SATHYABAMA

INSTITUTE OF SCIENCE AND TECHNOLOGY

(DEEMED TO BE UNIVERSITY)

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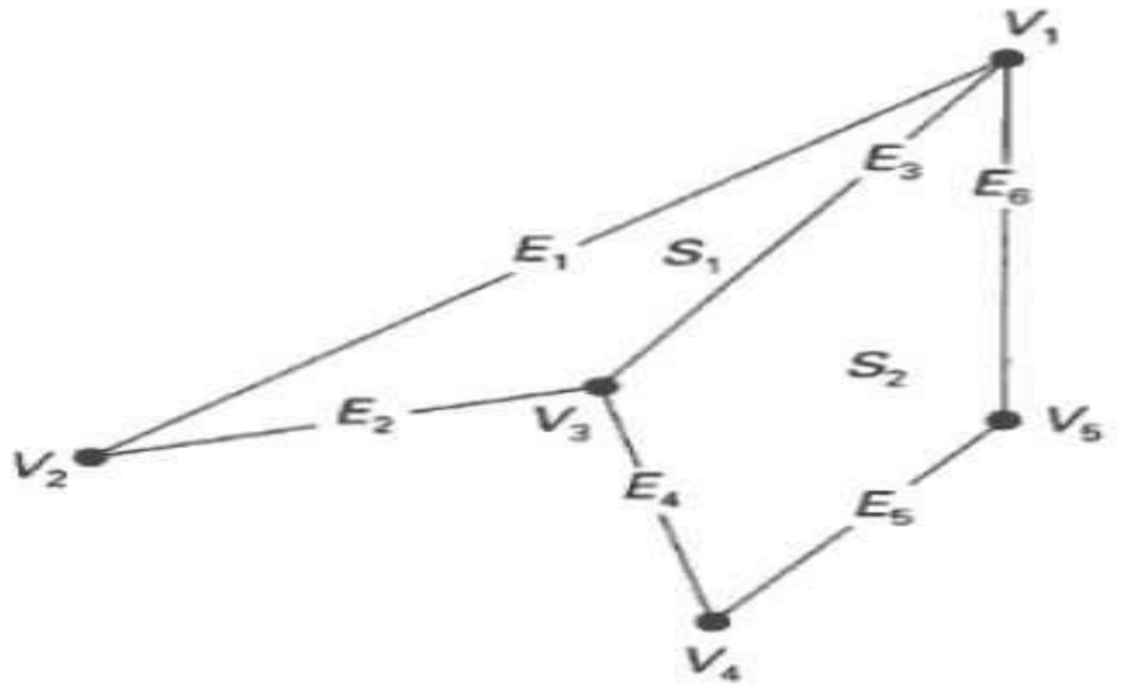
COMPUTER GRAPHICS & MULTIMEDIA SYSTEMS- SCS1302

UNIT III - Part I

3D object representation methods Polygon Surfaces, Polygon Tables

- **Boundary Representations (B-reps)** − It describes a 3D object as a set of surfaces that separates the object interior from the environment.
- **Space–partitioning representations** − It is used to describe interior properties, by partitioning the spatial region containing an object into a set of small, non-overlapping, contiguous solids (usually cubes).
- Each vertex stores x, y, and z coordinate information which is represented in the table as v_1: x_1, y_1, z_1.
- The Edge table is used to store the edge information of polygon. In the following figure, edge E_1 lies between vertex v_1 and v_2 which is represented in the table as E_1: v_1, v_2.
- Polygon surface table stores the number of surfaces present in the polygon. From the following figure, surface S_1 is covered by edges E_1, E_2 and E_3 which can be represented in the polygon surface table as S_1: E_1, E_2, and E_3.

Polygon Tables

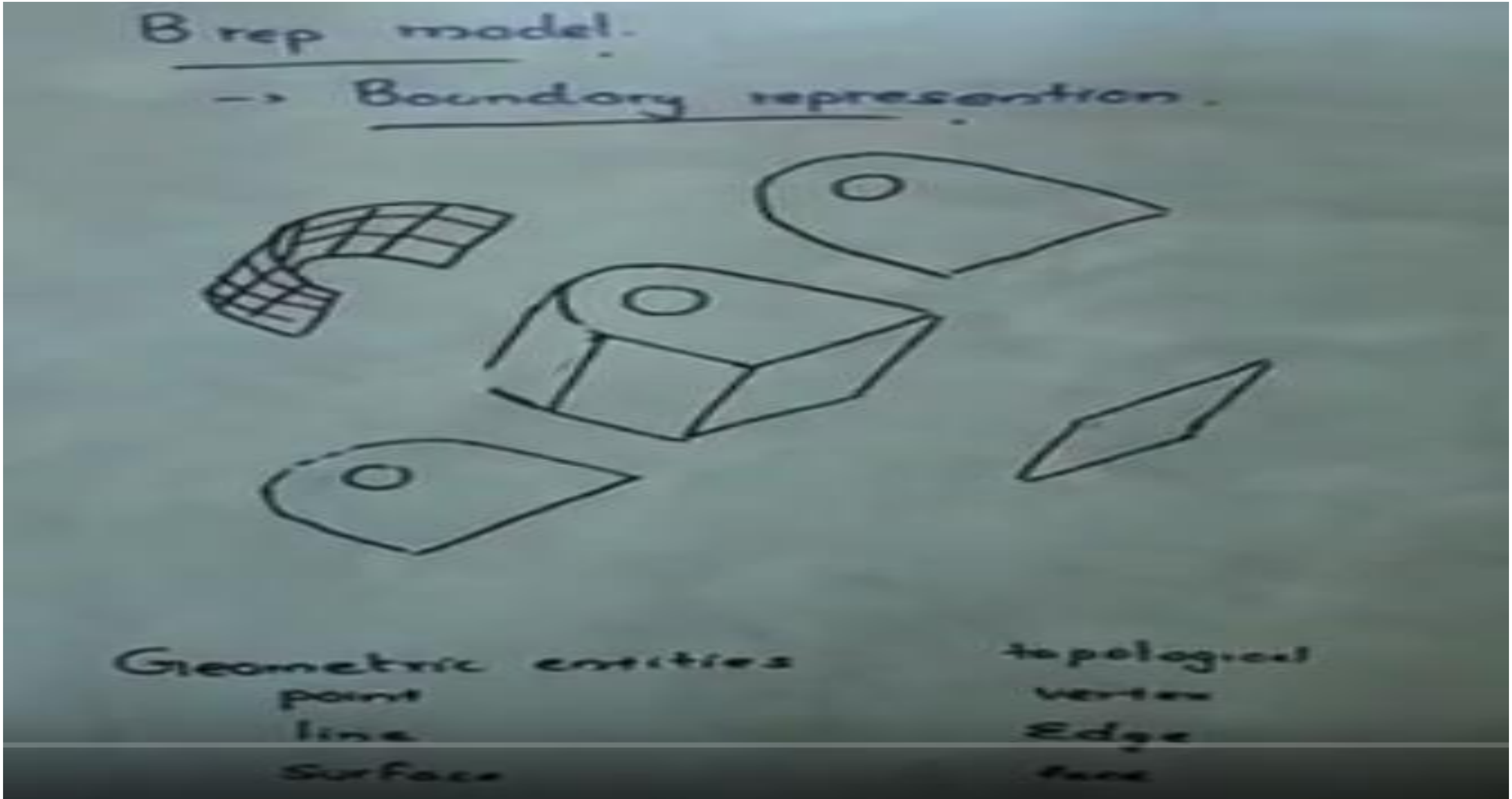


VERTEX TABLE	
V_1:	x_1, y_1, z_1
V_2:	x_2, y_2, z_2
V_3:	x_3, y_3, z_3
V_4:	x_4, y_4, z_4
V_5:	x_5, y_5, z_5

EDGE TABLE	
E_1:	V_1, V_2
E_2:	V_2, V_3
E_3:	V_3, V_1
E_4:	V_3, V_4
E_5:	V_4, V_5
E_6:	V_5, V_1

POLYGON-SURFACE TABLE	
S_1:	E_1, E_2, E_3
S_2:	E_3, E_4, E_5, E_6

https://www.youtube.com/watch?v=sXbRT439vRI



3D object representation methods

Plane Equations

The equation for plane surface can be expressed as:

$$Ax + By + Cz + D = 0$$

Where (x, y, z) is any point on the plane, and the coefficients A, B, C, and D are constants describing the spatial properties of the plane. We can obtain the values of A, B, C, and D by solving a set of three plane equations using the coordinate values for three non collinear points in the plane. Let us assume that three vertices of the plane are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3).

