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A lost sales (r, Q) inventory control model for perishables with fixed lifetime and lead time

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ABSTRACT

We consider a perishable inventory system that operates under stochastic demand, constant lifetime and a constant lead time. The system employs a continuous review (r, Q) inventory control policy where unfilled demands are lost. We investigate the properties of the cost function and present an approximation procedure to find the parameters r and Q that minimize the total cost. We then conduct a numerical analysis to examine the performance of the proposed model and study the sensitivity to changes in the system parameters. We demonstrate the suitability of the proposed approximations compared to optimal (r, Q) parameters obtained by simulation and show that our proposal outperforms another approximation procedure from the literature, in particular for increasing ordering cost and demand variability. The proposed model contributes to the literature by providing a simple and efficient algorithm to compute the best (r, Q) parameters that minimize the total cost. Besides, it can be used in automated store ordering systems.

1. Introduction

Economic sectors such as pharmaceutics, medical goods and consumer goods industries are concerned with the management of perishable products' inventory. In fact, drugs and food, for example, are produced to be consumed within a limited period of time and consequently the impact of perishability on inventory management cannot be disregarded. To illustrate such impact, Roberti (2005) noted that roughly 10% of all perishable goods go to waste before consumers purchase them. In the health care sector, 10.9% of blood platelets processed in the United States outdated without being transfused in 2006 (Fontaine et al., 2009). However, despite the growing literature on perishable inventory control, the fixed life perishability problem remains a complex problem when the product lifetime is longer than two units of time in a periodic review scenario (Nahmias, 1982). Indeed, determining the optimal ordering policy for perishables with deterministic lifetime requires a recursive solution of multi-dimensional dynamic programs. The multi-dimensionality is caused by the need to track the different age categories in stock. Besides, the computation of an optimal policy turns out to be impractical for a real-life situation. The investigations developed so far underline the complexity caused by tracking the different items in inventory by age. We refer to the following literature reviews that give further details pertaining to this difficulty: Goyal and Giri (2001), Karaesmen et al. (2011) and Nahmias (1982).

In this paper, we revisit the fixed life perishability problem and study continuous review (r, Q) inventory systems where products have deterministic lifetimes and excess demands are lost. We derive bounds on the expected number of perished units and expected lost sales and use these bounds to obtain approximations for the expected on-hand inventory level and the expected total cost. Based on the properties of the total cost function, we propose an algorithm to compute the parameters r and Q that minimize the approximated total cost. We show that, compared to similar existing studies, the model we propose performs very well.

Academic literature of inventory control for perishables with deterministic lifetime can be categorized into various classes depending on (i) whether the inventory is reviewed periodically or continuously, (ii) whether replenishment orders arrive instantaneously or after a positive lead time, (iii) the cost components considered, e.g., ordering, inventory holding, outdating and shortage costs. Under periodic review schemes, several heuristics dealing with deterministic lifetime were proposed to avoid the

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dimensionality of the dynamic problem. For example, Haijema et al. (2007, 2009) used a combination of dynamic programming and simulation to reduce the state space and provided "near optimal" order-up-to-level inventory policies. Haijema (2013) proposed an (r,S) policy for blood platelets where the order quantity is bounded by a minimum and a maximum. Haijema and Minner (2014) compared a variety of hybrid periodic review base stock and constant order policies using simulation based optimization. They proposed two modified based stock policies that achieve a gain of roughly 6% over the base stock policy. Minner and Transchel (2010) investigated a lost sales periodic-review inventory control system with positive lead time and negligible ordering cost. Their model operates both under the FIFO and LIFO issuing policies, i.e. the customers are first served either from the oldest or the youngest items in the inventory. Broekmeulen and van Donselaar (2009) and van Donselaar and Broekmeulen (2012) focused on the periodic review (r, nQ) policy and proposed an approximation for the outdating quantity by combining stochastic modeling, simulation and regression. In Kouki et al. (2014), a periodic review order up to level (T, S) policy is proposed where excess demand is either backordered or lost. Under the assumption of Poisson demand, exponential lifetime and constant lead, the inventory process of this model has Markov properties. Based on this setting, the authors derived the cost components and showed that the consideration of the lifetime variability leads to a significant improvement of the total optimal cost. A detailed literature review for periodic review inventory systems with perishable items is given in Karaesmen et al. (2011) and Nahmias (2011).

Existing studies for inventory control of perishables with continuous review are by Weiss (1980), who investigated the (r, S) policy under zero lead time and Poisson demand. Liu and Lian (1999) extended Weiss's model by considering a general renewal demand process. If the lifetimes and lead time are exponentially distributed, deriving the optimal control parameters becomes somewhat simple because of the application of the Markov renewal theory technique. Several papers appeared under such an assumption (see Kalpakam and Sapna, 1994; Liu and Yang, 1999; Karaesmen et al., 2011).

If a constant and deterministic lead time is introduced, finding an optimal or near optimal policy is analytically complex (both for a random and deterministic lifetime) because of the intractability of the different age categories of items in stock. In fact, there is a limited number of works dealing with continuous review perishable inventory systems with deterministic lead time (Karaesmen et al., 2011). In Table 1, we provide a comparison between the key papers dealing with perishable inventory systems. The initial work by Schmidt and Nahmias (1985) proposed a lost sales $(S-1, S)$ system with Poisson demand and fixed lifetime. Without considering ordering cost, the authors show some properties of the cost function regarding the variation of outdating and shortage cost. This model was extended by Olsson and Tydesj (2010) for backorders. Lian and Liu (2001) considered a general (r, S) perishable inventory model with batch demands. Assuming that the lead time is zero, they constructed a Markov renewal model and proposed a heuristic approach for dealing with the case of constant lead time. Under full backorders, Chiu (1995a) proposed an approximate (r, Q) policy. A similar model has also been studied by Kouki et al. (2013). Under constant lead time, the authors investigated the impact of perishability on the total cost by comparing three (r, Q) models for which they assume infinite, constant and variable lifetime provided by devices called Time Temperature technology (TTIs). Depending on the cost of this technology, the authors showed that considerable gain can be achieved in comparison with (r, Q) models without TTIs. Inventory models with lost sales are discussed in Bijvank and Vis (2011). Particularly for perishable items with deterministic lifetime, Tekin et al. (2001) introduced a (T, r, Q) policy in which a replenishment order of size Q is placed either when the inventory level drops to r, or when T units of time have elapsed since the last instance at which the inventory level hit Q, whichever occurs first. An exact analytical solution of the (T, r, Q) policy is obtained for Poisson demand (slow moving perishable items) and under the assumption that the aging of the batch Q begins after all units of the older batch have been depleted either through demand or by perishing. However, it is too complex to characterize the optimal parameters T, r and Q for general stochastic demand or for perishability starting when orders arrive to the stock.

In the particular case of Poisson demand, Berk and Grler (2008) analyzed the optimal total cost of the (r, Q) policy. They observed that the distribution of the remaining shelf life at epochs when the inventory level is equal to Q has Markovian properties and showed that the remaining shelf life constitutes an embedded Markov process. However, their analysis is only exact when at most one order is outstanding (i.e. $r < Q$). By analyzing this process, they derived a closed form expression for the total cost under the (r, Q) inventory policy with lost sales and positive lead time. Their analysis gives the exact cost expression of the (r, Q) policy under Poisson demand and does not apply for any other types of demand distribution, since then the Markov properties do not hold anymore. Berk and Grler (2008) compared their model to Chiu (1995a) with lost sales and Poisson demand and showed that Chiu's model performs worse if the ordering cost increases. The authors find that Chiu's model deviates from the benchmark (T, r, Q) policy by a maximum of 18% while the optimal (r, Q) policy exhibits a maximum difference of 3.5%. Our paper differs from existing works primarily by considering an (r, Q) inventory system with continuous demand distribution, constant lifetime and constant lead time. Exact analysis of such a system is provided in Berk and Grler (2008) under the assumption of Poisson demand. For other demand distributions, a heuristic solution is proposed in Chiu (1995a). In this paper, we improve Chiu's heuristic by deriving an approximate (r, Q) inventory policy under continuous demand and investigate the sensitivity of the cost function to changes in the system parameters. A similar (r, Q) inventory system has also been studied in Kouki et al. (2013). Their analysis focuses on the backorder case, while we assume that excess demand is lost. Additionally, they implicitly assumed that the inter-arrival time of the demand is discrete and the demand sizes follow a general distribution. In this paper, however, we treated demand as a continuous variable. Finally, our approach to finding the best inventory system parameters is based on determining upper bounds, rather than average values of the expected outdating quantity and expected lost sales.

In the remainder of this paper we formulate our model in Section 2. In Section 3, we investigate some analytical properties of the cost function and provide an approximation procedure for computing the best (r, Q) parameters. We conduct numerical experiments in Section 4 in order to first validate the proposed model, then perform a sensitivity analysis of the cost function, and compare the model we proposed with Chiu's model. In Section 5, we conclude the paper.

2. Model description

We study a single-stage perishable product inventory system. Products have a fixed lifetime of m units of time, which means that they are held in stock during a maximum of m units of time, after which, if they are not consumed, they are disposed of as shown in Fig. 1. The inventory is controlled by an (r, Q) continuous review system: an order of size $Q > 0$ is placed whenever the inventory

Table 1
Main contribution in literature of perishable inventory systems.

Article	Type of review	Replenishment policy	Lifetime distribution	Demand distribution	Excess demand	Lead time distribution	Planning horizon
Fries (1975)	Periodic	$(S-1,S)$	Constant	Continuous	Lost sales	0	Finite/ infinite
Nandakumar and Morton (1993)	Periodic	$(S-1,S)$	Constant	Continuous	Lost sales	0	Infinite
Williams and Patuwo (1999, 2004)	Periodic	$(S-1,S)$	Constant	Continuous	Lost sales	Deterministic	Finite
Nahmias (1975), Nahmias (1977)	Periodic	$(S-1,S)$	Constant	Continuous	Backorder	0	Finite
Chiu (1995b)	Periodic	(T,S)	Constant	General	Backorder	Deterministic	Infinite
Kouki et al. (2014)	Periodic	(T,S)	Exponential	Poisson	Lost sales	Deterministic	Infinite
Kouki and Jouini (2015)	Periodic	(T,r,Q)	Erlang	Poisson	Lost sales	Deterministic	Infinite
Broekmeulen and van Donselaar (2009), van Donselaar and Broekmeulen (2012)	Periodic	(r,nQ)	Constant	Gamma	Lost sales	Deterministic	Infinite
Nahmias (1978)	Periodic	(r,S)	Constant	Continuous	Backorder	0	Finite
Olsson and Tydesj (2010)	Continuous	$(S-1,S)$	Constant	Poisson	Lost sales	Deterministic	Infinite
Schmidt and Nahmias (1985)	Continuous	$(S-1,S)$	Constant	Poisson	Backorder	Deterministic	Infinite
Kalpakam and Sapna (1995), Kalpakam and Shanthi (2001)	Continuous	$(S-1,S)$	Exponential	Poisson	Lost sales	General	Infinite
Liu and Cheung (1997), Kalpakam and Shanthi (2000)	Continuous	$(S-1,S)$	Exponential	Poisson	Lost sales/ Backorder	Exponential	Infinite
Weiss (1980)	Continuous	(r,S)	Constant	Poisson	Backorder/ Lost sales	0	Infinite
Lian and Liu (1999)	Continuous	(r,S)	Constant	Batch Geometric	Backorder	0	Infinite
Liu and Lian (1999)	Continuous	(r,S)	Constant	Renewal	Backorder	0	Infinite
Lian and Liu (2001)	Continuous	(r,S)	Constant	Renewal	Backorder	Deterministic	Infinite
Kalpakam and Sapna (1994)	Continuous	(r,S)	Exponential	Poisson	Lost sales	Exponential	Infinite
Liu and Yang (1999)	Continuous	(r,S)	Exponential	Poisson	Backorder	Exponential	Infinite
Kalpakam and Shanthi (2006)	Continuous	(r,S)	Exponential	Renewal	Lost sales	Exponential	Infinite
Liu and Shi (1999), Lian et al. (2009)	Continuous	(r,S)	Exponential	Renewal	Backorder	0	Infinite
Grler and Ozkaya (2008)	Continuous	(r,S)	General	Renewal	Backorder	0	Infinite
Chiu (1995a)	Continuous	(r,Q)	Constant	General	Backorder/ Lost sales	Deterministic	Infinite
Kouki et al. (2013)	Continuous	(r,Q)	Constant	General	Backorder	Deterministic	Infinite
Berk and Grler (2008)	Continuous	(r,Q)	Constant	Poisson	Lost sales	Deterministic	Infinite
Tekin et al. (2001)	Continuous	(T,r,Q)	Constant	Poisson	Lost sales	Deterministic	Infinite
Proposed model	Continuous	(r,Q)	Constant	Continuous	Lost sales	Deterministic	Infinite

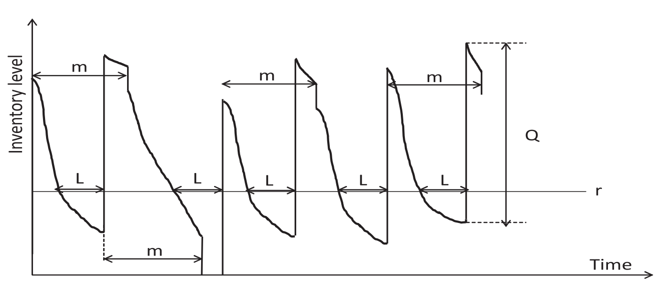

Fig. 1. An (r,Q) inventory policy for perishable products.

position (on hand inventory plus on order) drops to the reorder point r (the demand process is of unit size). Without loss of generality, we assume that the aging of products begins just at the time when replenishment orders are delivered (i.e. fresh units arrive in stock) and there is no decrease in the value of products during their usable lifetime. If a unit is not used by demand during its lifetime, it is discarded and a unit outdate cost is charged. The replenishment lead time L is assumed to be constant and known. The inventory is depleted according to a FIFO issuing policy, i.e. all orders are satisfied from the oldest units first, and all unmet demands are lost. Throughout this paper, we use the notations presented in Table 2.

