Int. J. Production Economics 157 (2014) 158–169

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Optimal ordering, issuance and disposal policies for inventory management of perishable products

Rene Haijema [a,b]

[a] TI Food and Nutrition, Wageningen, The Netherlands
[b] Operations Research and Logistics, Wageningen University, Hollandseweg 1, 6706 KN Wageningen, The Netherlands

ARTICLE INFO

Article history:
Received 1 October 2012
Accepted 12 June 2014
Available online 21 June 2014

Keywords:
Inventory management
Perishability
Food
Blood product
Disposal policy
Issuing policy
Withdrawal policy
Order policy
Simulation
Markov decision problem
Dynamic programming

ABSTRACT

Perishables, such as packed fresh food and pharmaceutical products (a.o. blood products), typically have a fixed shelf life set by a fixed use-by date or sell-by date. Despite their limited life time, orders in practice are usually based on the stock level irrespective of the ages of the products in stock. The management of inventories of such products can be improved by applying stock-age dependent ordering, issuing, and disposal policies.

This paper investigates cost reductions that can be achieved by an optimal stock-age dependent ordering, issuing, or disposal policy as obtained by Stochastic Dynamic Programming. Orders are made before the uncertain demand is revealed. When demand turns out to be relatively low, a disposal policy enables to get rid of excess (old) stock. Disposal decisions are an understudied area, but may be relevant to retailers for which displaying the freshest items is of high importance. Also blood banks prefer not to issue products that are about to expire as transfusion of younger blood products is more effective. This paper fills a research gap identified in Karaesmen et al. (2011): the paper appears to be the first to report optimal stock-age dependent disposal decisions, both under a base stock policy and under optimal stock-age dependent ordering. Results of optimal stock-age dependent ordering, disposal, and issuance are compared to a base stock policy, which is commonly used in practice.

Under FIFO issuance, the added value of an optimal disposal policy is high. An optimal disposal policy in combination with optimal ordering reduces the average costs only when issuing old products is penalized, e.g. by selling at a discounted price. Under LIFO issuance, an optimal disposal policy has significant impact when orders are set by a BSP, but not under optimal stock-age dependent ordering. When no penalty or discount applies, disposals reduce costs only in case of suboptimal ordering, e.g. by a BSP.

1. Introduction

The inventory control problem of perishable products consists of (at least) three optimization problems: the problem of determining an optimal order policy, an optimal issuing policy, and an optimal disposal policy. In a periodic review setting, the order policy prescribes how much to order. The issuing policy, in the literature also called the picking, dispatching, or withdrawal policy, sets the order in which products are taken from the shelf to meet the demand. The disposal policy prescribes how many products to remove from stock. In many studies, order policies are limited to base stock policies and issuing policies are either FIFO (=oldest first) or LIFO (=youngest first), or mixtures of FIFO and LIFO. Usually the disposal policy is to dispose products when the expiration date has exceeded, but one may decide to dispose prematurely some products, that is before the expiry date has elapsed. Practical reasons for disposing products prematurely are the following: (1) a retailer who sells at a high market segment likes to display only the freshest products, (2) the value of old products is perceived lower by customers, and thus the retailer provides a discount to the sales price, and (3) blood banks and hospitals may want to dispose the oldest blood products if stock levels are high, as transfusing younger blood products is more effective. The disposal decision does not result in much additional product waste, if the disposed products are kept behind in a back storage to meet the demand in case one runs out of stock, or when disposed products are sold at another market. In our analysis we have not included the selling of disposed products.

In this paper the focus is on a product with a fixed maximal shelf life of m days, after which a product cannot be sold. When information is available about the ages of the products in stock, an optimal policy is the so-called stock-age dependent. Nevertheless, in practice one finds mostly stock-level dependent policies, which

E-mail address: Rene.Haijema@wur.nl

http://dx.doi.org/10.1016/j.ijpe.2014.06.014
0925-5273/

do not minimize the operational and managerial costs of an inventory system. Many Automated Store Ordering (ASO) and Computer Assisted Ordering (CAO) systems apply base stock policies (BSP), also called the order-up-to S policies, for setting periodically the order quantity irrespective of the ages of the products in stock. We study how much one may improve the inventory control by stock-age dependent issuance or disposal, without changing such ordering systems, and compare the results with the case of optimal stock-age dependent ordering. We check the added value of a disposal policy by computing and simulating a combined ordering and disposal policy and an optimal ordering policy with no premature disposals.

Recently, Haijema (2011) shows that stock-age dependent issuance may not be FIFO in a lost sales inventory system. This result was first shown in Pierskalla and Roach (1972). In some cases of relatively high stock levels, it appears to be optimal to not issue the oldest products but to let them expire as this reduces holding cost and increases sales revenues by selling less products at discounted prices. Optimal issuance may result in less shortages without generating more outdating. Apparently optimal issuance results in a better timing of disposing of products. An optimal disposal policy is easier to implement than an age dependent issuing policy, as a disposal decision is made only once a day, whereas depending on the context the issuing decision is made multiple times a day. Moreover, stock-age dependent issuance is only applicable in cases in which the picking of products is controlled, like at blood banks and hospitals. In supermarkets the store manager has no full control on the issuing policy, as customers take products from the shelf themselves: their picking order is a mixture between FIFO and LIFO. By mirroring products on the shelf, i.e. display the oldest products at the front of the shelf, and keeping the freshest products in the back storage, store managers force the picking order to become close(r) to FIFO.

We investigate by how much the costs of a base stock policy (BSP) can be improved by an optimal ordering policy, an optimal issuing policy, and an optimal disposal policy. In particular, we are also interested whether an optimal ordering policy can be improved by a combined optimal ordering and disposal policy. Optimal stock-age dependent policies are computed numerically by Stochastic Dynamic Programming (SDP). Throughout this paper, the term ‘optimal’ policy stands for an optimal stock-age dependent policy as obtained by SDP.

Contribution: This paper is the first paper that studies integrally optimal stock-age dependent ordering, issuing, and disposal policies for perishable inventories. Therefore a Stochastic Dynamic Programming (SDP) model is developed that simultaneously optimizes the ordering and the disposal decision. Numerical experiments provide insights into the impact of stock-age dependent ordering, issuing and disposal policies on costs, waste, and shortages. Besides being of theoretical value, these insights are valuable to the practice of managing inventories of perishable products.

Outline: In the next section, we present related literature on the three optimization problems when dealing with perishable inventory management. The combined problem of optimal ordering and optimal disposal is formulated as a Markov decision problem (MDP) that can be solved by Stochastic Dynamic Programming (SDP) in Section 3. For a broad design of experiments, defined in Section 4, results are presented for both FIFO and LIFO issuance in Section 5. Finally, Section 6 concludes the paper by a discussion and a summary of the main findings and insights.

2. Literature

2.1. Order systems and order policies

In the last (two) decades Automated Store Ordering (ASO) and Computer Assisted Ordering (CAO) systems have become more and more in use to improve the efficiency and the effectiveness of inventory management at retailers (van der Vorst et al., 1998; van Donselaar et al., 2006). Using point of sales data the stock levels in the store are updated automatically, and either an order is placed automatically by an ASO system or an order quantity is proposed by a CAO system. In the latter case the order is to be processed further by a decision maker who is responsible for the ordering process. ASO systems are mainly found at supermarkets and retailers that offer a great variety of products. In settings with a smaller product assortment like blood banks and hospitals a CAO system can be used.

Mechanisms to set an order quantity are ordering policies that take as input the actual stock level (tracked by the system) and any information on future demand (based on historical information on demand realizations and predictions based on promotions). In case stock replenishment happens daily, often traditional periodic review ordering policies are implemented like a BSP. An example of a BSP is to order every weekday d up to an stock level S_d. In case of daily ordering, the review period R, that is, the time between two successive order moments, is 1. In some studies a batch size for ordering is included explicitly: throughout this paper the batch size is set to 1. As most ASO and CAO systems are designed for the inventory management of non-perishables, these systems do not acknowledge the aging of products in stock and thus do not anticipate outdating of old stock. In recent work of van Donselaar et al. (2006), Haijema et al. (2007), Broekmeulen and van Donselaar (2009) and Haijema (2013) new heuristics and ordering policies are proposed to improve the inventory management of perishables through such systems. These policies are not yet (widely) implemented in current ASO and CAO systems, which thus may result in unnecessary outdating of perishable products at retailers.

2.2. Issuing policies

With respect to the issuing policy, most studies found in the literature rely on a simple issuing policy, like FIFO (oldest product first) or LIFO (youngest product first), or a combination of the two. For an overview, see the reviews of Nahmias (1982) and Karaesmen et al. (2011). In case consumers may pick the products themselves, LIFO is a common way of modeling the selection behavior of customers, as many consumers prefer the youngest products available, that are products with the longest use-by-date. Store managers try to influence the selection behavior by putting the oldest product upfront. In case consumers do not pick themselves, products are usually issued by the supplier in the FIFO order as this reduces the number of products that outdate. Although FIFO issuance seems to be intuitively optimal, Pierskalla and Roach (1972) and Haijema (2011) show that in a lost sales inventory system, FIFO issuance may not be cost-optimal when a base stock policy is applied and old products are sold at discounted prices. In Haijema (2011) a Markov decision problem is formulated in which a day is split into multiple time interval, called the epochs. At the end of each epoch products are taken from stock to meet the demand for that epoch. In SDP terminology, the state is the tuple: day of the week, epoch, the number of products in stock in each of the m age categories, and the number of products demanded. The issuing action is the tuple: the number of products to issue from each of the m age categories. Stock is replenished according to a base stock policy, and only at the end of the day products that expire are disposed. The results in Haijema (2011) indicate that it may be occasionally better to dispose some products before they expire. That is, FIFO is not always optimal.

