T-Check
Accuracy Test for
Vector Network Analyzers
utilizing a Tee-junction

Application Note 1EZ43_0E

Products:

ZVR
ZVC
ZVRE
ZVCE
ZVRL

ROHDE&SCHWARZ

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ROHDE & SCHWARZ

T-Check

1   Abstract

The **measurement accuracy** of vector network analyzers should be tested simply, easily and quickly. For this purpose a simple DUT as shown in FIG. 1 can be used which consists of a common tee-junction with one port, e.g port 3, terminated with a resistive load Z.

The four S-parameters of resulting two-port 1-2 are measured with the vector network analyzer to be checked and then evaluated by means of the **T-Check** software. The result obtained is a coloured diagram (FIG. 2) with a curve as a function of frequency which is exactly 1 in the ideal case. A deviation from 1 indicates inaccuracies of the vector network analyzer under test, the quality of which can thus be easily checked.
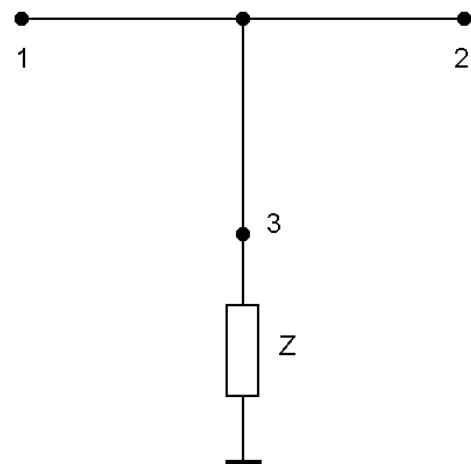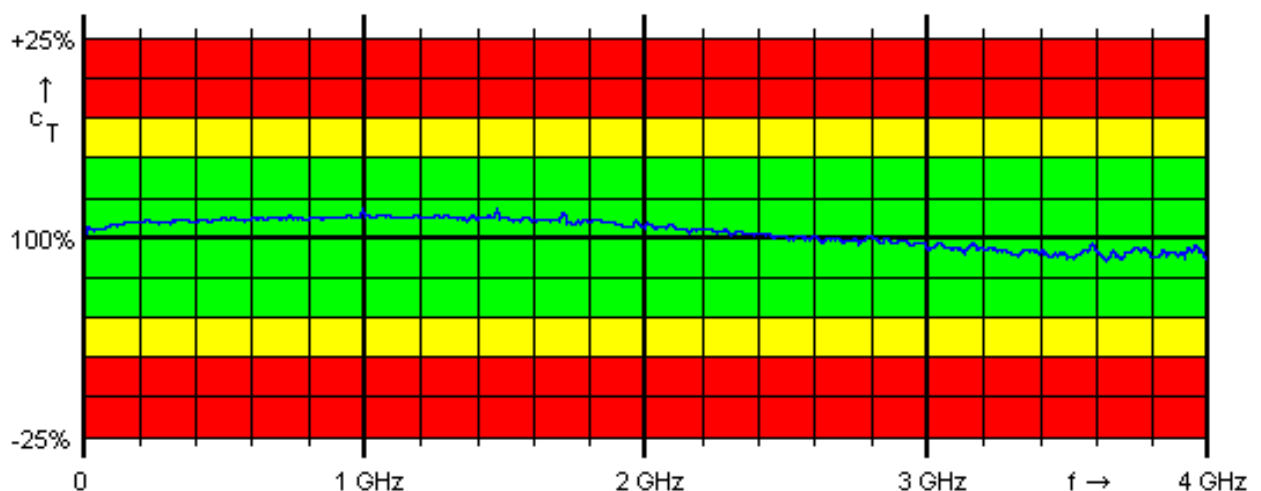
FIG.1 Principle structure of the *T-Checker*

FIG. 2 Typical result of **T-Check** for Vector Network Analyzer ZVR after system error calibration

2   Application

The **device under test (DUT)** is any lossless three-port, for example a common tee-junction. A resistive load Z, eg a matched load, is connected to one of the three ports. The DUT is thus a two-port. It is connected to the vector network analyzer to be checked which measures the four S-parameters of the DUT in magnitude and phase. These are stored as a table in a Touchstone®-compatible data format as a function of the frequency. For that the following syntax is required:

 # frequency unit   S   format   R   impedance
freq dbS₁₁ angS₁₁ dbS₂₁ angS₂₁ dbS₁₂ angS₁₂ dbS₂₂ angS₂₂
freq dbS₁₁ angS₁₁ dbS₂₁ angS₂₁ dbS₁₂ angS₁₂ dbS₂₂ angS₂₂
freq dbS₁₁ angS₁₁ dbS₂₁ angS₂₁ dbS₁₂ angS₁₂ dbS₂₂ angS₂₂

where *frequency unit* may be Hz, kHz, MHz or GHz, only DB is admissible as a *format*, i.e. magnitude in dB and phase in degrees, and the *impedance* may be any value in ohm, eg 50 or 75. As an example for the required syntax a table (which contains only a single frequency point at 1 GHz) is given:

```
#  Hz  S  DB  R   50
1.0E+09 -9.54 180.0 -3.52 0.0  -3.52 0.0  -9.54 180.0
```

If the vector network analyzer to be tested is any analyzer of the ZVR family, the suitable data format is defined via the following operating steps, a display of all the four S-parameters being necessary first:

[DISPLAY] QUAD CHANNEL QUAD SPLIT
[SAVE] SEL ITEMS TO SAVE ⇓: DISABLE ALL ITEMS: ASCII FILE: x1
[SAVE] ASCII FILE ⇓: TOUCHSTONE: dB MAG AND PHASE

The table is then stored in a file with the extension *.S2P which is read in and evaluated via the function "Open a Touchstone file" by means of the **T-Check** software. The result obtained is a graphical representation of the **T-Check** parameter c_T versus frequency (FIG. 2). This parameter should be exactly 1, i.e. 100%, in the ideal case. Any deviation from 100% indicates inaccuracies of the network analyzer. Deviations of up to ±10% are considered as minor (green range). Deviations between 10% and 15% are still acceptable (yellow range) and those more than 15% should not occur in a good vector network analyzer after careful system error calibration (red alert).

3   Theoretical Background

The fact is utilized that, due to fundamental relationships, the four S-parameters of the DUT cannot take on arbitrary values irrespective of each other. The values comply with a special relation which is similar to meeting the reciprocity condition $S_{21} = S_{12}$ of a reciprocal two-port; however, it goes beyond that and links all the four S-parameters of the two-port DUT.

The **theoretical relationship** is derived from the general scattering matrix (S) of an arbitrary three-port.

$$(S) = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad (1)$$

Each of the nine S-parameters is defined in such a way [1] that it describes the reflection S_{mm} at a port *m* or the transmission S_{mn} between two ports *n* and *m*. Unused ports are per definitionem ideally matched with reference impedance Z_o.

The present case involves a lossless three-port which is not necessarily reciprocal or symmetrical or matched. If port 3 of it is terminated with a resistive load Z, a resulting two-port is obtained, the properties of which have to be determined in the following. It is important to realize that the resistive load at port 3 needs not be ideal. Its impedance Z may deviate from the real reference impedance Z_o (eg Z_o = 50 Ω) and may be of any complex value. This can be seen by splitting the complex impedance Z = R + j X into its active component R and its reactive component X. One assumes that the active component R is generated from a resistor $R_o = Z_o$ with the aid of an ideal transformer. The reactive components of the matched load as well as the ideal transformer can be considered as an inner part of an extended lossless three-port where a perfectly matched load R_o is at port 3.

Keeping its general validity, the following consideration holds for a lossless three-port with its port 3 connected to the real matched load R_o and can then be applied to a lossless three-port with its port 3 connected to an arbitrary complex impedance Z.

The three-port used needs to be neither matched nor reciprocal nor symmetrical. The only condition to be fulfilled is the **losslessness of the tee-junction**.

The scattering matrix (S) of a non-dissipative three-port complies with unitarity condition [2], which means that the product of (S) multiplied by its transjugated $(S)^{*T}$ gives the unity matrix (E), i.e.:

$$(S) \cdot (S)^{*T} = (E) \qquad (2)$$

If one carries out the matrix multiplication from equation (2) for the scattering matrix (S) of the tee-junction according to equation (1), inter alia the following three relationships are obtained:

FIG. 3 Icon of the **T-Check** software

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \qquad (3)$$

$$|S_{21}|^2 + |S_{22}|^2 + |S_{23}|^2 = 1 \qquad (4)$$

$$S_{11} S_{21}^* + S_{12} S_{22}^* + S_{13} S_{23}^* = 0 \qquad (5)$$

Rearranging the equations:

From (3) $\Rightarrow$
$$|S_{13}|^2 = 1 - |S_{11}|^2 - |S_{12}|^2 \qquad (6)$$

From (4) $\Rightarrow$
$$|S_{23}|^2 = 1 - |S_{21}|^2 - |S_{22}|^2 \qquad (7)$$

From (5) $\Rightarrow$
$$S_{13} S_{23}^* = - (S_{11} S_{21}^* + S_{12} S_{22}^*) \qquad (8)$$

Formation of the magnitude of the terms of equation (8):

From (8) $\Rightarrow$
$$|S_{13}| \cdot |S_{23}| = |S_{11} S_{21}^* + S_{12} S_{22}^*| \qquad (9)$$

If one forms the quotient of equation (9) divided by the square root of the product of equation (6) and equation (7), this quotient should give exactly 1 which can be seen from the terms on the left side of the three equations (6), (7) and (9).

