Characterisation of Multipartite Entanglement

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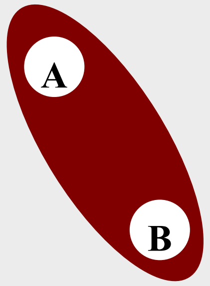
What is the Tangle?

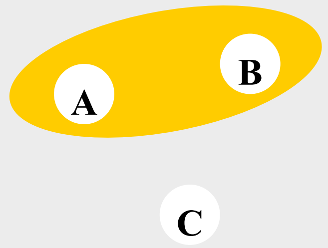


The tangle is a measure of how connected two qubits (the building blocks in quantum information theory, analogous to a computer bit but with special properties) are. The measure was created by Coffman, Kundu and Wootters in a landmark paper called “Distributed Entanglement”[1]

It has been a useful measure due to its simplicity for two qubits and is based on another entanglement measure which is known and generally accepted throughout quantum information theorists. Here, we hope to unveil some of its mysteries!

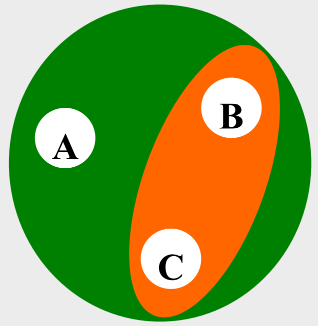
[1]arXiv:quant-ph/9907047v2

The Three Forms of the Tangle in a Three Qubit System



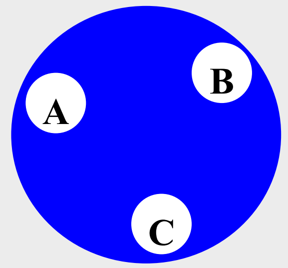
τ_{AB}

Between two qubits



$\tau_{A(BC)}$

With a pair of qubits



τ_{ABC}

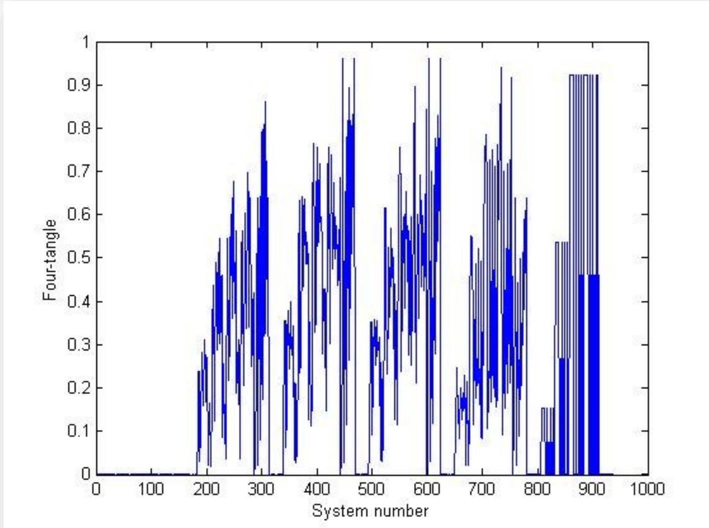
Inherent in the system

$$\tau_{A(BC)} = \tau_{AB} + \tau_{AC} + \tau_{ABC}$$

This equation refers to the three qubit system as shown above. It says that the tangle between qubit A and the rest of the system is the sum of the tangle between qubits A and B, the tangle between qubits A and C, plus a quantity which is inherent within the system: the tangle between all three qubits, or the so-called “residual tangle”.
The residual tangle has many papers written about it and its properties are getting more and more well-known. We know that it holds to the monogamy condition, so it is always positive, and also that it is symmetric. That is to say that if we were to focus on qubit C instead of qubit A, we would get the same value for the residual tangle.
This quantity is easy to calculate for pure states, which are states that we have all the information we could need about them. However, for mixed states, when we are missing some information, this residual tangle becomes extremely difficult to calculate

$$a\sqrt{p}|0000\rangle + b\sqrt{p}|1110\rangle + c\sqrt{1-p}|0011\rangle + d\sqrt{1-p}|0101\rangle + f\sqrt{1-p}|1001\rangle$$

Rather than attempting to check in the general four qubit case whether the four-tangle is always positive and symmetric, it was decided to take the state above and check whether monogamy holds in this case. This state was created by us as it reduces to a state on which information about its three-tangle is known from a published paper, which could have been helpful in the study.
By using known equalities and inequalities from some of the earliest papers on the tangle and its properties, whilst using some of the more recent ones for extra insight, we constructed an analytical expression for the four tangle of the above state, with conditions attached, before running a MATLAB simulation



Above is the result of the simulation. Using the equation for the four-tangle and by checking all of the conditions and evaluating each term, we plotted almost 1000 points of varying parameter values. The graph came back as above and it shows that all of the states created have a four-tangle of greater than or equal value to zero. Furthermore, they also have four-tangle less than one, as we should expect from its construction.
Note: In actual fact, a few values came back below zero, but this is believed to be due to rounding errors

$$\tau_{A(BCD)} = \tau_{AB} + \tau_{AC} + \tau_{AD} + \tau_{ABC} + \tau_{ABD} + \tau_{ACD} + \tau_{ABCD}$$

Little is known about the final term in the equation above and this study has aimed to shed some light on the matter.

Calculating the tripartite tangle of a pure, three qubit state is easy enough, but when you start mixing them, it becomes computationally difficult as there is no analytical expression for most states and they are calculated by taking a minimum over an infinite number of pure-state decompositions.

The numerical simulation seems to show that the four-tangle for this state is monogamous, but it would be better to know for certain. In the paper are the expressions and the conditions, which (with a bit of time) could be analysed further and the conjecture could be proved.
For now, we shall take the numerical results and say this:

The four-tangle for this state is monogamous