Submission by:

Tikhon Riazantsev

Agastya Heryudhanto

Exercise sheet 6

TASK 1: LAGRANGE MULTIPLIERS

Consider the optimization problem

$$\text{maximize } f(x_1, x_2) \text{ subject to } g(x_1, x_2) = 0$$

with

$$f(x_1, x_2) = -x_1^2 - 2x_2^2 + 7\frac{1}{2} \text{ and } g(x_1, x_2) = x_1 + x_2 - 1\frac{1}{2}.$$

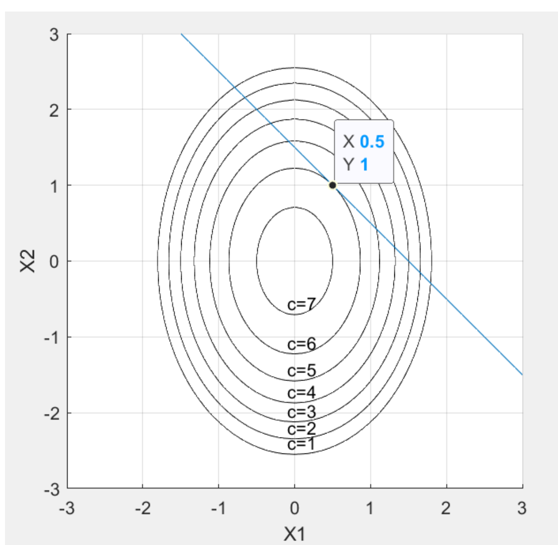
a) Write a MATLAB script which plots in one figure
   (i) the level contours of $f(x_1, x_2)$ for $c \in \{1, 2, \dots, 10\}$ and
   (ii) the function $g(x_1, x_2)$

   for $(x_1, x_2) \in [-3, 3] \times [-3, 3]$. Name the plotted contours and the axes. Zip your implementation and the plot and upload your archive to the Moodle course. (5 points)

b) Just from the plot, what can you say about the solution? (1 point)

c) Give the Lagrangian of the optimization problem and find the optimum of $f(x_1, x_2)$ under the given constraint. (4 points)

a) Completed in MatLab:



b) From the plot it is clear to see that the solution is at the point where the line, described by the function g(x1,x2)=0 is tangent to the curve, described by the function f(x1,x2)=c with c = 6.

The optimal solution can thus be derived as:

$$X = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$$

c) The Lagrangian is described as:

$$\Lambda(x_1, x_2, \alpha) = f(x_1, x_2) - \alpha(g(x_1, x_2) - c)$$

And for this problem as:

$$\Lambda(x_1, x_2, \alpha) = -x_1^2 - 2x_2^2 + 7.5 - \alpha x_1 - \alpha x_2 + 1.5\alpha$$

Let us now find all the partial derivatives of the lagrangian and solve the derived system of equations:

$$\frac{\partial \Lambda}{\partial x_1} = -2x_1 - \alpha = 0$$

$$\frac{\partial \Lambda}{\partial x_2} = -4x_2 - \alpha = 0$$

$$\frac{\partial \Lambda}{\partial \alpha} = -x_1 - x_2 + 1.5 = 0$$

$$\Downarrow$$

$$2x_1 = 4x_2 = -\alpha$$

$$-x_2 - 2x_2 + 1.5 = 0$$

$$\Downarrow$$

$$x_1 = 1$$

$$x_2 = 0.5$$

$$\alpha = -2$$

And optimal solution:

$$f(1, 0.5) = -1 - 0.5 + 7.5 = 6$$

Which matches our previously derived graphical solution.