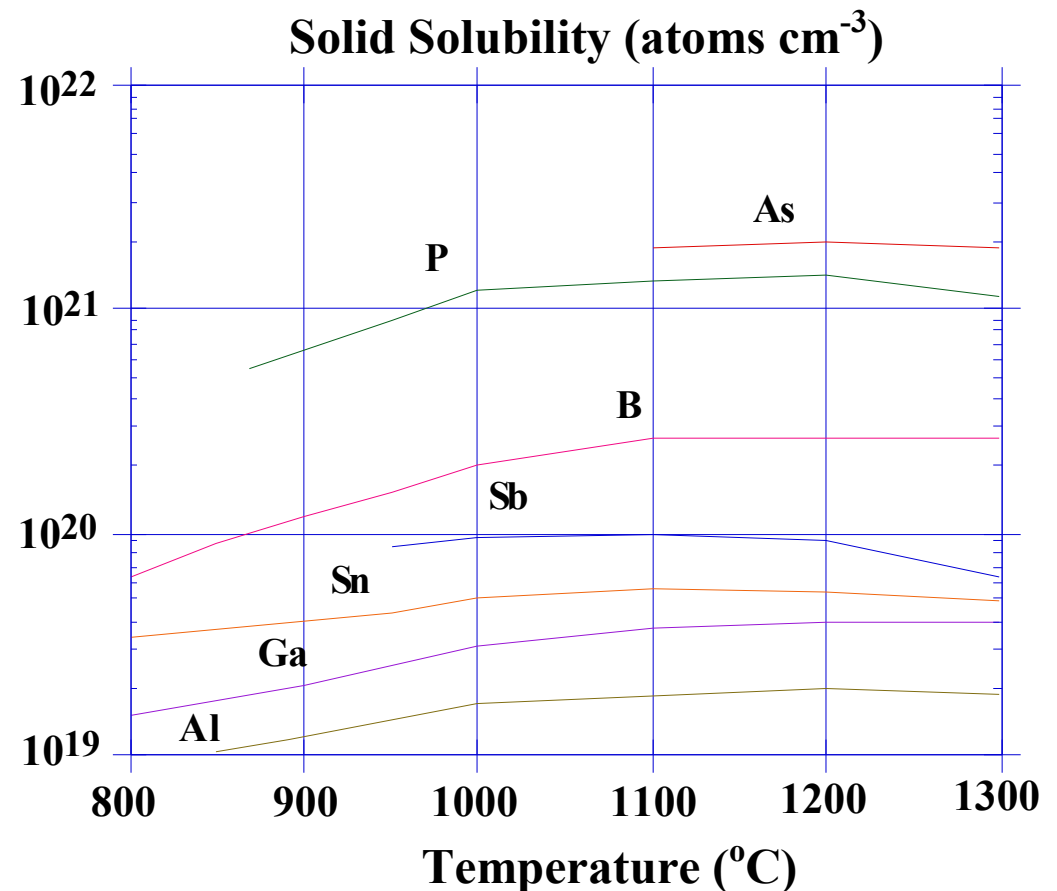
1. Change type
2. Change conductivity

- Concentration  $\leq C_{\text{sol}}$
- Electrically active Concentration:

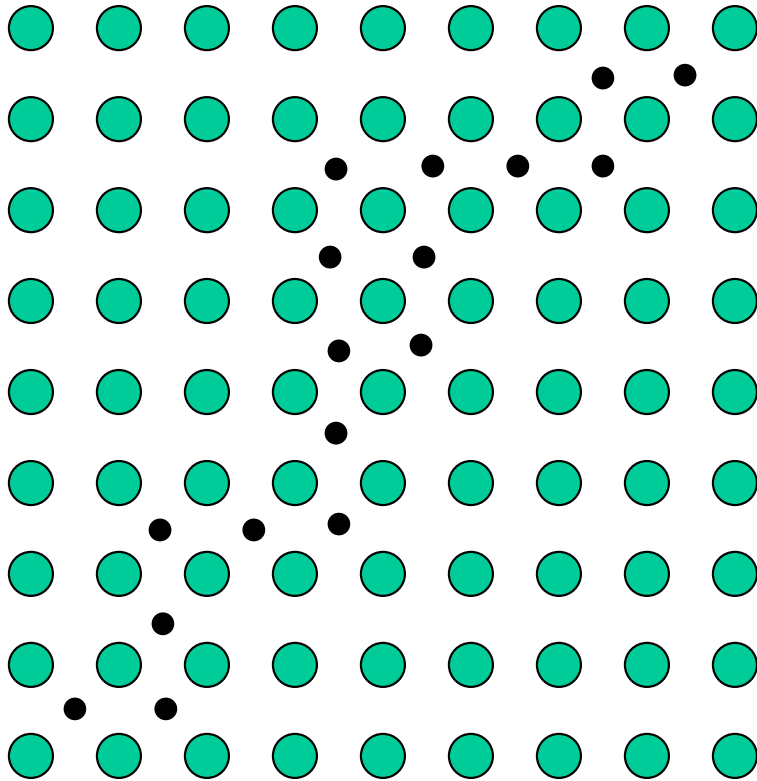
$$C_{\text{elec}} \leq C_{\text{chem}}$$

## Example: As-clustering

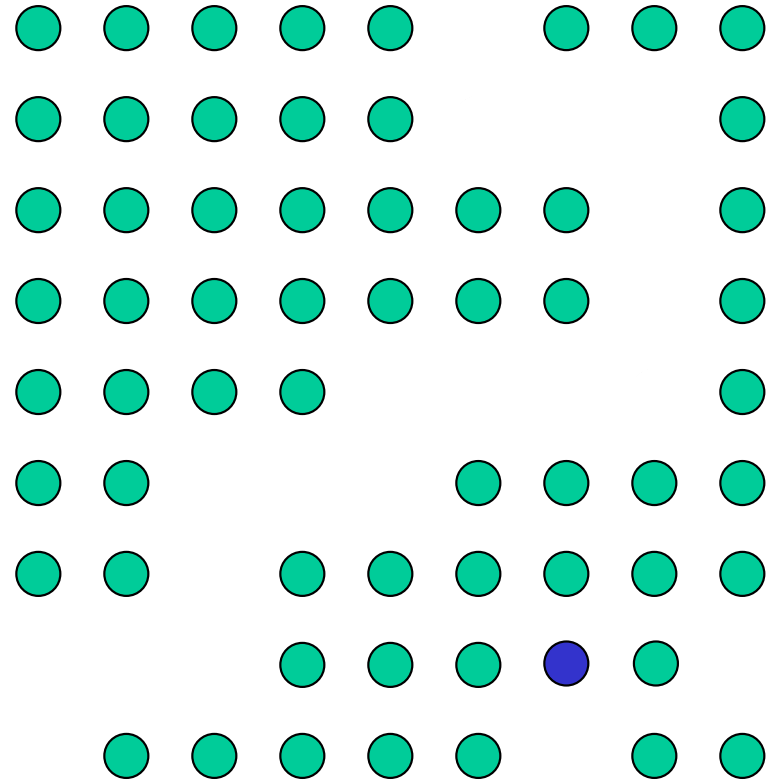
- $\text{As}_4\text{V}$
- $4\text{As} + \text{V} \leftrightarrow \text{As}_4\text{V}$
- $C_{\text{cluster}} = K C_{\text{As}}^4 C_{\text{V}}$
- Important at high doping
  - $C_{\text{chem}} < 2 \times 10^{21} / \text{cm}^3$
  - $C_{\text{elec}} < 2 \times 10^{20} / \text{cm}^3$



# Diffusion of Point Defects

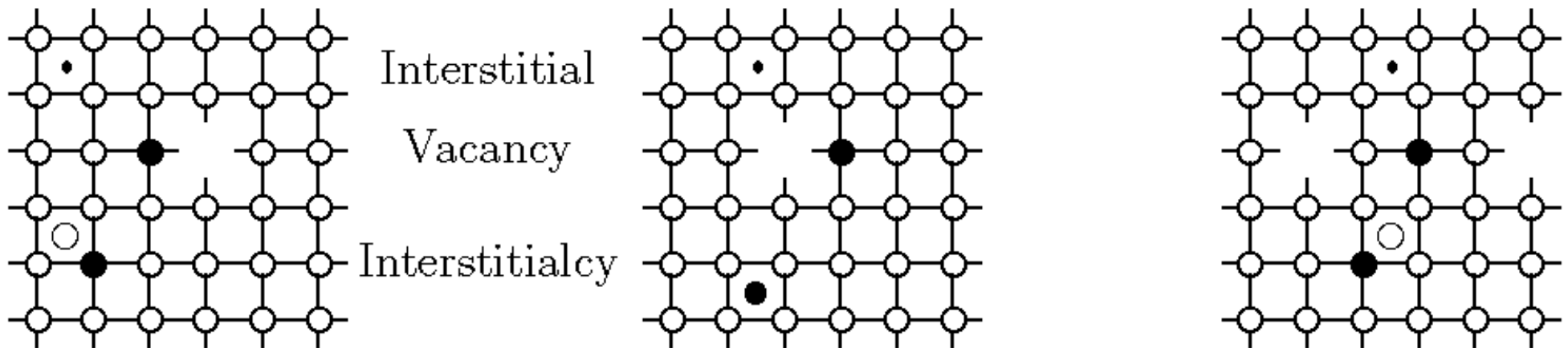


Interstitial diffusion  
Self interstitials  
Small foreign atoms



Vacancy diffusion  
Substitutional atoms  
Dopants

# Point Defect Assisted Diffusion

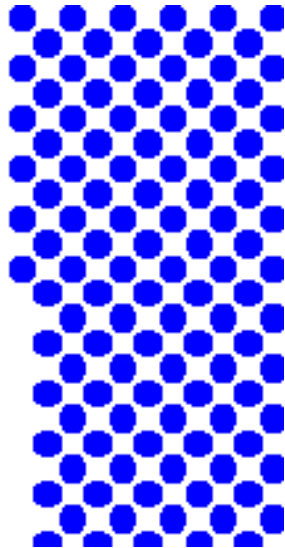


Diffusion of foreign atoms in Silicon can occur in several ways:

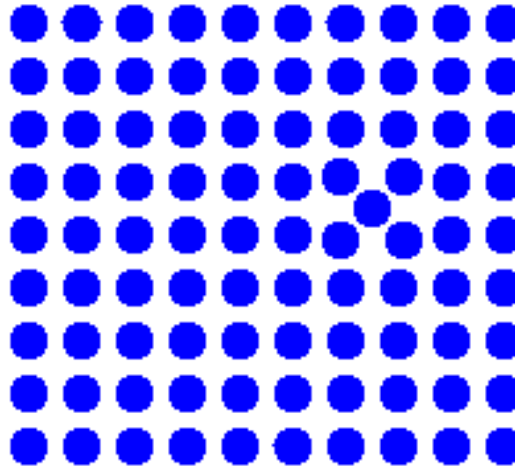
1. Direct Interstitial Diffusion: Small atoms, fast.
2. Vacancy assisted. Assumed for some dopants. Slow.
3. Interstitialcy – interstitial assisted diffusion. Slow.
  - Many different modes suggested.
  - Assumed for some dopants.

# Point Defect Assisted Diffusion

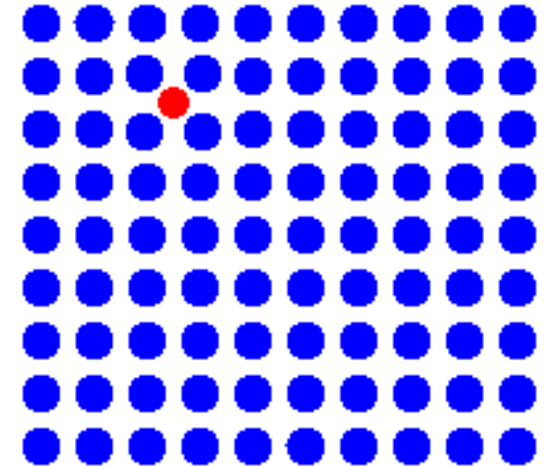
Vacancy  
diffusion



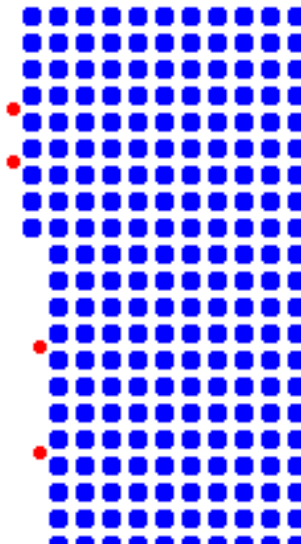
Indirect interstitial  
Mechanism



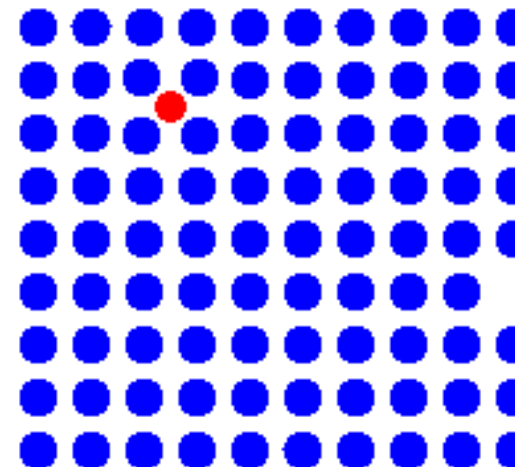
Kick-out mechanism



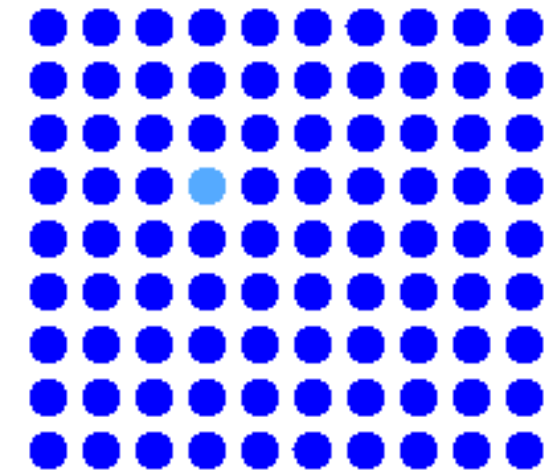
Direct  
interstitial  
mechanism



Frank Turnbull



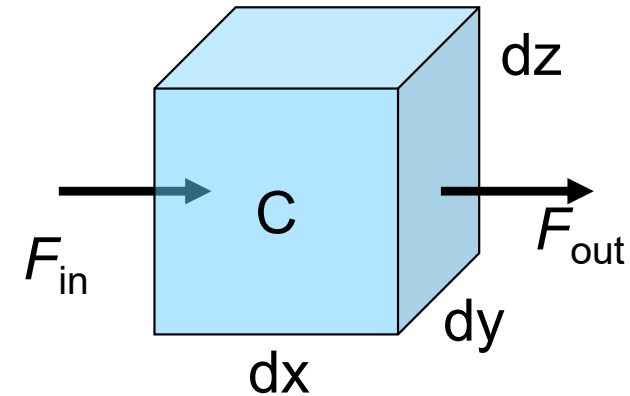
Exchange "rotate"



