Data Science

– SCSA 3016

Name: Mohnish Devaraj

Reg No: 39110636

Section: C1 (III Year)

Assignment-1

PART-A

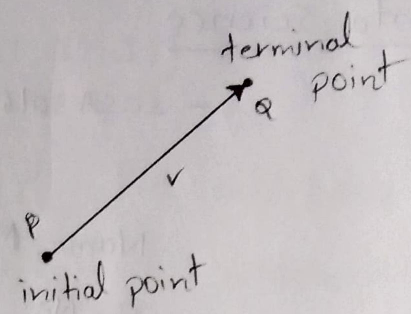
1. -2 det(A)
2. True
3. Square matrices only
4. Overdetermined
5. Plane

~~PART-B~~

Short Answers

1. A vector is an object that has both a magnitude and a direction. Geometrically, we can picture a vector as a directed line segment, whose length is the magnitude of the vector and with an arrow indicating the direction.

Vectors can be represented geometrically by arrows (directed line segments). The arrowhead indicating the direction of the vector, and the length of the arrow describes the magnitude of the vector.



A vector with initial point P (the tail of the arrow) and terminal point Q (the tip of the arrowhead) can be represented by

$$\overrightarrow{PQ}, v, \text{ or } \vec{v}.$$

(2) $A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$

$= (1 * 5) - (4 * 2)$

$= 5 - 8$

$\Rightarrow -3$ //

(4) <u>Null Space</u> :

The null space of any matrix A consist of all vectors B such that AB = 0 and B is not zero. It can also be thought as the solution obtained from AB = 0 where A is known matrix of size m x n and B is matrix to be found of size n x k. The size of the null space of the matrix provides us with the number of linear relations among attributes.

(5) The equation of a straight line is usually written:

$$y = mx + c$$

$y \rightarrow$ how far towards y-axis

$x \rightarrow$ how far towards x-axis

$m \rightarrow$ slope or gradient

$b \rightarrow$ value of y when $x = 0$

(3)

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 5 & 0 \\ 1 & 3 & 2 \end{bmatrix}$$

order, $n = 3$

Method 1

$$|A| = 1(10-0) - 2(6-0) + 1(9-5)$$

$$\Rightarrow 10 - 12 + 4$$

$$\Rightarrow 14 - 12$$

$$\Rightarrow 2 \neq 0$$

Hence,

the rank of a matrix, $r = 3$

Method 2

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 5 & 0 \\ 1 & 3 & 2 \end{bmatrix}$$

$R_1 \rightarrow 3R_1$

$$\begin{bmatrix} 3 & 6 & 3 \\ 3 & 5 & 0 \\ 1 & 3 & 2 \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -3 \\ 1 & 3 & 2 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -3 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_2 \rightarrow -R_2$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -3 \\ 0 & 0 & -2 \end{bmatrix}$$

Hence the rank of the matrix is 3.

Long Answer

1) $A = \begin{bmatrix} -6 & 4 \\ 3 & 5 \end{bmatrix}$

$\det[A - \lambda I] = 0$

$\left| \begin{bmatrix} -6 & 4 \\ 3 & 5 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$

$\left| \begin{bmatrix} -6 & 4 \\ 3 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$

$\left| \begin{bmatrix} -6-\lambda & 4 \\ 3 & 5-\lambda \end{bmatrix} \right| = 0$ —— (1)

$(-6-\lambda)(5-\lambda) - (4)(3) = 0$

$-30 + 6\lambda - 5\lambda + \lambda^2 - 12 = 0$

$\lambda^2 + \lambda - 42 = 0$

$\lambda^2 + 7\lambda - 6\lambda - 42 = 0$

$\lambda(\lambda+7) - 6(\lambda+7) = 0$

$(\lambda-6)(\lambda+7) = 0$

$\lambda = 6, -7$

The Eigen values of the matrix is

6, -7

$\underline{\lambda_1 = +6}$

from ①

$$\begin{bmatrix} -6-\lambda & 4 \\ 3 & 5-\lambda \end{bmatrix}$$

Sub λ in ①

$$\begin{bmatrix} -12 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-12x + 4y = 0$$

$$3x - y = 0$$

$$3x - y = 0$$

$$3x = y$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$\underline{\lambda_2 = -7}$

$$\begin{bmatrix} -6-\lambda & 4 \\ 3 & 5-\lambda \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 \\ 3 & 12 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x + 4y = 0$$

$$3x + 12y = 0$$

$$x + 4y = 0$$

$$x = -4y$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$$

$\therefore$ The Eigen values & Eigen vectors are

$$\lambda_1 = 6, \quad \lambda_2 = -7$$

$$\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \end{bmatrix}$$