The long run expected total cost per unit of time can be formulated as follows:

$$TC(r,Q) = \frac{\text{Expected cycle cost}}{\text{Expected cycle length}} = \frac{K + cQ + pE[S] + wE[O]}{E[T]} + hE[I]. \quad (1)$$

Next, we derive the required expressions for $E[O]$, $E[S]$, $E[T]$, and $E[I]$.

2.1. Expected outdating quantity

In this section, we derive an upper bound for the expected outdating quantity. To do so, we compute the upper bound once the order Q is delivered. Suppose that no item perishes during the lead time. This assumption follows common practice in perishable inventory models such as the one discussed by Chiu (1995a,b), who ignores the age distribution of the stored items. They only consider the information of the total inventory level at a decision epoch to derive approximate policies. Lian and Liu (2001) assume a zero lead time to analytically derive the policy parameters. They also suggested a heuristic to deal with positive lead time. Further references for similar treatments can be found in Liu and Lian (1999), Grler and Ozkaya (2008), van Donselaar and Broekmeulen (2012) and Haijema (2013). Based on this assumption, the batch Q finds $(r-x_L)^+$ items at the time of order arrival before it starts to be depleted by demand. Thus, if no item perishes during the lead time, the outdating quantity of the batch Q is

$$O = \left(Q + (r-x_L)^+ - x_m\right)^+. \quad (2)$$

Note that if $r<Q$, the inventory level is equal to the inventory position when an order is placed. However, if $r \geq Q$, we may have more than one order outstanding at instances where orders are placed. $(r-x_L)^+$ is an approximation of the number of items found by the batch Q at instances of order arrivals when $r \geq Q$. It is an approximation because if there is any order outstanding during the lead time, it can be either used or not by the demand during

Table 2
Notations.

x	Demand per unit of time: nonnegative continuous random variable
$f(x)$, $F(x)$	pdf and cdf of x, $f(x)>0$ for $x>0$ and $f(x)=0$ for $x\le 0$
μ, σ	Mean demand rate, standard deviation of demand
x_n, $n\ge 1$	Demand during n units of time: nonnegative random variable
f_n, F_n $n\ge 1$	pdf and cdf of the random variable x_n
K	Fixed ordering cost per order
h	Holding cost per unit of product held in stock per unit of time
c	Purchase cost per unit of product
p	Lost sales cost per unit of product
w	Outdate cost per unit of product that perishes
m	Product lifetime
L	Replenishment lead time
O	Outdating quantity associated with an order (random variable)
S	Lost sales quantity per cycle (random variable)
$E[I]$	The expected inventory level per unit of time
$E[T]$	The expected cycle length, i.e. the expected units of time that elapse between two successive instances where the inventory level reaches r
r^*, Q^*	The best reorder level and order quantity for the proposed model respectively
TC^*	Total optimal simulation cost

the lead time. Eq. (2) can be written as

$$O=\begin{cases}(r+Q-x_L-x_m)^+ & \text{if } x_L\le r;\\ (Q-x_m)^+ & \text{if } x_L>r.\end{cases} \tag{3}$$

Then, the expected outdating quantity per cycle is

$$\begin{aligned}E[O]&=E[(Q+(r-x_L)^+-x_m)^+]\\ &=\int_0^r\int_0^{r+Q-x_L}(r+Q-x_L-x_m)\,dF_m(x_m)\,dF_L(x_L)\\ &+\int_r^\infty\int_0^Q(Q-x_m)\,dF_m(x_m)\,dF_L(x_L).\end{aligned} \tag{4}$$

After integrating by parts twice, this expression is simplified to (see Appendix A)

$$E[O]=\int_0^Q F_m(x_m)\,dx_m+\int_0^r F_m(r+Q-x_L)F_L(x_L)\,dx_L. \tag{5}$$

Eq. (5) represents an approximation of the outdating quantity stemming from the batch Q, since the impact of perishability is ignored during the order lead times. If they had perished, then less items would be available at an order delivery, which results in less perished items of those Q units that got delivered.

2.2. Expected lost sales

The expected lost sales quantity in a cycle depends on the remaining lifetime of the r items. In Chiu (1995a), the expected lost sales are derived under the assumption that no items perish during the lead time. We relax this assumption and derive $E[S]$ by conditioning on whether or not perishability occurs during the lead time. A customer demand is lost when the demands plus the perished items (if any) during the lead time exceed r. Thus, to derive the expression of lost sales S, the remaining lifetime of the r units is needed, which is extremely complex. However, an upper bound of S is based on the assumption that the inventory level is zero when an order is delivered as shown in Figs. 2 and 3. Therefore, by making this assumption, we are assuming that only one age category is on hand at delivery. This assumption means that there is no order outstanding when an order is placed and the cycle length separating two successive orders is greater than m (cf. Figs. 2 and 3). Earlier works assume similar restrictions, such as in Liu and Lian (1999) and in Lian and Liu (2001), where an (r, S) policy is studied under zero lead time. A heuristic is then proposed for positive lead time. Tekin et al. (2001) investigated an age-based replenishment policy and assumed that units of the order Q do not perish before the inventory hits Q. That is, under the assumption

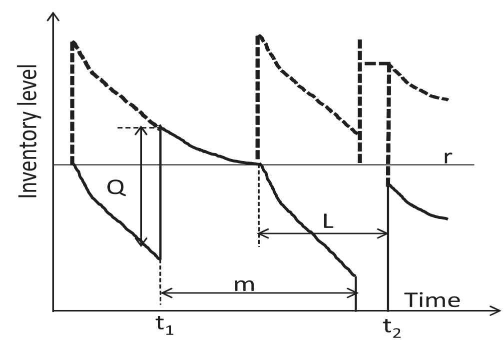

Fig. 2. Lost sales: case 1 ($r\ge Q$).

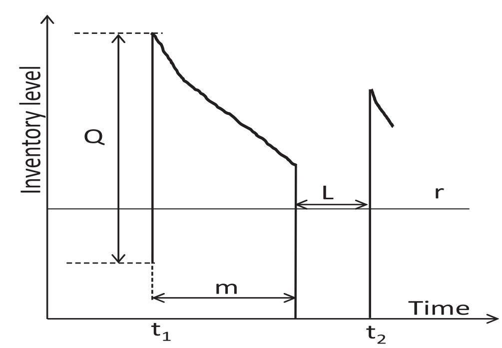

Fig. 3. Lost sales: case 2 ($r<Q$).

that the inventory level is zero when an order is delivered at time t_2 (cf. Figs. 2 and 3), we can write

$$S\approx\left(x_L-\left(r-(Q-x_m)^+\right)^+\right)^+. \tag{6}$$

In Eq. (6), $(Q-x_m)^+$ is the outdating quantity of the batch Q, $\left(r-(Q-x_m)^+\right)^+$ are the remaining non-perished items from r. The shortage occurs when the demand during the lead time exceeds $\left(r-(Q-x_m)^+\right)^+$. To derive the expected lost sales quantity, we distinguish between two cases:

Case 1: $r\ge Q$. In this case, $r-(Q-x_m)^+\ge 0$. Hence,

$$S=\begin{cases}(x_L-r+Q-x_m)^+ & \text{if } x_m\le Q;\\ (x_L-r)^+ & \text{if } x_m>Q.\end{cases} \tag{7}$$

The expected lost sales are

$$E[S]=\int_0^Q\int_{r-Q+x_m}^\infty(x_L-r+Q-x_m)\,dF_L(x_L)\,dF_m(x_m)$$

$$+\int_{Q}^{\infty}\int_{r}^{\infty}(x_L-r)\,dF_L(x_L)\,dF_m(x_m). \tag{8}$$

By double integration by parts, this expression becomes (see Appendix B)

$$E[S]=\mu L-r+\int_0^r F_L(x)\,dx_L+\int_0^Q F_m(x_m)\,dx_m - \int_0^Q F_L(r-Q+x_m)F_m(x_m)\,dx_m. \tag{9}$$

Case 2: $r<Q$. We can write S as

$$S=\begin{cases} x_L & \text{if } x_m\in[0,Q-r);\\ (x_L-r+Q-x_m)^+ & \text{if } x_m\in[Q-r,Q);\\ (x_L-r)^+ & \text{if } x_m\in[Q,\infty). \end{cases} \tag{10}$$

Consequently, the expected lost sales are

$$E[S]=\int_0^{Q-r}\int_0^{\infty}(x_L)\,dF_L(x_L)\,dF_m(x_m) + \int_{Q-r}^{Q}\int_{r-Q+x_m}^{\infty}(x_L-r+Q-x_m)\,dF_L(x_L)\,dF_m(x_m) + \int_{Q}^{\infty}\int_{r}^{\infty}(x_L-r)\,dF_L(x_L)\,dF_m(x_m). \tag{11}$$

This expression can be simplified to (see Appendix C)

$$E[S]=\mu L-r+\int_0^r F_L(x_L)\,dx_L+\int_{Q-r}^{Q} F_m(x_m)\,dx_m - \int_{Q-r}^{Q} F_L(r-Q+x_m)F_m(x_m)\,dx_m. \tag{12}$$

Eqs. (9) and (12) are the expected lost sales taking into account the perishability during the lead time. If the effect of perishability is ignored, $E[S]$ is reduced to the first three terms of (12), which expresses the well-known result of lost sales for non-perishable items, see Silver et al. (1998).

2.3. Expected cycle length and expected inventory level

First, it is easy to see that the expected cycle length when excess demands are lost is given by

$$E[T]=\frac{Q+E[S]-E[O]}{\mu}. \tag{13}$$

To derive the expected inventory level, we assume that items do not perish during the delivery lead time. The inventory held per cycle consists of two parts: before and after a new order Q is triggered. Therefore, to determine the expected inventory, we evaluate these two parts individually. This is denoted by the expected value of the areas A_1+A_2 and A_3 as shown in Fig. 4. To calculate the expected value of A_1+A_2, we need the expected value of the inventory level at the time of arrival of an order. Since

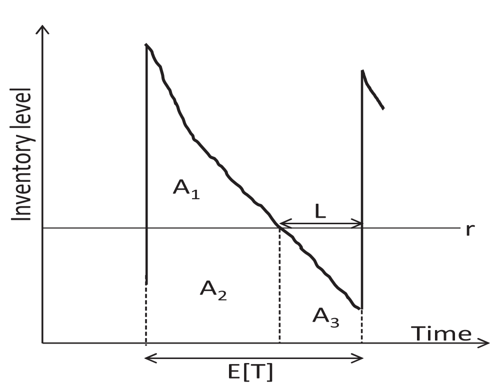

Fig. 4. Computing the expected inventory level.

the batch Q finds $(r-x_L)^+$ items on hand (by assumption), the inventory level just after the order arrives is $(r-x_L)^+ + Q$. One can approximate the net inventory level at the time of arrival of an order by $E[(r-x_L)^+]+Q-E[O]$. It should be noted that if orders are triggered either by perishability or by demand and lead to $E[O]>r$, the reorder point r does not play any role since orders are triggered by all items having perished and no item being available to satisfy demand. This situation is very unlikely to happen in a real situation. Therefore, we assume $E[O]\le r$. In addition, owing to the difficulty of tracking the age of items in stock, we withdraw the expected perished items from the net inventory once orders arrive and ignore $E[O]$ on estimating the inventory level during the cycle length. Based on this, the expected on-hand inventory level during $E[T]-L$ (the surfaces of the areas $A_1 + A_2$) is

$$\left(E[(r-x_L)^+]+Q-E[O]-r\right)\frac{E[T]-L}{2}+r(E[T]-L). \tag{14}$$

The first term in Eq. (14) expresses the expected value of the area A_1 and the second term represents the expected value of the area A_2. Similarly, the expected on-hand inventory level during L, which is the expected value of the area A_3, can be approximated by

$$\left(E[(r-x_L)^+]+r\right)\frac{L}{2}. \tag{15}$$

Summing up the above two equations and dividing by $E[T]$, we can write

$$E[I]=\frac{Q+r-E[O]+\int_0^r F_L(x_L)\,dx_L}{2}-\frac{\mu L(Q-E[O])}{2(Q-E[O]+E[S])}. \tag{16}$$

For tractability, we assume $\frac{Q-E[O]}{Q-E[O]+E[S]}\approx 1$, then $E[I]$ can be approximated by

$$E[I]=\frac{Q+r-E[O]+\int_0^r F_L(x_L)\,dx_L}{2}-\frac{\mu L}{2}. \tag{17}$$

We verified the accuracy of $E[I]$ using simulation and the results indicate that our approximation is quite good. The impact of using Eq. (17) instead of Eq. (16) in the total cost is provided in Appendix D.

3. Optimization procedure

Depending on whether or not the impact of perishability during the lead time is ignored, one can obtain two models. Model 1 assumes that items do not perish during the lead time and Model 2 takes the possibility of having items perish during the lead time into account. Our goal behind these two models is to test if neglecting perishability during the lead time affects the estimation of the best parameters (r^*, Q^*). *Model* 1 ignores the impact of perishability during the lead time, which means that $E[S]$ is expressed by the first three terms of (12):

$$E[S]=\int_r^{\infty}(x_L-r)\,dF_L(x_L)=\mu L-r+\int_0^r F_L(x_L)\,dx_L. \tag{18}$$

The total cost function of this model is given by Eq. (1) where $E[O]$, $E[S]$, $E[I]$ and $E[T]$ are provided in Eqs. (5), (18), (17) and (13) respectively. For *Model* 2, $E[O]$ is given by Eq. (5), $E[S]$ by Eq. (9) if $r\ge Q$ and by (12) if $r<Q$, $E[I]$ by Eq. (17) and $E[T]$ by Eq. (13). Model 2 is similar to Model 1, except that we relax the assumption that perishability does not occur during the lead time.

