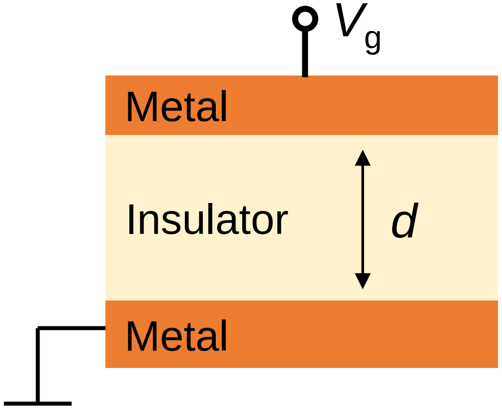
6.6 Quantum capacitor

Capacitance: the capacity to store charges

Parallel plate capacitor



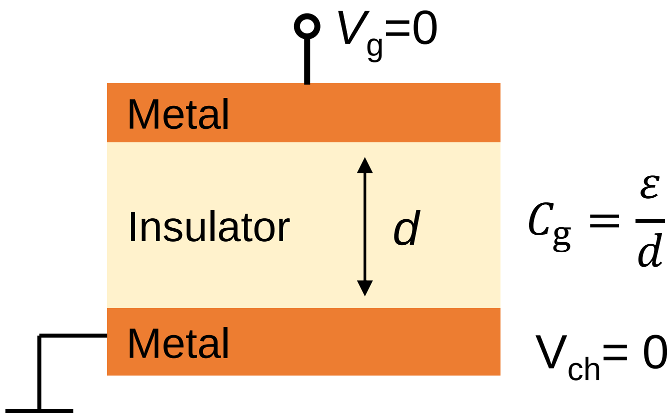
$$C_g = \frac{\varepsilon}{d}$$

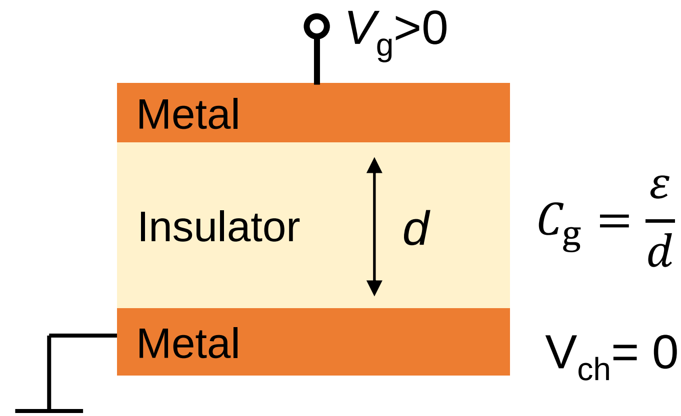
$V_{ch} = 0$

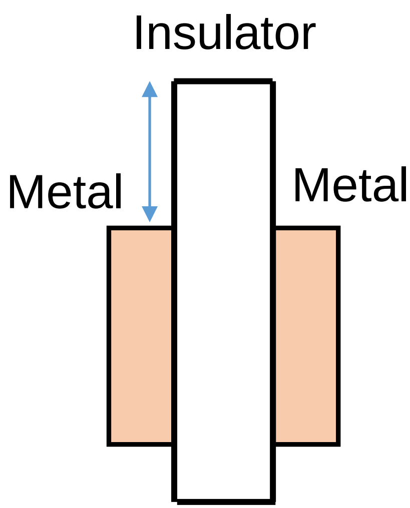
Applied voltage V_g, injected charges Q

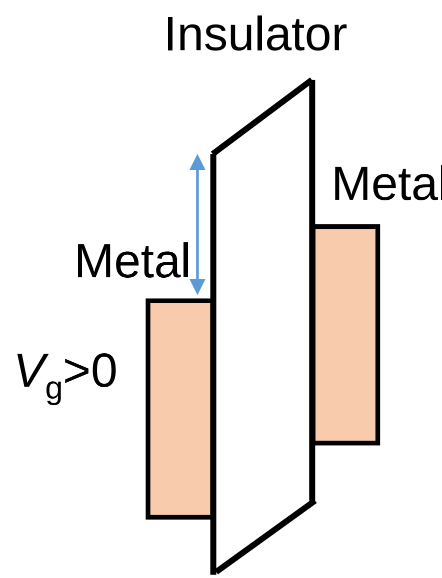
$$C_g = \frac{Q}{V_g}$$

Band diagram of the capacitor



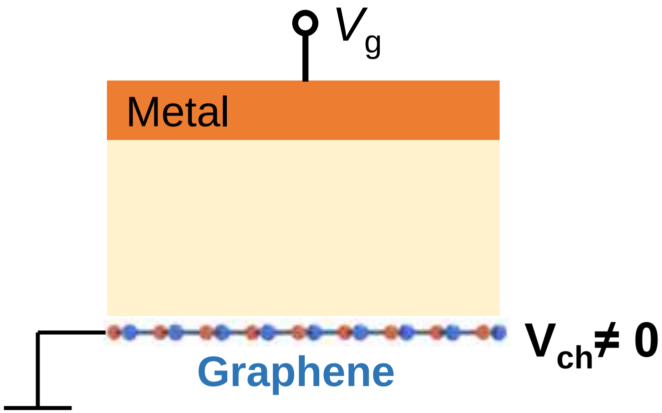


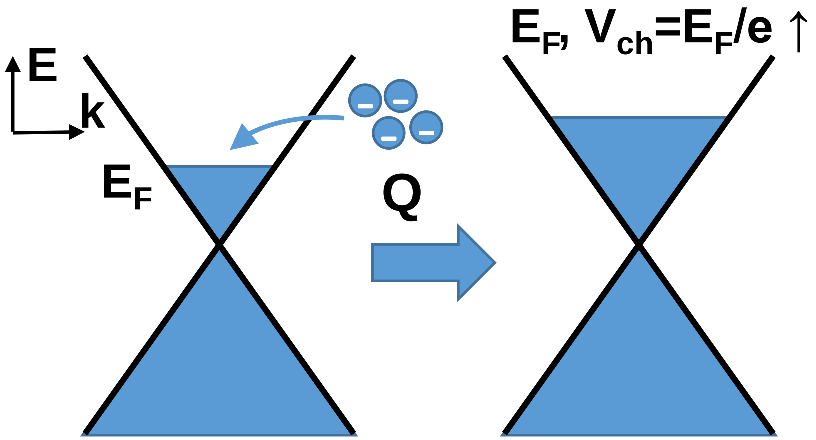




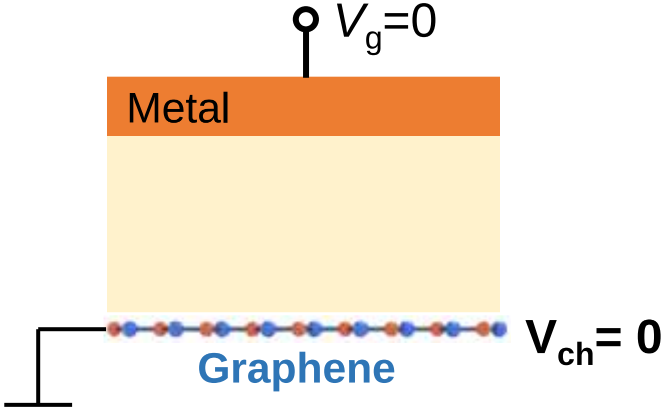
Quantum capacitance

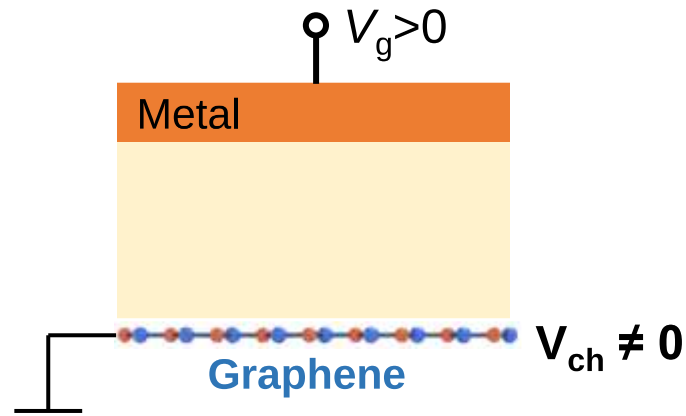
Semiconductor has a very low density of states near band edge or inside bandgap

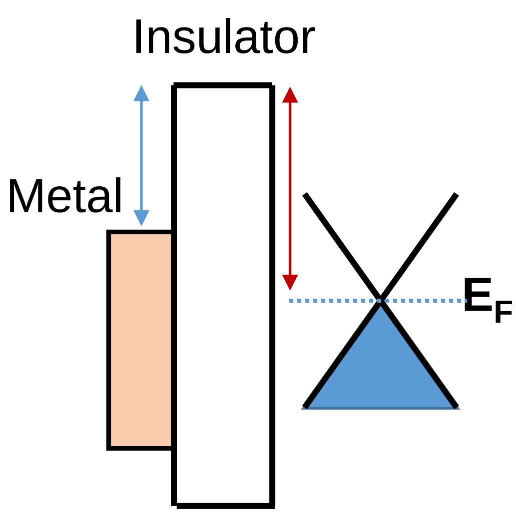


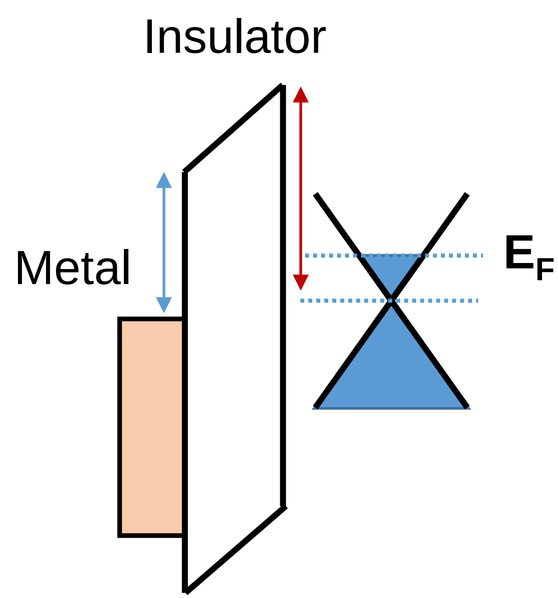


Quantum capacitance

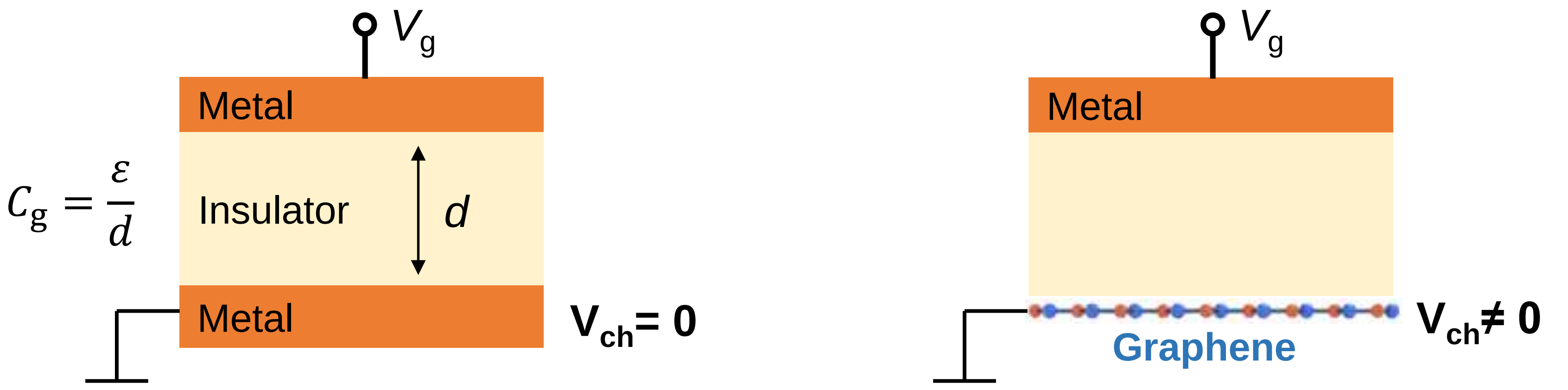


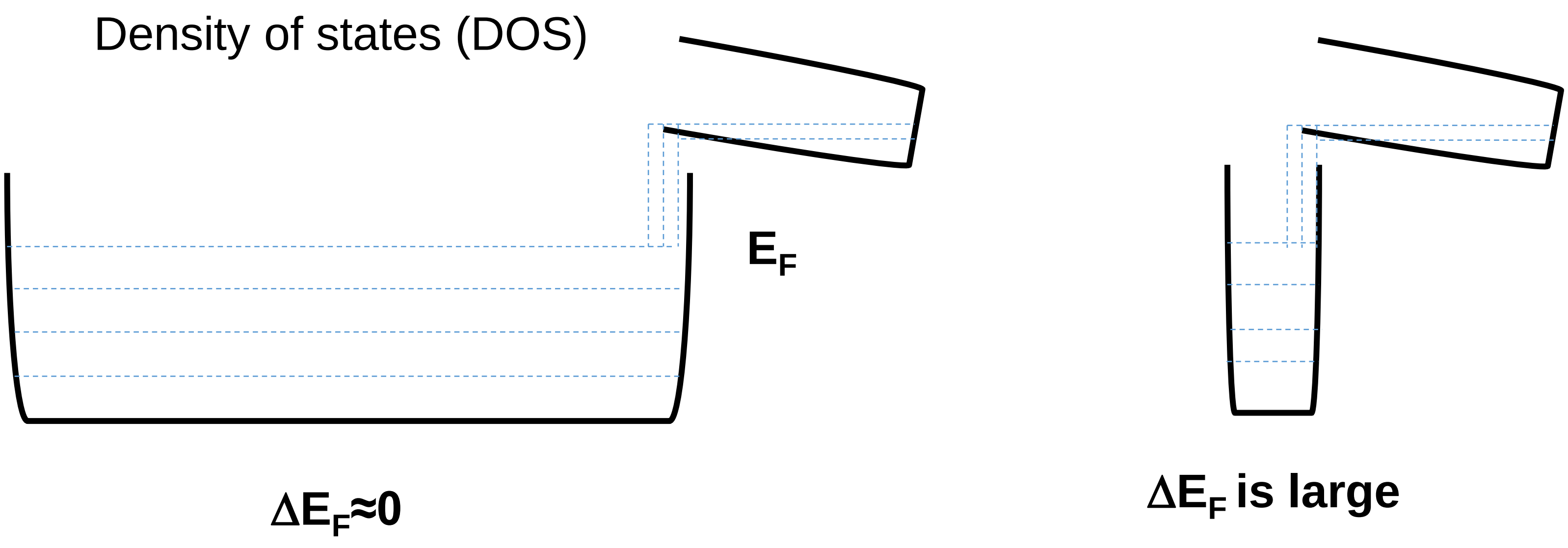






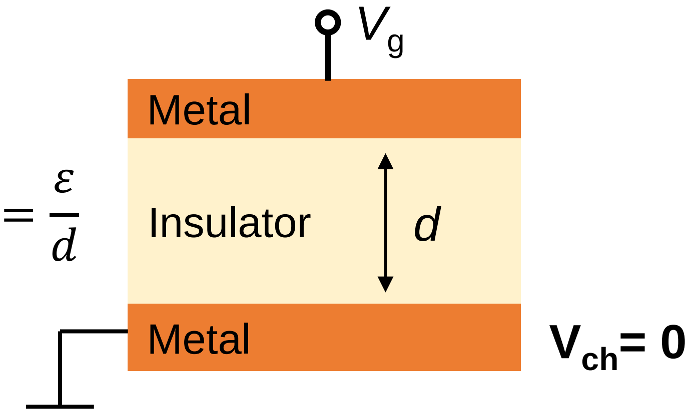
Quantum capacitance

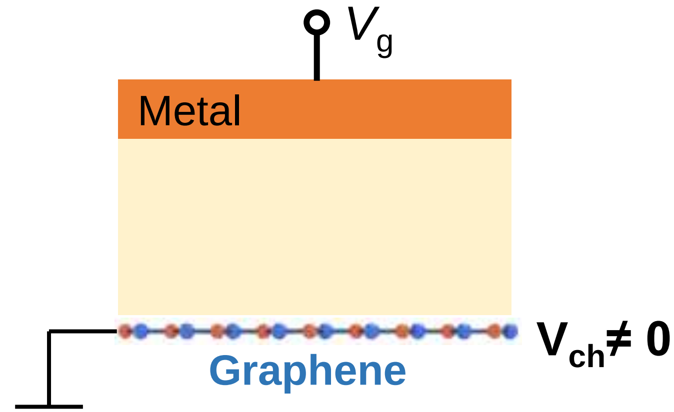




Quantum capacitance

$C_g = \frac{\varepsilon}{d}$





$$C_q = \frac{dQ}{dV_{ch}} = \frac{1}{e}\frac{dQ}{dE_F}$$

For conventional metal:

$C_q \to \infty$

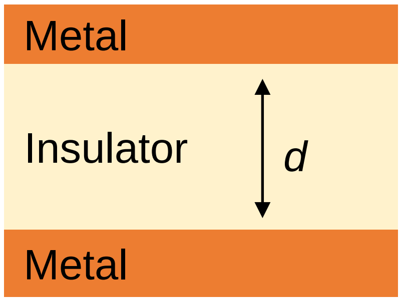
For graphene

C_q can be detected

Quantum capacitance: $C_q = e^2 \cdot \rho(E_F)$ Density of states

Quantum capacitance

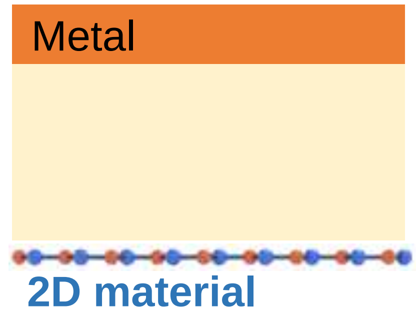
Parallel plate capacitor



Geometry capacitance

$$C_g = \frac{\varepsilon}{d}$$

Quantum capacitor



$$C_t^{-1} = \left(\frac{\varepsilon}{d}\right)^{-1} + \boldsymbol{C_q}^{-1}$$

Geometry capacitance

Quantum capacitance

Quantum capacitance: $C_q = e^2 \cdot \rho(E_F)$ ---→ Density of states

Metal - Large ρ, C_q> 100 μF/cm^2>> C_g 2D material - **Low ρ**, C_q~ **1 μF/cm^2**

Advantage of capacitance measurement

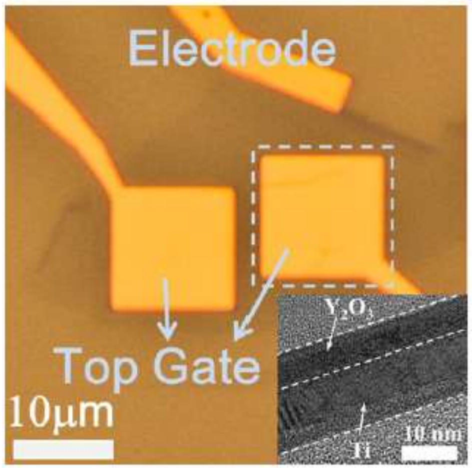
Capacitance $C_q = e^2 \cdot \rho(E_F)$

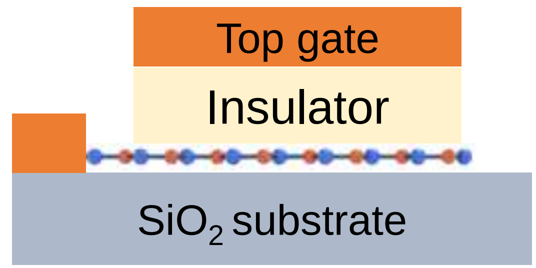
Transport $\sigma = e^2 \cdot D(E_\mathrm{F}) \cdot \rho(E_\mathrm{F})$

- ◆ Study intrinsic electronic states
- ◆ Detect impurity states
- ◆ Band structure
- ◆ Electron-electron interactions

Quantum capacitance in pristine graphene

Graphene/8nm-Y_2O_3/Au

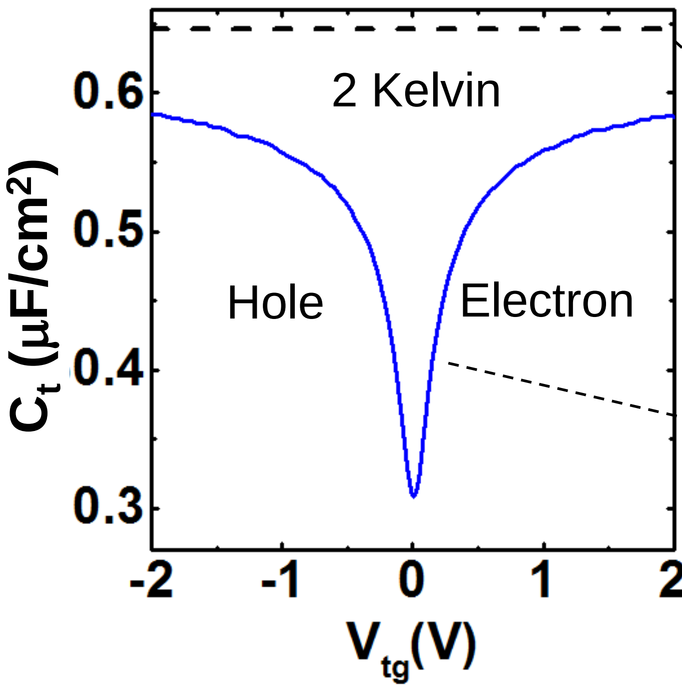


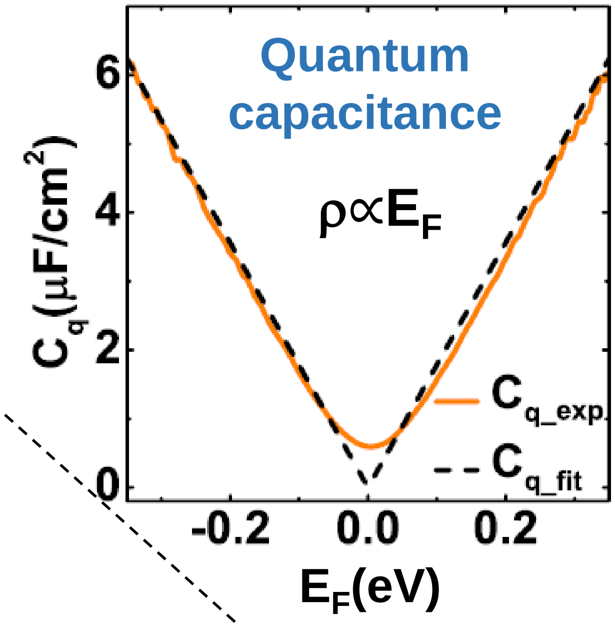


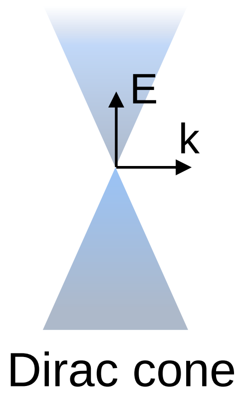
$$C_t^{-1} = \left(\frac{\varepsilon}{d}\right)^{-1} + \boldsymbol{C_q}^{-1}$$

Larger ε/d, better performance

Quantum capacitance in pristine graphene







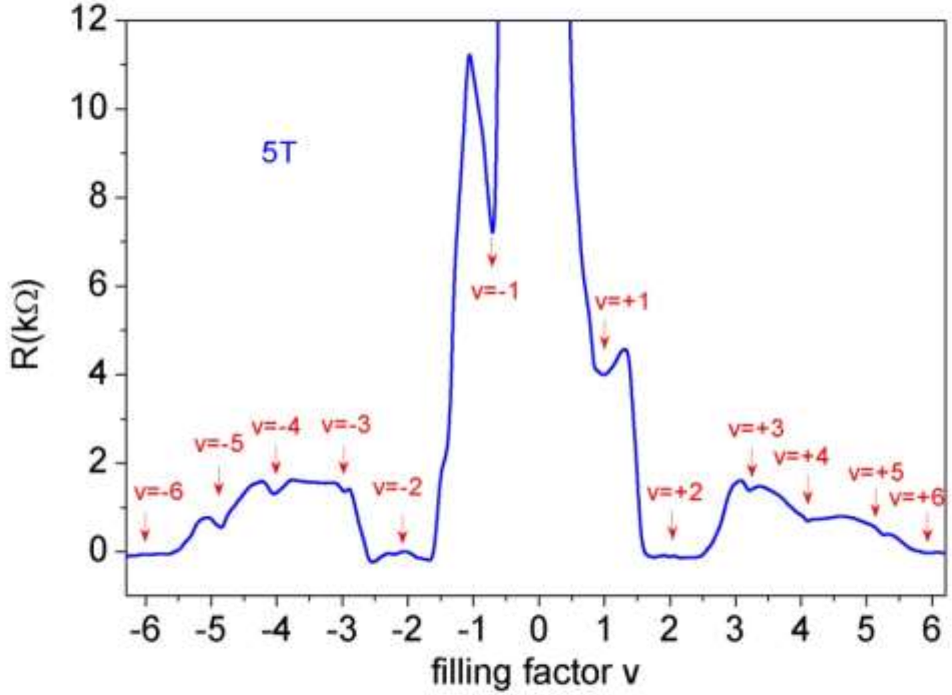
$$C_t^{-1} = \left(\frac{\varepsilon}{d}\right)^{-1} + \mathbf{C_q}^{-1}$$

Fermi energy

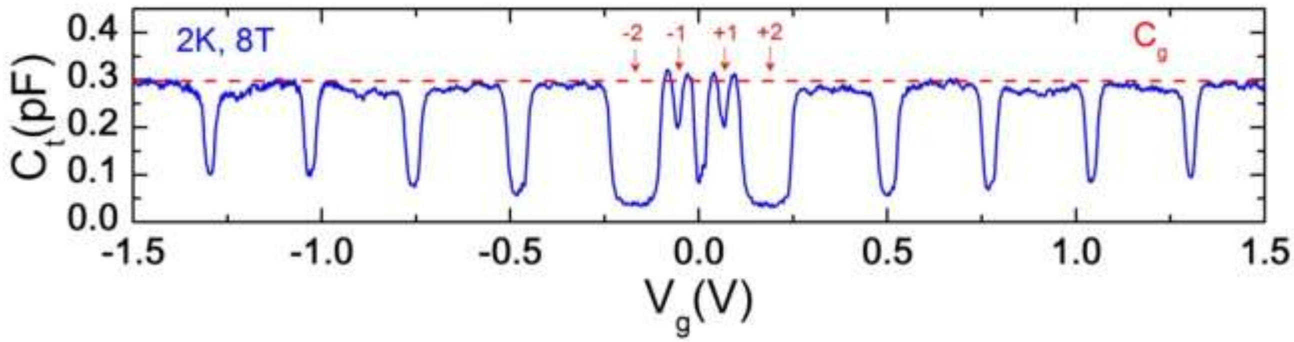
$$E_{\mathrm{F}} = e\int_0^{V_{tg}} \left(1 - \frac{C_t}{C_g}\right) dV_{tg}$$

Quantum phenonmena

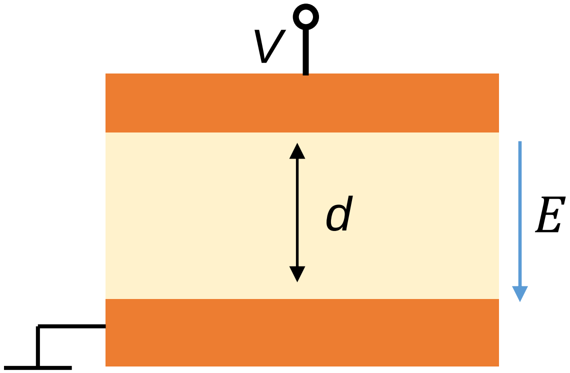
Transport under magnetic fields



Quantum capacitance measurements



6.7 Dielectric breakdown



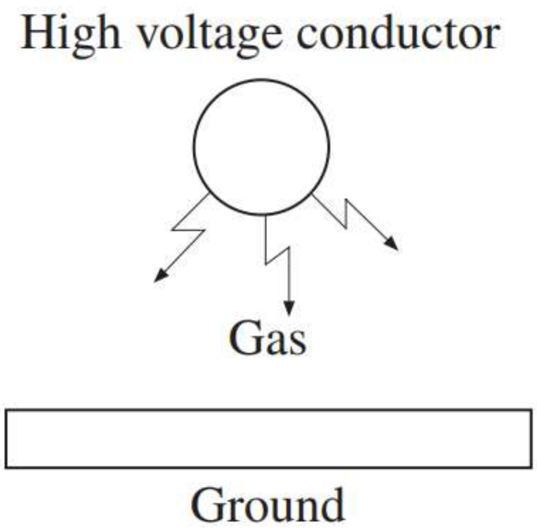
Electric field cannot be infinite large

When $E \geq E_{\mathrm{br}}$, there will be large current. This phenomenon is called **dielectric breakdown**

E_{br} is called **dielectric strength**

Dielectric Medium	Dielectric Strength	Comments
Atmosphere at 1 atm pressure	31.7 kV cm^{-1} at 60 Hz	1 cm gap. Breakdown by electron avalanche by impact ionization.
SF_6 gas	79.3 kV cm^{-1} at 60 Hz	Used in high-voltage circuit breakers to avoid discharges.
Polybutene	>138 kV cm^{-1} at 60 Hz	Liquid dielectric used as oil filler and HV pipe cables.
Transformer oil	128 kV cm^{-1} at 60 Hz	
Amorphous silicon dioxide (SiO_2) in MOS technology	10 MV cm^{-1} dc	Very thin oxide films without defects. Intrinsic breakdown limit.
Borosilicate glass	10 MV cm^{-1} duration of 10 μs	Intrinsic breakdown.
	6 MV cm^{-1} duration of 30 s	Thermal breakdown.
Polypropylene	295–314 kV cm^{-1}	Likely to be thermal breakdown or electrical treeing.