# Linear Predeposition - Model

Purpose: a controlled dopant dose  $Q$

- Control surface concentration  $C_s$ 
  - Obtain solid solubility  $C_{sol}$
- Control temperature ( $D=D(T)$ )
- Control time  $t$



1 D. Model, constant diffusivity:

Fick's 2. Law:  $\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2}$

Boundary conditions:  $C(0, t) = C_s, \quad C(\infty, t) = 0$

Initial conditions:  $C(x, 0) = 0$

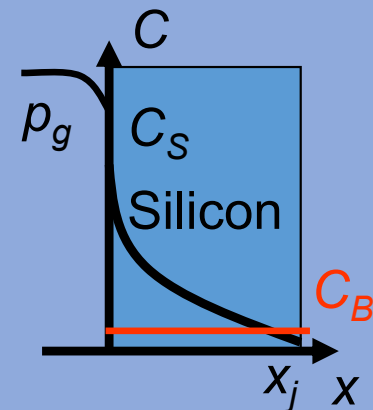
Solution:  $C(x, t) = C_s \operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)$

Dose:  $Q(t) = \int_0^\infty C(x, t) dx = \frac{2}{\sqrt{\pi}} C_s \sqrt{Dt}$

Junction depth :

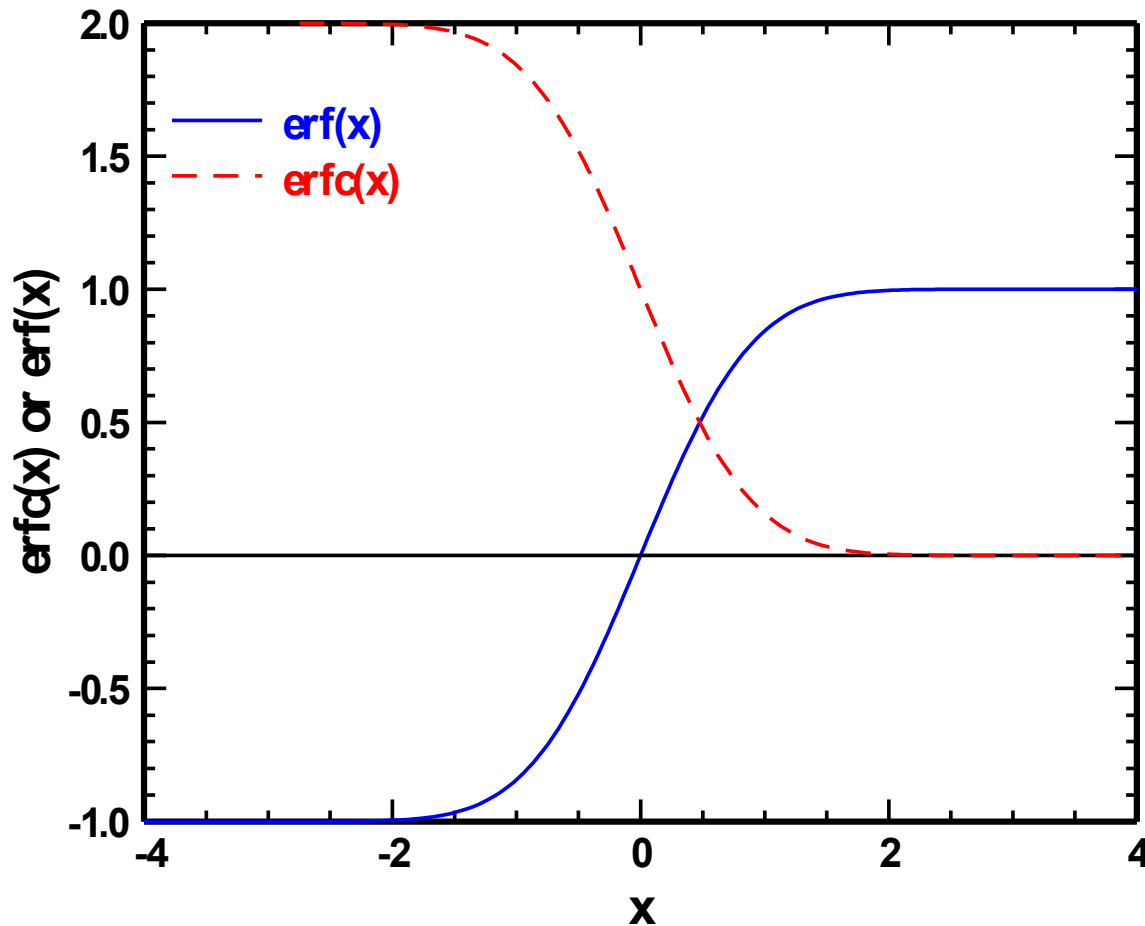
$$C(x_j, t) = C_B \Rightarrow x_j = 2\sqrt{Dt} \operatorname{erfc}^{-1}\left(\frac{C_B}{C_s}\right)$$

**Fick's second law** describes how the change in concentration in a volume element is determined by the fluxes in/out of the volume.





# Error Functions



$$\text{Definition: } \text{erf}(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x \exp(-u^2) du$$

$$\text{Definition: } \text{erfc}(x) \equiv 1 - \text{erf}(x)$$

$$\text{erf}(\pm\infty) = \pm 1, \quad \text{erf}(0) = 0$$

$$\text{erfc}(\infty) = 0, \quad \text{erfc}(0) = 1, \quad \text{erfc}(-\infty) = 2$$

$$\text{erf}(x) \approx \frac{2}{\sqrt{\pi}} x, \text{ for } x \ll 1$$

$$\text{erfc}(x) \approx \frac{1}{\sqrt{\pi}} \frac{\exp(-x^2)}{x}, \text{ for } x \gg 1$$

$$\frac{\partial \text{erf}(x)}{\partial x} = \frac{2}{\sqrt{\pi}} \exp(-x^2)$$

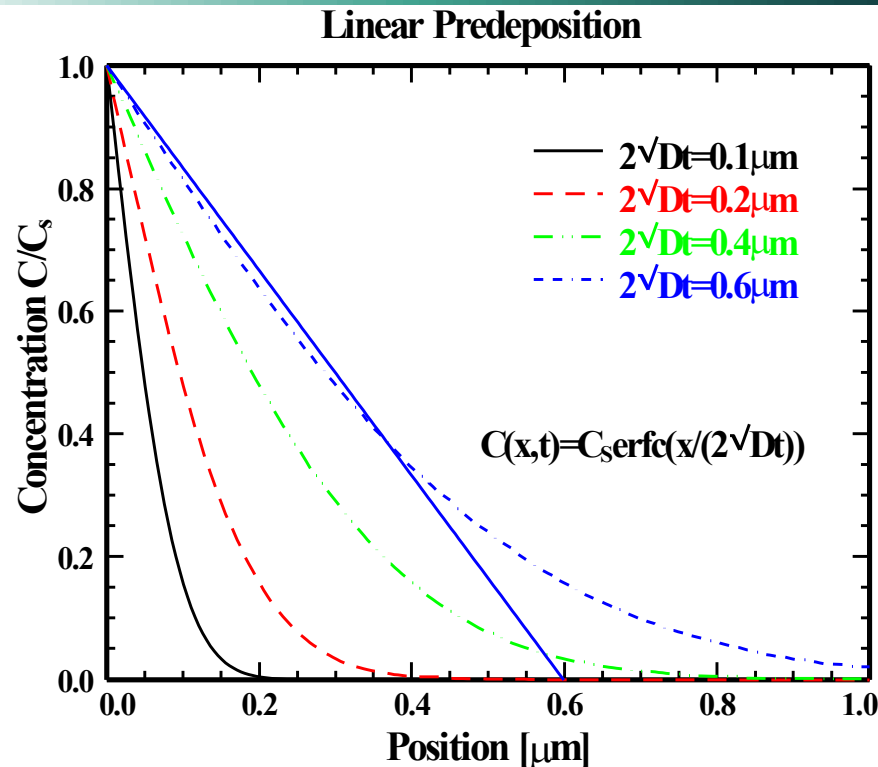
$$\int_0^x \text{erfc}(x') dx' = x \text{erfc}(x) + \frac{1 - \exp(-x^2)}{\sqrt{\pi}}$$

$$\int_0^\infty \text{erfc}(x) dx = \frac{1}{\sqrt{\pi}}$$





# Predeposition Profiles – constant $C_s$

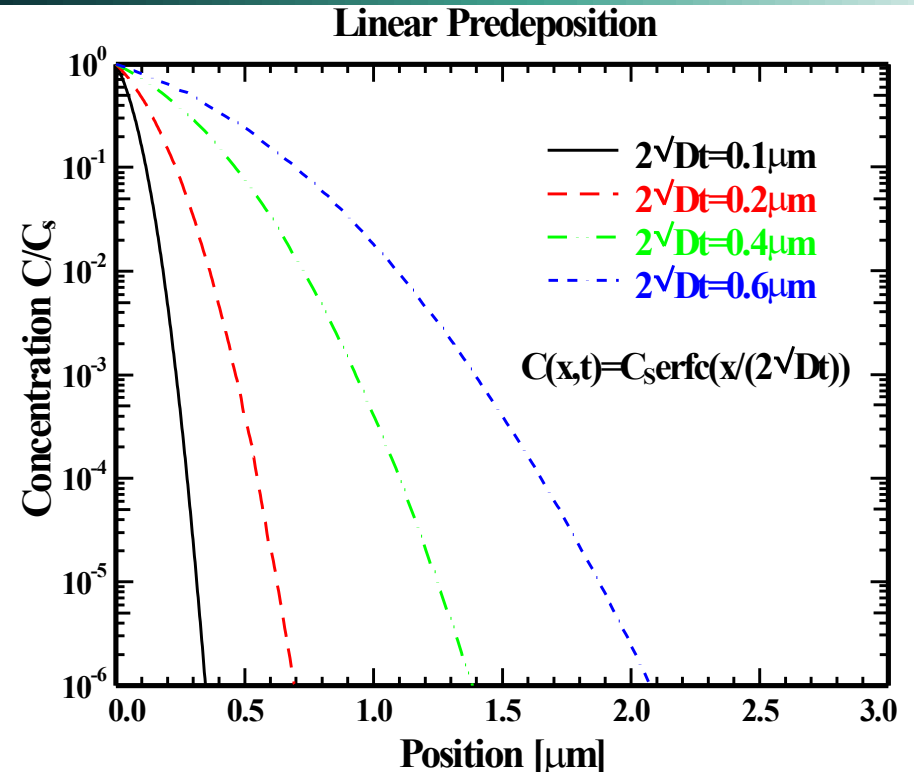


Almost triangular profile:

$$\text{Dose: } Q = \frac{2}{\sqrt{\pi}} C_s \sqrt{Dt} \approx C_s \sqrt{Dt}$$

$$\text{Surface gradient: } \frac{\partial C(0,t)}{\partial x} = \frac{C_s}{\sqrt{\pi Dt}} \approx \frac{C_s}{2\sqrt{Dt}}$$

$$\text{Gradient: } \frac{\partial C(0,t)}{\partial x} = \frac{C_s}{\sqrt{\pi Dt}} \exp\left(-\frac{x^2}{4Dt}\right)$$



