# Probability

Probability of an event = $\dfrac{\text{number of favourable outcomes}}{\text{Total no. of outcomes}}$

$$\boxed{P(A) = \frac{n(A)}{n(S)}}$$

## Axioms of probability

Let S be a Sample space and A be an event in S, then P(A) is called the probability of the event if the following conditions are satisfied.

i) $P(A) \geq 0$

ii) $P(S) = 1$

iii) If A and B are mutually exclusive events

$P(A \cup B) = P(A) + P(B)$

Note:

i) if $A \cap B = \phi$, then A and B are mutually exclusive events.

ii) Event: Any subset of Sample space is called an event.

Result 1): Prove that the probability of impossible event is zero. $P(\phi) = 0$.

Proof: The certain event S and the impossible event $\phi$ are mutually exclusive. $S \cup \phi = S$

$P(S \cup \phi) = P(S)$

$P(S) + P(\phi) = P(S) \Rightarrow P(\phi) = 0$

~~Number of favourable Outcomes~~

2) If $\bar{A}$ is the complementary event of A, then

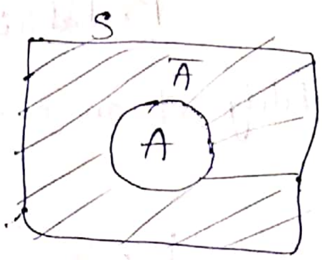
$P(\bar{A}) = 1 - P(A)$

Now, $A \cup \bar{A} = S$

$$P(A \cup \bar{A}) = P(S)$$

$$P(A) + P(\bar{A}) = \cancel{P(S)}\ 1$$

$$\boxed{P(\bar{A}) = 1 - P(A)}$$




## Addition theorem on Probability

**Statement**: If $A$ and $B$ are any two events, then

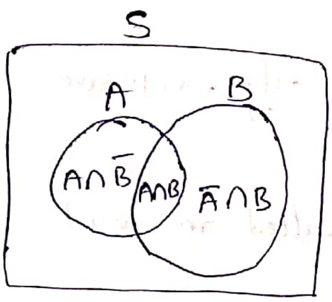
$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}.$$

**Proof**: Now $A \cup B = A \cup (\bar{A} \cap B)$

$$P(A \cup B) = P[A \cup (\bar{A} \cap B)]$$

$$\boxed{P(A \cup B) = P(A) + P(\bar{A} \cap B)} \text{ --- } (1)$$

($\because$ $A$ and $\bar{A} \cap B$ are mutually exclusive)




consider $B = [(A \cap B) \cup (\bar{A} \cap B)]$

$$P(B) = P[(A \cap B) \cup (\bar{A} \cap B)]$$

$$= P(A \cap B) + P(\bar{A} \cap B)$$

($\because$ $A \cap B$ and $\bar{A} \cap B$ are mutually exclusive events)

$$P(B) - P(A \cap B) = P(\bar{A} \cap B) \text{ --- } (2)$$

use (2) in (1)

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

**Result**: If $A$, $B$ and $C$ are any 3 events then,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C)$$
$$- P(C \cap A) + P(A \cap B \cap C)$$

→ The probability of three students A, B and C solving a problem in statistics are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. A problem is given to all the 3 students. What is the probability i) No one will solve the problem.

ii) only one will solve the problem.

iii) Atleast one will solve.

## Independent events

If A and B are independent events, then

$$\boxed{P(A \cap B) = P(A) \times P(B)}$$

Sol Given : $P(A) = \frac{1}{2}$ | $P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$

$P(B) = \frac{1}{3}$ | $P(\bar{B}) = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$

$P(C) = \frac{1}{4}$ | $P(\bar{C}) = 1 - P(C) = 1 - \frac{1}{4} = \frac{3}{4}$

i) $P(\text{No one will solve}) = P(\bar{A} \cap \bar{B} \cap \bar{C})$

$= P(\bar{A}) \times P(\bar{B}) \times P(\bar{C})$

($\because$ A, B, C are independent)

$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$

$= \frac{1}{4}$

ii) $P(\text{only one will solve}) = P(A) \times P(\bar{B}) \times P(\bar{C}) + P(\bar{A}) \times P(B) \times P(\bar{C}) + P(\bar{A}) \times P(\bar{B}) \times P(C)$

$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4}$

$= \frac{6+3+2}{24} = \frac{11}{24}$

iii) $P(\text{Atleast one will solve}) = 1 - P(\text{No one will solve})$

$$= 1 - \frac{1}{4}$$

$$= \frac{3}{4}$$

→ From 6 positive and 8 negative numbers, 4 numbers are chosen at random (without replacement) and multiplied. What is the probability that the product is positive.

Sol:

$$\boxed{{}^nC_r = \frac{n!}{r!(n-r)!}} \qquad \boxed{{}^nC_r = {}^nC_{n-r}}$$

$$S = 6(+ve) + 8(-ve)$$

$$\boxed{n(S) = 14}$$

$$P(\text{a number the product is } +ve) = \frac{{}^6C_4 + {}^8C_4 + {}^6C_2 \times {}^8C_2}{{}^{14}C_4}$$

$$= \frac{15 + 70 + 15 \times 28}{1001}$$

$$= \frac{505}{1001}$$

→ A bag contains 4 white and 6 black balls. Two balls are drawn at random. What is the probability that i) both are white

ii) both are black

iii) one white and one black.

Sol. There are 4 white & 6 Black balls.

Total no. of balls $= 4 + 6 = 10$

i) $P(\text{both are white}) = \frac{{}^4C_2}{{}^{10}C_2} = \frac{6}{45} = \frac{2}{15}$

ii) $P(\text{both are black}) = \frac{{}^6C_2}{{}^{10}C_2} = \frac{15}{45} = \frac{1}{3}$

iii) $P(\text{one white \& one black}) = \frac{{}^4C_1 \times {}^6C_1}{{}^{10}C_2}$

$= \frac{4 \times 6}{45} = \frac{8}{15}$

(*) → In a shooting test, the probability of hitting the target is $\frac{1}{2}$ for A, $\frac{2}{3}$ for B and $\frac{3}{4}$ for C. If all of them fire at the target. Find the probability that i) None of them hits the target.

ii) Atleast one of them will hit.

Given:

| | |
|---|---|
| $P(A) = \frac{1}{2}$ | $P(\bar{A}) = 1 - \frac{1}{2} = \frac{1}{2}$ |
| $P(B) = \frac{2}{3}$ | $P(\bar{B}) = 1 - \frac{2}{3} = \frac{1}{3}$ |
| $P(C) = \frac{3}{4}$ | $P(\bar{C}) = 1 - \frac{3}{4} = \frac{1}{4}$ |

$P(\text{None of them hit the target}) = P(\bar{A}\,\bar{B}\,\bar{C})$

$= P(\bar{A}) \times P(\bar{B}) \times P(\bar{C})$

$= \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} = \frac{1}{24}$

$P(\text{Atleast one}) = 1 - P(\text{None})$

$= 1 - \frac{1}{24}$

$= \frac{23}{24}$

→ What is the chance that a leap year selected at random will contain 53 Sundays?

Sol. Let A be the event that there are 53 Sundays in a leap year.

In a leap year there are 366 days.

$\Rightarrow$ There are 52 weeks and 2 days over.

The possible combinations are Sunday and Monday, Monday and Tuesday, Tue and Wed, Wed and Thu, Thu and Fri, Fri and Sat, Sat and Sun.

$\therefore$ Req. probability $= \frac{2}{7}$

$\rightarrow$ If A and B are independent events, then prove that $\bar{A}$ and $\bar{B}$ are also independent.

Sol

Since A and B are independent events

$$\Rightarrow P(A \cap B) = P(A) \times P(B)$$

To prove $\bar{A}$ and $\bar{B}$ are independent

$$(\overline{A \cup B}) = \bar{A} \cap \bar{B}$$

$$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$$

$$= 1 - P(A \cup B)$$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - [P(A) + P(B) - P(A) \times P(B)]$$

$$= 1 - [P(A) + P(B)[1 - P(A)]]$$

$$= (1 - P(A)) - P(B)[1 - P(A)]$$

$$= [1 - P(A)][1 - P(B)]$$

$$P(\bar{A} \cap \bar{B}) = P(\bar{A}) \times P(\bar{B})$$

$\Rightarrow$ $\bar{A}$ & $\bar{B}$ are independent.

→ If $P(A)=0.4$, $P(B)=0.7$, $P(A\cap B)=0.3$. Find $P(\bar{A}\cap\bar{B})$, $P(\bar{A}\cup\bar{B})$.

Sol.

i) $P(\bar{A}\cap\bar{B})$

W.K.T $(\overline{A\cup B}) = \bar{A}\cap\bar{B}$

$$P(\bar{A}\cap\bar{B}) = P(\overline{A\cup B})$$
$$= 1 - P(A\cup B)$$
$$= 1 - [P(A) + P(B) - P(A\cap B)]$$
$$= 1 - [0.4 + 0.7 - 0.3] = 0.2$$



ii) $P(\bar{A}\cup\bar{B})$

$$(\overline{A\cap B}) = \bar{A}\cup\bar{B}$$
$$P(\overline{A\cap B}) = P(\bar{A}\cup\bar{B})$$
$$P(\bar{A}\cup\bar{B}) = 1 - P(A\cap B)$$
$$= 1 - 0.3$$
$$= 0.7$$

→ If $A, B, C$ are any three events, such that
$P(A) = P(B) = P(C) = \frac{1}{4}$, $P(A\cap B) = P(B\cap C) = 0$, $P(C\cap A) = \frac{1}{8}$
$P(A) \geq 0$. Find the probability that atleast one of the events $A, B, C$ occurs.

Sol.