Dielectric breakdown in gas



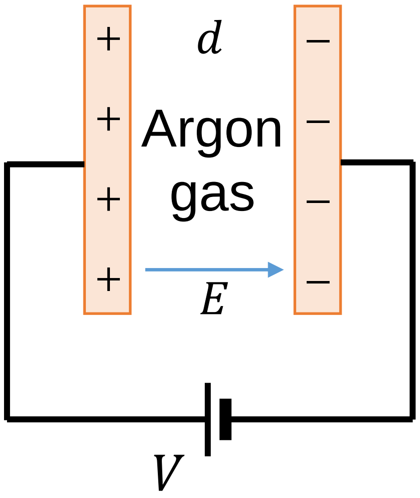
Cosmic radiation

Some free electrons in air

Free electrons are accelerated to a high energy, which can knock out other electrons from gas molecule

Avalanche effect: more and more electrons are knocked out

An example



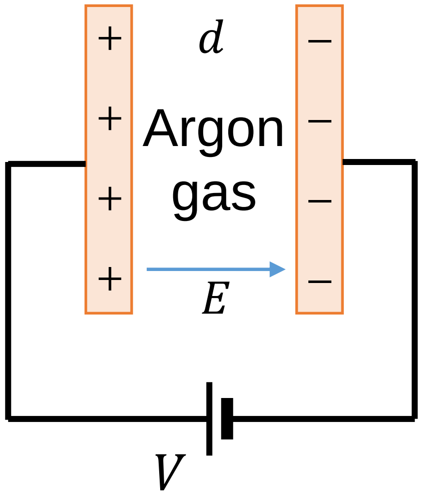
Argon ionization energy E_{I}

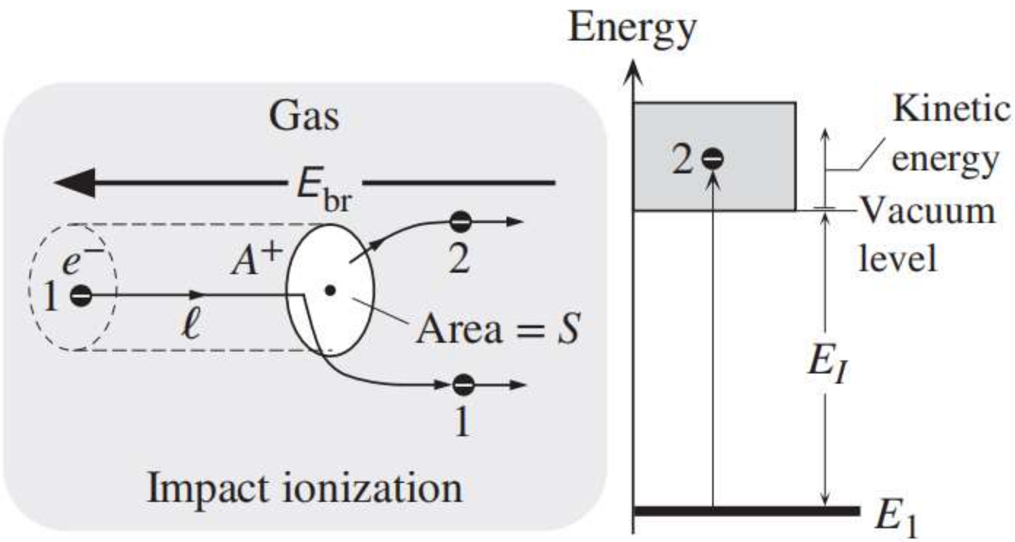
Argon gas pressure P

Ionization radius r_{i}

Temperature T

Breakdown voltage V_{br}





Electron mean free path parallel to *E*-field between two ionization collision l

To knock out electron: $eE_{\mathrm{br}}l = E_{\mathrm{I}}$

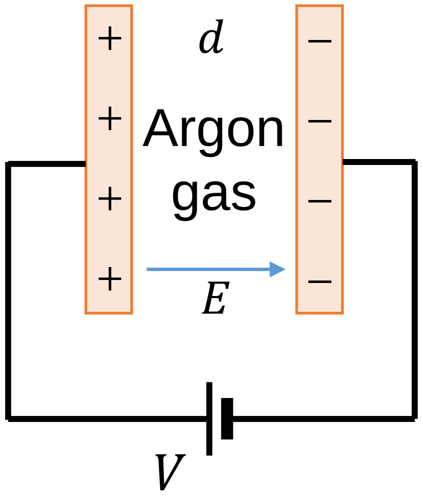
$$PV = NkT$$

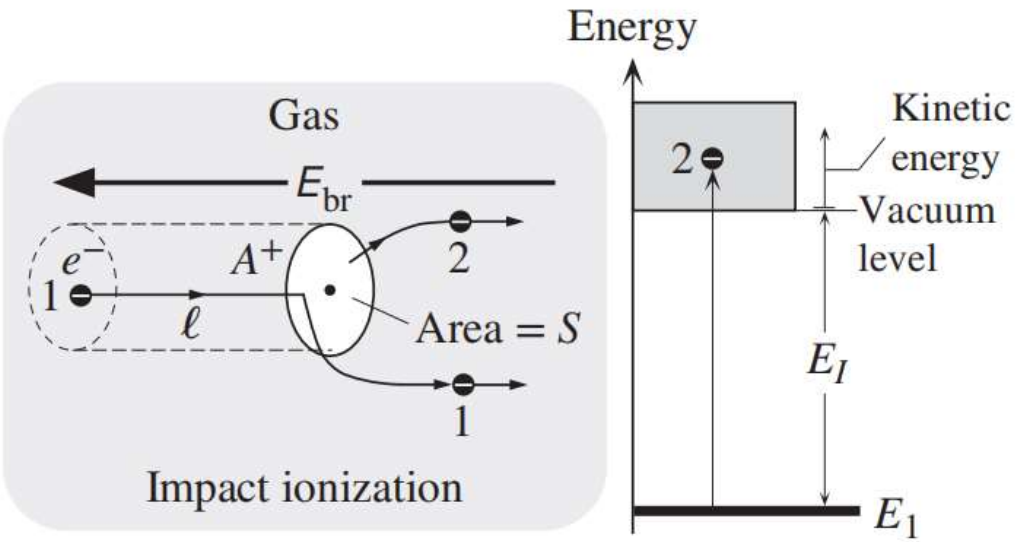
$$n_{\mathrm{gas}} = \frac{P}{kT}$$

$$l = \frac{1}{n_{\mathrm{gas}}(\pi r_{\mathrm{i}}^2)}$$

$$E_{\mathrm{br}} = \frac{\pi E_{\mathrm{I}} P r_{\mathrm{i}}^2}{ekT}$$

$$V_{\mathrm{br}} = E_{\mathrm{br}}d = \frac{\pi E_{\mathrm{I}} P r_{\mathrm{i}}^2 d}{ekT}$$





$$V_{\mathrm{br}} = \frac{\pi E_{\mathrm{I}} P r_{\mathrm{i}}^2 d}{ekT}$$

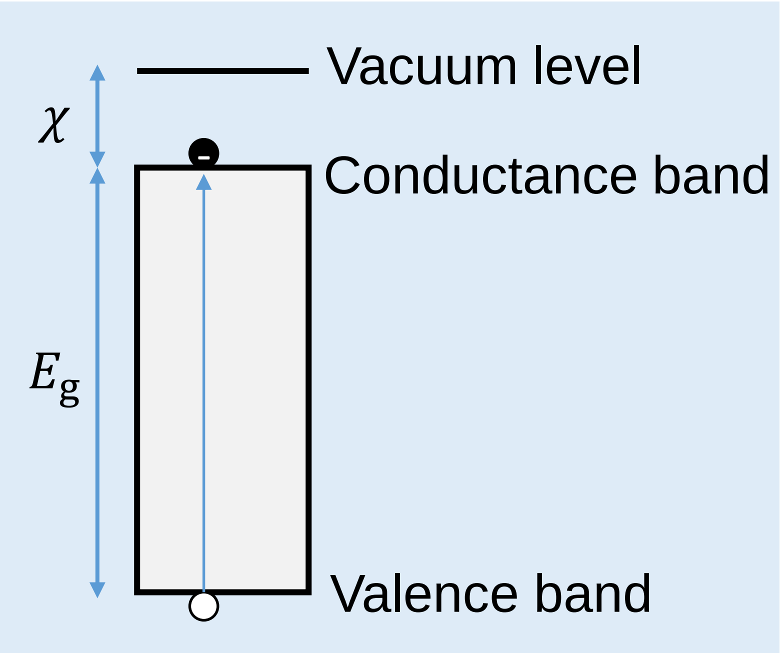
Higher pressure, larger breakdown voltage


宝宝心里苦

Dielectric breakdown in solids

Intrinsic/Electronic breakdown

Electron avalanche effect



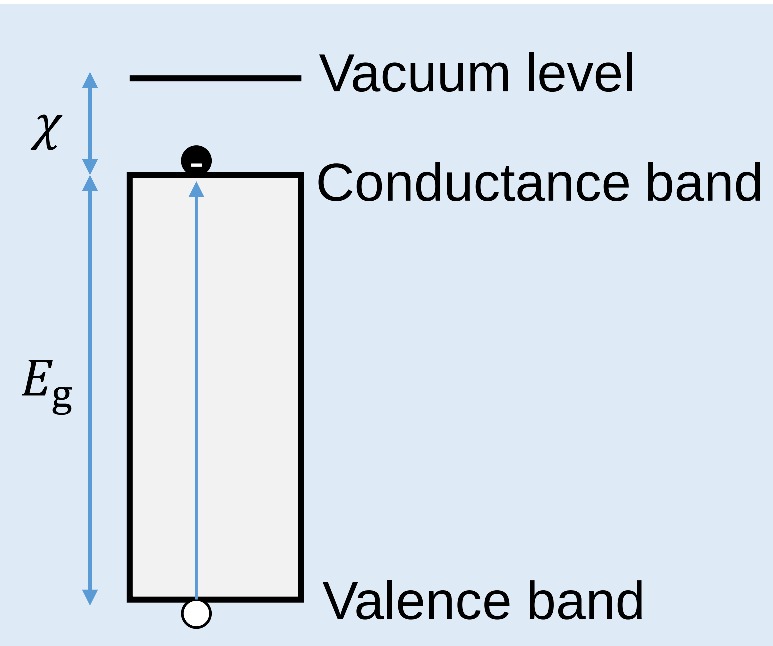
When $eE_{br}l \geq ??$

$$eE_{br}l \geq E_g$$

Electrons have enough energy to knock out other bonded electrons to the conductance band.

Large current

Intrinsic/Electronic breakdown



Amorphous SiO_2, $E_{\mathrm{br}} \sim 10\ \mathrm{MV/cm}$ for DC voltage.

If a MOSFET has a 10 nm-thick SiO_2, the breakdown voltage is 10V

If the dielectric material is thin enough, **quantum tunneling effect can occur before dielectric breakdown**.

Thermal breakdown

For high frequency AC voltage

Input power: $IV = j\omega C'^{V^2} + \dfrac{V^2}{R_{\mathrm{P}}}$

Joule heat per unit volume: $\dfrac{V^2}{R_{\mathrm{P}}}\dfrac{1}{\mathrm{Volume}} = \sigma_{\mathrm{P}}E^2$

If heat cannot be efficiently removed, local temperature will be very high

Conductance channel forms and dielectric breakdown

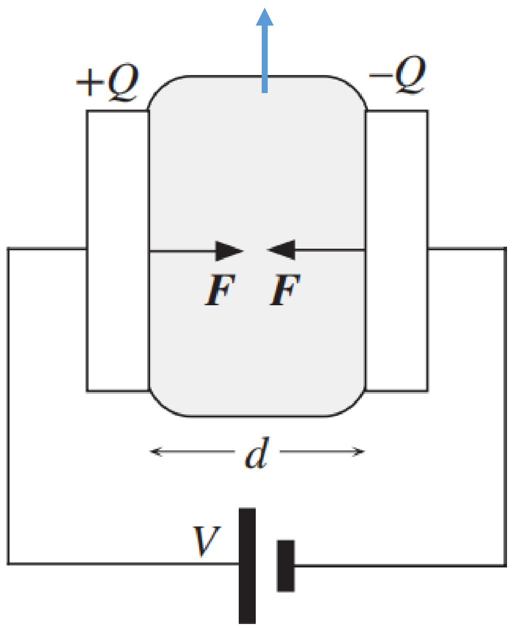
Thermal breakdown

Thermal breakdown depends on ambient temperatures

Higher temperature, lower dielectric strength E_{br}

Electromechanmical/Electrofracture breakdown

Small elastic modulus → Easy to deform 易变形



Apply a voltage V

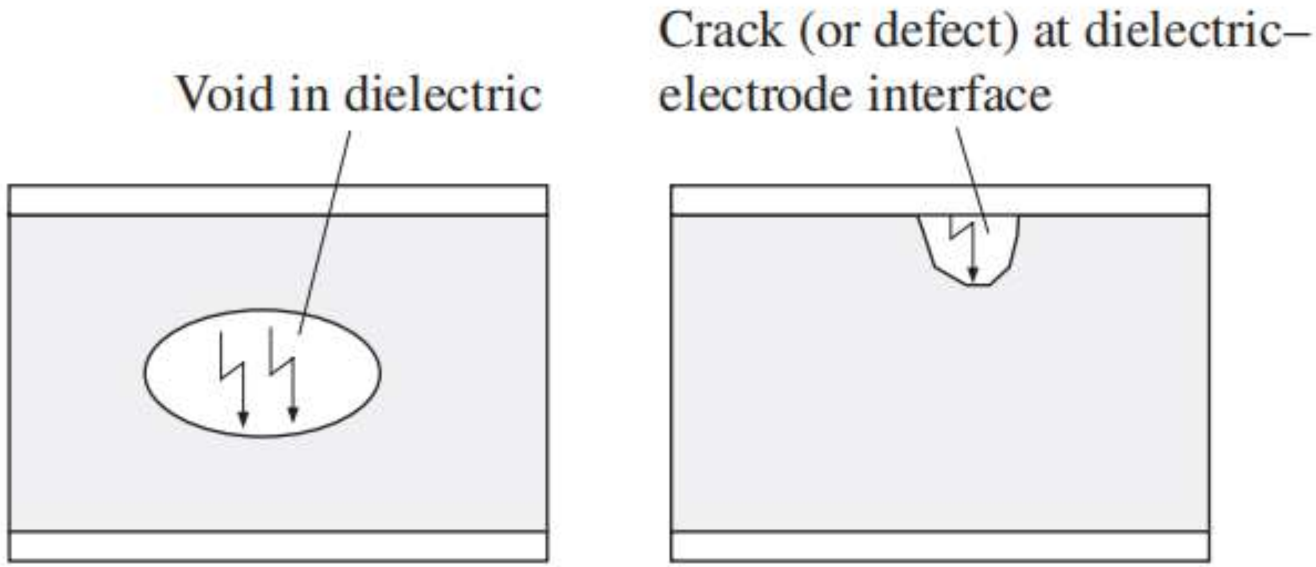
Two metal plates attract with each other

