$$E = \mu \sum_{i=1}^{N} \int_{\Omega} \theta(x) \left| \nabla H(\phi_i(x)) \right| dx \qquad (1) \qquad \text{where } x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Note:

$$\left| \nabla H(\phi) \right| = \sqrt{\left(\frac{\partial H}{\partial \phi} \frac{\partial \phi}{\partial x_1} \right)^2 + \left(\frac{\partial H}{\partial \phi} \frac{\partial \phi}{\partial x_2} \right)^2} = \underbrace{\frac{\partial H(\phi)}{\partial \phi}}_{\delta(\phi)} \left| \nabla \phi(x) \right| = \delta(\phi) \left| \nabla \phi(x) \right|$$

$$\overset{(1)}{\Longrightarrow} E = \mu \sum_{i=1}^{N} \int Q(x) \left| \nabla H(\phi_i(x)) \right| dx = \mu \sum_{i} \int Q(x)\, \delta(\phi_i) \left| \nabla \phi_i(x) \right| dx$$

E-L equation:

$$\frac{\partial L}{\partial \phi_i} - \sum_{j=1}^{2} \frac{\partial}{\partial x_j} \left(\frac{\partial L}{\partial \left(\frac{\partial \phi_i}{\partial x_j} \right)} \right) = 0$$

$$\frac{\partial L}{\partial \phi_i} = \mu Q(x) \frac{\partial}{\partial \phi_i} \left(\delta(\phi_i) \left| \nabla \phi_i(x) \right| \right) = 0$$

$$\sum_{j=1}^{2} \frac{\partial}{\partial x_j} \left(\frac{\partial L}{\partial \left(\frac{\partial \phi_i}{\partial x_j} \right)} \right) = \sum_{j=1}^{2} \frac{\partial}{\partial x_j} \left(\frac{\partial}{\partial \left(\frac{\partial \phi_i}{\partial x_j} \right)} \left(\mu Q(x)\, \delta(\phi_i) \left| \nabla \phi_i(x) \right| \right) \right)$$

$$= \sum_{j=1}^{2} \frac{\partial}{\partial x_j} \left(\mu Q(x)\, \delta(\phi_i) \frac{1}{2} \frac{2 \frac{\partial \phi_i}{\partial x_j}}{\left| \nabla \phi_i(x) \right|} \right)$$

$$= \mu\, \delta(\phi_i) \sum_{j=1}^{2} \frac{\partial}{\partial x_j} \left(Q(x) \frac{\frac{\partial \phi_i}{\partial x_j}}{\left| \nabla \phi_i \right|} \right)$$

$$= \mu\, \delta(\phi_i)\, \mathrm{div} \left(Q(x) \frac{\nabla \phi_i}{\left| \nabla \phi_i \right|} \right)$$

$$= \mu\, \delta(\phi_i) \left[\nabla Q \cdot \frac{\nabla \phi_i}{\left| \nabla \phi_i \right|} + Q\, \mathrm{div} \left(\frac{\nabla \phi_i}{\left| \nabla \phi_i \right|} \right) \right]$$

$$\Rightarrow \frac{\partial \phi_i}{\partial t} = - \left[\frac{\partial L}{\partial \phi_i} - \sum_{j=1}^{2} \frac{\partial}{\partial x_j} \left(\frac{\partial L}{\partial \left(\frac{\partial \phi}{\partial x_j} \right)} \right) \right] = \mu\, \delta(\phi_i)\, \mathrm{div} \left(Q \frac{\nabla \phi_i}{\left| \nabla \phi_i \right|} \right)$$

$$= \mu\, \delta(\phi_i) \left[\nabla Q \cdot \frac{\nabla \phi_i}{\left| \nabla \phi_i \right|} + Q\, \mathrm{div} \left(\frac{\nabla \phi_i}{\left| \nabla \phi_i \right|} \right) \right]$$