As a result from (6), (7) and (9) follows the so-called **T-Check formula** $\Rightarrow$

$$\frac{|S_{13}| \cdot |S_{23}|}{\sqrt{(|S_{13}|^2 \cdot |S_{23}|^2)}} = \frac{|S_{11} S_{21}^* + S_{12} S_{22}^*|}{\sqrt{(1 - |S_{11}|^2 - |S_{12}|^2)(1 - |S_{21}|^2 - |S_{22}|^2)}} \qquad (10)$$

The quotient given in equation (10) is defined as the **T-Check** parameter c_T. This parameter is evaluated by means of the **T-Check** software (freeware from R&S, FIG. 3 shows the icon of the software). The parameter c_T should be exactly 1 or 100% in the ideal case. Deviations from 1 indicate measurement inaccuracies of the network analyzer. The extent to which this identity is satisfied is considered as a quality criterion for the measurement accuracy of the checked network analyzer.

The tee-junction used does not need to meet any special conditions except the mentioned losslessness. Neither is the load impedance used at port 3 required to fulfill any condition. It may deviate from 50 Ω and have reactive components.

In addition to the explanation given, the above can be mathematically proven. To simplify matters, one assumes a concentric symmetrical tee-junction with the center port (3) connected to a load of any complex impedance Z. The S-parameters for the resulting two-port are:

$$S_{11} = S_{22} = -Z_o / (2 \cdot Z + Z_o) \qquad (11)$$

and $S_{21} = S_{12} = 2 \cdot Z / (2 \cdot Z + Z_o)$,
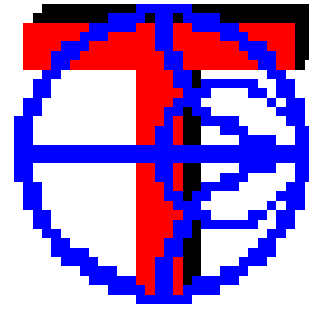
where Z_o is again the reference impedance.

By substituting this in equation (10) the following is obtained:

$$c_T = \left| S_{11} \cdot S_{21}^{\ *} + S_{21} \cdot S_{11}^{\ *} \right| / \left(1 - \left| S_{11} \right|^2 - \left| S_{21} \right|^2 \right) \tag{12}$$

$$= \frac{\left| (-Z_o / (2 \cdot Z + Z_o)) \cdot (2 \cdot Z / (2 \cdot Z + Z_o))^{*} + (-Z_o / (2 \cdot Z + Z_o))^{*} \cdot (2 \cdot Z / (2 \cdot Z + Z_o)) \right|}{1 - (-Z_o / (2 \cdot Z + Z_o)) \cdot (-Z_o / (2 \cdot Z + Z_o))^{*} - (2 \cdot Z / (2 \cdot Z + Z_o)) \cdot (2 \cdot Z / (2 \cdot Z + Z_o))^{*}}$$

$$= \frac{\left| -2 \cdot Z^{*} \cdot Z_o - 2 \cdot Z \cdot Z_o^{\ *} \right|}{(2 \cdot Z + Z_o) \cdot (2 \cdot Z + Z_o)^{*} - Z_o \cdot Z_o^{\ *} - 4 \cdot Z \cdot Z^{*}}$$

$$= 1 \qquad \text{q.e.d.}$$

4   The *T-Checker*

To enhance convenience for the practical use and to increase measurement accuracy, which might be affected by a dissipative T-junction, a special device for **T-Check** was designed. It incorporates the principle structure as already shown in FIG. 1 and can be used instead of the combination of a T-junction plus a resistive load. It is a compact coaxial device called the *T-Checker* (FIG. 4).
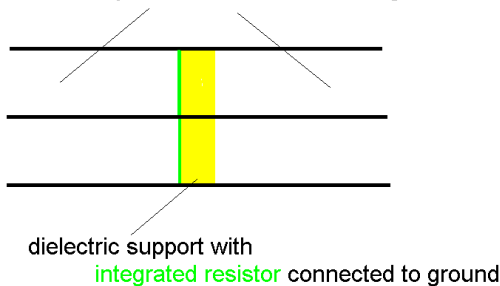

Compact and nearly lossless coaxial airline regions

dielectric support with
integrated resistor connected to ground

FIG. 4 Compact realization of the *T-Checker*

The *T-Checker* looks similar to a simple coaxial through-connection with a female and a male connector. It consists of coaxial outer and inner conducters. Both are gold plated to increase conductivity. The inner conductor is as usual fixed by a dielectric support. For the *T-Checker* this is made of three alumina substrates. Different to a usual support however an integrated thin film resistor is located on the surface of each of the substrates, thus realizing the wanted resitor to ground.