In this section, we obtain some analytical results that allow us to compute the best parameters r^* and Q^*. We focus on Model 1, before results are derived for Model 2 with a similar analysis. The complex form of total cost associated with Models 1 and 2 does not allow us to evaluate the Hessian matrix and analytically prove convexity. However, as in Liu and Yang (1999), Kalpakam and Shanthi (2006) and Olsson and Tydesj (2010), we conducted several numerical examples to verify that the total costs of the proposed models are jointly convex in r and Q. For Model 1, the following properties hold. Let

$$G(r,Q)=K+cQ+w\left(\int_0^Q F_m(x_m)\,dx_m+\int_0^r F_m(r+Q-x_L)F_L(x_L)\,dx_L\right)$$
$$+p\left(\mu L-r+\int_0^r F_L(x_L)\,dx_L\right), \tag{19}$$

$$CL(r,Q)=\frac{Q-r+\mu L-\int_0^Q F_m(x_m)\,dx_m-\int_0^r F_m(r+Q-x_L)F_L(x_L)\,dx_L+\int_0^r F_L(x_L)\,dx_L}{\mu}, \tag{20}$$

$$I(r,Q)=\frac{h\left(Q+r-\mu L-\int_0^Q F_m(x_m)\,dx_m-\int_0^r F_m(r+Q-x_L)F_L(x_L)\,dx_L+\int_0^r F_L(x_L)\,dx_L\right)}{2}. \tag{21}$$

Then,

$$TC(r,Q)=\frac{G(r,Q)}{CL(r,Q)}+I(r,Q),\quad \text{for } Q>0. \tag{22}$$

Lemma 1. *For any given r and Q, the expected cycle length of the* (r,Q) *policy is less than or equal to* $m+L$.

Proof. The proof is given in Appendix E.

Proposition 1. *For any given r, the best order quantity* Q^* *is finite and* $Q^*\le\mu(m+L)$.

Proof. The proof is given in Appendix F.

Proposition 2. *For any given Q, the best reorder point* r^* *is less than* $\tilde r$ *where* $\tilde r$ *is the solution of the following equation*:

$$\left.\frac{\partial G(r,Q)}{\partial r}\right|_{r=\tilde r}=-p+[p+wF_m(Q)]F_L(\tilde r)+w\int_0^{\tilde r}F_L(x_L)f_m(\tilde r+Q-x_L)\,dx_L=0. \tag{23}$$

Proof. The proof is given in Appendix G.

Fig. 5 is a typical plot of the left-hand side of Eq. (23) for a demand that follows a Normal distribution where $\mu=10$, $\sigma=2$, Q and the cost parameters p=20 and w=15 are fixed. $\tilde r$ is the point where the curve intersects with the horizontal axis.

Corollary 1. *For Model 2 the best reorder point* r^* *is less than* $\hat r$ *where* $\hat r$ *is the solution of the following*:

$$w\int_0^r F_L(x_L)f_m(r+Q-x_L)\,dx_L-p\left(1+\int_0^Q F_m(x_m)f_L(r-Q+x_m)\,dx_m\right)$$
$$+[p+wF_m(Q)]F_L(r)=0. \tag{24}$$

Proof. The proof is given in Appendix H.

To summarize the results of Proposition 2 and Corollary 1, we plot the behavior of (23) and (24) in Fig. 6. $\tilde r$ and $\hat r$ are the points where these two curves intersect with the horizontal axis. Intuitively, Eq. (23), as well as Figs. 5 and 6, is in line with the newsvendor concept that finds a balance between the marginal cost of an overstock $wF_m(Q)F_L(r)+w\int_0^r F_L(x_L)f_m(r+Q-x_L)\,dx_L$ and the marginal cost of an understock $p(1-F_L(r))$.

Due to the complexity of the total cost, explicit expressions of the best parameters r^* and Q^* cannot be obtained. However, from Propositions 1, 2 and Corollary 1, we know that $Q^*\le\mu(m+L)$, $r^*\le\tilde r$ for Model 1 and $r^*\le\hat r$ for Model 2. We suggest the following algorithm, initialized by $Q=\mu(m+L)$, to determine the best (r,Q) policy of the proposed models.

Algorithm for finding the best parameters r and Q.

Step 1: Determine $\tilde r$ and $\hat r$ by finding the solution of Eqs. (23) and (24) for Models 1 and 2 respectively.
Step 2: Find $r_1^*=\text{argmin}\{TC(r,Q),0\le r\le\tilde r\}$ for Model 1 and $r_2^*=\text{argmin}\{TC(r,Q),0\le r\le\hat r\}$ for Model 2.
Step 3. Decrease Q by one unit and repeat steps 1 and 2 until the minimum total cost of Models 1 and 2 is found.

4. Numerical analysis

In this section, we first test the validity of the proposed models by comparing them to the optimal (r,Q) policy obtained by simulation. Then we perform a sensitivity analysis with regard to cost parameters to illustrate changes in the best policy and its corresponding total cost. Finally, we evaluate the performance of Model 2 in comparison with Chiu (1995a). For all comparisons, we first compute the best (r^*,Q^*) policies and Chiu's model parameters. Using simulation, we compare the simulated cost of the models with the optimal cost obtained by simulation. Simulation experiments run on Arena software 12 with the following sequence of events: (1) an order arrives, (2) perished products are discarded, (3) demand is observed, (4) the inventory position is reviewed, and (5) an order is triggered. For each (r,Q), we run 10 replications, each of which lasts 20,000 units of time. In using these simulation parameters, we are able to obtain an estimation of the total operating cost within a precision of ± 0.01 with a confidence interval of 95%. As in Zhou et al. (2011), we use OptQuest, Arena's add-on optimizer, to obtain the optimal policy parameters and its corresponding optimal cost. The main algorithms that OptQuest uses are heuristics based on scatter search, tabu search and neural networks. In addition, we bounded r and Q

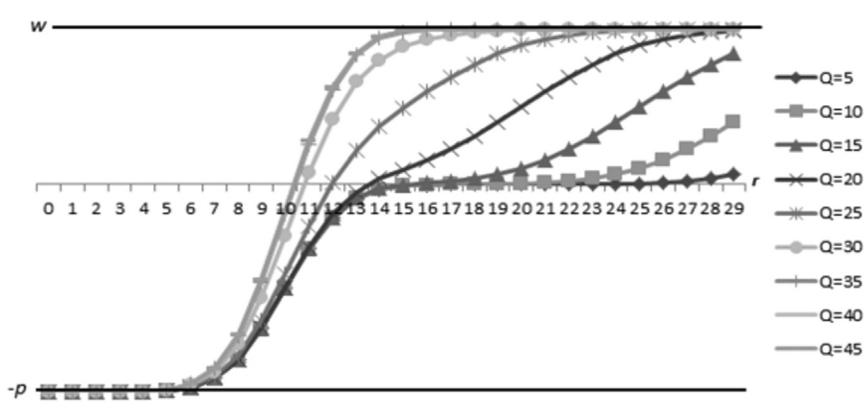

Fig. 5. Finding $\tilde r$ for a given Q.

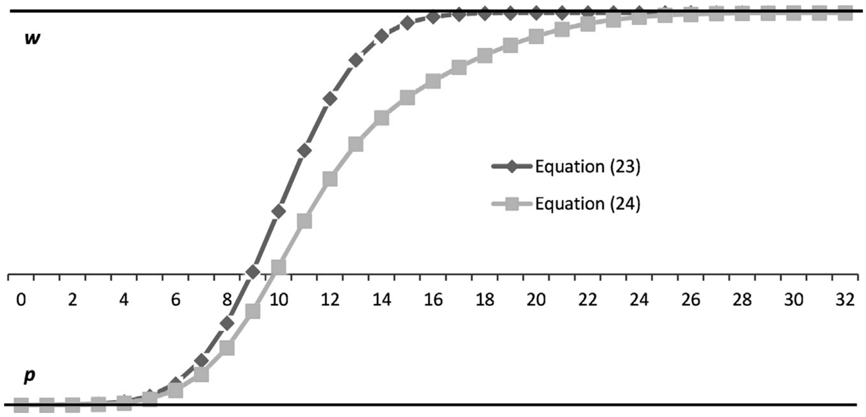

Fig. 6. Finding the best $\tilde{r}$ and $\hat{r}$ for a Normal demand distribution with $\mu=10$ and $\sigma=2$. $Q=26$ and the cost parameters $p=15$ and $w=30$.

Table 3
Running time (in seconds) of the algorithms for Gamma demand with mean 10.

Design point	Cost parameters				$L=1$								$L=2$							
					$cv^2=0.23$		$cv^2=0.4$		$cv^2=0.63$		$cv^2=1$		$cv^2=0.23$		$cv^2=0.4$		$cv^2=0.63$		$cv^2=1$	
	K	c	p	w	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1
1	10	5	20	5	23	18	20	17	19	21	19	23	34	23	34	24	33	37	85	63
2	50	5	20	5	23	33	19	18	23	32	34	41	34	25	51	41	33	23	83	64
3	100	5	20	5	20	14	19	15	19	15	19	14	34	23	48	36	33	52	87	58
4	10	5	40	5	20	14	20	15	20	15	47	36	36	25	40	31	35	51	35	46
5	50	5	40	5	20	15	20	15	42	36	51	40	36	25	36	40	35	25	35	25
6	100	5	40	5	20	17	20	18	20	26	53	42	36	25	35	25	35	25	100	64
7	10	15	20	15	18	24	18	18	37	24	45	32	31	22	30	21	32	45	71	51
8	50	15	20	15	18	13	18	17	17	14	42	35	31	21	30	21	29	21	28	42
9	100	15	20	15	21	20	17	20	17	13	42	37	31	21	30	28	29	21	28	20
10	10	15	40	15	19	14	38	26	19	14	48	41	33	23	33	22	32	28	32	22
11	50	15	40	15	19	14	42	23	19	14	48	41	33	23	32	22	55	46	66	46
12	100	15	40	15	19	14	21	26	19	20	47	39	33	23	39	43	32	22	75	47
13	10	5	20	15	18	13	20	20	17	13	17	32	33	21	31	26	50	46	68	47
14	50	5	20	15	18	16	31	23	23	30	16	13	32	25	33	26	29	32	66	46
15	100	5	20	15	18	17	41	30	34	37	45	37	43	25	30	37	29	21	60	46
16	10	5	40	15	19	34	30	28	23	40	18	15	50	55	40	57	32	22	70	50
17	50	5	40	15	19	27	46	31	19	30	18	14	33	29	33	61	55	23	43	49
18	100	5	40	15	19	23	19	28	18	30	44	40	34	25	32	22	47	54	31	29
19	10	15	20	5	20	14	19	20	19	15	50	39	34	23	36	24	33	24	57	28
20	50	15	20	5	20	15	20	26	20	40	53	41	34	23	46	39	34	24	74	48
21	100	15	20	5	20	18	19	15	19	15	46	41	34	24	35	35	78	55	88	59
22	10	15	40	5	26	33	34	32	36	39	52	40	36	25	37	26	36	26	93	64
23	50	15	40	5	20	18	20	15	20	23	52	37	36	27	42	56	36	25	91	67
24	100	15	40	5	20	14	20	21	20	15	53	40	36	25	41	33	98	58	39	66
25	200	5	20	5	20	17	19	26	20	43	49	42	34	23	36	33	93	64	85	61
26	200	5	40	5	20	29	31	34	28	43	53	43	37	25	36	25	86	68	97	70
27	200	5	20	15	18	19	18	21	17	28	46	38	31	21	30	21	81	55	75	52
28	200	5	40	15	19	17	19	14	19	21	51	37	34	23	32	22	78	53	83	61
29	200	15	20	5	24	32	20	15	37	22	54	37	34	23	34	37	55	66	92	63
30	200	15	40	5	23	33	31	28	48	35	49	41	37	27	82	52	36	28	99	58
31	200	15	20	15	18	20	19	28	17	23	43	32	31	21	66	46	78	52	34	28
32	200	15	40	15	19	14	19	14	19	14	47	33	33	23	73	49	83	57	65	52

and used 500 simulation scenarios where the selection of parameters r and Q are determined by OptQuest. Similar to Zhou et al. (2011), the optimal solution is found if the improvement in the current best value for the next 100 scenarios is less than 1%. If the optimal solution given by OptQuest contains one of the values of the bounds of r or Q, we increase the bounds and repeat the simulation run. These simulation parameters are chosen in order to have the most accurate estimations of system parameter values. The performance of the models is measured by the relative error denoted by Δ%. Regarding the mean of the demand, the lead time and the cost parameters, we use the setting from Berk and Grler (2008). That is, $\mu=10$, $L=\{1,2\}$, $m=3$, $K=\{10,50,100,200\}$, $h=1$, $c=\{5,15\}$, $w=\{5,15\}$ and $p=\{20,40\}$. We assume that demand follows a Gamma distribution, which allows us to evaluate the effect of changes in the coefficient of variation. Note that if the demand follows a Normal distribution, it can be approximated by a Gamma distribution using the relations: shape parameter$=(\mu/\sigma)^2$ and scale parameter$=\mu/\sigma^2$.

Performance of the proposed models: The algorithm is implemented in Matlab 7.1 and run on a computer with a processor intel core i5, 2.4 GHz. The run times (in seconds) are provided in Table 3. The computation of the parameters (r^*, Q^*) requires short

Table 4
Comparison between the proposed models, Chiu's model and simulation for Gamma demand with mean 10 and L=1.

