Materials and Methods

(Teori om encoding og decoding)

0.1 Encoding

Encoding for the code vector in systematic form:

$$X^{n-k}m(X) = q(X)g(X) + p(X) \tag{1}$$

By adding the redundancy polynomial to the shifted message polynomial, the encoded vector in systematic form is obtained.

$$c(X) = X^{n-k}m(X) + p(X) \tag{2}$$

A concrete example with the generator polynomial $g(X) = 1 + X^4 + X^6 + X^7 + X^8$ is used to encode the following message polynomial: $m(X) = 1 + X^2 + X^4 + X^6$, corresponding to $m = [1010101]$. The code is a cyclic code, $C_{cyc}(15, 7)$.
First the message is right shifted $n - k$ times.

$$X^{15-7}m(X) = X^8 + X^{10} + X^{12} + X^{14} \tag{3}$$

Find $p(X)$ by taking the remainder from $X^{n-k}m(x)$ divided by $g(X)$.

$$
\begin{array}{r|cccccccccc}
 & X^6 & +X^5 & +X^4 & +X^2 & +1 & & & & & \\
\cline{2-6}
X^8 \quad +X^7 \quad +X^6 \quad +X^4 \quad +1 & X^{14} & +X^{12} & +X^{10} & +X^8 & & & & & & \\
 & X^{14} & +X^{13} & +X^{12} & +X^{10} & +X^6 & & & & & \\
\cline{2-6}
 & & X^{13} & & +X^8 & +X^6 & & & & & \\
 & & X^{13} & +X^{12} & +X^{11} & +X^9 & +X^5 & & & & \\
\cline{3-7}
 & & & X^{12} & +X^{11} & +X^9 & +X^8 & +X^6 & +X^5 & & \\
 & & & X^{12} & +X^{11} & +X^{10} & +X^8 & & +X^4 & & \\
\cline{4-9}
 & & & & X^{10} & +X^9 & +X^6 & +X^5 & +X^4 & & \\
 & & & & X^{10} & +X^9 & +X^8 & +X^6 & +X^2 & & \\
\cline{5-9}
 & & & & & X^8 & +X^5 & +X^4 & +X^2 & & \\
 & & & & & X^8 & +X^7 & +X^6 & +X^4 & +1 & \\
\cline{6-10}
 & & & & p(X) = & & X^7 & +X^6 & +X^5 & +X^2 & +1
\end{array}
$$

Now $c(X)$ can be calculated:

$$c(X) = X^8m(X) + p(X) = 1 + X^2 + X^5 + X^6 + X^7 + X^8 + X^{10} + X^{12} + X^{14} \tag{4}$$

$$c = [101001111010101] \tag{5}$$

It is easily seen that the encoded vector is in systematic form, hence the message vector corresponds to the last 7 bits.

0.2 Decoding