$P(\text{Atleast any one of } A, B \text{ and } C) = P(A\cup B\cup C)$

$$P(A\cup B\cup C) = P(A) - P(B) - P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$$

$\Rightarrow P(A\cap B\cap C) = 0$

→ If A and B are independent events, then prove that (8)

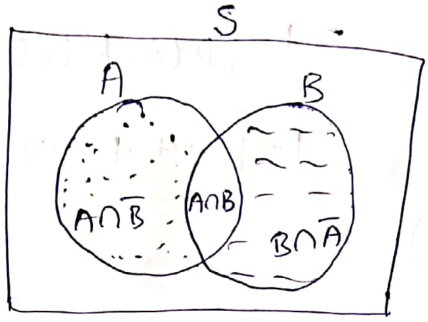
i) $\bar{A}$ and B are also independent.

ii) A and $\bar{B}$ are " "

To prove

$$P(\bar{A}\cap B) = P(\bar{A}) \times P(B)$$

$B = (A\cap B) \cup (B\cap \bar{A})$




$P(B) = P[(A\cap B)\cup(B\cap\bar{A})]$

$= P(A\cap B) + P(B\cap\bar{A})$

$= P(A) \times P(B) + P(B\cap\bar{A})$

$P(\bar{A}\cap B) = P(B) - P(A)\,P(B)$

$= P(B)[1-P(A)]$

$= P(\bar{A}) \times P(B)$

$\Rightarrow$ $\bar{A}$ & B are independent.

→ A lot consists of 10 good articles. 4 with minor defective and 2 with major defective. Two articles are chosen at random (without replacement). Find the probability that i) both are good

ii) both have major defective.

iii) Atleast 1 is good

iv) Atmost 1 is good.

v) Exactly 1 is good.

vi) neither has major defective

vii) neither is good.

Atleast $\geq$

Atmost
include Zero

Sol Total no. of articles = 10 g + 4 minor def + 2 major
= 16

i) $P(\text{both are good}) = \frac{{}^{10}C_2}{{}^{16}C_2} = \frac{45}{120} = \frac{3}{8}$

ii) $P(\text{both are major defective}) = \frac{{}^{2}C_2}{{}^{16}C_2} = \frac{1}{120}$

iii) $P(\text{atleast 1 is good}) = \frac{{}^{10}C_1 \times {}^{6}C_1 + {}^{10}C_2}{{}^{16}C_2} = \frac{7}{8}$

iv) $P(\text{atmost 1 is good}) = \frac{{}^{6}C_2 \times {}^{10}C_0 + {}^{6}C_1 \times {}^{10}C_1}{{}^{16}C_2} = \frac{5}{8}$

v) $P(\text{Exactly 1 is good}) = \frac{{}^{10}C_1 \times {}^{6}C_1}{{}^{16}C_2} = \frac{1}{2}$

vi) $P(\text{neither has major defective}) = \frac{{}^{10}C_1 \times {}^{4}C_1 + {}^{10}C_2 \times {}^{4}C_0 + {}^{4}C_2 \times {}^{10}C_0}{{}^{16}C_2}$
$= 1$

vii) $P(\text{neither is good}) = \frac{{}^{6}C_2}{{}^{16}C_2} = \frac{1}{8}$

→ A problem in statistics is given to 5 students A, B, C, D, E ; their chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$. What is the probability that,

i) No one will solve the problem.

ii) Atleast one will solve the problem.

Sol

$P(A) = \frac{1}{2}$ $\quad P(\bar{A}) = 1 - \frac{1}{2} = \frac{1}{2}$

$P(B) = \frac{1}{3}$ $\quad P(\bar{B}) = 1 - \frac{1}{3} = \frac{2}{3}$

$P(C) = \frac{1}{4}$ $\quad P(\bar{C}) = 1 - \frac{1}{4} = \frac{3}{4}$

$P(D) = \frac{1}{5}$ $\quad P(\bar{D}) = 1 - \frac{1}{5} = \frac{4}{5}$

$P(E) = \frac{1}{6}$ $P(\bar{E}) = 1 - \frac{1}{6} = \frac{5}{6}$

i) P(No one will solve) $= P(\bar{A}\bar{B}\bar{C}\bar{D}\bar{E})$

$= P(\bar{A}) \times P(\bar{B}) \times P(\bar{C}) \times P(\bar{D}) \times P(\bar{E})$

$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6}$

$= \frac{1}{6}$

ii) P(Atleast one will solve the problem)

$= 1 - $ p(none of them will solve)

$= 1 - P(\bar{A}\bar{B}\bar{C}\bar{D}\bar{E})$

$= 1 - \frac{1}{6}$

$= \frac{5}{6}$

(2M) **Conditional Probability**

The conditional probability of an event B. assuming that the event A has happened, is denoted by $P(B/A)$ and defined as $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$, provided $P(A) \neq 0$

similarly $P(A/B) = \frac{P(A \cap B)}{P(B)}$, $P(B) \neq 0$

For ex: when a die is rolled, the cond. prob. of 1 given that an odd number has been obtained:

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 3, 5\}$, $B = \{1\}$

$P(B/A) = \frac{n(A \cap B)}{n(A)} = \frac{1}{3}$

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## Baye's Theorem

Statement : Let $A_1, A_2, A_3, \ldots A_n$ be a set of exhaustive and mutually exclusive (disjoint) events associated with random experiment and B is another event then,

$$\boxed{P(A/B) = \frac{P(A_i) \times P(B/A_i)}{\sum_{i=1}^{n} P(A_i)\, P(B/A_i)}}$$

Theorem Total probability $P(B) = \sum_{i=1}^{n} P(A_i)\, P(B/A_i)$

→ In a belt factory machines A, B and C produce 25%, 35% and 40% of the total output. From their o/ps 5%, 4% and 2% are defective. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by the machines A, B and C.

Sol: Let $A_1, A_2, A_3$ be the probability of machines manufacturing both. Let B be the event of defective bolt.

| $P(A_i)$ | $P(B/A_i)$ | $P(A_i) \times P(B/A_i)$ |
|---|---|---|
| $P(A_1) = 0.25$ | $P(B/A_1) = 0.05$ | $P(A_1) \times P(B/A_1) = 0.25 \times 0.05 = 0.025$ |
| $P(A_2) = 0.35$ | $P(B/A_2) = 0.04$ | $P(A_2) \times P(B/A_2) = 0.014$ |
| $P(A_3) = 0.40$ | $P(B/A_3) = 0.02$ | $P(A_3) \times P(B/A_3) = 0.008$ |
| | | $\sum_{i=1}^{3} P(A_i)\, P(B/A_i) = 0.0345$ |

i) P (defective bolt manufacturing by machine $A_1$) $= P(A_1/B)$

$$= \frac{P(A_1)\,P(B/A_1)}{\sum_{i=1}^{3} P(A_i)\cdot P(B/A_i)} = \frac{0.0125}{0.0345} = 0.3623$$

ii) $P(A_2/B) = \frac{0.014}{0.0345} =$

iii) $P(A_3/B) = \frac{0.008}{0.0345} =$

→ Baye's Theorem

$$P(A_i/B) = \frac{P(A_i)\,P(B/A_i)}{\sum_{i=1}^{n} P(A_i)\,P(B/A_i)}$$

Total theorem of probability $P(A) = \sum_{i=1}^{n} P(A_i)\cdot P(B/A_i)$

*→ The first bag contains 3 white, 2 red and 4 black balls. $2^{nd}$ bag contains 2 white, 3 red and 5 black balls and $3^{rd}$ bag contains 3W, 4R and 2 Black balls. One bag is chosen at random and from it 3 balls are drawn out of 3 balls two balls are white and one is red. What ~~as~~ is the probability that they were taken from $1^{st}$, $2^{nd}$ and $3^{rd}$ bags?

Sol.

| I | II | III |
|---|---|---|
| 3W | 2W | 3W |
| 2R | 3R | 4R |
| 4B | 5B | 2B |
| Total = 9 balls | 10 balls | 9 balls. |

Let $A_1, A_2, A_3$ be the bag I, II and III.

Let B be the event that 3 balls are taken by 2 balls are white and 1 is red.

| $P(A_i)$ | $P(B/A_i)$ | $P(A_i)\,P(B/A_i)$ |
|---|---|---|
| $P(A_1)=\frac{1}{3}$ | $\frac{^3C_2 \times {^2C_1}}{^9C_3} = \frac{3\times 2}{84} = 0.0714$ | $= 0.0238$ |
| $P(A_2)=\frac{1}{3}$ | $\frac{^2C_2 \times {^3C_1}}{^{10}C_3} = \frac{1\times 3}{120} = 0.025$ | $= \cancel{0.01666}$ $0.0083$ |
| $P(A_3)=\frac{1}{3}$ | $\frac{^3C_2 \times {^4C_1}}{^9C_3} = \frac{3\times 4}{84} = 0.1428$ | $= 0.0476$ |
| Total $P(A) = \sum_{i=1}^{3} P(A_i)\cdot P(B/A_i)$ | | $= 0.0797$ |

$$P(A_1/B) = \frac{0.0238}{0.0797} = 0.2986$$

$$P(A_2/B) = \frac{0.0083}{0.0797} = 0.1041$$

$$P(A_3/B) = \frac{0.0476}{0.0797} = 0.5972$$

→ A bolt is manufactured by 3 machines A, B and C. A turns out twice as many items as B and machines B and C produce equal no. of items. 2% of bolts produced by A and B are defective and 4% of bolts produced by C are defective. All bolts are put into one box and one is chosen from this box. What is probability that it is defective?

Sol. Given

$P(B) = \frac{1}{4}$, $P(C) = \frac{1}{4}$, $P(A) = \frac{1}{2}$.

| A | B | C |
|---|---|---|
| $2x$ | $x$ | $x$ |

$$2x + x + x = 1$$
$$4x = 1$$
$$x = \frac{1}{4}$$

Let D be event of selecting defective bolt :

$$P(D/A) = \frac{2}{100}$$

$$P(D/B) = \frac{2}{100}$$

$$P(D/C) = \frac{4}{100}$$

By total theorem of probability.

$$P(D) = P(A) \times P(D/A) + P(B) \times P(D/B) + P(C) \times P(D/C)$$

$$= \frac{1}{2} \times \frac{2}{100} + \frac{1}{4} \times \frac{2}{100} + \frac{1}{4} \times \frac{4}{100}$$

$$= \frac{1}{100} + \frac{1}{200} + \frac{1}{100}$$

$$= \frac{2+1+2}{200}$$

$$= \frac{5}{200} = \frac{1}{40}$$

→ There are 3 true coins and 1 false coin with head on both sides. A coin is chosen at random and tossed 4 times. If head occurs all the 4 times. What is the probability that the false coin has been chosen and used?

sol: Total coins $= 3T + 1F$

$= 4$ coins

$$P(T) = \frac{3}{4}, \quad P(F) = \frac{1}{4}.$$

Let A be the event of ~~hear~~ getting all heads in 4 tosses.

$$P(A/T) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$$

$$P(A/F) = \frac{1}{1} \times \frac{1}{1} \times \frac{1}{1} \times \frac{1}{1} = 1$$

By Baye's theorem.

$$P(F/A) = \frac{P(F) \cdot P(A/F)}{P(T) \cdot P(A/T) + P(F) \cdot P(A/F)}$$

→ In a coin tossing experiment if the coin shows head, IS 1 die is throwing and result is recorded. But if the coin shows tail, 2 dice are thrown and their sum is recorded. what's the probability that the recorded number will be 2?

