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Survey of Literature on Continuously Deteriorating Inventory Models

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This paper presents a complete and up-to-date survey of published inventory literature for the deteriorating (decaying) inventory models. More specifically, those papers are addressed that consider the effect of deterioration as a function of the on-hand level of inventory. The basic features, extensions and generalization of various models are discussed. A classification scheme is presented along with suggestions for future research.

Key words: deteriorating inventory model, decaying models

INTRODUCTION

Inventory modelling is one of the most developed fields of operations management and much space has been devoted to this topic in the management science, operational research and practitioner-oriented journals. An interesting subset of inventory theory is the mathematical modelling of deteriorating items. The literature related to deteriorating items is scattered and no comprehensive, up-to-date discussion of these models is readily available. This paper presents a complete survey of the published literature in mathematical modelling of deteriorating inventory systems. More specifically, this survey considers those inventory models where deterioration is a function of the on-hand level of inventory.

One of the basic implicit assumptions of most inventory models has been the infinite shelf life of products while in storage. That is, a product once in stock remains unchanged and fully usable for satisfying future demand. If the rate of deterioration or decay is low and negligible its effect can be ignored; however, in many situations this effect plays a major role and its impact must be considered explicitly. Decay or deterioration is defined as any process that prevents an item from being used for its intended original use such as: (i) spoilage, as in perishable foodstuffs, fruits and vegetables; (ii) physical depletion, as in pilferage or evaporation of volatile liquids such as gasoline and alcohol; (iii) decay, as in radioactive substances, degradation, as in electronic components, or loss of potency as in photographic films and pharmaceutical drugs.

DETERIORATION CLASSIFICATION

The analysis of deteriorating inventory items involves different concepts of deterioration. First, there are situations in which all items remaining in inventory become simultaneously obsolete at the end of the planning horizon, such as style goods in fashion merchandizing, or the classic newsboy problem. Second, there are those situations in which the items deteriorate throughout their planning horizon. This category can be further divided into two classes: (1) items with a fixed shelf life such as blood, and (2) items with continuous decay (random lifetime) such as radioactive materials. Deteriorating items could also be classified with respect to their value or utility as a function of time. Constant-utility perishable goods undergo decay and face no appreciable decrease in value during their usable lifetime, for example prescription drugs. Decreasing-utility perishable goods lose value throughout their lifetime, for example fresh produce or fruits. Increasing-utility perishable goods increase in value, for example some wines or antiques that appreciate in value.

Significant research has been done to describe the optimal stocking policies for items with a fixed lifetime. In particular, Fries[1] and especially Nahmias[2] have studied this important class of models for products with arbitrary but fixed lifetimes under the assumptions of first-in-first-out (FIFO) issuing policy and fresh supply. This review does not consider this important class of

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perishable inventory models, since Nahmias[3] has already reviewed the fixed-life perishable inventory literature extensively. (In his survey he also reviewed a limited number of models that are subject to exponential decay.) The primary focus of this paper is to present an up-to-date and complete review of the literature for the continuously deteriorating mathematical inventory models. These models are also referred to as inventory models subject to exponential decay, or age-independent perishable inventory models. This paper does not cover the details of the mathematical derivations and only discusses the general nature and the primary concern of each type of model. Furthermore, the scope of coverage is limited to those papers that investigated the effect of deterioration as a function of the on-hand level of inventory.

CLASSIFICATION OF MODELS

There are a variety of inventory models that consider the effect of deterioration. Moreover, there are many different ways in which these models can be classified. Following an approach similar to that of Silver,[4] the following scheme is used to categorize the various models:

A(1, 2): Single vs multiple items

Almost all the published deteriorating inventory models are devoted to single items. Interactions of several items and the fact that they deteriorate and their impact on stocking policy are not considered implicitly or explicitly. For example, the effect of single replenishment cycle, quantity discount, storage space or budgetary constraints have not been considered. (The exception is the case where component parts that deteriorate are used in a single finished product; however, this is not the intended meaning for this subclassification.)