In some studies on the issuing policy, demand is specified per age category and the utility of a product depends on its age. The optimization problem is then to issue such that the long-run utility is maximized, allowing several degrees of substitution, see Derman and Klein (1958), Pierskalla and Roach (1972), and Deniz et al. (2010). Pierskalla and Roach (1972) show, by a single numerical example, that FIFO issuance is sub optimal for a lost sales inventory system of products with a fixed shelf life. Their results hold for a specific objective function that maximizes the utility of fulfilling the demand. In their study, the utility function assigns an increasing utility to meeting demand of customers with stronger age preferences: customer who requires the youngest products are valued higher than demand set by more flexible customers, who do mind to get somewhat old(er) products. Pierskalla and Roach do not present an optimal issuing policy, nor do they numerically investigate how big the difference between FIFO issuance and an optimal issuing policy could be. In our models, presented in the next section, we assume that the utility of a product is set by the age of products issued, rather than the customer class. This is in line with the discounting practice at Dutch supermarkets: all products have an equal profit margin, except for products that are discounted because they will expire by the end of the day. In blood banks and hospitals no discounting is applied although the utility is decreasing in the product age. In that setting we propose to set a fictitious penalty on issuing old products. The objective function in this study is slightly different than that in Pierskalla and Roach (1972), but again we will see that FIFO issuance is sub-optimal. In addition, we show for 43 cases the difference in utility between optimal issuance and FIFO issuance.

2.3. Disposal policies

One of the first studies of an optimal disposal policy for perishables is found in Veinott (1960), in which demand is deterministic. The first stochastic optimal disposal model is found in Martin (1986). For a perishable product with a fixed maximal lifetime and stochastic demand, Martin (1986) studied the optimal timing of disposition and the probability that a new product gets disposed. The disposal policy is constructed independent of the order policy and the salvage value, and the inventory holding cost. The disposal policy developed in our paper minimizes the operational and managerial costs, and considers next to the stock level, the ages of the products in stock, the cost structure, as well as the replenishment process set by the order policy. The optimal disposal policy will be explained in Section 3.

Disposal policies are studied also in remanufacturing systems: after re-using a number of times a component it is disposed to prevent a re-manufactured product to fail due to unreliable components. The combined ordering and disposal policy is studied mainly in the context of remanufacturing. The structure in those models is much different than that in our problem of a perishable product with a short maximal shelf life of m days.

The overview of Karaesmen et al. indicates that disposal policies are an understudied area in the literature on inventory management of perishable products (Karaesmen et al., 2011): "*When customers prefer fresher goods, disposal and outdating are key decisions that affect the age-composition (freshness) of inventory, and can influence the demand. Analysis of simple and effective disposal and outdating policies, coordination of disposal with replenishment policies* [...] *remain among the understudied research problems.*" In the next section we present an MDP model for deriving an cost-optimal stock-age dependent ordering and disposal policy. As such this paper contributes to a better understanding of the optimal control of inventories of perishables from a theoretical as well as a practical point of view.

3. Markov decision models

The three inventory optimization problems that we study are the problems of determining an optimal order policy, an optimal issuing policy, and an optimal disposal policy. Each of these problems can be formulated as a Markov decision problem (MDP). The objective function to minimize is a linear cost function consisting of inventory holding costs, shortage costs, waste costs (that covers unit ordering costs), and discounting or penalty costs for issuing old products that will expire at the end of the day. The penalty cost can be seen as a real or fictitious discount that a supplier gives to its consumers. In case old products show a decrease in utility, the unit penalty costs act as an incentive to issue not too many old products. In the MDP models below, we assume a single type of customer, whose demand can be fulfilled by products of any of the m age categories.

The computation of a cost-optimal order policy is well addressed in the literature: in virtually all studies one assumes a fixed issuing policy and no premature disposals (that is, only products that are expired will be disposed). Recently, the computation of an optimal stock-age dependent issuing policy is addressed in Haijema (2011), for a practical setting of issuing blood products at hospitals that order using a base stock policy. The results indicate that sometimes it is better to issue young products and let the oldest products in stock expire. As controlling the issuance may be impractical in many settings, especially at supermarkets, we check whether a similar cost reduction can be achieved by an optimal stock-age dependent disposal policy. As the results may strongly depend on the parameters of the base stock policy, we also consider the integral optimization of the ordering policy and the disposal policy. In this section, we present a Markov decision model that simultaneously optimizes over the ordering policy and the disposal policy. The model can be used also for optimizing the two policies in isolation of each other.

3.1. Discrete time view: decisions and inventory dynamics

First, we discuss the chronological order of events in discrete time: time is split into periods equal to 1 day as the maximal shelf life of all products is fixed to an integer number of m days (including any lead time). The model holds for a lead time between 0 and 1: $0 \le L \le 1$. Orders are generated at the start of every weekday (but may be not during weekends), and are delivered the same day: if $L=0.5$ products are delivered half way the opening hours; if $L=1$) product are delivered at or after closing time $L=1$.

What counts, with respect to the lead time, is not the exact time between ordering and replenishment, but the effective time during which the shop was open. If a shop is open 12 h a day, then $L=0.5$ ($L=1$) implies that the replenish arrives 6 (12) opening hours after ordering. The actual time between ordering and delivery may be much larger: e.g. the model with $L=0.5$ holds also for cases where the order is placed at the end of a day (after disposing of products that will expire that day), and the delivery happens the next day, half way its opening hours. Similarly, the results for $L=1$ also hold for cases where orders are placed at the end of day 0, and the actual replenishment happens at day 2 before the inventory point opens.

To ease the presentation of the model, we assume that orders are placed at the start of period and products expire by the end of a period. To model the general case of a fractional lead time, a period is split into two epochs. Epoch 1 lasts from the start of a day d till the replenishment at time $d+L$. Epoch 2 is related to the rest of the day. As in Haijema (2011), the periods are related to weekdays $d \in \{1, 2, \ldots, 7\}$, for Monday to Sunday. If the lead time L is 0.5, epoch 1 can be seen as the morning and epoch 2 as the afternoon (including the evening and night). (Alternatively, when L is set to 1, epoch 1 lasts a full day, and epoch 2 has length zero.) As products expire at the end of a day, the effective maximum shelf life is $m-L$ days.

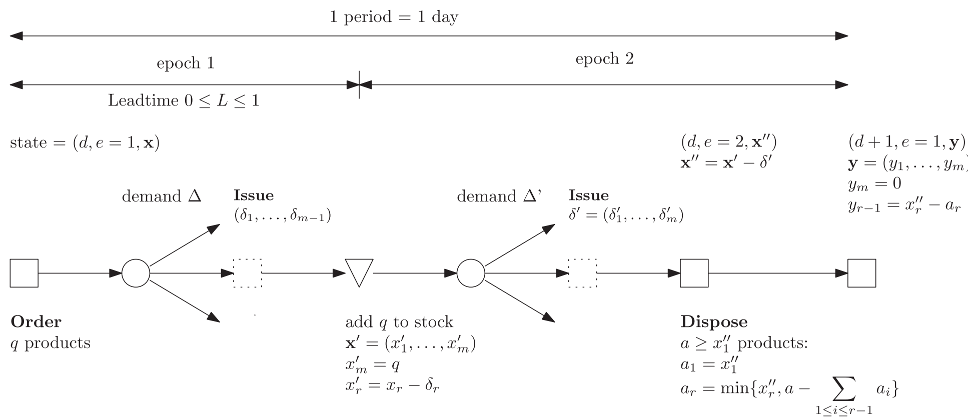

Fig. 1. The ordering, issuing, and disposal decisions in discrete time: issuing decisions is represented by dotted squares as in some settings it may be beyond control.

In Fig. 1, the discrete time view is presented. A square indicates a decision and a circle indicates a stochastic process. At the start of a period the order quantity q is set, next demand over epoch 1 occurs and products are issued. Any unmet demand is lost. The ordered products arrive at the end of epoch 1. During epoch 2 one again deals with stochastic demand. At the end of epoch 2 products are disposed. The figure also summarizes some more notations which we will explain later. The dotted squares are related to issuing decisions which cannot be fully controlled by the stock manager when customers pick the products themselves. The demand during each epoch is modeled by a discrete probability distribution with weekday and epoch dependent means $\mu_{d,e}$ and standard deviation $\sigma_{d,e}$. Issuing decisions are not included in the action space of the MDP model presented here. The reader interested in a model for determining optimal issuing decisions $(\delta_1, \ldots, \delta_m)$ is directed to Haijema (2011).