Let us solve the following simultaneous equations for ratios A/D, B/D, and C/D. You get the values of A, B, C, and D.

$$(A/D)\, x_1 + (B/D)\, y_1 + (C/D)\, z_1 = -1$$

$$(A/D)\, x_2 + (B/D)\, y_2 + (C/D)\, z_2 = -1$$

$$(A/D)\, x_3 + (B/D)\, y_3 + (C/D)\, z_3 = -1$$

To obtain the above equations in determinant form, apply Cramer's rule to the above equations.

$$A = \begin{bmatrix} 1 & y_1 & z_1 \\ 1 & y_2 & z_2 \\ 1 & y_3 & z_3 \end{bmatrix} B = \begin{bmatrix} x_1 & 1 & z_1 \\ x_2 & 1 & z_2 \\ x_3 & 1 & z_3 \end{bmatrix} C = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$$

$$D = -\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}$$

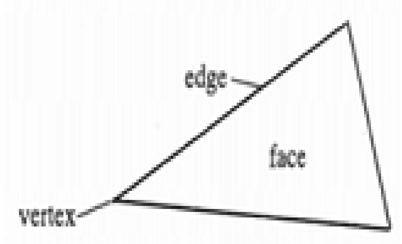
For any point (x, y, z) with parameters A, B, C, and D, we can say that -

- Ax + By + Cz + D ? 0 means the point is not on the plane.
- Ax + By + Cz + D < 0 means the point is inside the surface.
- Ax + By + Cz + D > 0 means the point is outside the surface.

3D object representation methods

- **B-Rep:**

1. B-Rep stands for Boundary Representation.
2. It is an extension to the wireframe model.
3. B-Rep describes the solid in terms of its surface boundaries: Vertices, edges and faces as shown below.

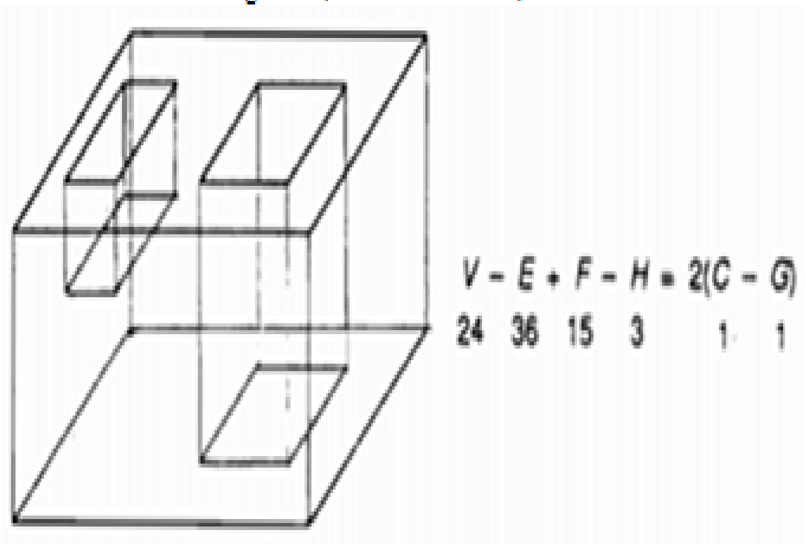


1. It is a method for representing shapes using the limits.
2. A solid is represented as a collection of connected surface elements, the boundary between solid and non-solid.
3. There are 2 types of information in a B – rep topological and geometric.
4. Topological information provides the relationships among vertices, edges and faces similar to that used in a wireframe model.
5. In addition to connectivity, topological information also includes orientation of edges and faces.
6. Geometric information is usually equations of the edges and faces.

1. The B-rep of 2 manifolds that have faces with holes satisfies the generalized Euler's formula:

$$V–E+F–H=2(C- G)$$

Where, V = Number of vertices, E = Number of edges, F = Number of faces.
H = Number of holes in the faces, C is the number of separate components (parts).
G is the genus (for a torus G = 1)



Geometric Modeling - Boundary Representations (BREP)

