AP HW6

Behavioural Finance

Assume Barberis, Huang, and Santos economy where investor receives utility from consumption as well as recent financial gain or loss. Use these parameters:

$$\delta = 0.99, \quad \gamma = 1, \quad \lambda = 2$$

Consumption growth has lognormal distribution:

$$\ln \tilde{g} = 0.02 + 0.02\tilde{\varepsilon}$$

where ε is standard normal random variable. Simulate probability distribution for consumption growth with (at least) 104 random draws from standard normal distribution.

With these parameters, risk-free rate is around 3% per year:

$$R_f = \frac{e^{0.0198}}{0.99} = 1.0303$$

Define x as one plus dividend yield for market portfolio:

$$x = \left(1 + \frac{P}{D}\right)\frac{D}{P} = 1 + \frac{D}{P}$$

and define error term:

$$e(x) = 0.99 b_0 E[v(x\tilde{g})] + 0.99x - 1$$

where utility from recent financial gain or loss is given by:

$$v(R) = \begin{cases} R - 1.0303 & \text{for } R \geq 1.0303 \\ v(R) = 2(R - 1.0303) & \text{for } R < 1.0303 \end{cases}$$

Solve for e(x) = 0 to find equilibrium value of x, using bisection search:

1. Set x– = 1 and x+ = 1.1, and use simulated distribution of consumption growth to confirm that e(x–) < 0 and e(x+) > 0 ⇒ solution must lie between x– and x+
2. Set x0 = 0.5*(x– + x+) and use simulated distribution of consumption growth to calculate e(x0)
3. If |e(x0)| < 10–5, then you have converged to solution
4. Otherwise if e(x0) < 0, then solution lies between x0 and x+ ⇒ repeat from step 2 with x– = x0
5. Otherwise if e(x0) > 0, then solution lies between x– and x0 ⇒ repeat from step 2 with x+ = x0

Repeat for b0 in range from 0 to 10, in increments of 0.1 (or less).

In [1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

In [2]:
```
# define all parameters
delta = 0.99
b0_values = np.arange(0, 10.1, 0.1)# from 0 to 10, step is 0.1
Rf = 1.0303
mu = 0.02
sigma = 0.02

np.random.seed(50)
num_samples = 10000
```

In [3]:
```
epsilon = np.random.normal(0, 1, num_samples)
log_g_tilde = mu + sigma * epsilon
g_tilde = np.exp(log_g_tilde)
```

In [4]:
```
def v(R):
    return np.where(R >= Rf, R - Rf, 2 * (R - 1.0303))

def e_x(x, b0):
    expected_v = np.mean(v(x * g_tilde))
    return delta * b0 * expected_v + delta * x - 1

# Dichotomy solution e(x) = 0
def bisection_search(b0, tol=1e-5, max_iter=1000):
    x_low, x_high = 1.0, 1.1
    for _ in range(max_iter):
        x_mid = 0.5 * (x_low + x_high)
        e_mid = e_x(x_mid, b0)
        if abs(e_mid) < tol:
            return x_mid
        elif e_mid < 0:
            x_low = x_mid
        else:
            x_high = x_mid
    return x_mid
```

Calculate price-dividend ratio for market portfolio:

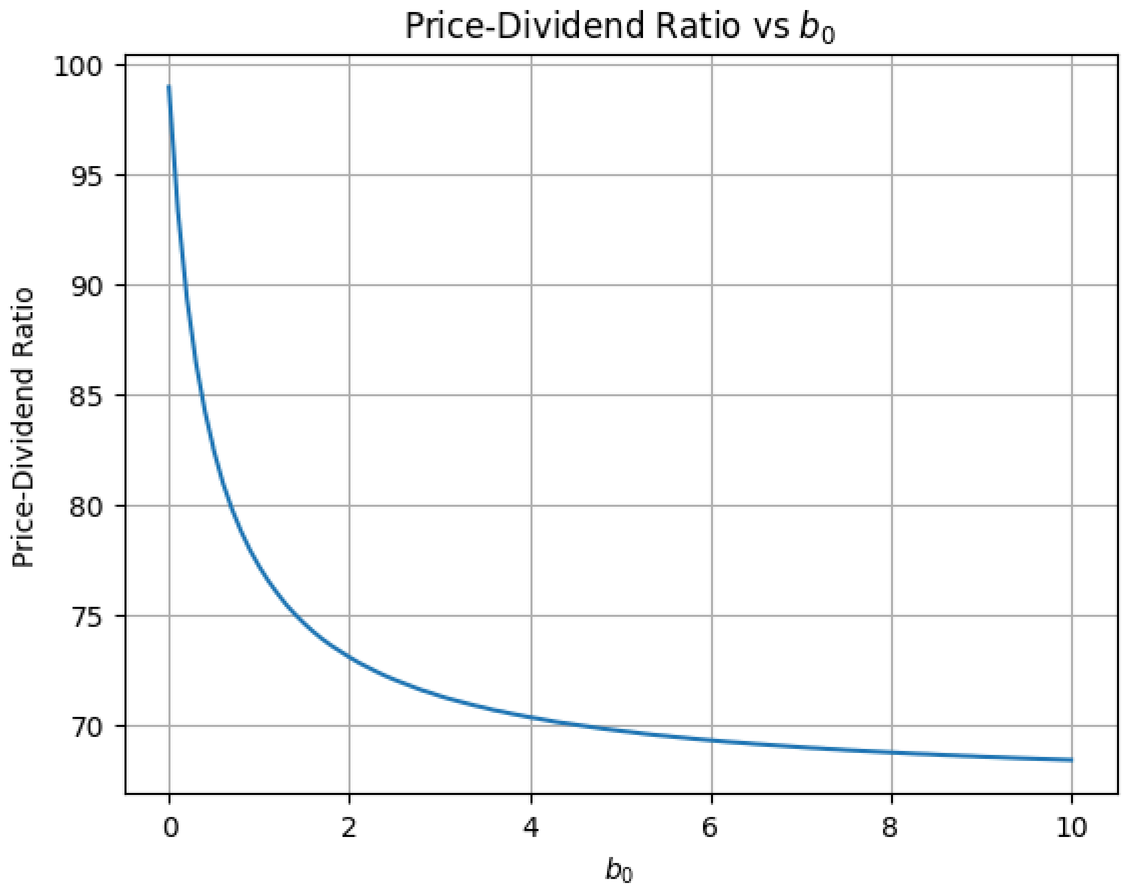
$$\frac{P}{D} = \frac{1}{x-1}$$

Plot price-dividend ratio (on vertical axis) vs b0.

In [5]:
```
price_dividend_ratios = []
for b0 in b0_values:
    x_star = bisection_search(b0)
    price_dividend_ratio = 1 / (x_star - 1)
    price_dividend_ratios.append(price_dividend_ratio)
```

In [6]:
```
plt.plot(b0_values, price_dividend_ratios)
plt.xlabel('$b_0$')
plt.ylabel('Price-Dividend Ratio')
```

```
plt.title('Price-Dividend Ratio vs $b_0$')
plt.grid(True)
plt.show()
```



Calculate expected market return:

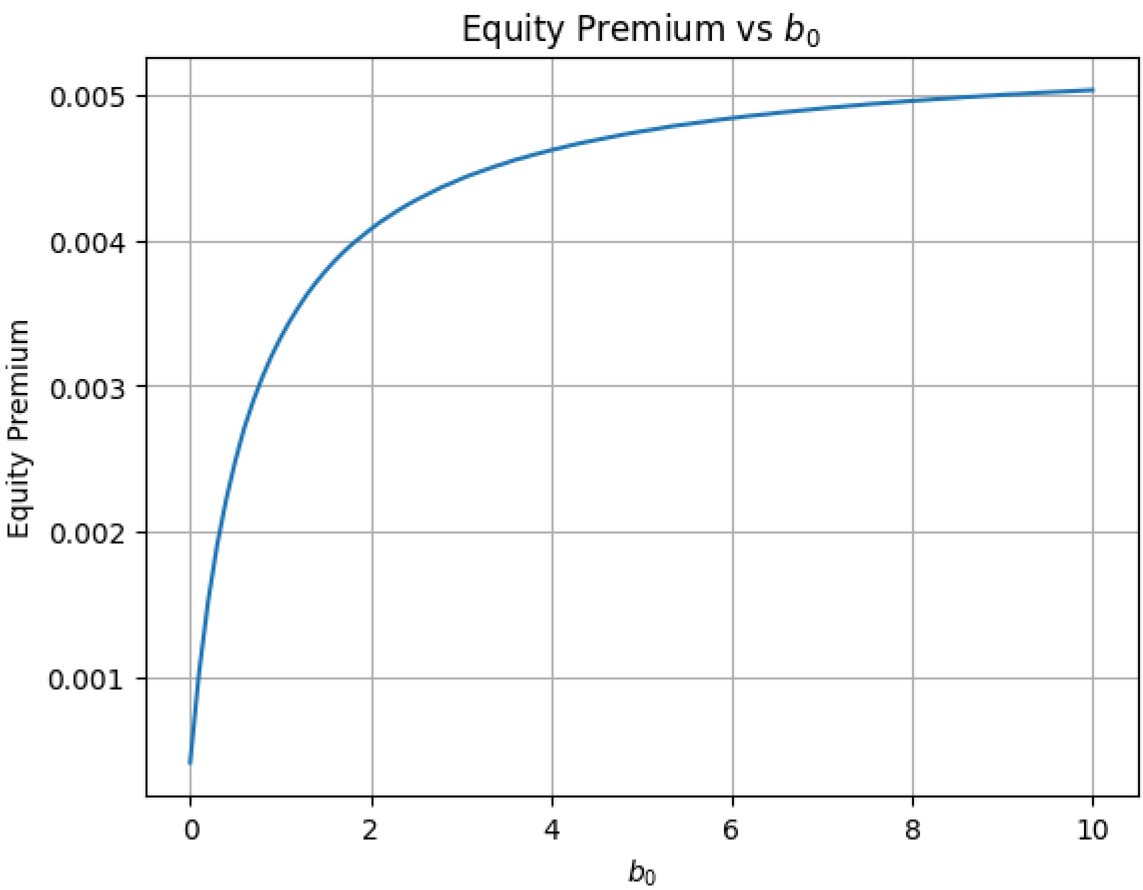
$$E\left(\tilde{R}_m\right) = E(x\tilde{g}) = xe^{0.0202}$$

Plot equity premium (on vertical axis) vs b0.

In [7]:
```
# calculate for expecation return
expected_market_returns = []
equity_premiums = []

for b0 in b0_values:
    x_star = bisection_search(b0)
    expected_market_return = x_star * np.exp(0.0202)
    expected_market_returns.append(expected_market_return)
    equity_premium = expected_market_return - Rf
    equity_premiums.append(equity_premium)

plt.plot(b0_values, equity_premiums)
plt.xlabel('$b_0$')
plt.ylabel('Equity Premium')
plt.title('Equity Premium vs $b_0$')
plt.grid(True)
plt.show()
```



In [10]:
```
data = {
    'price-dividend ratio': price_dividend_ratios,
    'expected_market_returns': expected_market_returns,
    'equity_premiums': equity_premiums
}
df = pd.DataFrame(data)

df
```

Out[10]:

	price-dividend ratio	expected_market_returns	equity_premiums
0	98.937198	1.030719	0.000419
1	93.302961	1.031342	0.001042
2	89.334787	1.031828	0.001528
3	86.413502	1.032214	0.001914
4	84.193217	1.032525	0.002225
...	...	...	...
96	68.423470	1.035318	0.005018
97	68.409186	1.035322	0.005022
98	68.394907	1.035325	0.005025
99	68.380634	1.035328	0.005028
100	68.366368	1.035331	0.005031

101 rows × 3 columns

Briefly describe (in words, without using mathematical equations or formulas) main characteristics of v(•) as well as economic significance and implications of b0 and λ.

> v(•) shows the investor's sensitivity to financial gains and losses, demonstrating loss aversion, which means losses have a greater emotional impact than equivalent gains. This makes investors more concerned about downside risk and more sensitive to potential losses.

> b0 represents how much weight an investor places on recent financial gains or losses. A higher value means the investor is more focused on short-term performance rather than solely on long-term wealth. A higher b0 will lead investors to demand a greater equity premium and can increase market volatility. During economic downturns or market declines, it may amplify price instability.

> λ represents the degree of loss aversion. A higher λ indicates that investors are more sensitive to losses compared to gains of the same magnitude. A higher λ will increase the demand for risk premiums, lowers equity prices, and amplify market volatility, especially during periods of economic uncertainty, as investors become more risk-averse.