The MDP model optimizes over both the order policy and the disposal policy. The issuing policy for an epoch is fixed to FIFO or LIFO. Note that, in the above representation, the decision about the order quantity q is separated from the disposal decision a, thus their optimization happens in two stages of a Stochastic Dynamic Program. We neglect the time interval between the disposal decision in period $d-1$ and the order decision in period d. This is a valid assumption when the inventory system is closed during this interval, or when orders are placed shortly after the disposal decision. In Section 3.3, we will discuss how sub-models are derived from the optimal ordering and the disposal model, by fixing either the ordering decision or the disposal decision to a fixed policy. If both the ordering and disposal decisions are fixed, the MDP model describes a Markov chain.

3.2. Optimal stock-age dependent ordering and disposal policy

A stock-age dependent ordering and disposal policy is computed by formulating the optimization problem as an infinite horizon Markov decision problem (MDP) that can be solved by value iteration, which is a special case of stochastic dynamic programming (SDP). Therefore, using the above notations and assumption, we define the state, the actions, the state transitions and transition probabilities, the direct contribution, and the value iteration algorithm for the numerical computation of an optimal combined ordering and disposal policy.

3.2.1. States

Relevant points in time for keeping track of the state are the time points at which a decision is to be made. As shown in Fig. 1 an ordering decision is made at the start of epoch $e=1$, and the disposal decision is made at the end of epoch $e=2$. Even when ordering and disposal decision are made simultaneously, in a dynamic program these decisions are modeled at two separate stages to avoid a high dimensional decision space. Both decision are based on the stock state, that is the actual number of products in stock of each of the m age groups. Further information that is needed is the day of the week d to anticipate non-stationary order moments (e.g. at blood banks during weekends no orders may be placed) or non-stationary demand, and the epoch e. Hence the state is the day of the week d, the epoch e and the stock state.

The stock state at the start of epoch 1 is denoted by the vector $\mathbf{x} = (x_1, \ldots, x_m)$, where x_r is the number of products with a residual shelf life of r days. At the start of epoch 1, x_m must be zero as we do not allow backlogging and products get one day older by the end of the day: products delivered today (at the end of epoch 1) will have the next morning a residual shelf life of $m-1$ days. The elements of $\mathbf{x}$ are limited by the order quantities set in the last $m-1$ periods. The stock state at the end of epoch 2 is denoted as $\mathbf{x}'' = (x''_1, \ldots, x''_m)$. All x''_1 products in age group 1 are to be disposed at the end of epoch 2.

So relevant states of the Markov decision problem are $(d, e = 1, \mathbf{x})$ at the start of epoch 1, and $(d, e = 2, \mathbf{x}'')$ at the end of epoch 2. As we consider a lost sales system, the state space in both epochs is finite, if the order quantity q is finite, which is the case if demand is finite. (In case of a system with backlogging instead of lost sales, the state space is more cumbersome as stock levels may become negative to indicate the number of backlogged products.)

3.2.2. Actions and action spaces

Given the actual state $(d, e = 1, \mathbf{x})$ in stage 1, that is at the start of epoch $e=1$, an optimal order quantity q is to be set. If the maximal demand over epochs 1 and 2 is respectively $\overline{\Delta}$, and $\overline{\Delta}'$, then $q \in \{0, 1, \ldots, \overline{\Delta} + \overline{\Delta}'\}$. Note we do not consider fixed ordering costs; in case of fixed order costs the upper bound on the action space is the maximal demand over m days.

At stage 2, that is at the end of epoch $e=2$, the number of products to dispose, a, depends on the state $(d, e = 2, \mathbf{x}'')$. The number of products to dispose from age group r, denoted by a_r, is set by assuming disposal happens in the FIFO order, that is the oldest products are disposed first. All products that are about to expire will be disposed: thus, $a_1 = x''_1$, and $a \geq x''_1$. The number of disposed products of age group $r > 1$ is $a_r = \min\{x''_r, a - \sum_{i=1}^{r-1} a_i\}$. The action space at stage 2 is finite: $\{x''_1, x''_1 + 1, \ldots, x''\}$, as a cannot exceed the total stock level $x'' = \sum_{r=1}^{m} x''_r$.

3.2.3. State transition and transition probabilities

The discussion of state transitions is split into two stages: the transition in stage 1 (from the moment of ordering to the moment of disposal) is different from the transition in stage 2 (from the moment of disposal to the moment of ordering).

State transition from $(d, e=1, \mathbf{x})$ at ordering to $(d, e=2, \mathbf{x}'')$ at disposal: In the transition from state $(d, e=1, \mathbf{x})$ at the start of epoch 1 to state $(d, e=2, \mathbf{x}'')$ at the end of epoch 2, a number of events happen: first, the stochastic demand for Δ products will be met, next, the replenishment arrives, and finally the stochastic demand over epoch 2 for Δ' products is met. With probability $P_{d,1}(\Delta)$, the demand during epoch 1 of day d is for $\Delta \in \{0, 1, \ldots \overline{\Delta}\}$ products, and $P_{d,2}(\Delta')$ denotes the probability that the demand is $\Delta' \in \{0, 1, \ldots \overline{\Delta}'\}$ during epoch 2 of day d.

Demand is met by a fixed issuing policy resulting in epoch 1 in issuing δ_r products from age group r, and in epoch 2, issuing δ'_r products from age group r. δ_r depends on the demand Δ, the stock state $\mathbf{x}$, and the issuing rule as follows. $\delta_m = 0$, as at the start of epoch 1, x_m is 0. In case of FIFO issuance, δ_1 equals $\min\{x_1, \Delta\}$, and for $2 \le r \le m-1$ holds $\delta_r = \min\{x_r, \Delta - \sum_{i=1}^{r-1} \delta_i\}$. In case of LIFO issuance, $\delta_r = \min\{x_r, \Delta - \sum_{i=r+1}^{m} \delta_i\}$ for $1 \le r \le m-1$. The number of products that inventory falls short to meet the demand of epoch 1 is $\Delta - \sum_r \delta_r$, and is considered as lost sales. The number of products of age group $1 \le r \le m-1$ left at the start of epoch 2 is $x_r - \delta_r$.

At the start of epoch 2, the products ordered in epoch 1 are added to stock, and are available to meet the demand for Δ' products in epoch 2. Under LIFO issuance, we get thus $\delta'_m = \min\{q, \Delta'\}$ and, for $1 \le r \le m-1$, $\delta'_r = \min\{x_r - \delta_r, \Delta' - \sum_{i=r+1}^{m} \delta'_i\}$. Under FIFO issuance, issuing happens according to $\delta'_1 = \min\{x_1 - \delta, \Delta'\}$, and for $2 \le r \le m$, $\delta'_r = \min\{x_r - \delta, \Delta' - \sum_{i=1}^{r-1} \delta'_i\}$. The shortage over epoch 2 equals $\Delta' - \sum_r \delta_r{}'$.

These events have to be taken all together to determine the stock state $(d, e=2, \mathbf{x}'')$ at the end of epoch 2, prior to the disposal decision. Thus we have $x''_m = q - \delta'_m$, and for $1 \le r \le m-1$, $x''{}_r = x_r - \delta_r - \delta'_r$. Instead of using state transition probabilities, we present the model using the event probabilities $P_{d,1}(\Delta)$ and $P_{d,2}(\Delta')$ for the demands in epoch 1 and epoch 2 respectively.

State transition from $(d, e=2, \mathbf{x}'')$ at disposal to $(d+1, e=1, \mathbf{y})$ at ordering: In the transition from state $(d, e=2, \mathbf{x}'')$ at the end of epoch 2 to state $(d+1, e=1, \mathbf{y})$ at the start of the next day, two events happen: (1) a products are removed from stock, and (2) the residual shelf life of all products carried over to the next day will be decreased by 1 day. The vector $\mathbf{y}$ contains the number of products in stock at the start of the next day. The state transition from the end of epoch 2 to the start of epoch 1 results in $y_m = 0$, and for $1 \le r \le m-1$, $y_r = x''_{r+1} - a_r$. The outdating of products is included in the disposal decision a. The total waste over a day is thus a products. In this definition of waste, we do discriminate between items that are prematurely disposed and those that have expired. Note, there is no uncertainty involved in this state transition. Further $d+1$ indicates the next day, and equals d mod $7+1$: it should be read as 1 (=Monday) if $d=7$ (Sunday).

3.2.4. Expected direct contributions

The objective is to minimize the average costs over an infinite horizon. The total costs over n days are separable into expected costs per day, also called the direct contribution (to the objective function). The costs per day is split into costs per stage. The first stage relates to the period from ordering until, but excluding, the disposal decision. EC is the sum of expected costs over stage 1: it consists of the holding costs (accounted for all products in stock at the start of epoch 1), the expected shortage costs, and discounting or penalty costs for issuing 'old' products. As older products having lower utility a unit penalty cost p_r is accounted for each product issued from age group r. For stage 1, the expected costs are thus

$$EC(d, \mathbf{x}, q) = h \cdot \sum_r x_r + \sum_{\Delta, \Delta'} P_{d,1}(\Delta) P_{d,2}(\Delta') \left(c \cdot \left(\Delta - \sum_r \delta_r + \Delta' - \sum_r \delta'_r \right) + \sum_r (\delta_r + \delta'_r) p_r \right) \tag{1}$$

Disposal costs are accounted for in the second stage, and are thus not included in EC. The second stage is from the moment of disposal up to, but excluding, the ordering decision of the next day. The second stage cost consists of the waste costs only: $w \cdot a$. Note, the disposal costs are irrespective of the age of the disposed products. The model can be generalized easily by making the disposal costs age dependent and thus taking into account a salvage value for disposing products with a positive residual shelf life.