Dielectric material is compressed

Electric field increase

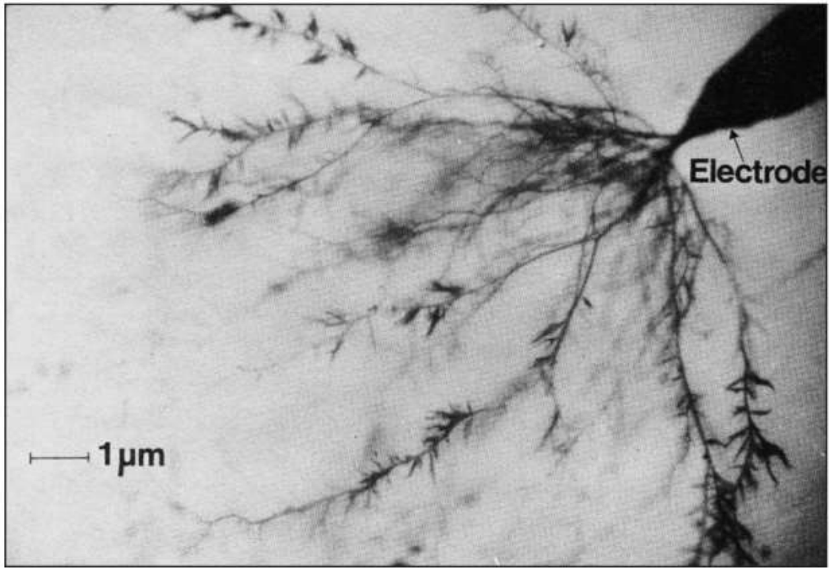
Dielectric breakdown

Internal discharge



Internal gas atmosphere region easily breakdown

Internal discharge



The void/crack

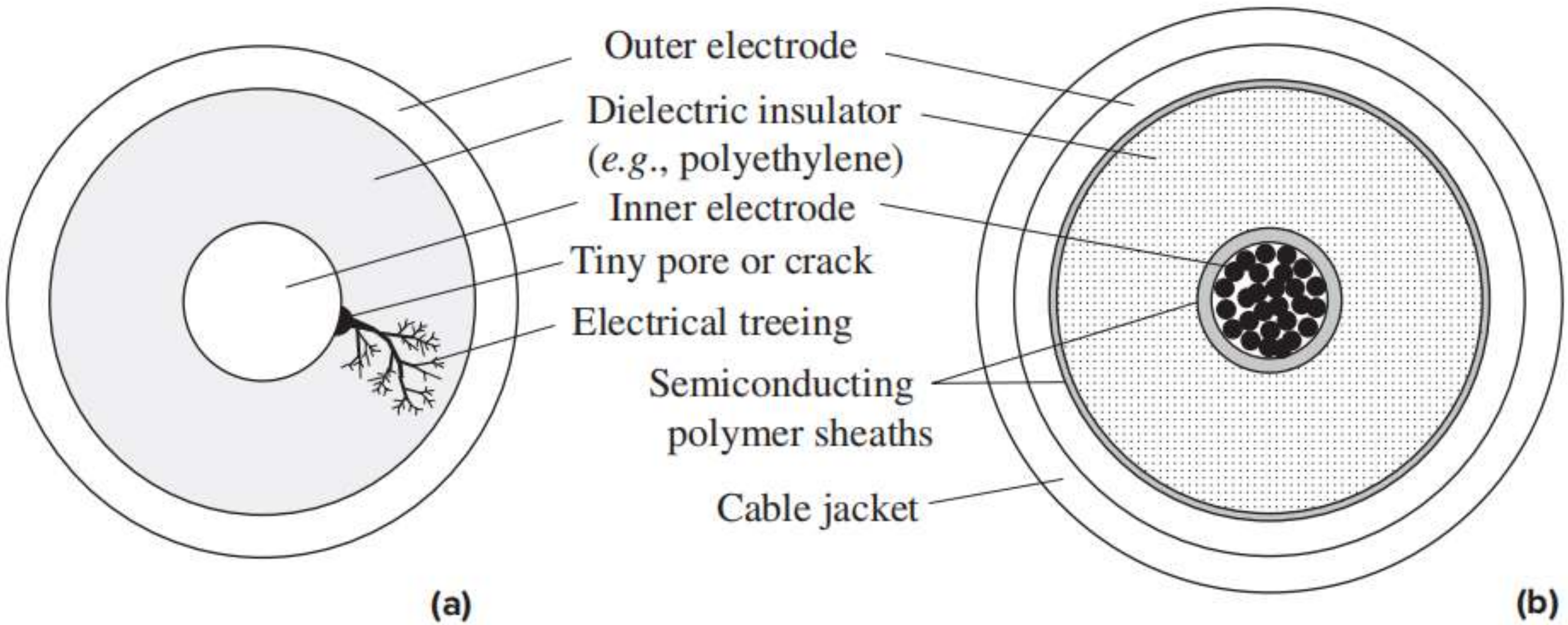
Locally breakdown

Melt dielectric or other chemical transformation

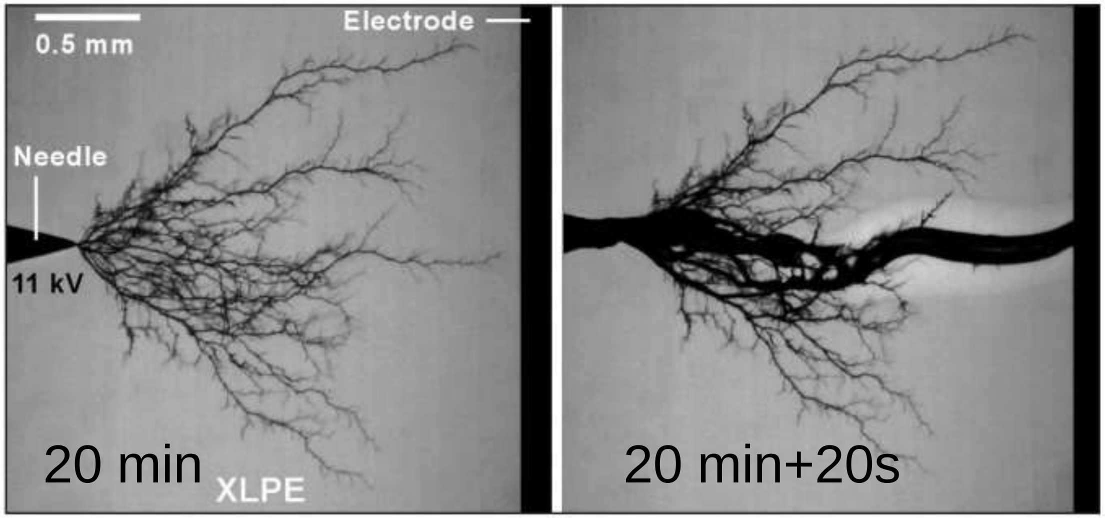
The breakdown region extends

Electrical tree 电树枝

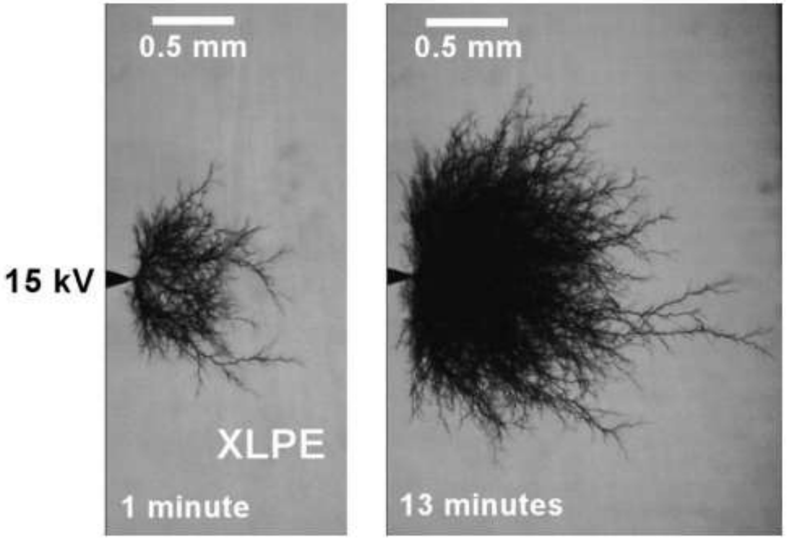
Breakdown in high voltage coaxial cable



Branch trees

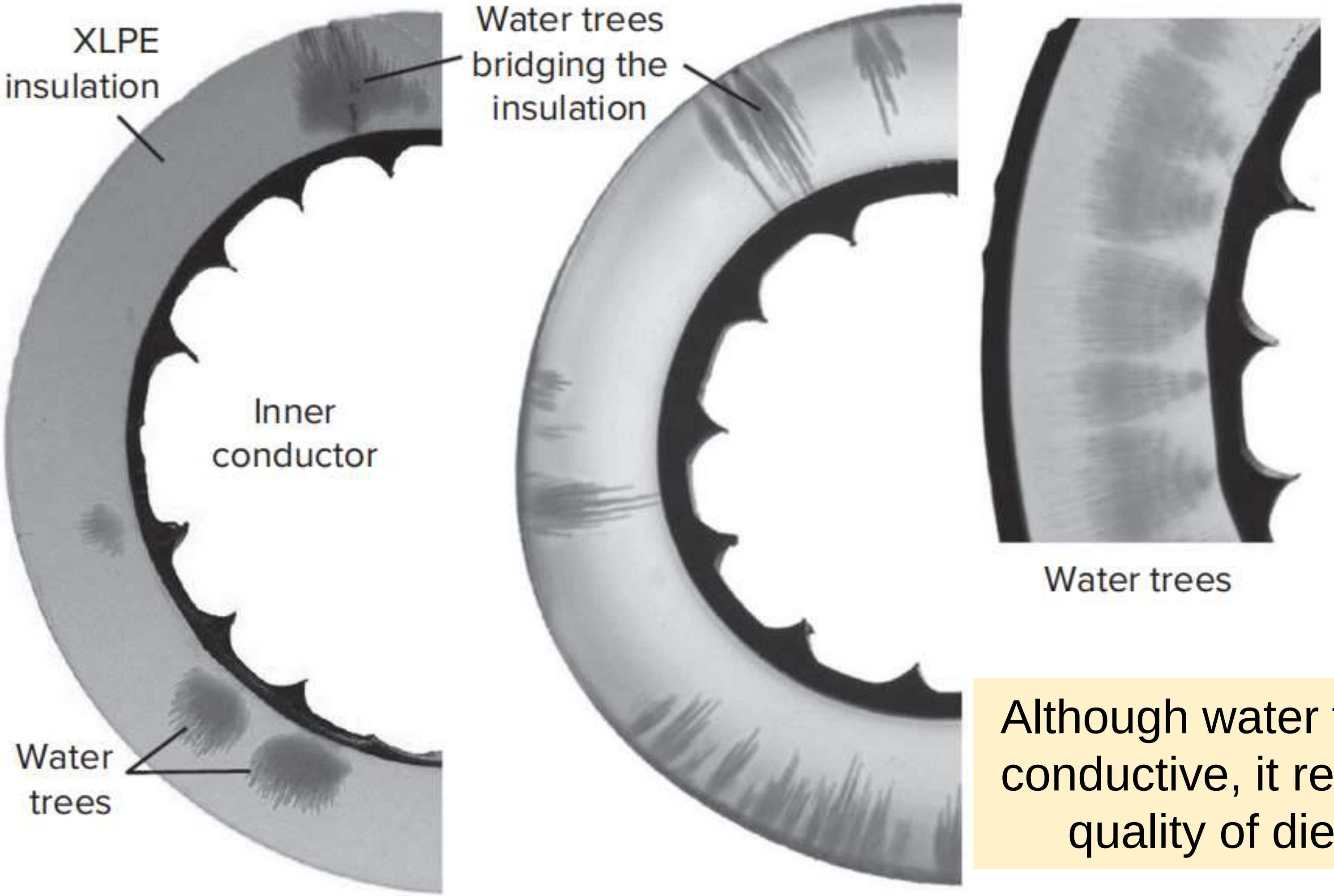


Bush trees



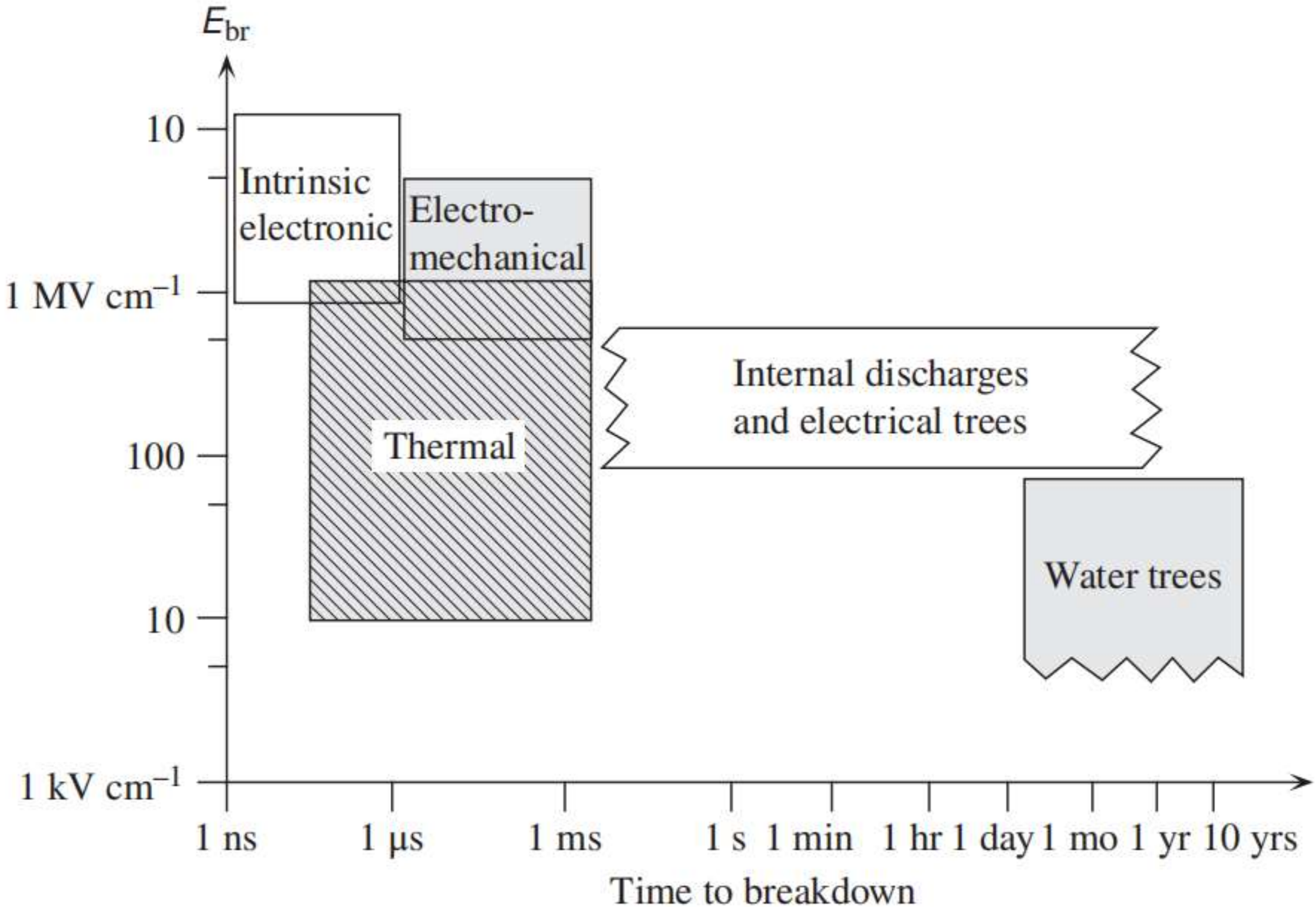
Insulation aging

Moist environment



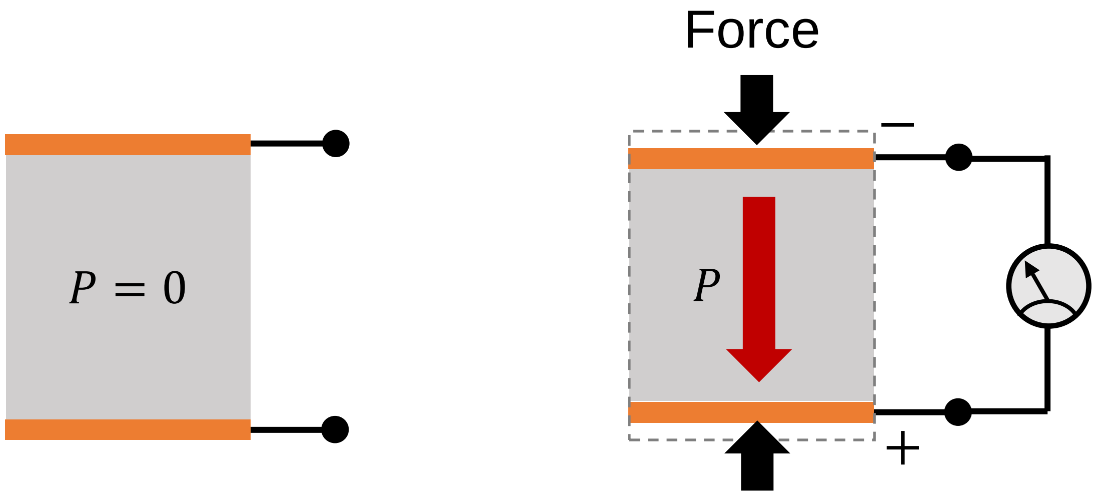
Although water tree is not conductive, it reduces the quality of dielectric.