Sol

i) when a single die is thrown $P(2) = \frac{1}{6}$.

ii) when 2 dice are thrown, the sum will be 2 if each die shows 1. $P(\text{getting 2 as sum with 2 dice}) = \frac{1}{36}$.

By theorem of total probability. $P(2) = P(H) \times P(2/H) + P(T) \times P(2/T)$

$$= \frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{36}$$

$$= \frac{1}{12} + \frac{1}{72}$$

$$= \frac{6+1}{72} = \frac{7}{72}$$

→ For a certain binary communication channel the probability that a transmitted '0' is received as '0' is 0.95 and the probability that a transmitted 1 is received as 1 is 0.90. If the probability that a '0' is transmitted is 0.4. Find the probability that

i) a '1' is received.

ii) a '1' is transmitted guess that a '1' was received.

Sol: Let A be the event of transmitted as '1'.

$\bar{A}$ be the event of transmitted as 0 and

Let B be the event of receiving as 1.

$\bar{B}$ be the event of receiving '0'.

Given:

| | |
|---|---|
| $P(\bar{A}) = 0.4$ | $\cancel{P(\bar{A}/\bar{B}) = 0.95}$ $P\left(\frac{B}{A}\right) = 0.90$ |
| $P(A) = 1 - 0.4 = 0.6$ | $\cancel{P(A/\bar{B}) = 0.05}$ $P(\bar{B}/\bar{A}) = 0.95$ |

and also ~~P(A/B) = 0.90~~ $P(B/\bar{A}) = 0.05$.

i) By theorem of total probability

$$P(B) = P(A) \times P(B/A) + P(\bar{A}) \cdot P(B/\bar{A}).$$

$$= 0.6 \times 0.90 + 0.4 \times 0.05$$

$$= 0.56$$

ii) By Baye's theorem

$$P(A/B) = \frac{P(A) \cdot P(B/A)}{P(B)}$$

$$= \frac{0.6 \times 0.90}{0.56}$$

## Random Variable

Continuous R.V

$f(x)$ : pdf

$f(x) \geq 0$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

Mean $= E(x)$

$var(x) = E(x^2) - [E(x)]^2$

i) if $x$ is discrete

$E(x) = \sum x\, p(x)$

ii) If $x$ is continuous

$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx.$$

Discrete R.V

→ A descrete r.v $x$ has the following probability distribution.

| $x$ : | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $P(x)$ : | a | 3a | 5a | 7a | 9a | 11a | 13a | 15a | 17a |

i) Find the value of 'a'. ii) Evaluate $P(x>3)$ and also Mean and Variance, cummulative distribution function. (c.d.f)

Sol

i) If $p(x)$ is probability mass function, then

$$\sum_{x=-\infty}^{\infty} p(x) = 1$$

$$\sum_{0}^{8} p(x) = 1$$

$p(0)+p(1)+p(2)+\cdots+p(8)=1$

$a+3a+5a+7a+9a+11a+13a+15a+17a=1$

$\therefore \quad 81a=1$

$$\boxed{a=\frac{1}{81}}$$

ii) $P(x>3)=1-P(x\leq 3)$

$=1-[P(0)+P(1)+P(2)+P(3)]$

$=1-[a+3a+5a+7a]$

$=1-16a$

$=1-16\left(\frac{1}{81}\right)$

$=\frac{81-16}{81}$

$=\frac{65}{81}$

(or)

$p(x>3)$

$=p(4)+p(5)+p(6)+p(7)+p(8)$

$=\frac{65}{81}$

To find Mean, Variance

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $P(x)$ | $\frac{1}{81}$ | $\frac{3}{81}$ | $\frac{5}{81}$ | $\frac{7}{81}$ | $\frac{9}{81}$ | $\frac{11}{81}$ | $\frac{13}{81}$ | $\frac{15}{81}$ | $\frac{17}{81}$ |

Mean $=E(x)$

$=\sum x\,p(x)$

$=0\times\frac{1}{81}+1\times\frac{3}{81}+2\times\frac{5}{81}+3\times\frac{7}{81}+4\times\frac{9}{81}$

$+5\times\frac{11}{81}+6\times\frac{13}{81}+7\times\frac{15}{81}+8\times\frac{17}{81}$

$E(x)=\frac{0+3+10+21+36+55+78+105+126}{81}$

$=\frac{444}{81}$

Now, $E(x^2)=\sum x^2 p(x)$

$=(0)^2\times\frac{1}{81}+(1)^2\times\frac{3}{81}+(2)^2\times\frac{5}{81}+(3)^2\left(\frac{7}{81}\right)+(4)^2\left(\frac{9}{81}\right)$

$$+ (5)^2\left(\frac{11}{81}\right) + (6)^2\left(\frac{13}{81}\right) + (7)^2\left(\frac{15}{81}\right) + (8)^2\left(\frac{19}{81}\right)$$

$$= \frac{2796}{81}$$

$\therefore$ Variance of $x$ = $Var(x) = E(x^2) - [E(x)]^2$

$$= \frac{2796}{81} - \left(\frac{444}{81}\right)^2$$

$$= 4.4725$$

Note:- If $x$ is discrete,

$$F(x) = P(x \leq x) = \sum_{-\infty}^{x} p(x)$$

If $x$ is continuous,

$$f(x) = \int_{-\infty}^{x} f(x)\,dx$$

To find c.d.f $f(x)$

$$f(x) = P(x \leq x)$$

$$f(0) = P(x \leq 0) = \frac{1}{81}$$

$$f(1) = P(x \leq 1) = P(0) + P(1) = \frac{1}{81} + \frac{3}{81} = \frac{4}{81}$$

$$f(2) = P(x \leq 2) = P(0) + P(1) + P(2) = \frac{9}{81}$$

$$f(3) = P(x \leq 3) = P(0) + P(1) + P(2) + P(3) = \frac{16}{81}$$

$$f(4) = \frac{25}{81}$$

$$f(5) = \frac{36}{81}$$

$$f(6) = \frac{49}{81}$$

$$f(7) = \frac{64}{81}$$

$$f(8) = \frac{81}{81} = 1$$

$$\left\{\begin{array}{l} f(x) = \text{pdf} \\ f(x) = \text{c.d.f} \\ \quad f(x) = f'(x) \end{array}\right.$$

→ In a continuous r.v the probability density fun (pdf) is given by $f(x) = kx(2-x); 0<x<2$. 1)

i) find K, mean & Variance. ii) find c.d.f

Sol. If $f(x)$ is pdf, then $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

$$\int_0^2 K\cdot x(2-x)\,dx = 1$$

$$K\int_0^2 (2x-x^2)\,dx = 1$$

$$K\left[\frac{\cancel{2}x^2}{\cancel{2}} - \frac{x^3}{3}\right]_0^2 = 1$$

$$K\left[\left((2)^2 - \frac{(2)^3}{3}\right) - 0\right] = 1$$

$$K\left[4 - \frac{8}{3}\right] = 1$$

$$\frac{4K}{3} = 1$$

$$\boxed{K = \frac{3}{4}}$$

∴ The p.d.f is $f(x) = \frac{3}{4}(2x-x^2); 0<x<2$.

i) <u>To find mean</u> $= E(x)$

$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_0^2 x\cdot\frac{3}{4}(2x-x^2)\,dx$$

$$= \frac{3}{4}\int_0^2 (2x^2-x^3)\,dx$$

$$= \frac{3}{4}\left[2\frac{x^3}{3} - \frac{x^4}{4}\right]_0^2$$

$$= \frac{3}{4}\left[\left(\frac{16}{3} - \frac{16}{4}\right) - 0\right] = \frac{3}{4}\left[\frac{64-48}{12}\right]$$

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$$= \frac{3}{4}\left[\frac{16}{12}\right] = \frac{4}{4} = 1$$

To find variance

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$= \int_0^2 x^2 f(x)\,dx$$

$$= \frac{3}{4}\int_0^2 (2x^3 - x^4)\,dx$$

$$= \frac{3}{4}\left[\frac{2x^4}{4} - \frac{x^5}{5}\right]_0^2$$

$$= \frac{3}{4}\left[\frac{32}{4} - \frac{32}{5}\right]$$

$$= \frac{3}{4} \times 32\left[\frac{1}{20}\right]$$

$$= \frac{6}{5}$$

$$Var(x) = E(x^2) - [E(x)]^2 = \frac{6}{5} - 1 = \frac{1}{5}$$

ii) To find c.d.f $F(x)$

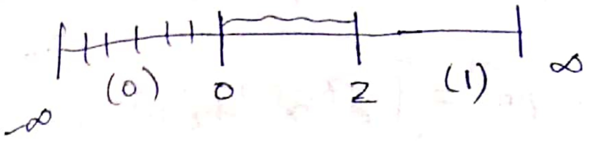
$$F(x) = P(X \le x)$$

$$= \int_{-\infty}^{x} f(x)\,dx$$

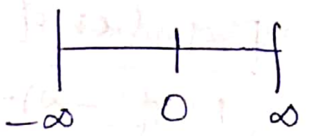
$$= \int_0^x \frac{3}{4}(2x - x^2)\,dx$$

$$= \frac{3}{4}\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^x$$

$$= \frac{3}{4}\left[x^2 - \frac{x^3}{3}\right]; \quad 0 < x < 2$$




$$F(x) = \begin{cases} 0 \; ; & x < 0 \\ \frac{3}{4}\left(x^2 - \frac{x^3}{3}\right) ; & 0 < x < 2 \\ 1 \; ; & x > 2 \end{cases}$$

→ 


$$F(-\infty) = 0$$

$$F(\infty) = 1$$

→ Check whether $f(x) = 3x^2, \; 0 < x < 1$ is p.d.f or not.