3.2.5. Value iteration

The optimal policy that minimizes the expected costs per week can be obtained from an approximation scheme based on the value iteration (VI). The computational scheme is similar to SDP: VI solves the problem backwardly. A single iteration of VI relates to a day that consists of two stages, as we have two decisions to optimize.

The computational scheme starts with setting the iteration number $n=1$, the weekday $d=7$, and setting initially the value of all stock states to zero. That is a vector $V_0(d=1, e=1, \mathbf{y})$ is set to zero for all stock states $\mathbf{y}$. Next, the scheme iteratively computes the value vector V_n for other weekdays d and epochs e. Per iteration n, only a single value of d is considered. At iteration 1, one computes V_1 for all states with $d=7$, at iteration 2, one computes V_2 for all states with $d=6$, etc. The relation between iteration number d and the day of the week d is $d = 7-(n-1) \bmod 7$. In the n-th iteration, V_n is computed for states with $d = 7-(n-1) \bmod 7$. The computation of $V_n(d, \cdot, \cdot)$ is divided into two stages related to subset of states with $e=2$ and $e=1$. First, the second stage disposal decisions are determined by computing $V_n(d, e=2, \mathbf{x}'')$ for all stock states $\mathbf{x}''$:

$$V_n(d, e=2, \mathbf{x}'') = \min_{x''_1 \le a \le \sum_r x''_r} (w \cdot a + V_{n-1}(d+1, 1, \mathbf{y})). \tag{2}$$

Next, the first stage ordering decisions are computed by computing $V_n(d, e=1, \mathbf{x})$ for all stock states $\mathbf{x}$:

$$V_n(d, e=1, \mathbf{x}) = \min_{0 \le q \le \overline{\Delta} + \overline{\Delta}'} \left(EC(d, \mathbf{x}, q) + \sum_{\Delta} \sum_{\Delta'} P_{d,1}(\Delta) P_{d,2}(\Delta') (V_n(d, 2, \mathbf{x}'')) \right). \tag{3}$$

At the end of an iteration, n is increased by 1, and the scheme is repeated. As n tends to infinity, the vector $D_n = V_n - V_{n-7}$ converges to a vector of constants equal to the average weekly costs of an optimal combined ordering and disposal policy. The difference between the maximal and the minimal value of D_n is called the span of D_n. When the span of D_n is smaller than a pre-specified accuracy ϵ the algorithm stops. For technical details on the convergence of VI, see Puterman (1994).

3.3. Sub models: optimal disposal policy and optimal order policy

If one does not allow premature disposals, the action space of a is set to a single value $\{x''_1\}$, the number of products that will expire. Then the above MDP model optimizes over the order quantity q only. Alternatively, one fixes the order policy and optimize only over a to get an optimal disposal policy. In the experiments in the next section, an optimal disposal policy is found by VI with the order quantities fixed to those set by a BSP with fixed weekday dependent order-up-to levels S_d. In the next sections, we report results for both sub-models next to the integral optimization over both policies. When both the ordering and the disposal policy are fixed, VI can be used to evaluate the average costs of the underlying Markov chain. Estimates of the expected weekly costs are obtained from the difference vector D_n.

3.4. An MDP model for optimal issuance

The MDP model described in this paper integrates decision making on the ordering and disposal of products, and assumes the issuing decision to be fixed, e.g. set to FIFO or LIFO. For completeness we also report results for an optimal issuing policy, which is of practical relevance only when the issuance of products can be fully controlled. An optimal issuing policy is derived by a similar MDP model but with a slightly different state description (see Haijema, 2011). The state of an optimal issuing model should contain next to the weekday d, epoch e, and stock state $\mathbf{x}$, the demand Δ to be met from stock. Hence the issuing decision is based on accumulated demand for an epoch. In practice of blood banks, the accumulated demand comes from regular orders from hospitals. Demand is accumulated till the moment of transport, which happens once or twice per day for big hospitals. Therefore a day is split into two epochs related to the two issuing moments. (In other settings with more issuing moments, a day is to be split into more than 2 epochs.)

The issuing decision $(\delta_1, \ldots, \delta_m)$ is a vector indicating the number of products to issue from each age group. Rather than enumerating all possible values of this vector to meet the demand, an efficient way for optimizing the issuing decision is to split the optimization of $(\delta_1, \ldots, \delta_m)$ into m stages. Each stage is related to deciding about the number of products to issue from a specific age group. For a detailed description of an MDP model for optimal ordering we refer to Haijema (2011). To not complicate the mathematical model for optimal ordering and disposal, the mathematical specification of the optimization of the issuing decision is not included in this paper. However, results for optimal issuance are included in the next section to investigate how much could be gained by optimal issuance compared to FIFO or LIFO issuance and optimal ordering and depletion.

4. Design of experiments

For both FIFO and LIFO inventory systems, we will evaluate five policies:

- OFN (or OLN): *O*ptimal ordering policy with *F*IFO (or *L*IFO) issuance and *N*o premature disposal of products.
- BFN (or BLN): *B*SP with *F*IFO (or *L*IFO) issuance and *N*o premature disposal.
- BFO (or BLO): *B*SP with *F*IFO (or *L*IFO) issuance and *O*ptimal disposal of products.
- BON: *B*SP with *O*ptimal issuance and *N*o premature disposal of products.
- OFO (or OLO): *O*ptimal ordering policy with *F*IFO (or *L*IFO) issuance and *O*ptimal disposal of products.

The order-up-to levels S_d of BSP are set to the average order-up-to level experienced in a long simulation of the optimal order policy (OFN or OLN), conform Haijema (2013). So an optimal order policy with FIFO issuance is computed first, such that the order-up-to levels S_d for the policies BFN, BFO, and BON are known. As the optimal policy is different for LIFO issuance, the policies BLN, BLO, and BON for LIFO issuance are based on different order-up-to levels. All policies, except BFN and BLN, require solving an SDP model by VI, which can be time consuming. BFN and BLN require simulation only. Results are reported in Section 5.

4.1. Value iteration and simulation

For each policy we evaluate their average daily costs, relative waste (=waste as a percentage of the production), and relative shortage (=the number products short as a percentage of demand). Cost figures are obtained exactly by VI, as $D_n = V_n - V_{n-7}$ converges to a constant vector containing the weekly costs. Also waste and shortages figures could be obtained by separate runs of VI by reducing the action space(s) in each state to a single value set by the disposal and ordering policies. For example, to get waste figures by VI one has to account only for the waste costs, and set all other costs to zero. As this would require multiple runs of VI, we prefer to use simulation to get many statistics simultaneously. To allow a fair comparison we use common random numbers. To get accurate results antithetic variates are used, and each simulation consists of 4000 runs of 54 weeks (including 2 weeks for warming up). The resulting 95% confidence intervals (not reported here) indicate that the statistics are accurate at +/−0.2%. For example, if the average costs for some policy is 200, then the (true) mean costs falls with 95% confidence in $[200 + / - 0.2\% \cdot 200] = [199.6, 200.4]$. Cost differences larger than 2∗0.2%=0.4% are statistically significant as the confidence intervals will be non-overlapping. The relative difference in performance between policies is in most experiment that we report in this section much larger than 0.4%, hence confidence intervals are omitted.

4.2. Data and design of experiments

We investigate the impact of each policy for a set of experiments that differ in the values of the maximal shelf life m, the lead time L, the mean daily demand μ, holding cost h, shortage costs c, and the unit penalty costs p_1 (for issuing from the oldest age category). Without loss of generality, we set $w=150$ in all experiments; all other cost parameters are set in relation to this figure. That is, the monetary unit is irrelevant. If each parameter could take 3 values a full factorial design of experiments would imply $3^6 = 729$ experiments, which would require too much time for solving all SDP models (4 per experiment). Therefore we split the design of experiments into three parts: two linear designs of experiments for FIFO and LIFO (each consisting of 13 experiments), and a partly full factorial design of 20 experiments for FIFO issuance. In a total 40 different experiments are thus considered, and thus 160 SDP models are solved and simulated.

For the linear design of experiments we define a base case for a product with $m=4$ days, $L=0.5$, Poisson distributed demand with a mean $\mu=4$ products/day, and cost parameters $w=150$, $h=0.1$, $p_1=105$, and $c=750$. Orders are placed on Monday to Friday morning before opening (ordering during weekends is investigated in Section 5.3). The cost figures are motivated from practical cases: a blood platelet concentrate costs 150 euros, alternatively one may think of a fresh food product with a purchase price of 150 euro cents. In this example w consists of the purchase price, but it may also include handling costs for disposal or a salvage value. The unit holding costs are 0.1, which implies holding a product in stock for 1 year costs 36.5, which is 24% of its purchase price, which is reasonable as keeping food fresh requires cooling and regular inventory inspections. The unit penalty or discount cost should be seen in relation to the sales price. Selling prices of blood platelet concentrates are over 450 euro per unit, hence the sales margin is more than 60%. Sales margins for fresh food products are in the order of 40–60% according to Ketzenberg and Ferguson (2008). Imposing a sales margin of 50% of the sales price implies a selling price of 300 and a sales margin of 150. Most Dutch supermarkets apply a discount of 35% to products sold on the date of expiration, thus we set p_1 to 105 (=35% of 300). Finally, the unit shortage cost c is set such that a service level (fill rate) of at least 97–98% is achieved, which is a requirement for many Dutch supermarkets and blood banks. After a few experiments it appears that $c=750$ results in the desired service level. A cost ratio of $c/w=5$ fits also to a blood bank environment, see Haijema et al. (2007).