Rapidly decaying tail:

$$C(x,t) = C_s \text{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)$$

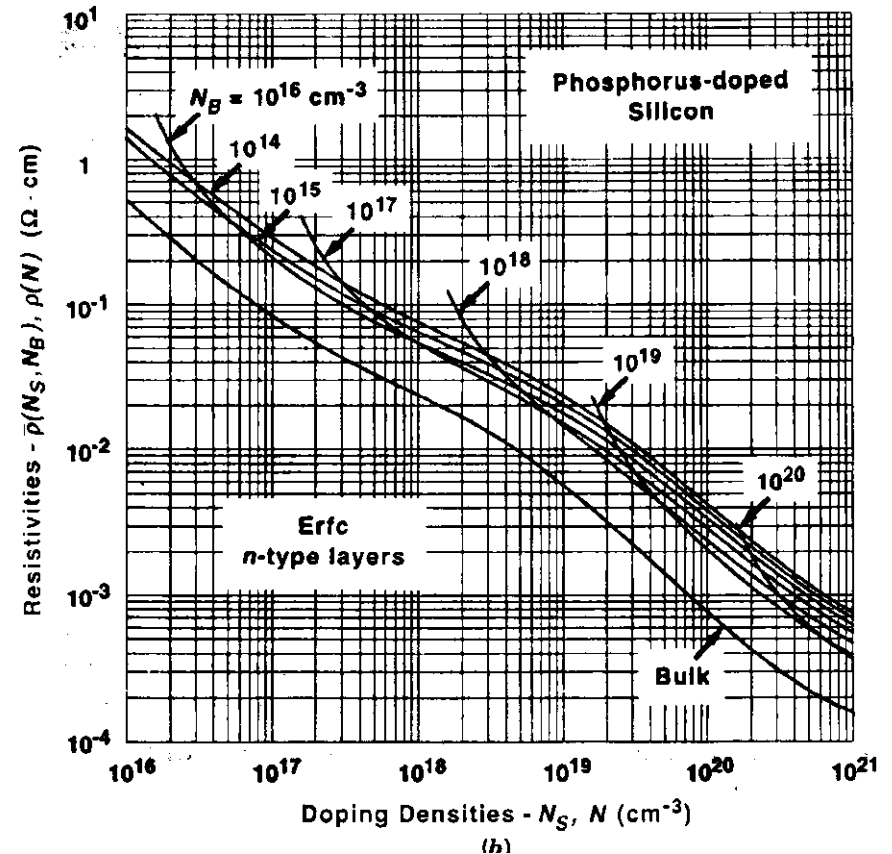
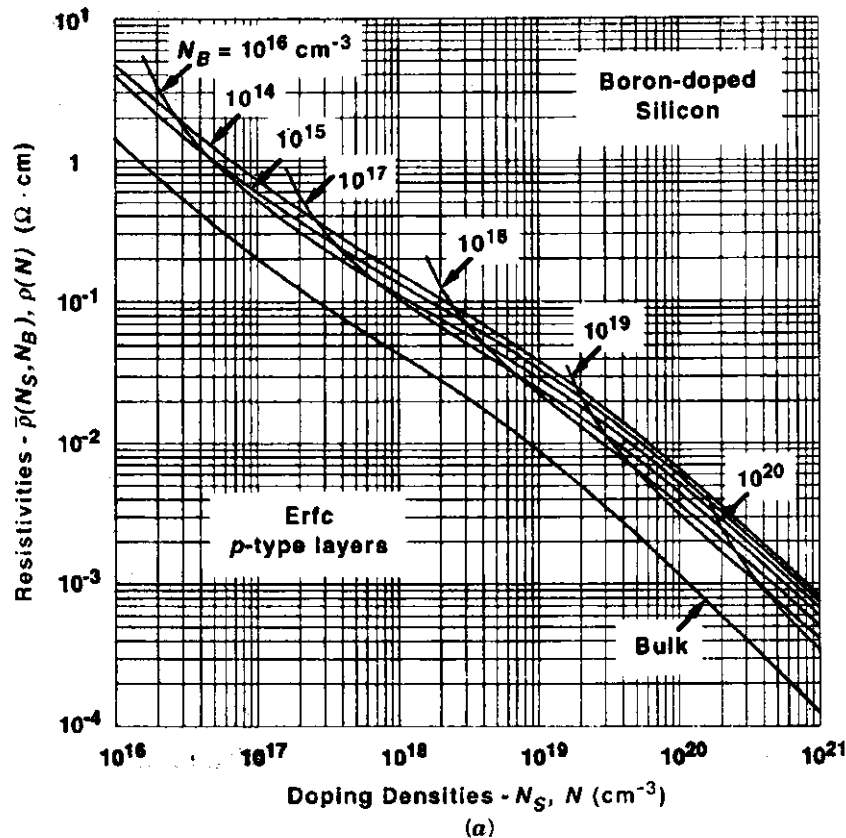
$$C(x,t) \approx C_s \frac{2\sqrt{Dt}}{\sqrt{\pi}x} \exp\left(-\frac{x^2}{4Dt}\right), \quad x \gg 2\sqrt{Dt}$$

$$\text{Definition: } \text{erfc}(x) \equiv 1 - \frac{2}{\sqrt{\pi}} \int_0^x \exp(-u^2) du$$



# Average Resistivity, Predep

## Irvins Graphs



$$\frac{1}{\sigma} = \frac{1}{\rho} = \frac{1}{x_j} \int_0^{x_j} \sigma dx \Rightarrow R_{\text{sh}} = \frac{\bar{\rho}}{x_j} = \frac{1}{\sigma x_j}$$

# Linear Drive-in Model

Purpose: Redistribute a fixed dose Q

- Heat treatment with closed surface (oxide covered)
- Control temperature  $T$ , ( $D=D(T)$ )
- Control time  $t$

1 D. Model, constant diffusivity :

$$\text{Fick's 2. Law : } \frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2}$$

$$\text{Boundary conditions : } \frac{\partial C(0,t)}{\partial x} = 0, \quad C(\infty,t) = 0$$

$$\text{Initial conditions : } C(x,0) = C_{\text{Predep}}(x) \approx Q \delta(x)$$

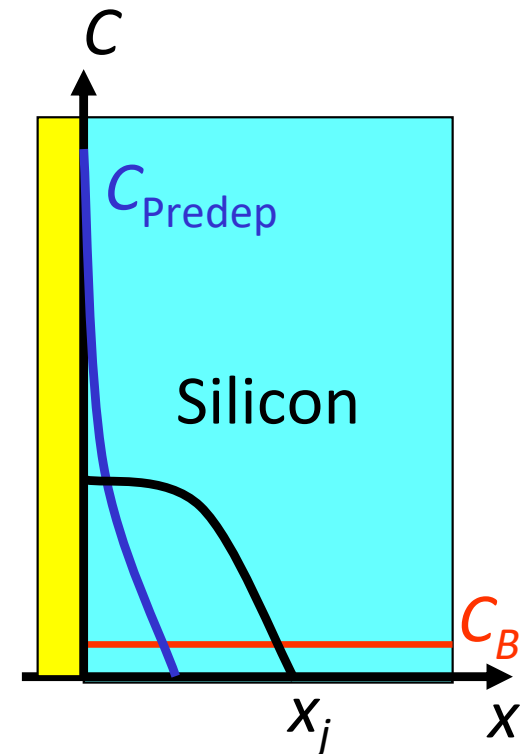
$$\text{Solution, a Gaussian : } C(x,t) = \frac{Q}{\sqrt{\pi Dt}} \exp\left(-\frac{x^2}{4Dt}\right)$$

$$\text{Surface concentration : } C_s(t) = \frac{Q}{\sqrt{\pi Dt}}$$

$$\text{Junction depth : } C(x_j, t) = C_B \Rightarrow x_j = 2\sqrt{Dt} \sqrt{\ln\left(\frac{C_s(t)}{C_B}\right)}$$

