Beizer curve

* Derivation of Quadratic Bezier curves

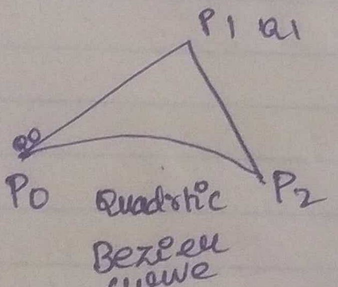
Q_0 + Q_1 has on the lines

$P_0 \rightarrow P_1$ and $P_1 \rightarrow P_2$

The Point on the Bezier curve lies on the line

$Q_0 \rightarrow Q_1 \Rightarrow P_0, P_1, P_2 \rightarrow$ control points.

when we join P_0, P_1, P_2 it is named as control polygon.



$$\left.\begin{aligned} x(t) &= (1-t)\,x_0 + t x_1 \\ y(t) &= (1-t)\,y_0 + t y_1 \end{aligned}\right\} \textcircled{1} \rightarrow$$

$0 \leq 1 \leq 1$

$$P(t) = (1-t)\,P_0 + t P_1 \rightarrow \textcircled{2}$$

$$\left.\begin{aligned} Q_0 &= (1-t)\,P_0 + t P_1 \\ Q_1 &= (1-t)\,P_1 + t P_2 \end{aligned}\right\} \rightarrow \textcircled{3}$$

$c(t)$ is a pt on the Bezier curve on the line $Q_0 \to Q_1$.

$$c(t) = (1-t)\,Q_0 + t\,Q_1 \rightarrow (4)$$

combining leads to.

$$c(t) = (1-t)^2 P_0 + 2t(1-t)\,P_1 + t^2 P_2$$

Sub (3) in (4) to get

$$c(t) = (1-t)\,[(1-t)\,P_0 + tP_1] + t[(1-t)\,P_1 + tP_2]$$

$$= (1-t)^2 P_0 + t(1-t)\cdot P_1 + t(1-t)\cdot P_1 + t^2 P_2.$$

$$= (1-t)^2 P_0 + 2t(1-t)\,P_1 + t^2 P_2.$$

* The derivation procedure is known as the Castelfau's algorithm.

$$c(t) = \underbrace{(1-t)^2 P_0}_{(b_0)} + \underbrace{2t(1-t)}_{(b_1)} P_1 + \underbrace{t^2 P_2}_{(b_2)} \rightarrow (5)$$

It can be expressed as

$$\sum p_i b_i = P_0 b_0 + P_1 b_1 + P_2 b_2$$

Eq (5) can be rewritten as

$$C(t) = (1-t)^2 P_0 + 2t(1-t) P_1 + t^2 P_2$$

$$= {}^2C_0 (1-t)^2 P_0 + {}^2C_1 t (1-t) P_1 + {}^2C_2 t^2 P_2.$$

$$= \sum_{i=0}^{i=2} {}^2C_i (1-t)^{2-i} t^i p_i \quad -(6)$$

It can be generalized as

$$c(t) = \sum_{i=0}^{i=n} {}^nC_i (1-t)^{n-i} t^i p_i \rightarrow (7)$$

Replaced by

$$C(t) = \sum_{i=0}^{n} b_{i,n}(t) p_i$$

$$b_{i,n}(t) = \binom{n}{i} t^i (1-t)^{n-i} \rightarrow$$

Basnein Polynomial

is ${}^nC_i$

$\binom{n}{i} \Rightarrow$ is the Binomial coeffi

$$\binom{n}{i} = \frac{n!}{i!\,(n-i)!}$$

The general form of a degree n Bezier curve

defined by the control point P_i

where $i = 0, 1, \ldots n$)

$$C(t) = \sum_{i=0}^{n} b_{i,n}(t)\, P_i .$$

The Binomial coeff is written using

$$\binom{n}{i} = \frac{n!}{i!(n-i)!}$$

$$C(t) = (1-t)^2 P_0 + 2t(1-t) P_1 + t^2 P_2$$

$$= (1^2 - 2t + t^2) P_0 + (2t - 2t^2) P_1 + t^2 P_2$$

$$= (t^2 - 2t + 1) P_0 + (-2t^2 + 2t) P_1 + t^2 P_2$$

This can be expressed in Matric form

$$C(t) = \begin{pmatrix} P_0 & P_1 & P_2 \end{pmatrix} \begin{pmatrix} 1 & -2 & 1 \\ -2 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} t^2 \\ t \\ 1 \end{pmatrix}$$