In the linear design of experiment, assume Poisson distributed demand with mean μ equal to 2, 4, and 6 products per day. The maximal shelf life equals $m=4$ or 5 days. Next to a lead time of 0.5 day, we consider a variation to the base case by setting the lead time to 1 day, resulting in an effective maximal shelf life of 3 days. Unit holding cost h is set to 0, 0.1 and 0.5. (At blood banks and hospitals, holding costs are higher than at supermarkets as more expensive equipment is used for storing blood products under the right conditions.) The effect of the penalty or discount costs is studied for the following values for p_1: 0, 60, 105, and 150, which are related to a discount of 0%, 20%, 35%, and 50% of a sales price of 300 respectively. A realistic sales price of a blood platelet concentrate is over 450 per product, while its unit production costs are about 150. The 'gross sales margin' should cover high fixed overhead costs related to expensive equipment, staff, buildings, and R&D activities. For blood banks a discount of $p_1=105$ is $105/450 \approx 23\%$ of the sales price; in current practice no discounts are given, hence p_1 serves as a penalty on issuing products that are about to expire.

A factorial design of another 20 experiments will be introduced in Section 5.1.

4.3. Computational complexity

To execute the computations of VI in finite time, the number of states, actions and state transitions to evaluate should be limited. The order quantity on day d should not exceed the maximal demand $\overline{\Delta}+\overline{\Delta}'$, but an optimal policy can be found faster if a stricter bound is provided. Therefore we limit the search for an optimal order quantity on day d to values q in $\{0,1,\ldots,Q_d-\sum_r x_r\}$, where $Q_d=\lceil (R+L)\mu+k\cdot\sqrt{(R+L)}\sigma\rceil$ with review period $R=1$ and safety factor $k=3.5$ for days 1–4 (=Monday to Thursday), and $R=3$ and $k=2$ for $d=5$ (=Friday). Consequently, stock levels will never exceed $Q=\max_d Q_d$, which may be seen as an artificial, non-binding storage capacity. To execute seven iterations of VI, Eqs. (2) and (3) are evaluated for $\mathcal{O}(Q^m)$ states, the state space is further reduced by eliminating states in epochs 1 and 2 which hold $\sum_{r=1}^m x_r \ge Q$ and $\sum_{r=1}^m x_r'' \ge Q$, respectively. The states space is thus finite as a consequence of finite order sizes and an artificial, non-binding storage capacity. In all experiments the optimal order-up-to levels of the BSP stay well below these bounds. Further we have verified that these bounds did not affect the (expected costs of an) optimal ordering policy.

The number of state transitions to consider from each state is set by the support of the demand distribution. Therefore the demand distributions $P_{d,1}$ and $P_{d,2}$ are truncated at a maximum demand level ($\overline{\Delta}$, and $\overline{\Delta}'$). The mean demand is split over epoch 1 and epoch 2, according to the lead time: the mean demand over epoch 1 is $L\cdot\mu$, and over epoch 2 the mean demand is $(1-L)\cdot\mu$. To get a finite number of transitions when evaluating Eq (3), the demand distributions are truncated and normalized. In all experiments the demand is Poisson distributed with a mean demand per epoch of either 0, 1, 2, 3, 4, or 6, and the maximal demand ($\overline{\Delta}$ and $\overline{\Delta}'$) is set to 0, 6, 7, 11, 14, or 17 products respectively. The demand distributions are normalized by adding the residual probability to the probability of a demand equal to the upper bounds. Thus, the mean daily demand and its standard deviations are virtually equal to μ and $\sqrt{\mu}$.

Computation times strongly depend on the CPU used, the programming environment, and the way in which VI is implemented. One should pay particular attention to the data structure to store the values of V_n, V_{n-1}, V_{n-7}, and EC. To speed up the computation time, one may compute and store EC prior to executing VI. But memory consumption may become a bottleneck, when all data is stored inefficiently in full matrices: remember that we can cut-off the state space by imposing an artificial storage capacity Q. Memory consumption can be reduced by using sparse arrays, such that only values of V_n, V_{n-1}, and V_{n-7} are stored for states that are in the state space. Instead of computing and storing EC once (before applying VI), one may decide to compute its values while executing VI. The online computation of values of EC slows down VI but saves memory usage. In our implementation in Matlab, the computations per experiment needed to optimize and simulate the five policies can be done in a few minutes up to over an hour, depending on the values of μ and m in each experiment. The most demanding experiments are those with $m=4$, $\mu=6$ and $L=1$, which required much more than 1 h computation time. The optimal combined ordering and disposal policy could not be computed for $m=5$, $\mu=4$ and $L=1$. For the same reason, cases with $m=5$ and $\mu=6$, could not be included in the design of experiments.

Experimenting with the model is limited to problem settings with a short maximal shelf life and a mean demand that is realistic but not very high. These are precisely the settings that we are interested in: we focus on fresh products for which shortages and product waste due to outdating is prevalent.

5. Results

The effect of optimal ordering and optimal disposal policy strongly depends on the issuance policy. Therefore we discuss the results for FIFO and LIFO in separate subsections.

5.1. Results for FIFO issuance

For FIFO issuance we consider both a linear design of 12 experiments, which is described in the previous section, and a factorial design of 20 experiments for $m=4$ and $m=5$. For each experiment, four SDP models are solved to derive an optimal issuance policy (BON), an optimal disposal policy (BFO), an optimal ordering policy (OFN), and an optimal combined ordering and disposal policy (OFO).

Linear design: FIFO base case and effects of m, μ, L, p_1, h, and c. The linear design of experiments is used for a ceteris paribus analysis that is compared to a base case, the effect of a parameter is investigate by changing the value of that parameter only. This gives us an impression of which parameters (factors) have greatest impact on the resulting difference between the five policies BFN, BON, BFO, OFN, and OFO. The definition of the policies is included in the header of Table 1: BFN is the base stock policy with FIFO issuance and No premature disposal of products. The costs of this policy are reported in the column named BFN, the costs of the other policies are reported in adjacent column as a percentage improvement over BFN.

The base case, introduced in the previous section, is presented on the first row below the headers. In the base case $c=750$, and the (nearly) optimal order-up-to levels on Monday to Friday are (12, 11, 11, 10, 18). In Table 2, one reads that at $c=750$, the service level of BFN is $100-2.1\%=97.9\%$, and the waste percentage is 9.9%. The first three rows of both Tables 1 and 2 show the effect of c on the expected costs, and the waste and shortage percentages. When shortage costs c would be set to the profit margin of 150, the service level would be only $100-9.03\%=91.0\%$. To achieve a service level (fill rate) of at least 97–98%, the shortage cost is thus set to $c=750$, which is five times the unit waste cost w. If the sales margin is 50% of the sales price, the profit margin is 150. The shortage cost is thus to be set much higher than the direct economic costs related to a shortage.

The column BON in Table 1 reports results for an optimal issuing policy in combination with a base stock policy and no premature disposal. For the base case, BON reduces costs by 10.5%

Table 1
Ceteris paribus analysis of reductions in average daily costs compared to BFN (=BSP, FIFO, No disposal).

								Policy	BFN	BON	BFO	OFN	OFO
								Order policy	BSP	BSP	BSP	SDP	SDP
								Issuing policy	FIFO	SDP	FIFO	FIFO	FIFO
								Disposal policy	–	–	SDP	–	SDP
	m	μ	L	p_1	h	w	c	BSP: $(S_1, \ldots, S_5)$	Costs	% Cost reductions achieved			
base	4	4	0.5	105	0.1	150	750	(12, 11, 11, 10, 18)	209.87	10.5	11.3	9.7	15.0
							150	(8, 9, 9, 9, 13)	104.60	4.8	3.4	3.7	6.8
							300	(9, 10, 10, 9, 15)	147.72	6.0	6.6	6.8	10.9
					0			(12, 11, 11, 10, 18)	209.08	10.8	11.3	9.7	15.1
					0.5			(12, 11, 11, 10, 18)	213.04	10.3	11.1	9.5	14.9
				0				(13, 12, 12, 13, 18)	108.30	2.7	2.9	6.2	6.2
				60				(12, 11, 11, 11, 18)	170.61	8.7	8.8	9.0	10.6
				150				(11, 10, 10, 10, 17)	239.54	11.4	12.7	8.2	17.1
			1					(14, 12, 12, 12, 19)	319.38	9.3	11.7	12.0	18.2
		2						(7, 6, 6, 6, 10)	162.42	8.1	9.9	8.9	12.5
		6						(16, 15, 15, 15, 25)	236.84	9.2	9.2	7.1	11.8
	5							(12, 12, 12, 12, 18)	99.34	7.3	7.0	2.7	9.0
Average										8.3	8.8	7.8	12.3

compared to BFN. A slightly higher cost reduction can be achieved by optimal premature disposal of products and FIFO issuance: 11.3% (see BFO). Optimal ordering with FIFO issuance and no premature disposal (OFN) results in slightly higher costs: a cost reduction of 9.7%. The highest cost reduction (15.0%) is obtained by combining optimal ordering and optimal disposal and FIFO issuance (OFO). For the base case, the cost reductions achieved by BON, BFO, and OFO are due to reducing shortages at the expense of somewhat more waste. Shortages can be reduced by a well timing of disposal, as a disposal lowers the stock level, and thus results in additional ordering the next ordering moment. OFN is the only policy that reduces both shortages and waste compared to BFN. Combining optimal disposal and optimal ordering reduces shortages at the expense of somewhat more outdating: a disposal is primarily to reduce discounting costs. Compared to OFN, shortages are reduced by OFO as the order policy now anticipates that the disposal policy can get rid of excess stock.