Due to its compact construction the *T-Checker* is a broadband device and the virtually ideal element to perform the **T-Check**. It is stable and rugged an allows a frequent use for checking the measurement accuracy of a vector network analyzer. Thus a **T-Check** can for instance be performed very easily as a final step after applying the system error calibration of a network analyzer. Also a confidence check for the measurement accuracy of the network analyzer can simply be carried out from time to time.

The following FIG. 5 shows a photograph of the realized *T-Checker* wich is available from R&S.

FIG. 5 The *T-Checker*

5   Error analysis

For the practice it is of importance to have an idea of the order of magnitude of the deviation from the ideal value 1. For this purpose, an **error analysis** is carried out to calculate the worst case value for c_T assuming the test deviations specified for the ZVR.

By examining equation (10) it can be seen that c_T takes on a maximum value when the numerator is maximum and the denominator is minimum. The denominator is minimum when all the four **magnitudes** $|S_{11}|$, $|S_{12}|$, $|S_{21}|$ and $|S_{22}|$ are at **maximum**. Starting with an ideal scattering matrix of a perfectly matched tee-junction at port 3,

$$(S) = \begin{bmatrix} -1/3 & 2/3 \\ 2/3 & -1/3 \end{bmatrix} \quad (13)$$

the numerator reduces to $|{-1/3} \cdot 2/3 - 2/3 \cdot 1/3|$ and is maximum when the **magnitude** of all four S-parameters also becomes maximum and their phase remains unchanged. Assuming an uncertainty of 0.4 dB with reflection measurements and 0.2 dB with transmission measurements as specified in the data sheet for Vector Network Analyzer ZVR, one obtains for the worst case c_T = 1.154, i.e. a deviation of approx. 15% from the ideal value of 1. This deviation is the limit between the yellow and the red range in the evaluation diagram.

Considering measurement deviations which amount to 1/10 of the specified data, one obtains for the worst case c_T = 1.014, i.e. a deviation of 1.4%. This value corresponds approximately to the deviations typically observed, but it should be noted that in practice there is no worst case addition of the different measurement errors.

At high frequencies, the attenuation increases at the two lateral tee-junction arms. This results, as can be simply seen, in c_T < 1. For example, if the attenuation is assumed to be 0.01 dB at each lateral tee-junction arm, c_T is reduced by approx. 1%.

6   Acknowledgements

7   References

[1]   B. Schiek: Meßsysteme der Hochfrequenztechnik, Hüthig, Heidelberg, 1984, pp. 3 ff. (High-frequency test systems)

[2]   H. Brand: Schaltungslehre linearer Mikrowellennetze, Hirzel, Stuttgart, 1970, pp. 150 ff. (Linear microwave networks)

[3]   B. Schiek, H.-J. Siweris: Rauschen in Hochfrequenzschaltungen, Hüthig, Heidelberg, 1990, p. 60. (Noise in high-frequency circuits)

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Rohde & Schwarz
3 June 1998

8 Further Application Notes

[1] O. Ostwald: 3-Port Measurements with Vector Network Analyzer ZVR, Appl. Note 1EZ26_1E.

[2] H.-G. Krekels: Automatic Calibration of Vector Network Analyzer ZVR, Appl. Note 1EZ30_1E.

[3] O. Ostwald: 4-Port Measurements with Vector Network Analyzer ZVR, Appl. Note 1EZ25_1E.

[4] T. Bednorz: Measurement Uncertainties for Vector Network Analysis, Appl. Note 1EZ29_1E.

[5] P. Kraus: Measurements on Frequency-Converting DUTs using Vector Network Analyzer ZVR, Appl. Note 1EZ32_1E.

[6] J. Ganzert: Accessing Measurement Data and Controlling the Vector Network Analyzer via DDE, Appl. Note 1EZ33_1E.

[7] J. Ganzert: File Transfer between Analyzers FSE or ZVR and PC using MS-DOS Interlink, Appl. Note 1EZ34_1E.

[8] O. Ostwald: Group and Phase Delay Measurements with Vector Network Analyzer ZVR, Appl. Note 1EZ35_1E.

