國立成功大學

工程科學學系

113 學年度第二學期
數值方法

HW 12

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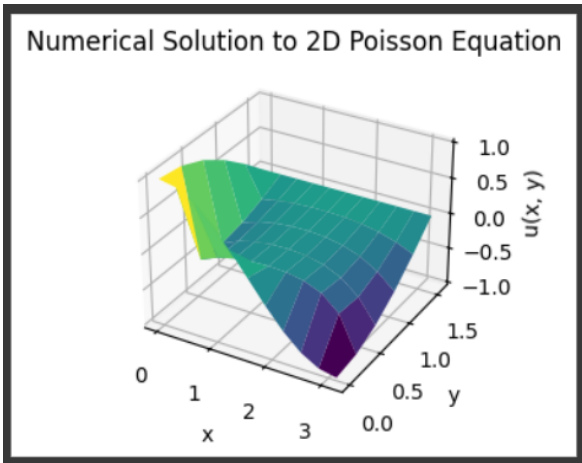
1. Given the problem

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=xy,\quad 0<x<\pi,\quad 0<y<\pi/2$$

$$u(0,y)=\cos y,\quad u(\pi,y)=-\cos y,\quad 0\le y\le \pi/2,$$

$$u(x,0)=\cos x,\quad u(x,\pi/2)=0,\quad 1\le y\le 2$$

To calculate $u(x,y)$ by using $h=k=0.1\pi$.



	0.000000	0.314159	0.628319	0.942478	1.256637	1.570796
0.000000	1.000000e+00	0.951057	0.809017	0.587785	0.309017	0.0
0.314159	9.510565e-01	-0.284370	-0.299716	-0.241930	-0.133551	0.0
0.628319	8.090170e-01	-0.183970	-0.193580	-0.158340	-0.088379	0.0
0.942478	5.877853e-01	-0.124381	-0.132294	-0.109470	-0.061625	0.0
1.256637	3.090170e-01	-0.092879	-0.101746	-0.085621	-0.048652	0.0
1.570796	6.123234e-17	-0.083764	-0.096189	-0.082615	-0.047363	0.0
1.884956	-3.090170e-01	-0.097336	-0.116631	-0.101287	-0.058184	0.0
2.199115	-5.877853e-01	-0.141588	-0.171710	-0.147720	-0.084087	0.0
2.513274	-8.090170e-01	-0.239119	-0.280902	-0.233795	-0.130444	0.0
2.827433	-9.510565e-01	-0.449901	-0.478984	-0.376114	-0.203894	0.0
3.141593	-1.000000e+00	-0.951057	-0.809017	-0.587785	-0.309017	0.0

2. Given the problem

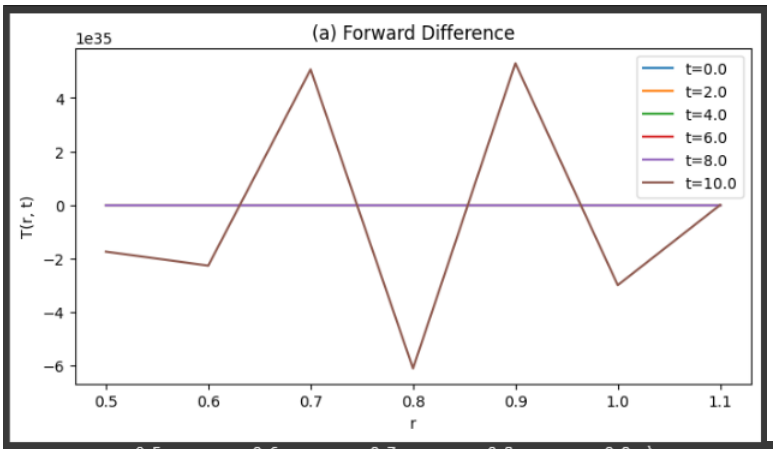
$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}=\frac{1}{4K}\frac{\partial T}{\partial t},\quad \frac{1}{2}\le r\le 1,\quad 0\le t,$$

$$T(1,t)=100+40t,\quad 0\le t\le 10;\quad \frac{\partial T}{\partial r}+3T=0 \quad \text{at} \quad r=\frac{1}{2}$$