Averaged over all experiments, the policies have similar effects. Most remarkable, and logical, whether optimal ordering (OFN) improves optimal disposal (BFO) depends strongly on the values of p_1. When the discount p_1 is 105 or 150, optimal disposal (BFO) is much better than optimal ordering (OFN), as a disposal policy allows for getting rid of old stock and thus saves on selling at discounted prices. If p_1 is 0, optimal ordering is much better than optimal disposal. At p_1 is equal to 60, the OFN and the BFO show almost identical costs. Under BFN and OFN, a higher value of p_1 triggers to keep lower stock levels, but when combined with a disposal policy the stock levels may be set higher and shortages may be reduced. Note, we have not simultaneously optimized the order-up-to levels S_d of BSP and the disposal policy. BFO reduces shortages by a good timing of disposals to reduce stock levels and thus increase the order quantity of the next order. That way the disposal policy enables to replace old products by younger products, which in turn reduces shortages in the coming days. Changes in the unit holding cost h hardly influence the results, as holding costs constitute only a minor part of the total costs. Averaged over all 12 experiments, OFO results in the lowest shortage percentage and the highest average waste percentage (11.8%).

Factorial design: combined effects of m, μ, L, and p_1. In a linear design of experiments, the experiments show a strong correlation to the base case. Hence to investigate the effect of relevant parameters a factorial design of experiments is preferred. The drawback of a (full) factorial design of experiment is that the number of experiments explodes in the number of experimental factors and the number of levels: if each factor would have 3 levels, and we have 6 factors, then the number of experiments is $3^6 = 729$, which is far beyond the number of experiments we can deal with. Thus we consider a factorial design of experiments with only 4 factors: the lead time, mean daily demand, and the maximal shelf life, and the unit discount or penalty cost.

As service levels for perishables in many Dutch supermarkets are set at 97–98% or even higher, all experiments are based on $c=750$. Holding costs play a minor role and thus h is kept at 0.1. A first full factorial design of experiments is considered for $m=4$, $\mu \in \{2, 4, 6\}$, $L \in \{0.5, 1\}$, and $p \in \{0, 105\}$. As computation times get too high for cases with $m=5$ and $\mu > 4$, a second full design of experiments for $m=5$ is smaller by excluding cases with $\mu=6$. Further the experiment with $m=5$, $\mu=4$, and $L=1$ is not complete: the combined optimal ordering and disposal policy (OFO) could not be computed, as available memory and CPU time was restrictive: in Table 3 it is indicated by N.A. (for Not Available). The full design of experiments consists thus of $3 \cdot 2 \cdot 2 + 2 \cdot 2 \cdot 2 = 20$ experiments.

In Table 3 the results are presented by separating the results for $p_1 = 0$ and $p_1 = 105$, as the value of the discount has a very strong impact. Averages of experiment with $p_1 = 0$ and $p_1 = 105$ are reported separately. Combining optimal disposal with optimal ordering (OFO) results for $p_1 = 0$ in virtually no improvement, as there is no incentive to dispose. The disposal policy in BFO has impact as it indirectly improves the order quantities: when many old products are in stock an optimal order policy would order a bit more than the BSP. By disposing of some products, order quantities under BSP will be higher, as less stock is left, and old products are thus effectively replaced by new products. For $p_1 = 0$, BFO improves BFN by 4% on average, but a greater cost reduction is achieved by optimal ordering (OFN).

For $p_1 = 105$, the effect of adding a disposal policy (BFO) is larger than replacing BSP by an optimal order policy (OFN). BFO reduces the average cost on average by 10.1%, while optimal ordering results in a lower cost reduction of 8.9%. The disposal policy in BFO takes care of getting rid of old stock, and indirectly improves the order quantities as explained before. When combining optimal ordering and optimal disposal, the cost is on average 14.4% lower than BFN, whereas OFO improves by 8.9%. Adding an optimal disposal policy is thus beneficial both under optimal ordering and under a base stock order policy, when the $p_1 = 105$.

Table 2
Impact of optimal ordering, disposal, and issuance on waste and shortages compared to BSP+FIFO+no disposal.

(a) Waste as a percentage of the total order volume

								Order policy	Waste percentage of policy				
								Issuing policy Disposal policy	BFN BSP FIFO –	BON BSP SDP –	BFO BSP FIFO SDP	OFN SDP FIFO –	OFO SDP FIFO SDP
	m	μ	L	p_1	h	w	c	BSP: $(S_1,\ldots,S_5)$					
base	4	4	0.5	105	0.1	150	750	(12, 11, 11, 10, 18)	9.9	10.9	11.8	9.2	11.8
							150	(8, 9, 9, 9, 13)	2.2	3.1	3.6	2.1	4.2
							300	(9, 10, 10, 9, 15)	4.5	5.4	6.2	4.4	6.9
					0			(12, 11, 11, 10, 18)	9.9	10.9	11.8	9.2	11.8
					0.5			(12, 11, 11, 10, 18)	9.9	10.9	11.8	9.2	11.7
				0				(13, 12, 12, 13, 18)	10.1	10.2	10.2	10.1	10.1
				60				(12, 11, 11, 11, 18)	9.8	10.3	10.1	9.4	9.8
				150				(11, 10, 10, 10, 17)	7.8	10.0	10.1	7.7	11.4
			1					(14, 12, 12, 12, 19)	13.1	13.8	16.0	13.0	18.5
		2						(7, 6, 6, 6, 10)	17.1	18.2	19.9	15.8	20.3
		6						(16, 15, 15, 15, 25)	5.9	6.6	7.1	5.8	7.8
	5							(12, 12, 12, 12, 18)	3.4	3.9	4.2	3.4	4.8
Average									8.6	9.5	10.2	8.3	10.8

(b) Shortage as a percentage of the total demand

									Shortage percentage of policy				
								Order policy Issuing policy Disposal policy	BFN BSP FIFO –	BON BSP SDP –	BFO BSP FIFO SDP	OFN SDP FIFO –	OFO SDP FIFO SDP
	m	μ	L	p_1	h	w	c	BSP: $(S_1,\ldots,S_5)$					
base	4	4	0.5	105	0.1	150	750	(12, 11, 11, 10, 18)	2.10	1.64	1.63	1.72	1.61
							150	(8, 9, 9, 9, 13)	9.03	9.82	9.07	8.63	7.87
							300	(9, 10, 10, 9, 15)	5.39	5.37	4.99	4.80	4.21
					0			(12, 11, 11, 10, 18)	2.10	1.64	1.63	1.72	1.61
					0.5			(12, 11, 11, 10, 18)	2.10	1.64	1.63	1.76	1.62
				0				(13, 12, 12, 13, 18)	1.37	1.13	1.24	1.14	1.14
				60				(12, 11, 11, 11, 18)	1.90	1.45	1.50	1.52	1.51
				150				(11, 10, 10, 10, 17)	2.96	2.54	2.54	2.41	1.87
			1					(14, 12, 12, 12, 19)	4.18	3.39	3.37	2.93	2.43
		2						(7, 6, 6, 6, 10)	3.45	2.69	2.72	2.85	2.76
		6						(16, 15, 15, 15, 25)	1.84	1.57	1.58	1.56	1.33
	5							(12, 12, 12, 12, 18)	1.29	1.24	1.25	1.22	1.04
Average									3.1	2.8	2.8	2.7	2.4

Table 3
Combined effect of parameters (m, μ, L, p_1) on cost reductions compared to BSP+FIFO+no disposal.