B(1, 2): Deterministic vs probabilistic demand

Where the lead-time demand is not known with certainty some attempts have been made to develop analytical solutions. Generally, the approach has been to use distribution free analysis for lead time demand with the exception of the work by Nahmias[5] and by Nahmias and Wang.[6,7] However, most published work assumes that the demand rate is deterministic and constant (See Addendum, Note 1).

C(1, 2): Static vs varying demand

A number of papers consider the changing nature of demand, for example, the case for power demand, where the behaviour of demand changes according to some specific function during the replenishment cycle. Situations have also been considered where demand is increasing (decreasing) during some specific time horizon, or where it is affected by price changes or where it is affected by the level of stocking.

D(1, 2): Single period vs multiple period

Multiple-period models are more appropriately addressed by the type of perishable inventory problems that are considered by Nahmias. For a single-period model, e.g. the newsboy problem, models that incorporate continuous deterioration have not been considered explicitly.

E(1, 2): Purchase vs production model

The delivery rate of products into stock is assumed to be either instantaneous or uniform during some time period. A number of papers consider this situation explicitly in the development of the mathematical models (See Addendum, Note 2).

F: Quantity discount

The purchase price of an item may depend on the size of the order (replenishment) quantity. There are no published models that consider the explicit effect of quantity discount on deteriorating inventory models.

G(1, 2): No shortage vs shortage

When an item is out of stock, demand can be back-ordered, considered to be lost or be partially back-ordered. A number of papers explicitly consider the case of back-ordering; however, there are only two papers that consider the effect of partial back-order with constant rate of deterioration.

H(1, 2): Constant vs changing deterioration rate

Decay or deterioration rate can be a constant fraction of the on-hand inventory, as is the assumption for most models, or it could be changing (increasing or decreasing) according to some function. The underlying deterioration distribution affects the nature of the deterioration rate. When the rate is not constant, the mathematical calculations become extremely difficult and closed-form solutions are generally not possible.

THE BASIC MODEL—THE EOQ MODEL WITH CONSTANT RATE OF DETERIORATION

The analysis of decaying inventory problems began with Ghare and Schrader,[8] who developed a simple economic order quantity model with constant rate of decay. This model can be classified as [A1, B1, C1, E1, H1]. They pointed out the importance of considering the effect of decay in inventory analysis by noting the potentials for cost saving measures and the improvements in inventory reordering policy. They formulated the change in inventory level, $I(t)$, as:

$$I(t)' = -\theta I(t) - d,$$

where θ is constant rate of decay and d is the constant demand rate. They derived an equation for the optimum order quantity, Q, as a function of the inventory cycle time, T, that is,

$$Q^* = (d/\theta)\{\exp(\theta T) - 1\}.$$

By approximating the exponential function by the first three terms of the Taylor series, and using the standard economic order quantity analysis, they obtained the following implicit equation for determining the inventory cycle time:

$$(C_1 d/2) + (C_1 d T\theta/2) + (C_4 d\theta/2) + (C_3/T) = 0,$$

where C_1, C_3, and C_4 are inventory carrying cost, ordering cost and deteriorating cost. The optimal inventory cycle, T^*, can then be obtained by solving the above equation. Subsequent authors expanded on this model and have developed numerous models that consider the effect of deterioration.

Given the multitude of models and approaches to this class of inventory problems, it is difficult to place all the published deteriorating inventory models in one systematic classification. However, the scheme presented here describes the evolutionary enhancements of various models and provides a focused view that should aid future work by allowing researchers to embellish the existing models.