$$T(r,0)=200(r-0.5),\quad 0.5\le r\le 1,$$

and use $\Delta t=0.5$, $\Delta r=0.1$, and $K=0.1$ to calculate $T(r,t)$

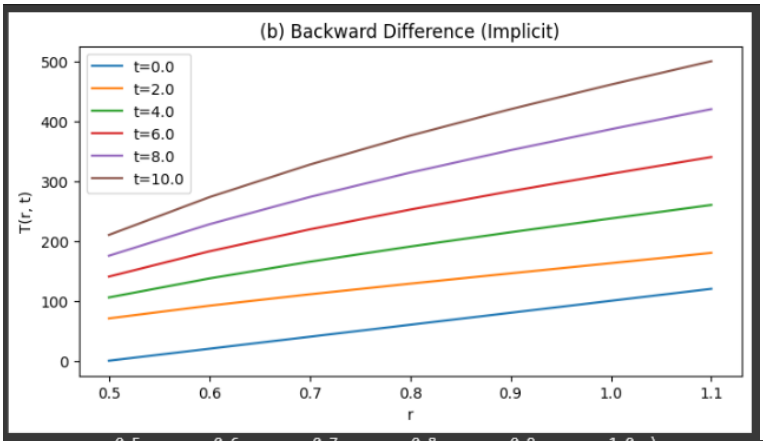
(a) the forward-difference method



	0.5	0.6	0.7	0.8	0.9
0.0	0.000000e+00	2.000000e+01	4.000000e+01	6.000000e+01	8.000000e+01
0.5	6.666667e+01	8.666667e+01	9.714286e+01	1.100000e+02	1.244444e+02
1.0	-4.078144e+01	-5.301587e+01	1.780952e+02	1.758730e+02	1.800000e+02
1.5	3.983607e+03	5.178689e+03	-4.161587e+03	3.052381e+02	-1.584857e+04
2.0	-1.685414e+05	-2.191038e+05	2.650184e+05	-4.267162e+05	1.232001e+06
2.5	8.613222e+06	1.119719e+07	-2.354871e+07	4.779104e+07	-8.611181e+07
3.0	-6.069256e+08	-7.890032e+08	2.150441e+09	-4.135265e+09	5.925469e+09
3.5	5.095157e+10	6.623705e+10	-1.871329e+11	3.275123e+11	-4.086554e+11
4.0	-4.387446e+12	-5.703680e+12	1.554642e+13	-2.496565e+13	2.837928e+13
4.5	3.683432e+14	4.788461e+14	-1.247214e+15	1.868216e+15	-1.986241e+15
5.0	-2.995770e+16	-3.894501e+16	9.756740e+16	-1.384533e+17	1.400423e+17
5.5	2.371993e+18	3.083591e+18	-7.495250e+18	1.020497e+19	-9.937910e+18
6.0	-1.839774e+20	-2.391706e+20	5.682593e+20	-7.497102e+20	7.090917e+20
6.5	1.405155e+22	1.826702e+22	-4.266907e+22	5.496176e+22	-5.082333e+22
7.0	-1.060999e+24	-1.379298e+24	3.181090e+24	-4.023549e+24	3.655996e+24
7.5	7.943432e+25	1.032646e+26	-2.358970e+26	2.942538e+26	-2.637631e+26
8.0	-5.909327e+27	-7.682126e+27	1.742319e+28	-2.150393e+28	1.907332e+28
8.5	4.375136e+29	5.687677e+29	-1.282971e+30	1.570646e+30	-1.381752e+30
9.0	-3.227582e+31	-4.195856e+31	9.425541e+31	-1.146731e+32	1.002432e+32
9.5	2.374509e+33	3.086861e+33	-6.912473e+33	8.369710e+33	-7.280567e+33
10.0	-1.743271e+35	-2.266252e+35	5.062648e+35	-6.107396e+35	5.292416e+35