Loop A
in => C
next => B

Loop B
in => F
next =>-1

Loop C
in => D
next => E

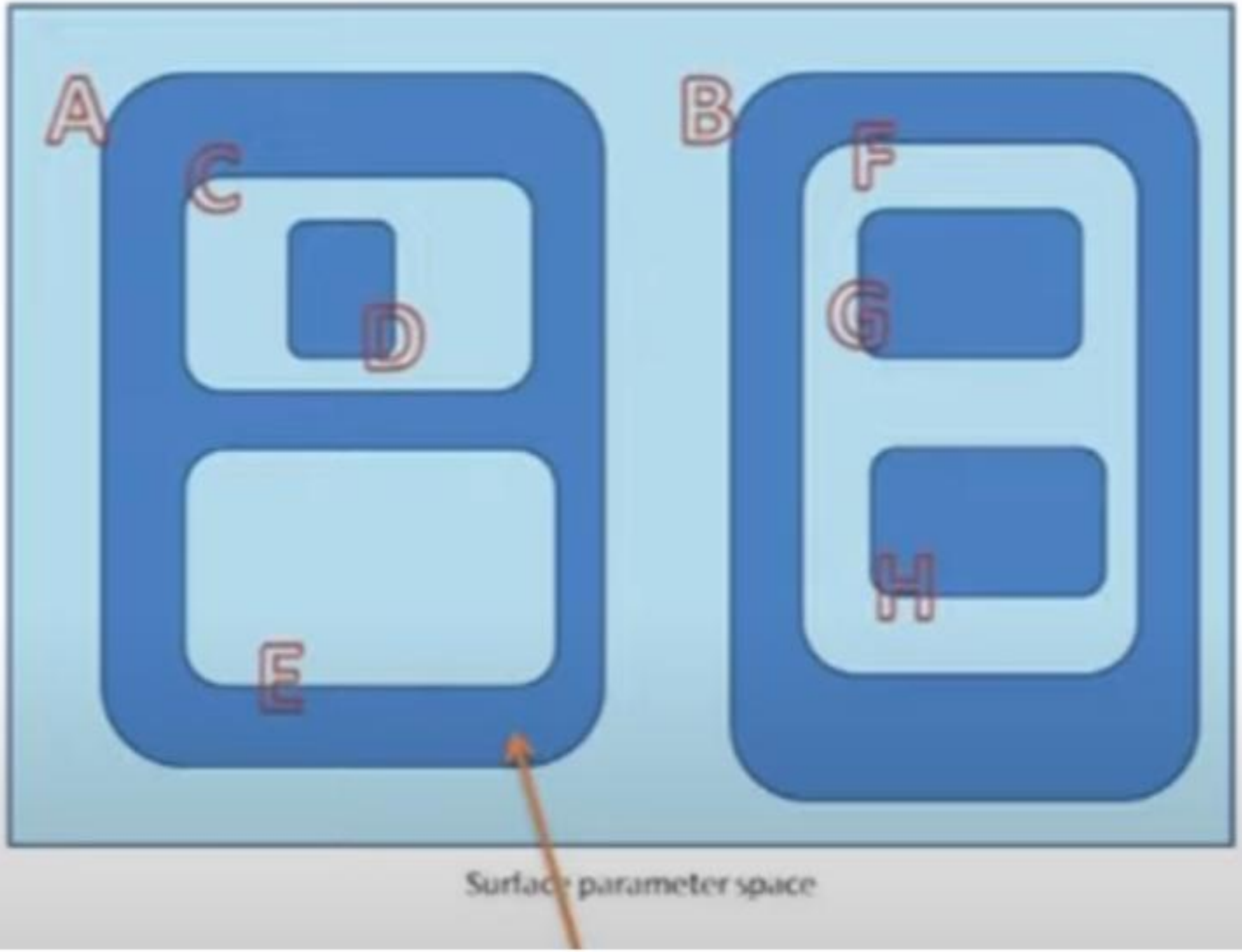
Loop D
in => -1
next => -1

Loop E
in => -1
next => -1

Loop F
in => G
next => -1

Loop G
in => -1
next => H

Loop H
in => -1
next => -1

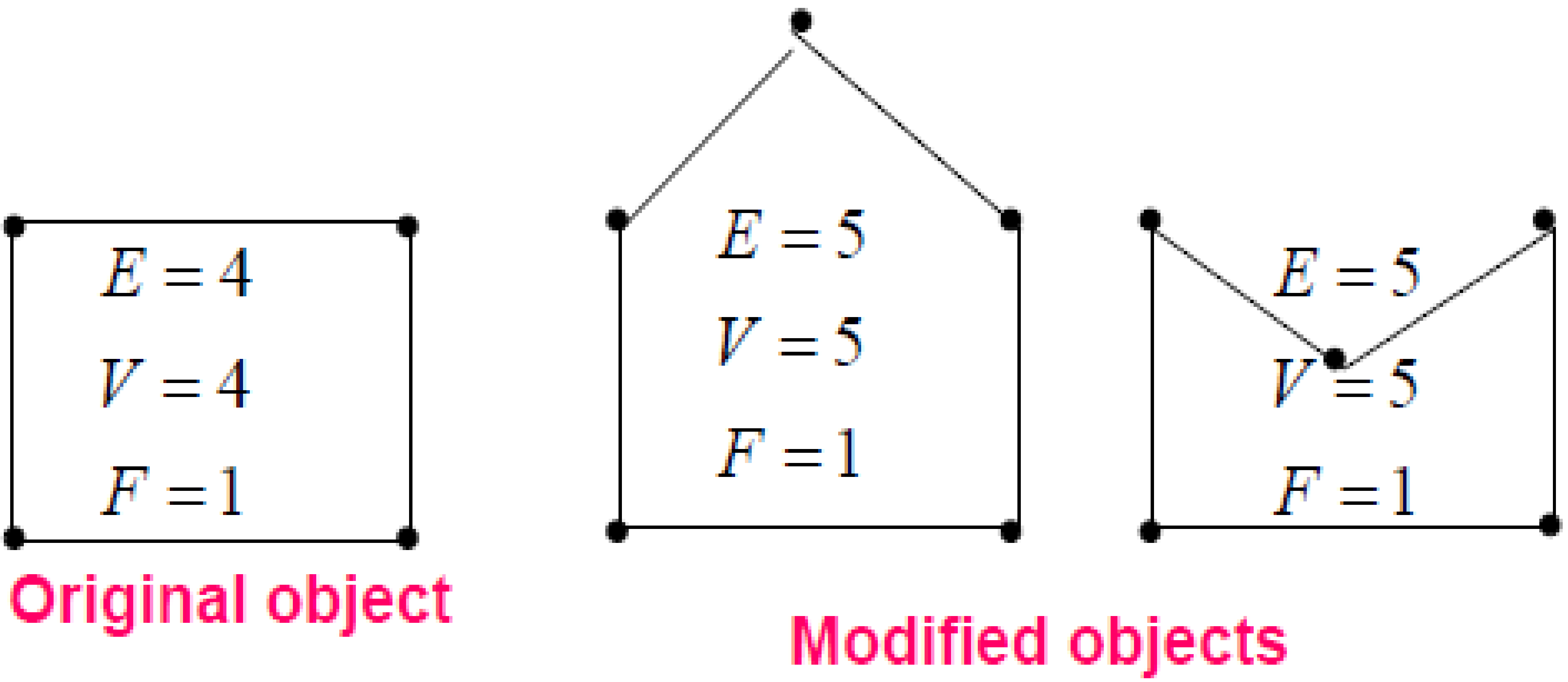


Object Modeling with B-rep

Both polyhedra and curved objects can be modeled using the following primitives

- **Vertex**: A unique point (ordered triplet) in space.
- **Edge** :A finite, non-selfintersecting directed space curve bounded by two vertices that are not necessarily distinct.
- **Face** :Finite, connected, non-selfintersecting region of a closed, orientable surface bounded by one or more **loops**.
- **Loop** :An ordered alternating sequence of vertices and edges. A loop defines non-self intersecting piecewise closed space curve which may be a boundary of a face.
- **Body** :An independent solid. Sometimes called a shell has a set of faces that bound single connected closed volume. A minimum body is a point (vortex) which topologically has one face one vortex and no edges. A point is therefore called a seminal or singular body.
- **Genus** :Hole or handle.

Boundary Representation



Euler-Poincare Law

- **Euler (1752)** a Swiss mathematician proved that polyhedra that are homomorphic to a sphere are topologically valid if they satisfy the equation:

$F - E + V - L = 2(B - G)$	General
$F - E + V = 2$	Simple Solids
$F - E + V - L = B - G$	Open Objects

F=Face	E=Edge	V=Vertices
B=Bodies	L=Faces' inner Loop	G=Genus

Euler Operations

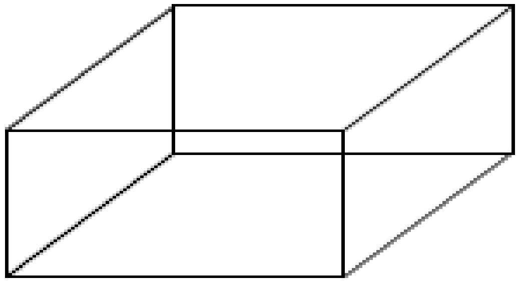
- A connected structure of vertices, edges and faces that always satisfies Euler's formula is known as **Euler object**.
- The process that adds and deletes these boundary components is called an **Euler operation**

Applicability of Euler formula to solid objects:

- At least three edges must meet at each vertex.
- Each edge must share two and only two faces
- All faces must be simply connected (homomorphic to disk) with no holes and bounded by single ring of edges.
- The solid must be simply connected with no through holes