Now $\int_{-\infty}^{\infty} f(x)\,dx = \int_0^1 3x^2\,dx = \left[3 \times \frac{x^3}{3}\right]_0^1$

$$= 1 - 0$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$\Rightarrow f(x)$ is p.d.f.

→ A random variable $x$ has the p.d.f $f(x)$ is given by

$$f(x) = \begin{cases} c\,x e^{-x} \; ; & x > 0 \\ 0 \; ; & x \le 0 \end{cases}$$

i) Find the value of c.

ii) Find the cummulative distribution function.

Sol:

Since $f(x)$ is p.d.f

$$\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_0^{\infty} c \cdot x e^{-x}\,dx = 1$$

$$c\int_0^\infty e^{-x} x^{2-1}\,dx = 1$$

$$\Gamma n = \int_0^\infty e^{-x} x^{n-1}\,dx.$$

$$c\,\Gamma 2 = 1$$

$$c\,\lfloor 1 = 1$$

Note : $e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0.$

$$c \times 1 = 1$$

$$\boxed{c=1}$$

(or)

properties of c.d.f

i) $f(-\infty)=0$

ii) $f(\infty)=1$

iii) $f(x) = F'(x)$

$$c\int_0^\infty x e^{-x}\,dx = 1$$

| | |
|---|---|
| $u = x$ | $v = e^{-x}$ |
| $u' = 1$ | $v_1 = e^{-x}$ |
| $u'' = 0$ | $v_2 = e^{-x}$ |

$$c\left[uv_1 - u'v_2\right] = 1$$

$$c\left[-xe^{-x} - 1\,e^{-x}\right]_0^\infty = 1$$

$$c\left[(0-0)-(0-1)\right] = 1$$

$$\boxed{c=1}$$

$\therefore$ The p.d.f is $f(x) = 1 \times x \times e^{-x}$

$$= xe^{-x};\ x>0$$

To find c.d.f $F(x) = P(x \le x)$

$$= \int_{-\infty}^{\infty} f(x)\,dx$$

$$= \int_0^x x e^{-x}\,dx$$

$$= \left[uv_1 - u'v_2 + \cdots\right]_0^x$$

$$= \left[-xe^{-x} - e^{-x}\right]_0^x$$

$$= \left\{\left[xe^{-x} + e^{-x}\right] - (0+1)\right\}$$

$= 1 - e^{-x}(1+x) \; ; \; x \geq 0$



→ The distribution function of R.V $x$ is given by
$f(x) = 1 - (1+x)e^{-x} \; ; \; x \geq 0$

i) Find the probability density fun (p.d.f).

ii) Find the mean and Variance.

Sol. Given $F(x) = 1 - (1+x)e^{-x} \; ; \; x \geq 0$

W.K.T, $f(x) = F'(x)$

$$= 0 - \{-(1+x)e^{-x} + e^{-x}(0+1)\}$$

$$= (1+x)e^{-x} - e^{-x}$$

$$= e^{-x} + xe^{-x} - e^{-x}$$

∴ p.d.f is $f(x) = xe^{-x} \; ; \; x \geq 0$

To find mean $= E(x)$

$$= \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_{0}^{\infty} x \cdot x e^{-x}\,dx$$

$$= \int_{0}^{\infty} e^{-x} x^2\,dx \quad \left(\because \int_{0}^{\infty} e^{-x} x^{n-1}\,dx = \Gamma n\right)$$

$$= \int_{0}^{\infty} e^{-x} x^{3-1}\,dx$$

$$= \Gamma 3 = \lfloor 2 = 1 \times 2 = 2$$

If $x$ discrete $E(x) = \sum_{x=-\infty}^{\infty} x p(x)$

If $x$ continuous $E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

To find Variance of $x = Var[x]$

$$Var[x] = E[x^2] - [E(x)]^2$$

$= 6-(2)^2$

$= 6-4 = 2$

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$= \int_0^{\infty} x^2 \cdot x e^{-x}\,dx$$

$$= \int_0^{\infty} e^{-x} x^3\,dx = \int_0^{\infty} e^{-x} x^{4-1}\,dx = \Gamma 4$$

$$= \underline{|3} = 1\times 2\times 3 = 6$$

→ A continuous R.V $x$ has the p.d.f $f(x) = \frac{K}{1+x^2}$; $-\infty < x < \infty$

i) find $K$, dist funcn of $x$.

ii) Evaluate $p(x \le 0)$

Sol. Given $f(x) = \frac{K}{1+x^2}$; $-\infty < x < \infty$.

Since $f(x)$ is p.d.f

$$\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$K\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx = 1$$

$$K\left[\operatorname{Tan}^{-1}(x)\right]_{-\infty}^{\infty} = 1$$

$$K\left[\frac{\pi}{2} - \operatorname{Tan}^{-1}(-\infty)\right] = 1$$

$$K\left[\frac{\pi}{2} + \operatorname{Tan}^{-1}(\infty)\right] = 1$$

$$K\left[\frac{\pi}{2} + \frac{\pi}{2}\right] = 1$$

$$K[\pi] = 1 \Rightarrow K = \frac{1}{\pi}$$

$\therefore$ p.d.f is $f(x) = \frac{1}{\pi}\left(\frac{1}{1+x^2}\right)$; $-\infty < x < \infty$

To find c.d.f

$$F(x) = p(x \le x)$$

$$= \int_{-\infty}^{x} f(x)\,dx = \frac{1}{\pi}\int_{-\infty}^{x} \frac{1}{1+x^2}\,dx$$

$$= \frac{1}{\pi}\left[\tan^{-1}(x)\right]_{-\infty}^{x} = \frac{1}{\pi}\left[\tan^{-1}(x) + \frac{\pi}{2}\right]$$

Since $P(X \le x) = \int_{-\infty}^{x} f(x)\,dx$

$$P(X \le 0) = \int_{-\infty}^{0} f(x)\,dx = \frac{1}{\pi}\int_{-\infty}^{0} \frac{1}{1+x^2}\,dx$$

$$= \frac{1}{\pi}\left[\tan^{-1}(x)\right]_{-\infty}^{0}$$

$$= \frac{1}{\pi}\left[0 - \tan^{-1}(-\infty)\right]$$

$$= \frac{1}{\pi} \times \frac{\pi}{2} = \frac{1}{2}$$

(*) If the p.d.f of a cont. R.V $x$

$$f(x) = \begin{cases} ax ; & 0 \le x \le 1 \\ a ; & 1 \le x \le 2 \\ 3a - ax ; & 2 \le x \le 3 \\ 0 ; & x > 3 \end{cases}$$

i) find $a$ ii) find c.d.f

Sol

Since $f(x)$ is p.d.f

$$\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{0}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx + \int_{2}^{3} f(x)\,dx = 1$$

$$\int_{0}^{1} ax\,dx + \int_{1}^{2} a\,dx + \int_{2}^{3} (3a - ax)\,dx = 1$$

$$a\left\{\left[\frac{x^2}{2}\right]_0^1 + \left[x\right]_1^2 + \left[3x - \frac{x^2}{2}\right]_2^3\right\} = 1$$

$$a\left[\frac{1}{2}+(2-1)+\left(9-\frac{9}{2}\right)-\left(6-\frac{4}{2}\right)\right]=1$$

$$a\left[\frac{1}{2}+1+9-\frac{9}{2}-6+2\right]=1$$

$$2a=1$$

$$\boxed{a=\frac{1}{2}}$$

## Unit – II

### Probability Distributions

* Note

$=$ → Exactly

$\geq$ atleast

$\leq$ almost

i) $P(x\geq n)=1-P(x<n)$

ii) $P(x>n)=1-P(x\leq n)$

① Discrete distributions

i) Binomial distribution

ii) Poisson distribution

iii) Geometric distribution

② Continuous distributions

i) Uniform distribution (or) Rectangular distribution

ii) Exponential Distribution

iii) Normal distribution

### Binomial distribution

A random variable $x$ is said to follow a binomial if its assumes only non-negative values and its probability mass function is given by

$$\boxed{P(X=x)={}^nC_x\, p^x q^{n-x}} \quad x=0,1,2,3,\ldots n$$

where $\boxed{p+q=1}$

**Result1:** In a binomial frequency distribution function

$$f(x)=N\,P(X=x)=N\left({}^nC_x\, p^x q^{n-x}\right)$$

Result 2

Mean $= np$

Variance $= npq$

Standard deviation $S.D = \sqrt{npq}$

properties

i) There must be a fixed no. of trials.

ii) All trials must have identical probability of success (P).

iii) The trial must be independent of each other.

problems

→ The mean of binomial distribution is 5 and S.D is 2. Determine the distribution and also find $P(x \geq 2)$.

Given :

Mean $= 5$

$\Rightarrow \boxed{np = 5}$

$S.D = 2$

$\sqrt{npq} = 2$

$npq = 4$

$5q = 4$

$\Rightarrow \boxed{q = \frac{4}{5}}$

$p+q = 1$

$p = 1-q$

$p = 1 - \frac{4}{5}$

$\Rightarrow \boxed{p = \frac{1}{5}}$

∴ The binomial dist is $P(X=x) = {}^{n}C_{x} p^{x} q^{n-x}$

$= {}^{25}C_{x} \left(\frac{1}{5}\right)^{x} \left(\frac{4}{5}\right)^{25-x}$

$\Rightarrow np = 5$

$n\left(\frac{1}{5}\right) = 5$

$\boxed{n = 25}$

$x = 0, 1, 2, 3, \ldots 25.$

$P(X \geq 2) = 1 - P(X < 2)$

$= 1 - \left[P(X=0) + P(X=1)\right]$

$= 1 - \left[{}^{25}C_{0}\left(\frac{1}{5}\right)^{0}\left(\frac{4}{5}\right)^{25-0} + {}^{25}C_{1}\left(\frac{1}{5}\right)^{1}\left(\frac{4}{5}\right)^{25-1}\right]$

$= 1 - \left[1 \times 1 \times \left(\frac{4}{5}\right)^{25} + 25 \times \frac{1}{5} \times \left(\frac{4}{5}\right)^{24}\right]$

$= 1 - \left(\frac{4}{5}\right)^{24}\left[\frac{4}{5} + 5\right] = 1 - \frac{29}{5}\left[\frac{4}{5}\right]^{24}$

→ Comment on the following,

The mean of a binomial is 3 and Variance 4.

