Chapter 15
Managing Perishable and Aging Inventories: Review and Future Research Directions

Itir Z. Karaesmen, Alan Scheller-Wolf, and Borga Deniz

15.1 Introduction

Over the years, several companies have emerged as exemplary of "best practices" in supply chain management; for example, Wal-Mart is frequently cited as using unique strategies to lead its market. One significant challenge for Wal-Mart is managing inventories of products that frequently *outdate*: A significant portion of Wal-Mart's product portfolio consists of *perishable* products such as food items (varying from fresh produce to dairy to bakery products), pharmaceuticals (e.g., drugs, vitamins, cosmetics), chemicals (e.g., household cleaning products), and cut flowers. Wal-Mart's supply chain is not alone in its exposure to outdating risks – to better appreciate the impact of perishability and outdating in society at large, consider these figures: In a 2003 survey, overall unsalable costs at distributors to supermarkets and drug stores in consumer packaged goods *alone* were estimated at $2.57 billion, and 22% of these costs, over 500 million dollars, were due to expiration in *only* the branded segment (Grocery Manufacturers of America 2004). In the produce sector, the $1.7 billion US apple industry is estimated to lose $300 million annually to spoilage (Webb 2006). Note also that perishability and outdating are a concern not only for these consumer goods, but for industrial products (for instance, Chen (2006), mentions that adhesive materials used for plywood lose strength within 7 days of production), military ordnance, and blood – one of the most critical resources in health care supply chains. According to a nationwide survey on blood collection and utilization, 5.8% of all components of blood processed for transfusion were outdated in 2004 in the USA (AABB 2005).

In this chapter, we provide an overview of research in supply chain management of products that are perishable or that outdate, i.e., products that *age over time*. Thus, we largely exclude single-period models which are commonly used to represent perishable items (with an explicit cost attached to expected future outdates).

I.Z. Karaesmen (✉)
Kogod School of Business, American University, 4400 Massachusetts Avenue NW, Washington DC 20016
e-mail: karaesme@american.edu

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While these newsvendor-type models can convey important insights, they are often too simple to provide answers to some of the more complex questions that arise as inventory levels, product characteristics, markets and customer behaviors change over time. Furthermore, we exclude research that models decay or deterioration of inventories assuming certain functional forms (e.g., exponential decay). The reader can refer to Goyal and Giri (2001) and Raafat (1991), which is supplemented by Dave (1991), for a bibliography and classification of research on that topic. Finally, we also exclude the research on management, planning or allocation of *capacity* which is commonly referred as perishable inventory, for instance, in the airline revenue management literature (see Talluri and van Ryzin (2004), for information on revenue management). Our emphasis is on models where the inventory level of a product must be controlled over a horizon taking into account demand, supply, and a finite shelf-life (which may be fixed or random).

Modeling in such an environment implies that at least one or both of the following holds: First, demand for the product may change over time as the product ages; this could be due to a decrease in the utility of the product because of the reduced lifetime, lessened quality, or changing market conditions. Second, operational decisions can be made more than once (e.g., inventory can be replenished by ordering *fresher* products, or prices can be marked down) during the lifetime of the product. Either of these factors makes the analysis of such systems a challenge, owing to the expanded state space.

Our goal in this chapter is not to replicate surveys of past work (such as Nahmias (1982), Prastacos (1984), and Pierskalla (2004); in fact we refer to them as needed in the remainder of this chapter) but to discuss in more detail *directions for future research.* We provide a selective review of the existing research and focus on those papers which, in our view, constitute crucial stepping stones or point to promising directions for future work. We also refer to several papers in the supply chain management literature that do not specifically study perishable inventories in order to highlight analogous potential research areas in the supply chain management of perishable goods.

The chapter is organized as follows: We first provide a discussion of common challenges in production planning of perishables in Sect. 15.2. We review the research on single product, single location models in inventory management of perishables in Sect. 15.3. We next focus on multi-echelon and multi-location models in Sect. 15.4. Research with novel features such as multiple products, multiple-types of customers and different demand models are reviewed in Sect. 15.5. We conclude by detailing a number of open research problems in Sect. 15.6.

15.2 Challenges in Production Planning

One of the most comprehensive studies on production planning and perishable goods is the doctoral dissertation of Lütke Entrup (2005), which focuses on the use of leading advance planning and scheduling (APS) systems (such as

PeopleSoft's EnterpriseOne and SAP's APO) to manage products with short shelf-lives. According to Lütke Entrup (2005), the use of APS in perishable supply chains remains low in contrast to the supply chains of non-perishable goods. He describes how shelf-life is integrated into the current APS, and identifies particular weaknesses of APS systems for perishables by carefully studying the characteristics and requirements of the supply chains of three different products. Based on these case studies, he proposes customized solutions which would enable the APS systems to better match the needs of the fresh produce industry. We refer the reader to this resource for more information on practical issues in production planning for perishable goods. In the remainder of this section, we provide an overview of the *analytical* research in production planning, focusing mainly on the general body of knowledge in operations management and management science disciplines.

Capacity planning is one of the key decisions in the production of perishables. Research on this topic is primarily focused on agricultural products: Kazaz (2004) describes the challenges in production planning by focusing on long-term capacity investments, yield uncertainty (which is a common problem in the industries that involve perishables), and demand uncertainty. He develops a two-stage problem where the first stage involves determining the capacity investment and the second-stage involves the production quantity decision. Jones et al. (2001) analyze a production planning problem where after the initial production there is a second chance to produce, still facing yield uncertainty; Jones et al. (2003) describe the real-life capacity management problem for a grower in more detail. Allen and Schuster (2004) present a model for agricultural harvest risk. Although these papers do not model the aging of the agricultural product once it is produced, they highlight the long term investment and planning challenges. We refer the reader to Kazaz (2004) for earlier references on managing yield uncertainty and to Lowe and Preckel (2004) for research directions within the general domain of agribusiness.

In addition to capacity planning and managing yield uncertainty, product-mix decisions are integral to the management of perishables. Different companies in the supply chain (producer, processor, distributor, and retailer) face this problem in slightly different ways. For example, in agribusiness, a producer decides on the use of farm land for different produce, hence deciding on the capacity-mix. A supermarket can offer a fresh fruit or vegetable as is or as an ingredient in one of their ready-to-eat products (e.g., pre-packed fruit salad). Similarly, fresh produce can be used in ready-to-eat, cooked products (e.g., pre-cooked, frozen dishes) or it can be sold as an uncooked, frozen product. Note that, in these examples, one perishable component can be used to produce products that have different shelf lives. Consider another example from blood inventories: whole blood outdates in 42 days, whereas a critical blood component, platelets, outdate in 5–7 days. Given their prevalence in practice, the strategic management of interrelated products with explicitly different shelf-lives stands as an important open problem. We discuss the limited research in this area in Sect. 15.5.

For perishables, both capacity planning and product-mix decisions are typically driven by target levels of supply for a final product. In that respect, production plans are tightly linked to inventory control models. We therefore discuss research in inventory control for perishables in the rest of this chapter, treating single location models in Sect. 15.3, the multi-location and multi-echelon models in Sect. 15.4, and modeling novelties in Sect. 15.5. Once again, we attempt to emphasize those areas most in need for further research.

15.3 Managing Inventories at a Single Location

Single-location inventory models form the basic building blocks for more complex models with multiple locations and/or echelons, information flows, knowledge of market or customer behavior, and/or additional logistics options. The work is pioneered by Veinott (1960), Bulinskaya (1964), and Van Zyl (1964), who consider discrete review problems without fixed cost under deterministic demand, stochastic demand for items with a one-period lifetime, and stochastic demand for items with a two-period lifetime, respectively. We first discuss the discrete review setting, before expanding our scope to continuous review systems. There is a *single product* in all the models reviewed in this section, and inventory is depleted starting from the oldest units in stock, i.e., *first-in-first-out (FIFO) inventory issuance* is used.

15.3.1 Discrete Review Models

We provide an overview of research by classifying the work based on modeling assumptions: Research assuming no fixed ordering costs or lead times is reviewed in Sect. 15.3.1.1, followed by research on models with fixed ordering costs but no lead times in Sect. 15.3.1.2. Research on models with positive lead times, which complicate the problem appreciably, is reviewed in Sect. 15.3.1.3.

Table 15.1 provides a high-level overview of some of the key papers within the discrete review arena. Categorization of the papers and the notation used in Table 15.1 are described below:

- Replenishment policy: Papers have considered the optimal control policy (Opt), base stock policies that keep a constant order-up-to-level for total items in system (TIS), summed over all ages, or only new items in system (NIS) (see Sect. 15.3.1.1), other heuristics (H), and, when fixed ordering costs are present, the (s,S) policy. When (s,S) policy is annotated with a ‡, it is implied to be the optimal replenishment policy based on earlier research (however a formal proof of optimality of (s,S) has not appeared in print for the model under discussion).
- Excess demand: Excess demand is either backlogged (B) or lost (L).
- Problem horizon: Planning horizon is either finite (F) or infinite (I).
- Replenishment lead time: All papers, save for one, assume zero lead time. The exception is a paper that allows a deterministic (det) replenishment lead time.
- Product lifetime: Most papers assume deterministic lifetimes of general length (D) although two papers assume the lifetime is exactly two time periods (2).

Table 15.1 A summary of discrete time single item inventory models

Article	Replenishment policy	Excess demand	Planning horizon	Lead time	Life-time	Demand distribution	Costs
Nahmias and Pierskalla (1973)	Opt	*B*/*L*	*F*/*I*	0	2	cont	*p*,*m*
Fries (1975)	Opt	*L*	*F*/*I*	0	*D*	cont	*c*,*h*,*b*,*m*
Nahmias (1975a)	Opt	*B*	*F*	0	*D*	cont	*c*,*h*,*p*,*m*
Cohen (1976)	TIS	*B*	*I*	0	*D*	cont	*c*,*h*,*p*,*m*
Brodheim et al. (1975)	NIS	*L*	*I*	0	*D*	disc	
Chazan and Gal (1977)	TIS	*L*	*I*	0	*D*	disc	
Nahmias (1975b,1975c)	*H*	*B*	*F*	0	*D*	E_k	*c*,*h*,*p*,*m*
Nahmias (1976)	TIS	*B*	*F*	0	*D*	cont	*c*,*h*,*p*,*m*[1]
Nahmias (1977a)	*H*	*B*	*F*	0	*D*	cont	*c*,*h*,*p*,*m*[1]
Nahmias (1977c)	TIS	*B*	*F*	0	disc[7]	cont	*c*,*h*,*p*,*m*[1]
Nandakumar and Morton (1993)	*H*	*L*	*I*	0	*D*	cont	*c*,*h*,*b*,*m*
Nahmias (1978)	(*s*,*S*)	*B*	*F*	0	*D*	cont	*c*,*h*,*p*,*m*,K[1]
Lian and Liu (1999)	(*s*,*S*)[‡]	*B*	*I*	0	*D*	batch geo	*h*,*p*,*b*,*m*,K
Lian et al. (2005)	(*s*,*S*)[‡]	*B*	*I*	0	PH[2]	batch PH	*h*,*p*,*b*,*m*,K
Williams and Patuwo (1999,2004)	*H*	*L*	*F*	det	2	cont	*c*,*h*,*b*,*m*

[1]Denotes papers in which holding and shortage costs can be general convex functions.
[2]Denotes use of a *disaster* model in which all units perish at once.
[7]Denotes the assumption that items, even though they have random lifetimes, will perish in the same sequence as they were ordered.
[‡]Denotes the case when (*s*,*S*) policy is implied to be the optimal replenishment policy based on earlier research (however, a formal proof of optimality of (*s*,*S*) has not appeared in print for the model under discussion).

