國立成功大學

工程科學學系

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數值方法

HW 4

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1. Determine the values $\int_1^2 e^x \sin(4x)dx$ with $h = 0.1$ by

a. Use the composite trapezoidal rule

b. Use the composite Simpsons' method

c. Use the composite midpoint rule

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--------------------第一題--------------------
Composite trapezoidal rule :   0.39614759221490675
Composite Simpsons' method :   0.3856635960237503
Composite midpoint rule :   0.38080479837729925
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2. Approximate $\int_1^{1.5} x^2 \ln xdx$ using Gaussian Quadrature with $n = 3$ and

$n = 4$. Then compare the result to the exact value of the integral.

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--------------------第二題--------------------
Gaussian Quadrature with n = 3 :   0.1922593772651118
Gaussian Quadrature with n = 4 :   0.1922593578041556
Absolute error between Gaussian Quadrature with n = 3 and exact :   1.9532315764791974e-08
Absolute error between Gaussian Quadrature with n = 4 and exact :   7.135955715220632e-11
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3. Approximate $\int_0^{\pi/4} \int_{\sin x}^{\cos x} (2y \sin x + \cos^2 x)dydx$ using

a. Simpson's rule for $n = 4$ and $m = 4$

b. Gaussian Quadrature, $n = 3$ and $m = 3$

c. Compare these results with the exact value.

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--------------------第三題--------------------
(a) Simpson's rule for n = 4 and m =  4 :   0.5119875440121252
(b) Gaussian Quadrature for n = 3 and m = 3 :   0.5118655400009525
(c) Exact value :   0.5118446353109126
    Absolute error between Simpson's rule and exact :   0.00014290870121258514
    Absolute error between Gaussian Quadrature and exact :   2.0904690039902363e-05
```

4. Use the composite Simpson's rule and $n = 4$ to approximate the

improper integral a) $\int_0^1 x^{-1/4} \sin xdx$, b) $\int_1^\infty x^{-4} \sin xdx$ by use the transform

$t = x^{-1}$

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--------------------第四題--------------------
(a) Improper integral ∫ x^(-1/4) sin(x) dx :   0.5259312819330653
(b) Improper integral ∫ x^(-4) sin(x) dx by using the transform t=x^(-1) :   0.2744816193780423
```