Validity Checking for Simple Solids

$$F - E + V = 2 \quad \text{Simple Solids}$$

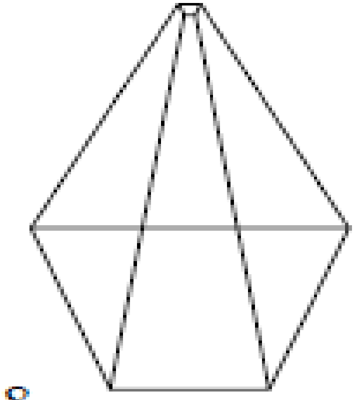


$E = 12$

$V = 8$

$F = 6$

$\boxed{6 - 12 + 8 = 2}$

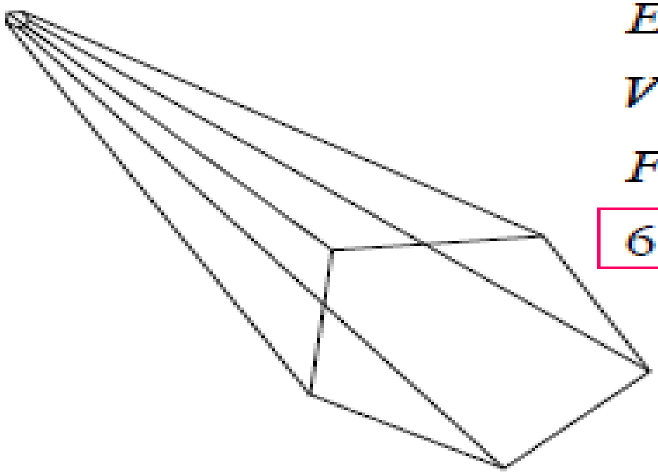


$E = 8$

$V = 5$

$F = 5$

$\boxed{5 - 8 + 5 = 2}$

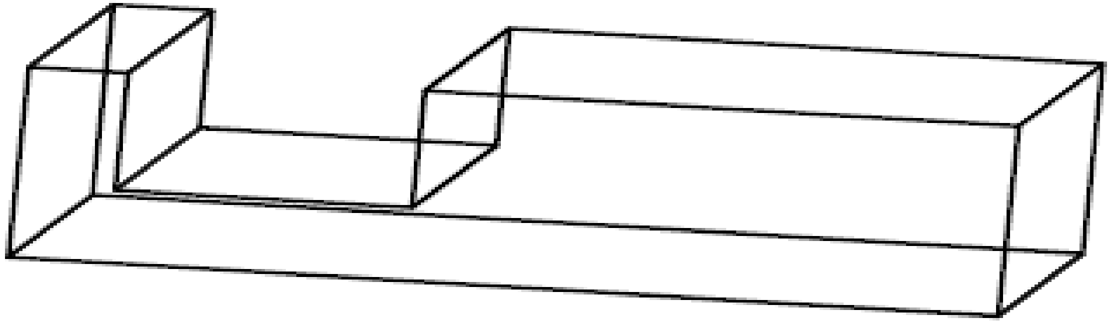


$E = 10$

$V = 6$

$F = 6$

$\boxed{6 - 10 + 6 = 2}$



$E = 24$

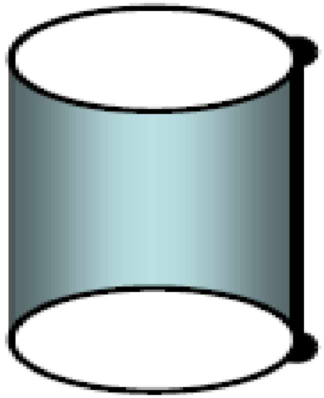
$V = 16$

$F = 10$

$\boxed{10 - 24 + 16 = 2}$

Validity Checking for Simple Solids

$$F - E + V = 2 \quad \text{Simple Solids}$$

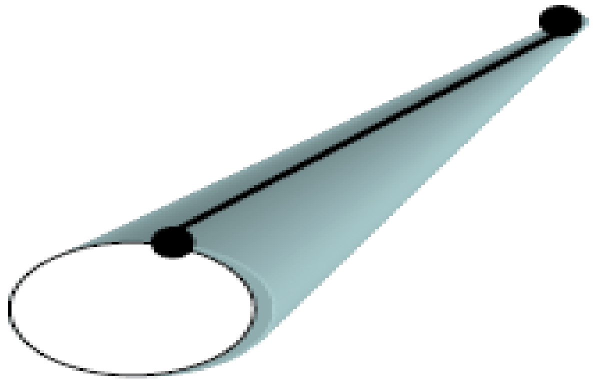


$E = 3$

$V = 2$

$F = 3$

$\boxed{3 - 3 + 2 = 2}$

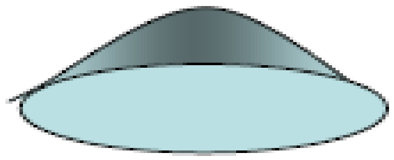


$E = 2$

$V = 2$

$F = 2$

$\boxed{2 - 2 + 2 = 2}$



$E = 2$

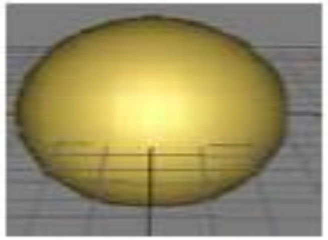
$V = 2$

$F = 2$

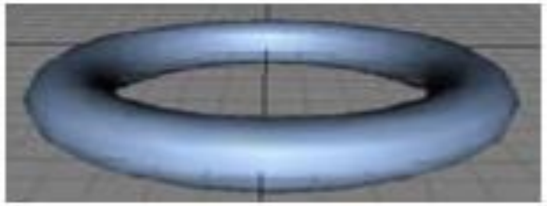
$\boxed{2 - 2 + 2 = 2}$

Loops (rings),Genus & Bodies

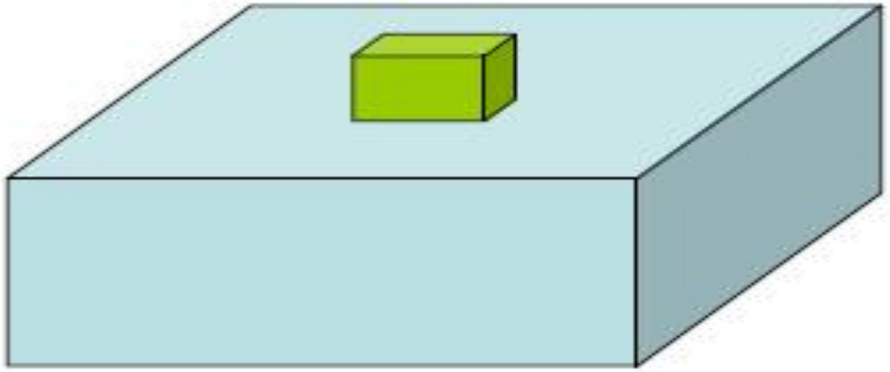
- Genus zero



- Genus one



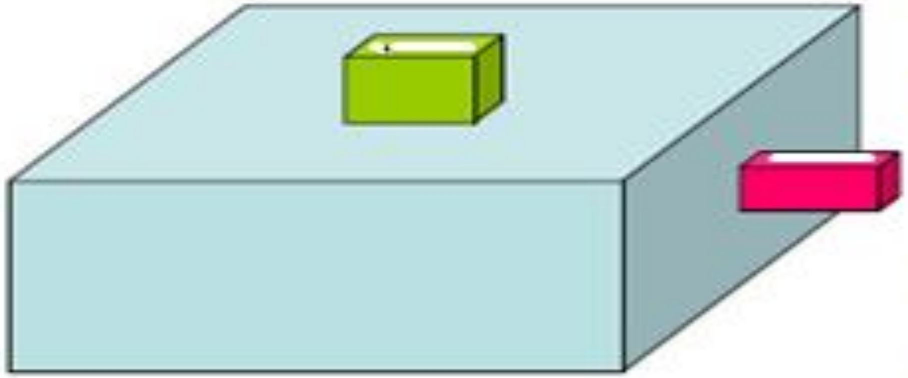
- Genus two



- One inner loop

Validity Checking for Polyhedra with inner loops

$$F - E + V - L = 2(B - G)$$ General



$E = 36$

$F = 16$

$V = 24$

$L = 2$

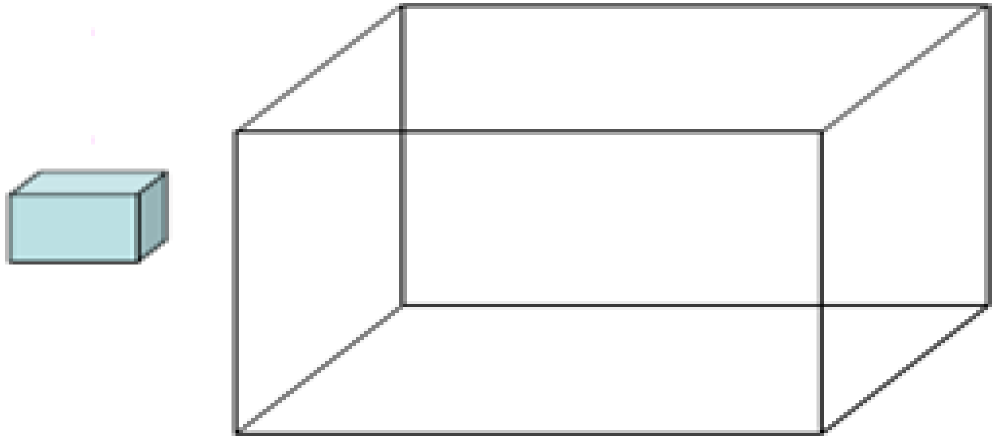
$B = 1$

$G = 0$

$$\boxed{16 - 36 + 24 - 2 = 2(1 - 0) = 2}$$

Validity Checking for Polyhedra with holes