Sol. Given:

Mean = 3 | variance = 4

$np = 3$ | $npq = 4$

$3q = 4$

$\boxed{q = \frac{4}{3}} > 1$

which is false ($\because p+q=1$)

⇒ Binomial dist can not be constructed.

→ Four Coins are tossed simultaneously. What is the probability of getting i) 2 heads ii) atleast 2 heads iii) atmost 2 heads.

Sol Let X denote the event of selecting a head.

$p = \frac{1}{2},\ q = \frac{1}{2},\ n = 4.$

The binomial dist is $p(X=x) = {}^nC_x p^x q^{n-x}$

$$P(X=x) = {}^4C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{n-x};$$

$x = 0, 1, 2, 3 \ldots n.$

i) P(exactly 2 heads) $= P[x=2]$

$$= {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2}$$

ii) P[Atleast 2 heads] $= P[X \geq 2]$

$$= P(X=2) + P(X=3) + P(X=4)$$

(or)

$$= 1 - [P(X<2)]$$

$$= 1 - [P(X=0) + P(X=1)]$$

$$= 1 - \left[{}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{4-0} + {}^4C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{4-1}\right]$$

$$= 1 - \left[1 \times 1 \times \left(\frac{1}{2}\right)^4 + 4 \times \frac{1}{2^4}\right]$$

$$= 1 - \frac{5}{16}$$

$$= \frac{11}{16}$$

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ii) $P[\text{atmost 2 heads}] = P[X \leq 2]$

$$= P(X=0) + P(X=1) + P(X=2)$$

## Discrete Random Variables

### Binomial Random Variables

A random variable X is said to follow a binomial distribution (assume only non-ve values), if it's probability mass funcn (pmf) is given by $P(X=x) = {}^nC_x p^x q^{n-x}$ ; $x = 0, 1, 2, \ldots n$.

In a frequency dist: $f(x) = N \cdot P(X=x) = N \cdot {}^nC_x p^x q^{n-x}$

Result: Mean of Binomial dist $= np$

Variance $= npq$

$S.D = \sqrt{npq}$

2. Poisson distribution

If X is discrete r.v that assumes only non-neg ve values such that it's probability mass funcn is $P(X=x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$ ; $x = 0, 1, 2, \ldots$

Result

In a frequency distrib is $f(x) = N \cdot p(X=x)$

$$= \frac{N \cdot e^{-\lambda}\lambda^x}{\lfloor x}$$

Result

→ Mean = Variance = $\lambda$

$$\boxed{\lambda = np}$$

Note: poisson distribution is a limiting case of binomial dist under the following conditions,

i) The no. of trials $n$ is infinitely large.

ii) The probability of success for each trial is very small.

→ If 2% of the electric bulbs manufactured by a company are defective. Find the probability that in a sample of 200 bulbs exactly 10 bulbs are defective.

Sol Let X denote no. of defective bulbs.

$$p = 2\% = \frac{2}{100} = 0.02$$

$$\lambda = np = 200 \times \frac{2}{100} = 4$$

The poisson dist is $P(X=x) = \dfrac{e^{-\lambda}\lambda^{x}}{x!}$ ; $x = 0, 1, 2, ---$

$$= \frac{e^{-4}(4)^{x}}{x!}$$

$$P(\text{Exactly 10 defective}) = P(X=10) = \frac{e^{-4}(4)^{10}}{4!}$$

$$= 0.0053$$

(*)

→ The no. of monthly breakdown of a computer is a random variable having a poisson dist with mean equal to 1.8. Find the probability that this computer will funcn for a month i) without a breakdown ii) only one breakdown iii) atleast 1 breakdown

Sol Let X denote no. of breakdowns

Given : Mean = 1.8 (or) $\lambda = 1.8$

The poisson distribution $P(X=x) = \dfrac{e^{-\lambda}\lambda^{x}}{x!}$ ; $x = 0, 1, 2, ---$

$$P(X=x) = \frac{e^{-1.8}(1.8)^{x}}{x!} \ ; \ x = 0, 1, 2, ----$$

i) $P[\text{without breakdown}] = P[X=0]$

$$= \frac{e^{-1.8}(1.8)^{0}}{0!} = e^{-1.8} = 0.1653$$

ii) $P[\text{only one breakdown}] = P[X=1] = \dfrac{e^{-1.8}(1.8)^{1}}{1!} = \dfrac{0.1653 \times 1.8}{1}$

$$= 0.2975$$

iii) $P[\text{atleast 1 breakdown}] = P[X \geq 1]$

$$= 1 - P[X < 1]$$

$$= 1 - P[X = 0]$$

$$= 1 - \frac{e^{-1.8}(1.8)^0}{\underline{|0}}$$

$$= 1 - \frac{e^{-1.8}(1.8)^0}{1} = 1 - \frac{e^{-1.8} \times 1}{1}$$

$$= 1 - 0.1653 = 0.8347$$

**

→ Out of 800 families with 4 children each, how many families would be expected to have,

i) 2 boys and 2 girls.

ii) Atleast one boy.

iii) Atmost 2 girls.

iv) Children of both sex.

Sol

Let X denote the event of selecting boys.

Let p denote the probability of selecting a boy $\Rightarrow p = \frac{1}{2}$

$$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

$$n = \text{no. of children} = 4$$

The binomial distribution $P(X = x) = {}^nC_x\, p^x q^{n-x}$ ; $x = 0, 1, 2, \ldots$

$$= {}^4C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x}$$

$x = 0, 1, 2, \ldots$

i) $P[2 \text{ boys } \& \text{ } 2 \text{ girls}] = P[X = 2] = {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2}$

$$= 6 \times \frac{1}{2^4} = \frac{6}{16} = \frac{3}{8}$$

∴ Out of 800 families, the probability of selecting 2 boys & 2 girls $= N \cdot P(X = x)$

$$= 800 \times P(X = 2)$$

$$= 800 \times \frac{3}{8} = 300$$

ii) $P[\text{Atleast } 1 \text{ boy}] = P[X \geq 1]$

$$= 1 - P[X < 1]$$

$$= 1 - P[X = 0]$$

$$= 1 - {}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{4-0}$$

$$= 1 - 1 \times 1 \times \frac{1}{16}$$

$$= \frac{15}{16}$$

Out of 800 families, the expected no. of families having atleast 1 boy $= 800 \times \frac{15}{16} = 750$

iii) $P[\text{Atmost } 2 \text{ girls}] = P[\text{Atleast } 2 \text{ boys}]$

$$= P[X \geq 2]$$

$$= 1 - P[X < 2]$$

$$= 1 - [P(X=0) + P[X=1]]$$

$$= 1 - \left[{}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{4-0} + {}^4C_1 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-1}\right]$$

$$= 1 - \frac{5}{16} = \frac{11}{16}$$

Out of 800, the expected no. of families having atmost 2 girls $= 800 \times \frac{11}{16} = 550$

iv) $P[\text{children of both sex}] = 1 - P[\text{children of same sex}]$

$$= 1 - [P(X=0) + P(X=4)]$$

$$= 1 - \frac{2}{16} = \frac{14}{16}$$

(*)
→ 6 dice are thrown 729 times, How many times do you expect atleast 3 dice to show 5 or 6.

Let X denote event of getting a number 5 or 6.

$p = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$ ; $q = 1-p = 1 - \frac{1}{3} = \frac{2}{3}$.

$n = 6$.

$\therefore$ The binomial dist $P(X=x) = {}^nC_x\, p^x q^{n-x}$ ; $x = 0,1,2,\dots$

$$= {}^6C_x \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{6-x}$$

$P[\text{atleast 3 dice}] = P[X \geq 3] = 1 - P[X < 3]$

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= \frac{233}{729}.$$

$\therefore$ Out of 729; The expected no. of times to show 5 or 6 is $= 729 \times \frac{233}{729} = 233$.

→ If $X$ is poisson distribution with $P(X=1) = P(X=2)$. Find $P(X \geq 3)$.

Since $X$ is poisson distribution,

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{\lfloor x} = \frac{e^{-2}(2)^x}{\lfloor x}$$

<u>Given</u>: $P(X=1) = P(X=2)$

$$\frac{e^{-\lambda}\lambda^1}{\lfloor 1} = \frac{e^{-\lambda}\lambda^2}{\lfloor 2}$$

$$2\lambda = \lambda^2$$

$$\boxed{\lambda = 2}$$

$$\therefore P(X=x) = \frac{e^{-2}(2)^x}{\lfloor x}$$

$P(X \geq 3) = 1 - P(X < 3)$

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= 1 - \left[\frac{e^{-2}(2)^0}{\lfloor 0} + \frac{e^{-2}(2)^1}{\lfloor 1} + \frac{e^{-2}(2)^2}{\lfloor 2}\right]$$

$= 1 - e^{-2}(1+2+2) = 1 - 5 \times e^{-2} = 0.3233$

→ If $X$ is poisson r.v such that $P(X=1) = \frac{3}{8}$ and $P(X=2) = \frac{1}{5}$. Find i) $P(X=0)$ ii) $P(X=3)$.