THE BASIC MODEL WITH VARYING RATE OF DETERIORATION

This class of models extends the basic model by considering that the deterioration rate is non-constant. These models can be classified as [A1, B1, C1, E1, H2]. Covert and Philip[9] extended Ghare and Schrader's model and obtained an economic order quantity model for a variable rate of deterioration by assuming a two-parameter Weibull distribution. (A constant rate of decay implies that the underlying deteriorating distribution is an exponential distribution which is a special case of the Weibull distribution.) The differential equation that describes this model is given by

$$I(t)' = -g(t)I(t) - d,$$

where

$$g(t) = \alpha\beta t^{(\beta - 1)}.$$

($g(t)$ is the Weibull distribution's instantaneous deterioration rate function with α and β the scale and shape parameters of the distribution). ($g(t)$ is analogous to the hazard function in the reliability theory terminology.) The inventory lot size for this model is given by

$$Q = \int_0^T [d \exp(\alpha t^\beta)]\, dt = d \sum_{n=0}^{\infty} \alpha^n T^{((n\beta+1))}/[n!(n\beta + 1)].$$

In this model, it was further assumed that the average total inventory may be approximated by $Q/2$, and thereby the following equation was obtained:

$$[C_1 d \exp(\alpha T^\beta)/2] + (C_4 d)\left[\sum_{n=0}^{\infty} \frac{\alpha^n n\beta T^{(n\beta-1)}}{(n\beta+1)n!}\right] + (C_3/T) = 0,$$

which is an implicit function of T and which must be solved numerically to obtain T^*. Philip[10] extended this model by assuming a three-parameter Weibull distribution which takes into account the impact of the already deteriorated items that are received into an inventory system as well as those items that may start deteriorating in the future. In his model $g(t)$ was defined as follows:

$$g(t) = \alpha\beta(t - \gamma)^{(\beta-1)},$$

where γ is the location parameter of the Weibull distribution.

Tadikamalla[11] considered the same model with no backlogging and assumed a gamma distribution for the deterioration time of items in inventory. In comparing Weibull and gamma distributions for variable rates of deterioration, he observed that even where these two distributions have 'similar' shapes their instantaneous deterioration rate functions are significantly different. This implies that it is essential that the underlying deteriorating distribution be identified for a more accurate assessment of the cost implications. Shah[12] developed a similar model (which also included backlogging) by considering an arbitrary, well-behaved deterioration function and derived expressions for Q and T. In all the above models it was assumed that the average level of inventory can be estimated linearly (i.e. that $I(t)$ is a linear function). Aggarwal[13] corrected this assumption for Shah's model and obtained the exact expression for the average carrying inventory. As a specific case, he explicitly considered an inventory model with a constant rate of deterioration. Goel[14] also examined a general variable rate of deterioration for an inventory model with shortages and partial lost sales and derived expressions for Q and T. Additionally, Raafat[15] obtained the appropriate equations for the Weibull and Raleigh distributions for this model with no backlogging. Furthermore, Raafat[16] considered the case where deterioration rate function was a monotonically increasing function of the form

$$g(t) = [a/(b - t)], \qquad a, b > 0,$$

and obtained a closed form solution for the exact average total cost equation.

THE FINITE PRODUCTION RATE MODELS WITH DETERIORATION

These models can be classified as [A1, B1, C1, E2, G(1, 2), H(1, 2)]. Misra[17] developed the first production lot size model in which both a constant and variable rate of deterioration were considered and obtained approximate expressions for the production lot size with no backlogging. For the case of Weibull distribution deterioration, no closed expression for the lot size and the average total cost is possible. However, for the case of exponential distribution (i.e. constant deterioration rate), through a series of approximations, he calculated the optimal production lot size to be

$$Q = \{1 + [(C_4 d\theta)/(C_1 p)]\}^{-0.5} Q_E,$$

where $Q_E = \{(2C_3 dp)/[C_1(p - d)]\}^{0.5}$, Q_E is the production lot size for items without decay and p is the constant production rate. Shah and Jaiswal[18] derive results similar to those of Misra for a constant deterioration rate and extend his model to include backlogging. By assuming the average carrying inventory to be approximately one-half the maximum level of inventory, they obtained the following expression for the production lot size as a function of inventory cycle time:

$$Q = (p/\theta) \ln\{1 + (d/p)[\exp(\theta T) - 1]\}.$$

However, the optimal value of T in the above expression has to be determined through a numerical analysis of a rather complicated equation which when solved would then allow for calculating Q^*. Hwang[19] considered Misra's model and developed an inventory model with Weibull distribution deterioration and LIFO issuing policy, while Mak[20] obtained approximate expressions for the optimum production lot size, the production cycle time and the inventory cycle time for the same model with backlogging. Deb and Chaudhuri[21] consider this same model with the Raleigh deterioration distribution, which is a special case of Weibull distribution. Their approach is the same as in Raafat's[15] for the non-production (purchase) model. Hark and Hwan[22] and Raafat[23] also present an alternative methodology for obtaining the optimum characteristics of the production-lot size inventory model with constant rate of deterioration. They obtained the exact average total cost expression and then by utilizing a computerized search routine obtained the optimum production lot size and the inventory cycle time. Elsayed and Teresi[24] also considered the same inventory model and obtained similar results.

Chowdhury and Chaudhuri[25] considered an order-level inventory model with finite production rate and constant rate of deterioration, with shortages allowed, and obtained expressions for both the deterministic and the probabilistic demand. Later on, Sachan[26] extended this model for the deterministic case to include the effect of partial loss of demand rather than total backlog. His model was solved for maximizing the average total profit rather than the usual minimization of the average total cost.

Park[27] developed a production inventory model for decaying raw materials which was based on Goyal's[28] integrated inventory model for a single product system. This model considers the inventory problem of decaying raw materials and a single finished product. It was assumed the finished product does not decay and that it is produced in batches and that only the raw materials decay at a constant rate; the effect of in-process deterioration was not considered. Raw materials are ordered from outside suppliers and the arrival of fresh supplies coincides with the start of the production run. Equations for obtaining the average annual operating cost and the optimum inventory cycle time were then derived. Raafat[29] extended this model to allow the final finished product also to deteriorate at a constant rate. Furthermore, Raafat[30] examined the above model for a situation where the deterioration was assumed to be a monotonically-increasing function of time rather than being constant. Gunasekaran[31] and Gunasekaran *et al.*[32] also expanded the Park model by considering the effect of decaying in-process inventories and derived expressions for the order quantity, Q, and the total annual variable cost. In the latter paper the authors used a computer search technique to obtain the optimal value of the order quantity.

PROBABILISTIC MODELS WITH DETERIORATION

These models can be classified as [A1, B(1, 2), E1, G(1, 2), H(1, 2)]. Shah and Jaiswal[33] developed an order-level inventory model by assuming instantaneous delivery and constant rate of deterioration. In this model the scheduling period is a prescribed constant and lead time is zero. First, they developed a model with a constant demand rate, and then they extended it to include a stochastic demand. They further assumed that the function describing the average carrying inventory can be approximated as a linear function and proceeded to derive a complex cost expression as a function of order level, S. Aggarwal[34] dropped the linearity assumption and obtained the exact expressions for the inventory characteristics of this model.

Shah and Jaiswal[35] also examined a periodic review inventory model for deteriorating items with no shortages and the review period a prescribed constant, i.e. (s_p, Q) policy. This was an extension of one of Naddor's[36] many models in which they derived the equations for an arbitrary deterioration function. They then considered Weibull and exponential deterioration distributions as specific cases. As in their previous model, they assumed that the average carrying inventory function is linear and then proceeded to derive fairly complex expressions to describe the expected total cost equation of the inventory system. Furthermore, Jani *et al.*[37] developed a similar model for a probabilistic reorder point inventory model with an (s, Q_p) policy and constant rate of deterioration. In this model, the lot size and review period were assumed to be constant and the reorder point was the decision variable. This model, as in Shah,[12] has the same assumptions and shortcomings with respect to inventory carrying cost function and the final cost structure. In a later work, Shah and Jaiswal[38] also considered a probabilistic scheduling inventory model with an arbitrary deterioration rate function with no shortages and zero lead time, and derived the characteristic expressions for Q and T. Similarly, Goel[14] considered an arbitrary deterioration function for an inventory model with backorders and partial lost sales. Both deterministic and probabilistic models were considered with constant deterioration rate and 100% backlogging as particular cases of the more general models.