One paper allows for general discrete phase-type lifetimes, (PH), and assumes that all items perish at the same time, i.e., a *disaster* model. This latter is denoted by [2] in Table 15.1. Another permits general discrete lifetimes, but assumes items perish in the same sequence as they were ordered. This is denoted by [7] in the table.

- Demand distribution: In most cases, demand in each period has a continuous density (cont) function. In others, it is discrete (disc), Erlang – which may include exponential – (E_k) or batch demand with either geometric (geo) or general discrete phase-type renewal arrivals (PH). Oftentimes the continuous demand is assumed for convenience – results appear to generalize to general demand distributions.
- Costs: Costs include unit costs for ordering (c), holding per unit time (h), perishing (m), shortage per unit time (p), or one-time cost for shortage (b) and fixed cost for ordering (K). The annotation [1] in Table 15.1 indicates papers that allow the unit holding and shortage costs to be generalized to convex functions. Any paper that does not list cost parameters is concerned with the general properties of the model, such as expected outdates, without attaching specific costs to these.

15.3.1.1 Discrete Review Models Without Fixed Ordering Cost or Lead Time

Early research in discrete time models without fixed ordering costs focus on characterizing optimal policies. Fundamental characterization of the (state dependent) optimal ordering policy is provided by Nahmias and Pierskalla (1973) for the two period lifetime problem, when penalty costs per unit short per unit time, and unit outdating costs are present. Fries (1975) and Nahmias (1975a), independently, characterize the optimal policy for the general lifetime problem. Both of these papers have per unit outdating costs, per unit ordering costs, and per unit per period holding costs. In addition, Fries (1975) has per unit shortage costs, and Nahmias (1975a) per unit per period shortage costs; this difference arises because Fries (1975) assumes lost sales and Nahmias (1975a) backlogging of unsatisfied demand. The cost structure in Nahmias (1975a) is the "standard" for discrete time models without fixed ordering cost, and we refer to it as such within the rest of this section, although in later papers the per unit per period penalty and holding costs are extended to general convex functions. The incorporation of separate ordering and outdating costs allows modeling flexibility to account for salvage value; use of only one or the other of these costs is essentially interchangeable.

Treatment of the costs due to outdating is the crucial differentiating element within the perishable setting. In fact, while Fries (1975) and Nahmias (1975a) take alternate approaches for modeling the costs of expiration – the former paper charges a cost in the period items expire, while the latter charges an expected outdating cost in the period items are ordered – Nahmias (1977b) shows that these two approaches were essentially identical, modulo end of horizon effects. The policy structures outlined in Fries (1975) and Nahmias (1975a) are quite complex; perishability destroys the simple base-stock structure of optimal policies for discrete review models without fixed ordering costs in the absence of perishability.

This complexity of the optimal policy is reinforced by Cohen (1976), who characterizes, the stationary distribution of inventory for the two-period problem with the *standard* costs, showing that the optimal policy for even this simple case was quite complex, requiring state-dependent ordering. To all practical extents, this ended academic study of optimal policies for the discrete review problem under FIFO issuance; to find an optimal policy dynamic programming would be required, and implementation of such a policy would be difficult owing to its complex, state dependent nature (with a state vector tracking the amount of inventory in system *of each age*). Therefore many researchers turned to the more practical question of seeking effective heuristic policies that would be (a) easy to define, (b) easy to implement, and (c) close to optimal.

Very soon after the publication of Nahmias (1975a), a series of approximations appeared for the backorder and lost sales versions of the discrete review zero-fixed ordering cost problem. Initial works (Brodheim et al. 1975; Nahmias 1975b) examine the use of different heuristic control policies; Brodheim et al. (1975) propose a fascinating simplification of the problem – making order decisions based only on the amount of new items in the system; what we call the NIS heuristic. They show that if a new order size is constant, Markov chain techniques can be used to derive exact expressions, or alternately very simple bounds, on key system statistics (which is the focus of their paper). Nahmias (1975b) used simulation to compare multiple heuristics for the problem with *standard* costs, including the "optimal" TIS policy, a piecewise linear function of the optimal policy for the non-perishable problem, and a hybrid of this with NIS ordering. He found that the first two policies outperform the third, effectively disrupting further study of NIS and its variants for a few decades.

Nahmias (1975c, 1976, 1977a), explicitly treats the question of deriving good approximation policies (and parameters) for the problem with the *standard* costs; Nahmias (1977a) can be considered as the culmination of these initial heuristic efforts. This latter computationally compares the heuristic that keeps only two states in the inventory vector, new inventory, and inventory over one day old, with both the globally optimal policy (keeping the entire inventory vector) and the optimal TIS policy, which is easily approximated using the techniques in Nahmias (1976). For Erlang or exponential demand and a lifetime of three periods, the performance of both heuristics is exceptional – always within 1% of optimal, with the reduced state-space heuristic uniformly outperforming the optimal TIS policy. Note also that using the two-state approximation eliminates any need to track the age of inventory other than new versus old, significantly reducing both the computational load and complexity of the policy.

All of these heuristics consider backorder models. The first heuristic policy for discrete time lost sales models is provided by Nandakumar and Morton (1993). They incorporate the properties and bounds on the expected outdates under TIS in lost sales systems, originally provided by Chazan and Gal (1977), into heuristics following the framework of Nahmias (1976) for the backorder problem. Nandakumar and Morton (1993) compare these heuristics with alternate "near-myopic" heuristics which use a newsvendor-type logic, for the problem with *standard* costs. They found that all of the heuristics performed within half a percentage of the optimal, with the near myopic heuristics showing the best performance, typically within 0.1% of the optimal. (These heuristics could likely be improved even further, with the use of even tighter bounds on expected outdates derived by Cooper (2001).)

Note that both the backorder and lost sales heuristics should be quite robust with respect to items having even longer lifetimes than those considered in the papers: Both Nahmias (1975a) and Fries (1975) observed that the impact of newer items on ordering decisions is greater than the impact of older items. (This, in fact, is one of the factors motivating the heuristic strategies in Nahmias (1977a).) Thus, the *standard* single-item, single-location, discrete-time, fixed-lifetime perishable inventory model, with backorders or lost sales has for all practical purposes been solved – highly effective heuristics exist that are well within our computational power to calculate. One potential extension would be to allow items to have random lifetimes; if items perish in the same sequence as they were ordered, many of the fixed lifetime results continue to hold (Nahmias 1977c), but a discrete time model with random lifetimes that explicitly permits items to perish in a different sequence than the one they were ordered remains an open, and likely challenging question. The challenge arises from the enlarged state space required to capture the problem characteristics – in this case the entire random lifetime vector – although techniques from continuous time models discussed in Sect. 15.3.2.1 could prove useful in this endeavor.

15.3.1.2 Discrete Review Models Without Lead Times Having Positive Fixed Ordering Cost

Nahmias (1978) was the first to analyze the perishable inventory problem with no lead time but positive fixed ordering cost in addition to the *standard* costs described in Sect. 15.3.1.1. In his paper, *for the one-period problem*, Nahmias (1978) established that the structure of the optimal policy is (*s,S*) only when the lifetime of the object is two; lifetimes of more than two periods have a more complex, non-linear structure. Extensions of the two-period (*s,S*) result to the multi-period problem appeared quite difficult, given the analytical techniques of the time. The fact that the costs *excluding* the fixed ordering cost are not convex appears to render hopes of extending the K-convexity of Scarf (1960) to the perishable domain to be in vain. Nevertheless, Nahmias (1978) reports extensive computational experiments supporting the conjecture that the general ((*s,S*) or non-linear) optimal policy structure holds for the multi-period problem. One possible avenue to prove such structure could be through the application of recent proof techniques involving decomposition ideas, such as those in Muharremoglu and Tsitsiklis (2003).

Lian and Liu (1999) consider the discrete review model with fixed ordering cost, per unit per period holding cost, per unit *and* per unit per unit time shortage costs, and per unit outdating costs. Crucially, their model is comprised of discrete time epochs where demand is realized or units in inventory expire (hence discrete time refers to distinct points in time where change in the inventory levels occur) as opposed to distinct time periods in Nahmias (1978). Lian and Liu (1999) analyze an (s,S) policy which was shown to be optimal under continuous review by Weiss (1980) for Poisson demand. The instantaneous replenishment assumption combined with the discrete time model of Lian and Liu (1999) ensures that the optimal reorder level s will be no greater than -1 because any value larger than -1 will add holding costs without incurring any shortage cost in their model. The zero lead time assumption and cost structure of Lian and Liu (1999) are common in the continuous review framework for perishable problems (see Sect. 15.3.2.3); it was a desire to develop a discrete time model to approximate the continuous review that motivated Lian and Liu (1999). In the paper they use matrix analytic methods to analyze the discrete time Markov chain and establish numerically that the discrete review model is indeed a good approximation for the continuous review model, especially as the length of the time intervals gets smaller.

Lian et al. (2005) follow Lian and Liu (1999), sharing similar costs and replenishment assumptions. This later paper allows demands to be batch with discrete phase type interdemand times and lifetimes to be general discrete time phase type, but requires that all items of the same batch perish at the same time. They again use matrix analytic methods to derive cost expressions, and numerically show that the variability in the lifetime distribution can have a significant effect on system performance. Theirs is a quite general framework, and provides a powerful method for the evaluation of discrete time problems in the future, provided the assumption of the instantaneous replenishments is reasonable in the problem setting.

Of great practical importance again is the question of effective heuristic policies. Nahmias (1978) establishes, computationally, that while not strictly optimal, (s,S) type policies perform very close to the optimal. Moreover, he also presents two methods of approximating the optimal (s,S) parameters. Both of these are effective: On average their costs are within 1% of the globally optimal cost for the cases considered, and in all cases within 3%. Thus, while there certainly is room for further refinements of these heuristic policies (along the lines of the zero fixed cost case, for example), the results in Nahmias (1978) are already compelling.

15.3.1.3 Discrete Review Models with Positive Lead Time

As optimal solutions to the zero lead time case require the use of dynamic programming, problems with lead times are in some sense no more difficult, likewise requiring dynamic programming, albeit of a higher dimension. Thus problems considering optimal policies for discrete review problems with lead times are also often considered in the context of other generalizations to the model, for example, different selling prices depending on age (Adachi et al. 1999). Interestingly, there is very little work on extending discrete review heuristics for the zero lead time model to the case of positive lead times, although many of the methods for the zero lead time case should in principle be applicable, by expanding the vector of ages of inventory kept to include those items on order, but which have not yet arrived.

One possible direction is provided in the work of Williams and Patuwo (1999, 2004) who derive expressions for optimal ordering quantities based on system recursions for a one-period problem with fixed lead time, lost sales, and the following costs: per unit shortage costs and per unit outdating costs, per unit ordering costs and per unit per period holding costs. Williams and Patuwo (2004) in fact state that their methods can be extended to finite horizons, but such an extension has not yet appeared. Development of such positive lead time heuristics, no matter what their genesis, would prove valuable both practically and theoretically, making this a potentially attractive avenue for future research. The trick, essentially, is to keep enough information so as to make the policy effective, without keeping so much as to make the policy overly cumbersome. More complex heuristics have been proposed for significantly more complex models with lead times, such as in Haijema et al. (2005, 2007), but these likely are more involved than necessary for standard problem settings.

15.3.2 Continuous Review Models

We now turn our attention to continuous review models. These are becoming increasingly important with the advent of improved communication technology (such as radio frequency identification, RFID) and automated inventory management and ordering systems (as a part of common enterprise resource planning, ERP, systems). These two technologies may eventually enable management of perishables in real time, potentially reducing outdating costs significantly. We first consider continuous review models without fixed ordering costs or lead times in Sect. 15.3.2.1, models without fixed ordering costs but positive lead times in Sect. 15.3.2.2, and finally the models which incorporate fixed ordering costs in Sect. 15.3.2.3.

