HW7: Written

Tuesday, December 11, 2018 6:26 PM

CS_188_Fall_2018_Written_HW7

CS 188 Fall 2018 | Introduction to Artificial Intelligence | Written HW 7

Due: Monday 10/22/2018 at 11:59pm (submit via Gradescope).

Leave self assessment boxes blank for this due date.

Self assessment due: Monday 10/29/2018 at 11:59pm (submit via Gradescope)

For the self assessment, **fill in the self assessment boxes in your original submission** (you can download a PDF copy of your submission from Gradescope – be sure to delete any extra title pages that Gradescope attaches). For each subpart where your original answer was correct, write "correct." Otherwise, write and explain the correct answer. **Do not leave any boxes empty.**

If you did not submit the homework (or skipped some questions) but wish to receive credit for the self-assessment, we ask that you first complete the homework without looking at the solutions, and then perform the self assessment afterwards.

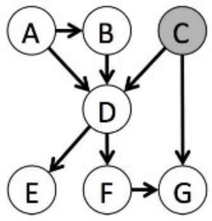
Policy: Can be solved in groups (acknowledge collaborators) but must be written up individually

Submission: Your submission should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question 1 begins on page 2, etc.). **Do not reorder, split, combine, or add extra pages.** The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

First name	
Last name	
SID	
Collaborators	

Q1. Variable Elimination

(a) For the Bayes' net below, we are given the query $P(A, E \mid -c)$. All variables have binary domains. Assume we run variable elimination to compute the answer to this query, with the following variable elimination ordering: B, D, G, F.



Complete the following description of the factors generated in this process:

After inserting evidence, we have the following factors to start out with:

$$P(A), P(B|A), P(+c), P(D|A, B, -c), P(E|D), P(F|D), P(G|+c, F)$$

When eliminating B we generate a new factor f_1 as follows:

$$f_1(A, +c, D) = \sum_b P(b|A)P(D|A, b, -c)$$

This leaves us with the factors:

$$P(A), P(+c), P(E|D), P(F|D), P(G|+c, F), f_1(A, +c, D)$$

When eliminating D we generate a new factor f_2 as follows:

$$f_2(F, E, A, +c) = \sum_d P(F|d)P(E|d)f_1(A, +c, d)$$

This leaves us with the factors:

$$P(A)P(c)P(G|F, c)f_2(F, E, A, +c)$$

When eliminating G we generate a new factor f_3 as follows:

$$f_3(F, c) = \sum_g P(g|F, c)$$

This leaves us with the factors:

$$P(A)P(c)f_3(F, c)f_2(F, E, A, +c)$$

B

$f_1(A, c, D) = \sum_b P(b|A)P(D|A, b, c)$

$P(A)P(c)P(E|D)P(F|D)P(G|c,F)f_1(A,D,c)$

G

$f_2(F) = \sum_g P(g|c, F)$

$P(A)P(c)P(E|D)P(F|D)f_1(A,D,c)f_2(F, c)$

F

$f_3(D, c) = \sum_f f_2(f, c)P(f|D)$

$P(A)P(c)P(E|D)f_1(A,D,c)f_3(D, c)$

D

$f4(E,A, c) = \sum_d P(E|d)f_1(A, d, c)f_3(d, c)$

$P(A)P(c)P(E|D)f_1(A,D,c)f_3(D, c)$

When eliminating F we generate a new factor f_4 as follows:

$$f_4(A, E, c) = \sum_f f_3(f) f_2(f, E, +c, \boldsymbol{A})$$

This leaves us with the factors:

$$P(A)P(c)f_4(E, A, c)$$

Self assessment If correct, write "**correct**" in the box. Otherwise, **write and explain** the correct answer.

(b) Write a formula to compute $P(A, E \mid +c)$ from the remaining factors.

$$\alpha P(A)P(c)f_4(E, c)$$

Self assessment If correct, write "**correct**" in the box. Otherwise, **write and explain** the correct answer.

(c) Among f_1, f_2, f_3, f_4, which is the largest factor generated, and how large is it? Assume all variables have binary domains and measure the size of each factor by the number of rows in the table that would represent the factor.

factor 2 has 3 active variable and is size 2^3 = 8

Self assessment If correct, write "**correct**" in the box. Otherwise, **write and explain** the correct answer.

(d) Find a variable elimination ordering for the same query, i.e., for $P(A, E \mid +c)$, for which the maximum size factor generated along the way is smallest. Hint: the maximum size factor generated in your solution should have only 2 variables, for a size of $2^2 = 4$ table. Fill in the variable elimination ordering and the factors generated into the table below.

Variable Eliminated	Factor Generated
B	
G	
F	
D	

B

$f_1(A, c, D) = \sum_b P(b|A)P(D|A,b,c)$

$P(A)P(c)P(E|D)P(F|D)P(G|c,F)f_1(A,D,c)$

G

$f_2(F) = \sum_g P(g|c,F)$

$P(A)P(c)P(E|D)P(F|D)f_1(A,D,c)f_2(F, c)$

F

$f_3(D, c)=\sum_f f_2(f,c)P(f|D)$

$P(A)P(c)P(E|D)f_1(A,D,c)f_3(D, c)$

D

$f4(E,A, c)=\sum_d P(E|d)f_1(A,d,c)f_3(d,c)$

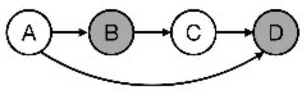
$P(A)P(c)P(E|D)f_1(A,D,c)f_3(D, c)$

For example, in the naive ordering we used earlier, the first row in this table would have had the following two entries: B, $f_1(A, +c, D)$.