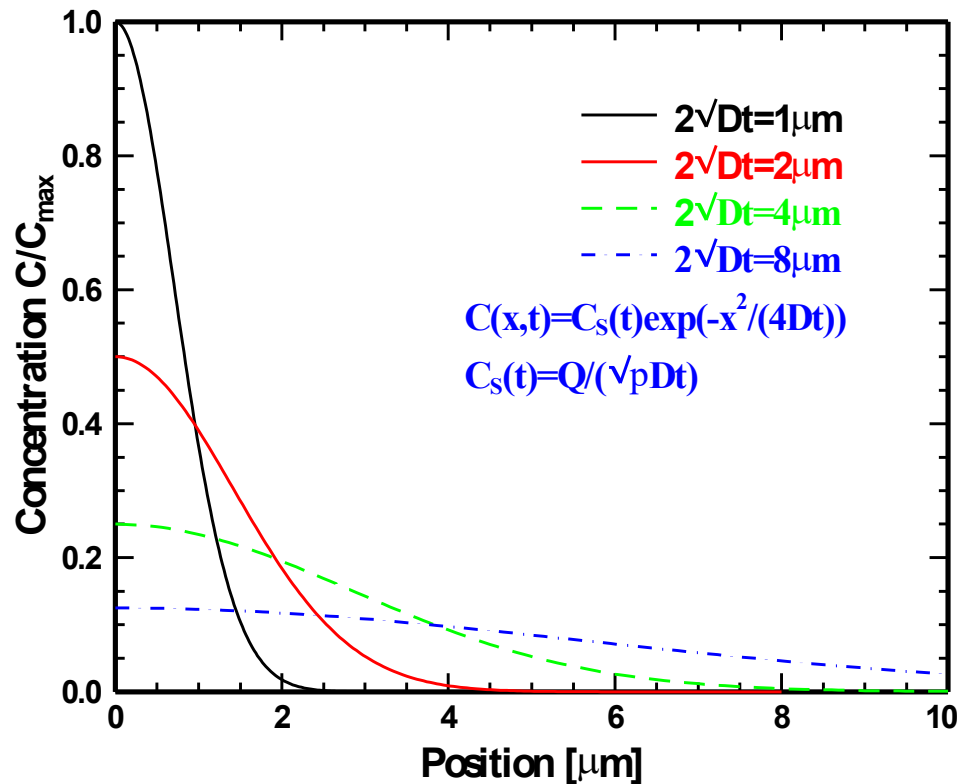
**Repeated Drive - in's**

$$(Dt)_{\text{eff}} = \sum Dt = \int D dt$$

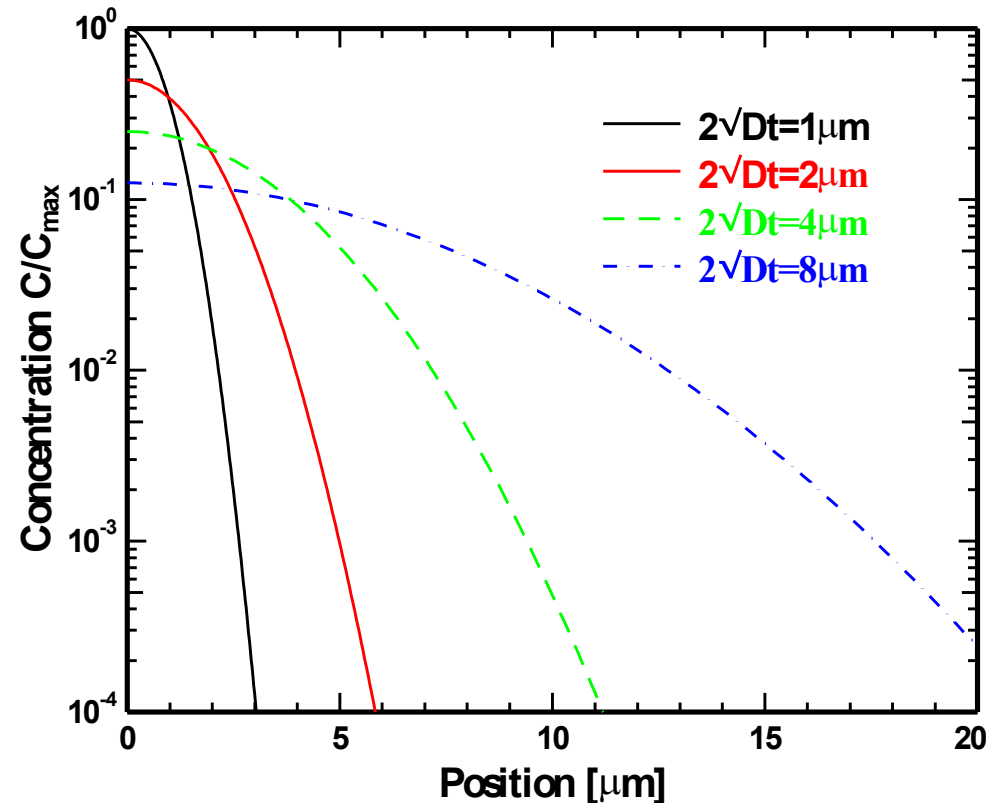


# Drive-in Profiles: constant $Q_T$

Linear Drive-in



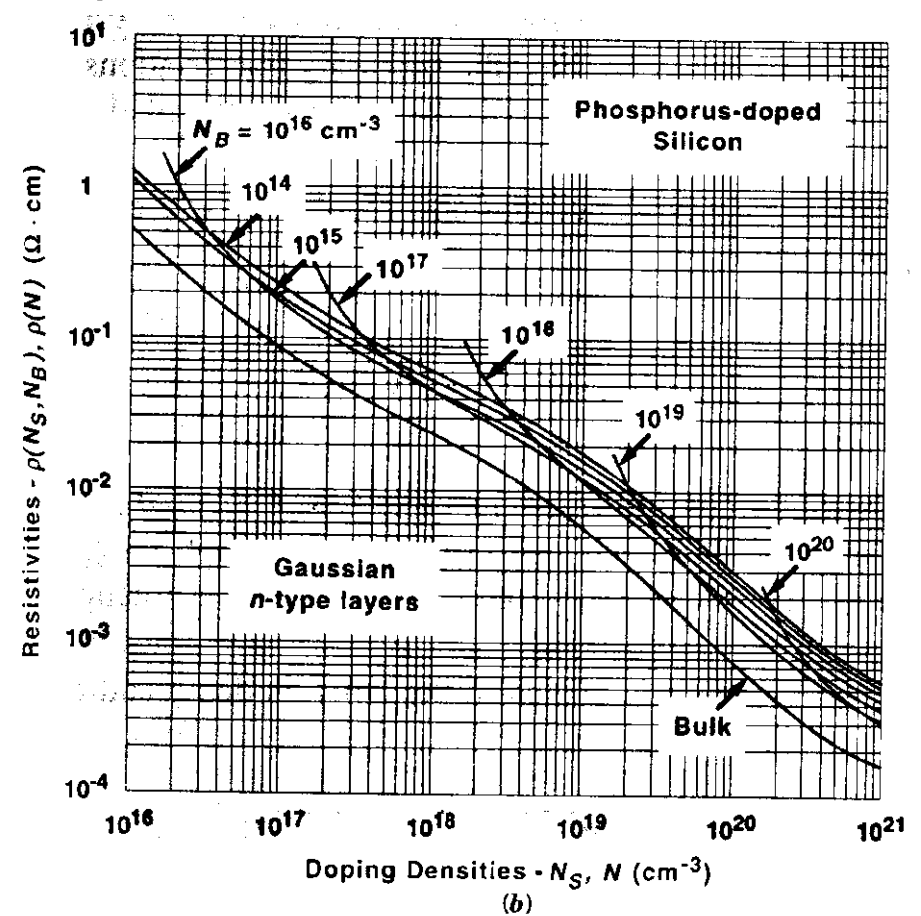
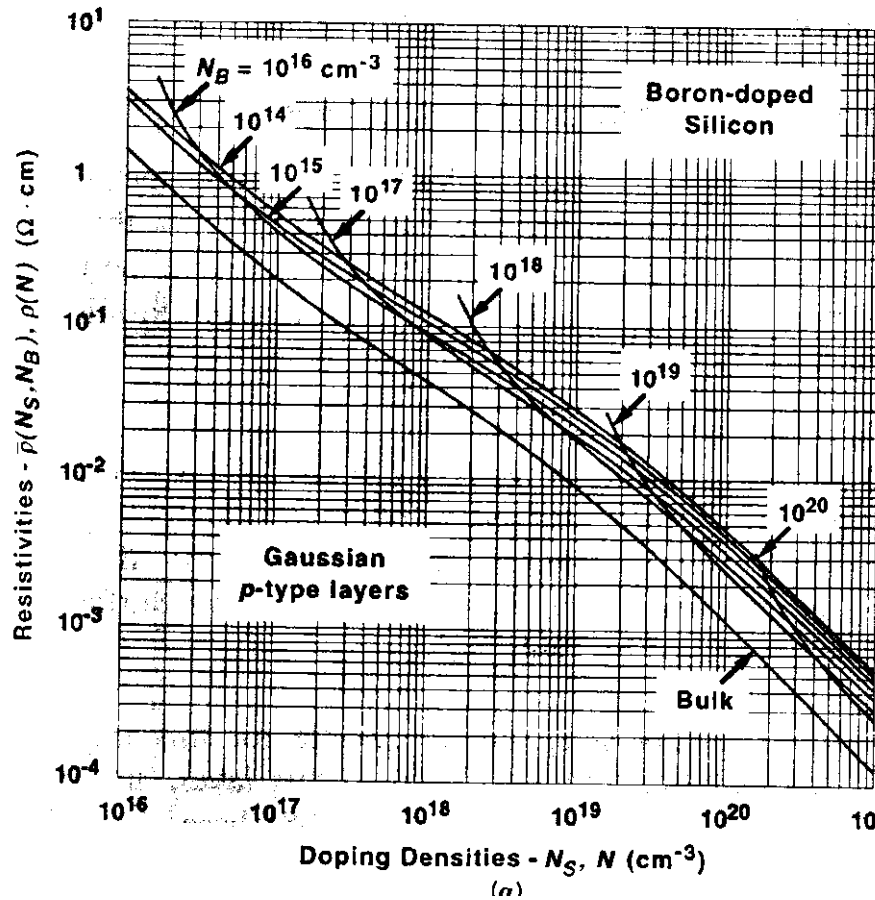
Linear Drive-in



- Zero gradient at the surface
- Time decaying surface concentration
- Rapidly decaying tail

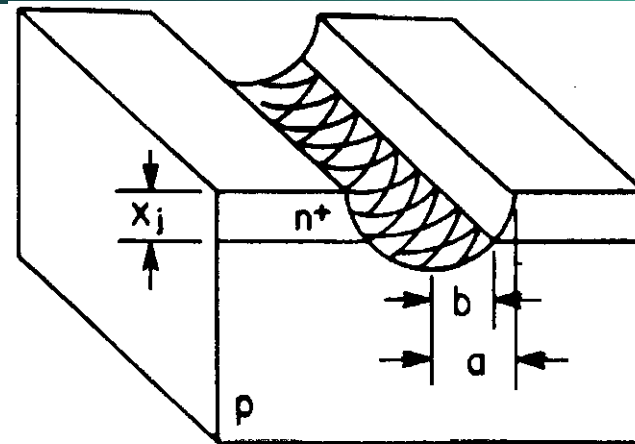
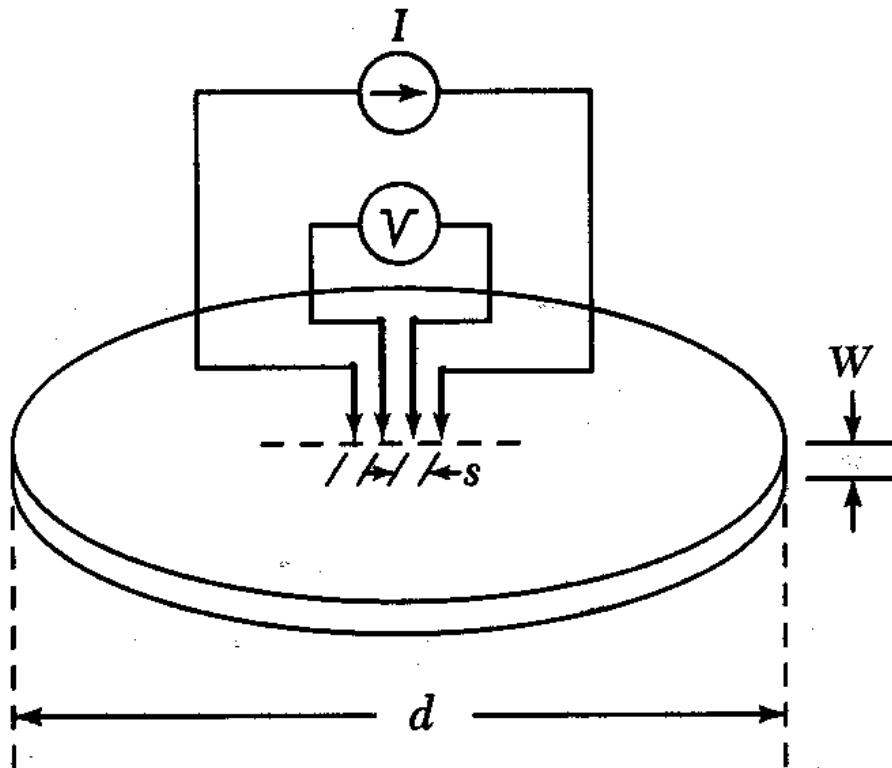
# Average Resistivity, Drive-in

## Irvin's Graphs

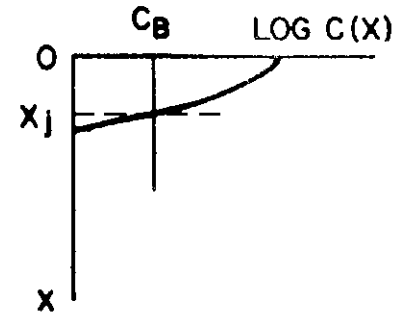


$$\bar{\sigma} = \frac{1}{\rho} = \frac{1}{x_j} \int_0^{x_j} \sigma dx \Rightarrow R_{\text{sh}} = \frac{\bar{\rho}}{x_j} = \frac{1}{\bar{\sigma} x_j}$$

# Evaluation of Diffused Layers



(a)



(b)

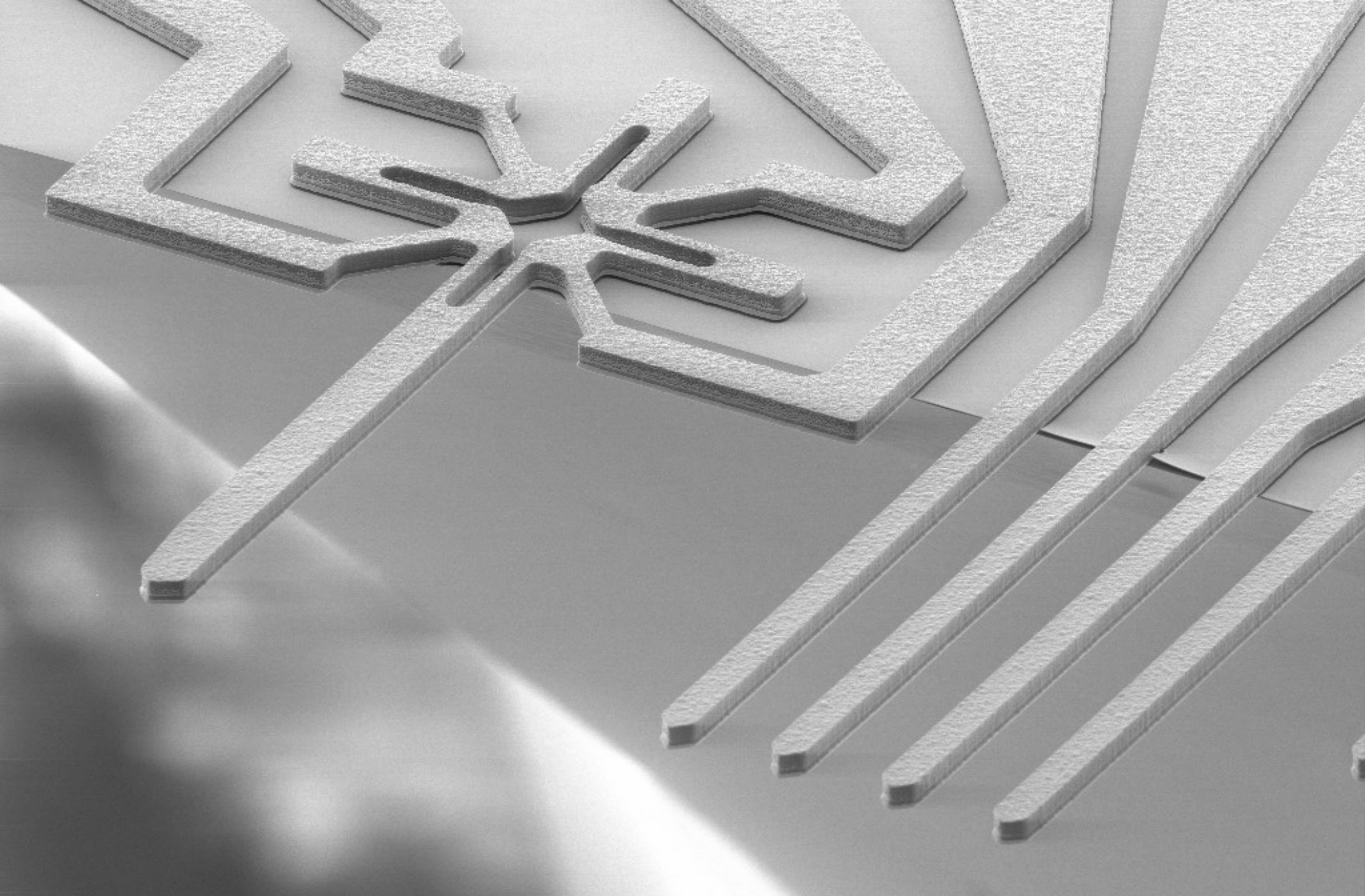
Junction depth measurement.  
Groove, mechanical grinding.  
Stain p-region ( $\text{HNO}_3 + \text{HF} + h\nu$ )

## Four Point Probe :

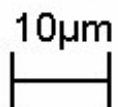
$$R_{sh} = \frac{\pi}{\ln 2} \frac{V}{I} = 4.532 \frac{V}{I}$$

Simple low-tech measurements  
Four point probe a routine check.  
Groove & stain useful for large  $x_j$ .