Design point	Cost parameters				$cv^2 = 0.23$								$cv^2 = 0.4$							
					Model 2		Model 1		Chiu		Simulation		Model 2		Model 1		Chiu		Simulation	
	K	c	p	w	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	Δ(%)	(r^*, Q^*)	Δ(%)	(r^*, Q^*)	TC^*	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	TC^*
1	10	5	20	5	(12,15)	0.0	(12,15)	0.0	(12,15)	0.0	(12,14)	67.0	(13,15)	0.3	(13,15)	0.3	(13,15)	0.3	(12,15)	67.8
2	50	5	20	5	(11,25)	0.0	(11,25)	0.0	(11,25)	0.0	(11,25)	85.2	(11,24)	0.0	(11,24)	0.0	(11,24)	0.0	(11,24)	87.0
3	100	5	20	5	(11,26)	0.5	(10,27)	0.0	(10,27)	0.0	(10,27)	104.3	(11,25)	0.2	(10,26)	0.0	(10,27)	0.2	(10,26)	106.9
4	10	5	40	5	(13,15)	0.6	(13,15)	0.6	(13,15)	0.6	(12,15)	67.4	(14,15)	0.5	(14,15)	0.5	(14,15)	0.5	(13,15)	68.6
5	50	5	40	5	(12,24)	0.0	(12,24)	0.0	(12,24)	0.0	(11,25)	86.3	(13,22)	0.4	(13,22)	0.4	(13,22)	0.4	(12,23)	88.4
6	100	5	40	5	(12,25)	0.7	(11,27)	0.3	(11,27)	0.3	(11,26)	105.9	(12,25)	0.0	(12,25)	0.0	(12,25)	0.0	(12,25)	109.3
7	10	15	20	15	(11,15)	0.1	(11,15)	0.1	(11,15)	0.1	(10,15)	165.9	(11,15)	0.0	(11,15)	0.0	(11,15)	0.0	(11,15)	166.6
8	50	15	20	15	(9,25)	0.0	(9,25)	0.0	(9,25)	0.0	(9,25)	183.9	(8,24)	0.0	(8,24)	0.0	(8,25)	0.2	(9,23)	185.1
9	100	15	20	15	(0,26)	0.0	(0,26)	0.0	(0,35)	19.1	(0,26)	201.7	(0,25)	0.0	(0,25)	0.0	(0,33)	11.8	(0,25)	202.9
10	10	15	40	15	(12,15)	0.0	(12,15)	0.0	(12,15)	0.0	(12,15)	167.3	(13,15)	0.0	(13,15)	0.0	(13,15)	0.0	(13,15)	168.3
11	50	15	40	15	(12,23)	0.3	(12,23)	0.3	(12,23)	0.3	(11,24)	186.3	(12,22)	0.0	(12,22)	0.0	(12,22)	0.0	(12,22)	188.9
12	100	15	40	15	(11,25)	0.0	(11,25)	0.0	(11,25)	0.0	(11,25)	206.7	(11,24)	0.0	(11,24)	0.0	(11,24)	0.0	(11,24)	210.4
13	10	5	20	15	(12,15)	0.0	(12,15)	0.0	(12,15)	0.0	(12,14)	67.0	(13,15)	0.3	(13,15)	0.3	(13,15)	0.3	(12,15)	67.8
14	50	5	20	15	(11,24)	0.0	(11,24)	0.0	(11,24)	0.0	(11,24)	85.5	(11,23)	0.0	(11,23)	0.0	(11,23)	0.0	(11,23)	87.4
15	100	5	20	15	(10,26)	0.0	(10,26)	0.0	(10,26)	0.0	(10,26)	104.9	(10,25)	0.0	(10,25)	0.0	(10,26)	0.1	(10,25)	108.0
16	10	5	40	15	(13,15)	0.6	(13,15)	0.6	(13,15)	0.6	(12,15)	67.4	(14,15)	0.5	(14,15)	0.5	(14,15)	0.5	(13,15)	68.6
17	50	5	40	15	(12,23)	0.1	(12,23)	0.1	(12,23)	0.1	(11,24)	86.5	(13,21)	0.5	(13,21)	0.5	(13,21)	0.5	(12,22)	89.0
18	100	5	40	15	(12,25)	0.7	(11,26)	0.0	(11,26)	0.0	(11,25)	106.7	(12,24)	0.0	(12,24)	0.0	(12,24)	0.0	(12,24)	110.7
19	10	15	20	5	(11,15)	0.1	(11,15)	0.1	(11,15)	0.1	(10,15)	165.9	(11,15)	0.0	(11,15)	0.0	(11,15)	0.0	(11,15)	166.6
20	50	15	20	5	(9,25)	0.0	(9,25)	0.0	(9,26)	0.0	(9,25)	183.8	(9,24)	0.0	(9,24)	0.0	(8,25)	0.1	(9,24)	184.9
21	100	15	20	5	(0,27)	0.0	(0,27)	0.0	(0,35)	13.0	(0,27)	201.4	(0,26)	0.0	(0,26)	0.0	(0,34)	10.2	(0,26)	202.5
22	10	15	40	5	(12,15)	0.0	(12,15)	0.0	(12,15)	0.0	(12,15)	167.3	(13,15)	0.0	(13,15)	0.0	(13,15)	0.0	(13,15)	168.3
23	50	15	40	5	(12,23)	0.3	(12,23)	0.3	(12,23)	0.3	(11,24)	186.0	(12,22)	0.0	(12,22)	0.0	(12,22)	0.0	(12,22)	188.5
24	100	15	40	5	(11,25)	0.0	(11,26)	0.0	(11,26)	0.0	(11,26)	206.2	(11,24)	0.0	(11,25)	0.0	(11,25)	0.0	(11,25)	209.7
25	200	5	20	5	(10,28)	0.0	(9,29)	0.1	(8,31)	3.8	(10,28)	140.2	(9,28)	0.1	(9,29)	0.4	(6,32)	7.2	(10,28)	143.3
26	200	5	40	5	(11,27)	0.1	(11,28)	0.0	(11,28)	0.0	(11,28)	143.4	(11,26)	0.2	(11,28)	0.3	(11,28)	0.3	(11,27)	148.2
27	200	5	20	15	(10,27)	0.3	(9,28)	0.0	(7,31)	7.3	(9,28)	141.4	(9,27)	0.0	(9,27)	0.0	(0,37)	30.2	(9,27)	145.2
28	200	5	40	15	(11,26)	0.2	(11,27)	0.0	(11,27)	0.0	(11,27)	145.0	(11,26)	0.0	(11,26)	0.0	(11,26)	0.0	(11,26)	150.3
29	200	15	20	5	(0,28)	0.0	(0,28)	0.0	(0,37)	15.5	(0,28)	228.5	(0,27)	0.0	(0,27)	0.0	(0,36)	12.4	(0,27)	230.3
30	200	15	40	5	(10,27)	0.0	(10,28)	0.1	(10,28)	0.1	(10,27)	243.6	(10,26)	0.1	(10,27)	0.0	(10,27)	0.0	(10,27)	248.6
31	200	15	20	15	(0,27)	0.0	(0,27)	0.0	(0,36)	19.3	(0,27)	229.1	(0,27)	0.0	(0,26)	0.0	(0,35)	15.2	(0,26)	231.1
32	200	15	40	15	(10,27)	0.0	(10,27)	0.0	(10,27)	0.0	(10,27)	244.5	(10,26)	0.0	(10,26)	0.0	(10,26)	0.0	(10,26)	250.0

	$cv^2 = 0.63$								$cv^2 = 1$							
	Model 2		Model 1		Chiu		Simulation		Model 2		Model 1		Chiu		Simulation	
	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	TC^*	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	TC^*
1	(13,15)	0.0	(13,15)	0.0	(13,15)	0.0	(13,15)	69.1	(14,14)	0.0	(14,14)	0.0	(14,14)	0.0	(14,15)	71.0
2	(12,22)	0.0	(11,23)	0.0	(11,23)	0.0	(11,23)	89.2	(12,21)	0.0	(11,22)	0.3	(11,22)	0.3	(12,22)	92.2
3	(11,25)	0.0	(10,26)	0.2	(10,26)	0.2	(11,25)	109.9	(10,24)	0.3	(10,25)	0.3	(10,26)	0.6	(11,24)	113.6
4	(15,14)	0.0	(15,14)	0.0	(15,14)	0.0	(14,15)	70.2	(16,13)	0.0	(16,13)	0.0	(16,13)	0.0	(16,3)	72.7
5	(13,22)	0.0	(13,22)	0.0	(13,22)	0.0	(13,22)	91.4	(14,20)	0.0	(14,20)	0.0	(14,20)	0.0	(14,20)	95.4
6	(12,24)	0.1	(12,25)	0.2	(12,25)	0.2	(13,24)	113.5	(13,23)	0.0	(12,24)	0.9	(12,24)	0.9	(13,23)	118.7
7	(11,15)	0.5	(11,15)	0.5	(11,15)	0.5	(11,15)	166.6	(11,15)	0.0	(11,15)	0.0	(11,15)	0.0	(11,14)	168.7
8	(8,22)	0.0	(8,23)	0.0	(7,24)	0.3	(8,22)	186.7	(7,21)	0.0	(7,21)	0.0	(4,25)	1.8	(8,21)	188.5
9	(0,24)	0.0	(0,24)	0.0	(0,32)	9.6	(0,24)	204.4	(0,23)	0.0	(0,23)	0.0	(0,31)	8.2	(0,23)	206.3
10	(14,14)	0.0	(14,14)	0.0	(14,14)	0.0	(14,14)	169.5	(15,13)	0.0	(15,13)	0.0	(15,13)	0.0	(15,13)	172.7
11	(12,20)	0.0	(12,21)	0.0	(12,21)	0.0	(12,21)	192.3	(13,18)	0.0	(13,18)	0.0	(13,18)	0.0	(13,19)	197.1
12	(11,23)	0.0	(11,23)	0.0	(11,23)	0.0	(11,23)	215.3	(11,22)	0.1	(11,22)	0.1	(11,22)	0.1	(12,21)	221.6
13	(13,15)	0.0	(13,15)	0.0	(13,15)	0.0	(13,15)	69.1	(14,14)	0.0	(14,14)	0.0	(14,14)	0.0	(14,14)	71.3
14	(11,22)	0.0	(11,22)	0.0	(11,22)	0.0	(11,22)	90.1	(11,21)	0.2	(11,21)	0.2	(11,21)	0.2	(12,20)	93.6
15	(10,24)	0.0	(10,24)	0.0	(10,25)	0.1	(10,24)	111.5	(10,23)	0.0	(10,23)	0.0	(9,25)	1.0	(11,23)	115.9
16	(15,14)	0.0	(15,14)	0.0	(15,14)	0.0	(14,15)	70.3	(16,12)	0.0	(16,12)	0.0	(16,12)	0.0	(15,12)	73.1
17	(13,20)	0.0	(13,20)	0.0	(13,20)	0.0	(13,20)	92.5	(14,18)	0.1	(14,18)	0.1	(14,18)	0.1	(14,19)	97.3
18	(12,23)	0.0	(12,23)	0.0	(12,23)	0.0	(12,23)	115.4	(13,21)	0.2	(12,22)	0.5	(12,22)	0.5	(13,22)	121.7
19	(11,15)	0.5	(11,15)	0.5	(11,15)	0.5	(11,15)	166.6	(11,15)	0.0	(11,15)	0.0	(11,15)	0.0	(11,15)	168.6
20	(8,23)	0.0	(8,23)	0.0	(7,25)	0.4	(8,23)	186.3	(7,22)	0.0	(7,22)	0.0	(5,25)	1.1	(8,21)	188.0
21	(0,25)	0.0	(0,25)	0.0	(0,33)	8.1	(0,25)	203.9	(0,24)	0.0	(0,24)	0.0	(0,31)	5.2	(0,24)	205.5
22	(14,15)	0.0	(14,15)	0.0	(14,15)	0.0	(14,15)	170.0	(15,13)	0.0	(15,13)	0.0	(15,13)	0.0	(15,13)	172.3
23	(12,21)	0.0	(12,21)	0.0	(12,21)	0.0	(12,21)	191.6	(13,19)	0.0	(13,19)	0.0	(13,19)	0.0	(13,19)	196.0
24	(12,23)	0.0	(11,24)	0.1	(11,24)	0.1	(12,23)	214.1	(12,22)	0.0	(11,23)	0.2	(11,23)	0.2	(12,23)	219.6
25	(9,27)	0.1	(9,28)	0.1	(0,37)	17.2	(10,28)	146.9	(8,27)	0.4	(8,28)	0.5	(0,37)	14.6	(10,27)	151.1
26	(12,26)	0.0	(11,27)	0.4	(11,28)	0.9	(12,26)	153.6	(12,25)	0.2	(11,27)	1.1	(11,27)	1.1	(13,26)	160.4
27	(9,26)	0.0	(8,27)	0.3	(0,36)	23.4	(9,27)	149.6	(8,26)	0.2	(8,26)	0.2	(0,35)	18.0	(9,26)	154.5
28	(11,25)	0.0	(11,26)	0.0	(11,26)	0.0	(11,26)	157.0	(11,24)	0.5	(11,25)	0.3	(10,26)	1.8	(12,25)	164.8
29	(0,27)	0.0	(0,26)	0.0	(0,35)	9.6	(0,26)	232.4	(0,26)	0.0	(0,26)	0.0	(0,34)	7.7	(0,25)	234.8
30	(11,25)	0.1	(10,26)	0.1	(9,28)	1.4	(11,26)	254.2	(10,25)	0.2	(10,26)	0.2	(9,27)	1.1	(11,25)	261.0
31	(0,26)	0.0	(0,26)	0.0	(0,34)	11.9	(0,26)	233.5	(0,25)	0.0	(0,25)	0.0	(0,33)	9.7	(0,24)	236.2
32	(10,25)	0.0	(10,25)	0.0	(9,27)	1.0	(10,25)	256.3	(10,24)	0.0	(10,24)	0.0	(8,27)	2.2	(11,24)	264.1

Table 5
Comparison between the proposed models, Chiu's model and simulation for Gamma demand with mean 10 and L=2.

