1. Summary of the questions

- We have a portfolio of:
  - A SOFR swap in which we are the fixer payer (float receiver).
  - Equity: 4 U.S stocks.
- Data given:
  - Historical SOFR curve.
  - Historical stock prices.
- Task:
  1. Calculate Parametric VaR.
  2. Calculate: Monte Carlo VaR using a) full revaluation, 2) risk-based method.
  3. Calculate: Historical VaR using a) full revaluation, 2) risk-based method.

2. General idea

- With the assumption of normal distribution, the major step is to calculate the mean and variance of the whole portfolio in terms of dollar amount, then using these values to calculate VaR.
- Calculating VaR is effectively finding a value at a certain percentile threshold.
- To calculate the mean changes in the portfolio value we use the formula:

$$\overline{\Delta P} = \overrightarrow{w_i} \cdot \overrightarrow{\mu_i}$$

  with $\overrightarrow{\mu_i}$ represents the mean changes of the risk factors in the portfolio, and $\overrightarrow{w_i}$ represents the weights (loading) of those factors.
- The risk factors of the portfolios are:
  1) the changes in the SOFR rate of the swap tenors, and
  2) the changes in stock prices.
- To find $\overrightarrow{w_i}$, note that our portfolio have two major sub-portfolios: the swap and the equity, each of which requires different methods.
  a) For SOFR tenors, we calculate $DV01$, which is the impact on the swap value of 1bps increase in the SOFR of a tenor.
  b) For the equity, their weights are simply their dollar position, which is $1 million each in this project. Note that the weight here are the dollar amount, not the percentages weights because we are evaluating the sensitivity of the whole

portfolio in terms of dollar amount. (multipling these $1 million with their daily changes will provide the impacts of their daily changes on the whole portfolio)

- To calculate portfolio variance, we can use vectorization:

$$\overrightarrow{w_i}^T \cdot \Sigma \cdot \overrightarrow{w_i}$$

Σ is the covariance of the daily changes of the risk factors in the portfolio.

- Following this, we can calculate the VaR models, either manually or by some packages.

2. Data prepation

2.1. Swap preparation

- We can refer to our lectures in QG609/Risk Analysis and QF605/Fixed Income for swap valuation. In particular:
  - ***Calculate the discount factor for each tenor for each given day*** i in the data set using the formula (assuming continuous compounding):

    $$D(i,T) = e^{-r_{i,T} \times T}$$

    E.g. for the 1M tenor, which is corresponding to T = 1/12 = 0.083333 as shown in the 2nd column, we then have a series of the SOFR rates in each given day. For each of this we calculate the discount factor:

    1) For 20221031 we have 0.038721. Thus $D(20221031, 1M) = e^{-0.038721 \times \frac{1}{12}} = 0.996778$.
    2) For 20221101 we have 0.039023: Thus $D(20221101, 1M) = e^{-0.039023 \times \frac{1}{12}} = 0.996773$.
    3) And so on.
  - ***Calculate the PV of the floating leg***: the formula is $PV_{float} = 1 - D(T_{max})$ (1 minus the discount factor of the longest tenor). Thus, for each given day i, following the calculation of the discount factors above for each tenor, we do $1 - D(i, 40Y)$. This is the present value of the floating leg for each given day.
  - ***Calculate the PV of the fixed leg***: it is essentially the SUMPRODUCT of the discount factors with the par swap rate. For each given day:

    $$PV_{fixed,i} = Par\ swap\ rate\ \times \sum_{T=1}^{40} D(i,T)$$

(Note: one way to think of it is to consider that we have a notional amount of \$1, hence we calculate the cash flow for each tenor by doing $1 \times par\ swap\ rate \times discount\ factor$, then we sum these values up. We do not use the SOFR rates here because this is the fixed leg, which is subjected to the par swap rate)

- ***Calculate the PV of the swap payer by netting the PV of the two legs***. It is effectively the NPV of the swap payer, so for each day, we take difference between the PV of the cash inflow (the floating leg) and the PV of the cash outflow (the fixed leg).

$$PV_{swap\ payer} = PV_{float} - PV_{fixed}$$

- Having calculated the swap value of each given day, we now can do sensitivity analysis. In particular, we estimate $\Delta PV_{swap\ payer}$ for 1 basis point change in each tenor (partial DV01). This is useful because it helps to understand which tenors have the biggest impact on the swap's value if the yield curve shifts.

$$DV01 = \frac{PV_{new\ swap\ payer} - PV_{baseline\ swap\ payer}}{0.0001}$$

To do tasks: For each tenor:

1) Increase the SOFR rate for all given days by 1 bps. Those of the remaining tenors remain unchanged.
2) Re-calculate the $PV_{swap\ payer}$ to obtain $PV_{new\ swap\ payer}$.
3) Calculate $DV01$ using the formula above.