Mag = 1.60 K X

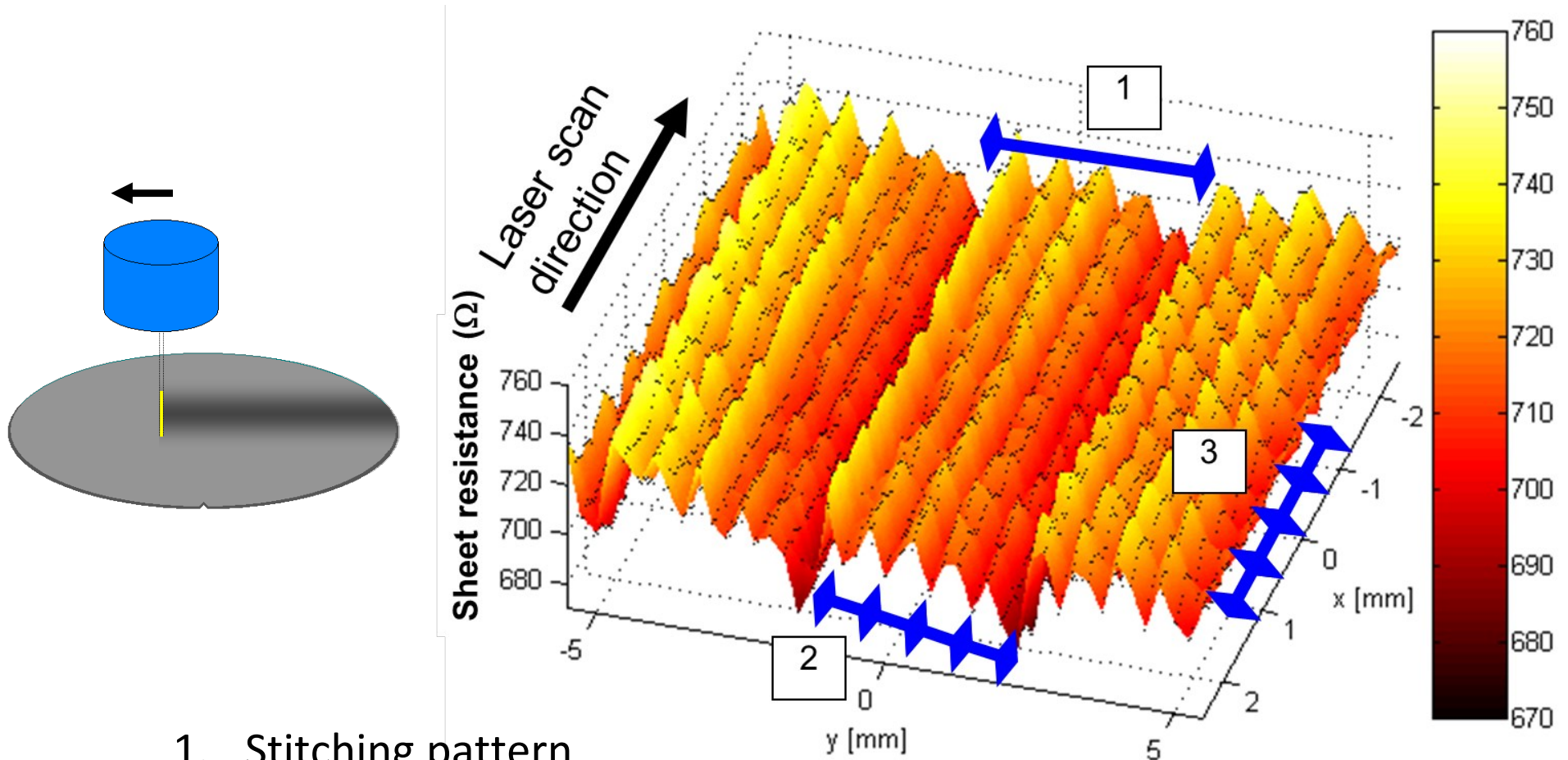


EHT = 5.00 kV  
WD = 10 mm

Signal A = SE2  
Photo No. = 4678

Date :11 Sep 2008  
Time :19:33

# Laser annealed USJ: Sheet resistance



1. Stitching pattern
2. Spatial laser power density variations
3. Temporal laser power fluctuations

D. H. Petersen *et al.* JVST B **26**, 362 (2008).  
W. Vandervorst *et al.* MRS 2008 Spring meeting (2008).



# Temperature Effect

**Intrinsic diffusion coefficient increases as temperature increases**

$$D = D_o \exp\left(-\frac{E_a}{kT}\right)$$

- $E_a$  for interstitial diffusion is the energy required for dopants to move from one interstitial site to another (around 0.5 to 2 eV)
- $E_a$  for vacancy diffusion is related to the energies of dopant motion and vacancy formation (around 3 to 5 eV)

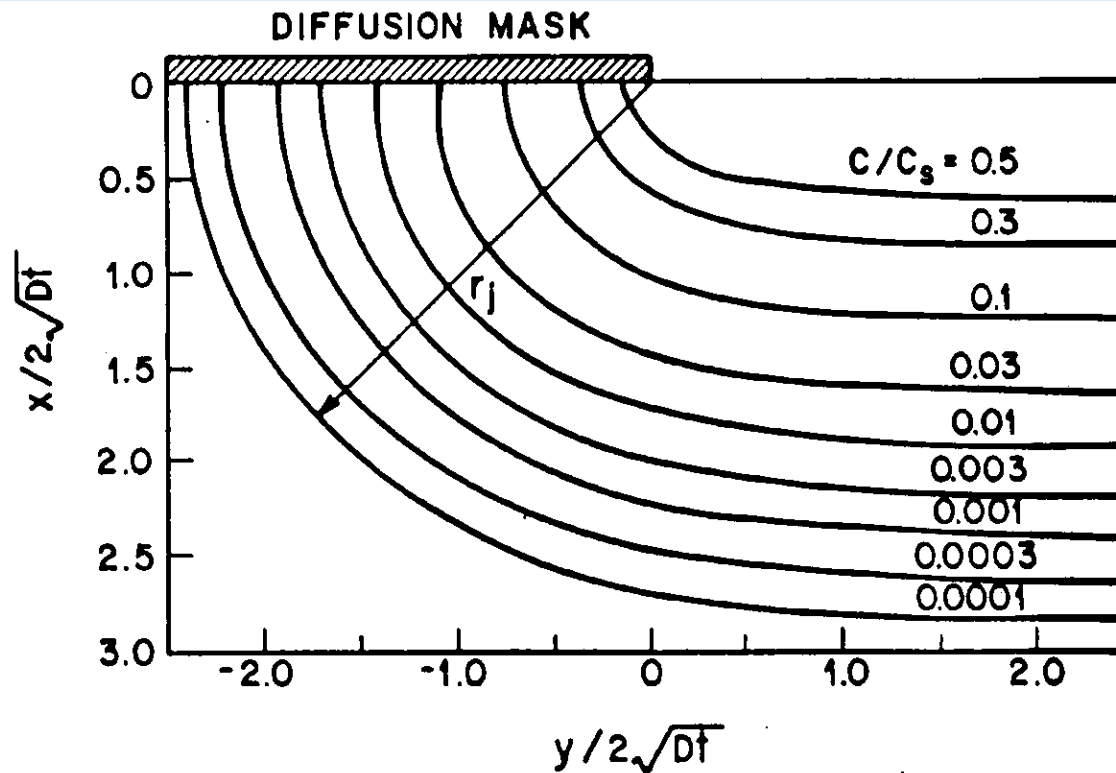
	<b>Si</b>	<b>B</b>	<b>In</b>	<b>As</b>	<b>Sb</b>	<b>P</b>	<b>Units</b>
$D^0$	560	1.0	1.2	9.17	4.58	4.70	cm <sup>2</sup> sec <sup>-1</sup>
$E_A$	4.76	3.5	3.5	3.99	3.88	3.68	eV

- Note that  $n_i$  is very large at process temperatures, so "intrinsic" actually applies under many conditions.
- Note the "slow" and "fast" diffusers. Solubility is also an issue in choosing a particular dopant.



# Masking

- Required Mask Thickness in Predeposition?
- Dopant Profile after Predeposition & Drive-in? A 2D / 3D problem.



Dopant iso-concentration contours after a masked **Predeposition**.  
Lateral junction depth  $\sim$  80% of vertical junction depth .

# Required Mask Thickness

Constant surface concentration  
& Interface segregation

Diffusion problem: Make  $x_j=0$

**Fick's 2. Law :**  $\frac{\partial C_i}{\partial t} = D_i \nabla^2 C_i$

**Initial :**  $C(x,0) = 0$

**Boundary :**  $C(\infty, t) = 0, \quad C(-x_{ox}, t) = C_0$

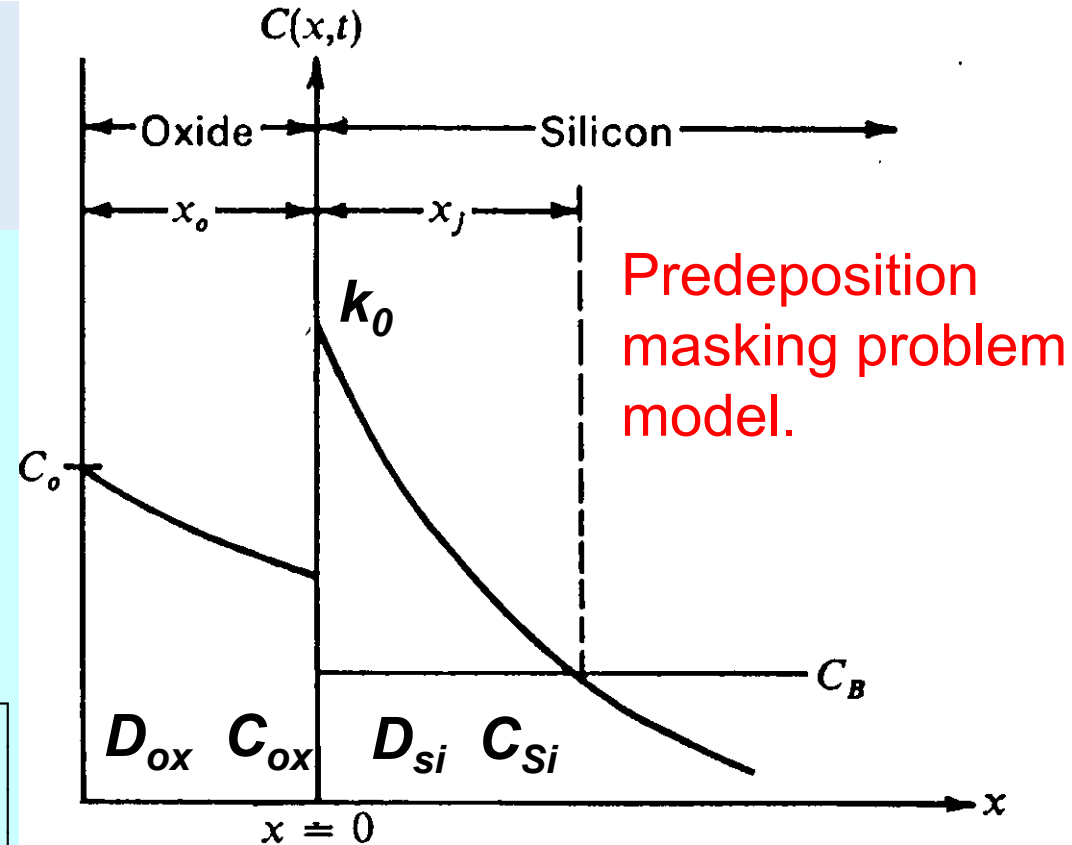
**Interface segregation :**  $C_{Si}(0, t) = k_0 C_{ox}(0, t)$

**Continuous flux :**  $D_{ox} \nabla C_{ox} = D_{Si} \nabla C_{Si}$

**Approximate solution :**

$$C_{ox} \approx C_0 \left[ \operatorname{erfc} \left( \frac{x_{ox} + x}{2\sqrt{D_{ox}t}} \right) - \frac{k_0 - \frac{D_{ox}}{D_{Si}}}{k_0 + \frac{D_{ox}}{D_{Si}}} \operatorname{erfc} \left( \frac{x_{ox} - x}{2\sqrt{D_{ox}t}} \right) \right]$$