Design point	Cost parameters				$cv^2 = 0.23$								$cv^2 = 0.4$							
					Model 2		Model 1		Chiu		Simulation		Model 2		Model 1		Chiu		Simulation	
	K	c	p	w	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	TC^*	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	TC^*
1	10	5	20	5	(23,15)	0.5	(23,15)	0.5	(23,15)	0.5	(22,15)	67.8	(24,15)	0.4	(24,15)	0.4	(24,15)	0.4	(23,14)	69.1
2	50	5	20	5	(21,25)	0.0	(21,25)	0.0	(21,25)	0.0	(21,25)	86.3	(22,23)	0.0	(22,23)	0.0	(22,23)	0.0	(21,24)	88.6
3	100	5	20	5	(21,26)	0.4	(20,27)	0.0	(20,27)	0.0	(20,27)	105.3	(21,25)	0.3	(20,26)	0.1	(20,27)	0.1	(21,26)	108.2
4	10	5	40	5	(24,15)	0.5	(24,15)	0.5	(24,15)	0.5	(23,15)	68.6	(25,15)	0.3	(25,15)	0.3	(25,15)	0.3	(24,15)	70.3
5	50	5	40	5	(23,23)	0.5	(23,23)	0.5	(23,23)	0.5	(22,24)	87.7	(23,22)	0.2	(23,22)	0.2	(24,21)	0.6	(23,24)	90.8
6	100	5	40	5	(22,25)	0.2	(22,26)	0.0	(22,26)	0.0	(22,26)	107.7	(23,24)	0.3	(22,25)	0.1	(22,25)	0.1	(22,26)	111.6
7	10	15	20	15	(21,15)	0.1	(21,15)	0.1	(21,15)	0.1	(20,15)	166.6	(21,16)	0.1	(21,16)	0.1	(22,15)	0.2	(20,15)	167.5
8	50	15	20	15	(18,25)	0.0	(18,25)	0.0	(19,25)	0.1	(18,25)	184.3	(17,24)	0.0	(17,24)	0.0	(18,24)	0.0	(18,24)	185.7
9	100	15	20	15	(0,26)	0.0	(0,26)	0.0	(0,43)	41.5	(0,26)	201.3	(0,25)	0.0	(0,25)	0.0	(0,41)	34.4	(0,25)	202.2
10	10	15	40	15	(23,16)	0.0	(23,16)	0.0	(23,16)	0.0	(23,15)	168.4	(24,15)	0.1	(24,15)	0.1	(25,14)	0.3	(24,14)	169.9
11	50	15	40	15	(22,22)	0.1	(22,22)	0.1	(22,22)	0.1	(22,23)	188.1	(23,20)	0.0	(23,20)	0.0	(23,20)	0.0	(22,21)	191.7
12	100	15	40	15	(21,25)	0.0	(21,25)	0.0	(21,25)	0.0	(21,25)	208.6	(22,23)	0.1	(21,24)	0.0	(21,24)	0.0	(21,24)	213.4
13	10	5	20	15	(23,15)	0.5	(23,15)	0.5	(23,15)	0.5	(22,15)	67.8	(24,15)	0.5	(24,15)	0.5	(24,15)	0.5	(23,14)	69.1
14	50	5	20	15	(21,24)	0.0	(21,24)	0.0	(21,24)	0.0	(21,24)	86.7	(22,22)	0.6	(22,22)	0.6	(22,22)	0.6	(21,23)	89.4
15	100	5	20	15	(20,26)	0.0	(20,26)	0.0	(20,26)	0.0	(20,26)	106.2	(20,25)	0.0	(20,25)	0.0	(20,25)	0.0	(20,25)	109.7
16	10	5	40	15	(24,15)	0.5	(24,15)	0.5	(24,15)	0.5	(23,15)	68.6	(25,14)	0.2	(25,14)	0.2	(25,15)	0.3	(24,15)	70.3
17	50	5	40	15	(23,22)	0.5	(23,22)	0.5	(23,22)	0.5	(22,23)	88.2	(23,21)	0.0	(23,21)	0.0	(23,21)	0.0	(23,21)	92.0
18	100	5	40	15	(22,24)	0.2	(22,24)	0.2	(22,24)	0.2	(22,25)	109.0	(22,23)	0.3	(22,24)	0.0	(22,24)	0.0	(22,24)	113.6
19	10	15	20	5	(21,15)	0.1	(21,15)	0.1	(21,15)	0.1	(20,15)	166.6	(21,16)	0.1	(21,16)	0.1	(22,15)	0.2	(20,15)	167.5
20	50	15	20	5	(19,25)	0.0	(19,25)	0.0	(19,25)	0.0	(18,25)	184.2	(18,24)	0.0	(18,24)	0.0	(18,25)	0.1	(18,25)	185.4
21	100	15	20	5	(0,27)	0.0	(0,27)	0.0	(0,43)	28.8	(0,27)	201.1	(0,26)	0.0	(0,26)	0.0	(0,41)	23.7	(0,26)	202.0
22	10	15	40	5	(23,16)	0.0	(23,16)	0.0	(23,16)	0.0	(23,15)	168.4	(24,15)	0.0	(24,15)	0.0	(25,14)	0.2	(24,14)	169.9
23	50	15	40	5	(22,23)	0.0	(22,23)	0.0	(22,23)	0.0	(22,23)	187.7	(23,21)	0.1	(23,21)	0.1	(23,21)	0.1	(22,23)	191.0
24	100	15	40	5	(21,25)	0.1	(21,25)	0.1	(21,25)	0.1	(21,26)	207.9	(22,23)	0.2	(22,23)	0.2	(22,24)	0.0	(22,24)	212.3
25	200	5	20	5	(19,28)	0.2	(19,29)	0.0	(17,32)	6.9	(20,29)	140.6	(19,28)	0.0	(19,28)	0.0	(15,33)	9.9	(19,28)	144.0
26	200	5	40	5	(21,27)	0.4	(21,28)	0.0	(21,28)	0.0	(21,28)	144.6	(22,26)	0.7	(21,28)	0.2	(21,28)	0.2	(22,28)	149.6
27	200	5	20	15	(19,27)	0.4	(19,28)	0.0	(16,31)	7.7	(19,28)	142.1	(18,27)	0.2	(18,27)	0.2	(0,46)	59.1	(19,27)	146.2
28	200	5	40	15	(21,26)	0.5	(21,27)	0.0	(21,27)	0.0	(21,27)	146.8	(21,25)	0.9	(21,26)	0.2	(21,26)	0.2	(21,27)	152.9
29	200	15	20	5	(0,28)	0.0	(0,27)	0.0	(0,45)	29.5	(0,28)	222.5	(0,27)	0.0	(0,27)	0.0	(0,44)	26.7	(0,27)	223.8
30	200	15	40	5	(20,27)	0.1	(20,27)	0.1	(20,28)	0.0	(20,28)	245.1	(20,26)	0.2	(20,27)	0.0	(20,27)	0.0	(21,27)	250.5
31	200	15	20	15	(0,27)	0.0	(0,27)	0.0	(0,45)	42.8	(0,27)	222.9	(0,26)	0.0	(0,26)	0.0	(0,43)	35.9	(0,26)	224.3
32	200	15	40	15	(20,26)	0.3	(20,27)	0.0	(20,27)	0.0	(20,27)	246.3	(20,25)	0.3	(20,26)	0.0	(19,27)	0.4	(20,26)	252.5

	$cv^2 = 0.63$								$cv^2 = 1$							
	Model 2		Model 1		Chiu		Simulation		Model 2		Model 1		Chiu		Simulation	
	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	TC^*	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	Δ (%)	(r^*, Q^*)	TC^*
1	(24,15)	0.1	(24,15)	0.1	(25,14)	0.4	(24,14)	70.8	(25,14)	0.1	(25,14)	0.1	(25,14)	0.1	(21,22)	73.3
2	(22,22)	0.7	(22,22)	0.7	(22,22)	0.7	(22,23)	91.4	(22,21)	1.4	(22,21)	1.4	(22,21)	1.4	(22,23)	94.9
3	(20,25)	0.4	(20,26)	0.2	(20,26)	0.2	(21,26)	111.4	(20,24)	0.6	(20,25)	0.2	(19,27)	1.3	(20,25)	115.4
4	(26,14)	0.1	(26,14)	0.1	(26,14)	0.1	(26,14)	72.5	(28,12)	0.0	(28,12)	0.0	(28,12)	0.0	(28,12)	76.1
5	(24,21)	0.0	(24,21)	0.0	(24,21)	0.0	(23,24)	95.3	(25,19)	0.3	(25,19)	0.3	(25,19)	0.3	(23,24)	100.9
6	(23,23)	1.3	(23,24)	0.3	(23,24)	0.3	(23,25)	116.1	(24,22)	2.5	(23,23)	2.5	(23,24)	0.7	(23,24)	121.8
7	(21,15)	0.0	(21,15)	0.0	(22,15)	0.2	(21,13)	168.6	(21,14)	0.1	(21,14)	0.1	(22,14)	0.2	(18,19)	170.2
8	(16,23)	0.0	(16,23)	0.0	(16,25)	0.6	(17,23)	187.4	(0,22)	1.9	(0,22)	1.9	(0,35)	19.5	(17,21)	189.3
9	(0,24)	0.0	(0,24)	0.0	(0,39)	27.2	(0,24)	203.5	(0,23)	0.0	(0,23)	0.0	(0,38)	23.8	(1,23)	204.9
10	(25,13)	0.0	(25,13)	0.0	(26,13)	0.1	(25,13)	172.4	(26,12)	0.0	(26,12)	0.0	(26,12)	0.0	(21,22)	176.3
11	(23,19)	0.0	(23,19)	0.0	(23,19)	0.0	(23,19)	196.5	(23,18)	0.1	(23,18)	0.1	(23,18)	0.1	(21,22)	202.9
12	(22,22)	0.7	(21,23)	0.1	(21,23)	0.1	(22,23)	219.3	(21,21)	1.4	(21,21)	1.4	(21,22)	0.2	(21,22)	226.5
13	(24,14)	0.0	(24,14)	0.0	(24,14)	0.0	(24,14)	71.0	(25,13)	0.0	(25,13)	0.0	(25,13)	0.0	(21,22)	73.8
14	(22,21)	0.8	(22,21)	0.8	(22,21)	0.8	(21,22)	92.8	(21,20)	1.3	(21,20)	1.3	(22,20)	1.1	(21,22)	97.2
15	(20,24)	0.1	(20,24)	0.1	(20,24)	0.1	(20,25)	113.7	(19,23)	0.7	(19,23)	0.7	(19,24)	0.4	(20,24)	118.5
16	(26,13)	0.0	(26,13)	0.0	(26,13)	0.0	(26,13)	72.8	(27,12)	1.2	(27,12)	1.2	(27,12)	1.2	(27,12)	76.0
17	(24,19)	0.0	(24,19)	0.0	(24,19)	0.0	(24,19)	96.8	(24,18)	0.4	(24,18)	0.4	(25,17)	0.1	(22,23)	103.3
18	(23,22)	1.0	(23,22)	1.0	(23,22)	1.0	(23,24)	119.5	(23,21)	1.9	(23,21)	1.9	(23,21)	1.9	(22,23)	127.0
19	(21,16)	0.1	(21,16)	0.1	(22,15)	0.1	(21,14)	168.6	(21,15)	0.2	(21,15)	0.2	(22,14)	0.1	(18,19)	170.0
20	(17,23)	0.0	(17,23)	0.0	(16,25)	0.3	(17,23)	187.0	(0,22)	2.1	(0,22)	2.1	(13,26)	1.6	(17,22)	188.8
21	(0,25)	0.0	(0,25)	0.0	(0,40)	20.7	(0,25)	203.1	(0,24)	0.0	(0,24)	0.0	(0,38)	16.1	(1,24)	204.3
22	(25,14)	0.0	(25,14)	0.0	(26,13)	0.0	(25,13)	172.2	(26,12)	0.0	(26,12)	0.0	(27,11)	0.1	(21,22)	175.8
23	(23,20)	0.0	(23,20)	0.0	(23,20)	0.0	(23,20)	195.6	(23,19)	0.2	(23,19)	0.2	(23,19)	0.2	(22,23)	201.2
24	(22,23)	0.1	(22,23)	0.1	(22,23)	0.1	(22,24)	217.3	(22,22)	1.2	(22,22)	1.2	(21,23)	0.4	(22,23)	223.6
25	(18,27)	0.6	(18,28)	0.5	(0,47)	37.8	(20,28)	147.6	(17,27)	0.9	(17,28)	1.0	(0,46)	32.1	(19,28)	151.8
26	(22,25)	1.2	(21,27)	0.8	(21,28)	0.8	(22,28)	155.3	(22,25)	1.4	(21,27)	1.5	(21,28)	1.8	(22,26)	161.9
27	(18,26)	0.3	(17,27)	0.7	(0,45)	51.4	(19,27)	150.7	(17,26)	0.3	(17,26)	0.3	(0,44)	43.2	(18,26)	155.9
28	(21,25)	0.8	(21,25)	0.8	(21,26)	0.3	(22,26)	159.7	(21,24)	1.3	(21,25)	0.8	(20,26)	1.8	(21,25)	168.3
29	(0,26)	0.0	(0,26)	0.0	(0,43)	23.8	(0,26)	225.3	(0,25)	0.0	(0,25)	0.0	(0,41)	18.9	(1,25)	227.0
30	(20,25)	0.5	(20,26)	0.1	(19,28)	0.9	(21,26)	256.5	(20,24)	0.7	(19,26)	0.6	(18,28)	2.0	(20,26)	263.6
31	(0,25)	0.0	(0,25)	0.0	(0,42)	32.0	(0,25)	226.1	(0,24)	0.0	(0,24)	0.0	(0,40)	25.4	(1,24)	228.0
32	(20,25)	0.1	(20,25)	0.1	(19,26)	0.3	(20,26)	259.3	(19,24)	0.6	(19,25)	0.4	(17,28)	3.4	(20,25)	267.4

Table 6
Average error of the approximate costs from simulation.