Since $X$ is poisson r.v.

$$\Rightarrow P(X=x) = \frac{e^{-\lambda}\lambda^x}{\lfloor x}\ ;\ x = 0, 1, 2, \ldots$$

Given

$P(X=1) = \frac{3}{10}$ | $P(X=2) = \frac{1}{5}$

$\frac{e^{-\lambda}\lambda^1}{\lfloor 1} = \frac{3}{10}$ —— ① | $\frac{e^{-\lambda}(\lambda)^2}{\lfloor 3} = \frac{1}{5}$ —— ②

$$\frac{②}{①} \Rightarrow \frac{\frac{e^{-\lambda}\lambda^2}{\lfloor 2}}{\frac{e^{-\lambda}\lambda}{\lfloor 1}} = \frac{\frac{1}{5}}{3/10} \Rightarrow \frac{\lambda}{2} = \frac{2}{3}$$

$$\Rightarrow \lambda = \frac{4}{3}$$

The poisson distribution $P(X=x) = \frac{e^{-4/3}\left(\frac{4}{3}\right)^x}{\lfloor x}$

$P(X=0) = \frac{e^{-4/3}\left(\frac{4}{3}\right)^0}{\lfloor 0}$, $P(X=3) = \frac{e^{-4/3}\left(\frac{4}{3}\right)^3}{\lfloor 3}$

→ Fit a poisson distribution for the following observations

| $x$: | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $f$: | 142 | 156 | 64 | 27 | 5 | 1 |

To find $\lambda$ = Mean

| $x$ | $f$ | $fx$ |
|---|---|---|
| 0 | 142 | 0 |
| 1 | 156 | 156 |
| 2 | 64 | 128 |
| 3 | 27 | 81 |
| 4 | 5 | 20 |
| 5 | 1 | 5 |

$$\bar{x} = \lambda = \frac{\Sigma fx}{\Sigma f} = \frac{400}{400}$$

$$\boxed{\lambda = 1}$$

$\Sigma f = 400$ , $\Sigma fx = 400$

$\therefore$ The Poisson distribution is $p(X=x) = \frac{e^{-\lambda} \lambda^x}{\lfloor x}$ $x = 0, 1, 2..$

$\therefore p(X=x) = \frac{e^{-1}(1)^x}{\lfloor x}$

Theoretical Distribution

$\cancel{P(X(F(x)))}\ f(x) = N \cdot P(X=x)$

$f(x) = 400 \times \frac{e^{-1}(1)^x}{\lfloor x}$

$f(0) = 400 \times \frac{e^{-1}(1)^0}{\lfloor 0} = 147.1517 \simeq 147$

$f(1) = \frac{400 \times e^{-1}(1)^1}{\lfloor 1} = 147.1517 \simeq 147$

$f(2) = \frac{400 \times e^{-1}(1)^2}{\lfloor 2} = 73.57 \simeq 74$

$f(3) = \frac{400 \times e^{-1}(1)^3}{\lfloor 3} = 24.52 \simeq 25$

$f(4) = \frac{400 \times e^{-1}(1)^4}{\lfloor 4} = 6.133 \simeq 6$

$f(5) = \frac{400 \times e^{-1}(1)^5}{\lfloor 5} = 1.226 \simeq 1$

Hence theoretical distribution is

| $x$: | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $f(x)$ | 147 | 147 | 74 | 25 | 6 | 1 |

Geometric Distribution (We must get success) Don't include Zero.

A ~~geometric~~ random variable $X$ is said to have a geometric distribution, if it assumes only non-neg values and its p.m.f is given by

$P(X=x) = q^{x-1} P$ ; $x = 1, 2, 3..$

which gives the probability that the $1^{st}$ success occurs only at the $x^{th}$ trial and the proceeding $(x-1)$ are failure. 36

**Result**: Mean $= \frac{p}{q}$ ; Variance $= \frac{p}{q^2}$

2M

(*) **Memory less property**

$$P(X>s+t / X>t) = P(X>s)$$

→ The probability of student passing a subject is 0.8. what is the probability that he will pass the subject

i) on his third attempt ?

ii) before third attempt ?

Let $X$ denotes no. of trials req to get success.

Given $p = 0.8$, $q = 1-p = 1-0.8 = 0.2$

By Geometric distribution

$$P(X=x) = q^{x-1} p = (0.2)^{x-1}(0.8)$$

$$x = 1, 2, 3, \ldots$$

i) $P(X=3) = (0.2)^{3-1} \times 0.8 = \cancel{0.0064}\ 0.032$

ii) $P(X<3) = P(X=1) + P(X=2)$

$$= (0.2)^{1-1}(0.8) + (0.2)^{2-1} \times (0.8)$$

$$= 1 \times 0.8 + 0.2 \times 0.8 = 0.96$$

→ If the probability that an applicant for a driver's ~~becaus~~ license will pass the road test on any given trial is 0.8. What is the probability that he will finally pass the test a) on the $4^{th}$ trial.

b) in fewer than 4 trials ?

Let x denote no. of trials req to get ~~the~~ pass the test.



Given: $p = 0.8 \quad q = 1-p = 1-0.8 = 0.2$

The G.D is $P(X=x) = q^{x-1} p \quad x = 1, 2, 3, \ldots$

a) $P(X=4) = (0.2)^{4-1}(0.8)$

$= (0.2)^3 (0.8) = 0.0064$

~~b) $P(X=4) = (0.2)^{4-1}(0.8) = 0.992$~~

b) $P(X=3) + P(X=2) + P(X=1) = P(X<4)$

$= 0.032 + 0.96$

$= 0.992$

→ In a certain factory turning razar blades there is a small chance $\frac{1}{500}$ for any blades to be defective. The blades are in packet of 10. To calculate the approximate no. of packets containing i) No defective ii) one defective iii) 2 defective blades in a consignment of 10,000 packets.

Let X denote event of getting defective item.

The poisson dist $P(X=x) = \frac{e^{-\lambda} \cdot \lambda^x}{\lfloor x}, \quad x = 0, 1, 2, \ldots$

Given: $p = \frac{1}{500}, \; n = 10, \; \lambda = np = 10 \times \frac{1}{500} = 0.2$

$P(X=x) = \frac{e^{-(0.02)} (0.02)^x}{\lfloor x}, \quad x = 0, 1, 2, \ldots$

i) $P(X=0) = \frac{e^{-(0.02)} (0.02)^0}{\lfloor 0} = 0.9802$.

Out of 10,000 packets, the prob. of no detective

$= 10{,}000 \times 0.9802 = 9802$.

ii) $P(X=1)$ iii) $P(X=2)$

** → It is known that the probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20. Find the no. of packets containing atleast, exactly and atmost 2 defective items in a consignment of 1000 packets by using Binomial and poisson distribution.

Sol - Let x denote the event of getting defective items.

$p = 0.05, \; n = 20 \qquad p+q=1$

$q = 1-p$

$= 1-0.05$

$q = 0.95$

By Binomial distribution: $P(X=x) = {}^nC_x \, p^x q^{n-x}$ $\quad x = 0,1,2,\dots n$

$= 20C_x (0.05)^x (0.95)^{20-x} \quad x = 0,1,2,\dots$

i) $P[\text{atleast 2 def}] = P[X \geq 2]$

$= 1 - P[X < 2]$

$= 1 - [P(X=0) + P(X=1)]$

$= 1 - \left[{}^{20}C_0 (0.05)^0 (0.95)^{20-0} + {}^{20}C_1 (0.05)^1 (0.95)^{20-1}\right]$

$= 1 - \left[1 \times 1 \times (0.95)^{20} + 20 \times (0.05)(0.95)^{19}\right]$

$= 1 - (0.95)^{19}[0.95 + 1]$

$= 0.26416$

∴ Out of 1000, the no. of packets containing atleast 2 defective $= 1000 \times 0.26416 = 264.16$

$\simeq 264$

ii) P[Exactly 2 defective] = P(X = 2)

$$= {}^{20}C_2 (0.05)^2 (0.95)^{20-2}$$

$$= 0.188676$$

out of 1000, the no. of packets containing exactly 2 defective $= 1000 \times 0.188676 \simeq 189$

iii) P[atleast 2 def] = $P[X \le 2]$

$$= P[X=0] + P[X=1] + P[X=2]$$

$$= {}^{20}C_0 (0.05)^0 (0.95)^{20-0} + {}^{20}C_1 (0.05)^1 (0.95)^{19}$$

$$+ {}^{20}C_2 (0.05)^2 (0.95)^{20-2}$$

$$= 0.9246$$

Using $\lambda$ (poisson distribution)

$\lambda = np = 20 \times 0.05$

$$\boxed{\lambda = 1}$$

$\therefore$ The poisson distribution is $P(X=x) = \dfrac{e^{-\lambda} \lambda^x}{\lfloor x}$, $x = 0, 1, 2, \ldots$

$$= \frac{e^{-1} (1)^x}{\lfloor x} = \frac{e^{-1}}{\lfloor x}$$

P[atleast 2 def]

$$= P[X \ge 2] = 1 - P[X < 2] = 1 - (P[x=0] + P[x=1])$$

$$= 1 - \left( \frac{e^{-1}}{\lfloor 0} + \frac{e^{-1}}{\lfloor 1} \right)$$

$$= 1 - 2e^{-1}$$

$$= 0.26416$$

**Continuous Distribution**

1) Uniform Distribution (or) Rectangular Distribution

A continuous random variable X is said to follow a uniform distribution over an interval (a, b) if it's

Probability density function is given by

$$f(x) = \begin{cases} \frac{1}{b-a} & ; a<x<b \\ 0 & ; \text{otherwise} \end{cases}$$

Result: Mean $= \frac{a+b}{2}$

Variance $= \frac{1}{12}(a-b)^2$

→ If X is uniformly distributed over $(\overset{a}{0}, \overset{b}{10})$ find
i) $P(X<4)$ ii) $P(X>6)$ iii) $P(2<X<5)$.

Since X is uniform distribution

$$\therefore f(x) = \frac{1}{b-a}; \quad a<x<b.$$

$$f(x) = \frac{1}{10-0}; \quad 0<x<10$$

$$\therefore \boxed{f(x) = \frac{1}{10}; \quad 0<x<10}$$

i) $P(X<4) = \int_0^4 f(x)\,dx$

$$= \int_0^4 \frac{1}{10}\,dx = \frac{1}{10}[x]_0^4 = \frac{4}{10} = \frac{2}{5}$$

ii) $P(X>6) = \int_6^{10} f(x)\,dx = \int_6^{10} \frac{1}{10}\,dx = \frac{1}{10}[x]_6^{10}$

$$= \frac{10-6}{10} = \frac{4}{10} = \frac{2}{5}$$

iii) $P(2<X<5) = \int_2^5 f(x)\,dx = \int_2^5 \frac{1}{10}\,dx = \frac{1}{10}[x]_2^5$

$$= \frac{5-2}{10} = \frac{3}{10}$$

→ A random variable X has a uniform distribution over the interval $(\overset{a}{-3}, \overset{b}{3})$. Compute
i) $P(X=2)$ ii) $P(|x-2|<2)$ iii) Find K such that $P(X>K) = \frac{1}{3}$.