Tables 15.2 and 15.3 provide a high-level overview of some of the key papers within the continuous review framework, segmented by whether a fixed ordering cost is present in the model (Table 15.3) or not (Table 15.2). Categorization of the papers and the notation used in these tables are described below:

- Replenishment policy: In Table 15.2, base stock policy is denoted by a z. Papers which focus on characterizing system performance have a blank in the replenishment policy column. In Table 15.3, all papers either use the (s, S) policy, annotated with a $*$ when proved to be optimal, with a ‡ when only implied to be the optimal policy based on earlier research (however a formal proof of optimality of (s, S) has not appeared in print for the model under concern) or the (Q, r) or (Q, r, T) policies when batch sizes are fixed. The (Q, r, T) policy orders when inventory is depleted below r or when items exceed T units of age.
- Excess demand: Papers may assume simple backlogging (B), some sort of generalized backlogging in which not all backlogged customers wait indefinitely (B^g), or lost sales (L). Two papers assume all demand must be satisfied; this field is blank in that case.
- Problem horizon: Planning horizon is either finite (F) or infinite(I).

Table 15.2 A summary of continuous time single item inventory models without fixed inventory cost

Article	Replenishment policy	Excess demand	Planning horizon	Lead time	Life-time	Demand distribution	Costs
Graves (1982)		B/L	I	M	D	batch M	
Kaspi and Perry (1983)		L	I	M	D	M	
Kaspi and Perry (1984)		B^g/L	I	renewal	D	M	
Perry and Posner (1990)		L	I	M^3	D	M^3	c,h,b,c_s
Perry (1997)		L	I	M^3	D	M^3	c,h,b,c_s^8
Perry and Stadje (1999)		B^g/L	I	M^4	exp/D	M^4	c,h,p,m,r,h',p'
Perry and Stadje (2000a)		L	I	M	D^0	M	
Perry and Stadje (2000b)		L	I	M	gen	M	
Perry and Stadje (2001)		L	I	M	D/M^2	M	
Nahmias et al. (2004a)		L	I	M^4	D	M^4	
Nahmias et al. (2004b)		L	I	batch M	D	renewal	c,b,m,r^9
Pal (1989)	z	B	I	exp	exp	M	h,p,b,m
Liu and Cheung (1997)	z	B^g/L	I	exp/ample	exp	M	constraints
Kalpakam and Sapna (1996)	z	L	I	exp	exp	renewal	c,h,b,m
Kalpakam and Shanthi (2000)	z	B^g/L	I	M^4	exp	M	c,h,p,b,m
Kalpakam and Shanthi (2001)	z	L	I	cont	exp	M	c,h,b,m
Schmidt and Nahmias (1985)	z	L	I	det	D	M	c,h,b,m
Perry and Posner (1998)	z	B^g/L	I	det	D	M	

[g]Denotes a generalized backorder model.
[0]Denotes the characteristic that after items perish from a first system they have use in a second system.
[2]Denotes use of a *disaster* model in which all units perish at once.
[3]Denotes the ability to control the arrival and/or demand rates.
[4]Denotes state-dependent arrival and/or demand.
[8]Denotes slightly modified cost definitions to fit within the context of a Brownian control framework.
[9]Denotes actuarial valuations based on expected costs and age-dependent revenues.

Table 15.3 Continuous time single item inventory models fixed ordering cost

Article	Replenishment policy	Excess demand	Planning horizon	Lead time	Life-time	Demand distribution	Costs
Weiss (1980)	(s,S)*	B/L	I	0	D	M	c,h,p,m,r,K[1]
Kalpakam and Arivarignan (1988)	(s,S)‡	L	I	0	exp	M	c,h,m,K
Liu (1990)	(s,S)‡	B	I	0	exp	M	h,p,b,m,K
Moorthy et al. (1992)	(s,S)‡	L	F/I	0	E_k	batch iid	
Liu and Shi (1999)	(s,S)‡	B	I	0	exp	renewal	h,p,b,m,K
Liu and Lian (1999)	(s,S)‡	B	I	0	D	renewal	h,p,b,m,K
Lian and Liu (2001), Gürler and Özkaya (2003)	(s,S)‡ / (s,S)	B	I	0 / det	D	batch renewal	h,p,b,m,K
Gürler and Özkaya (2006)	(s,S)	B	I	0 / det	gen[2]	batch renewal	h,p,b,m,K
Kalpakam and Sapna (1994)	(s,S)	L	I	exp	exp	M	c,h,b,m,K
Ravichandran (1995)	(s,S)	L	I	cont[5]	D/D[6]	M	c,h,b,m,K
Liu and Yang (1999)	(s,S)	B	I	exp	exp	M	h,p,b,m,K
Nahmias and Wang (1979)	(Q,r)	B	I	det	decay	cont	h,b,m,K
Chiu (1995)	(Q,r)	B	I	det	D	general	c,h,b,m,K
Tekin et al. (2001)	(Q,r,T)	L	I	det	D[6]	M	h,m,K,constraint
Berk and Gürler (2006)	(Q,r)	L	I	det	D[2]	M	h,b,m,K

[1] Denotes papers in which holding and shortage costs can be general convex functions.
[2] Denotes use of a *disaster* model in which all units from the same order perish at once.
[5] That only one outstanding order is allowed at a time.
[6] The assumption that product aging starts only once all inventory from the previous lot has been exhausted or expired.
*Denotes the case when (s,S) policy is proved to be optimal.
‡Denotes the case when (s,S) policy is implied to be the optimal replenishment policy based on earlier research (however, a formal proof of optimality of (s,S) has not appeared in print for the model under concern).

- Replenishment lead time: In Table 15.2, many papers assume that items arrive according to a Poisson process, *outside of the control of the system manager*; these are denoted by (M). This may be generalized to the case of batch replenishments (batch), the case when the replenishment rates can be controlled (M^5), or are state dependent (M^6). Other papers in the no fixed ordering cost regime assume either exponential (exp) with a single or ample (ample) servers, renewal (renewal), continuous (cont) or fixed (det) lead times. Most papers in Table 15.3 assume either zero lead time or a deterministic (det) lead time, although some allow exponential (exp), or general continuously distributed (cont) lead times.
- Product lifetime: Most lifetimes are deterministic (D), possibly with the assumption that the item can be used for a secondary product after it perishes (D^0), that all perish at once, a *disaster* model, indicated by (D^2), or that items within a lot only begin to age after all items from the previous lot have left inventory (D^6). Item lifetimes may also be exponentially distributed (exp), generally distributed (gen), Erlang (E_k) and in one case they decay (decay).
- Demand distribution: As we are within a continuous model, this column describes the assumptions on the demand *interarrival* distribution. Most are Poisson (M), although in some cases they may have rates that can be controlled by the system manager (M^3) or which are state-dependent (M^4). Selected papers allow independent and identically distributed arrivals (iid), continuously distributed interarrivals (cont), some allow arrivals in batches (batch), and others have renewal (renewal), or general (general) demand interarrival distributions.
- Costs: Typically, the costs introduced in Sect. 15.3.1, Table 15.1 are used. When demand and interarrival rates can be controlled, there is a cost (c_s) for adjusting these rates. Some models calculate profits, via using a unit revenue (r). One paper not only charges the standard per unit time holding and penalty costs (h) and (p), but also per unit time costs based on the average age of the items being held (h') or backlogged (p'). Two papers optimize subject to constraints (possibly with other costs), and several (but not all) that concern themselves only with characterizing system performance have this column left blank. In addition, one paper utilizes a Brownian control model with two barriers; it defines shortage and outdate costs slightly differently to account for the infinite number of hits of the barriers. This is denoted by ^{8}. One paper calculates *actuarial* valuations of costs and expected future revenues, where the latter depend on the age of an item when it is sold. This is denoted with a ^{9}. Finally, when annotated with a ^{1}, a paper allows the unit holding and shortage costs to be generalized to convex functions.

15.3.2.1 Continuous Review Without Fixed Ordering Cost or Lead Time

The continuous review problem without fixed ordering costs is unique in that under certain modeling assumptions, direct parallels can be drawn between the perishable inventory problem and stochastic storage processes in general, and queueing theory in particular. These parallels provide structural results as well as powerful analytical tools.

One of the earliest papers to make these connections is written by Graves (1982), who focuses on characterization of the system behavior, as in the earlier papers. Graves (1982), without an explicit cost structure or replenishment policy, shows that the virtual waiting time in an M/M/1 queue with impatient customers and a M/D/1 with finite buffer can be used to model the inventory in a perishable inventory system with Poisson demands of exponential, or unit size, respectively, with either lost sales or backlogging. A crucial assumption is that items are replenished according to what Graves (1982) calls "continuous production"; the arrival of items into inventory is modeled as a Poisson process, essentially out of the control of the inventory manager. Under this convention, Graves (1982) notes that the key piece of information is the age of the oldest item currently in stock, an observation that is used by many subsequent researchers. For example, Kaspi and Perry (1983, 1984) model systems with Poisson demand and Poisson or renewal supply, as might be the case, for example in blood banks that rely on donations for stock. For their analysis Kaspi and Perry (1983 1984) track what they call the virtual death process, which is the time until the next death (outdate) if there would be no more demand. This, of course, is just a reformulation of the age of the oldest item in stock, as used by Graves (1982).

These papers mark the start of a significant body of work by Perry (sometimes in conjunction with others) on continuous review perishable inventory systems with no fixed ordering cost, some sort of Poisson replenishment, and nearly all using the virtual death process. Only Perry and Posner (1990) and Perry and Stadje (1999) contain explicit cost functions; the rest of the papers concern themselves solely with performance characteristics, as in Graves (1982). Perry and Posner (1990) develop level crossing arguments for storage processes to capture the effects of being able to control supply or demand rates within the model of Kaspi and Perry (1983); the limiting behavior of this system was analyzed by Perry (1997) using a diffusion model and martingale techniques. Perry and Stadje (1999) depart from the virtual death process, using instead partial differential equations to capture the stationary law of a system which now may have state-dependent arrival and departure rates with deterministic or exponential lifetimes and/or maximum waiting times, as well as finite storage space. This work is generalized in Nahmias et al. (2004a) using the virtual death process. Likewise using the virtual death process, Perry and Stadje (2000a, b, 2001) evaluate systems where after perishing the item can be used for a secondary product (such as juice for expired apples); items may randomly perish before the expiration date; or, in addition to their fixed lifetime, items may all perish before their expiration date (due to disasters, or obsolescence). Still using the virtual death process, Nahmias et al. (2004b) provide actuarial valuations of the items in system and future sales, when item values are dependent on age. Recently Perry and Stadje (2006) solved a modified M/G/1 queue and showed how it related, again, to the virtual death process in a perishable system, this time with lost sales. Thus work on extensions to what can reasonably be called the *Perry model* continues.

Note that the work in these papers is concerned with performance analysis of systems with random input and output. This is in contrast to the models in Sect. 15.3.1

which typically assume input is completely controllable. Thus most of the papers in this section do not focus on optimization. However, a few do include the ability to control the input rate (typically at a cost); see Table 15.2.

15.3.2.2 Continuous Review Without Fixed Ordering Cost Having Positive Lead Time

A defining characteristic of the *Perry model*, mentioned in Sect 15.3.2.1, is that the supply of perishable items arrives according to a (possibly state dependent) Poisson process. When replenishment decisions and lead times must be included in a more explicit manner, researchers need to develop other analytical methods.