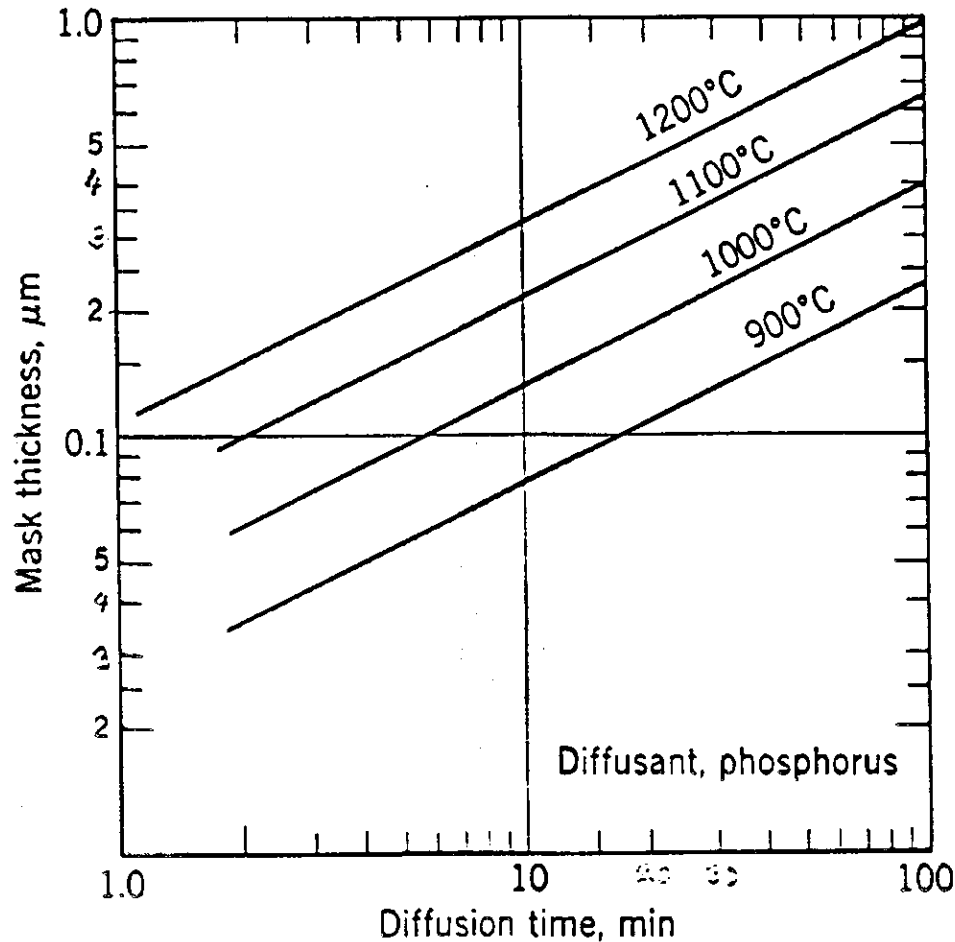
$$C_{Si} \approx C_0 \frac{2k_0 \frac{D_{ox}}{D_{Si}}}{k_0 + \frac{D_{ox}}{D_{Si}}} \operatorname{erfc} \left( \frac{x_{ox}}{2\sqrt{D_{ox}t}} + \frac{x}{2\sqrt{D_{Si}t}} \right)$$



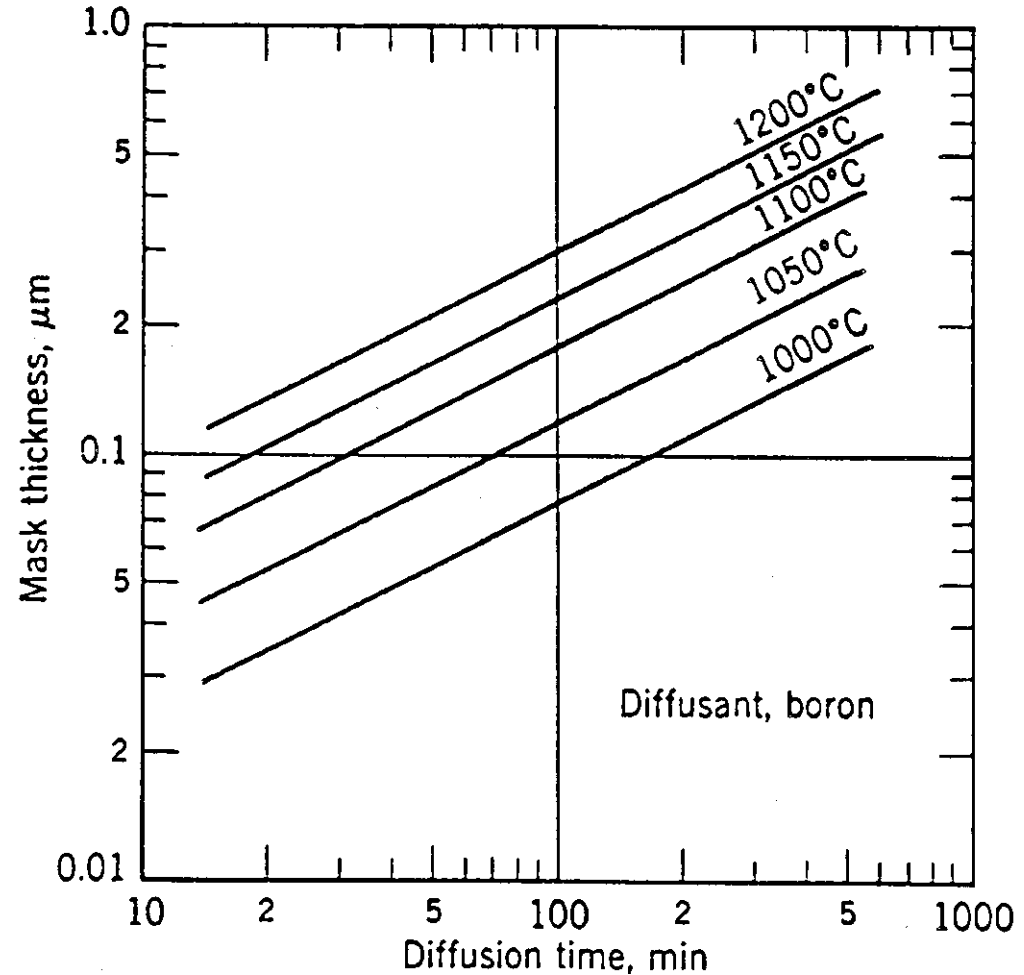
$$x_{ox} > 2\sqrt{D_{ox}t} \operatorname{argerfc} \left( \frac{C_B}{C_0} \frac{k_0 + \frac{D_{ox}}{D_{Si}}}{2k_0 \frac{D_{ox}}{D_{Si}}} \right)$$



# Required Mask Thickness



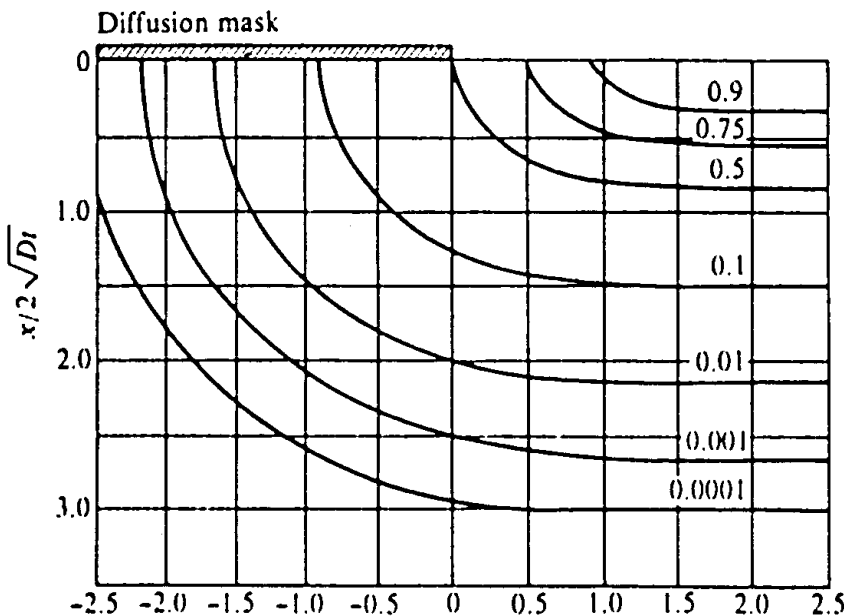
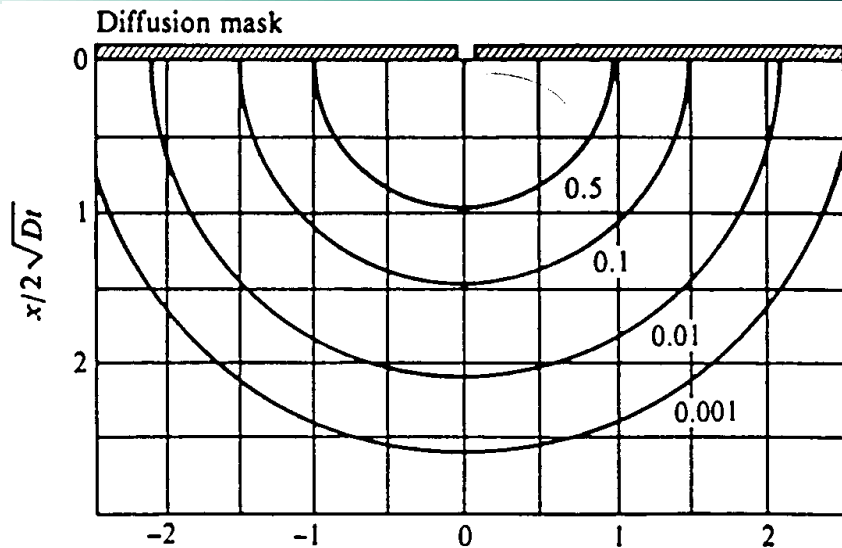
Thick masks due to high diffusivity and  $k_0=10$ .



Thin masks due to low Diffusivity and  $k_0=0.3$ .  
Note, most graphs are wrong!



# Drive-in Profiles



## Drive - in : Line source

Fick's 2. Law:  $\frac{\partial C}{\partial t} = D\nabla^2 C = D\left(\frac{\partial^2 C}{\partial r^2} + \frac{\partial C}{r\partial r}\right)$

Initial :  $C(x, y, 0) = Q'\delta(x)\delta(y)$

Boundary :  $C(r \rightarrow \infty, t) = 0, \quad \frac{\partial C(0, y, t)}{\partial x} = 0$

Gaussian Solution :  $C(r, t) = \frac{Q'}{2\pi Dt} \exp\left(-\frac{r^2}{4Dt}\right)$

## Drive-in: Half plane source

Fick's 2. Law:  $\frac{\partial C}{\partial t} = D\nabla^2 C = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right)$

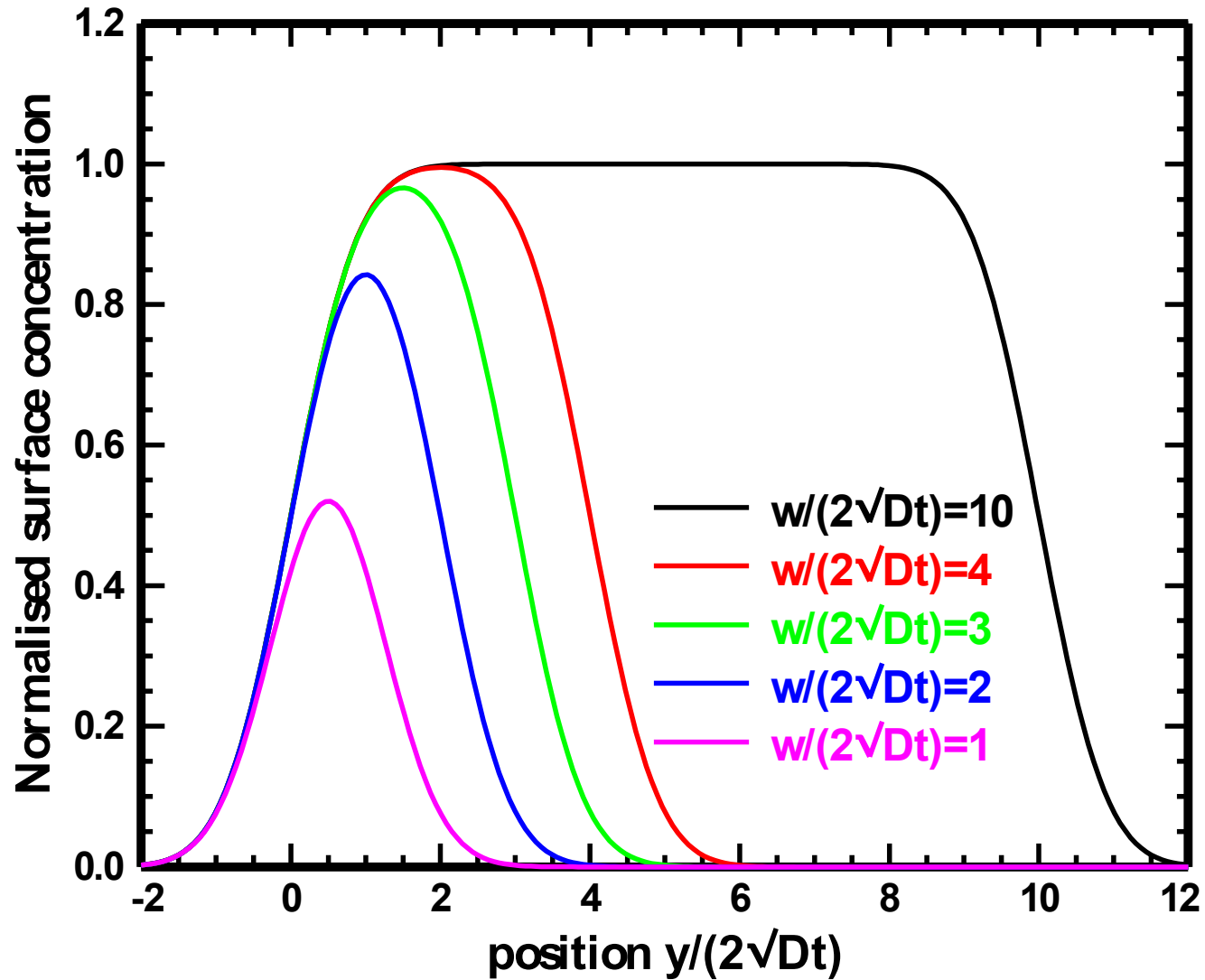
Initial:  $C(x, y, 0) = Q\delta(x)h(y)$

Boundary:  $C(x \rightarrow \infty, y, t) = 0, \quad \frac{\partial C(0, y, t)}{\partial x} = 0$