Aggarwal and Goel[39] developed an order level (S, Q) inventory model with a power demand pattern with constant rate of deterioration and they analysed deterministic as well as probabilistic cases of demand both without and with shortages. This paper, essentially, showed the mathematical extension of Naddor's[36] non-deteriorating model with no discussions or analysis about the implication of the results of the model. Elsayed and Teresi[24] presented an additional model in their paper which assumed, as in Naddor,[36] that demand is normally distributed and that deterioration is given by a two-parameter Weibull distribution. By using numerical iterations they obtained optimal values for Q and T.

DISCRETE-IN-TIME INVENTORY MODELS WITH DETERIORATION

These models can be classified as [A1, B(1, 2), C(1, 2), E1, G(1, 2), H1]. Dave[40] presented an interesting paper in which he considered time to be in discrete units. The model was developed under the assumptions of deterministic demand, no shortages, and constant rate of deterioration. The order quantity as a function of inventory cycle time was determined to be:

$$Q = (d/\theta)\{(1 - \theta)^{-T} - 1\}.$$

Along the same lines, Dave[41] developed an order level inventory model with constant demand rate and constant deterioration. The solution technique used for these models was based on the use of the calculus of finite differences rather than the ordinary differential equations, and in this respect, it was an interesting and novel approach for solving this class of problem. Rengarajan and Vartak[42] generalized Dave's[41] model by assuming power demand, i.e. demand varying according to some function of time during the inventory cycle, and were able to show that the optimal value of the order quantity, Q, is not dependent on the nature of the demand pattern.

Dave and Jaiswal[43] presented a probabilistic scheduling, discrete-in-time inventory model with constant rate of deterioration and no shortages. This was a generalization of Dave's[40] model and they provided sensitivity analysis of potential cost savings in comparison with the case which has no deterioration. Furthermore, Dave[44] developed an inventory model for deteriorating items that operate for exactly m-scheduling periods ($m > 2$), under the assumption of probabilistic demand, constant decay rate and no returns in the first (m-1) scheduling periods. Dave and Shah[45] also presented a discrete-in-time probabilistic model with constant deterioration, no shortages, and lead time equal to one scheduling period. This is the only model that considers the effect of lead-time on the inventory policy for deteriorating items for this class of models.

PRICING AND FINANCIAL INVENTORY MODELS WITH DETERIORATION

These models can be classified as [A1, B1, C1, E(1, 2), G(1, 2), H1]. A number of papers have incorporated deteriorating inventory models into their own specific field of interests. For example, Cohen[46] considered the problem of simultaneously setting selling price and order quantity for an exponentially decaying product under known demand, $d(x)$, which is a function of unit selling price, x. The optimal inventory cycle time as a function of unit selling price was approximated to be

$$T_x = \{(2C_3)/[d(x)(C_4\theta + C_1)]\}^{0.5}$$

The parametric changes to the ordering–pricing decision indicated a non-monotonic response for the optimal price to changes in product deterioration. Goel and Aggarwal[47] extended this model by developing expressions for the optimal pricing and ordering policy for a three-parameter Weibull deterioration distribution for cases of both no-shortages and backlogging. This is a mixture of Philip's and Cohen's model. Moreover, Kang and Kim[48] extended this model by considering a finite production rate (i.e. their model is a combination of Misra's and Cohen's model with constant rate of deterioration and no shortages). The maximum profit–price decisions were computed with changes in product deterioration. The result of their analysis indicates that the trade-off between revenue and loss due to deterioration leads to unexpected patterns of pricing and production decisions.

