d) Considering the following two dimensional data samples xi with the labels yid) Considering the following two dimensional data samples xi with the labels yi =

| i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $x_i$ | $\binom{2}{4}$ | $\binom{4}{6}$ | $\binom{5}{5}$ | $\binom{7}{7}$ | $\binom{6}{1}$ | $\binom{6}{2}$ | $\binom{8}{4}$ | $\binom{9}{6}$ |
| $y_i$ | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 |

a Quadratic Programming solver gives the dual solution $a^* = (0;\ 0;\ 0;\ 0{:}41;\ 0;\ 0{:}03;\ 0;\ 0{:}38)$. With this solution:

i) Name all support vectors, (1 points)
Support Vectors:

From the dual solution, $\alpha = (0; 0; 0; 0{,}41; 0; 0.03; 0; 0.38)$,

the corresponding support vectors are datapoints in $x_i$ where the lagrange multipliers are non-zero. These are x4, x6 and x8 (where $a_4 = 0.41$, $a_6 = 0.03$ and $a_8 = 0.38$)

$$x4 = \binom{7}{7}$$

$$x6 = \binom{6}{2}$$

$$x8 = \binom{9}{6}.$$

ii) Calculate the primal solution, (2 points)

Primal Solution: $w^* = \sum_{i=1}^{N} \alpha_i y_i x_i$

$$w = 0.41 \cdot \binom{7}{7} + 0.03 \cdot \binom{6}{2} + 0.38 \cdot \binom{8}{4}$$

$$w = [1.99 \quad 1.06].$$

iii) Calculate the value d* and…? (2 points)

Calculate d* with the formula:

$$d^* = \frac{| \mathrm{w}_* \cdot \mathrm{x}_i + b^* |}{\| \mathrm{w}_* \|}$$

Find b*, where $b^* = y_i - (w^*)^T * x_i$

Substitute values $y_i$ and $x_i$ with $y_4$ and $x_4$

$$\begin{aligned} b &= -1 - [1.99 \quad 1.06] \cdot \begin{pmatrix} 7 \\ 7 \end{pmatrix} \\ &= -1 - 14.63 \\ b &= -15.63. \end{aligned}$$

Inserting b and $x_4$ in the previous equation to obtain d*:

$$d^* = \frac{| \, w_* \cdot x_i + b^* \, |}{\| \, w * \|}$$

$$d^* = \frac{| \, [1.99 \quad 1.06] \cdot \binom{7}{7} - 15.63 \, |}{\| \, [1.99 \quad 1.06] \, \|} = \frac{0.28}{2.27}$$

iv) calculate the margin. (1 points)

The margin is twice the value of d*:
margin = 2d

$$Margin = 2 * \frac{0.28}{2.27} = \frac{0.56}{2.27}$$