$$F - E + V - L = 2(B - G)$$ General



$E = 24$

$F = 12$

$V = 16$

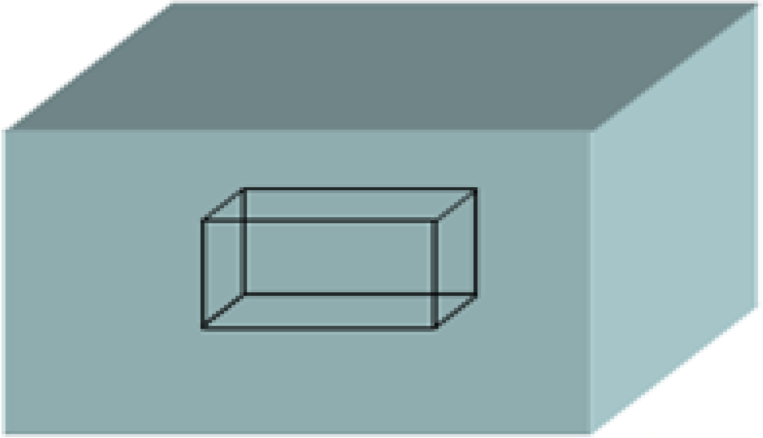
$L = 0$

$B = 2$

$G = 0$

Filled, separate components

$$12 - 24 + 16 - 0 = 2(2 - 0) = 4$$



$E = 24$ $F = 11$

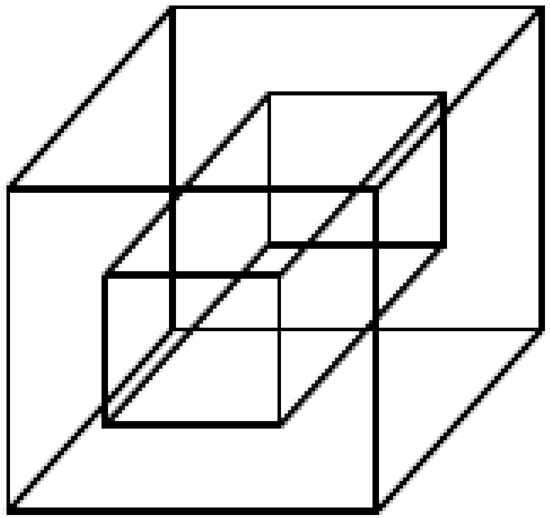
$V = 16$ $L = 1$

$B = 1$ $G = 0$

$$11 - 24 + 16 - 1 = 2(1 - 0) = 2$$

Validity Checking for Polyhedra with through holes (handles)

$$F - E + V - L = 2(B - G) \qquad \text{General}$$



$E = 24$

$F = 10$

$V = 16$

$L = 2$

$B = 1$

$G = 1$

$$\boxed{10 - 24 + 16 - 2 = 2(1 - 1) = 0}$$

Sweep

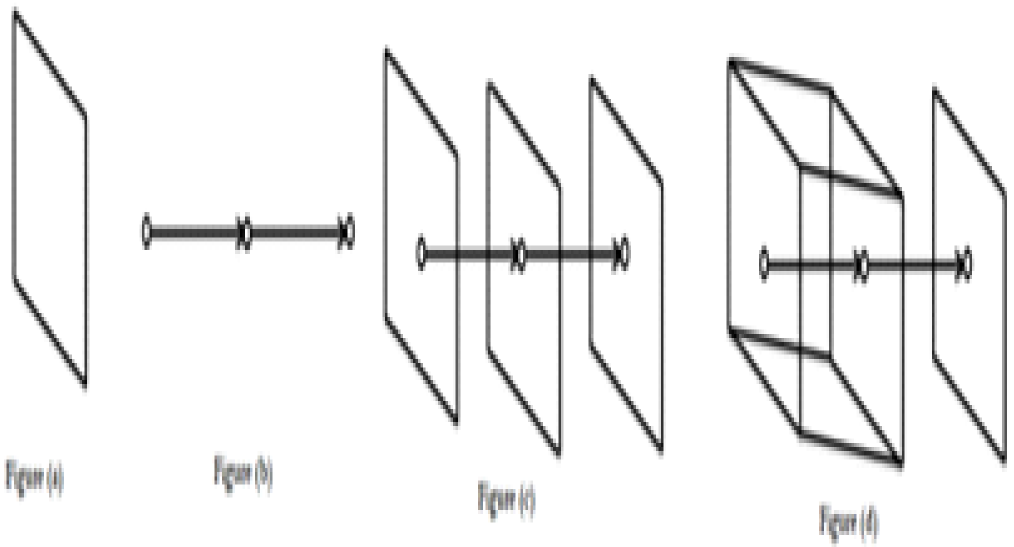
- **Translational sweep:**

i. Define a shape as a polygon vertex table as shown in figure 2 (a).

ii. Define a sweep path as a sequence of translation vectors figure 2 (b).

iii. Translate the shape; continue building a vertex table figure 2 (c).

iv. Define a surface table figure 2 (d).



Rotational sweep:

i. Define a shape as a polygon vertex table as shown in figure 3 (a).

ii. Define a sweep path as a sequence of rotations.

iii. Rotate the shape; continue building a vertex table as shown in figure 3 (b).

iv. Define a surface table as shown in figure 3 (c).

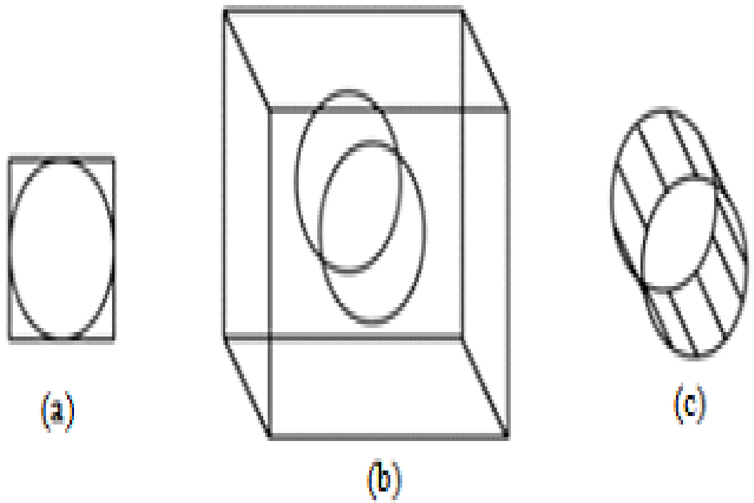


Fig. 3

SWEEP REPRESENTATIONS

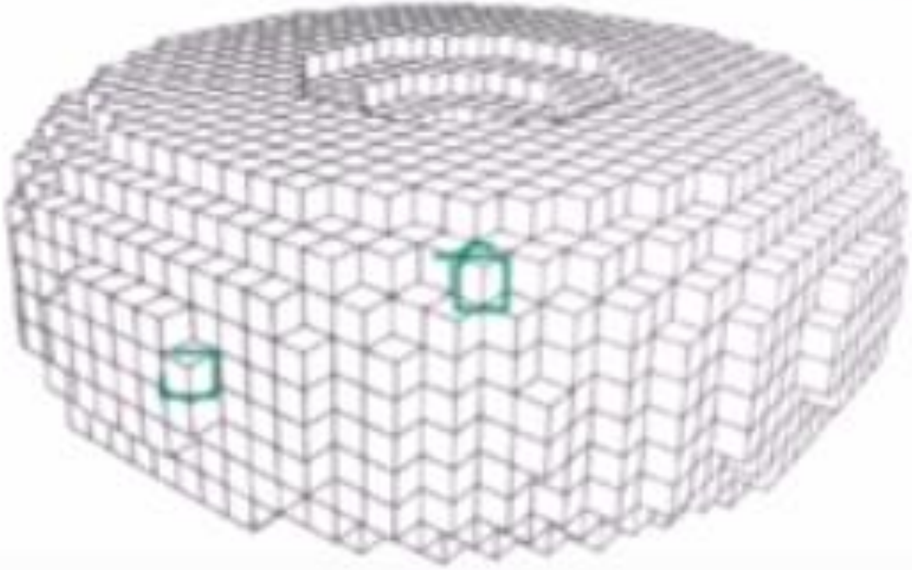
https://www.youtube.com/watch?v=k_3lISNgkAo