Comparison of various breakdown mechanism



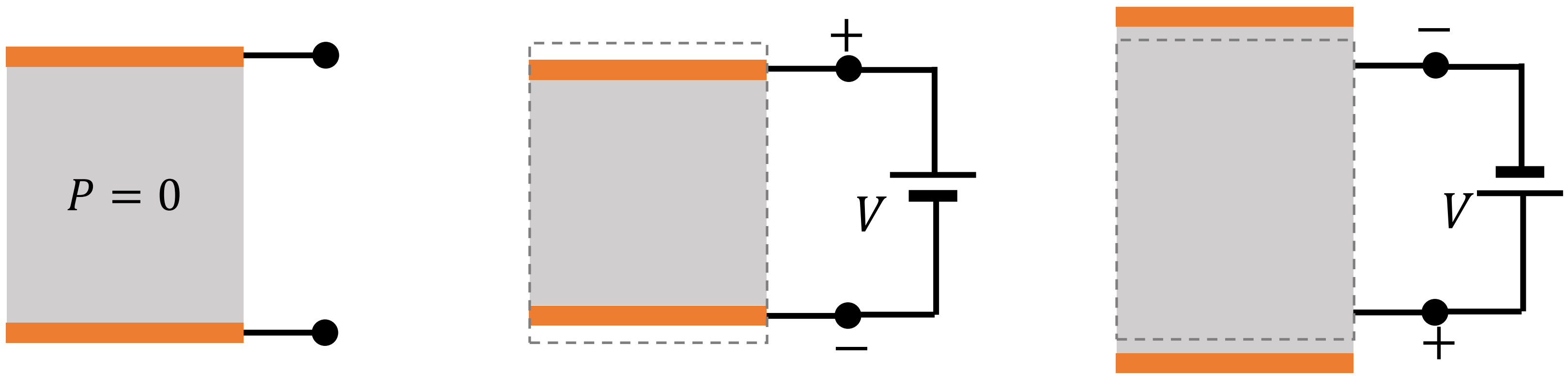
6.8 Piezoelectricity 压电效应

Crystals, such as quartz (crystalline SiO_2) and $BaTiO_3$, become polarized when they are mechanically stressed.



Charges appear on the surface under stress, and results in a voltage difference between two surfaces.

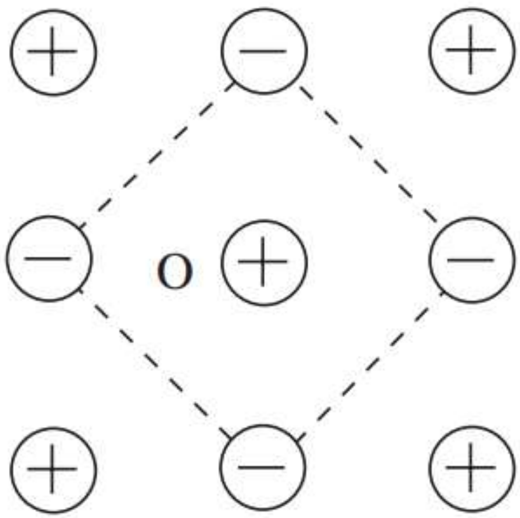
When apply an external electric field, the crystal also experiences mechanical deformation (extension or compression).



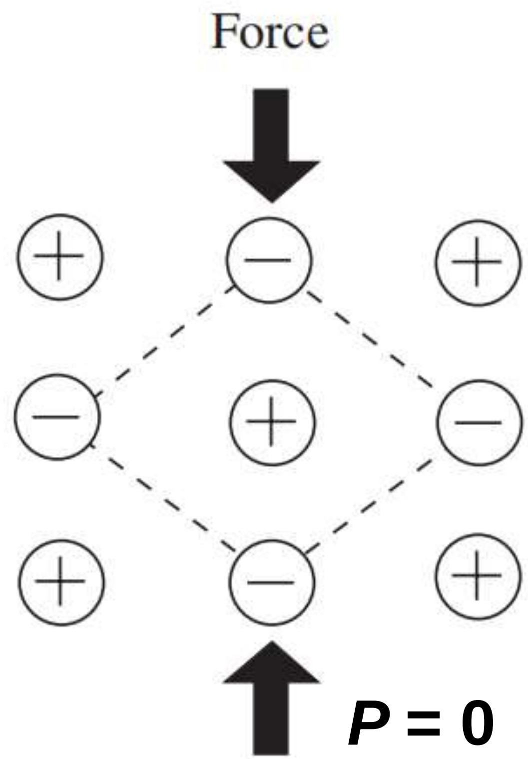
These two effects are called **piezoelectricity**.

Physical mechanisms

(1) When crystalline structure has center of symmetry



极化矢量P = 0

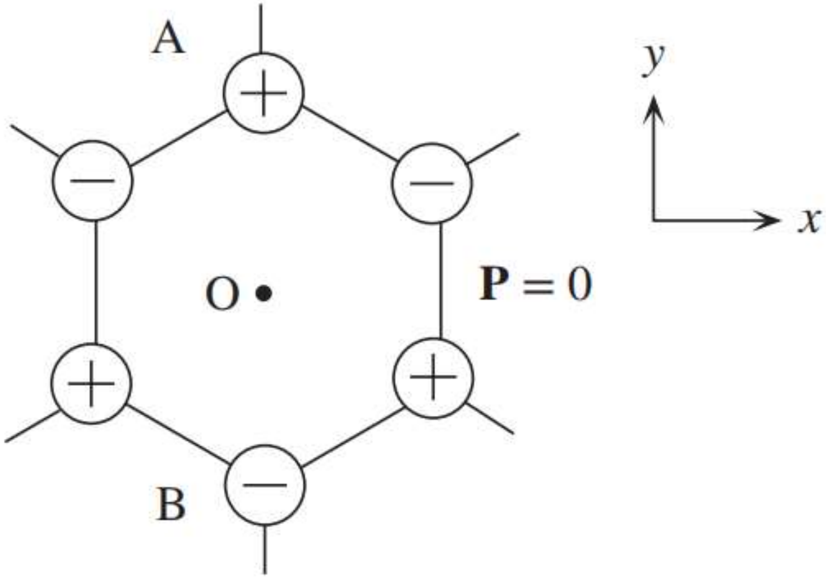


P = 0

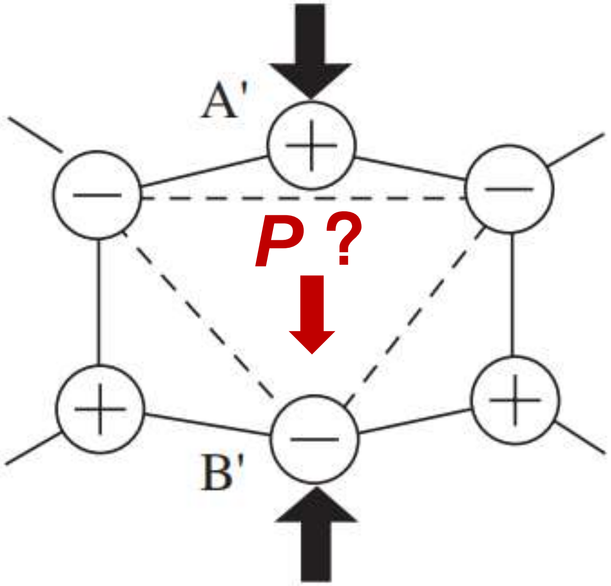
No piezoelectricity!

(2) When crystalline structure is noncentrosymmetric

Hexagonal cell



When stress is along y direction

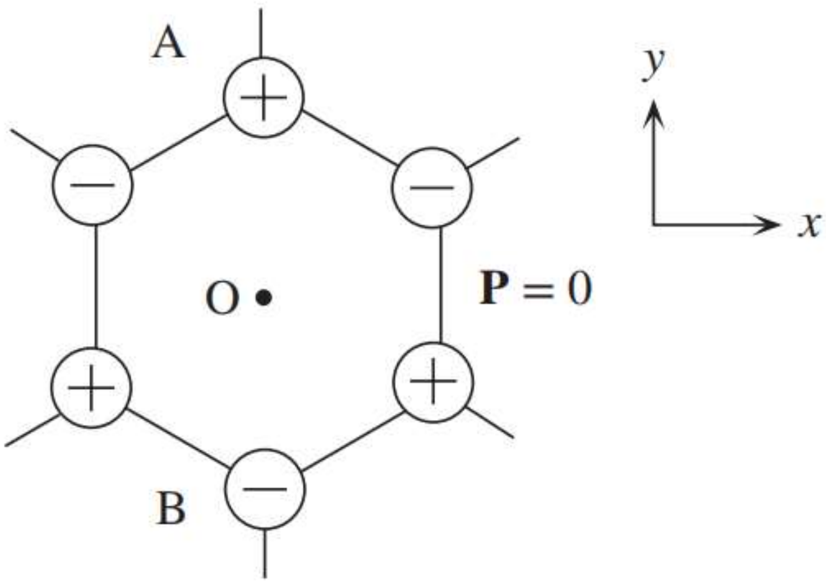


$$P_x = 0$$

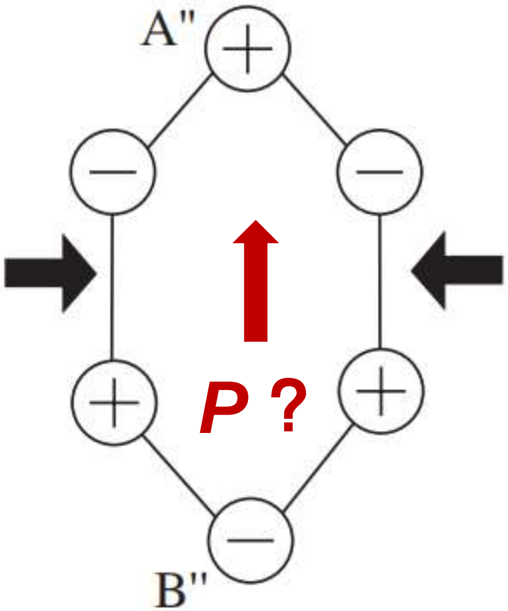
$$P_y < 0$$

(2) When crystalline structure is noncentrosymmetric

Hexagonal cell



When stress is along x direction



$P_x = 0$ $P_y > 0$

An applied stress in one direction can give rise to induced polarization in other crystal directions.

Induced polarization along *i*-direction $P_\mathrm{i} = d_\mathrm{ij} T_\mathrm{j}$ Mechanical stress along *j*-direction

d_ij: piezoelectric coefficients

d_ij: has a unit of m/V

T_j: has a unit of Pa

Induced strain (应变) along *i*-direction $S_\mathrm{i} = d_\mathrm{ij} E_\mathrm{j}$ Applied electric field along *j*-direction

Electromechanical coupling factor机电耦合系数

$$k^2 = \frac{\text{Electrical energy coverted to mechanical energy}}{\text{Input of electrical energy}}$$

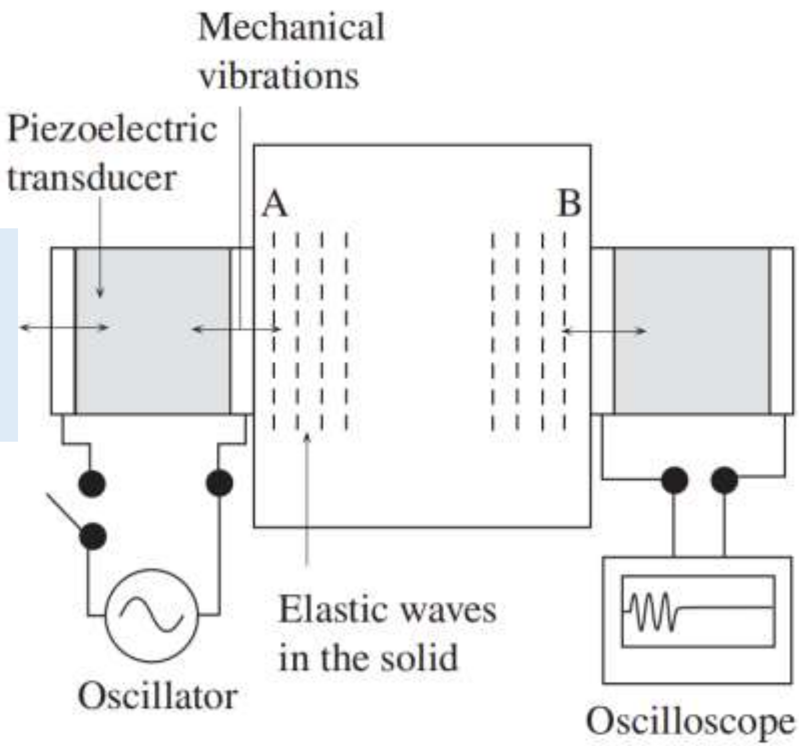
or

$$k^2 = \frac{\text{Mechanical energy coverted to electrical energy}}{\text{Input of mechanical energy}}$$

Crystal	d (m V^{-1})	k	Comment
Quartz (crystal SiO_2)	2.3×10^{-12}	0.1	Crystal oscillators, ultrasonic transducers, delay lines, filters
Rochelle salt ($NaKC_4H_4O_6 \cdot 4H_2O$)	350×10^{-12}	0.78	
Barium titanate ($BaTiO_3$)	190×10^{-12}	0.49	Accelerometers
PZT, lead zirconate titanate ($PbTi_{1-x}Zr_xO_3$)	480×10^{-12}	0.72	Wide range of applications including earphones, microphones, spark generators (gas lighters, car ignition), displacement transducers, accelerometers
Polyvinylidene fluoride (PVDF)	18×10^{-12}	—	Must be poled; heated, put in an electric field and then cooled. Large area and inexpensive

Piezoelectric transducer 压电换能器

An AC electric energy converts to mechanical vibration

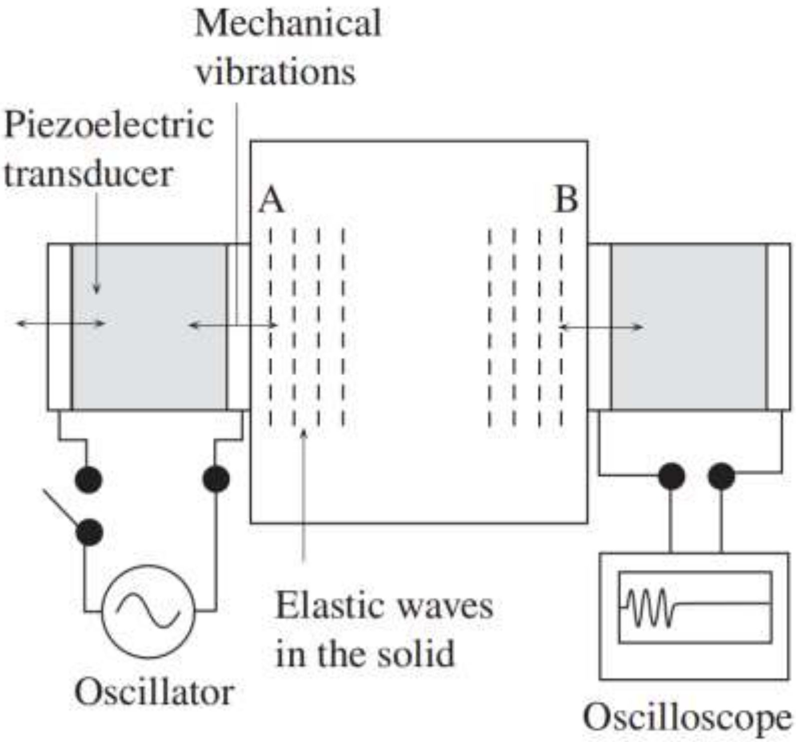


Mechanical vibration converts to electric signal, and are detected

The transducer is coupled with solids using grease油脂

An elastic waves is generated in solids, and are usually in ultrasonic wave region 超声波.

