EXERCISE SHEET 4

Task 1: Optimization and Maximum Margin (8 points)

Submission of:
Tikhon Riazantsev 382715
Agastya Heryudhanto 286824

a) Imagine you have an algorithm for solving maximization problems. How can this algorithm be used to solve a minimization problem? (2 point)

Any maximization problem can be turned into a minimization problem by negating the function in question, e.g., finding the maximum of f(x) is the same as finding the minimum of -f(x).

b) How is the margin defined? What is a maximum margin seperating plane? (2 points)

The margin is defined as the distance between the separating hyperplane and the closest point of each separated class. Maximizing the margin leads to better class separation. Maximum margin separating plane is such hyperplane

$$w^T x + d = 0$$

that admits a canonical representation with respect to the closest point above the line **p** of one class and the closest point **n** below it from another class

$$w^T p + d = 1$$

$$w^T n + d = -1$$

and which has the smallest possible value $||\bar{w}||$. Margin H in this case can be calculated as:

$$\frac{1}{||\bar{w}||}$$

c) Is it possible to find a maximum margin separating plane using 1) Linear Programming (LP) and 2) a Perceptron? (2 points)

1) It is impossible to find a maximum margin separating plane using Linear Programming because it can only give separating lines which are connected with the solution on one of the corner points of the feasible region. That means that the separating plane will always go through two of the points, one from each class and the margin would always be 0.
2) While it is not necessarily impossible to get to a separating plane with the maximum margin using a Perceptron, it is unlikely to happen as there is no constraint in its algorithm that would set the requirements for the margin to be maximal.

d) Show that the hyperplane H : (2, 0, 1, 2)x−3 = 0 separates the points p = (1, 1, 1, 1)T and q = (−1, −1, −1, −1)T . Decide if H is the maximum margin hyperplane to separate p and q. Give a reason for your decision. (2 points)

We will find the distance from each point to the separating plane:

$$\mathrm{d}(\mathbf{p},\mathrm{H}) = \frac{\left|\mathbf{w}^{\mathrm{T}}\mathbf{p} + \mathrm{d}\right|}{||\mathbf{w}||} = \frac{|2 * 1 + 1 * 1 + 2 * 1 - 3|}{\sqrt{4+1+4}} = \frac{2}{3}$$

which means that p is above the separating hyperplane.

$$\mathrm{d}(\mathbf{n},\mathrm{H}) = \frac{\left|\mathbf{w}^{\mathrm{T}}\mathbf{n} + \mathrm{d}\right|}{||\mathbf{w}||} = \frac{|2 * (-1) + 1 * (-1) + 2 * (-1) - 3|}{\sqrt{4+1+4}} = -\frac{8}{3}$$

which means that n is below the separating hyperplane.

As we have found out, the hyperplane does indeed separate the two points. But as we can see, the distances between these points and the hyperplane are not equal, which means that H is not the maximum margin hyperplane.