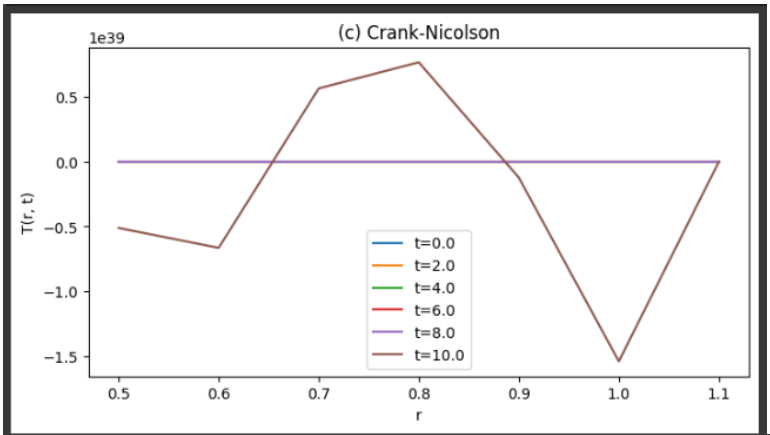
	1.0	1.1
0.0	1.000000e+02	120.0
0.5	1.400000e+02	120.0
1.0	-5.755556e+02	140.0
1.5	2.880667e+04	160.0
2.0	-1.421223e+06	180.0
2.5	7.883948e+07	200.0
3.0	-4.710860e+09	220.0
3.5	2.963075e+11	240.0
4.0	-1.932044e+13	260.0
4.5	1.292704e+15	280.0
5.0	-8.815403e+16	300.0
5.5	6.098812e+18	320.0
6.0	-4.266739e+20	340.0
6.5	3.011303e+22	360.0
7.0	-2.140051e+24	380.0
7.5	1.529259e+26	400.0
8.0	-1.097561e+28	420.0
8.5	7.904419e+29	440.0
9.0	-5.708053e+31	460.0
9.5	4.130762e+33	480.0
10.0	-2.994305e+35	500.0

(b) the backward-difference method



	0.5	0.6	0.7	0.8	0.9	1.0	1.1
0.0	0.000000	20.000000	40.000000	60.000000	80.000000	100.000000	120.0
0.5	40.174469	52.226810	65.399726	79.186895	93.157870	106.904751	120.0
1.0	52.061376	67.679789	82.321797	96.590932	110.819371	125.223275	140.0
1.5	61.652200	80.147860	96.948933	112.875062	128.460152	144.075939	160.0
2.0	70.654353	91.850658	110.866252	128.645384	145.817118	162.825646	180.0
2.5	79.465142	103.304685	124.533901	144.208201	163.032614	181.506451	200.0
3.0	88.206910	114.668983	138.108980	159.690651	180.190474	200.157903	220.0
3.5	96.922869	125.999730	151.649142	175.142382	197.325991	218.797848	240.0
4.0	105.629068	137.317788	165.176065	190.582417	214.452967	237.433383	260.0
4.5	114.331563	148.631032	178.697959	206.018006	231.576691	256.067236	280.0
5.0	123.032651	159.942447	192.217943	221.451904	248.699179	274.700449	300.0
5.5	131.733205	171.253167	205.737200	236.885161	265.821196	293.333420	320.0
6.0	140.433556	182.563622	219.256182	252.318173	282.943036	311.966298	340.0
6.5	149.133829	193.873978	232.775059	267.751092	300.064807	330.599141	360.0
7.0	157.834073	205.184295	246.293896	283.183976	317.186553	349.231971	380.0
7.5	166.534306	216.494597	259.812718	298.616846	334.308288	367.864796	400.0
8.0	175.234534	227.804894	273.331534	314.049712	351.430020	386.497619	420.0
8.5	183.934761	239.115189	286.850348	329.482575	368.551751	405.130441	440.0
9.0	192.634987	250.425484	300.369162	344.915438	385.673481	423.763262	460.0
9.5	201.335214	261.735778	313.887974	360.348301	402.795211	442.396084	480.0
10.0	210.035440	273.046071	327.406787	375.781163	419.916941	461.028906	500.0

© the Crank-Nicolson algorithm.