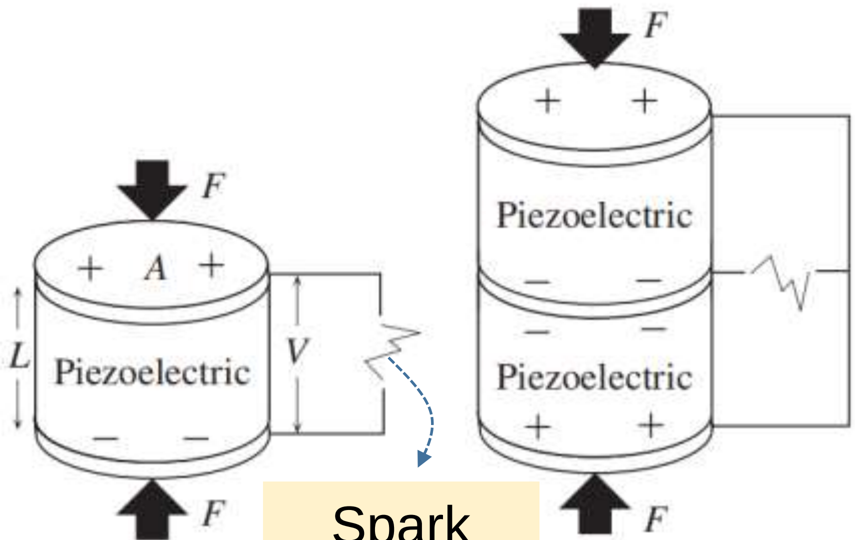
Piezoelectric transducer 压电换能器



Can be used to determine the Young's modulus of solids, and imperfections (such as cracks) in solids.

Piezoelectric spark generator压电火花发生器

Widely used in lighters打火机 and car ignitions汽车点火.



Spark gap in air

If we know:

Piezoelectric coefficient *d*

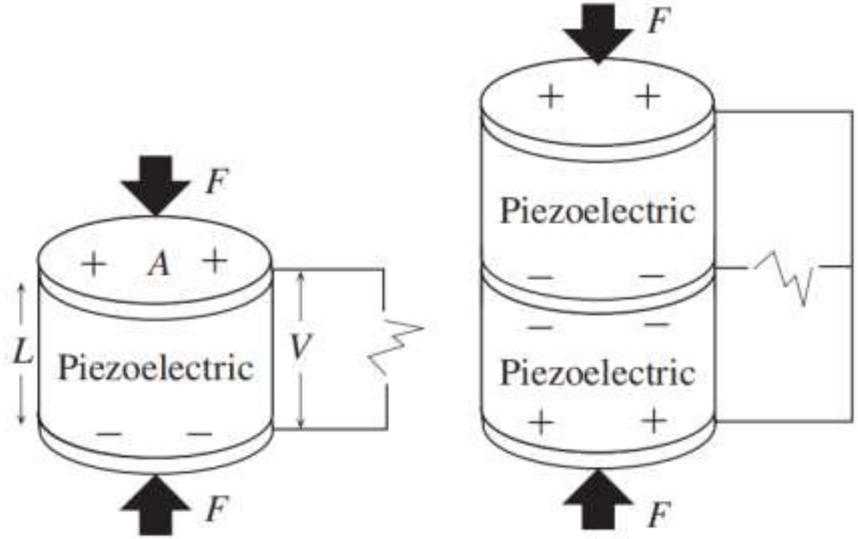
Relative dielectric constant ε_r

Surface area *A*

Thickness *L*

Breakdown voltage of the spark gap in air V_{br}

Ask: the minimum force needed to generate the spark.

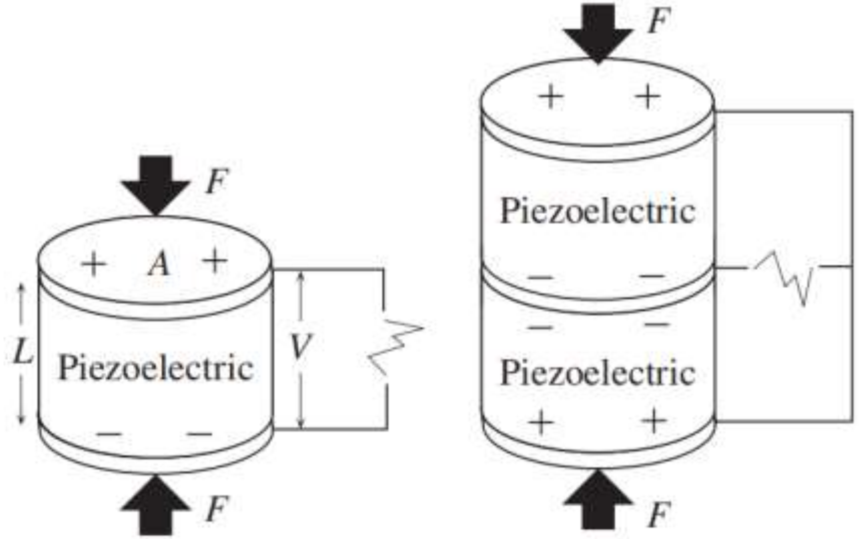


(a) Left figure

Induced polarization: $P = dT = d\dfrac{F}{A}$

Induced voltage: $V = \dfrac{Q}{C} = \dfrac{PA}{\varepsilon_{\mathrm{r}}\varepsilon_0 A/L} = \dfrac{PL}{\varepsilon_{\mathrm{r}}\varepsilon_0} = \dfrac{FLd}{A\varepsilon_{\mathrm{r}}\varepsilon_0}$

Minimum force: $F = \dfrac{A\varepsilon_{\mathrm{r}}\varepsilon_0}{LdV_{\mathrm{br}}}$



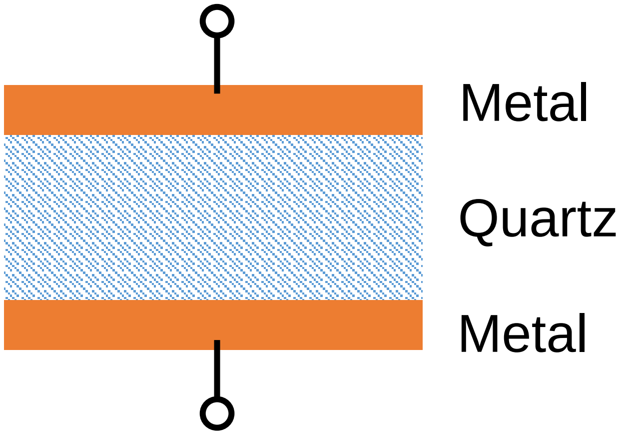
(b) Right figure

Induced polarization: $P = dT = d\dfrac{F}{A}$

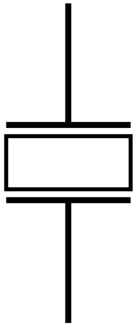
Induced voltage: $V = \dfrac{Q}{C} = \dfrac{2PA}{\varepsilon_{\mathrm{r}}\varepsilon_0 A/L} = \dfrac{2PL}{\varepsilon_{\mathrm{r}}\varepsilon_0} = \dfrac{2FLd}{A\varepsilon_{\mathrm{r}}\varepsilon_0}$

Minimum force: $F = \dfrac{A\varepsilon_{\mathrm{r}}\varepsilon_0}{2LdV_{\mathrm{br}}}$

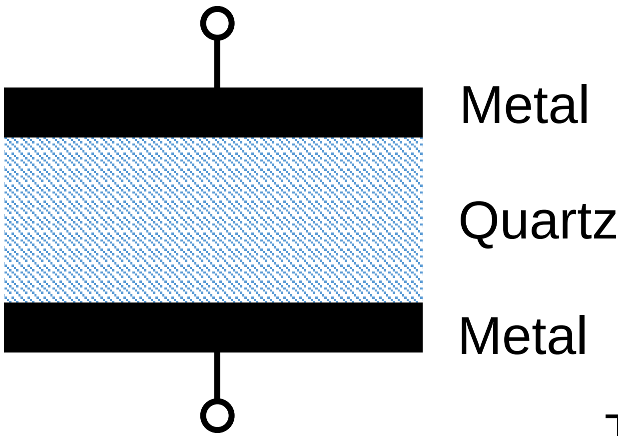
Quartz oscillators and filters



Electronic circuit symbol



Piezoelectric vibration 压电振荡



1) Apply a AC voltage across quartz;

2) Mechanical vibration in quartz;

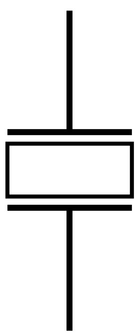
The amplitude of mechanical vibration is small

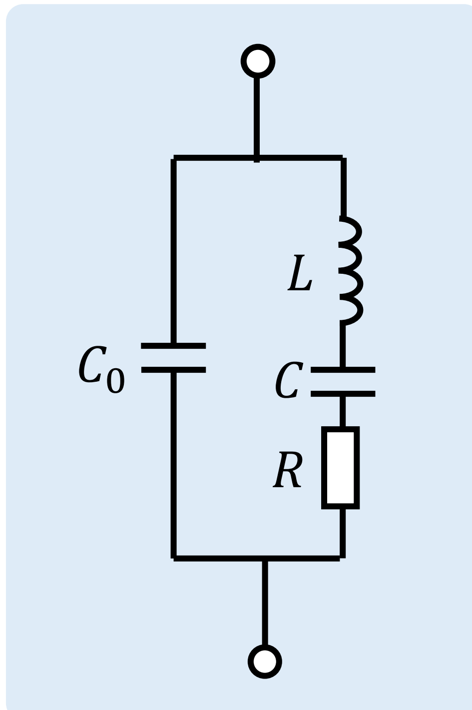
3) When AC voltage frequency equals to the intrinsic frequency of quartz.

4) The amplitude of vibration is very large, oscillation is resonant.

Intrinsic frequency: 固有频率，is also called resonant frequency 共振频率

Equivalent circuit 等效电路





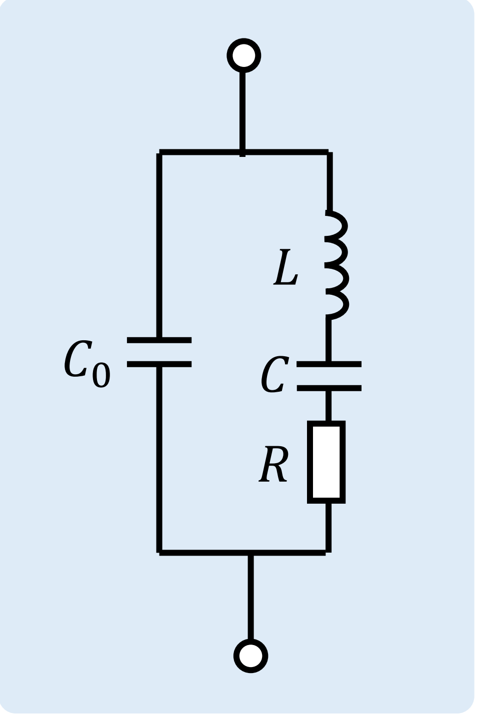
C_0: Static capacitance

L: Inertia of mechanical vibration 机械振动的惯性

$C \ll C_0$: Elastic capacitance

R: Friction dissipation 摩擦损耗

Equivalent circuit 等效电路



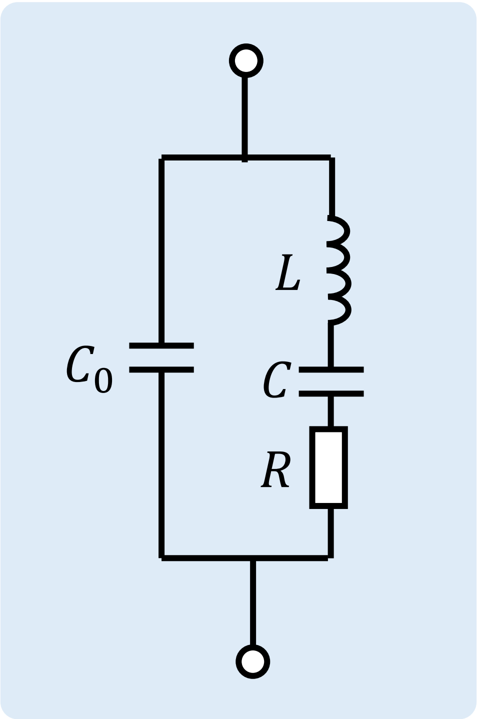
C_0: Determine by area and thickness of quartz

L is large: 10 H ~ 10 mH

C is small: 0.01 ~ 0.1 pF

R is small: ~100 Ω

Equivalent circuit 等效电路

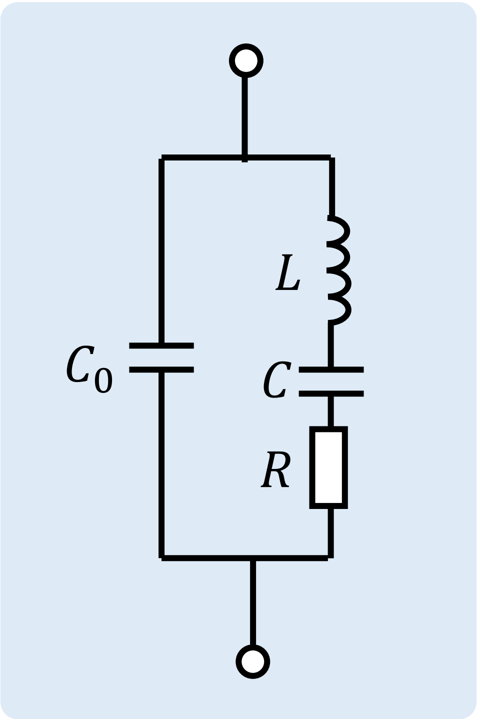


Quality factor Q is large: $10^4 \sim 10^6$

Frequency stability $\Delta f / f_0$ is very good:

- RC oscillator $\Delta f / f_0$: 10^{-2}
- LC oscillator $\Delta f / f_0$: $10^{-3} \sim 10^{-4}$
- Quartz oscillator $\Delta f / f_0$: $10^{-9} \sim 10^{-11}$

Frequency response of impedance



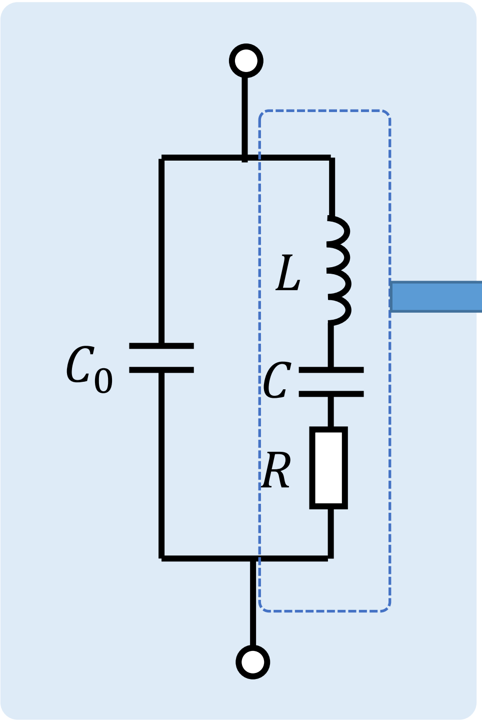
Ignore R: R=0

Impedance 阻抗 $\dot{Z}$= reactance 电抗 $\dot{X}$

$$\dot{X} = \frac{\frac{1}{\mathrm{j}\omega C_0}(\mathrm{j}\omega L + \frac{1}{\mathrm{j}\omega C})}{\frac{1}{\mathrm{j}\omega C_0} + \mathrm{j}\omega L + \frac{1}{\mathrm{j}\omega C}}$$

$$= \frac{1 - \omega^2 LC}{\mathrm{j}\omega C_0(\frac{C}{C_0} + 1 - \omega^2 LC)}$$

Frequency response of impedance



$$\dot{X} = \frac{1-\omega^2 LC}{\mathrm{j}\omega C_0\left(\frac{C}{C_0}+1-\omega^2 LC\right)}$$

(a) When $1-\omega^2 LC = 0$, $\dot{X} = 0$

Series resonance 串联谐振

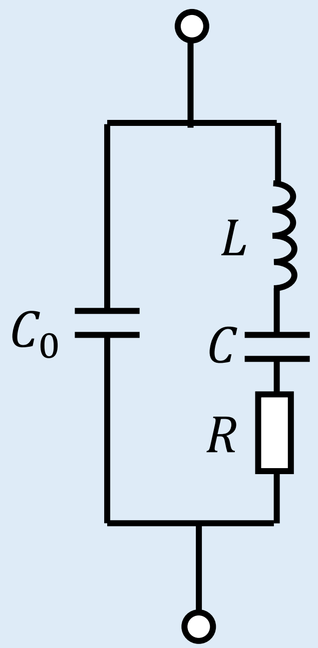
Series resonant frequency:

$$f_s = \frac{1}{2\pi\sqrt{LC}}$$

When $f < f_s$: circuit is capacitive 电路呈容性

When $f > f_s$: circuit is inductive 电路呈感性

Frequency response of impedance



$$\dot{X} = \frac{1-\omega^2 LC}{\mathrm{j}\omega C_0(\frac{C}{C_0}+1-\omega^2 LC)}$$

(b) When $\frac{C}{C_0}+1-\omega^2 LC = 0$, $\dot{X} \to \infty$

Parallel resonance 并联谐振

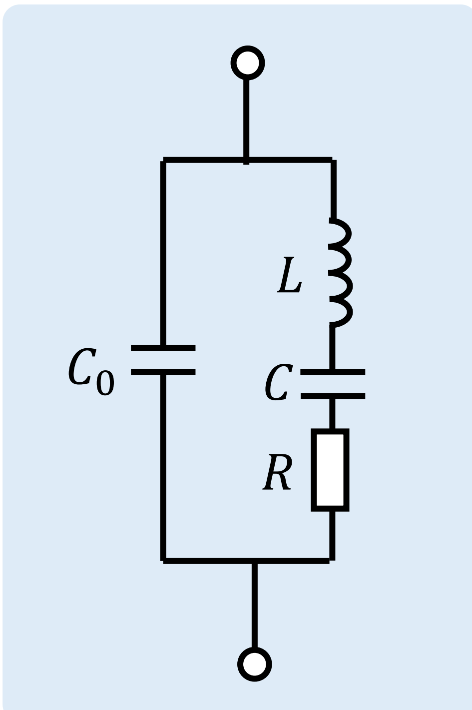
Parallel resonant frequency:

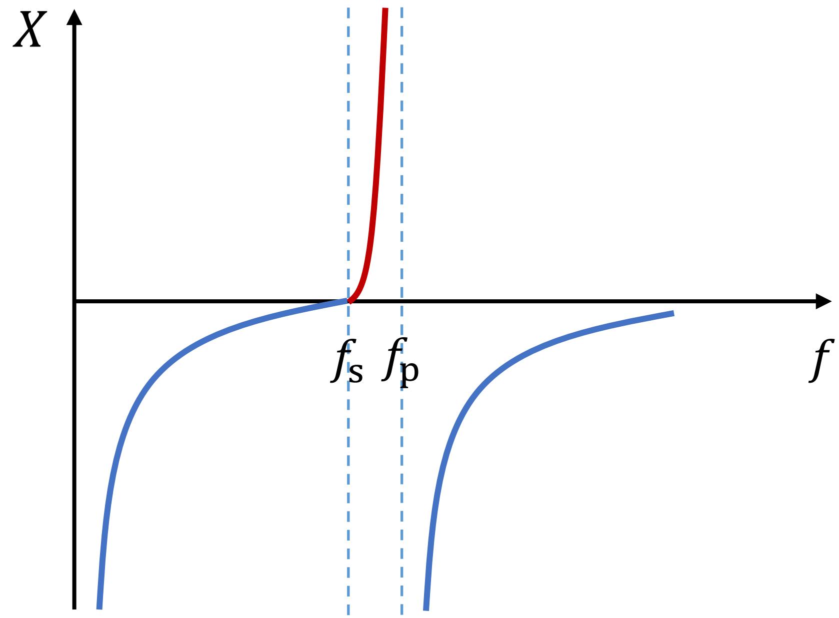
$$f_\mathrm{p} = \frac{1}{2\pi\sqrt{LC||C_0}} > f_\mathrm{s}$$

$$C \ll C_0\text{: } f_\mathrm{p} \approx f_\mathrm{s}$$

When $f > f_\mathrm{p}$: circuit is capacitive 电路呈容性

Frequency response of impedance





When $f < f_s$: circuit is capacitive

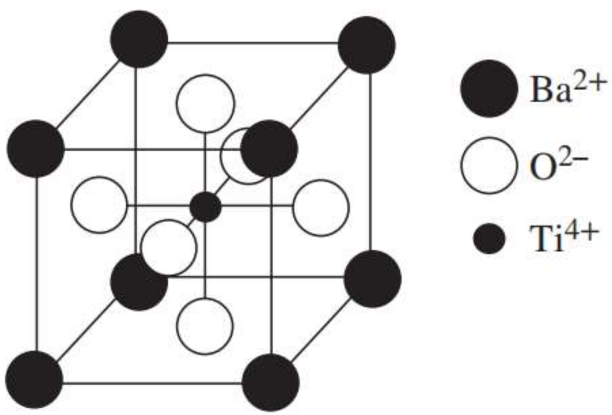
When $f_s < f < f_p$: circuit is inductive

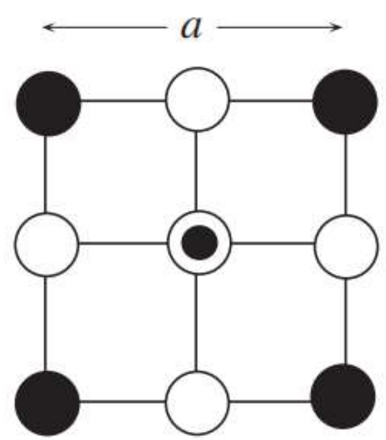
When $f > f_p$: circuit is capacitive

6.8 Ferroelectric and pyroelectric crystals铁电和热释电晶体

Ferroelectric crystals: crystals are permanently polarized in the absence of an applied stress.

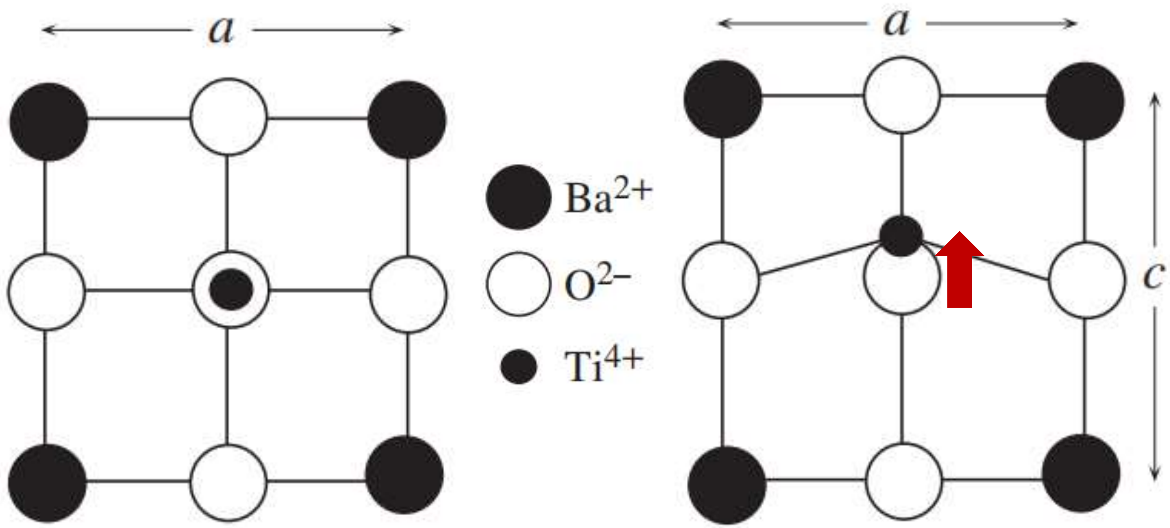
$BaTiO_3$ cubic crystal structure above 130 ^oC





No polarization!

Above 130 ^{o}C Below 130 ^{o}C

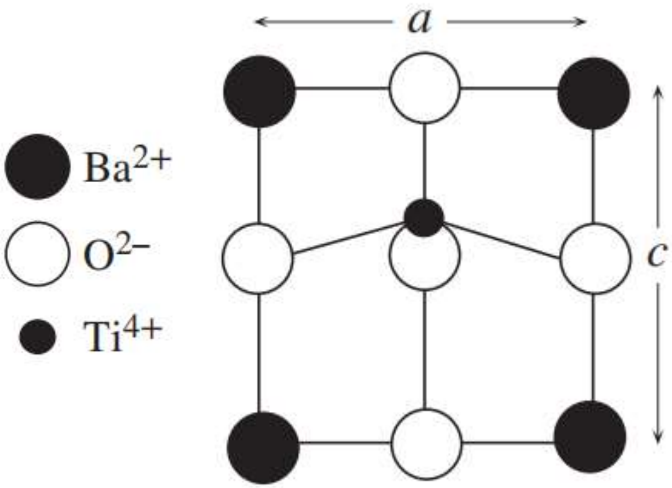


No polarization! Polarization!

Non-ferroelectric! Ferroelectric!

The transition temperature T_c=130 ^{o}C is called **Curie temperature**.

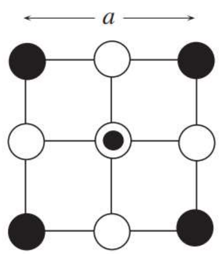
Below 130 °C



a=0.4 nm

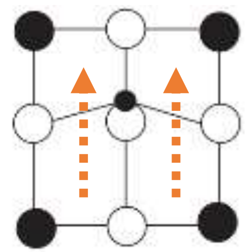
Displacement of Ti^{4+} atoms is around 0.012 nm.

Poling 极化



Above 130 ^{o}C

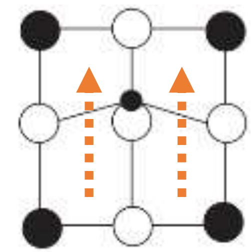
External field ΔE



Above 130 ^{o}C

$$\Delta P = \varepsilon_0(\varepsilon_r - 1)\Delta E$$

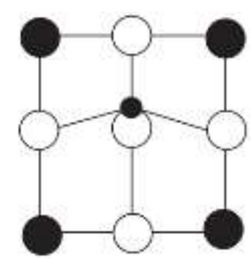
Cooling down to below 130 ^{o}C



Below 130 ^{o}C

$$P = \Delta P + P_{\text{permanent}}$$

Remove electric field



Below 130 ^{o}C

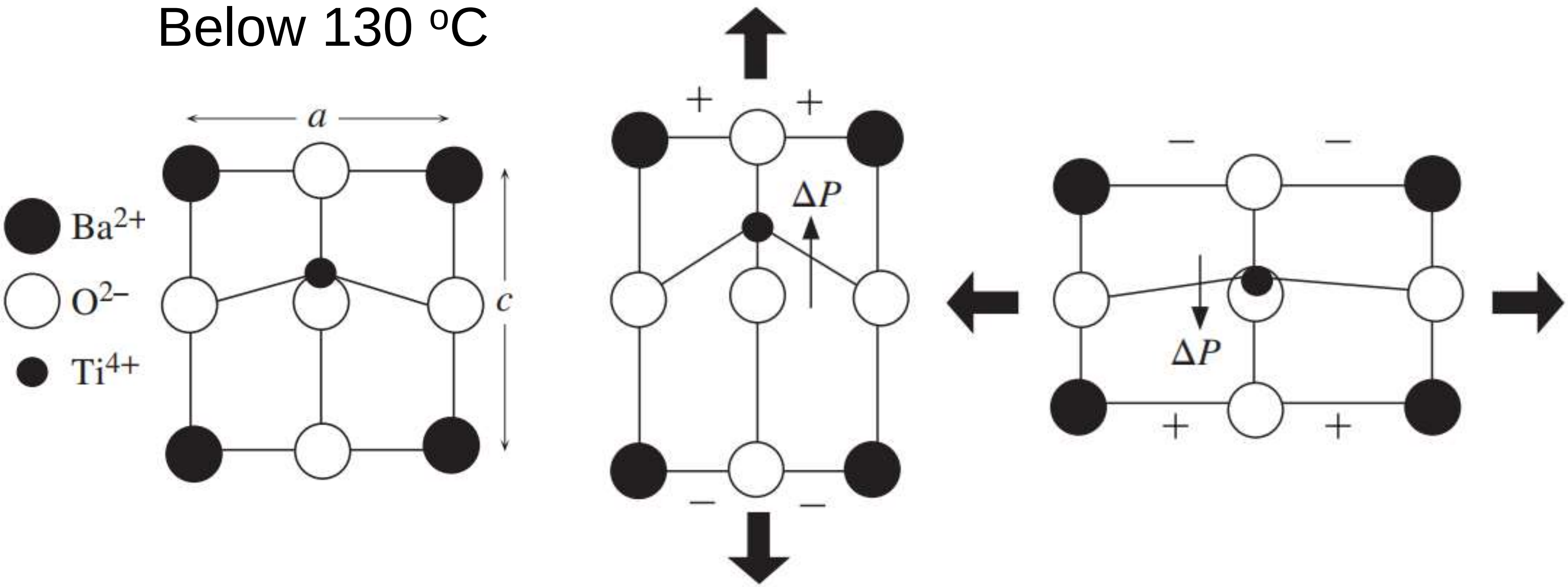
$$P = \Delta P + P_{\text{permanent}}$$

The axis along which P develops is called the **ferroelectric axis**.