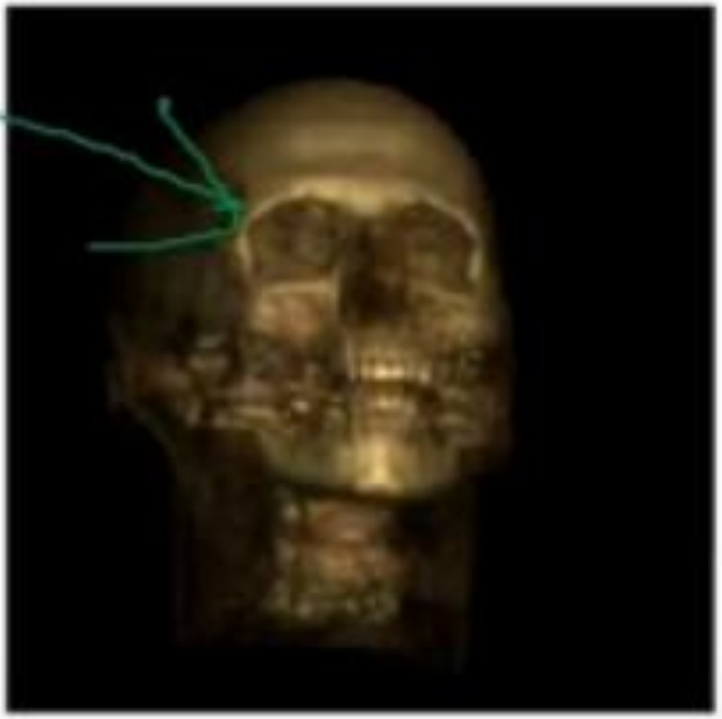
https://www.youtube.com/watch?v=021P5-Vxl2o

http://www.dailyfreecode.com/code/creats-3d-solid-object-translational-654.aspx

Voxels

- Uniform Grid of Volumetric Samples
  - Acquired from CAT, MRI, etc.



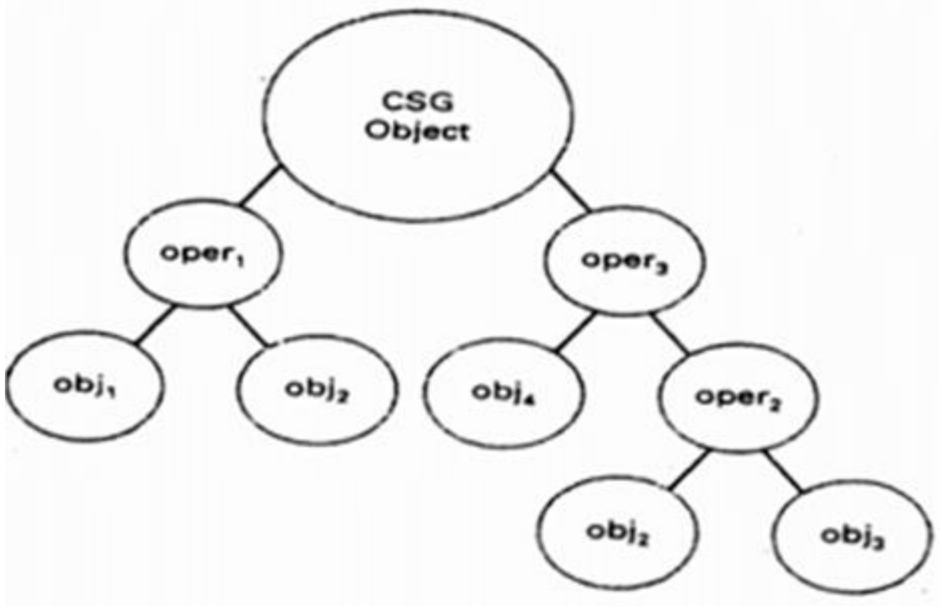


Sweep Representations:

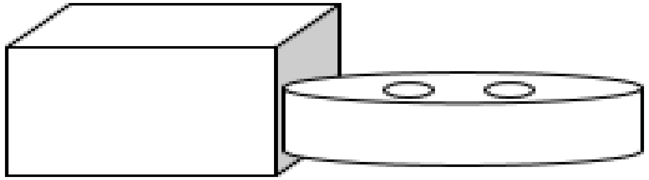
Sweep representations are used to construct 3D object from 2D shape that have some kind of symmetry.

• CSG:

- CSG stands for Constructive Solid Geometry.
- It is based on set of 3D solid primitives and regularized set theoretic operations.
- Traditional primitives are: Block, cones, sphere, cylinder and torus.
- Operations: union, intersection, difference + translation and rotation.
- A complex solid is represented using with a binary tree usually called as CSG tree.
- CSG tree is shown below.



- **CSG Example:**



(a) Given image

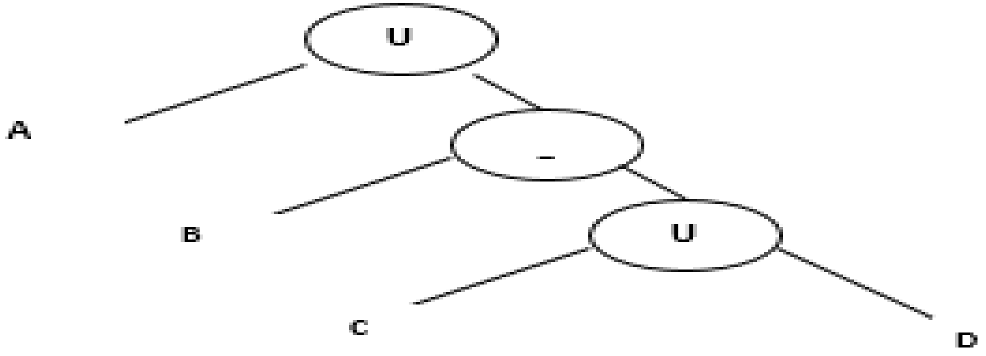
(b) Identification of Primitives used

Fig. 10. (a) Given Image (b) Identification of Primitives used (c) Tree Structure