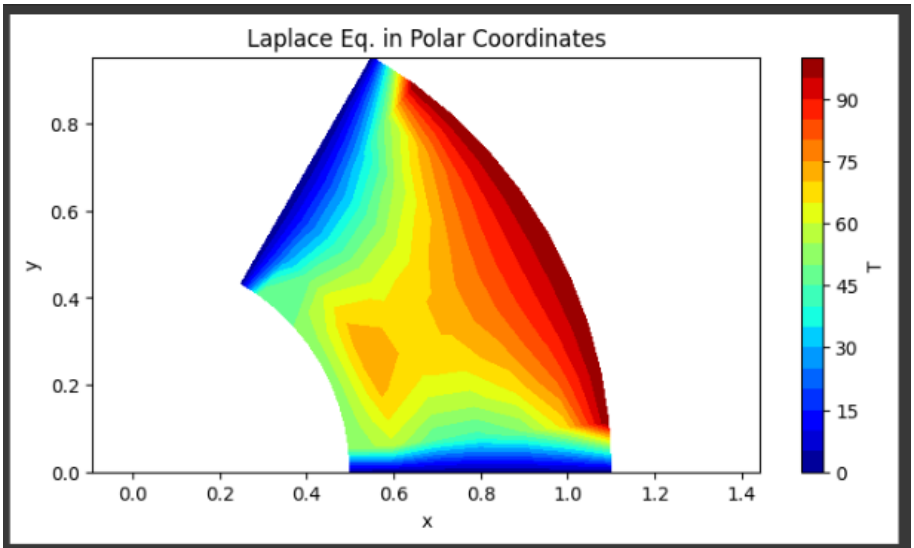


	0.5	0.6	0.7	0.8	0.9	1.0	1.1
0.0	0.000000e+00	2.000000e+01	4.000000e+01	6.000000e+01	8.000000e+01	1.000000e+02	120.0
0.5	9.411440e+02	1.223487e+03	1.265792e+03	1.150196e+03	9.155130e+02	5.741606e+02	120.0
1.0	-5.473060e+04	-7.114978e+04	-7.206342e+04	-6.447336e+04	-5.068384e+04	-3.009696e+04	140.0
1.5	3.381725e+06	4.396242e+06	4.369103e+06	3.921904e+06	3.193264e+06	2.023756e+06	160.0
2.0	-2.092791e+08	-2.720629e+08	-2.621493e+08	-2.360646e+08	-2.022304e+08	-1.396622e+08	180.0
2.5	1.301542e+10	1.692004e+10	1.550709e+10	1.400503e+10	1.293820e+10	9.998996e+09	200.0
3.0	-8.163535e+11	-1.061260e+12	-8.963252e+11	-8.107369e+11	-8.386919e+11	-7.459180e+11	220.0
3.5	5.192761e+13	6.750589e+13	4.980053e+13	4.492307e+13	5.527120e+13	5.806710e+13	240.0
4.0	-3.376970e+15	-4.390061e+15	-2.570061e+15	-2.283967e+15	-3.715898e+15	-4.708073e+15	260.0
4.5	2.269325e+17	2.950123e+17	1.125323e+17	9.424978e+16	2.556404e+17	3.957103e+17	280.0
5.0	-1.595003e+19	-2.073503e+19	-2.716524e+18	-1.351215e+18	-1.803479e+19	-3.426177e+19	300.0
5.5	1.185105e+21	1.540636e+21	-2.211389e+20	-3.433236e+20	1.305507e+21	3.036121e+21	320.0
6.0	-9.359479e+22	-1.216732e+23	4.945547e+22	6.193477e+22	-9.685666e+22	-2.737673e+23	340.0
6.5	7.839802e+24	1.019174e+25	-6.354884e+24	-7.709809e+24	7.343469e+24	2.500047e+25	360.0
7.0	-6.903358e+26	-8.974366e+26	6.972163e+26	8.463668e+26	-5.665260e+26	-2.303888e+27	380.0
7.5	6.313216e+28	8.207181e+28	-7.123283e+28	-8.752213e+28	4.423755e+28	2.136918e+29	400.0
8.0	-5.925599e+30	-7.703279e+30	7.006932e+30	8.758938e+30	-3.475825e+30	-1.991260e+31	420.0
8.5	5.653837e+32	7.349989e+32	-6.743637e+32	-8.597215e+32	2.730533e+32	1.861775e+33	440.0
9.0	-5.446192e+34	-7.080049e+34	6.405958e+34	8.337118e+34	-2.129414e+34	-1.745039e+35	460.0
9.5	5.272221e+36	6.853888e+36	-6.037080e+36	-8.021978e+36	1.634281e+36	1.638712e+37	480.0
10.0	-5.114409e+38	-6.648732e+38	5.662240e+38	7.678585e+38	-1.219687e+38	-1.541142e+39	500.0

3. Given the problem

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}=0,\quad \frac{1}{2}\le r\le 1,\quad 0\le t\le \pi/3,$$

$$T(r,0)=0,\quad T(r,\pi/3)=0,\quad T(1/2,\theta)=50,\quad T(1,\theta)=100.$$



	0.000000	0.104720	0.209440	0.314159	0.418879	0.523599	\
0.5	0.0	50.000000	50.000000	50.000000	50.000000	50.000000	
0.6	0.0	58.101790	65.408976	70.905679	73.544129	73.017372	
0.7	0.0	44.959877	57.063193	64.696671	68.260068	67.903125	
0.8	0.0	43.163248	57.594001	66.384993	70.489924	70.477958	
0.9	0.0	50.093847	65.625366	74.158771	77.978265	78.181389	
1.0	0.0	66.957974	80.275086	86.141825	88.535360	88.726185	
1.1	0.0	100.000000	100.000000	100.000000	100.000000	100.000000	