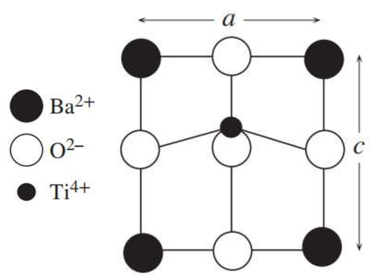
Since ε_r along a-axis ~4200 is much larger than that along c-axis ~160, the displacement of Ti atom will be more efficient along a-axis.

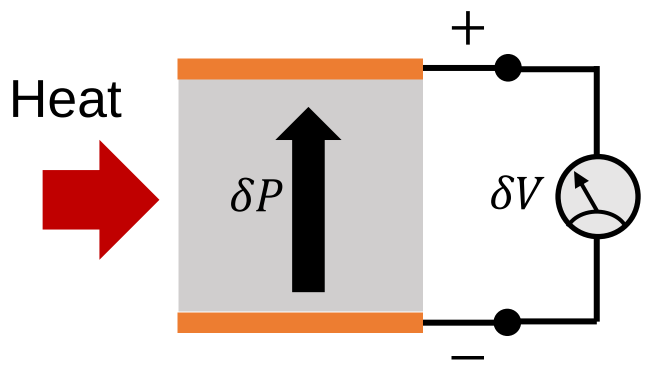
All ferroelectric crystals are also piezoelectric, but the reverse is not true.

Below 130 ^{o}C



Pyroelectricity 热释电





Temperature ↑ → Crystal expands → Polarization change δP

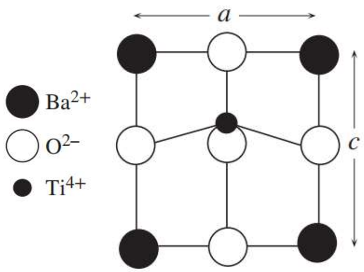
↓

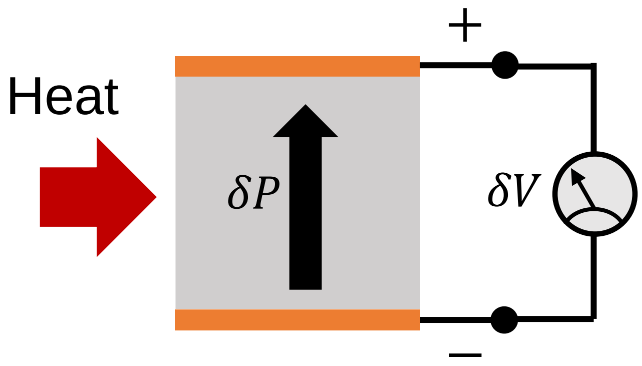
Electric field change

$$\delta E = \frac{\delta P}{\varepsilon_0(\varepsilon_r - 1)}$$

← Potential change δV

A temperature change induces a change in polarization: **pyroelectricity**.

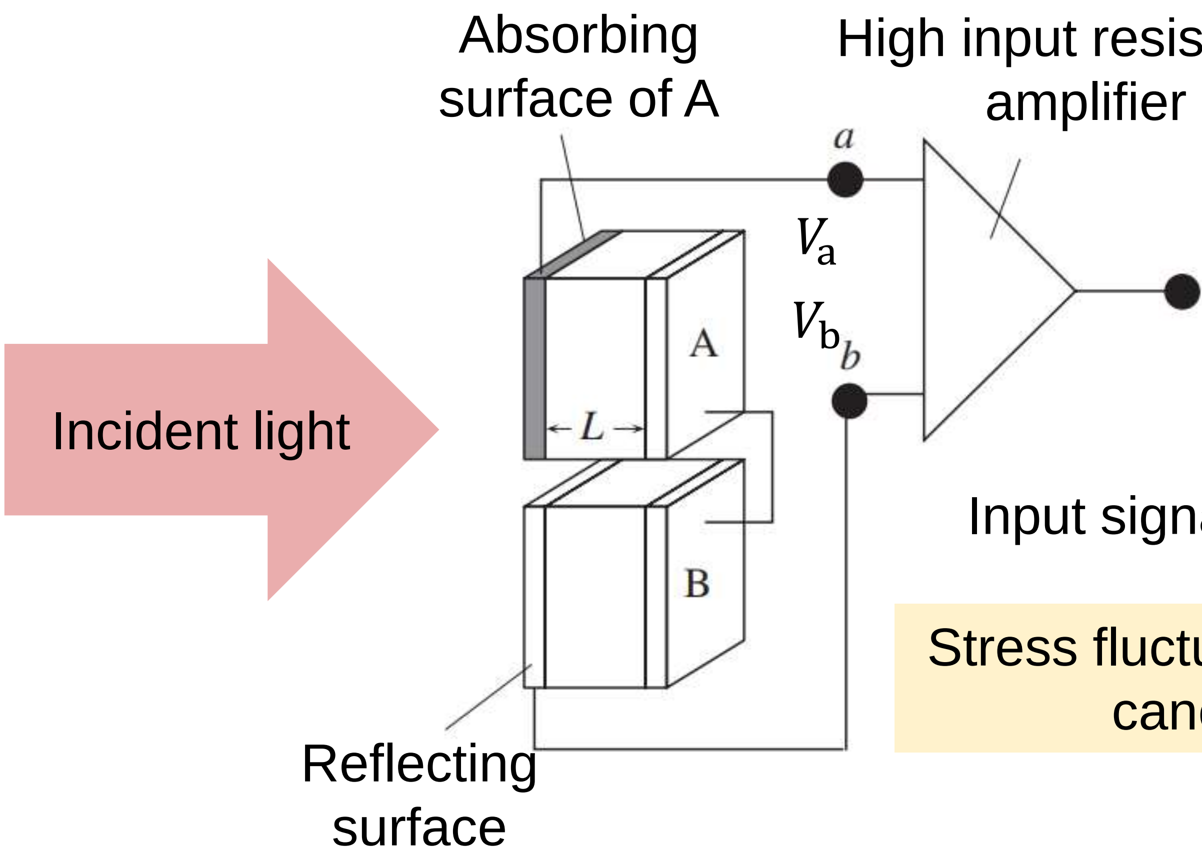




Pyroelectric coefficient:

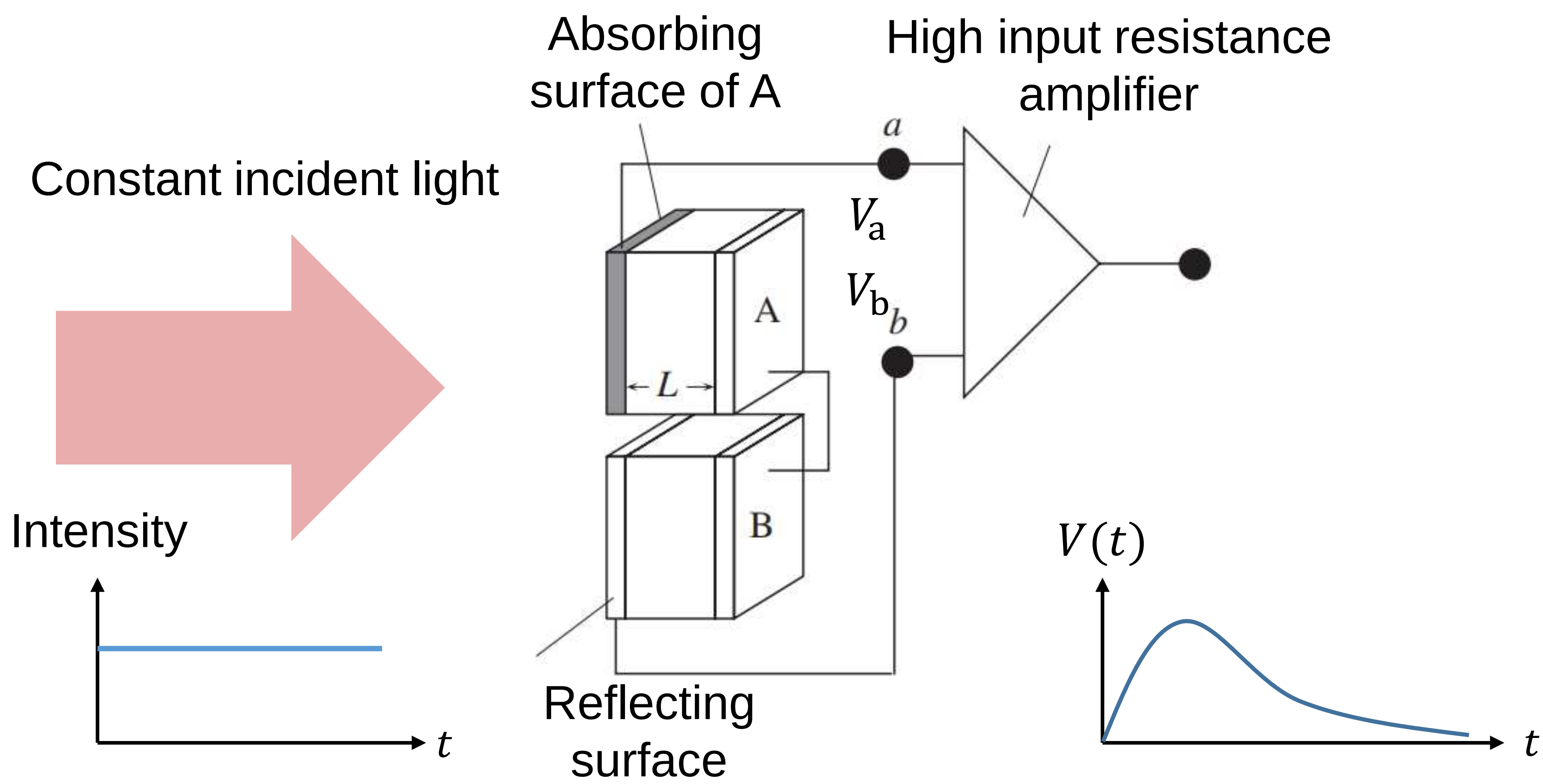
$$p = \frac{dP}{dT}$$

Pyroelectric detector



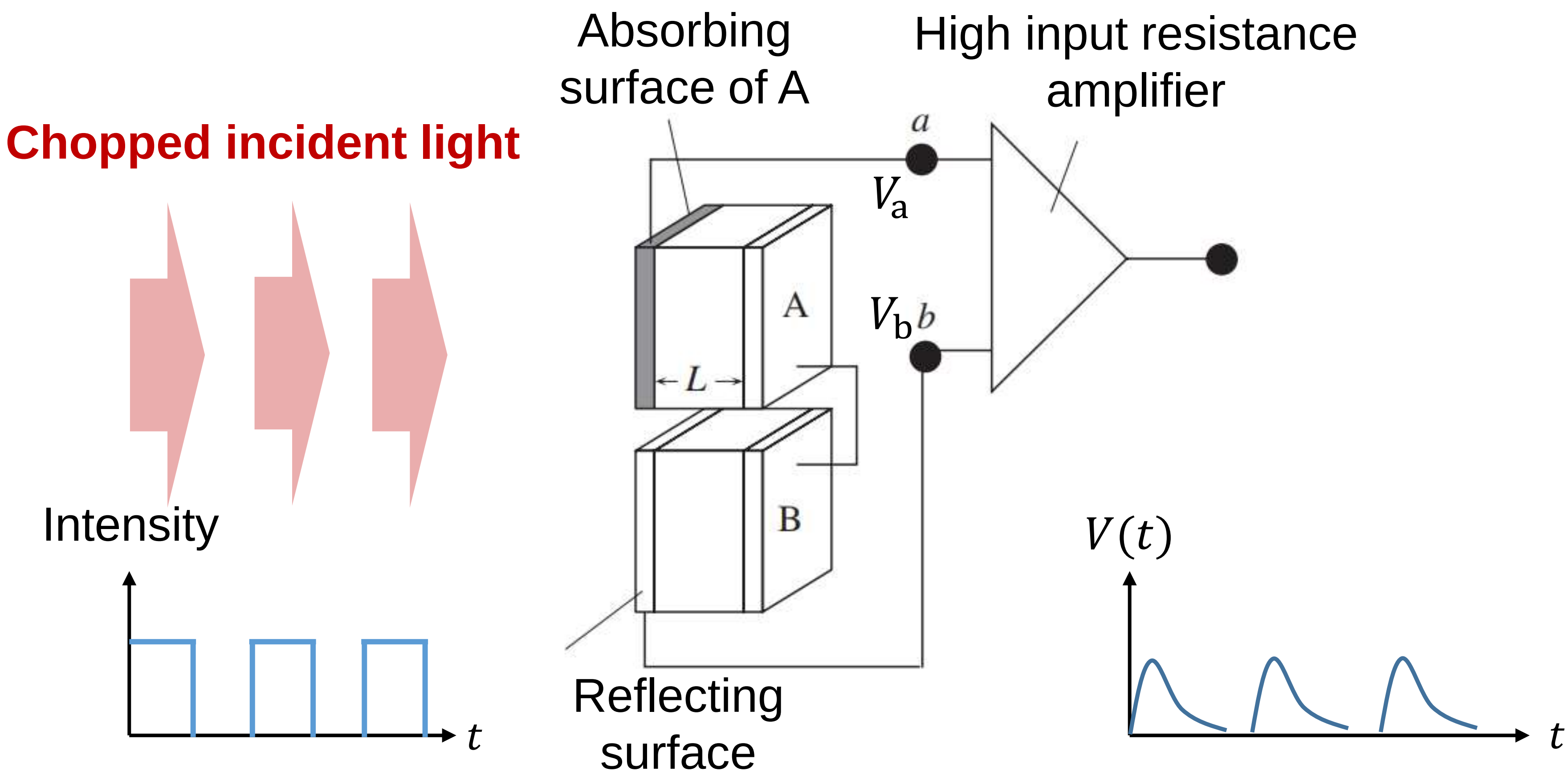
Input signal: $V_a - V_b$

Stress fluctuations can be cancelled.



After incident light, charges start accumulate at surface, and voltage increases.

When temperature is steady, charges at surface will slowly become neutralized or leak.



Chopped incident light
Absorbing surface of A
High input resistance amplifier
a
V_a
V_b b
A
B
L
Intensity
t
Reflecting surface
V(t)
t

Pyroelectric current density

$$J_{\mathrm{p}} = \frac{dP}{dt} = p\frac{dT}{dt}$$

Pyroelectric current responsivity

$$R_{\mathrm{I}} = \frac{J_{\mathrm{p}}}{\text{Input radiation power}}$$

Pyroelectric voltage responsivity

$$R_{\mathrm{V}} = \frac{\text{Pyroelectric output voltage}}{\text{Input radiation power}}$$