cv^2	L	Model 2 (%)				Model 1 (%)			
		$E[O]$	$E[S]$	$E[I]$	TC^*	$E[O]$	$E[S]$	$E[I]$	TC^*
0.23	1	−11.3	−12.4	−0.2	0.0	−14.5	−8.1	−0.2	0.0
0.4	1	−8.6	−10.5	0.0	0.0	−12.0	−4.5	0.1	0.1
0.63	1	−5.8	−6.6	0.2	0.2	−6.9	0.3	0.3	0.4
1	1	−9.2	−2.7	0.8	0.2	−10.4	5.5	0.9	0.6
0.23	2	−23.0	−7.5	2.7	0.0	−28.2	−5.7	2.7	0.0
0.4	2	−21.5	−6.6	3.1	−0.1	−24.7	−4.4	3.2	0.0
0.63	2	−11.8	−5.0	3.9	0.1	−16.8	−1.8	3.9	0.2
1	2	−15.0	−3.2	4.9	0.2	−16.4	0.1	5.2	0.4

run times. The algorithms take a maximum of 100 s to find the best parameters r^* and Q^*. In most instances, less than 1 min is required to find the best (r^*, Q^*) parameters. This demonstrates the efficiency of the proposed algorithm. Moreover, the run time increases with the variability of the demand but the increase is still acceptable since it is around 1 min. The results of comparisons between our models and the simulation model are shown in Tables 4 and 5. The results show that the proposed models perform very well. The percentage difference of the total cost stemming from our models deviates by a maximum of 1.1% and 2.5% from the simulation model for $L=1$ and $L=2$ respectively. In addition, the proposed models provide near optimal r and Q parameters. In all parameter settings and when $L=1$, we see that r^* and Q^* are equal to or deviate slightly from the optimal reorder point and the order Q stemming from the simulation. While for $L=2$, Model 2 provides performance similar to Model 1, it gives a more accurate estimation of the parameters (r, Q) in some instances. Integrating the impact of perishability during the lead time yields a better estimation of the best (r, Q) policy and its associated cost (i.e. Model 2). This is particularly true as the demand variability increases. When the demand variability is low, Model 2 can be worse than Model 1. This is partly due to the fact that a low demand variability will reduce the risk of outdating during the lead time and consequently, items will probably not perish during the lead time. However, Model 2 is based on the assumption that perishability may occur during the lead time, which will affect the accuracy of the policy parameters and yield better performance of Model 1.

Finally we tested the accuracy of each operating cost in comparison with the true cost obtained from simulation. The results are presented in Table 6 where the average deviations over the 32 test problems of each cost ($E[O]$, $E[S]$, $E[I]$) are reported. We observe that, on average, the bounds of $E[O]$ and $E[S]$ achieve an overestimation while the expected inventory level $E[I]$ provides an underestimation of the true corresponding costs. This underestimation is partly due to the assumption of ignorance of $E[O]$ on the calculation of $E[I]$. These errors, however, are still acceptable since their impact on the total operating cost is negligible.

Sensitivity analysis: To study the effects of changing parameter values on the best policy, a sensitivity analysis is performed by changing the value of each of the parameters from −60% to 60% in steps of 10%, changing one parameter at a time and keeping the remaining parameters unchanged. The sensitivity analysis is examined for a Gamma demand distribution with $cv^2=0.63$, $K=200$, $c=10$, $p=20$ and $w=10$. The results are plotted in Figs. 7–14. We observe that the best minimum cost is sensitive to changes in c and K, whereas it is insensitive to changes in w. Its sensitivity, however, grows as the lost sales cost p gets smaller. Similarly, when we explore the sensitivity of the best minimum total cost to changes in demand variability, we observe that it is significantly sensitive to changes in σ while keeping μ fixed. In fact, it appears that an increase in demand variance results in a higher dispersion of demand and consequently increases the cost. Q^* is highly sensitive to changes in σ (cf. Tables 4 and 5) and in the ordering cost K (cf. Fig. 14) and slightly sensitive to changes in p, c and w. However, r^* is most sensitive to p and K (cf. Figs. 11 and 12) and to σ (cf. Tables 4 and 5).

Comparison with Chiu's model: Results of our comparison are shown in Tables 4 and 5, where the relative error is also calculated by comparing the optimal simulated total cost to the optimal cost evaluated by simulation but with the best (r^*, Q^*) parameters stemming from approximate models (the proposed models and Chiu's model). Apart from this, we reported the sensitivity of Chiu's model to the cost parameters in Figs. 7–14. We see that the

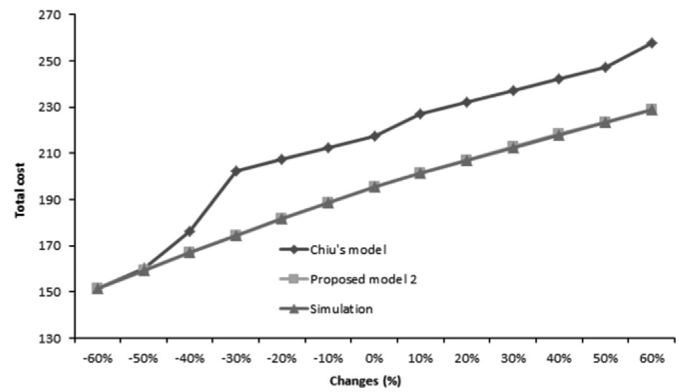

Fig. 7. Impact of the ordering cost K.

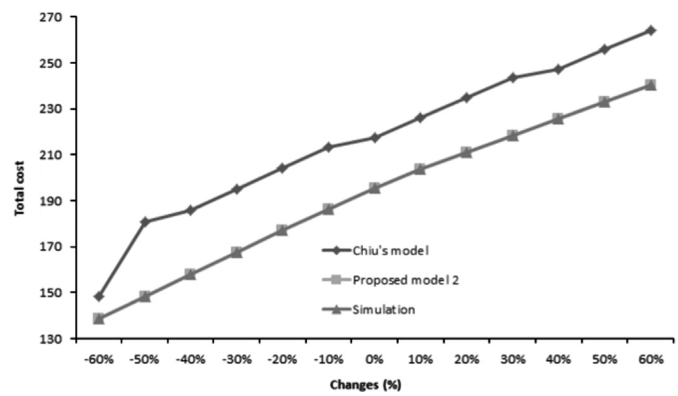

Fig. 8. Impact of the purchasing cost c.

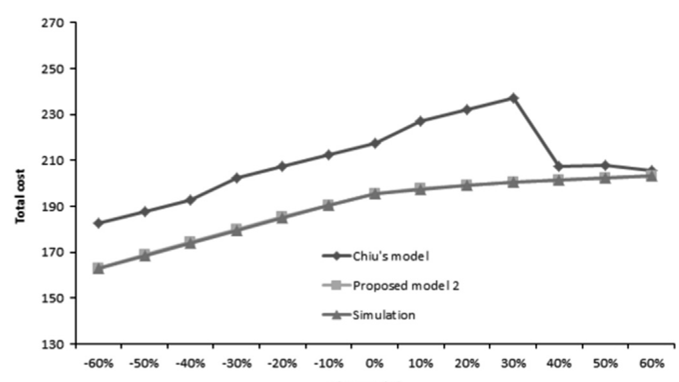

Fig. 9. Impact of the lost sales cost p.

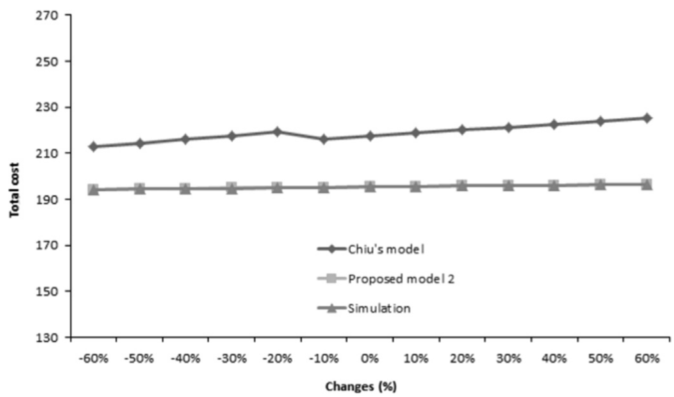

Fig. 10. Impact of the outdating cost w.

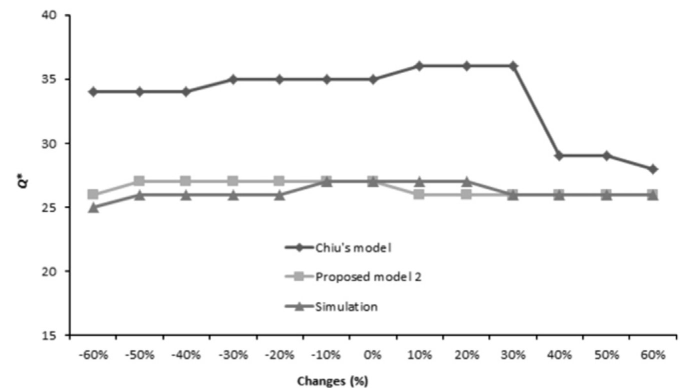

Fig. 13. Changes in Q^* w.r.t the lost sales cost p.

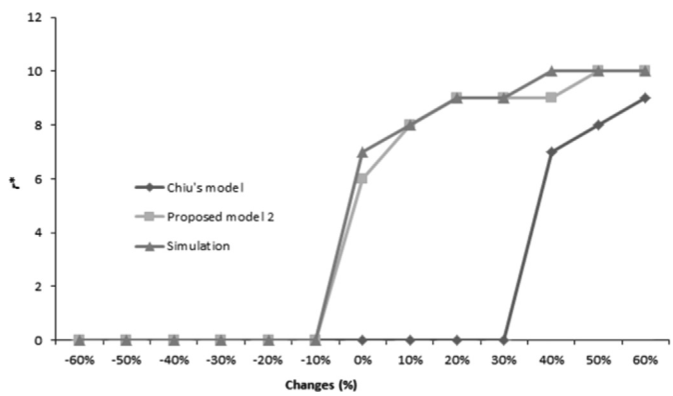

Fig. 11. Changes in r^* w.r.t the lost sales cost p.

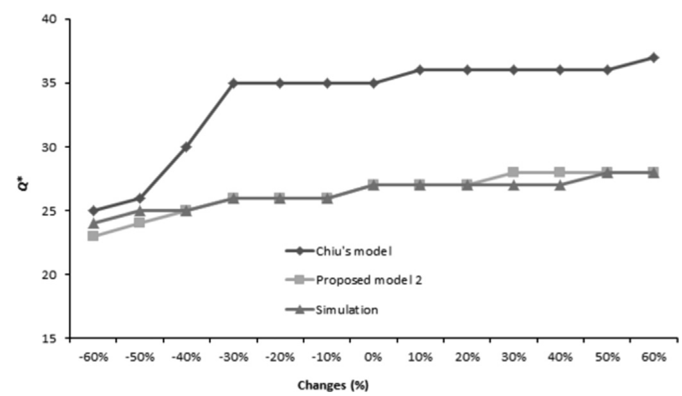

Fig. 14. Changes in Q^* w.r.t the ordering cost K.

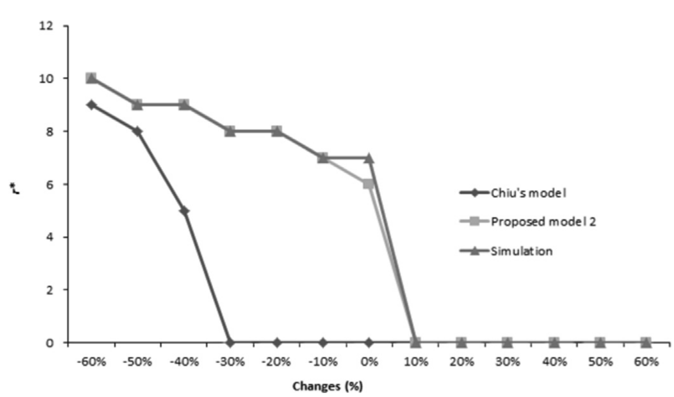

Fig. 12. Changes in r^* w.r.t the ordering cost K.

cost reduction when using the proposed, rather than Chiu's model, is negligible for small values of K, whereas for larger values of K, the cost reduction is quite significant. Moreover, it appears that Chiu (1995a) performs worse not only when the ordering cost increases, but also when lost sales cost decreases (cf. Figs. 7 and 9). The gain using our model is more significant when the ordering cost K increases or when the lost sales cost p decreases with respect to the other cost components. The larger errors of Chiu's model are due to the underestimation of the expected outdating quantity. In fact, when K increases or p decreases, it is more profitable to lose demand than to hold items in stock. Thus, Chiu's approximation underestimates the real expected outdating quantity, since it considers the demand during the lead time that might be fully or partially lost. This underestimation allows Chiu's model to call for a larger Q^* (e.g., test problem number 29 in Table 5). In addition, one can observe that Chiu's model is also more sensitive to demand variability than the proposed models. In fact, from Tables 4 and 5, we see that the performance of Chiu's model worsens as the coefficient of variation of the demand increases. One may conclude from this section that the proposed model provides a good approximation of the optimal (r, Q) policy, while Chiu's model is unsuitable especially when (i) the ordering cost K is relatively high, (ii) the lost sales cost p is relatively small with respect to the remaining cost components or (iii) for highly variable demand.

5. Conclusion

We proposed a new (r, Q) model under lost sales and deterministic lead time. Based on bounding the best parameters r and Q, an approximate long-run expected total cost is derived and a simple algorithm is designed. Our numerical study showed that the obtained parameters are very close to the optimal (r, Q) policy. A sensitivity analysis is performed, showing that the best minimum cost is significantly affected by changes in K, c, p, μ and σ. Finally, significant cost savings are achieved using our model instead of Chiu's model, particularly for higher ordering cost or demand variability. This work could be extended to the case of multi-echelon inventory systems for perishables. It might be particularly interesting for one-warehouse multi-retailer systems controlled by (r, Q) policies. In this direction, academic research has been very limited so far.

Table 7
Comparison between the proposed models and simulation for Gamma demand with mean 10, $cv^2=1$ and $L=2$.