When lifetimes and lead times are exponentially distributed the problem is simplified somewhat, as this allows the application of renewal theory, transform methods, and Markov or semi-Markov techniques, often on more complex versions of the problem. Pal (1989) looks at the problem with exponential lead times and lifetimes, Kalpakam and Sapna (1996) allow renewal demands with lost sales, Kalpakam and Shanthi (2000, 2001) consider state dependent Poisson lead times and then general continuous lead times, and Liu and Cheung (1997) are unique in that they consider fill rate and waiting time constraints. All of these papers consider base-stock, or $(S-1, S)$ inventory control; Kalpakam and Sapna (1996) and Kalpakam and Shanthi (2001) consider per unit shortage costs and per unit outdating costs, per unit ordering costs and per unit per period holding costs. To these Kalpakam and Shanthi (2000) add a per unit per period shortage cost, while Pal (1989) also adds the per unit per unit time shortage cost, but disregards the per unit ordering cost. Liu and Cheung (1997) take a different approach, seeking to minimize the inventory subject to a service level constraint. In all of these cases, the cost function appears to be unimodal in S, but no formal proofs have been provided, owing to the difficulty in proving unimodality. Furthermore, nowhere has the performance of base-stock policies been formally benchmarked against the optimal, possibly due to the difficulty in dealing with a continuous state space – time – within the dynamic programming framework. This remains an open question.

The case of fixed lead times and lifetimes is arguably both more realistic and analytically more difficult. Schmidt and Nahmias (1985) consider a system operating under a base-stock policy with parameter S, lost sales, Poisson demand, per unit shortage, outdating and ordering costs, and per unit per period holding costs. They define and solve partial differential equations for the S-dimensional stochastic process tracking the time since the last S replenishment orders. Numerical work shows that again cost appears to be monotonic in S (in fact convex), although surprisingly, the optimal value of S is *not* monotonic in item lifetime.

Perry and Posner (1998) generalize this work to allow for general types of customer impatience behavior, using level crossing arguments to derive the stationary distribution of the vector of times until each of the S items in the system outdate (reminiscent of their *virtual death process*). They also show that the distribution of the differences between the elements of this vector follows that of uniform order statistics, which enables them to derive expressions for general customer behavior. These expressions may, as a rule, require numerical evaluation.

Perry and Posner (1998) are concerned with general system characteristics; they do not include explicit costs in their paper. While Perry and Posner (1998) provide rich material for future research – for example exploring how different customer behavior patterns affect different echelons of a supply chain for perishable products, – there is still a need for research following the work of Schmidt and Nahmias (1985), with the aim of minimizing costs in the continuous review setting under fixed lead times and fixed lifetimes.

15.3.2.3 Continuous Review with Fixed Ordering Cost

Within the continuous review fixed ordering cost model we make a distinction between those models that assume fixed batch size ordering, leading to (Q,r) type models, and those that assume batch sizes can vary, leading to (s,S) type models. We consider the (s,S) models first.

Initial work in this setting assumed zero lead time, which simplifies the problem considerably, as there is no need to order until all the items are depleted. In this case when considering fixed ordering costs, unit revenues, unit ordering and outdate costs, and convex holding and penalty costs per unit time, the optimal policy structure under Poisson demand was found by Weiss (1980) who showed that for the fixed lifetime problem over an infinite horizon, with lost sales or back ordering, an (s,S) policy is optimal. Weiss (1980) also established that the optimal s value is zero in the *lost sales* case, and in the *backorder* case no larger than -1 (you never order if you have items in stock). Thus the optimal policy structure was established, but the question of efficiently finding the optimal parameters is open.

The publication of Weiss (1980) initiated a series of related papers, all having in common the assumption of immediate supply. Kalpakam and Arivarignan (1988) treat the lost sales model as Weiss (1980), but assume exponential lifetimes, consider only costs (not revenues), and of these costs disregard the shortage costs as they are irrelevant, as Weiss shows that the optimal is $s = 0$. They also show that in this case the cost, assuming an optimal s value, is convex in S. Liu (1990) takes the setting of Weiss (1980), assumes exponential lifetimes, disregards the unit ordering costs and revenues, but considers penalty costs per unit and per unit time. His focus is on providing closed-form expressions for system performance, based on transform analysis. Moorthy et al. (1992) perform a similar analysis using Markov chains theory under Erlang lifetimes, assuming no shortage is permitted – or equivalently lost sales, as in this case Weiss (1980) shows that it is optimal not to allow any shortage. Liu and Shi (1999) follow Liu (1990), but now allow for general renewal demand. They focus their analysis on the reorder cycle length, using it as a vehicle to prove various structural properties of the costs with respect to the parameters. The assumption of exponential lifetimes is crucial here – the results would be very unlikely to hold under fixed lifetimes. Liu and Lian (1999) consider the same problem as Liu and Shi (1999) and Lian but with fixed, rather than exponential lifetimes. They permit renewal demands, and derive closed-form cost expressions, prove unimodality of costs with respect to parameters, and show that the distribution of inventory level is uniform over (s,S) (as in the non-perishable case).

All of the previous papers make the zero lead time assumption of Weiss (1980), which simplifies the problem. A few papers include positive *fixed* lead times, Lian and Liu (2001) treat the model of Lian and Liu (1999) incorporating batch demands to provide a heuristic for the fixed lead time case, but a proof in that paper contained a flaw, which was fixed by Gürler and Özkaya (2003). These papers show how to efficiently find good (s,S) parameters for the fixed lead time problem, but do not provide any benchmark against the optimal. Thus while efficient heuristics exist for the fixed lead time case, they have not as yet been benchmarked against more complex control policies.

Random lead times have appeared in several papers within the continuous review, fixed ordering cost framework. Kalpakam and Sapna (1994) allow exponential lead times, while also assuming exponential lifetimes. As in the fixed lead time case, this implies that the (s,S) policy is no longer necessarily optimal. They nevertheless assume this structure. In addition to fixed ordering costs, they account for unit purchasing, outdate and shortage costs, as well as holding costs per unit per unit time. Under the assumption of only one outstanding order at a time, they derive properties of the inventory process and costs. Ravichandran (1995) permits general continuous lead times, deriving closed-form expressions for costs under the assumption that only one order is outstanding at a time, and items in an order only begin to perish after all items from the previous order have left the system. Liu and Yang (1999) generalize Kalpakam and Sapna (1994) to allow for backlogs and multiple outstanding orders, using matrix analytical methods to generate numerical insights under the assumption of an (s,S) policy. For random lifetimes, still within the (s,S) structure, the most comprehensive work is by Gürler and Özkaya (2006), who allow a general lifetime distribution, batch renewal demand, and zero lead time with a heuristic for positive lead time. Their cost structure follows Lian and Liu (2001); Gürler and Özkaya (2006) can be thought of as generalizing Lian and Liu (2001) to the random lifetime case. Gürler and Özkaya (2006) argue that the random lifetime model is important for modeling lifetimes at lower echelons of a supply chain, as items arriving there will have already begun to perish. To this end they demonstrate the importance of modeling the variability in the lead time distribution on costs, including a comparison to the fixed lifetime heuristic of Lian and Liu (2001).

In general, within the fixed ordering cost model with variable lot sizes, for the zero lead time case research is quite mature, but for positive lead times and/or batch demands there are still opportunities for research into both the optimal policy structure and effective heuristics. These models are both complex and practically applicable, making this yet another problem that is both challenging and important.

If lot sizes must be fixed, initial work for "decaying" goods by Nahmias and Wang (1979), using a (Q,r) policy, was followed two decades later by Chiu 1995), who explicitly considers perishable (rather than decaying) items by approximating the outdating and inventory costs to get heuristic (Q,r) values. Nahmias and Wang (1979) consider unit ordering and shortage costs, and holding costs per unit per unit time, as well as unit outdate costs. To these Chiu (1995) adds unit ordering costs.

Tekin et al. (2001) take a slightly different approach; they disregard unit ordering costs and consider a service level constraint, rather than a shortage cost. They also simplify the problem by assuming that a lot only starts aging after it is put into use, for example moved from a deep freezer. To combat the effects of perishability, they advocate placing an order for Q units whenever the inventory level reaches r or when T time units have elapsed since the last time a new lot was unpacked, whichever comes first, giving rise to a (Q,r,T) policy. Not surprisingly, they find that inclusion of the T parameter is most important when service levels are required to be high or lifetimes are short. Berk and Gürler (2006) define the "effective shelf life" of a lot: The distribution of the remaining life at epochs when the inventory level hits Q. They show that this constitutes an embedded Markov process (as they assume Poisson demand), and thus via analysis of this process they are able to derive *optimal* (Q,r) parameters, when facing unit outdate and penalty costs, holding costs per unit per unit time, and fixed ordering costs. They compare the performance of their policy with that of Chiu (1995) and also with the modified policy of Tekin et al. (2001). Not surprisingly, the optimal (Q,r) policy outperforms the heuristic of Chiu (1995), sometimes significantly, and is outperformed by the more general modified policy of Tekin et al. (2001). Nevertheless, this latter gap is typically small, implying that use of the more simple (Q,r) policy is often sufficient in this setting.

Note that throughout these papers the (Q,r) structure has only been assumed, and once again there does not appear to be any benchmarking of the performance of the (Q,r) policy against the optimal, as optimal policies are difficult to establish given the increased problem complexity the continuous time setting with perishability causes. If such benchmarking were done, it would help identify those problem settings for which the (Q,r) or (Q,r,T) policy is adequate, and those which would most benefit from further research into more complex ordering schemes. Essentially, the underlying question of how valuable lifetime information is, in what degree of specificity, and when it is most valuable, remains.

15.4 Managing Multi-Echelon and Multi-Location Systems

Analysis of multi-echelon inventory systems dates back to the seminal work of Clark and Scarf (1960); the reader can refer to Axsäter (2000) for a unified treatment of the research in that area, and Axsäter (2003) for a survey of research on serial and distribution systems. In serial and distribution systems, each inventory location has one supplier. In contrast, multi-location models consider flow of products from various sources to a particular location (possibly including transshipments). See, for instance, Karmarkar (1981) for a description of the latter problem. Managing multi-echelon and/or multi-location systems[1] with aging products is a challenge because of the added complexity in:

- *replenishment and allocation decisions,* where the *age* of goods replenished at each location affects the age-composition of inventory and the system performance, and the *age* of goods supplied/allocated downstream may be as important as the *amount* supplied,
- *logistics-related decisions such as transshipment, distribution, collection*, which are complicated by the fact that products at different locations may have different remaining lifetimes,
- *centralized vs. decentralized planning,* where different echelons/locations may be managed by different decision-makers with conflicting objectives, operating rules may have different consequences for different decision-makers (e.g., retailers may want to receive LIFO shipments but suppliers may prefer to issue their inventory according to FIFO), and system-wide optimal solutions need not necessarily improve the performance at each location.

Given the complexity in obtaining or characterizing optimal decision structures, analytical research in multi-echelon and multi-location systems has mainly focused on particular applications (modeling novelties) and heuristic methods. We review analysis of replenishment and allocation decisions in Sect. 15.4.1, logistics and distribution related decision in Sect. 15.4.2, and centralized vs. decentralized planning in Sect. 15.4.3.

15.4.1 Research on Replenishment and Allocation Decisions

Research on multi-echelon systems has been confined to two-echelons[2] except in the simulation-based work (e.g., van der Vorst et al. (2000)). Motivated by food supply chains and blood banking, the upstream location(s) in the two-echelon models typically involve the supplier(s), the distribution center(s) (DC), or the blood banks, and the downstream location(s) involve the retailer(s), the warehouse(s), or the hospital(s). In this section, we use the terms *supplier* and *retailer* to denote the upstream and downstream parties, respectively. We call the inventory at the retail locations the *field inventory*.

We first classify the research by focusing on the nature of the decisions and the modeling assumptions. Table 15.4 provides a summary of the *analytical* work

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[1] Note that multi-location models we review here are different from the two-warehouse problem described in Section 10 of Goyal and Giri (2001); that model considers a decision-maker who has the option of renting a second storage facility if he/she uses up the capacity of his/her own storage.