Append the results all of tenors, and we should have a vector of $DV01$. This is also the weights for these risk factors, which are applied later for portfolio evaluation.

- Propose modelling procedures in Python:
  1) Define a funtion to calculate the baseline $PV_{swap\ payer}$. (prefer vectorization over loop for computational effciency)
  2) Define a function for sensitivity analysis: $DV01$.

2.2. Equity preparation

- To evaluate the sensitivity of the portfolio to the daily changes of the stocks, we calculate the mean daily return of each stock over the period, then multiplying with the value of the positions.

$$\Delta Equity = \sum_{i=1}^{4} r_i \times w_i$$

with r_i is the daily mean return of the stock i, and w_i is \$1 million for each stock in this project.

- ***To-do tasks***:
  1) Calculate the daily return for each stocks.
  2) Calculate the mean return for each stocks.

2.3. Portfolio preparation

- Our main task here is to prepare: portfolio mean changes; covariance matrix of the daily changes; portfolio variance.
- ***To-do task: Calculating the mean changes***:
  1) ***Generate the weight vector of the risks factors.*** Particularly, the weights for the swaps are the DV01 and the weights for the stock are \$1 million for each. Note that they are weights because when we multiply them with the respective changes in the SOFR/stock returns, we obtain their impacts on the portfolio value; our target here is the absolute change in the portfolio value, not the return.
  2) ***Generate a matrix (data table) of the daily changes*** in the SOFR rate and the stock returns: the columns are the risk factors, while the rows are date time. We already calculated the daily return of the stocks above. Here, we use the SOFR data to calculate the daily change of the rates, then merge it (or generate a new table) with the daily changes of the stocks. We have to do this separately because the daily changes in the rates and the stocks are calculated differently.
  3) ***Calculate the mean changes of the new data table.*** This provides the mean fluctuation in the SOFR and the stocks.
  4) ***Calculating the mean change of the whole portfolio.*** Take the dot product of the mean changes of the risk factors with their weights.

$$\overline{\Delta P} = \overrightarrow{w_\iota} \cdot \overrightarrow{\mu_\iota}$$

- ***To do task: Calculating the portfolio variance***:
  1) Use the matrix of daily changes to calculate the covariance matrix, Σ.
  2) Calculate the portfolio variance using vectorization as

$$var_P = \overrightarrow{w_\iota}^T \cdot \Sigma \cdot \overrightarrow{w_\iota}$$

3. Parametric VaR

- We can find the 95th percentile by using:

$$Parametric\ Var\ 95 = \overline{\Delta P} + \mathrm{z} \times \sigma_P$$

4. Historical VaR

- ***Risk-based approach***: basically it is the dot product of the weights with the daily changes, then find the 95% percentile. (linear assumption)
- ***Full revaluation approach***:
  - In stead of doing linear combination to estimate the sensitivity and VaR of the portfolio, we re-calculate the swap position and the equity position, then take the sum of these values to have the portfolio values.
  - The assumption in historical VaR is the recurrence of the history. E.g. the 1st data point in the given historical data correspond to the 1st day in the revaluation period, the 2nd data point in the given historical data correspond to the 2nd day in the revaluation period, and so on. Therefore, we generate a new set of SOFR by adding the historical daily changes to to initial SOFR, and use this set to calculate the new swap values.
  - Baseline portfolio value = current swap value + $ 4 million of equity.
  - Calculate the new portfolio value:
    1. For the swap, use the new SOFR set to re-calculate the swap values as in 2.1.
    2. For each of the position, multiple the dollar weight ($1 million) with the new stock price.
    3. Sum up these values to obtain the new portfolio value.
  - Subtract the baseline value from the new value to obtain the daily changes.
  - Find the 95% percentile of the daily changes.

5. Monte Carlo VaR

- The assumption is that the historical changes are applied as future scenarios. Hence, we use the sample historical data to simulate the changes in the risk factors. E.g. We can use the Numpy function random.multivariate_normal.

- The approach for risk-based and revaluation approach is similar to Historical VaR. The only difference is that we now use the simulated set instead of the historical set.

6. Result discussion

- We may compare the VaR among the methods to check which is the least and most conservative approach. (the higher VaR, the more conservative)
- Can also consider to add the advantages and limitations of the methods.

7. Taks allocation