Cost parameters				Model 1				Model 2				Simulation	
K	c	p	w	(r^*,Q^*) with (17)	Δ (%)	(r^*,Q^*) with (16)	Δ (%)	(r^*,Q^*) with (17)	Δ (%)	(r^*,Q^*) with (16)	Δ (%)	(r^*,Q^*)	TC^*
10	15	20	15	(21,14)	−0.10	(22,14)	−0.20	(21,14)	−0.10	(22,14)	−0.20	(21,13)	170.16
50	15	20	15	(0,22)	−1.90	(16,21)	0.10	(0,22)	−1.90	(16,21)	0.00	(16,21)	189.33
50	5	20	15	(21,20)	−1.30	(22,20)	−0.70	(21,20)	−1.30	(22,19)	−0.90	(21,22)	97.25
100	5	20	15	(19,23)	−0.70	(20,23)	0.30	(19,23)	−0.70	(20,23)	−0.20	(20,24)	118.5
50	5	40	15	(24,18)	−0.40	(25,17)	−0.20	(24,18)	−0.40	(25,17)	−0.10	(25,18)	103.3
10	15	20	5	(21,15)	−0.20	(22,14)	−0.10	(21,15)	−0.20	(22,14)	−0.10	(21,13)	170.03
50	15	20	5	(0,22)	−2.10	(17,22)	0.10	(0,22)	−2.10	(17,22)	0.00	(17,22)	188.76
10	15	40	5	(26,12)	0.00	(27,11)	−0.10	(26,12)	0.00	(27,11)	−0.10	(26,12)	175.76
200	15	40	5	(19,26)	−0.60	(20,25)	−0.30	(20,24)	−0.70	(20,24)	−0.70	(21,26)	263.62

Appendix A. Simplification of Eq. (4) to Eq. (5)

Using integration by parts twice, we can rewrite the first and the second term of Eq. (4) as

$$\int_0^r \int_0^{r+Q-x_L} (r+Q-x_L-x_m)\, dF_m(x_m)\, dF_L(x_L) = \int_0^r \underbrace{[(r+Q-x_L-x_m)F_m(x_m)]_0^{r+Q-x_L}}_{=0} dF_L(x_L) + \int_0^r \int_0^{r+Q-x_L} F_m(x_m)\, dx_m dF_L(x_L). \quad (25)$$

Using again integration by parts with respect to the variable x_L, (25) becomes

$$\int_0^r \int_0^{r+Q-x_L} F_m(x_m)\, dx_m\, dF_L(x_L) = \left[F_L(x_L) \int_0^{r+Q-x_L} F_m(x_m)\, dx_m\right]_0^r + \int_0^r F_m(r+Q-x_L)F_L(x_L)\, dx_L = F_L(r)\int_0^Q F_m(x_m)\, dx_m + \int_0^r F_m(r+Q-x_L)F_L(x_L)\, dx_L. \quad (26)$$

Similarly, a first integration by part with respect to the variable x_m of the second term of (4) gives

$$\int_r^\infty \int_0^Q (Q-x_m)\, dF_m(x_m)\, dF_L(x_L) = \int_r^\infty \left(\underbrace{[(Q-x_m)F_m(x_m)]_0^Q}_{=0} + \int_0^Q F_m(x_m)\, dx_m \right) dF_L(x_L), \quad (27)$$

and by integration by parts with respect to the variable x_L, (27) is simplified to

$$\int_r^\infty \int_0^Q F_m(x_m)\, dx_m\, dF_L(x_L) = \left[F_L(x_L)\int_0^Q F_m(x_m)\, dx_m\right]_r^\infty = (1-F_L(r))\int_0^Q F_m(x_m)\, dx_m. \quad (28)$$

Summing up (26) and (28), we obtain Eq. (5).

Appendix B. Simplification of Eq. (8) to Eq. (9)

First, we can write

$$\int_{r-Q+x_m}^\infty (x_L-r+Q-x_m)\, dF_L(x_L) = \mu L - r + Q - x_m + \int_0^{r-Q+x_m} (r-Q+x_m-x_L)\, dF_L(x_L), \quad (29)$$

and

$$\int_r^\infty (x_L-r)\, dF_L(x_L) = \mu L - r + \int_0^r (r-x_L)\, dF_L(x_L). \quad (30)$$

Rewriting Eq. (8) using (29) and (30), we obtain

$$E[S] = \mu L - r + \int_0^Q (Q-x_m)\, dF_m(x_m) + \int_0^Q \int_0^{r-Q+x_m} (r-Q+x_m-x_L)\, dF_L(x_L)\, dF_m(x_m) + \int_Q^\infty \int_0^r (r-x_L)\, dF_L(x_L)\, dF_m(x_m). \quad (31)$$

Second, as in Appendix A, we use integration by parts to simplify Eq. (31). The last two terms of (31) can be simplified to

$$\int_0^Q \int_0^{r-Q+x_m} (r-Q+x_m-x_L)\, dF_L(x_L)\, dF_m(x_m) = \int_0^Q \underbrace{[(r-Q+x_m-x_L)F_L(x_L)]_0^{r-Q+x_m}}_{=0} dF_m(x_m) + \int_0^Q \int_0^{r-Q+x_m} F_L(x_L)\, dx_L\, dF_m(x_m), \quad (32)$$

and

$$\int_Q^\infty \int_0^r (r-x_L)\, dF_L(x_L)\, dF_m(x_m) = \int_Q^\infty \underbrace{[(r-x_L)F_L(x_L)]_0^r}_{=0} dF_m(x_m) + \int_Q^\infty \int_0^r F_L(x_L)\, dx_L\, dF_m(x_m) = (1-F_m(Q))\int_0^r F_L(x_L)\, dx_L. \quad (33)$$

Integrating (32) by parts with respect to x_m, we get

$$\int_0^Q \int_0^{r-Q+x_m} F_L(x_L)\, dx_L\, dF_m(x_m) = \left[F_m(x_m)\int_0^{r-Q+x_m} F_L(x_L)\, dx_L\right]_0^{r-Q+x_m} - \int_0^Q F_L(r-Q+x_m)\, dx_L F_m(x_m)\, dx_m = F_m(Q)\int_0^r F_L(x_L)\, dx_L - \int_0^Q F_L(r-Q+x_m)\, dx_L F_m(x_m)\, dx_m. \quad (34)$$

Third, we simplify the first integral of Eq. (31) to

$$\int_0^Q (Q-x_m) dF_m(x_m) = \int_0^Q F_m(x_m)\, dx_m. \quad (35)$$

Finally, substituting Eqs. (33), (34) and (35) into (31) yields Eq. (9).

Appendix C. Simplification of Eq. (11) to Eq. (12)

The first term of Eq. (11) can be written as

$$\int_0^{Q-r} \int_0^\infty x\, dF_L(x_L)\, dF_m(x_m) = \mu L F_m(Q-r). \quad (36)$$

Using (29) from Appendix B, the second term of Eq. (11) can be written as

$$\int_{Q-r}^{Q}\int_{r-Q+x_m}^{\infty}(x_L-r+Q-x_m)\,dF_L(x_L)\,dF_m(x_m)=\int_{Q-r}^{Q}(\mu L-r+Q-x_m)\,dF_m(x_m)+\int_{Q-r}^{Q}\int_{0}^{r-Q+x_m}(r-Q+x_m-x_L)\,dF_L(x_L)\,dF_m(x_m). \quad (37)$$

By integration by parts, the first term of (37) becomes

$$\int_{Q-r}^{Q}(\mu L-r+Q-x_m)\,dF_m(x_m)=(\mu L-r)F_m(Q)-\mu LF_m(Q-r)+\int_{Q-r}^{Q}F_m(x_m)\,dx_m. \quad (38)$$

By integration by parts, the second term of (37) is simplified to

$$\int_{Q-r}^{Q}\int_{0}^{r-Q+x_m}(r-Q+x_m-x_L)\,dF_L(x_L)\,dF_m(x_m)=\int_{Q-r}^{Q}\int_{0}^{r-Q+x_m}F_L(x_L)\,dx_L\,dF_m(x_m)+\int_{Q-r}^{Q}\underbrace{[(r-Q+x_m-x_L)F_L(x_L)]_0^{r-Q+x_m}}_{=0}\,dF_m(x_m). \quad (39)$$

Using again integration by parts, (39) can be written as

$$\int_{Q-r}^{Q}\int_{0}^{r-Q+x_m}F_L(x_L)\,dx_L\,dF_m(x_m)=\left[F_m(x_m)\int_0^{r-Q+x_m}F_L(x_L)\,dx_L\right]_{Q-r}^{Q}-\int_{Q-r}^{Q}F_L(r-Q+x_m)F_m(x_m)\,dx_m$$
$$=F_m(Q)\int_0^r F_L(x_L)\,dx_L-\int_{Q-r}^{Q}F_L(r-Q+x_m)F_m(x_m)\,dx_m. \quad (40)$$

Similarly, using (30) and integration by parts, the last term of Eq. (11) can be simplified to

$$\int_{Q}^{\infty}\int_{r}^{\infty}(x_L-r)\,dF_L(x_L)\,dF_m(x_m)=(\mu L-r)(1-F_m(Q))+\int_0^r F_L(x_L)\,dx_L-F_m(Q)\int_0^r F_L(x_L)\,dx_L. \quad (41)$$

Summing up Eqs. (36), (40) and (41), we obtain (12).

Appendix D. Impact of using (17) instead of (16)

The cost parameters that yield to better performance of the models with Eq. (17) than with Eq. (16) are reported in Table 7. For all other cost combinations taken from Tables 4 and 5, the models provide the same policy. We observe that there are only 2 cases where the best total cost of the proposed models with $E[I]$ given by Eq. (16) achieves a better performance. This occurs when the cost parameters are $K=50, c=15, p=20$ and $w=\{5,15\}$. The error from the simulation model is 2% less than the proposed models with $E[I]$ expressed by Eq. (17). For all other cases, the models provide either the same policy parameters or a slightly different policy with roughly the same performance (an error of 1% from simulation cost).

Appendix E. Proof of Lemma 1

Proof. Suppose that at time t an order of size Q is placed. At time $t+m+L$, the order Q must be depleted by demand or else it perishes because of reaching its usable lifetime. Thus, if no demand occurs during $m+L$, Q will be totally outdated and a new order is triggered at $t+m+L$. If a demand occurs during $m+L$, then an order will be placed before $t+m+L$. This proves that at any time the cycle length is less than or equal to $m+L$. Therefore, the expected cycle length is also less than or equal to $m+L$.□

Appendix F. Proof of Proposition 1

Proof. The proof consists of two parts: first we prove that Q^* is finite, then we show that $Q^* \le \mu(m+L)$. From Lemma 1, the cycle length must be less than or equal to $m+L$. Suppose that there exists a $\tilde{Q}$ minimizing the total cost such that the cycle length is equal to $m+L$. Then, evaluating Eq. (22) on $\tilde{Q}$ yields

$$TC(r,\tilde{Q})=\frac{G(r,\tilde{Q})}{m+L}+I(r,\tilde{Q}). \quad (42)$$

Using the first-order conditions, we derive the following equations:

$$\frac{\partial}{\partial Q}\left(\frac{G(r,Q)}{m+L}\right)=\frac{c+w\left(F_m(Q)+\int_0^r F_L(x_L)f_m(r+Q-x_L)\,dx_L\right)}{m+L}>0, \quad (43)$$

$$\frac{\partial I(r,Q)}{\partial Q}=\frac{h}{2}\left(1-F_m(Q)-\int_0^r F_L(x_L)f_m(r+Q-x_L)\,dx_L\right), \quad (44)$$

and

$$\frac{\partial^2 I(r,Q)}{\partial Q^2}=\frac{-h}{2}\left([1-F_L(r)]f_m(Q)+\int_0^r f_L(x_L)f_m(r+Q-x_L)\,dx_L\right)\le 0. \quad (45)$$

Eq. (45) implies that $I(r,Q)$ is concave in Q for a fixed r and attains its maximum when $\frac{\partial I(r,Q)}{\partial Q}=0$. Since $\lim_{Q\to+\infty}\frac{\partial I(r,Q)}{\partial Q}=0$, then $\frac{\partial I(r,Q)}{\partial Q}>0$ and $I(r,Q)$ is an increasing function for all finite Q. From Eq. (43), $\frac{G(r,Q)}{m+L}$ also increases in Q. Consequently, if $Q^*>\tilde{Q}$, then the cycle length is also $m+L$ and since Eq. (42) is an increasing function on Q, Q^* must satisfy $Q^*\le\tilde{Q}$. This proves that Q^* is finite.