[2] The model of Lin and Chen (2003) has three echelons in its design: A cross-docking facility (central decision maker) orders from multiple suppliers according to the demand at the retailers and the system constraints, and allocates the perishable goods to retailers. However, the replenishment decisions are made for a single echelon: The authors propose a genetic algorithm to solve for the single-period optimal decisions that minimize the total system cost.

Table 15.4 A summary of the analytical models on replenishment and/or allocation decisions in multi-echelon and multi-location systems

Article	Replenishment policy		Inventory issuance	Excess demand	Allocation rule	Trans-shipments	No. of retailer(s)	Fixed costs	Centralized planning
	Supplier	Retailer							
Yen (1975)	TIS	TIS	FIFO	Backlog	Proportional, fixed	–	2	–	Yes
Cohen et al. (1981a)	TIS	TIS	FIFO	Backlog	Proportional, fixed	–	2	–	Yes
Lystad et al. (2006)	TIS	TIS	FIFO	Backlog	Myopic	–	>2	–	Yes
Fujiwara et al. (1997)	TIS (per cycle)	TIS (per period)	FIFO = LIFO	Lost at supplier, expediting for retailer	–	–	2	–	Yes
Kanchanasuntorn and Techanitisawad (2006)	(s,S)	TIS	FIFO	Backlog at supplier, lost at retailer	FIFO	–	2	Supplier	Yes
Prastacos (1978)†	–	–	FIFO	Lost at retailer	Myopic	Rotation	>2	–	Yes
Prastacos (1981)	–	–	FIFO	Lost at retailer	Myopic	Rotation	> 2	–	Yes
Prastacos (1979)†	–	–	LIFO	Lost at retailer	Segregation	Rotation	>2	–	Yes
Nose et al. (1983)†	–	–	LIFO	Lost at retailer	Based on convex programming	Rotation	>2	–	Yes
Federgruen et al. (1986)†	–	–	FIFO	Lost at retailer	Based on convex programming	Rotation	>2	–	Yes
Abdel-Malek and Ziegler (1988)	EOQ	EOQ	–	–	–	–	1	Retailer, supplier	Yes
Ketzenberg and Ferguson (2006)	Order 0 or Q	Order 0 or Q	FIFO	Expediting for Supplier, lost at retailer	–	–	1	–	Yes, no

†Indicates single-period decision making.

that determines heuristic replenishment policies for a supplier and retailer(s), and/or effective allocation rules to ship the goods from the supplier to the retailers; analytical research has been rather limited, modeling assumptions have varied, and some problems (such as replenishment with fixed costs or decentralized planning) have received very little attention.

Notice that the centralized, multi-echelon models with perishables are no different from the serial or distribution systems studied in classical inventory theory; the stochastic models involve the one-supplier, multi-retailer structure reminiscent of Eppen and Schrage (1981) or the serial system of Clark and Scarf (1960). For the one-supplier, multi-retailer system with non-perishables, it is known that order-up-to policies are optimal under the "balance assumption" (when the warehouse has insufficient stock to satisfy the demand of the retailers in a period, available stocks are allocated such that retailers achieve uniform shortage levels across the system; this might involve "negative shipments" from the warehouse, i.e., transshipments between retailers at the end of that period to achieve system balance), and optimal policy parameters can be determined by decomposing the system and solving a series of one-dimensional problems (see e.g., Diks and de Kok (1998)). There is no equivalent of this analysis with perishable inventories, mainly due to the complexity of the optimal ordering policy at a single location. Similarly, there is no work that investigates continuous replenishment policies with perishables for multi-echelons or multi-locations, although this is a well-studied problem for single-location models with perishables (see Sect. 15.3.2) and for non-perishables in multi-echelon supply chains.

15.4.1.1 Analytical Research on Allocation Decisions

All the models listed in Table 15.4 involve no capacity restrictions, and have a single supplier who receives the freshest goods upon replenishment (although the goods may not have the maximal lifetime at the time of arrival when lead time is positive). However, the shipments from the supplier to the retailers can involve stock of any age depending on the supplier's inventory. Allocation and/or transshipment decisions are simplified, for instance, when there is a single retailer (see Table 15.4) or when the supplier's inventory consists of goods of the same age. The latter happens in the following two cases: (a) The goods start perishing at the retailer but not at the supplier, i.e., all retailers are guaranteed to receive fresh goods from the supplier (see Fujiwara et al. (1997) and Abdel-Malek and Ziegler (1988)). (b) The lifetime of the product is equal to the length of the periodic review interval. In that case, all the goods at any location are of the same age and the goods perish at the end of one cycle, as is the case for the supplier in the model of Fujiwara et al. (1997).

The focus on multi-echelon problems is on allocation decisions when the supplier is assumed to receive a random amount of supply at the beginning of every period (Prastacos 1978, 1979, 1981) – we refer to this as the *Prastacos model*. The random supply assumption is motivated by the application area – blood products which rely on donations for supply. Two special distribution systems are considered in these papers: (a) A *rotation* (or *recycling*) system where all unsold units that have not expired at the retailers are returned to the supplier at the end of each period – these units are distributed among the retailers along with the new supply of freshest goods at the beginning of the next period; (b) A *retention* system where each retailer keeps all the inventory allocated. In the *Prastacos model*, the supplier does not stock any goods, i.e., all the inventory is allocated and shipped to the retailers at the beginning of a period, there are no inventory holding costs at any location, and the goal is to minimize shortage and outdating costs that are uniform across all the retailers.

In a rotation system, the total number of units to outdate in a period depends only on how the units with only one period of lifetime remaining are allocated in the previous period, and the total amount of shortage depends only on how the inventory is allocated, regardless of the age. Based on these observations, Prastacos (1978) proposes the following *myopic allocation policy that minimizes one-period system-wide outdate and shortage costs*: Starting with the oldest, the stocks of a given age are allocated across the retailers so that the probability that the demand at each location exceeds the total amount allocated to that retailer up to that point in the algorithm are equalized, and this is repeated iteratively for items of all ages. Prastacos (1978) also analyzes a retention system where the supplier only ships the fresh supply to the retailers at the beginning of each period. In the one-period analysis of the retention system, the amount to outdate at the end of a period depends on the amount of oldest goods in stock and demand at each retailer, but does not depend on the supply allocated in that period. Based on this observation, Prastacos (1978) suggests a myopic allocation rule that equalizes the one-period shortage probability at each retailer to minimize the one-period system-wide shortage and outdate costs. Prastacos (1981) extends the analysis of the *Prastacos model* to the multi-period setting and shows that the myopic allocation policy preserves some of the properties of the optimal allocation that minimizes expected long-run average shortage and outdating costs, and is, in fact, optimal in numerical examples with two retailers and product lifetime of two periods. Since the cost parameters are the same for all the retailers, the allocation resulting from the myopic rule is *independent* of the unit costs of shortage and outdating.

Prastacos (1979) analyzes essentially the same single-period model assuming LIFO issuance, as opposed to FIFO in Prastacos (1978). In case of LIFO, the optimal myopic allocation policies in both rotation and retention systems depend on the unit costs of shortage and outdating. In addition, the optimal allocation policies *segregate* the field inventory by age as opposed to a *fair* allocation where each retailer receives goods of each age category. Under segregation, some retailers have only newer goods and some only older goods so that system-wide expected outdates are minimized. The optimal myopic allocation policy under LIFO for both rotation and retention systems can be determined by solving a dynamic program with stages corresponding to retail locations. Prastacos (1979) obtains the optimal allocation rule for specific demand distributions. For rotation, he proposes a heuristic: First, allocate the stock in order to equalize the probability of shortage at each retailer and then swap the inventory among retailers to obtain field inventories that are segregated by age.

There are several practical extensions of the *Prastacos model*: The assumption on uniform outdating and shortage costs can be relaxed, shipments/transportation costs can be added, penalties can be incurred on leftover inventory (e.g., end of period holding costs), transshipments among retailers can be enabled, and the supplier may keep inventory as opposed to shipping all units downstream. The first three of these issues have been addressed in the literature for only single-period decision-making: Nose et al. (1983) and Federgruen et al. (1986) generalize the FIFO model of Prastacos (1978) by assuming that there is a unit transportation cost for each item shipped from the supplier to the retailers, and the unit outdating, shortage, and transportation costs are retailer-specific. They both develop convex programming formulations for the single-period inventory allocation problem and propose algorithms based on the Lagrangean relaxation to determine the optimal allocation. Their models rely on the observation that the costs are only a function of the amount of old vs. fresh (i.e., stock that will outdate in one period vs. the inventory that has more than one period of lifetime remaining) goods allocated to each retailer. In the rotation system considered by Nose et al. (1983), the retailers are also charged per unit inventory returned to the supplier at the end of each period (i.e., there is an end-of-period penalty on leftover inventory at each location).

In the model of Yen (1975) and Cohen et al. (1981a), the supplier uses the FIFO rule to determine which goods are to be shipped downstream and then uses one of the following two allocation rules to determine the age-composition of the shipments to the retailers in each period: (a) *proportional allocation* and (b) *fixed allocation*. In proportional-allocation, each retailer receives a proportion of goods of each age category based on their share of the total demand. In *fixed allocation*, each retailer receives a pre-determined fraction of goods in each period. Both of these allocation rules are *fair* in that the shipments to retailers involve goods of each age category. Yen (1975) and Cohen et al. (1981a) explore the optimality conditions for the parameters associated with these allocation rules. They show that, under certain conditions, there exists a fixed allocation rule that yields the same expected shortage, outdating, and holdings costs as a system operating under the optimal proportional allocation rule. Their analysis can be extended to include multiple (>2) retailers. Prastacos (1981) shows that his myopic allocation rule is the same as proportional allocation for certain probability distributions of demand.

15.4.1.2 Analytical Research on Replenishment and Allocation Decisions

Analysis of optimal replenishment policies for a serial, two-echelon system is presented in Abdel-Malek and Ziegler (1988) assuming *deterministic* demand, zero lead times, and price that linearly decreases with the age of the product. They determine the economic order quantities (EOQs) for the retailer and the supplier by restricting the order cycle lengths to be no more than the product lifetime.

Under demand uncertainty, several heuristic, discrete review replenishment policies have been considered and the focus has been on determining the optimal parameters for these restricted policies. These heuristic replenishment policies include the

TIS policies (Yen (1975), Cohen et al. (1981a), Lystad et al. (2006), Fujiwara et al. (1997) and Kanchanasuntorn and Techanitisawad (2006)), a "zero-or-fixed quantity" ordering policy in Ketzenberg and Ferguson (2006) – denoted as "0 or Q" in Table 15.4, – and (*s,S*) policy for the retailers in Kanchanasuntorn and Techanitisawad (2006), analysis which is restricted to the case where the retailers' demand is Normal. The replenishment lead times are no longer than one period (i.e., goods are received no later than the beginning of the next period) with exceptions being Lystad et al. (2006) and Kanchanasuntorn and Techanitisawad (2006); the latter assumes that the lifetime of the product is a multiple of the replenishment cycle lengths of the retailers and the supplier, and that the supplier responds to retailers' orders in a FIFO fashion (hence allocation decisions are trivial).