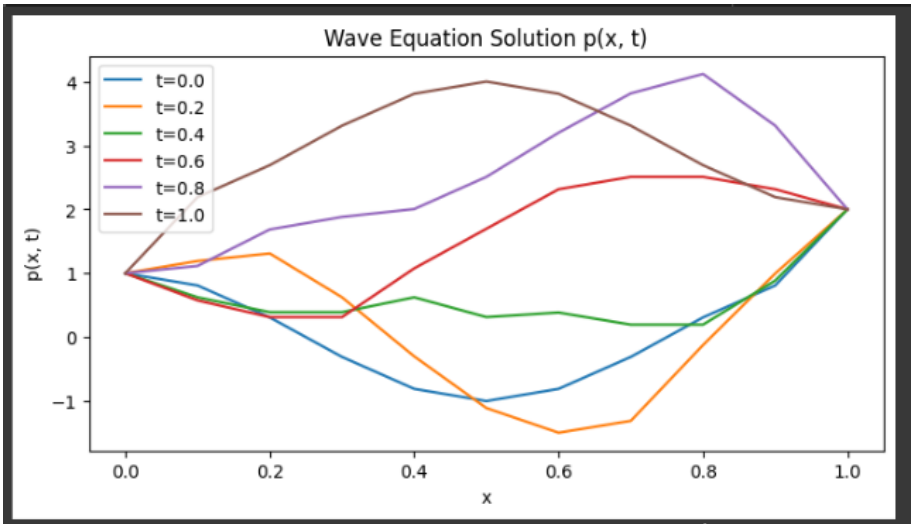
	0.628319	0.733038	0.837758	0.942478	1.047198
0.5	50.000000	50.000000	50.000000	50.000000	0.0
0.6	68.993099	60.825092	47.420432	27.333594	0.0
0.7	63.511959	54.714148	41.037398	22.382191	0.0
0.8	66.415691	57.892396	44.152688	24.564504	0.0
0.9	74.960114	67.692024	54.744371	33.163398	0.0
1.0	86.921211	82.517585	73.428911	53.210252	0.0
1.1	100.000000	100.000000	100.000000	100.000000	0.0

4. Given the problem

$$\frac{\partial^2 p}{\partial t^2} = \frac{\partial^2 p}{\partial x^2}, \quad 0 \le x \le 1, \quad 0 \le t$$

$$p(0,t) = 1, \quad p(1,t) = 2, \quad p(x,0) = \cos(2\pi x), \quad \frac{\partial p}{\partial t}(x,0) = 2\pi \sin(2\pi x), \quad 0 \le x \le 1$$

To calculate p by using $\Delta x = \Delta t = 0.1$.



	0.0	0.1	0.2	0.3	0.4	0.5	0.6	\
0.0	1.0	0.809017	0.309017	-0.309017	-0.809017	-1.000000	-0.809017	
0.1	1.0	1.424288	1.000530	0.194603	-0.685656	-1.304017	-1.424288	
0.2	1.0	1.191513	1.309874	0.623891	-0.300397	-1.109944	-1.495530	
0.3	1.0	0.885586	0.814874	0.814874	0.199603	-0.491910	-0.995530	
0.4	1.0	0.623361	0.390586	0.390586	0.623361	0.314017	0.384729	
0.5	1.0	0.505000	0.199073	0.199073	0.505000	1.500000	1.505000	
0.6	1.0	0.575712	0.313487	0.313487	1.075712	1.695983	2.314344	
0.7	1.0	0.808487	0.690126	1.190126	1.504470	1.890056	2.699603	
0.8	1.0	1.114414	1.685126	1.881109	2.004470	2.508090	3.199603	
0.9	1.0	1.876639	2.305397	2.499470	2.884729	3.314017	3.623361	
1.0	1.0	2.190983	2.690983	3.309017	3.809017	4.000000	3.809017	

	0.7	0.8	0.9	1.0
0.0	-0.309017	0.309017	0.809017	2.0
0.1	-1.000530	-0.194603	1.185656	2.0
0.2	-1.309874	-0.123891	0.996380	2.0
0.3	-0.618891	-0.118891	0.690453	2.0
0.4	0.195453	0.195453	0.884729	2.0
0.5	1.199073	1.199073	1.505000	2.0
0.6	2.508620	2.508620	2.314344	2.0
0.7	3.623891	3.623891	3.003620	2.0
0.8	3.814874	4.118891	3.309547	2.0
0.9	3.694603	3.500530	3.115271	2.0
1.0	3.309017	2.690983	2.190983	2.0