Thompson[49] considered inventory models with deterioration and obsolescence as part of a larger presentation of inventory management and capital budgeting. Under some limiting conditions, he provided the expression for the discounted present value of profits (this can be considered as an inventory model with decreasing utility function!). Gurnani[50,51] developed a number of models for present worth analysis of deterministic periodic cashflows among which are three specific cases that consider the effect of constant decay for cashflows. These models may be considered deteriorating only in the sense that the 'value' of items (i.e. money) in inventory is decreasing with time.

Hwang and Sohn[52] examined the effect of inflation and deterioration on inventory and concluded that inflation prolongs the inventory cycle time while deterioration shortens it. Sensitivity analysis of their model indicated that there are no significant cost increases when existence of decay is ignored unless the inflation rate is high.

DETERIORATING INVENTORY MODELS AND OPTIMAL CONTROL THEORY

These models can be classified as [A1, B1, C1, E(1, 2), G1, H(1, 2)]. Bensoussan *et al.*[53] applied optimal control theory to obtain optimum replenishment policy under the assumption of quadratic cost functions for inventory models with constant and variable rate of deterioration. Optimization of the model then yields linear replenishment policies. Choi and Hwang,[54] using the same approach, developed a production inventory model with quadratic production and inventory costs and constant decay rate.

DETERIORATING INVENTORY MODELS WITH VARYING DEMAND RATE

These models can be classified as [A1, B(1, 2), C2, E(1, 2), G(1, 2), H1]. Aggarwal and Goel[39,55] developed an order level (S, Q) inventory model with a power demand pattern with a constant rate of deterioration. They considered deterministic as well as probabilistic cases of demand without and with shortages. No general discussion of the model was provided and the papers showed only the mathematical derivations of Naddor's[36] non-deteriorating models. Rengarajan and Vartak[42] extended Dave's[41] discrete-in-time model to include the power demand pattern and they showed that the optimality conditions for the optimal order quantity, Q, is not dependent on the nature of demand pattern.

Dave and Patel[56] developed an inventory model in which the demand rate is changing linearly with time (time proportional) with no backlogging allowed. They assumed the deterioration rate to be constant with finite planning horizon and equal replenishment cycles. They obtained expressions for determining the optimum number of replenishments. Sachan[57] corrected some of the approximation errors of this work and extended it to allow for backlogging. Bahari-Kashani[58] considered the same model with no backlogging and relaxed the requirement of equal replenishment cycles. He argued that since demand rate increases over time, both order quantity and order frequency need to increase to minimize costs. He obtained a solution through an heuristic, which though not optimal provides for a lower average total cost by allowing the order size and order cycles to vary. He was able to obtain savings of over 3% in comparison to Dave and Patel's model.

Padmanabhan and Vrat[59] examined a case where the demand rate is a function of inventory level and used the term 'stock dependent consumption rate' to describe this type of model. For a simple EOQ model with constant deterioration rate and no backlogging they assumed a demand rate that is given by the following equation:

$$d = \alpha + \beta Q^{\gamma},$$

where α, β and γ are positive constants. Then by substituting into the equations from Ghare and Schrader they derived an equation for the order quantity and then performed sensitivity analysis on the model. Mandal and Phaujdar[60] also looked at a stock-dependent model where demand is a linear function of stocking level (i.e. $\gamma = 1$). By allowing for a constant production rate and backlogging, they developed equations for Q for models with constant and linear deterioration rate functions. Since the resultant cost equations are quite complex, they use a numerical method to provide a sensitivity analysis for α and β of an example problem.

Hollier and Mak[61] developed replenishment policies for deteriorating items in a declining market. Specifically, they derived expressions for the optimal number of replenishment cycles, the cycle time and the size of each replenishment for two inventory models with constant deterioration rate and negative exponentially declining demand rate. While Hollier and Mak were concerned with a finite-horizon model, Cheng[62] developed the closed-form solution for the infinite-horizon case of the same problem by using a dynamic programming approach. In these two models no shortages were allowed.