Research that analyzes replenishment and allocation decisions jointly is confined to the work of Yen (1975), its extension in Cohen et al. (1981a), and Lystad et al. (2006). Yen (1975) and Cohen et al. (1981a) explore the structural properties of the expected total cost function that includes expected holding, outdating, and shortage costs, when both the supplier and the retailer use TIS policies to replenish inventory. They investigate fixed and proportional allocation rules and identify conditions on the existence of unique target inventory levels. Yen (1975) also identifies conditions for the optimality of the proportional allocation rule for this system. These conditions are satisfied, for instance, when the lifetime of the product is restricted to two or three periods, or when the demand at each location and each period is i.i.d and the target inventory levels of the retailers are the same. However, the analysis relies on one simplification: the finite lifetime of the perishable product is not taken into account explicitly, and goods that remain in inventory beyond their lifetime can be used to satisfy excess demand, but are charged a unit outdating cost. Analysis of replenishment *and* allocation policies that explicitly take these factors into account remains an open problem of theoretical interest.

Lystad et al. (2006) use the myopic allocation rule of Prastacos (1981), and propose heuristic echelon-based TIS policies. For a given system, they first determine the best TIS policy via simulation. Then, they do a regression analysis to establish the relationship between the order-up-to levels of this best policy and two heuristic order-up-to levels: One heuristic is based on the newsvendor-based, approximate echelon-stock policies for non-perishables and the other is the single-location heuristic of Nahmias (1976) for perishables. The resulting regression model is then used in computational experiments to study the effect of the lifetime of a product on system costs, and to compare the performance of the approximation against policies that are derived assuming the product is non-perishable. Thus Lystad et al. (2006) take a first step in developing approximate echelon-based policies for perishables, and this topic deserves more attention.

Notice that the effectiveness of the proposed allocation rules combined with good replenishment policies have not been benchmarked in any of these studies, and various simplifying assumptions have been made to derive the policies. There is a need for further research on the analysis of replenishment and allocation decisions in multi-echelon systems. Interesting research directions include investigation of the "balancing" of echelon inventories for perishables (as in Eppen and Schrage (1981)), analysis of systems without making simplifying assumptions on inventory recursions (as in Yen (1975)), analysis of different system designs (e.g. rotation and retention systems have been studied to some extent), or incorporation of different cost parameters to the models (e.g. the *Prastacos model* excludes holding costs).

15.4.1.3 Simulation Models of Multi-Echelon Inventory Systems

In addition to the analytical research, simulation models have also been used in analyzing multi-echelon, multi-location systems with perishable goods. For this complex problem, simulation models present more opportunities in terms of model richness, which we highlight in this section.

The earlier research (e.g., Yen (1975), Cohen and Pierskalla (1979), Cohen et al. (1981b)) is motivated mainly by managing regional blood centers; see also Prastacos (1984) and Pierskalla (2004).

More recently, van der Vorst et al. (2000) describe a discrete-event simulation model to analyze a fresh produce supply chain with three echelons. Among other factors, van der Vorst et al. (2000), test the system performance – measured in terms of inventory levels at the retailers and distribution centers, and product freshness – using several scenarios. Katsaliaki and Brailsford (2007) present results of a project to improve procedures and outcomes by modeling the entire supply chain for blood products in the UK. Their simulation model includes a serial supply chain with the product flow that includes collection of supply, processing/testing and storage at a service center, and shipment to a hospital where blood is crossmatched/transfused[3] for patients use. The model includes multiple products with different shelf-lives. Six different policies varying in (a) the type of products that are stocked at the hospital, (b) the target inventory levels, (c) the time between crossmatching and release which can influence the amount of unused and still usable inventory that is returned, (d) the order trigger points for expedited deliveries, (e) the inventory issuance rules for releases and returns, (f) the order and delivery lead times, and (g) the number of daily deliveries to the hospital. Performance is measured in terms of number of expired units, mismatched units, amount of shortage, and number of routine and expedited deliveries. Note that allocation decisions are not included in Katsaliaki and Brailsford (2007) because they model a serial supply chain. Mustafee et al. (2006) provide the technical details of the simulation model and the distributed simulation environment used in this latter project.

In contrast to Katsaliaki and Brailsford (2007), the simulation model of the single supplier, multiple-retailer system in Yen (1975) includes returns of unused units

[3] One common practice in managing blood inventories is cross-matching, which is assigning units of blood from inventory to particular patients. Jagannathan and Sen (1991) report that more than 50% of blood products held for patients are not eventually transfused (i.e., used by the patient). The release of products that are cross-matched enable re-distribution of inventories in a blood supply chain. See Prastacos (1984), Pierskalla (2004), and Jagannathan and Sen (1991) for more information on cross-matching.

from retailers to the supplier, variations of the fixed and proportional allocation rules, expedited shipments to retailers, transshipments between retailers, and limited supply at the supplier. In addition to analyzing the impact of inventory levels, allocation and transshipment rules on system costs, Yen (1975) also looks at the impact of magnitude of demand at the retailers, and observes that the system cost in his centralized model is more sensitive to the sizes of the retailers rather than the number of retailers. Cohen et al. (1981b) also study different allocation rules in a centralized system: In the first one, the supplier chooses a retailer and fills its demand and goes on to fill the demand of the next retailer until all stock is depleted or all demand is satisfied. In the second one, the supplier uses the proportional allocation rule, and in the third one, the myopic allocation rule. Cohen et al. (1981b) suggest using the second method in a practical setting because it has less information needs (i.e., does not need the probability distribution of demand at each retailer like the third method) and advise against using the first method in a centralized system because it will lead to an imbalanced distribution of aging inventory. In addition, the outdate probabilities of the retailers will vary significantly with the first method; this increases the possibility of costly transshipments which are discussed in the next section.

15.4.2 Logistics: Transshipments, Distribution, and Routing

Other than replenishment and allocation decisions, three of the critical logistics activities in managing perishables in multi-location systems are transshipments, distribution and collection (particularly for blood). Research that focuses on these three decisions has been limited. Within the analytical work cited in Table 15.4, the rotation system is the only form in which excess inventory is exchanged among the retailers, and the exchange happens with a one period delay. Rotation systems are also the basis for the goal programming model developed by Kendall and Lee (1980). Prastacos and Brodheim (1980) develop a mathematical programming model for a hybrid rotation-retention system to efficiently distribute perishables in a centralized system. Both of these papers are motivated by operations of regional blood centers; see Prastacos (1984) for a review of these and other earlier work on the distribution and transshipment problems.

Note that rotation encompasses only indirect transshipments among the retailers. The simulation model of Yen (1975) includes transshipments between retailers after each location satisfies its own demand and serves as a guideline for the practical inventory control and distribution system described in Cohen et al. (1981b). Cohen et al. (1981b) suggest using transshipments in this practical setting if (a) the supplier is out-of-stock, one retailer has an emergency need, and transshipping units from other retailers do not significantly increase the probabilities of shortage at those retailers, (b) the difference between the shortage probabilities of two retailers when a unit is transshipped from one to the other is greater than the ratio of the unit transportation cost to the shortage cost, or (c) the difference between outdate probabilities of retailers when a unit is shipped from one to the other is greater than the ratio of transportation cost to the outdate cost. They use the terms *emergency*, *shortage-anticipating* and *outdate-anticipating* transshipments, respectively, to denote these three cases. Cohen et al. (1981b) point out that when all the retailers use optimal TIS policies to manage their inventories, the amount of transshipments is insignificant based on simulation results. This emphasizes the need for effective replenishment policies in multi-echelon and multi-location systems.

Federgruen et al. (1986), in addition to their analysis of the allocation decision, consider the distribution of goods from the supplier to the retailers by formulating a combined routing and inventory allocation problem. The decisions involve assigning each location to a vehicle in the fleet and allocating fresh vs. old products among the locations. The allocations do not affect the transportation costs and the routes of vehicles do not affect shortage and outdating costs. They propose exact and heuristic solution methods. They also compare their combined routing and allocation approach to a more hierarchical one where the allocation problem is solved first, and its solution is used as an input to the distribution problem. Based on computational experiments, the combined approach provides significant savings in terms of total transportation costs, although these savings may not lead to a significant decrease in total costs depending on the magnitude of the inventory related costs. However, the combined approach has significant benefits when the number of vehicles used is few (where the hierarchical approach may yield an infeasible solution). In addition to inventory levels and allocation, the simulation-based research conducted by Gregor et al. (1982) also examines the impact of the number of vehicles used in distribution on system-wide costs.

Or and Pierskalla (1979) consider daily vehicle routing decisions as a part of a regional location-allocation problem where they also determine the optimal number and location of blood centers, and the assignment of hospitals to the blood centers that supply the hospitals on a periodic basis. They develop integer programming models and propose heuristic solution methods. However, their model is designed at an aggregate level and age of inventory is not considered. A similar problem is studied by Hemmelmayr et al. (2006) where periodic delivery schedules and vehicle routes are determined to distribute blood across a region.

Recent research on supply chain scheduling has addressed the need to effectively distribute time-sensitive goods; Chen (2006) provides a survey of research in this growing area. However, perishability is not modeled explicitly in this literature, rather production orders are assumed to come from customers along with information on delivery time windows and delivery due dates. Similarly, there are several articles that model and solve real-life distribution problems of perishable products such as dairy products, or food (e.g., Adenso-Diaz et al. (1998), Golden et al. (2001)) where aging or perishability is not explicitly modeled but is implicit in the time-windows. More recently, Yi (2003) developed a model for daily vehicle routing decisions to bring back supply (blood) from collection sites to a central location in order to meet the daily target level of platelets (that can only be extracted within 8 hours of blood donation); this is a *vehicle routing problem with time windows and time-dependent rewards.*

Notice that the research in this area has been confined to a single product, or multiple products without age considerations. Interestingly, the distribution problem posed by Prastacos (1984) still remains open: *How can a distribution plan for a centralized system be created to include shipments for multiple products, each with a different lifetime and supply-demand pattern?* Katsaliaki and Brailsford's (2007) simulation model provides only a partial answer to this question; their model involves only one supplier and one retailer. Although challenging, analysis of the centralized problem with multiple retailers and multiple products definitely deserves attention.

15.4.3 Information Sharing and Centralized/Decentralized Planning

Information technology paved the way for various industry-wide initiatives including Efficient Consumer Response in the grocery industry; these initiatives aim to decrease total system costs and inventories while improving availability of products and customer satisfaction. A critical component of these initiatives is the sharing of demand and product flow information among the suppliers, distributors, and retailers. Fransoo and Wouters (2000) discuss the benefit of sharing electronic point of sale (EPOS) information for supply chains of two perishable products (salads and ready-made pasteurized meals). Their empirical analysis suggests that the benefit of EPOS would be higher for the supply chain of salad because of the magnitude of the bullwhip effect observed. The reason for the higher bullwhip effect appears to be the larger fluctuations in the demand for salad (e.g., when there is a sudden increase in temperature, there is a spike in demand), associated shortage-gaming by the retail franchisees, and the additional order amplification by the DC.

Information sharing and the value of information has been widely studied in the general supply chain literature. Chen (2002) provides a survey of research on this topic by focusing on where the information is coming from (such as the point-of-sale data from *downstream*, or capacity information from *upstream* in the supply chain), quantity, accuracy and speed of information, and centralized vs. decentralized planning in the supply chain (specifically he considers incentives for sharing information and whether the environment is competitive or not). However, this rich literature studies non-perishable goods or single-period models; the unique characteristics of perishables are ignored, except by Ferguson and Ketzenberg (2006) and Ketzenberg and Ferguson (2006).

Ferguson and Ketzenberg (2006), motivated by the grocery industry, investigate the value of information for a retailer managing inventory. They focus on the retailer's replenishment problem and consider an infinite-horizon periodic review inventory model for a single product with finite lifetime, lost sales, one-period lead time and no outdating cost (see Sect. 15.3.1.1). The age of all units in a replenishment are the same. The age of stock at the supplier is a random variable, and its distribution is known to the retailer. In case of information sharing, the retailer knows exactly the age of stock prior to giving an order. Ferguson and Ketzenberg

(2006) quantify the value of information on the age of stock under heuristic replenishment policies with FIFO, LIFO or random issuing of inventory. Numerical experiments reveal profits increase and outdates decrease, on the average, when information is shared. An interesting finding is that investments that extend the product lifetime provides a greater benefit than information sharing.

