AP HW5

Stochastic Discount Factor

In [1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

> Simulate consumption growth
>
> $$\ln \tilde{g} = 0.02 + 0.02\tilde{\epsilon} + \tilde{\nu}$$
>
> Here $\tilde{\epsilon}$ is a standard normal random variable, while $\tilde{\nu}$ is an independent random variable that has value of either zero (with probability of 98.3%) or ln(0.65) (with probability of 1.7%).

In [2]:
```
samples = int(1e4)
epsilon = np.random.normal(0, 1, samples)
uniform = np.random.uniform(0, 1, samples)
nu = np.where(uniform > 0.983, np.log(0.65), 0)
```

In [3]:
```
ln_g = 0.02 + 0.02 * epsilon + nu
g = np.exp(ln_g)
```

> Use the simulated distribution of consumption growth to find the simulated distribution of the pricing kernel for power utility:
>
> $$\tilde{M} = 0.99\tilde{g}^{-\gamma}$$
>
> Calculate the mean (μM) and standard deviation (σM) of pricing kernel for each value of γ, and plot the volatility ratio (σM/μM) on the vertical axis vs γ on the horizontal axis.

In [4]:
```
gamma = np.arange(1, 4.1, 0.1)
```

In [5]:
```
all_mu_M = []
all_sigma_M = []
all_volatility_ratio = []

for i in gamma:
    M = 0.99 * g ** (-i)
    mu_M = np.mean(M)  # mean
    sigma_M = np.std(M)  # sd
    volatility_ratio = sigma_M / mu_M


    all_mu_M.append(mu_M)
    all_sigma_M.append(sigma_M)
    all_volatility_ratio.append(volatility_ratio)
```

```python
data = pd.DataFrame({
    'gamma': gamma,
    'mu_M': all_mu_M,
    'sigma_M': all_sigma_M,
    'volatility_ratio': all_volatility_ratio
})

data
```

Out[5]:

	gamma	mu_M	sigma_M	volatility_ratio
0	1.0	0.980491	0.073790	0.075258
1	1.1	0.979815	0.082782	0.084487
2	1.2	0.979196	0.092121	0.094079
3	1.3	0.978637	0.101825	0.104047
4	1.4	0.978139	0.111908	0.114409
5	1.5	0.977704	0.122387	0.125178
6	1.6	0.977336	0.133282	0.136372
7	1.7	0.977036	0.144608	0.148007
8	1.8	0.976807	0.156387	0.160100
9	1.9	0.976652	0.168637	0.172668
10	2.0	0.976573	0.181380	0.185731
11	2.1	0.976574	0.194636	0.199305
12	2.2	0.976657	0.208429	0.213411
13	2.3	0.976825	0.222783	0.228068
14	2.4	0.977082	0.237720	0.243296
15	2.5	0.977432	0.253267	0.259115
16	2.6	0.977877	0.269451	0.275547
17	2.7	0.978421	0.286299	0.292613
18	2.8	0.979069	0.303840	0.310335
19	2.9	0.979824	0.322104	0.328736
20	3.0	0.980691	0.341121	0.347838
21	3.1	0.981674	0.360926	0.367664
22	3.2	0.982777	0.381551	0.388238
23	3.3	0.984006	0.403033	0.409584
24	3.4	0.985364	0.425408	0.431726
25	3.5	0.986859	0.448714	0.454689
26	3.6	0.988494	0.472992	0.478498
27	3.7	0.990275	0.498284	0.503177
28	3.8	0.992209	0.524633	0.528752
29	3.9	0.994302	0.552084	0.555248
30	4.0	0.996559	0.580686	0.582691

> Find the smallest value of γ (in your data) for which $\frac{\sigma M}{\mu M} > 0.4.$