Sol $f(x) = \frac{1}{3+3} = \frac{1}{6}$

i) $P(X=2) = 0$ $[\because X$ is variance over $(a,b)]$

ii) $P[|X-2|<2] = P[-2<(x-2)<2]$

$= P[2-2<x<2+2]$

$= P[0<x<4]$

$= \int_0^4 f(x)\,dx$

$= \int_0^4 \frac{1}{6}\,dx = \frac{1}{6}[x]_0^4 = \frac{4}{6} = \frac{2}{3}$

iii) $P(x>K) = \frac{1}{3}$

$\int_K^3 f(x)\,dx = \frac{1}{3}$

$\int_K^3 \frac{1}{6}\,dx = \frac{1}{3}$

$\frac{1}{6}[x]_K^3 = \frac{1}{3}$

$\frac{3-K}{6} = \frac{1}{3}$

$9-3K = 6$

$3K = 3$

$\boxed{K=1}$

* → Buses arrive at a specified stop at 15 mins interval starting at 6 AM (i.e) they arrive at 6 AM, 6:15 AM, 6:30 AM and so-on. If a passenger arrives at the stop at a time that is uniformly distributed b/w 6 and 6:30 AM.

Find the probability that he waits

i) Less than 5 min

ii) More than 10 min for a bus.

Let $X$ denote the no. of mins after 6 AM that the passenger arrives at the stop.

Since $X$ is uniformly distributed over $(0,30)$

$f(x) = \frac{1}{b-a} = \frac{1}{30-0} = \frac{1}{30}$ ; $0 < x < 30$.

Since the buses arrive at 15 min interval starting with 6 AM, a passenger has to wait less than 5 min. If he comes to stop b/w 6:10 and 6:15 (or) b/w 6:25 and 6:30.

Required probability $= P(10 < X < 15) + P(25 < X < 30)$

$$= \int_{10}^{15} f(x)\,dx + \int_{25}^{30} f(x)\,dx$$

$$= \int_{10}^{15} \frac{1}{30}\,dx + \int_{25}^{30} \frac{1}{30}\,dx$$

$$= \frac{1}{30}[x]_{10}^{15} + \frac{1}{30}[x]_{25}^{30}$$

$$= \frac{5}{30} + \frac{5}{30}$$

$$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

The passenger has to wait more than 10 min if he comes to stop b/w 6 and 6:05 or b/w 6:15 and 6:20

Required probability $= P(0 < X < 5) + P(15 < X < 30)$

$$= \int_{0}^{5} f(x)\,dx + \int_{15}^{20} f(x)\,dx$$

$$= \int_{0}^{5} \frac{1}{30}\,dx + \int_{15}^{20} \frac{1}{30}\,dx$$

$$= \frac{1}{30}[x]_{0}^{5} + \frac{1}{30}[x]_{15}^{20}$$

$$= \frac{5}{30} + \frac{5}{30}$$

$$= \frac{1}{6} + \frac{1}{6}$$

$$= \frac{1+1}{6} = \frac{2}{6} = \frac{1}{3}$$

## Exponential Distribution

A continuous random variable $X$ is said to follow an exponential distribution with parameter $\lambda > 0$, if it's probability density function is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda} & ; x \geq 0 \\ 0 & ; \text{otherwise} \end{cases}$$

Result :- Mean $= \frac{1}{\lambda}$

Variance $= \frac{1}{\lambda^2}$

Note :- $f(x)$ is exponential $\Rightarrow \int_0^\infty f(x)\,dx = 1$

→ The milage which car owners get with a certain kind of radial tire is a random variable having an exponential distribution with mean 40,000 Km. Find the probability that one of these tires will last

i) Atleast 20,000 Km.

ii) Atmost 30,000 Km.

Since $X$ is exponentially distributed

$$\therefore f(x) = \lambda e^{-\lambda};\ x \geq 0$$

Given

Mean = 40,000

$$\frac{1}{\lambda} = 40,000$$

$$\Rightarrow \lambda = \frac{1}{40,000}$$

$$\therefore \boxed{f(x) = \frac{1}{40,000} e^{-\frac{1}{40,000}x}}, \quad x \geq 0$$

i) $P(\text{atleast } 20,000) = P(X > 20,000)$

$$= \int_{20,000}^{\infty} f(x)\,dx = \frac{1}{40,000}\left[x\right]_{20,000}^{\infty}$$

$$= \int_{20,000}^{\infty} \frac{1}{40,000} e^{-\frac{1}{40,000}x} \, dx$$

$$= \frac{1}{40,000} \left[ \frac{e^{-\frac{1}{40,000}}}{-\frac{1}{40,000}} \right]_{20,000}^{\infty}$$

$$= -\left[ e^{-\infty} - e^{-\frac{20,000}{40,000}} \right]$$

$$= -\left[ 0 - e^{-0.5} \right]$$

$$= e^{-0.5} = 0.6065$$

ii) $P[\text{atmost } 30,000] = P[X \leq 30,000]$

$$= \int_0^{30,000} f(x) \, dx$$

$$= \int_0^{30,000} \frac{1}{40,000} e^{-\frac{1}{40,000}x} \, dx$$

$$= \frac{1}{40,000} \left[ \frac{e^{-\frac{1}{40,000}x}}{-\frac{1}{40,000}} \right]_0^{30,000}$$

$$= -\left[ e^{-\frac{30,000}{40,000}} - e^{0} \right] = -\left[ e^{-3/4} - 1 \right]$$

$$= 1 - e^{-3/4} = 0.5276.$$

→ The time (in hours) req to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$. (Avg time taken to repair machine is 2 hrs).

a) What is the probability that the repair exceeds 2 hrs?

b) What is the conditional probability that a repair takes atleast 10 hrs given that it's duration exceeds 9 hrs.

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

Since $X$ is exponentially distributed

$\Rightarrow$ It's pdf is given by,

$$f(x) = \lambda e^{-\lambda x} ; x \geq 0$$

$$\int e^{ax} dx = \frac{e^{ax}}{a} + c$$

$$= \frac{1}{2} e^{-\frac{1}{2}x} ; x \geq 0$$

a) $P(X>2) = \int_2^\infty f(x) dx = \frac{1}{2} \int_2^\infty e^{-\frac{1}{2}x} dx$.

$$= \cancel{\frac{1}{2}} \left[ \frac{e^{-\frac{1}{2}x}}{-\cancel{\frac{1}{2}}} \right]_2^\infty$$

$$= -\left[ e^{-\infty} - e^{-\frac{1}{2}(2)} \right]$$

$$= -\left[ 0 - e^{-1} \right]$$

$$= e^{-1}$$

$$= 0.3679$$

b) $P[X>10 / X>9] = P(X>9+1 / X>9)$

by memoryless property $\Longrightarrow P(X>1) = \int_1^\infty f(x) dx$

$$= \int_1^\infty \frac{1}{2} e^{-\frac{1}{2}x}$$

$$= \cancel{\frac{1}{2}} \left[ \frac{e^{-\frac{1}{2}x}}{-\cancel{\frac{1}{2}}} \right]_1^\infty$$

$$= -\left[ e^{-\infty} - e^{-1/2} \right]$$

$$= -\left[ 0 - e^{-0.5} \right]$$

$$= e^{-0.5} = 0.6065$$

$\rightarrow$ The daily consumption of milk in excess of 20,000 gallons is appr-exponentially distributed with $\lambda = \frac{1}{3000}$.

The city has a daily stock of 35,000 gallons. What is the probability that of two days selected at random, the

Stock is insufficient for both days? 

Let $x$ denote the daily consumption of milk in excess of 20,000 ltr.

Since $X$ is given exponentially distributed

$\Rightarrow$ It's pdf is given by

$$f(x) = \lambda e^{-\lambda x} ; x \geq 0$$

Given $\lambda = \frac{1}{3000}$

$$f(x) = \frac{1}{3000} e^{-\frac{1}{3000}x} ; x > 0$$

Let $y$ denotes the daily assumption, then

$$X = Y - 20,000$$

P[The stock is insufficient on any day] = P(The consumption exceeds 35,000 lts)

$$= P[Y \rightarrow 35,000]$$

$$= P[X + 20,000 > 35,000]$$

$$= P[X > 15,000]$$

$$= \int_{15,000}^{\infty} f(x)\, dx$$

$$= \int_{15,000}^{\infty} \frac{1}{3000} e^{-\frac{1}{3000}x} dx$$

$$= \frac{1}{\cancel{3000}} \left[ \frac{e^{-\frac{1}{3000}}}{-\frac{1}{\cancel{3000}}} \right]_{15,000}^{\infty}$$

$$= e^{-5}$$

Hence, P[The stock is sufficient any two days]

$$= e^{-5} \times e^{-5}$$

$$= e^{-10}$$

## Normal Distribution

A continuous r.v $x$ is said to follow a normal distrib. with parameters $\mu$ and $\sigma$ if its pdf is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} ; \quad -\infty < x < \infty$$
$$-\infty < \mu < \infty$$

Symbolically $X \sim N(\mu, \sigma^2)$

Note: Mean $= \mu$, S.D $= \sigma$.

Standard normal probability distribution.

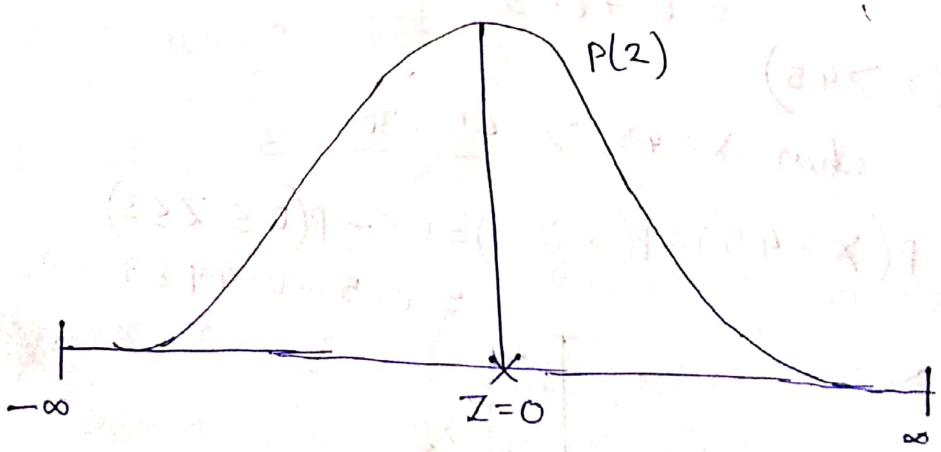
If $X$ is normally distributed with mean $\mu$ and S.D $\sigma$, then $\boxed{Z = \frac{X-\mu}{\sigma}}$ is called standard normal r.v.