There are complications associated with different parties operating by different rules in managing supply chains of perishables: For instance, the supplier can presumably induce the retailers to order more frequently by adapting an issuance and/or replenishment policy that leads to more frequent outdates (e.g., using FIFO issuance and/or having older stock in its inventory will enable the supplier to sell goods that have a smaller shelf-life to the retailer). This is only mentioned in Ketzenberg and Ferguson (2006) – but not analyzed – and is ignored by other researchers. In that paper, Ketzenberg and Ferguson (2006) study the value of information in a two-echelon setting with one retailer and one supplier. Both parties replenish inventory heuristically, the retailer's order quantity in each period is either 0 or Q (which is an exogenous fixed batch size) and issues inventory in a FIFO fashion, whereas the supplier uses the same quantity Q in giving orders but need not give an order every period. In fact, the supplier determines the timing of his replenishments by considering a safety lead time. The retailer knows the supplier's inventory state – including the age of items in stock – and the supplier knows the retailer's replenishment policy. Ketzenberg and Ferguson (2006) quantify (a) the value of information regarding the inventory state and replenishment policies in a decentralized system via numerical examples, and (b) the value of centralized planning. The value of information for perishables can be significant, and increase in supply chain profit, due to centralization, is not always Pareto improving for both parties.

Note that all the papers we introduced so far focus on a single decision maker. Likewise, Hahn et al. (2004) derive the optimal parameters of a TIS policy for a retailer under two different contracts offered by the supplier; however, the supplier's optimal decisions are disregarded. Among the few studies that mention decentralized decision-making, Popp and Vollert (1981) provide a numerical comparison of centralized vs. decentralized planning for regional blood banking. Problems that involve multiple decision-makers, decentralized planning (vs. centralized) and coordination of supply chains have been widely studied for non-perishables and/or using single-period models (see, Chen (2002) and Cachon (2003)). In practice, perishable products share the same supply chain structure as many non-perishables, and decentralized planning and/or coordination issues are just as critical. Furthermore, there are more challenges for perishables due to cost of outdating, and possibly declining revenues due to aging. However, research in this area has been scarce, and this issue remains as one of the important future research directions.[4]

[4] There is research on coordination issues in supply chains with deteriorating goods: A *permissible delay in payment* agreement between a retailer and a supplier is proposed in the deterministic model of Yang and Wee (2006) to coordinate the supply chain. Chen and Chen (2005) study centralized and decentralized planning for the joint replenishment problem with multiple deteriorating goods.

15.5 Modeling Novelties: Demand and Product Characteristics, Substitution, Pricing

The research we have reviewed so far includes models where the inventory of a single product is depleted either in a LIFO or FIFO manner. Analysis of single location models in Sect. 15.3 is confined to FIFO. Earlier research on single location models has shown the difficulty in characterizing stationary distribution of stock levels under LIFO even when the lifetime of the product is only two periods (see the references and comments in Nahmias (1982)). Nahmias (1982) mentions that when the lifetime is two periods, the order up to level in a TIS policy is insensitive to the choice of FIFO vs. LIFO despite the difference in total system costs. Therefore, the replenishment policies/heuristics developed under FIFO can also be used effectively for LIFO. However, replenishment and issuance decisions may be interconnected – hence a more a careful analysis is needed – under more general demand models.

Typically, excess demand is treated via backlogging or lost sales, with some papers incorporating expedited delivery in their models (e.g., Fujiwara et al. (1997), Ketzenberg and Ferguson (2006), Yen (1975), Bar-Lev et al. (2005) and Zhou and Pierskalla (2006)). In practice, there is another way to fulfill the excess demand for a product: *Substitution*. In the case of perishables, products of different ages often coexist in the market place, and inventory can be issued using rules more complicated than FIFO or LIFO, allowing items of different ages or shelf-lives to be used as substitutes for each other. This idea first appeared in the perishable inventory literature in Pierskalla and Roach (1972) who assume there is demand for any category (age) and that the demand of a particular category can be satisfied from the stocks of that category or using items that are fresher. Pierskalla and Roach (1972) show that FIFO is optimal in this model with respect to two objectives: FIFO minimizes total backlog/lost sales and minimizes outdates. The model has an important simplification: The demand and supply (replenishment) are assumed to be independent of the issuing policy. Since issuing can potentially affect demand – fresher goods could lead to more loyal customers – the study of models in which this assumption of independence is relaxed will be important.

The motivation for many of the papers that involve substitution and age-dependent demand streams come from health care. Cohen et al. (1981a) mention hospitals doing special surgeries get higher priority for fresh blood. Haijema et al. (2005, 2007) mention that platelets have 4–6 days of effective shelf-life, and 70% of the patients requiring platelets suffer from platelet function disorder and need a fresh supply of platelets (no older than 3 days) on a regular basis whereas the remaining 30% of the patients who may lack platelets temporarily due to major trauma or surgery do not have a strong preference with respect to the age of the platelet up to the maximal shelf-life. For supply chains involving perishable goods other than blood, substitution usually depends on customers' choice and/or a retailer/supplier's ability to influence customers' purchasing decisions. In their empirical research, Tsiros and Heilman (2005) study the effect of expiration dates on the purchasing behavior of grocery store customers. They conducted surveys to investigate consumer behavior across different perishable product categories. They find that consumers check the expiration dates more frequently if their perceived risk (of spoilage or health issues) is greater. They also determine that consumers' willingness to pay decreases as the expiration date nears for all the products in this study; again finding that the decrease varies across categories in accordance with customer perceptions. Tsiros and Heilman's (2005) findings support the common practice of discounting grocery goods that are aging in order to induce a purchase. However, they find that promotions should differ across categories and across customer groups in order to exploit the differences in customers' tendencies to check the expiration dates and the differences in their perceived risks across categories. In light of these motivating examples and empirical findings, we provide below an overview of analytical research that considers substitution, multiple products and pricing decisions.

15.5.1 Single Product and Age-Based Substitution

Research like that of Pierskalla and Roach (1972), where products of different shelf-lives are explicitly modeled is limited. For a single product with a limited shelf-life, substitution has been considered to sell goods of different ages: Parlar (1985) analyzes the single-period problem for a perishable product that has two periods of lifetime, where a fixed proportion of unmet demand for new items is fulfilled by unsold old items and vice-versa, but his results do not extend to longer horizons. Goh et al. (1993) consider a two-stage perishable inventory problem. Their model has random supply and separate, Poisson-distributed demand streams for new and old items. Their analysis relies on an approximation and they *computationally* compare a restricted policy (where no substitution takes place) and an unrestricted policy (where stocks of new items are used to fulfill excess demand for old). Considering only shortage and outdating costs they conclude that the unrestricted policy is less costly, unless the shortage cost for fresh units is very high. Ferguson and Koenigsberg (2007) study a problem in a *two-period setting* with pricing and internal competition/substitution. In their model, the demand for each product in the second period is given by a linear price-response curve which is a function of the price of both products as well as the quality deterioration factor of the old product, and their decisions include the prices of both products as well as the number of leftover units of old product to keep in the market. They investigate whether a company is better off by carrying both or only the new product in the second period.

Ishii (1993) models two types of customers (high and low priority) that demand only the *freshest* products or products of *any age,* respectively, and obtains the optimal target inventory level that maximizes the expected profits in a single period for a product with finite lifetime. The demand of high priority customers is satisfied from the freshest stock first, and then inventory is issued using FIFO in this model. Ishii and Nose (1996) analyze the same model under a warehouse capacity constraint. More recently, Haijema et al. (2005, 2007) study a finite horizon problem for blood platelet production with a demand model of two types of customers similar to Ishii (1993) and Ishii and Nose (1996). Haijema et al. (2005, 2007) formulate a Markov Decision Process (MDP) model to minimize costs associated with holding, shortage, outdating and substitution (incurred when the demand for a "fresh" item is fulfilled by older stock) costs. They assume inventory for the any-age demand is issued in a FIFO manner from the oldest stock and fresh-demand is issued using LIFO from the freshest stock. Haijema et al. (2005, 2007) propose a TIS and a combined TIS-NIS heuristic, i.e., there is a daily target inventory level for total inventory in stock and also the new items in stock. Computational experiments show that the hybrid TIS-NIS policy is an improvement over TIS and that these heuristics provide near-optimal inventory (production) policies.

Deniz et al. (2008) provide a detailed analysis of the interplay between replenishment policies and inventory issuance. Their infinite horizon, periodic review formulation for a product with two periods of lifetime includes lost sales, holding, outdating as well as substitution costs (both new-to-old and old-to-new). They assume two separate demand streams for new and old items; demand can be correlated across time or across products of different ages. Deniz et al. (2008) study different substitution options: The excess demand for a new item is satisfied from the excess stock of old, and/or the excess demand for an old item is satisfied from the excess stock of new, or not, including the no-substitution case. Both LIFO and FIFO inventory issuance, as is common in the literature, can be represented using this substitution model. The inventory is replenished using either TIS or NIS in Deniz et al. (2008), and they identify conditions for the cost parameters under which the supplier would indeed benefit from restricted (only old-to-new, only new-to-old, or no substitution) or unrestricted forms of substitution while using a practical replenishment policy. They show that even when substitution costs are zero, substitution can be economically inferior to no-substitution for a supplier using a TIS policy. Alternately, even when substitution costs are very high, no-substitution is not guaranteed to be superior for a supplier using TIS. These counter-intuitive properties are the side-effects of the TIS policy which constrains reordering behavior. In contrast, more intuitive results under the NIS policy exist and conditions on cost parameters establish the economic benefit of substitution for this replenishment policy.

Deniz et al. (2008) and Deniz (2007) do extensive computational experiments to quantify the benefits of substitution and to compare TIS and NIS. Deniz et al. (2008) and Deniz (2007) find that NIS – the policy that uses *no information* on the level and age of inventory – proves more effective than TIS and provides lower long-run average costs except when the demand for new items is negligible. The effectiveness of NIS in their model is in contrast with the observations in earlier research papers. This is because inventory is depleted in a FIFO manner – there is no demand for new items as long as old items are available – in the classical literature. Similar to the observation of Cohen et al. (1981b) on the limited need for transshipments (see our discussion in Sect. 15.4.2), Deniz et al. (2008) show that the amount of substitution is small when inventory is replenished using effective policies. This latter paper really only *begins* the consideration of managing age-dependent demand and effect of different inventory issuance rules (via substitution) for perishable items

– items with longer lifetimes, other issuance rules, or substitution between different perishable products within the same category (e.g., different types of fruit) remain to be investigated. Note that substitution is only modeled as a *recourse* in these papers, and dynamic substitution where one strategically sells a product of an age diggerent from that requested before the stocks of the requested item are depleted, has not been studied.

15.5.2 Multiple Products

Nahmias and Pierskalla (1976) study the optimal ordering policies for an inventory system with two products, one with a fixed, finite shelf-life and the other with an infinite lifetime. The problem is motivated by the operation of a blood bank storing frozen packed red cells.[5] Demand is satisfied from the inventory of perishable product first in a FIFO manner, any remaining demand is fulfilled from the inventory of non-perishable products. Nahmias and Pierskalla (1976) analyze the structural properties of the expected cost function in a finite-horizon, dynamic, discrete review system and show that the optimal ordering policy in each period is characterized by three choices: Do not order, order only product with the finite lifetime, or order both products. Their results include monotonicity of order-up-to level of the perishable product, e.g., the decrease in order up-to-level is higher with the increase in the stock levels of newer items as opposed to old ones. They also show that if it is optimal to order both products in a given period, then it is optimal to bring the total system-wide inventory up to a level that does not vary with the on-hand inventory levels, but with the time remaining until the end of the planning horizon.