From (b), the following set operations are identified,

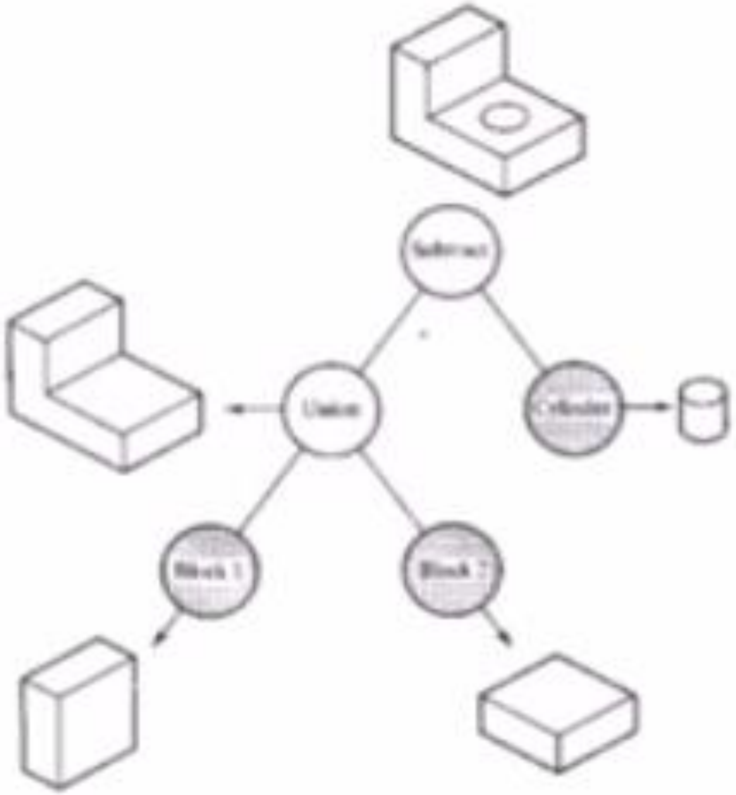
(1) (C U D) (Object C union Object D)
(2) [B – (C U D)]
(3) A U [B – (C U D)]

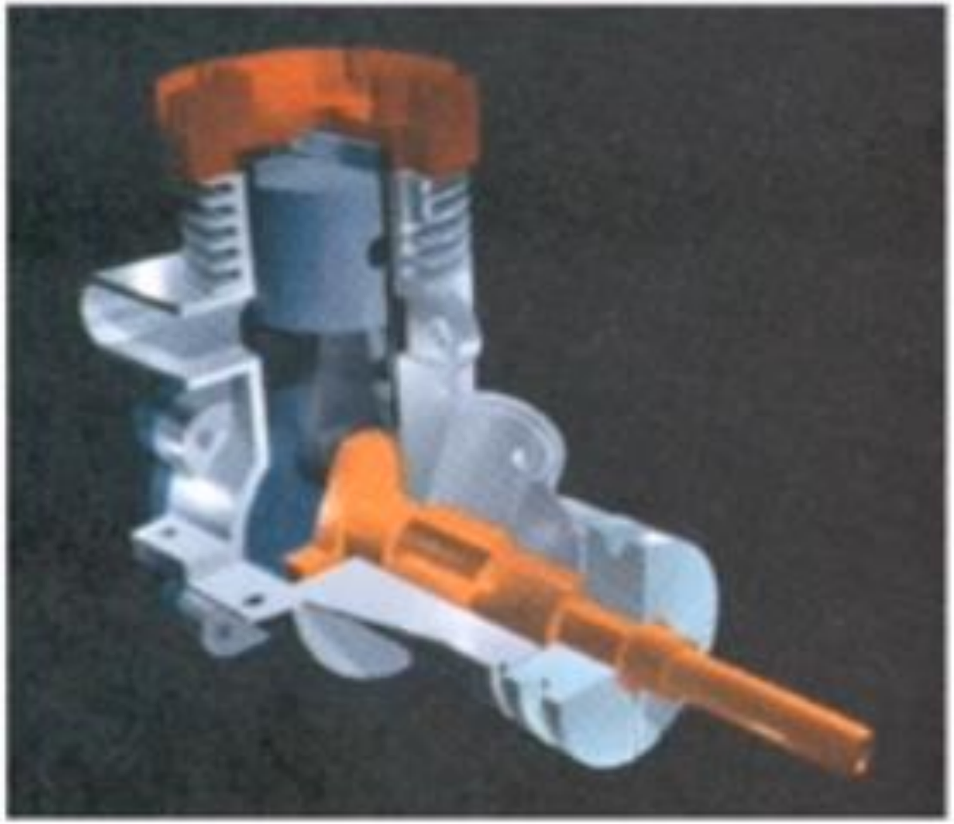
Now, the tree structure is formed from the previous step.



CSG

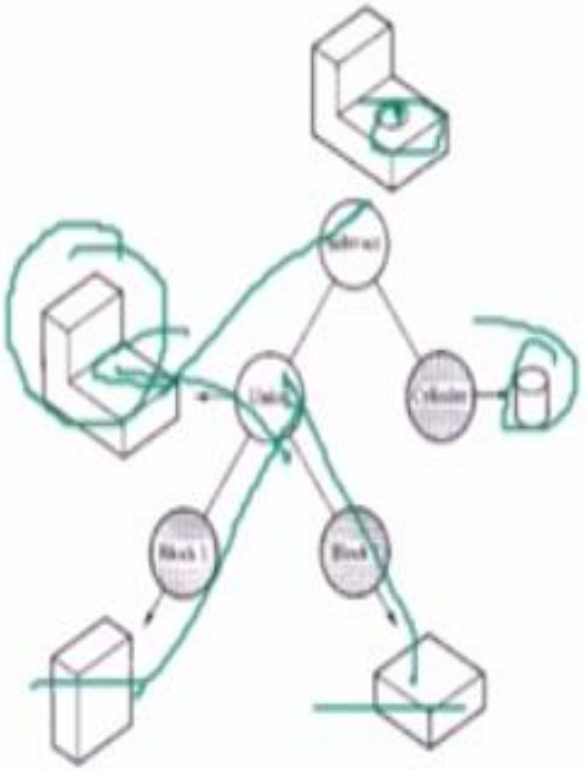
- Hierarchy of Boolean Set Operations (Union, Difference, Intersect) Applied to Simple Shapes

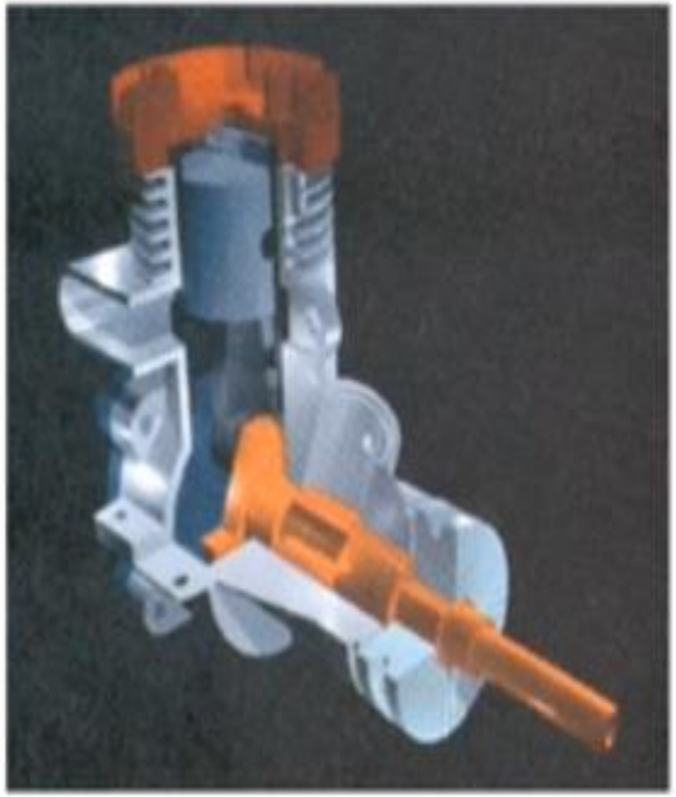




CSG

- Hierarchy of Boolean Set Operations (Union, Difference, Intersect) Applied to Simple Shapes





Qn- Create CSG model

1] Primitives used.

2] Set operations.