Properties of Normal distrib

i) The normal curve is symmetrical with $p=q$.

ii) The normal curve is a single peaked curve.

iii) The mean, median & mode coincide and lower and upper quartiles are equidistant from the median.




→ If $X$ is normal variable with mean 30 and S.D 5. Find the probability such that

i) $P(26 \le X \le 40)$ ii) $P(X \ge 45)$

iii) $P(|X-30| > 5)$

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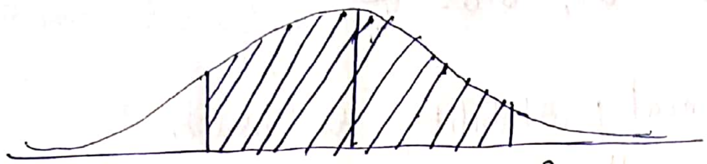
Given: $X \sim N(\mu, \sigma^2)$

Mean = 30 | S.D = 5

$\mu = 30$ | $\sigma = 5$

The normal r.v $Z = \frac{X-\mu}{\sigma} = \frac{X-30}{5}$

i) $P(26 \leq X \leq 40)$




when $X = 26$, $Z = \frac{26-30}{5} = -0.8$

when $X = 40$, $Z = \frac{40-30}{5} = 2$

$P(26 \leq X \leq 40) = P(-0.8 \leq Z \leq 2)$

$= P(-0.8 \leq Z \leq 0) + P(0 \leq Z \leq 2)$

$= P(0 \leq Z \leq 0.8) + P(0 \leq Z \leq 2)$

$= 0.2881 + 0.4772$

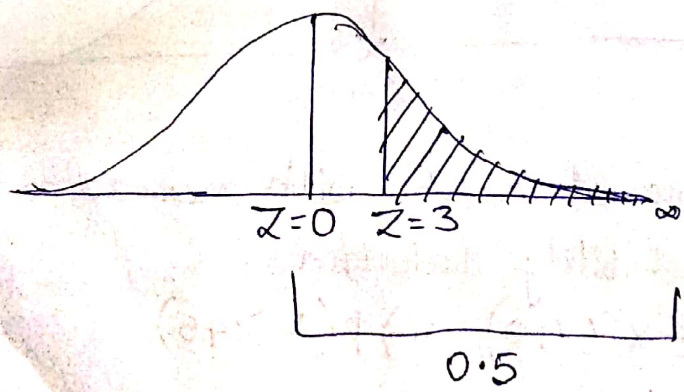
$= 0.7653$

ii) $P(X \geq 45)$

when $X = 45$, $Z = \frac{45-30}{5} = 3$

$\therefore P(X \geq 45) = P(Z \geq 3) = 0.5 - P(0 \leq Z \leq 3)$

$= 0.5 - 0.49 87$



iii) $P(|X-30|>5) = 1-P(|X-30|\leq 5)$

$= 1-P(-5\leq (X-30)\leq 5)$

$= 1-P(25\leq X\leq 35)$

when $X=25$, $Z=\frac{25-30}{5}=-1$

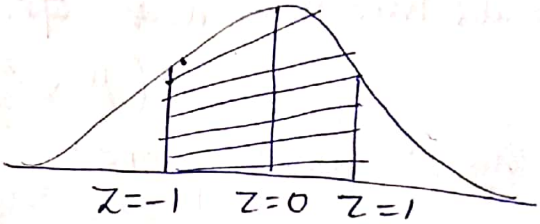
$X=35$, $Z=\frac{35-30}{5}=1$

$= 1-P(-1\leq Z\leq 1)$

$= 1-2P(0\leq Z\leq 1)$

$= 1-2\times 0.3413$

$= 0.3174$




→ The weekly wages of 1000 workers are normally distributed around a mean of Rs. 70 and S.D of Rs. 5. Estimate the no. of workers whose weekly wages will be

i) b/w Rs. 69 and Rs. 72 $[P(64\leq X\leq 72)]$

ii) < Rs. 69 $(P(X\leq 69))$

iii) more than Rs. 72 $(P(X\geq 72))$

Given $\mu=70$, $\sigma=5$

$\therefore Z=\frac{X-M}{\sigma}=\frac{X-70}{5}$

i) $P(69\leq X\leq 72)=P(-0.2\leq Z\leq 0.4)=P(-0.2\leq Z\leq 0)+P(0\leq Z\leq 0.4)$

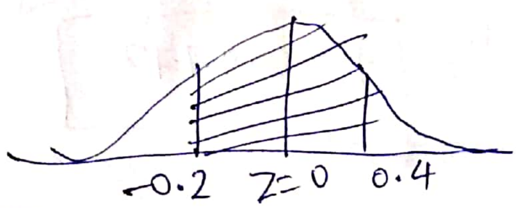
when $X=69$, $Z=\frac{69-7}{5}=-0.2$

$X=72$, $Z=\frac{72-70}{5}=0.4$

$= P(0\leq Z\leq 0.2)+P(0\leq Z\leq 0.4)$

$= 0.0793+0.1554$

$= 0.2347$.



Hence, the no. of workers getting salary b/w

69 & 72 $= 1000 \times 0.2347$

$= 234.7 \simeq 235$

→ The marks obtained by a no. of students in a certain Subject are approx normally distributed with mean 65 and SD 5. If 3 students are selected at random from this group. what is the probability that atleast 1 of them would have score above 75?

$X \sim N(\mu, \sigma^2)$

Given: $\mu = 65$ | SD $= 5$, $\sigma = 5$

∴ The normal variable $Z = \frac{X - \mu}{\sigma}$

$$\boxed{Z = \frac{X - 65}{5}}$$

i) P(a student score above 75) $= P(X > 75)$

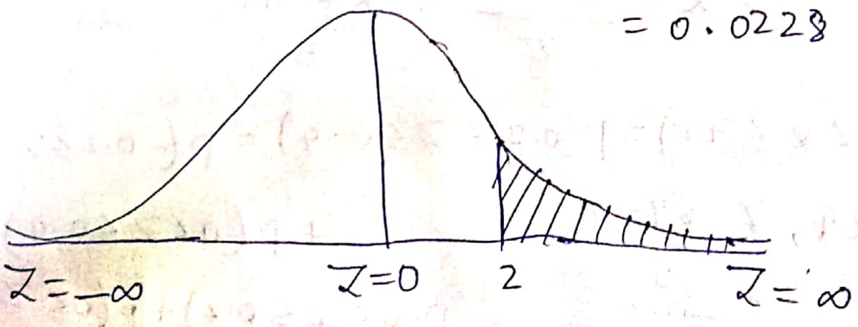
When $X = 75$, $Z = \frac{75 - 65}{5} = 2$

$= P(Z > 2)$

$\phi(Z) = 0.5 - P(0 < Z < 2)$

$= 0.5 - 0.4772$

$= 0.0228$




$p = 0.0228$, $n = 3$

$q = 1 - p = 1 - 0.0228$

$$\boxed{q = 0.9772}$$

∴ The binomial distribution

$$P(Y=y) = {}^{n}C_y\, p^y q^{n-y} \qquad y=0,1,2,\dots n$$

$$= {}^{3}C_y (0.0228)^y (0.9772)^{2-y}$$

Hence, $P$(atleast 1 student score above 75)

$$= P(Y \geq 1)$$

$$= 1 - P(Y<1)$$

$$= 1 - P(Y=0)$$

$$= 1 - {}^{3}C_0 (0.0225)^0 \times (0.9972)^{2-0}$$

$$= 1 - 1\times 1 \times (0.9772)^3$$

→ In a normal distrib 31% of the items are under 45 and 8% are over 64. Find Mean and standard deviation.

$P(-\infty < Z < Z_1) = 0.31$

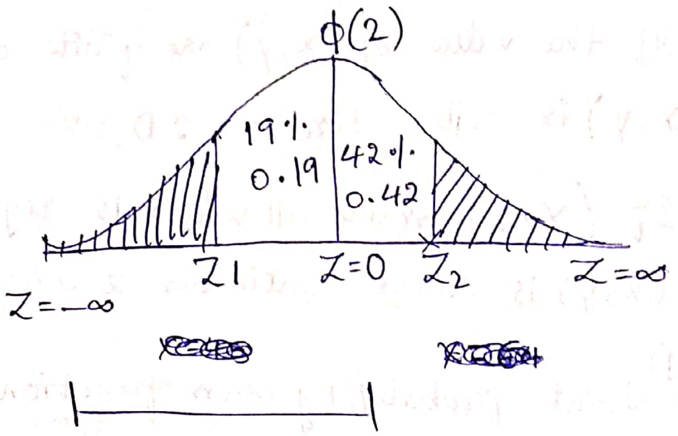
$P(Z_2 < Z < \infty) = 0.08$

$P(Z_1 < Z < 0) = 0.19$

$\Rightarrow Z_1 = -0.49$ (from table)

$P(0 < Z < Z_2) = 0.42$

$\Rightarrow Z_2 = 1.4$




$$Z = \frac{X-\mu}{\sigma}$$

when $X=45$, $Z_1 = \dfrac{45-\mu}{\sigma}$

$$-0.49 = \frac{45-\mu}{\sigma}$$

$$\mu - 0.49\sigma = 45 \quad \text{—①}$$

when $X=64$, $Z_2 = \dfrac{64-\mu}{\sigma}$ $\qquad 1.4 = \dfrac{64-\mu}{\sigma}$

$$\mu + 1.4\sigma = 64 \quad \text{—②}$$

$\sigma = 9.7436 \simeq 10 \qquad \mu = 49.9 \simeq 50$

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