Multiple perishable products are also considered by Deuermeyer (1979, 1980). Deuermeyer (1979) determines the one-period optimal order-up-to-levels for two products. In his model, the products are produced by two processes, one of which yields both products and the other yields only one product. Deuermeyer (1980) determines the single-period, optimal order-up-to levels for multiple perishable products, each with a different lifetime. A critical assumption in the latter is the *economic substitution* assumption where the marginal total cost of a product is assumed to be nondecreasing in the inventory levels of other products. Using the resulting properties of the single-period expected total cost function, Deuermeyer (1980) is able to obtain the monotonicity results on order-up-to-levels for the single-period, multi-product problem. His results mimic that of Fries (1975) and Nahmias (1975a) for the single product problem. Specifically, these results show that the optimal order-up-to level of a product is more sensitive to changes in stock levels of newer items (as discussed above for Nahmias (1976)), and that the optimal order quantity decreases with an increase in the on-hand stock levels, while the optimal target inventory level remains nondecreasing.

[5] We refer the reader to Prastacos (1984) for earlier, simulation-based research on the effect of freezing blood products on inventory management.

15.5.3 Pricing of Perishables

Pricing, in general, has become one of the most widely studied topics in the operations management literature in the last decade. There is a significant body of research on dynamic pricing and markdown optimization for "perishables." One well-studied research problem in that domain involves determining the optimal price path for a product that is sold over a finite horizon given an initial replenishment opportunity and various assumptions about the nature of the demand (arrival processes, price-demand relationship, whether customers expect discounts, whether customers' utility functions decrease over time etc.). We refer the reader to the book by Talluri and van Ryzin (2004) and survey papers by Elmaghraby and Keskinocak (2003), and Bitran and Caldentey (2003), for more information on pricing of perishable products. In that stream of research, all the items in stock at any point in time are of the same age because there is only one replenishment opportunity. In contrast, Konda et al. (2003), Chandrashekar et al. (2003), and Chande et al. (2004, 2005) combine pricing decisions with periodic replenishment of a perishable commodity that has a fixed lifetime, and their models include items of different ages in stock in any period. They provide MDP formulations where the state vector includes the inventory level of goods of each age. Their pricing decisions are simplified, i.e., they only decide whether to promote all the goods in stock in a period or not. Chande et al. (2004) suggest reducing the size of the state of space by aggregating information of fresher goods (as opposed to aggregation of information on older goods as in Nahmias (1977a)). Performance of this approximation is discussed via numerical examples in Chande et al. (2004, 2005), and sample look-up tables for optimal promotion decisions are presented for given inventory vectors.[6]

Based on Tsiros and Heilman (2005) observations on customers' preferences and close-substitutability of products in fresh-produce supply chains, it is important to analyze periodic promotion/pricing decisions across age-groups of products. There are several research opportunities in this area in terms of demand management via pricing to minimize outdates and shortages across age-groups of products and product categories.

15.6 Summary and Future Research Directions

We presented a review of research on inventory management of perishable and aging products, covering single-location inventory control, multi-echelon and multi-location models, logistics decisions and modeling novelties regarding demand and

[6] Another paper that considers prices of perishable products is by Adachi et al. (1999). Items of each age generate a different revenue in this model, demand is independent of the price, and the inventory is issued in a FIFO manner. The work entails obtaining a replenishment policy via computation of a profit function given a price vector.

product characteristics. We identified or re-emphasized some of the important research directions in Sect. 15.2 to 15.5, ranging from practical issues such as product-mix decisions and managing inventories of multiple, perishable products, to technical ones such as the structure of optimal replenishment policies with fixed costs in the single-product, single-location problem. We provide some final comments on possible research topics below.

Multiple products (joint replenishment and product-mix): The research within the perishable domain has largely been confined to inventory management of a *single product* as the survey in this chapter shows. However, grocery or blood supply chains involve multiple perishable products with possibly differing lifetimes. Joint replenishment is a typical practice in these industries, and analytical research that studies the interaction between multiple items in ordering decisions – focusing on economies of scale, or substitution/complementarity effects of products with different lifetimes and in different categories – has not been studied. These interactions provide opportunities for more complex control policies, which make such problems both more challenging analytically, and potentially more rewarding practically.

Considering multiple products, another problem that has not attracted much attention from the research community is determining the optimal product-mix when one type of perishable product can be used as a raw material for a second type of product, possibly with a different lifetime, as we mentioned in Sect. 15.2. Decisions regarding when and how much of a base product to sell/stock as is, vs: how much to process in order to obtain a final product with a different lifetime or different potential value/revenue are quite common in blood and fresh produce supply chains. Consideration of multiple products will lead to more realistic decision problems, for which practical and effective solutions are needed.

Capacity, freshness, disposal, and outdating: A significant majority of the research on inventory management or distribution of perishable goods disregards capacity constraints. Models with limited capacity are better representative of the challenges in practice and require innovative heuristic policies.

The practical decision of when (if at all) to *dispose of the aging inventory* has not received much attention even in single location models, possibly because capacity is assumed to be unlimited and/or demand is assumed to be satisfied with FIFO inventory issuance. However, disposal decisions are especially critical when *capacity is constrained* (e.g., a retailer has limited shelf-space to display the products), and customers choose the products based on their (perceived) *freshness/quality*. Veinott (1960), in his deterministic model, included disposal decisions for a retailer of perishable products with fixed lifetime. Martin (1986) studied optimal disposal policies for a perishable product where demand is stochastic. His queueing model considers the trade-off between retaining a unit in inventory for potential sales vs. salvaging the unit at a constant value. Vaughan (1994) models an environment where a retailer decides on the optimal parameter of a TIS policy and also a "sell-by" date that establishes an effective lifetime for the product with a random shelf life; this may be considered a joint ordering and outdating policy. Vaughan (1994) discusses that his model would be useful for retailers if they were to select suppliers based on their potential for ordering and outdating, but no analysis is provided.

When customers prefer fresher goods, disposal and outdating are key decisions that affect the age-composition (freshness) of inventory, and can influence the demand. Analysis of simple and effective disposal and outdating policies, coordination of disposal with replenishment policies, and analysis of inventory models where customers (retailers) choose among suppliers and/or consider risk of supply/freshness remain among the understudied research problems.

Inventory issuance and demand models: The majority of research on perishables assumes demand for a product is either independent of its age, or that the freshest items are preferred. These typical assumptions motivate the primary use of FIFO and LIFO issuance in inventory control models. However, one can question how realistic these issuance policies are especially in a business-to-business (B2B) setting. A service level agreement between a supplier (blood center) and its retailer (hospital) may not be as strict as "freshest items must be supplied" (motivating LIFO) or as loose as "items of any age can be supplied" (motivating FIFO), but rather "items that will not expire within a specified time-window must be supplied".[7] Faced with such a demand model, and possibly with multiple retailers, a supplier can choose his/her optimal issuance policy which need not necessarily be LIFO or FIFO. Similarly, Pierskalla (2004) notes that for a regional blood supply chain, FIFO issuance may not be the most appropriate for a supplier who distributes blood to multiple locations; if certain locations receive shipments infrequently, then it is better to use LIFO for those locations to extend the lifetime of the product. To the best of our knowledge, few researchers have shown an awareness of the heterogeneity among the customers/locations (see Sect. 15.5.1). For future research on perishables to be of more practical use, we need demand models and inventory issuance rules that are representative of the more general business rules and policies today.

Competition: While problems that involve competition (among retailers, suppliers, or supply chains) have received a lot of attention in the last decade (see, for e.g., Cachon (1998)), models that include competition involving perishable and aging products have not appeared in the literature. One distinct feature of competition in a perishable commodity supply chain is that suppliers (retailers) may compete not only on availability and/or price but also on freshness.

Contracting: In the produce industry, a close look at the relationship between suppliers and buyers reveals several practical challenges. Perosio et al. (2001) present survey results that indicate that about 9% of the produce in the USA. is sold through spot markets, and about 87.5% of product purchases are made under contracts with suppliers. Perosio et al. (2001) make the following observation which is essentially a call for further research: "*Despite a number of considerable disadvantages, in general, todays buyers and sellers alike appear to be won over by the greater price certainty that contracting makes possible...However, high*

[7] We thank Feryal Erhun from Stanford University for bringing this practical issue, which she has witnessed in blood supply chains, to our attention.

degrees of product perishability, weather uncertainty and resulting price volatility, and structural differences between and among produce buyers and sellers create significant challenges to the design of the produce contract."

Recently, Burer et al. (2006) introduced different types of contracts used in the agricultural seed industry and investigated – via single-period models – whether the supply chain can be coordinated using these contracts. In their ongoing work, Boyabatli and Kleindorfer (2006) study the implications of a proportional product model (where one unit of input is processed to produce proportional amounts of multiple agricultural outputs) on the optimal mix of long-term and short-term (spot) contracting decisions. We believe further analysis of supplier-retailer relations, and the design of contracts to improve the performance of a supply chain that involves perishable products remain fruitful research topics.

Pricing and blood supply chains: Pricing was mentioned as one of the important research directions by Prastacos (1984) to encourage collaboration between hospitals and blood banks/centers; this is also echoed in Pierskalla (2004). There seems to be almost no research in this direction to date. According to a recent survey in the U.S., the mean cost of 250 ml of fresh frozen plasma to a hospital varied from $20 to $259.77, average costs of blood components were higher in Northeastern states compared to the national average, and hospitals with higher surgical volume typically paid less than the national average for blood components in 2004 (AABB, 2005). Given the importance of health care both for the general welfare and the economy, there is a pressing need for further research to understand what causes such variability in this environment, and whether pricing can be combined with inventory management to better match demand for perishable blood components with the supply. The potential relevance of such work reaches well beyond the health care industry.

Technology: Advances in technology have increased the efficiency of conventional supply chains significantly; for perishable goods, technology can potentially have an even greater impact. Not only is there the potential to enable information flow among different parties in a supply chain, as has proved to be valuable in conventional chains, but there is also the possibility of detecting and recording the age of the products in stock (e.g., when RFID is implemented). This information can be used to affect pricing decisions, especially of products nearing their usable lifetimes. Moreover, advances in technology can potentially increase the freshness and extend the lifetime of products (e.g., when better storage facilities or packaging equipment is used). The relative magnitudes of these benefits calibrated to different product and market characteristics remains an important open problem.

The majority of the work on the analysis of inventory management policies assume that the *state* of the system is known completely; i.e., inventory levels of each age of product at each location are known. However, this may not be the case in practice. While effective heuristic policies, such as TIS and NIS, for inventory management at a single location reduce the information need and do not require a complete characterization of the state of the inventory, availability of information can be pivotal for applications with features that are not represented in typical single-location models (e.g., models that emphasize freshness of inventory and/or consider disposal). Cohen et al. (1981a) discuss the need for detailed demand and inventory information to apply shortage or outdate anticipating transshipment rules in a centralized system, and argue that the system would be better without transshipments between the retailers if accurate information is not available. Chande et al. (2005) presents an RFID architecture for managing inventories of perishable goods in a supply chain. They describe how the profile of current on-hand inventory, including the age, can be captured on a real-time basis, and conclude by stating that *"there is a need for measures and indicators ... to determine ... (a) whether such development would be beneficial, and (b) when implemented, how the performance of the system compares to the performance without auto ID enhancements."*

Final Note: With the acceleration of product life cycles, the line between "perishable" and "durable" products continues to be blurred. Strictly speaking, goods such as computers and cell phones obsolesce rather than perish, but many of the same questions we raised about "perishable" inventory currently are, and will continue to become increasingly relevant in this category as well.

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