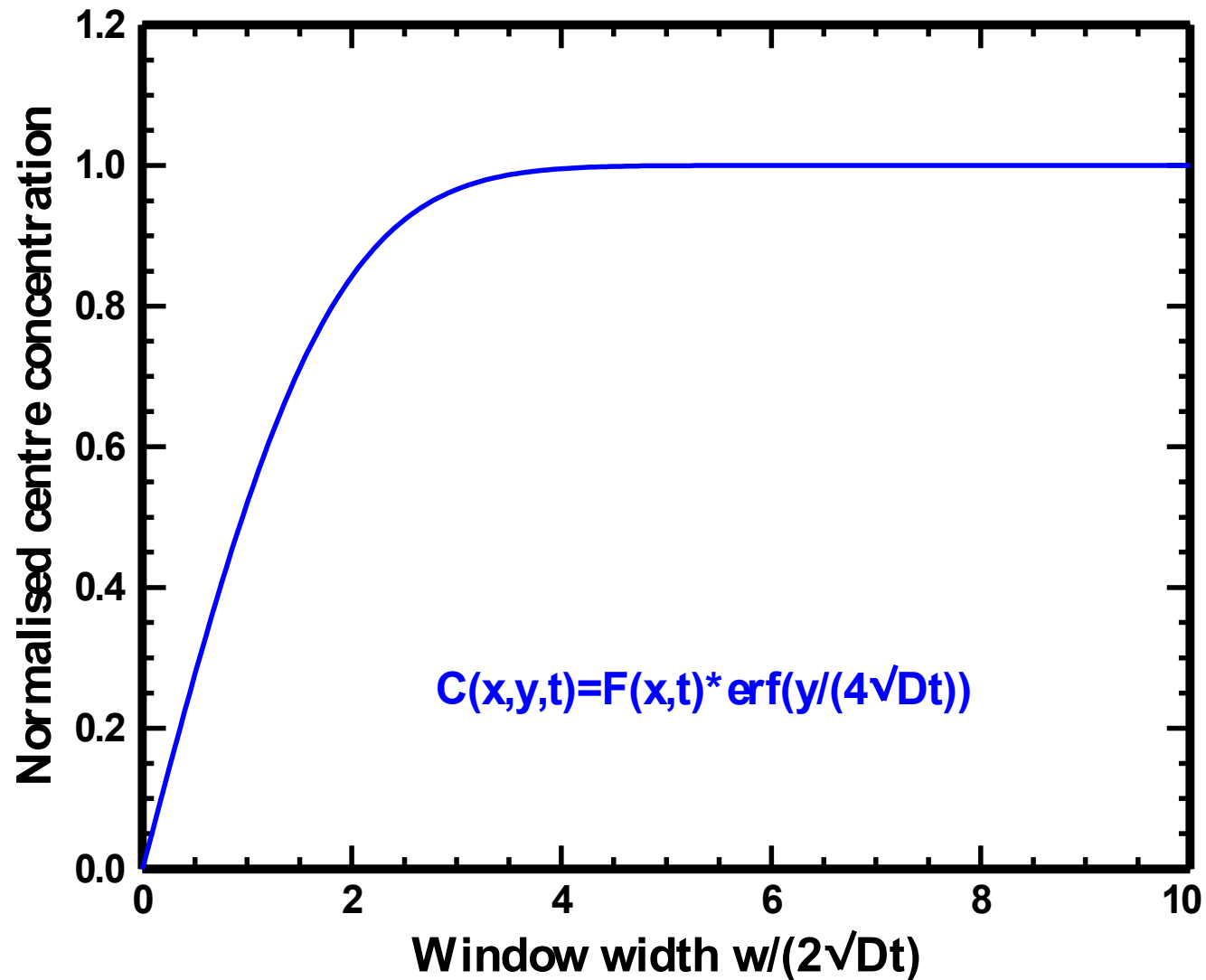
Solution:  $C(x, y, t) = \frac{Q}{\sqrt{\pi Dt}} \exp\left(-\frac{x^2}{4Dt}\right) \frac{\operatorname{erfc}\left(\frac{-y}{2\sqrt{Dt}}\right)}{2}$



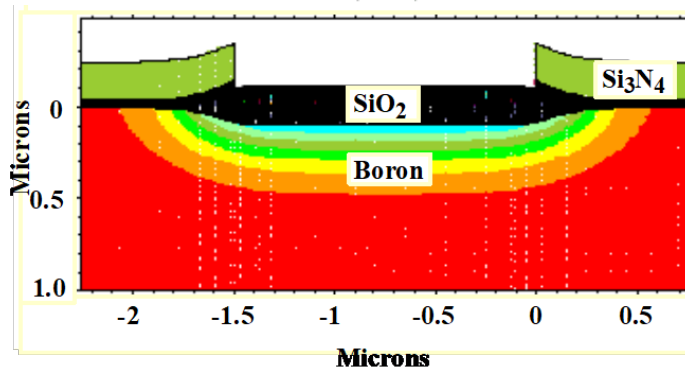
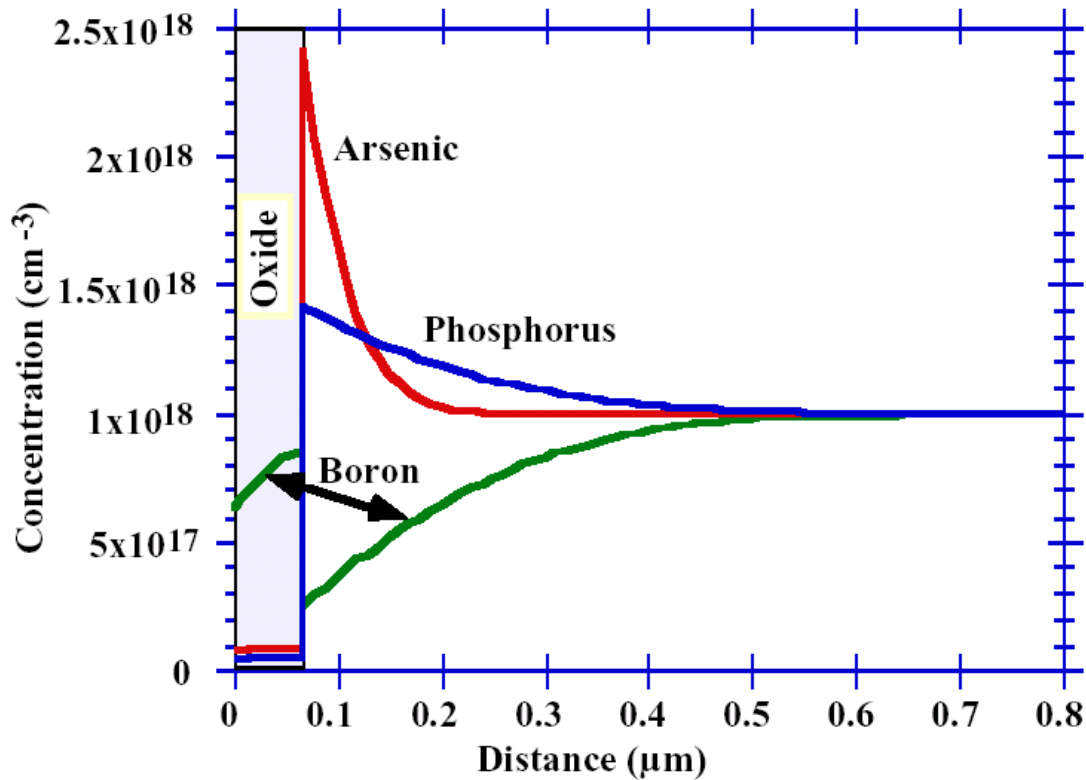
# Masked Drive-in Diffusion



# Masked Drive-in Diffusion



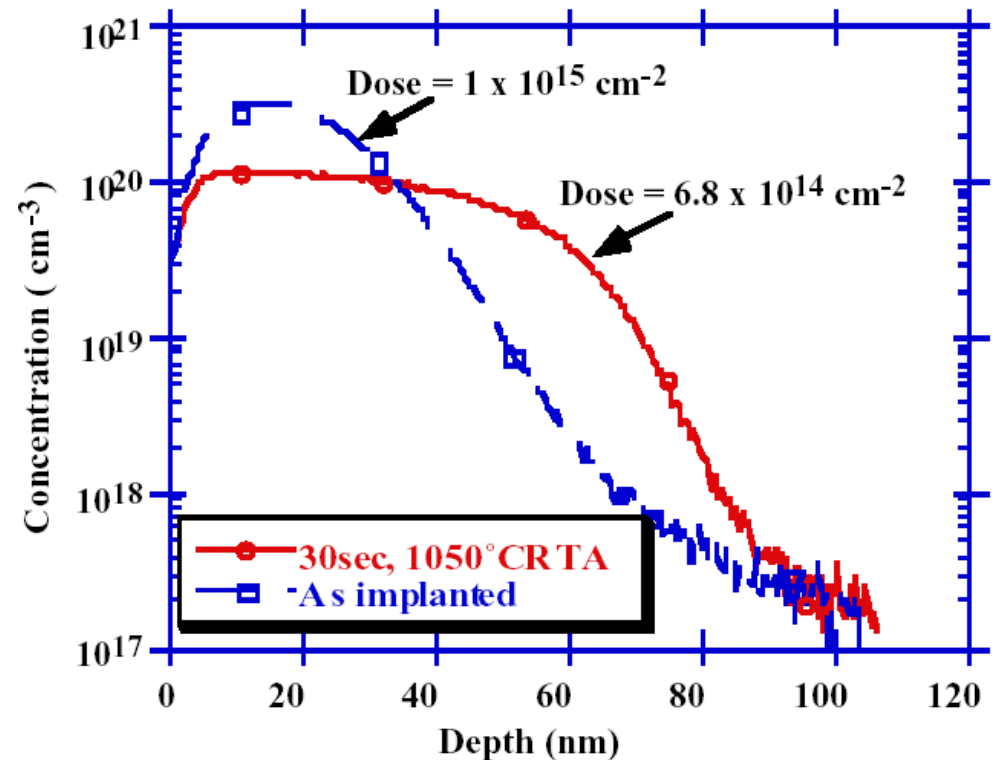
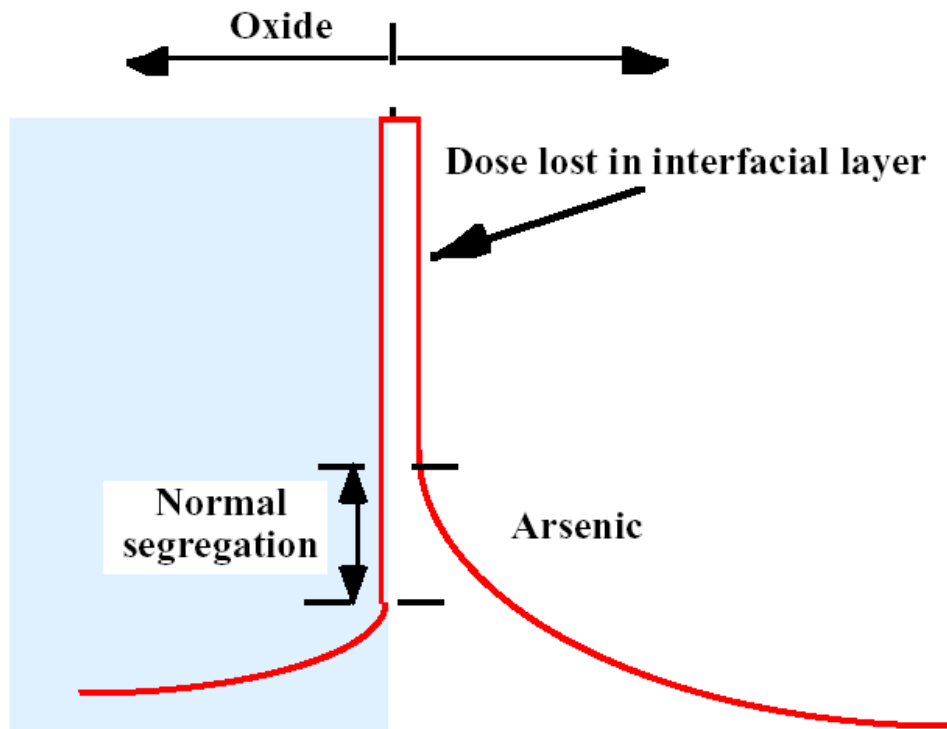
# Dopant Segregation



- **Solubility in oxide and silicon different, (2 Phases):**
  - Equilibrium:  $C_{Si} = k_0 C_{Ox}$
  - $k_0$ : Segregation coefficient
  - B:  $k_0 \approx 0.3$
  - P, As, Sb:  $k_0 \approx 10$
- **Diffusivity in oxide and silicon different**
  - Mostly:  $D_{Si} \gg D_{Ox}$
  - Dopant redistribution during oxidation
  - Complicated moving boundary condition @ interface