Dave and Pandya[63] extended Naddor's[64] inventory returns and special sales model to include the constant rate of deterioration. Two cases of infinite and finite-horizon planning period were considered in which the deterioration was assumed to be a constant fraction of the on-hand inventory. Later, Dave[65] extended this work to allow for backlogging. These models are appropriate for those who suddenly realize that they have excess inventory which is subject to deterioration. This may happen as a result of an unexpected decline in the demand for the stock item.

MISCELLANEOUS INVENTORY MODELS WITH DETERIORATION

Emmons[66] investigated the production and inventory of radioactive nuclide generators which are used for diagnosis and treatment of patients, which due to their nature are subject to continuous decay. He developed models that are uniquely interesting in describing the relationship between the parent and daughter nuclide generators. Specifically, he considered the problem of exponential decay in which one product decayed at one rate into a new product that decayed at a faster rate. The model was based upon a simple deterministic (s, S) inventory model. He assumed inventory holding cost is negligible with respect to replenishment costs and that the inventory is replenished by S or $(S\text{-}s)$ units whenever the on-hand inventory falls below s.

Nahmias and Wang[6] calculated the expected number of shortages during the lead time for an exponentially decaying product under the assumption that the lead time demand is normally distributed. Nahmias and Wang[7] also developed a heuristic lot size reorder point model (Q, r) for exponentially decaying inventories. This inventory model is a continuous review model with random demand and positive lead time. They developed approximate expressions that under simulated conditions gave a worst-case error rate of 2.77% of the optimum cost value. This model was also discussed and presented in Nahmias.[5]

Kumaraswamy and Subramanian[67] developed a continuous-review inventory model with Poisson demand in which the lifetime distribution of items in inventory had a negative exponential distribution (i.e. the failure rate was constant). The transient and steady-state probability distributions of the stock levels were derived and the optimal decision rules for the long run were calculated. Menipaz[68] and Pandit and Rao[69] also presented models with similar assumptions and derived the steady-state expressions for stocking level and demand.

Sarma[70] developed a model for a single deteriorating item which is stored in two different warehouses and obtained the expression for the optimal beginning stock for the period. In this order-level inventory model with constant deterioration rate and fixed scheduling time, the second warehouse charges a higher holding cost but with a better preserving facility, resulting in a lower rate of deterioration than the first warehouse.

Although the inventory models in this section are structurally different in terms of approach and assumptions from those presented in this paper, nevertheless, because they meet some of the basic requirements of this survey, they are stated for the sake of completeness.

CONCLUSION

In this paper we have provided a complete and up-to-date review of published articles for mathematical inventory models of deteriorating items. The primary goal was to bring together all the work that has been done in this area. In presenting this review, papers were classified into a number of categories. Of course, many of the papers could clearly be stated and combined under a different heading. It is difficult to unite so many works with varied contents and emphasis under a homogeneous category. Nevertheless, it is believed that the list of references in itself can be a valuable tool for further model development in this area.

Future models could be developed by incorporating the effect of constant decay into the variety of existing inventory models. More complicated deterioration functions do not seem to be useful or practical; except in the paper by Tadikamalla[11] they do not seem to have provided any additional insights. Generally, almost all of the reviewed papers provide complicated equations for determining the optimal values for order quantity, Q, inventory cycle time, T, and replenishment policy. Except for the basic model, no simple decision rules are given. Simpler heuristics or approximations are needed in this regard. Furthermore, additional deteriorating inventory models need to be developed to consider the effects of quantity discount and multiple-item stocking environments.

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ADDENDUM

Material below came to the attention of the author at the time of printing of the final draft of this paper.

Note 1. Pal[1] presents a model in which the lead time is exponentially distributed with a constant rate of deterioration.

Note 2. Shiue[2] presents a restrictive inventory model that considers the effect of quality discount.

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