國立成功大學

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數值方法

HW 3

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1. Use the Lagrange interpolating polynomials of degree one, two, three and four to approximate $\cos(0.750) = 0.7317$ if $\cos(0.698) = 0.7661$, $\cos(0.733) = 0.7432$, $\cos(0.768) = 0.7193$, $\cos(0.803) = 0.6946$.

   Find the error bound.

----------第一題----------

True cos(0.75) = 0.731689

Degree = 1
  Approximation = 0.731591
  Error Bound = 1.530000e-04

Degree = 2
  Approximation = 0.731716
  Error Bound = 2.652000e-06

Degree = 3
  Approximation = 0.731704
  Error Bound = 3.513900e-08

Degree = 4
  因資料點僅有4點，故結果將與Degree = 3相同

2. Use iterated inverse interpolation to find an approximation to the solution $x - e^{-x} = 0$ using the data $e^{-0.3} = 0.740818$, $e^{-0.4} = 0.670320$, $e^{-0.5} = 0.606531$, $e^{-0.6} = 0.548812$.

----------第二題----------

Iterated Inverse Interpolation:

Step	x_n	e^{-x_n}	x_{n+1}
1	0.5650000000	0.5683601468	0.5651191275
2	0.5651191275	0.5682924434	0.5652381426
3	0.5652381426	0.5682248121	0.5653570434
4	0.5653570434	0.5681572537	0.5654758285
5	0.5654758285	0.5680897691	0.5655944962
6	0.5655944962	0.5680223592	0.5657130449
7	0.5657130449	0.5679550249	0.5658314730
8	0.5658314730	0.5678877671	0.5659497788
9	0.5659497788	0.5678205866	0.5660679607
10	0.5660679607	0.5677534844	0.5661860172
11	0.5661860172	0.5676864614	0.5663039467
12	0.5663039467	0.5676195184	0.5664217475
13	0.5664217475	0.5675526563	0.5665394180
14	0.5665394180	0.5674858760	0.5666569568
15	0.5666569568	0.5674191784	0.5667743621
16	0.5667743621	0.5673525642	0.5668916325
17	0.5668916325	0.5672860345	0.5670087663
18	0.5670087663	0.5672195900	0.5671257620
19	0.5671257620	0.5671532316	0.5672426181

Converged : x = 0.5671

3. A car travelling along a straight road is clocked at a number of points. The data from the observations are given in the following table, where the time ***T*** is in seconds, the distance ***D*** is in feet, and the speed ***V*** is in feet per second.

T	0	3	5	8	13
D	0	200	375	620	990
V	75	77	80	74	72

   a. Use a Hermite polynomial to predict the position of the car and its speed when $t = 10$ s.

   b. Use the derivative of the Hermite polynomial to determine whether the car ever exceeds a 55 mi/h speed limit on the road. If so, what is the first time the car exceeds this speed?

   c. What is the predicted maximum speed for the car ?

----------第三題----------

(a)
D(10) = 777.0615 (ft)
V(10) = 87.1844 (ft/s)

(b)
The car exceeds a 55 mi/h speed limit !
First time over limit = 3.9429 (sec)

(c)
Max speed = 87.3464 (ft/s)
Time of max speed = 9.8639 (sec)