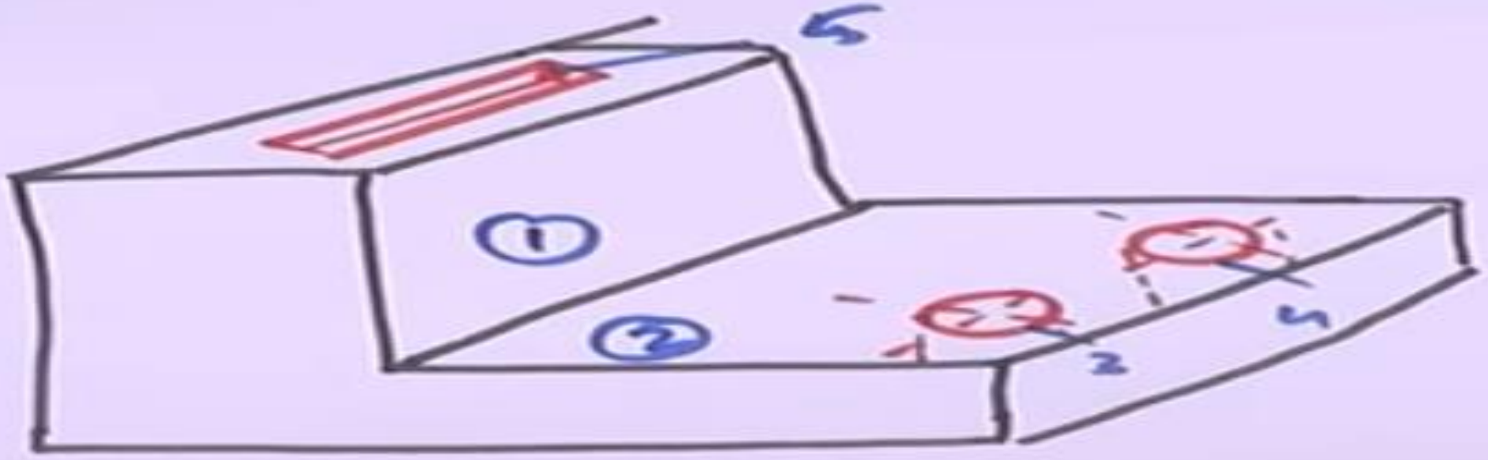
i] $(C \cup D)$.

ii] $(B - [C \cup D])$.

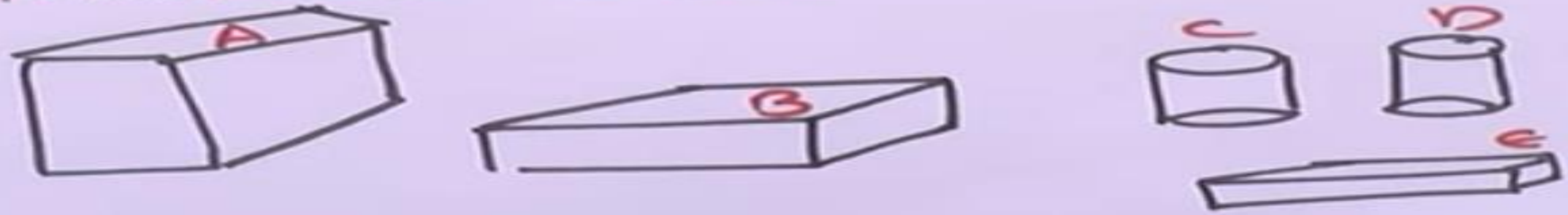
iii] $A \cup [B - [C \cup D)]$

3] Tree Structure

Qn- Create CSG model



1] Primitives used



2] Set operations.

i] $C \cup D$

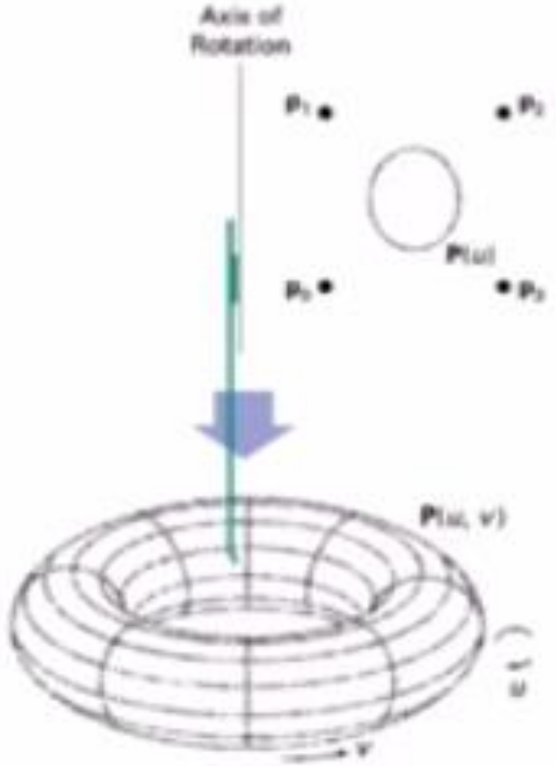
ii] $B - [C \cup D]$

iii] $A - E$

iv] $(A - E) \cup [B - [C \cup D]$

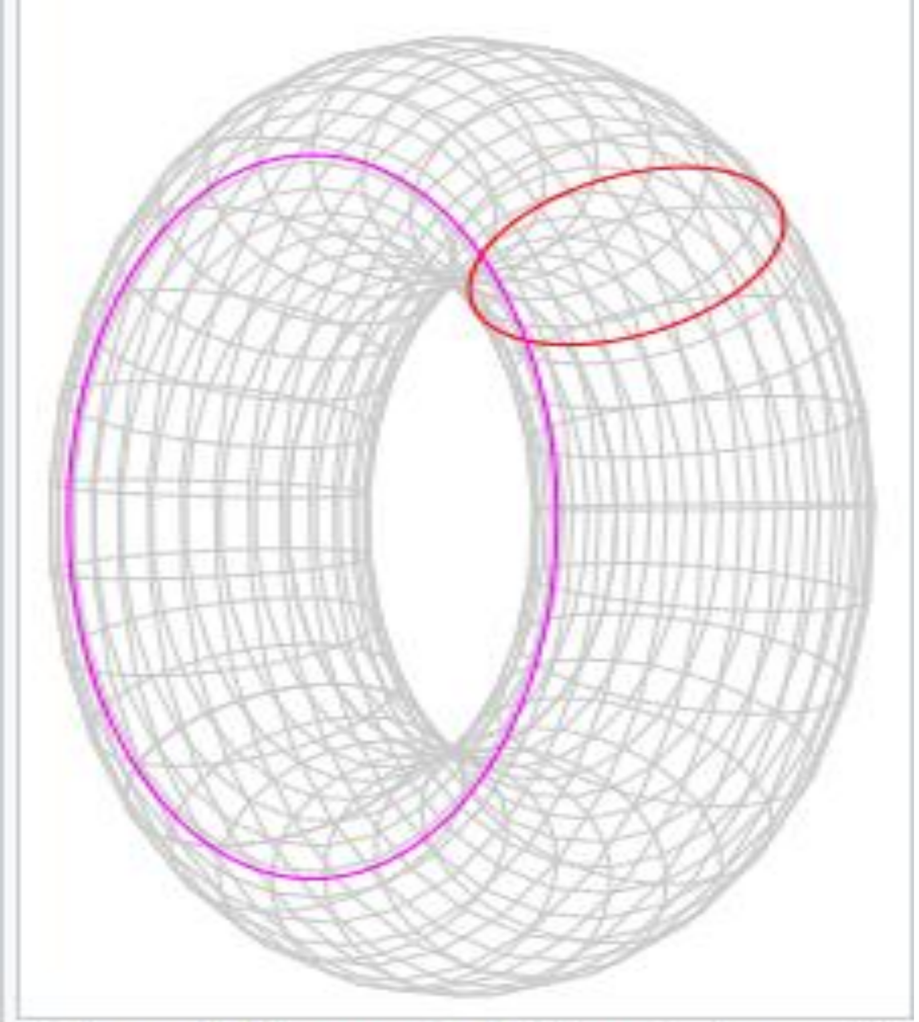
Sweep

- Solid Swept by Curve Along Trajectory



Constructing a Torus using Rotational Sweep

Genus, Torus



A torus is the product of two circles, only one of which is shown in this