							Policy	BFN	BON	BFO	OFN	OFO
							Order policy	BSP	BSP	BSP	SDP	SDP
							Issuing policy	FIFO	SDP	FIFO	FIFO	FIFO
							Disposal policy	–	–	SDP	–	SDP
m	μ	L	p_1	h	w	c	BSP: $(S_1, \ldots, S_5)$	Costs	% Cost reductions achieved			
4	2	0.5	0	0.1	150	750	(7, 7, 7, 8, 10)	101.35	3.9	4.7	9.4	9.4
4	2	1	0	0.1	150	750	(9, 8, 8, 9, 11)	148.07	4.4	7.0	14.6	14.6
4	4	0.5	0	0.1	150	750	(13, 12, 12, 13, 18)	108.30	2.7	2.9	6.2	6.2
4	4	1	0	0.1	150	750	(16, 14, 14, 15, 20)	174.54	4.0	5.2	11.1	11.3
4	6	0.5	0	0.1	150	750	(18, 17, 18, 20, 26)	114.91	2.3	1.7	3.1	3.4
4	6	1	0	0.1	150	750	(22, 20, 21, 22, 29)	195.91	4.7	4.7	8.4	8.9
5	2	0.5	0	0.1	150	750	(8, 8, 8, 8, 10)	61.84	0.8	3.9	10.4	10.4
5	2	1	0	0.1	150	750	(9, 9, 9, 9, 12)	84.35	3.0	5.4	10.6	10.6
5	4	0.5	0	0.1	150	750	(14, 14, 14, 14, 19)	51.93	0.6	0.8	2.1	2.2
5	4	1	0	0.1	150	750	(16, 16, 16, 17, 21)	84.29	−0.1	2.2	4.9	N.A.
Average[a]									2.9	4.0	8.4	8.6
4	2	0.5	105	0.1	150	750	(7, 6, 6, 6, 10)	162.42	8.1	9.9	8.9	12.5
4	2	1	105	0.1	150	750	(8, 7, 7, 7, 11)	233.73	9.1	13.5	14.7	17.5
4	4	0.5	105	0.1	150	750	(12, 11, 11, 10, 18)	209.87	10.5	11.3	9.7	15.0
4	4	1	105	0.1	150	750	(14, 12, 12, 12, 19)	319.38	9.3	11.7	12.0	18.2
4	6	0.5	105	0.1	150	750	(16, 15, 15, 15, 25)	236.84	9.2	9.2	7.1	11.8
4	6	1	105	0.1	150	750	(20, 17, 17, 17, 28)	391.94	11.9	13.2	12.2	18.9
5	2	0.5	105	0.1	150	750	(7, 7, 7, 6, 10)	95.41	7.3	7.7	5.4	12.1
5	2	1	105	0.1	150	750	(8, 9, 8, 7, 11)	133.34	6.0	7.7	7.2	14.7
5	4	0.5	105	0.1	150	750	(12, 12, 12, 12, 18)	99.34	7.3	7.0	2.7	9.0
5	4	1	105	0.1	150	750	(14, 15, 13, 13, 20)	158.39	7.3	7.9	5.4	N.A.
Average[a]									8.7	10.1	8.9	14.4

[a] Averages are calculated excluding the last experiment ($m=5$, $\mu=4$, and $L=1$), which is incomplete.

For 18 out of all 20 experiments, BFO is better than BON (=optimal issuance). OFN (=optimal ordering instead of a BSP) is better than BFO in only 2 out of the 10 experiments with $p_1 = 105$, whereas OFN is better than BFO for all 10 experiments with $p_1 = 0$. Overall, we observe that the difference between BFO and OFN gets smaller, and BFO may become even better than OFN, when the lead time L is shorter, or when μ, m, and p are higher. In the given design of experiments the strongest effect is set by parameter p_1. Whether BFO is better than OFN depends on the actual values of all parameters. In all cases the best results are obtained by combining optimal ordering and optimal disposal (OFO). For $p_1 = 105$, OFO results in a cost reduction of 14.4% compared to BFN, whereas optimal ordering with no premature disposal (OFN) results in a cost reduction of 8.9%.

5.2. Results for LIFO issuance

The linear design of experiments for FIFO issuance is also applied to LIFO issuance. Table 4 reports the effects of m, μ, L, p_1, h, and c on the average daily costs. The base case is again reported in the first row, and in each row below, the value of one parameter is changed. In the last two rows we report waste and shortage percentages averaged over all 12 experiments: both are significantly higher under LIFO than under FIFO issuance. Compared to FIFO issuance, LIFO issuance results in a doubling of the waste; shortages are, under LIFO, about 1.5 times as high. LIFO issuance is thus far from cost-optimal, hence optimal issuance (BON) greatly reduces the average daily costs: by 44.7% for the base case and 42.9% over all experiments. The order-up-to levels S_d of a base stock policy under LIFO issuance are slightly different from those under FIFO issuance, as these are re-optimized. As a result, the waste under BON is now a bit higher than that reported in Table 2 (11.4% versus 9.5%), while shortages are now a bit lower (2.5% versus 2.8%).

An optimal disposal policy combined with ordering by a BSP (BLO) results in a cost reduction compared to BLN of 13.2% on average. The disposal decision reduces stock levels prior to ordering and thus results in more fresh products in stock. Thus shortages are reduced at the expense of more outdating. Optimal ordering (OLN) results in a greater cost reduction than optimal disposal (BLO): on average 17.5% versus 13.2%. Optimal ordering reduces shortages without generating more waste than BLN, or BLO. Surprisingly, the combined optimal ordering and disposal policy (OLO) shows virtually no further cost reduction in case of LIFO issuance (18.7% vs 18.6%, averaged over all 12 experiments), whereas under FIFO issuance a significant cost reduction from, on average, 7.8% to 12.3% was achieved. We cannot conclude that a disposal policy is not of added value in general under LIFO issuance and optimal ordering: in some cases it provides a marginal improvement. Anyway let us discuss some intuitive explanation of why the observed effect is virtually zero. Next to a small reduction of holding cost, the main benefit of disposing an (old) product is to have a lower chance of fulfilling demand at a discounted price. A detriment of disposing an old product prematurely is the risk of falling short. Under LIFO issuance the oldest products are used only as a last resort to meet the demand. Hence the benefit of a disposal, that is not having to offer a discount, may be undone by the costs of falling short. Under optimal ordering we thus see hardly no disposal of products.

When ordering by a BSP, optimal disposal does significantly reduce average costs, also under LIFO issuance. The reason why an optimal disposal in BLO has impact while it has no impact in OLO is that the disposal decision indirectly improves the order decision by lowering the stock level. For LIFO issuance, implementing an optimal order policy is preferred over implementing a BSP with an optimal disposal policy. A deeper analysis of the effect of the parameters by a factorial design is left behind. Pure LIFO is not found in many systems, if at all, as not all consumers are selective, and inventory managers try to influence their picking behavior towards FIFO.

Table 4
Ceteris paribus analysis of reductions in average daily costs compared to BSP+LIFO+no disposal (=BLN).

	m	μ	L	p_1	h	w	c	Policy Order policy Issuing policy Disposal policy BSP: $(S_1,\ldots,S_5)$	BLN BSP LIFO – Costs	BON BSP SDP – % Cost reductions achieved	BLO BSP LIFO SDP	OLN SDP LIFO –	OLO SDP LIFO SDP
base	4	4	0.5	105	0.1	150	750	(12, 10, 10, 10, 19)	354.88	44.7	14.5	18.7	18.7
							150	(8, 7, 7, 6, 13)	159.18	25.3	1.1	6.1	6.1
							300	(9, 9, 8, 8, 16)	233.55	34.4	5.5	11.1	11.1
					0			(12, 10, 10, 10, 19)	354.15	44.9	14.5	18.7	18.8
					0.5			(12, 10, 10, 10, 19)	357.81	44.1	14.4	18.6	18.6
				0				(12, 11, 10, 10, 19)	317.79	60.8	15.9	21.2	21.2
				60				(12, 11, 10, 10, 19)	341.58	50.6	15.4	19.6	20.2
				150				(12, 10, 10, 10, 19)	371.64	40.7	14.3	18.3	18.4
			1					(14, 13, 12, 13, 22)	428.54	29.7	16.3	21.3	21.4
		2						(7, 6, 6, 6, 10)	238.65	37.5	11.8	16.2	16.3
		6						(16, 14, 14, 14, 27)	458.93	41.0	17.0	20.0	20.7
	5							(11, 14, 10, 9, 20)	218.07	45.4	6.9	10.2	11.1
Average costs reduction (%)										42.9	13.2	17.5	17.6
Average waste (%)									20.5	11.6	22.0	20.0	20.1
Average shortage (%)									6.2	2.5	4.1	4.2	4.1

Table 5
Effect of ordering and deliveries during weekends on average daily costs: variations to the base case ($m=4$, $\mu=4$, $L=0.5$, $p_1=105$, $h=0.1$, $w=150$, and $c=750$).