Finally, from the analogy with queues with impatient customers, the $(r,\tilde{Q})$ inventory model can be seen as a $D^{\tilde{Q}}/G/1+D$ queue where a batch of size $\tilde{Q}$ arrives after $m+L$ units of time and has a deterministic patience m. $\tilde{Q}$ is depleted by the demand (the service G) and any item still in stock after m units of time is discarded from the queue. Under light and heavy traffic regimes, the offered load ρ of a $D^{\tilde{Q}}/G/1+D$ queue must be less than or equal to 1. Thus,

$$\rho=\left(\frac{\tilde{Q}}{m+L}\right)\frac{1}{\mu}\le 1 \Rightarrow Q^*\le\tilde{Q}\le\mu(m+L).\square \quad (46)$$

Appendix G. Proof of Proposition 2

Proof. For a given Q, taking the first derivative on r of $G(r,Q)$, $CL(r,Q)$ and $I(r,Q)$, we get

$$\frac{\partial G(r,Q)}{\partial r}=-p+[p+wF_m(Q)]F_L(r)+w\int_0^r F_L(x_L)f_m(r+Q-x_L)\,dx_L, \quad (47)$$

$$\frac{\partial}{\partial r}\left(\frac{1}{CL(r,Q)}\right)=\frac{1-(1-F_m(Q))F_L(r)+\int_0^r F_L(x_L)f_m(r+Q-x_L)\,dx_L}{\mu[CL(r,Q)]^2}\ge 0, \quad (48)$$

and

$$\frac{\partial I(r,Q)}{\partial r}=\frac{h}{2}\left(1+F_L(r)-\int_0^r f_L(x_L)F_m(r+Q-x_L)\,dx_L\right)>0. \quad (49)$$

Thus, for $y>0$, we have

$$TC(\tilde{r}+y,Q)=\frac{G(\tilde{r}+y,Q)}{CL(\tilde{r}+y,Q)}+I(\tilde{r}+y,Q)\ge\frac{G(\tilde{r},Q)}{CL(\tilde{r}+y,Q)}+I(\tilde{r}+y,Q)$$

$$\geq \frac{G(\tilde{r},Q)}{CL(\tilde{r},Q)} + I(\tilde{r},Q). \tag{50}$$

Eq. (50) shows that the total cost function increases for all values $r \geq \tilde{r}$. Thus, r^* is less than or equal to $\tilde{r}$. Moreover, we have

$$\lim_{r\to 0}\frac{\partial G(r,Q)}{\partial r} = -p < 0 \quad \text{and} \quad \lim_{r\to+\infty}\frac{\partial G(r,Q)}{\partial r} = wF_m(Q) > 0. \tag{51}$$

Since

$$\frac{\partial^2 G(r,Q)}{\partial r^2} = w\int_0^r f_L(x_L) f_m(r+Q-x_L)\, dx_L + [p+wF_m(Q)]f_L(r) \geq 0, \tag{52}$$

there must exist a unique solution $\tilde{r}$ that satisfies $\frac{\partial G(r,Q)}{\partial r} = 0$.□

Appendix H. Proof of Corollary 1

Proof. *Case when* $r > Q$: In Model 1, the expected lost sales are expressed by (9) and we can rewrite the total cost of Model 1 as

$$TC(r,Q) = \frac{G(r,Q) + p\left(\int_0^Q (1-F_L(r-Q+x_m))F_m(x_m)\, dx_m\right)}{CL(r,Q) + \int_0^Q (1-F_L(r-Q+x_m))F_m(x_m)\, dx_m} + I(r,Q). \tag{53}$$

Using the same reasoning as Proposition 2, we can write for $r > Q$

$$\frac{\partial}{\partial r}\left[G(r,Q) + p\left(\int_0^Q (1-F_L(r-Q+x_m))F_m(x_m)\, dx_m\right)\right] = [p+wF_m(Q)]F_L(r) + w\int_0^r F_L(x_L) f_m(r+Q-x_L)\, dx_L - p\left(1 + \int_0^Q F_m(x_m) f_L(r-Q+x_m)\, dx_m\right), \tag{54}$$

and

$$\frac{\partial^2}{\partial r^2}\left[G(r,Q) + p\left(\int_0^Q (1-F_L(r-Q+x_m))F_m(x_m)\, dx_m\right)\right] = wF_m(Q)f_L(r) + pf_L(r) + w\int_0^r f_L(x_L) f_m(r+Q-x_L)\, dx_L - p\left(\int_0^Q F_m(x_m)\frac{\partial}{\partial r} f_L(r-Q+x_m)\, dx_m\right). \tag{55}$$

Since, $F_m(x_m) \leq 1$ for all $0 \leq x_m \leq Q$, then

$$\int_0^Q F_m(x_m)\frac{\partial}{\partial r} f_L(r-Q+x_m)\, dx_m \leq \int_0^Q \frac{\partial}{\partial r} f_L(r-Q+x_m)\, dx_m = f_L(r). \tag{56}$$

From (56), Eq. (55) ≥ 0, consequently (54) is an increasing function on r. In addition, we have

$$\lim_{r\to 0}\frac{\partial}{\partial r}\left[G(r,Q) + p\left(\int_0^Q (1-F_L(r-Q+x_m))F_m(x_m)\, dx_m\right)\right] = -p\left(1 + \int_0^Q F_m(x_m) f_L(-Q+x_m)\, dx_m\right) < 0, \tag{57}$$

and

$$\lim_{r\to+\infty}\frac{\partial}{\partial r}\left[G(r,Q) + p\left(\int_0^Q (1-F_L(r-Q+x_m))F_m(x_m)\, dx_m\right)\right] = wF_m(Q) > 0. \tag{58}$$

Therefore, there is a unique solution of r to (54). Finally, we can easily see that

$$\frac{\partial}{\partial r}\left(CL(r,Q) + \int_0^Q (1-F_L(r-Q+x_m))F_m(x_m)\, dx_m\right)^{-1} > 0 \quad \text{and} \quad \frac{\partial I(r,Q)}{\partial r} > 0.$$

As in (50), the best reorder point $r^* \leq \hat{r}$, where $\hat{r}$ is the reorder point at which (54) vanishes.

Case when $r \leq Q$: When calculating $\hat{r}$, one may find $\hat{r} \geq Q$ so that $\hat{r} \geq Q > r^*$. If the calculation of $\hat{r}$ leads to $\hat{r} < Q$, the best solution is of the type $r^* < Q$. Since,

$$\frac{\partial}{\partial r}\left[G(r,Q) + p\left(\int_{Q-r}^Q (1-F_L(r-Q+x_m))F_m(x_m)\, dx_m\right)\right]\Bigg|_{r=\hat{r}} = pF_m(Q-\hat{r}) \geq 0, \tag{59}$$

and

$$\frac{\partial}{\partial r}\left[G(r,Q) + p\left(\int_{Q-r}^Q (1-F_L(r-Q+x_m))F_m(x_m)\, dx_m\right)\right] = [p+wF_m(Q)]F_L(r) + w\int_0^r F_L(x_L) f_m(r+Q-x_L)\, dx_L - p\left(1 - F_m(Q-r) + \int_{Q-r}^Q F_m(x_m) f_L(r-Q+x_m)\, dx_m\right) \leq \frac{\partial}{\partial r}\left[G(r,Q) + p\left(\int_0^Q (1-F_L(r-Q+x_m))F_m(x_m)\, dx_m\right)\right]. \tag{60}$$

Therefore, r^* is less than or equal to $\hat{r}$.□

References

Berk, E., Gürler, U., 2008. Analysis of the (Q,r) inventory model for perishables with positive lead times and lost sales. Oper. Res. 56 (5), 1238–1246.
Bijvank, M., Vis, I., 2011. Lost-sales inventory theory: a review. Eur. J. Oper. Res. 215 (1), 1–13.
Broekmeulen, R.A., van Donselaar, K., 2009. A heuristic to manage perishable inventory with batch ordering, positive lead-times, and time-varying demand. Comput. Oper. Res. 36 (11), 3013–3018.
Chiu, H., 1995a. An approximation to the continuous review inventory model with perishable items and lead times. Eur. J. Oper. Res. 87 (1), 93–108.
Chiu, H., 1995b. A heuristic (R, T) periodic review perishable inventory model with lead times. Int. J. Prod. Econ. 42 (1), 1–15.
Fontaine, M., Chung, Y., Rogers, W., Sussmann, H., Quach, P., Galel, S., Goodnough, L., Erhun, F., 2009. Improving platelet supply chains through collaborations between blood centers and transfusion services. Transfusion 49 (10), 2040–2047.
Fries, B., 1975. Optimal order policy for a perishable commodity with fixed lifetime. Oper. Res. 23 (1), 46–61.
Goyal, S., Giri, B., 2001. Recent trends in modeling of deteriorating inventory. Eur. J. Oper. Res. 134 (1), 1–16.
Gürler, U., Özkaya, B., 2008. Analysis of the (s, S) policy for perishables with a random shelf life. IIE Trans. 40 (8), 759–781.
Haijema, R., 2013. A new class of stock-level dependent ordering policies for perishables with a short maximum shelf life. Int. J. Prod. Econ. 143 (2), 434–439.
Haijema, R., Minner, S., 2014. Stock-Level Dependent Ordering of Perishables: a Comparison of Hybrid Base-Stock and Constant Order Policies. Working Paper.
Haijema, R., van der Wal, J., van Dijk, N., 2007. Blood platelet production: optimization by dynamic programming and simulation. Comput. Oper. Res. 34 (3), 760–779.
Haijema, R., van Dijk, N.M., van der Wal, J., Sibinga, C., 2009. Blood platelet production with breaks: optimization by SDP and simulation. Int. J. Prod. Econ. 121 (2), 464–473.
Kalpakam, S., Sapna, K., 1994. Continuous review (s, S) inventory system with random lifetimes and positive leadtimes. Oper. Res. Lett. 16 (2), 115–119.
Kalpakam, S., Sapna, K., 1995. (S-1, S) perishable systems with stochastic lead times. Math. Comput. Model. 21 (6), 95–104.
Kalpakam, S., Shanthi, S., 2000. Perishable system with modified base stock policy and random supply quantity. Comput. Math. Appl. 39 (12), 79–89.
Kalpakam, S., Shanthi, S., 2001. A perishable inventory system with modified (S-1, S) policy and arbitrary processing times. Comput. Oper. Res. 28 (5), 453–471.
Kalpakam, S., Shanthi, S., 2006. A continuous review perishable system with renewal demand. Ann. Oper. Res. 143 (1), 211–225.
Karaesmen, I., Scheller-Wolf, A., Deniz, B., 2011. Managing perishable and aging inventories: review and future research directions. In: Kempf, K., Keskinocak, P., Uzsoy, R. (Eds.), Planning Production and Inventories in the Extended Enterprise: A State of the Art Handbook. International Series in Operations Research and Management Science, vol. 151. Springer, New York, pp. 393–436.
Kouki, C., Sahin, E., Jemaï, Z., Dallery, Y., 2013. Assessing the impact of perishability and the use of time temperature technologies on inventory management. Int. J. Prod. Econ. 143 (1), 72–85.

Kouki, C., Sahin, E., Jemaï, Z., Dallery, Y., 2014. Analysis of a periodic review inventory control system with perishables having random lifetime. Int. J. Prod. Res. 52 (1), 283–298.
Kouki, C., Jouini, O., 2015. On the effect of lifetime variability on the performance of inventory systems. Int. J. Prod. Econ. 167 (1), 23–34.
Lian, Z., Liu, L., 1999. A discrete-time model for perishable inventory systems. Ann. Oper. Res. 87, 103–116.
Lian, Z., Liu, L., 2001. Continuous review perishable inventory systems: models and heuristics. IIE Trans. 33 (9), 809–822.
Lian, Z., Liu, X., Zhao, N., 2009. A perishable inventory model with Markovian renewal demands. Int. J. Prod. Econ. 121 (1), 176–182.
Liu, L., Cheung, K., 1997. Service constrained inventory models with random lifetimes and lead times. J. Oper. Res. Soc. 48 (10), 1022–1028.
Liu, L., Lian, Z., 1999. (s, S) continuous review models for products with fixed lifetimes. Oper. Res. 47 (1), 150–158.
Liu, L., Shi, D., 1999. An (s, S) model for inventory with exponential lifetimes and renewal demands. Naval Res. Logist. 46 (1), 39–56.
Liu, L., Yang, T., 1999. An (s, S) random lifetime inventory model with a positive lead time. Eur. J. Oper. Res. 113 (1), 52–63.
Minner, S., Transchel, S., 2010. Periodic review inventory-control for perishable products under service-level constraints. OR Spectr. 32 (4), 979–996.
Nahmias, S., 1975. Optimal ordering policies for perishable inventory. Oper. Res. 23 (4), 735–749.
Nahmias, S., 1977. Higher order approximations for the perishable inventory problem. Oper. Res. 25 (4), 630–640.
Nahmias, S., 1978. The fixed charge perishable inventory problem. Oper. Res. 26 (3), 464–481.
Nahmias, S., 1982. Perishable inventory theory: a review. Oper. Res. 30 (4), 680–708.
Nahmias, S., 2011. Perishable Inventory Systems. Springer, New York.
Nandakumar, P., Morton, T., 1993. Near myopic heuristics for the fixed-life perishable problem. Manag. Sci. 39 (12), 1490–1498.
Olsson, F., Tydesjö, P., 2010. Inventory problems with perishable items: fixed lifetimes and backlogging. Eur. J. Oper. Res. 202 (1), 131–137.
Roberti, M., 2005. RFID will Help Keep Perishables Fresh. Technical Report. Also available as ⟨http://www.rfidjournal.com/article/view/1775/1/1⟩.
Schmidt, C., Nahmias, S., 1985. (S-1, S) policies for perishable inventory. Manag. Sci. 31 (6), 719–728.
Silver, E., Pyke, D., Peterson, R., 1998. Inventory Management and Production Planning and Scheduling, 3rd edition. John Wiley and Sons, New York.
Tekin, E., Gürler, U., Berk, E., 2001. Age-based vs. stock level control policies for perishable inventory system. Eur. J. Oper. Res. 124 (2), 309–329.
van Donselaar, K., Broekmeulen, R., 2012. Approximations for the relative outdating of perishable products by combining stochastic modeling, simulation and regression modeling. Int. J. Prod. Econ. 140 (2), 660–669.
Weiss, H., 1980. Optimal ordering policies for continuous review perishable inventory models. Oper. Res. 28 (2), 365–374.
Williams, C.L., Patuwo, B., 1999. A perishable inventory model with positive order lead times. Eur. J. Oper. Res. 116 (2), 352–373.
Williams, C.L., Patuwo, B., 2004. Analysis of the effect of various unit costs on the optimal incoming quantity in a perishable inventory model. Eur. J. Oper. Res. 156 (1), 140–147.
Zhou, D., Leung, L.C., Pierskalla, W.P., 2011. Inventory management of platelets in hospitals: Optimal inventory policy for perishable products with regular and optional expedited replenishments. Manuf. Serv. Oper. Manag. 13 (4), 420–438.

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Erratum

Corrigendum to "A Lost Sales (r,Q) Inventory Control Model for Perishables with Fixed Lifetime and Lead Time" [International Journal of Production Economics, 168 (2015) 143–157]

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