Self assessment If correct, write "**correct**" in the box. Otherwise, **write and explain** the correct answer.

Q2. Bayes Nets: Sampling

Consider the following Bayes Net, where we have observed that $B = +b$ and $D = +d$.



$P(A)$	
$+a$	0.5
$-a$	0.5

$P(B\|A)$		
$+a$	$+b$	0.8
$+a$	$-b$	0.2
$-a$	$+b$	0.4
$-a$	$-b$	0.6

$P(C\|B)$		
$+b$	$+c$	0.1
$+b$	$-c$	0.9
$-b$	$+c$	0.7
$-b$	$-c$	0.3

$P(D\|A,C)$			
$+a$	$+c$	$+d$	0.6
$+a$	$+c$	$-d$	0.4
$+a$	$-c$	$+d$	0.1
$+a$	$-c$	$-d$	0.9
$-a$	$+c$	$+d$	0.2
$-a$	$+c$	$-d$	0.8
$-a$	$-c$	$+d$	0.5
$-a$	$-c$	$-d$	0.5

(a) Consider doing Gibbs sampling for this example. Assume that we have initialized all variables to the values $+a, +b, +c, +d$. We then unassign the variable C, such that we have $A = +a$, $B = +b$, $C = ?$, $D = +d$. Calculate the probabilities for new values of C at this stage of the Gibbs sampling procedure.

$P(C = +c$ at the next step of Gibbs sampling$) =$ (0.1*0.6)/(0.1*0.6+0.9*0.1)

$P(C = -c$ at the next step of Gibbs sampling$) =$ 0.9

Self assessment If correct, write "**correct**" in the box. Otherwise, **write and explain** the correct answer.
P(c) = $\sum_b$ P(b, c)

(b) Consider a sampling scheme that is a hybrid of rejection sampling and likelihood-weighted sampling. Under this scheme, we first perform rejection sampling for the variables A and B. We then take the sampled values for A and B and extend the sample to include values for variables C and D, using likelihood-weighted sampling.

(i) Below is a list of candidate samples. Mark the samples that would be rejected by the rejection sampling portion of the hybrid scheme.

☐ $-a$ $-b$
☐ $+a$ $+b$
☐ $+a$ $-b$
☐ $-a$ $+b$

Self assessment If correct, write "**correct**" in the box. Otherwise, **write and explain** the correct answer.

(ii) To decouple from part (i), you now receive a *new* set of samples shown below. **Fill in** the weights for these samples under our hybrid scheme.

				Weight
a	$+b$	c	$+d$	0.5
$+a$	$+b$	$-c$	$+d$	0.1
$+a$	$+b$	$-c$	$+d$	0.1
a	$+b$	$+c$	$+d$	0.2
$+a$	$+b$	$-c$	$+d$	0.6

$P(D \mid A, C)$			
$+a$	$+c$	$+d$	0.6
$+a$	$+c$	$-d$	0.4
$+a$	$-c$	$+d$	0.1
$+a$	$-c$	$-d$	0.9
$-a$	$+c$	$+d$	0.2
$-a$	$+c$	$-d$	0.8
$-a$	$-c$	$+d$	0.5
$-a$	$-c$	$-d$	0.5

Self assessment If correct, write "**correct**" in the box. Otherwise, **write and explain** the correct answer.

(iii) Use the weighted samples from part (ii) to calculate an estimate for $P(+a \mid +b, +d)$.

The estimate of $P(+a \mid +b, +d)$ is (0.1+0.1+0.6)/(0.5+0.1+0.1+0.2+0.6)=0.5333

Self assessment If correct, write "**correct**" in the box. Otherwise, **write and explain** the correct answer.

(c) We now attempt to design an alternative hybrid sampling scheme that combines elements of likelihood-weighted and rejection sampling. For each proposed scheme, indicate whether it is valid, i.e. whether the weighted samples it produces correctly approximate the distribution $P(A, C \mid +b, +d)$.

(i) *First collect a likelihood-weighted sample for the variables A and B. Then switch to rejection sampling for the variables C and D. In case of rejection, the values of A and B and the sample weight are* ***thrown away****. Sampling then restarts from node* ***A****.*

◯ Valid ◯ Invalid

(ii) *First collect a likelihood-weighted sample for the variables A and B. Then switch to rejection sampling for the variables C and D. In case of rejection, the values of A and B and the sample weight are* ***retained****. Sampling then restarts from node* ***C****.*

◯ Valid ◯ Invalid

Self assessment If correct, write "**correct**" in the box. Otherwise, **write and explain** the correct answer.

we collect weighted-sample(A,B)
Next sample C and D, given assignments to A, b=b
if rejection occurs, restart at a

ii is invalid because it doesn't penalize samples for making d less likely.