	Frequency of ordering and delivery	Policy Order policy Issuing policy Disposal policy BSP: $(S_1,\ldots,S_7)$	BFN BSP FIFO – Costs	BON BSP SDP – % Cost reductions achieved	BFO BSP FIFO SDP	OFN SDP FIFO –	OFO SDP FIFO SDP
FIFO	Mon to Fri	(12, 11, 11, 10, 18)	209.87	10.5	11.3	9.7	15.0
	Mon to Sat	(11, 11, 11, 11, 10, 14)	129.53	11.3	7.8	2.7	11.5
	Mon to Sun	(10, 10, 10, 10, 10, 10, 10)	100.93	11.2	7.2	1.3	15.0
Average				11.0	8.8	4.6	13.8
LIFO	Mon to Fri	(12, 10, 10, 10, 19)	354.88	44.7	14.5	18.7	18.7
	Mon to Sat	(10, 11, 10, 11, 10, 15)	294.71	59.1	12.8	18.1	18.2
	Mon to Sun	(10, 10, 10, 10, 10, 10, 10)	270.03	66.8	12.5	18.1	18.2
Average				57.8	13.1	18.6	18.7

5.3. *Ordering and delivery frequency*

In all 40 experiments orders are placed every Monday to Friday and delivered either halfway the opening hours ($L=0.5$) or at the end of the day or the start of the next day ($L=1$). As blood banks produce hardly any blood products during weekends, ordering on Monday to Friday is the base case. Orders placed on Friday morning are delivered either the same day ($L=0.5$), or before opening on Saturday ($L=1$). In the base case the next order will be placed Monday. Large supermakets may have an additional order moment and replenishment during the weekend.We now investigate the effect of the possibility to order on Saturday and/or Sunday and having the delivery the same day. Delivery on 6 or 7 days a week was not included in the factorial design of experiments, as we had to keep the number of experiments low.

Table 5 shows the effect of more order moments during the weekend. We cannot draw general conclusion, as alternative ordering and delivery frequencies are evaluated only for the base case (under FIFO and LIFO issuance). Nevertheless, we believe that it is worth to briefly discuss the results. For LIFO issuance we still observe hardly any added value of a disposal policy in combination with optimal ordering, but in combination with a BSP the added value of a disposal policy is significant as observed and explained before.

Under FIFO issuance and ordering by a BSP, the added value of a disposal policy gets less as orders can be placed more frequently. The effect of an optimal disposal policy, when combining it with an optimal ordering policy (OFO), remains high even when orders can be placed every day. When orders are placed more frequently, the average cost of a BSP (BFN) gets closer to that of an optimal ordering policy (OFN). Hence, a BSP fits well to a case with FIFO issuance and daily ordering.

A disposal policy seems to have less impact when combined with a BSP than when combined with an optimal order policy. This may be explained as follows. In some states of the stock, it may be optimal to order a bit more than BSP (with no premature disposals) would suggest. In that state, the decision to dispose some products results in ordering more products the next order moment, as the disposal lowers the inventory level. BFO generates some more waste in these states to compensate for bad ordering. OFO is not restricted to a fixed order up to level and thus does not need premature disposals to deviate from it, its main reason for premature disposals is to sell less at discounted prices.

6. Discussion and conclusion

Disposal decisions may be of practical relevance to hospitals as transfusing old blood products is less effective and thus not preferred. For a supermarket that serves a high market segment, disposal can be relevant for displaying mostly fresh products. Although we have assumed that disposed products do not return to the inventory system, in practice one could decide to keep them behind to resolve shortages or to sell them to another market segment. Although sub optimal, base stock policies (BSP) are commonly used for ordering perishables with a short maximal shelf life. An optimal policy depends on the number of products in stock of each age group. Due to uncertain demand and a positive lead time, the number of products in each age group fluctuates. Consequently stock levels of some age groups are sometimes relatively high, which we call the 'overstocking'. Having many old products in stock causes waste due to outdating. A base stock policy does not anticipate product outdating and thus may result in more outdating than a stock-age dependent ordering policy. Throughout this paper we have investigated policies that minimize the costs of managing perishable inventories by considering a unit holding cost per day, a unit shortage cost, a unit waste cost, and a penalty or discount cost for issuing products that expire by the end of the day. Optimal policies that we report are policies that depend on the ages of the products in stock. We have investigated cost reductions that may be achieved by combining a BSP with an optimal disposal policy or an optimal issuing policy. The results are compared with results for an optimal ordering policy and a combined optimal ordering and disposal policy. A disposal policy reduces 'overstocking' when the demand has been relatively low compared to demand predictions. The disposal decision implies the removal of the oldest products from stock.

We have formulated the optimal order and disposal problem as a Markov Decision Problem (MDP), and solved it by value iteration. The main conclusions we draw based on simulation of the solutions of 160 MDP models (related to 40 experiments) are the following:

- Under FIFO issuance, the added value of an optimal disposal policy is high. When no penalty or discount applies, disposals reduce costs only in case of suboptimal ordering, e.g. by a BSP.
- Under LIFO issuance, the added value of an optimal disposal policy is high when orders are placed by a BSP, but a disposal policy is insignificant when orders are placed by an optimal order policy.
- FIFO issuance is sub-optimal, as was observed earlier by Pierskalla and Roach (1972).
- An optimal disposal policy reduces the number of old products issued (if the penalty or discount cost p_1 is high enough), and may compensate for suboptimal ordering by a BSP.
- An optimal ordering policy may perform worse than a BSP with an optimal disposal policy, especially in case of FIFO issuance and if the penalty or discount cost p_1 is high enough.
- The combination of optimal ordering and disposal results in lowest costs and lowest shortages, but may result in a high waste percentage.

Solving an MDP is time consuming, when the number of states is large as it requires to find an optimal decision for each possible state. The number of states to consider grows exponentially in the maximal shelf life m. Long computation times could hamper the direct application of the models in practice. Nevertheless the models (and results) are of practical value: (1) the results indicate how much may be gained by using an optimal ordering, issuance or depletion policy next to, or instead of, ordering by a base stock policy, and (2) the results on optimal disposal policies may trigger research in practical strategies to improve inventory management in practice. Disposals reduce the issuance of old products thus reducing the loss of sales margins due to discount. If disposals are optimal in some setting, than it may be even better to keep old products behind in a back storage rather than disposing them immediately, and only issue them when needed. Further research could be directed to include the return of products from a back storage to the shop floor when running out of stock.

Similar models can be developed to decide on the number of young products to keep behind in the back storage, such that one forces consumers to first pick the older products in stock and thus waste may be reduced. Next to extending the models in these directions, one may develop more simple and easier to implement disposal and issuing policies.

Acknowledgments

The author thanks TIFN (project RE-002) for funding the research, two anonymous reviewers for their constructive comments, and Michal Bilinski for coding the SDP algorithm for computing an optimal issuing policy.

References

Broekmeulen, R.A.C.M., van Donselaar, K.H., 2009. A heuristic to manage perishable inventory with batch ordering, positive lead-times, and time-varying demand. Comput. Oper. Res. 36 (11), 3013–3018, http://dx.doi.org/10.1016/j.cor.2009.01.017.

Deniz, B., Karaesmen, I., Scheller-Wolf, A., 2010. Managing perishables with substitution: inventory issuance and replenishment heuristics. Manuf. Serv. Oper. Manag. 12 (2), 319–329.

Derman, C., Klein, M., 1958. Inventory depletion management. Manag. Sci. 4 (4), 450–456.

Haijema, R., 2011. Optimal issuing of perishables with a short fixed shelf life. In: Jahn, C., Shi, X., Stahlbock, R., Voss, S., Böse, J.W., Hu, H. (Eds.), Lecture Notes in Computer Science, Computational Logistics (ICCL 2011). Springer-Verlag, Berlin, Heidelberg, pp. 160–169, ISBN 978-3-642-24263-2.

Haijema, R., June 2013. A new class of stock level dependent ordering policies for perishables with a short maximum shelf life. Int. J. Prod. Econ. 143 (2), 434–439, http://dx.doi.org/10.1016/j.ijpe.2011.05.021.

Haijema, R., van der Wal, J., van Dijk, N.M., March 2007. Blood platelet production: optimization by dynamic programming and simulation. Comput. Oper. Res. 34 (3), 760–779, http://dx.doi.org/10.1016/j.cor.2005.03.023.

Karaesmen, I., Scheller-Wolf, A., Deniz., B., 2011. Planning production and inventories in the extended enterprise, in: International Series in Operations Research & Management Science, Managing Perishable and Aging Inventories: Review and Future Research Directions, vol. 151. Springer, US, pp. 393–436.

Ketzenberg, M., Ferguson, M.E., 2008. Managing slow-moving perishables in the grocery industry. Prod. Oper. Manag. 17 (5), 513–521, http://dx.doi.org/10.3401/poms.1080.0052, ISSN 1937-5956.

Martin, G.E., 1986. An optimal decision model for disposal of perishable inventories. Int. J. Prod. Res. 24 (1), 73–79.

Nahmias, S., 1982. Perishable inventory theory: a review. Oper. Res. 30, 680–708.

Pierskalla, W.P., Roach, C.D., 1972. Optimal issuing policies for perishable inventory. Manag. Sci. 18, 603–614.

Puterman, M.L., 1994. Markov Decision Processes: Discrete Stochastic Dynamic Programming. Wiley Series in Probability and Mathematical Statistics.

van der Vorst, J.G.A.J., Beulens, A.J.M., de Wit, W., van Beek, P., 1998. Supply chain management in food chain: improving performance by reducing uncertainty. Int. Trans. Oper. Res. 5 (6), 487–499.

van Donselaar, K., van Woensel, T., Broekmeulen, R., Fransoo, J., 2006. Inventory control of perishables in supermarkets. Int. J. Prod. Econ. 104, 462–472.

Veinott, A.F., Jr., Optimal Ordering, Issuing, and Disposal of Inventory with Known Demand (unpublished Ph.D. dissertation). Columbia University, 1960.