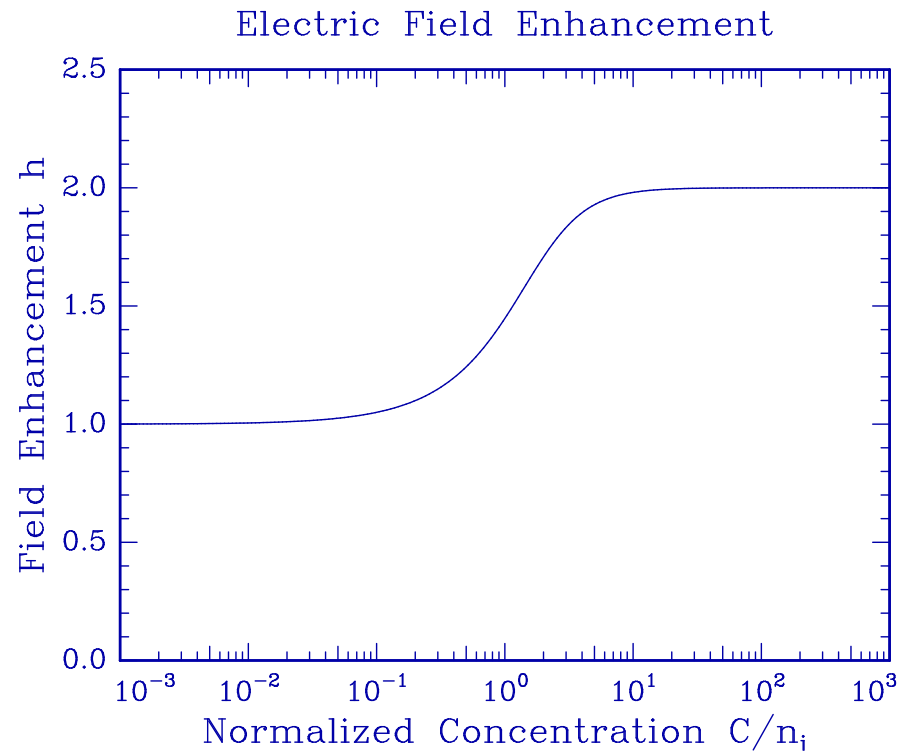
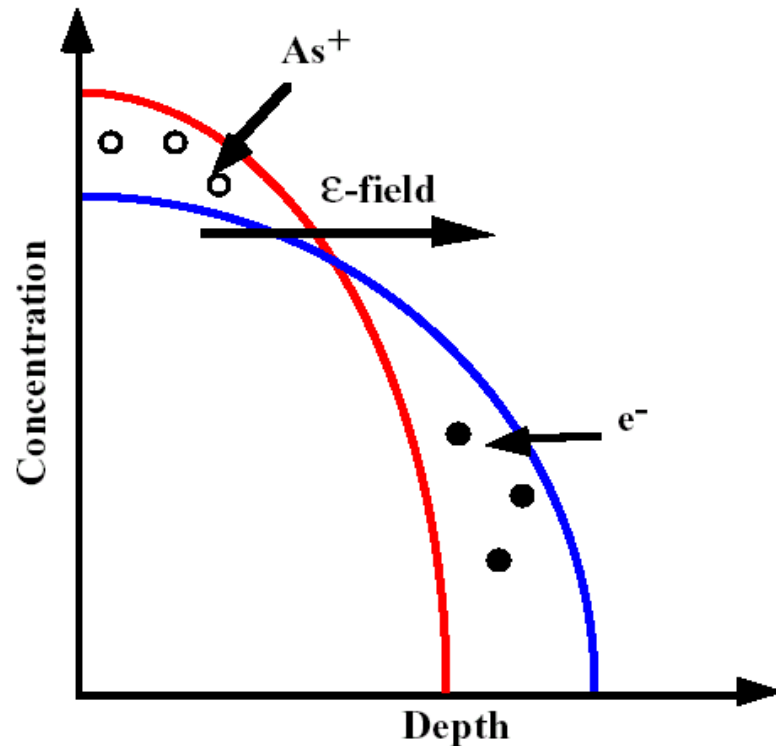
# Interfacial Dopant Pile Up



The surface is really a separate phase.  
A very different solubility might apply.  
Inactive dopant may accumulate ~ 0.5ML.

Experiment: Implanted annealed As  
Dopant loss during anneal ~30%  
Important for group V elements.

# Electric Field Effects



**Ionised Dopants, Dopant Gradients & Fast Electrons :**

Built in electric field :  $\vec{E} = -\frac{kT}{q} \nabla \ln \frac{n}{n_i}$ , from  $\vec{J}_n = 0$

$$F = -D \nabla C + \frac{qD}{kT} \vec{E} C = -D_{\text{eff}} \nabla C = -h D \nabla C$$

$$\text{Field enhancement factor: } h \cong 1 + \frac{C}{\sqrt{C^2 + 4n_i^2}}$$

Important at high doping:  
 Criterion:  $C > n_i \Rightarrow h \approx 2$   
 $C < n_i \Rightarrow h \approx 1$

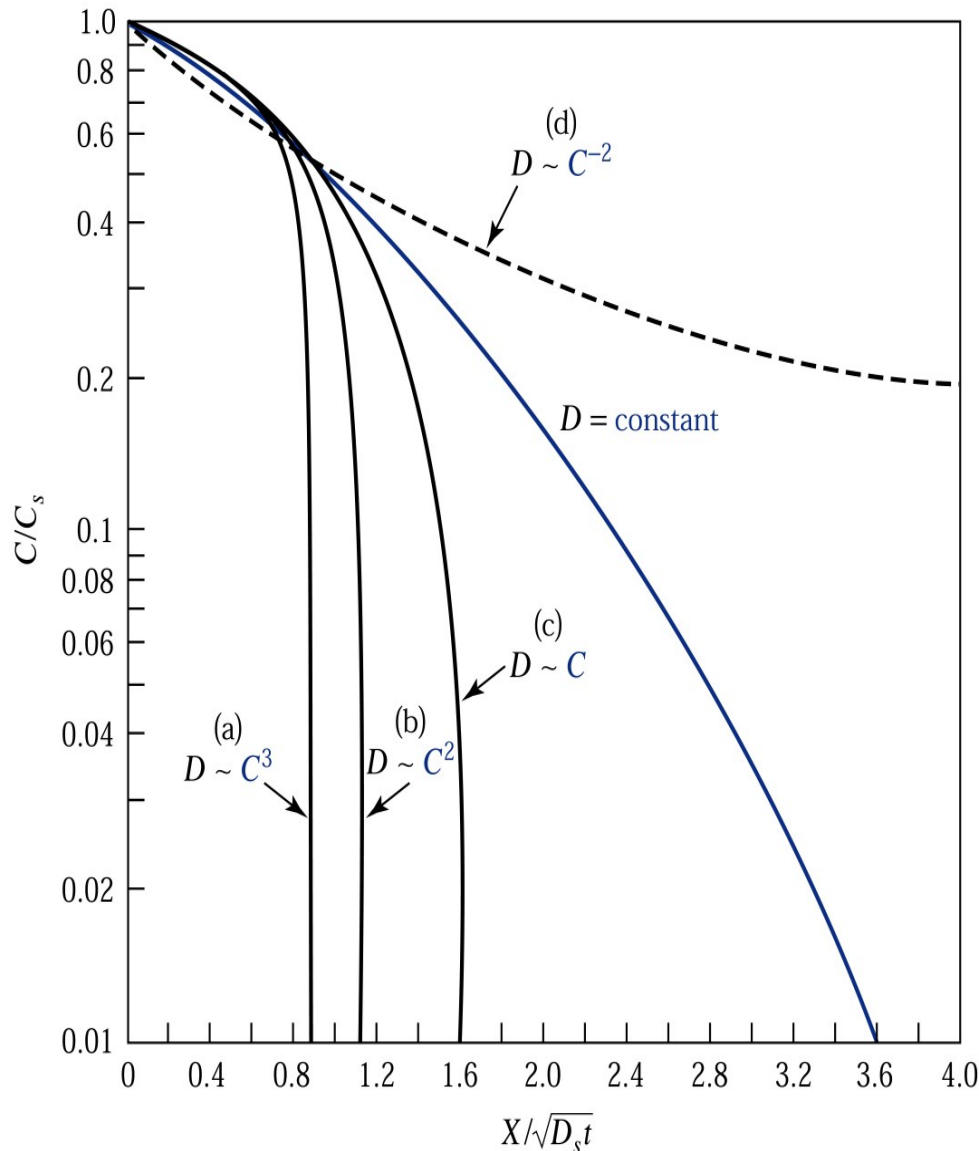
# Concentration Dependent Diffusivity

## Constant-Surface-Concentration Diffusion

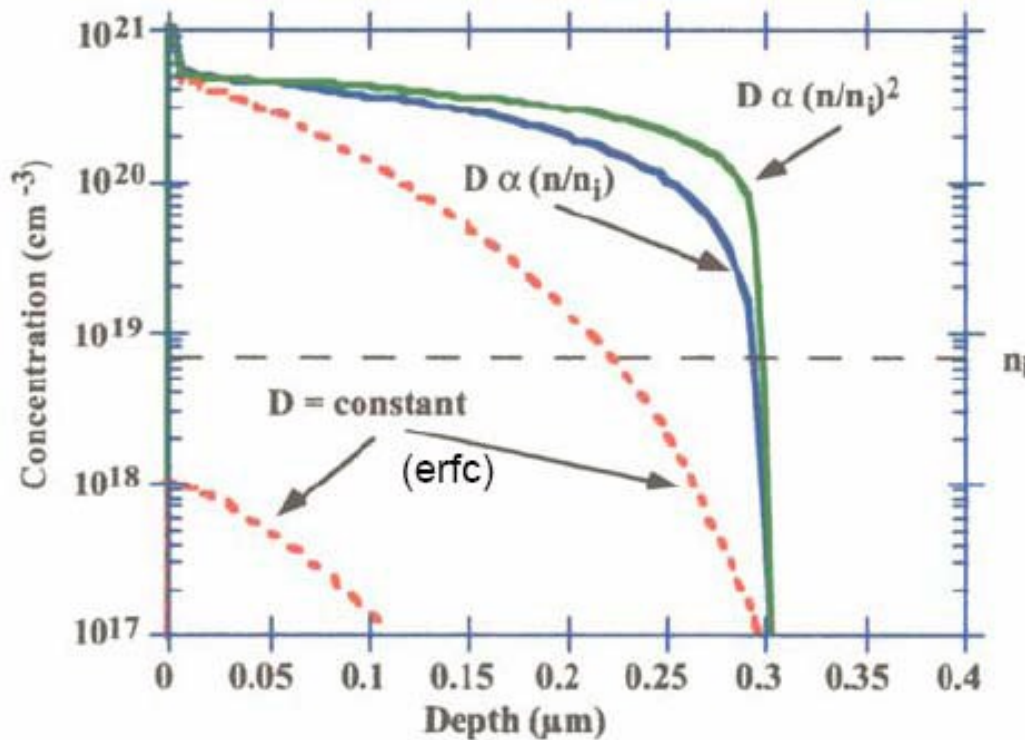
$$D = D_s \left( \frac{C}{C_s} \right)^\gamma$$

$D_s$ : diffusion coefficient at the surface  
 $C_s$ : the surface concentration

- for  $\gamma > 0$  (B or As in Si, Zn in GaAs), the diffusion coefficient decreases as concentration drops
- due to sharp drop of the dopant concentration, abrupt junction is formed for  $\gamma > 0$  with wide range of background doping (**good for devices**)
- for  $\gamma < 0$  (Au and Pt), dopant can penetrate deep into substrate due to increased diffusion coefficient

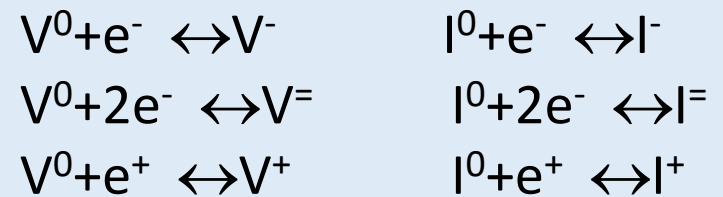


# Concentration Dependent Diffusivity



$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left( D_A^{eff} \frac{\partial C}{\partial x} \right)$$

Assumed reaction schemes:



All point defect charge states are contributing:

$$D = D_{i0} + D_{i-} \left( \frac{n}{n_i} \right) + D_{i=} \left( \frac{n}{n_i} \right)^2 \quad \text{for N type dopants}$$

$$D = D_{i0} + D_{i+} \left( \frac{p}{n_i} \right) + D_{i++} \left( \frac{p}{n_i} \right)^2 \quad \text{for P type dopants}$$

At low doping densities:  $n = n_i$

$$D_i = D_{i0} + D_{i-} + D_{i=}$$

$$D = D_0 \exp \left( - \frac{D \cdot E}{kT} \right)$$

# Experimental Diffusivities

	Si	B	In	As	Sb	P
$D_{00}$ [cm <sup>2</sup> /s]	560	0.05	0.6	0.011	0.214	3.85
$E_{A0}$ [eV]	4.76	3.5	3.5	3.44	3.65	3.66
$D_{0+}$ [cm <sup>2</sup> /s]		0.95	0.6			
$E_{A+}$ [eV]		3.5	3.5			
$D_{0-}$ [cm <sup>2</sup> /s]				31.0	15.0	4.44
$E_{A-}$ [eV]				4.15	4.08	4.0
$D_{0--}$ [cm <sup>2</sup> /s]						44.2
$E_{A--}$ [eV]						4.37



# HW

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Calculate the effective diffusion coefficient at 1000 °C for two different box shaped arsenic profiles grown by silicon epitaxy, one doped  $1 \times 10^{18} \text{cm}^{-3}$ , and the other doped at  $1 \times 10^{20} \text{cm}^{-3}$ .

