

國立成功大學

工程科學學系

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數值方法

HW 5

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1. The initial-value problem

$y' = 1 + (y/t) + (y/t)^2$, $1 \leq t \leq 2$, $y(1) = 0$ has the exact

solution $y(t) = t \tan(\ln t)$.

a. Use Euler's method with $h = 0.1$ to approximate the solution, and

compare it with the actual values of y .

b. Use Taylor's method of order 2 with $h = 0.1$ to approximate the

solution, and compare it with the actual values of y .

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-----第一題-----

=== Q1 (a) Euler Method ===

      t |      Euler |      Exact y |      Euler Err |
1.000000 | 0.0000000 | 0.0000000 | 0.0000000 |
1.100000 | 0.1000000 | 0.1051598 | 0.0051598 |
1.200000 | 0.2099174 | 0.2212428 | 0.0113254 |
1.300000 | 0.3304706 | 0.3491211 | 0.0186506 |
1.400000 | 0.4623535 | 0.4896817 | 0.0273281 |
1.500000 | 0.6062855 | 0.6438753 | 0.0375899 |
1.600000 | 0.7630415 | 0.8127527 | 0.0497113 |
1.700000 | 0.9334750 | 0.9974941 | 0.0640191 |
1.800000 | 1.1185367 | 1.1994386 | 0.0809019 |
1.900000 | 1.3192926 | 1.4201158 | 0.1008232 |
2.000000 | 1.5369433 | 1.6612818 | 0.1243385 |

=== Q1 (b) Taylor 2nd Order ===

      t |      Taylor |      Exact y |      Taylor Err |
1.000000 | 0.0000000 | 0.0000000 | 0.0000000 |
1.100000 | 0.1050000 | 0.1051598 | 0.0001598 |
1.200000 | 0.2209192 | 0.2212428 | 0.0003236 |
1.300000 | 0.3486124 | 0.3491211 | 0.0005087 |
1.400000 | 0.4889538 | 0.4896817 | 0.0007279 |
1.500000 | 0.6428828 | 0.6438753 | 0.0009925 |
1.600000 | 0.8114382 | 0.8127527 | 0.0013146 |
1.700000 | 0.9957867 | 0.9974941 | 0.0017074 |
1.800000 | 1.1972517 | 1.1994386 | 0.0021869 |
1.900000 | 1.4173435 | 1.4201158 | 0.0027723 |
2.000000 | 1.6577947 | 1.6612818 | 0.0034871 |

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2. The system of initial-value problems

$$u_1' = 9u_1 + 24u_2 + 5\cos t - \frac{1}{3}\sin t, \quad u_1(0) = \frac{4}{3},$$

$$u_2' = -24u_1 - 52u_2 - 9\cos t + \frac{1}{3}\sin t, \quad u_2(0) = \frac{2}{3},$$

has the unique solution

$$u_1 = 2e^{-3t} - e^{-39t} + \frac{1}{3}\cos t, \quad u_2 = -e^{-3t} + 2e^{-39t} - \frac{1}{3}\cos t.$$

Try $h = 0.05$ and $h = 0.1$ in Runge-Kutta method, and compare their results with the exact value.

-----第二題-----

=== Q2 RK4 h = 0.1 ===

t	u1(h)	u1(exact)	u1 error	u2(h)	u2(exact)	u2 error
0.000000	1.333333	1.333333	0.000000	0.666667	0.666667	0.000000
0.100000	-3.0524371	1.7930626	4.8454997	8.9893053	-1.0320025	10.0213078
0.200000	-23.8477949	1.4239024	25.2716973	51.1927040	-0.8746810	52.0673850
0.300000	-130.1652017	1.1315765	131.2967782	269.2691932	-0.7249986	269.9941917
0.400000	-680.2314851	0.9094086	681.1408937	1399.3685835	-0.6082142	1399.9767977
0.500000	-3531.2995854	0.7387878	3532.0383732	7258.2418388	-0.5156577	7258.7574965
0.600000	-18312.7950522	0.6057096	18313.4007619	37634.9554830	-0.4404108	37635.3958937
0.700000	-94951.3319073	0.4998603	94951.8317675	195131.8717354	-0.3774038	195132.2491392
0.800000	-492306.4656395	0.4136715	492306.8793110	1011721.8720780	-0.3229535	1011722.1950315
0.900000	-2552513.6238674	0.3416143	2552513.9654818	5245578.8265899	-0.2744088	5245579.1009987
1.000000	-13234278.7891679	0.2796749	13234279.0688428	27197287.2065870	-0.2298878	27197287.4364748

=== Q2 RK4 h = 0.05 ===

t	u1(h)	u1(exact)	u1 error	u2(h)	u2(exact)	u2 error
0.000000	1.333333	1.333333	0.000000	0.666667	0.666667	0.000000
0.050000	1.7218803	1.9120586	0.1901784	-0.4995993	-0.9090766	0.4094772
0.100000	1.7269150	1.7930626	0.0661475	-0.8325977	-1.0320025	0.1994047
0.150000	1.6171606	1.6019668	0.0151939	-0.8903730	-0.9614587	0.0710857
0.200000	1.4816873	1.4239024	0.0577849	-0.8610421	-0.8746810	0.0136389
0.250000	1.3489450	1.2676456	0.0812994	-0.8075045	-0.7952208	0.0122838
0.300000	1.2270633	1.1315765	0.0954868	-0.7503406	-0.7249986	0.0253421
0.350000	1.1174781	1.0129986	0.1044796	-0.6958859	-0.6630596	0.0328263
0.400000	1.0195255	0.9094086	0.1101169	-0.6457318	-0.6082142	0.0375176
0.450000	0.9319767	0.8186295	0.1133471	-0.5999342	-0.5593892	0.0405450
0.500000	0.8535405	0.7387878	0.1147527	-0.5580925	-0.5156577	0.0424348
0.550000	0.7830173	0.6682747	0.1147426	-0.5197063	-0.4762247	0.0434815
0.600000	0.7193370	0.6057096	0.1136274	-0.4842903	-0.4404108	0.0438795
0.650000	0.6615603	0.5499094	0.1116509	-0.4514071	-0.4076353	0.0437717
0.700000	0.6088677	0.4998603	0.1090074	-0.4206726	-0.3774038	0.0432688
0.750000	0.5605468	0.4546947	0.1058521	-0.3917541	-0.3492955	0.0424586
0.800000	0.5159800	0.4136715	0.1023086	-0.3643647	-0.3229535	0.0414112
0.850000	0.4746326	0.3761577	0.0984749	-0.3382586	-0.2980760	0.0401825
0.900000	0.4360426	0.3416143	0.0944283	-0.3132261	-0.2744088	0.0388173
0.950000	0.3998123	0.3095830	0.0902293	-0.2890893	-0.2517387	0.0373506
1.000000	0.3655998	0.2796749	0.0859249	-0.2656980	-0.2298878	0.0358102