[9] O. Ostwald: Multiport Measurements using Vector Network Analyzer, Appl. Note 1EZ37_1E.

[10] O. Ostwald: Frequently Asked Questions about Vector Network Analyzer ZVR, Appl. Note 1EZ38_3E.

[11] A. Gleißner: Internal Data Transfer between Windows 3.1 / Excel and Vector Network Analyzer ZVR, Appl. Note 1EZ39_1E.

[12] A. Gleißner: Power Calibration of Vector Network Analyzer ZVR, Appl. Note 1EZ41_2E

[13] O. Ostwald: Pulsed Measurements on GSM Amplifier SMD ICs with Vector Analyzer ZVR, Appl. Note 1EZ42_1E.

[14] O. Ostwald: Zeitbereichsmessungen mit dem Netzwerkanalysator ZVR, Appl. Note 1EZ44_1D.

9 Ordering Information

Order designation	Type	Frequency range	Order No.
Vector Network Analyzers (test sets included) *			
3-channel, unidirectional, 50 Ω, passive	ZVRL	9 kHz to 4 GHz	1043.0009.41
3-channel, bidirectional, 50 Ω, passive	ZVRE	9 kHz to 4 GHz	1043.0009.51
3-channel, bidirectional, 50 Ω, active	ZVRE	300 kHz to 4 GHz	1043.0009.52
4-channel, bidirectional, 50 Ω, passive	ZVR	9 kHz to 4 GHz	1043.0009.61
4-channel, bidirectional, 50 Ω, active	ZVR	300 kHz to 4 GHz	1043.0009.62
3-channel, bidirectional, 50 Ω, active	ZVCE	20 kHz to 8 GHz	1106.9020.50
4-channel, bidirectional, 50 Ω, active	ZVC	20 kHz to 8 GHz	1106.9020.60
Alternative Test Sets *			
75 Ω SWR Bridge for ZVRL (instead of 50 Ω) [1]			
75 Ω, passive	ZVR-A71	9 kHz to 4 GHz	1043.7690.18
75 Ω SWR Bridge Pairs for ZVRE and ZVR (instead of 50 Ω) [1]			
75 Ω, passive	ZVR-A75	9 kHz to 4 GHz	1043.7755.28
75 Ω, active	ZVR-A76	300 kHz to 4 GHz	1043.7755.29
Options			
AutoKal	ZVR-B1	0 to 8 GHz	1044.0625.02
Time Domain	ZVR-B2	same as analyzer	1044.1009.02
Mixer Measurements [2]	ZVR-B4	same as analyzer	1044.1215.02
Reference Channel Ports	ZVR-B6	same as analyzer	1044.1415.02
Power Calibration [3]	ZVR-B7	same as analyzer	1044.1544.02
3-Port Adapter	ZVR-B8	0 to 4 GHz	1086.0000.02
Virtual Embedding Networks [4]	ZVR-K9	same as analyzer	1106.8830.02
4-Port Adapter (2xSPDT)	ZVR-B14	0 to 4 GHz	1106.7510.02
4-Port Adapter (SP3T)	ZVR-B14	0 to 4 GHz	1106.7510.03
Controller (German) [5]	ZVR-B15	-	1044.0290.02
Controller (English) [5]	ZVR-B15	-	1044.0290.03
Ethernet BNC for ZVR-B15	FSE-B16	-	1073.5973.02
Ethernet AUI for ZVR-B15	FSE-B16	-	1073.5973.03
IEC/IEEE-Bus Interface for ZVR-B15	FSE-B17	-	1066.4017.02
Generator Step Attenuator PORT 1	ZVR-B21	same as analyzer	1044.0025.11
Generator Step Attenuator PORT 2 [6]	ZVR-B22	same as analyzer	1044.0025.21
Receiver Step Attenuator PORT 1	ZVR-B23	same as analyzer	1044.0025.12
Receiver Step Attenuator PORT 2	ZVR-B24	same as analyzer	1044.0025.22
External Measurements, 50 Ω [7]	ZVR-B25	10 Hz to 4 GHz (ZVR/E/L) 20 kHz to 8 GHz (ZVC/E)	1044.0460.02

[1] To be ordered together with the analyzer.
[2] Harmonics measurements included.
[3] Power meter and sensor required.
[4] Only for ZVR or ZVC with ZVR-B15.
[5] DOS, Windows 3.11, keyboard and mouse included.
[6] For ZVR or ZVC only.
[7] Step attenuators required.

* Note:
Active test sets, in contrast to passive test sets, comprise internal bias ne tworks, eg to supply DUTs.