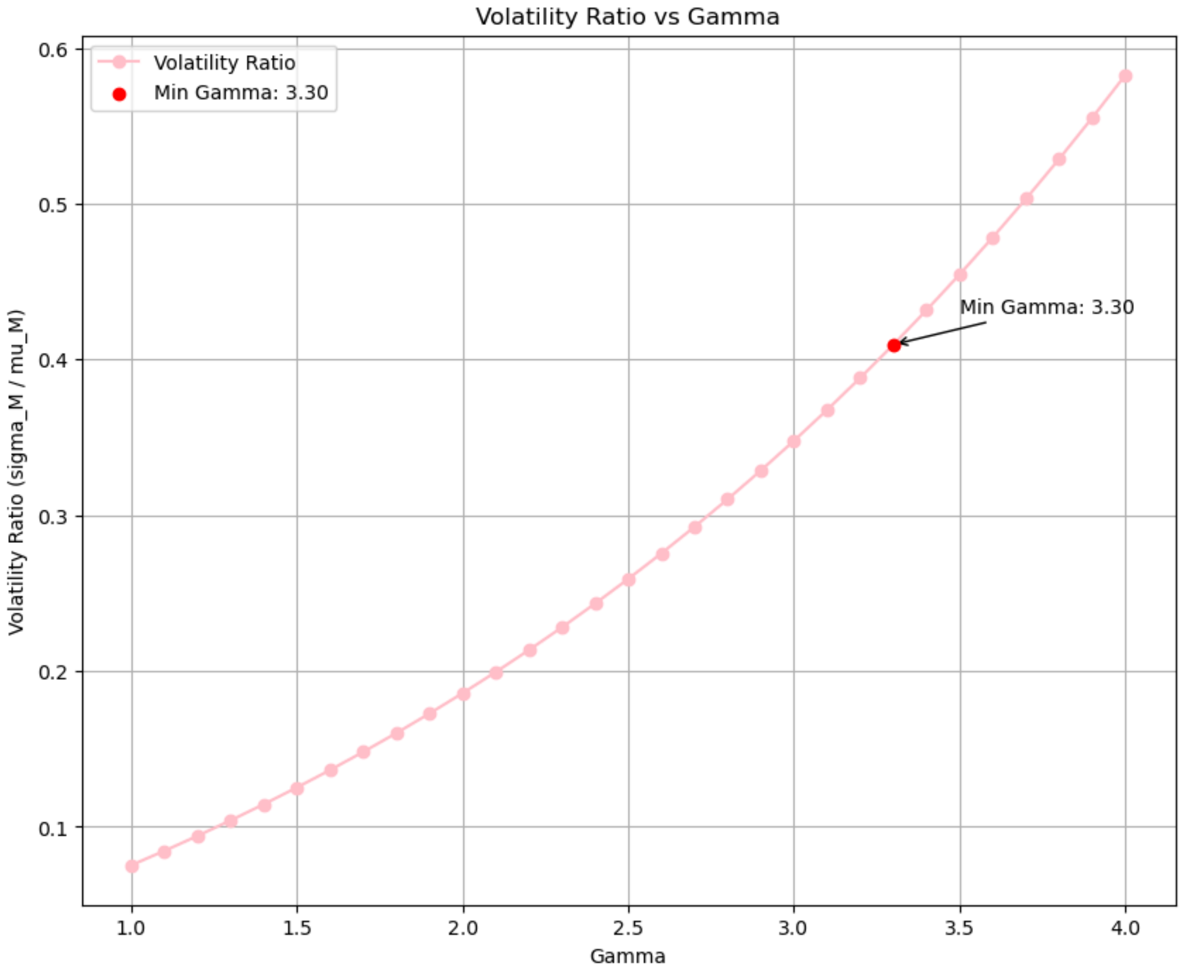
In [6]:
```
gamma_above_threshold = gamma[np.array(all_volatility_ratio)> 0.4]
min_gamma = np.min(gamma_above_threshold)
print(f'The smallest gamma for which volatility ratio > 0.4 is: {min_gamma}
```

```
The smallest gamma for which volatility ratio > 0.4 is: 3.3000000000000002
```

In [7]:
```
plt.figure(figsize=(10, 8))
plt.plot(gamma, all_volatility_ratio, marker='o', linestyle='-', color='pinl

min_volatility_ratio_index = np.where(gamma == min_gamma)[0][0]
min_volatility_ratio = all_volatility_ratio[min_volatility_ratio_index]
plt.scatter(min_gamma, min_volatility_ratio, color='red', zorder=5, label=f'
plt.annotate(f'Min Gamma: {min_gamma:.2f}',
             xy=(min_gamma, min_volatility_ratio),
             xytext=(min_gamma + 0.2, min_volatility_ratio + 0.02),
             arrowprops=dict(facecolor='black', arrowstyle='->'))

plt.title('Volatility Ratio vs Gamma')
plt.xlabel('Gamma')
plt.ylabel('Volatility Ratio (sigma_M / mu_M)')
plt.grid(True)
plt.legend()
plt.show()
```



> Explanation

γ represents the investors' sensitivity to risk. From the provided results and the accompanying graph, we observe that the smallest γ is approximately 3.3 when the volatility ratio $\frac{\sigma_M}{\mu_M}$ surpasses 0.4. Investors with high risk aversion will pay more attention to catastrophic events, which are low probability but high impact.

Investors are more concerned about the effects of rare catastrophic events on the future retrun of their investment when $\gamma > 3.3$. Therefore, they require a hiher return rate not only to offset potential losses from these future catastrophic events but also to compensate for the uncertainty and psychological pressure associated with holding risky assets under such conditions.

It means that inverstors will not concerned about the rare catastrophic events too much when $\gamma < 3.3$, so they do not consider this kinds of the risk when deciding their investment strategy.As γ increases towards the critical point of 3.3, investors become more conservative in their investment strategies. After this threshold, they take rare disaster risks more seriously and require higher returns to offset the perceived risk.