

## WORKED OUT EXAMPLES

### OPERATOR PRECEDENCE PARSING

**1. Given a grammar,**

$$E \rightarrow E + T \mid T$$

$$T \rightarrow T * F \mid F$$

$$F \rightarrow (E) \mid id$$

**Input String : id + id \* id**

**Verify whether the given input string is valid or not using Operator Precedence parsing method.**

**Soln:**

The given grammar is an operator grammar

$$E \rightarrow E + T$$

$$E \rightarrow T$$

$$T \rightarrow T * F$$

$$T \rightarrow F$$

$$F \rightarrow (E)$$

$$F \rightarrow id$$

1. COMPUTATION OF LEADING
2. COMPUTATION OF TRAILING
3. CONSTRUCTION OF PRECEDENCE TABLE
4. PARSING THE GIVEN STRING

### (1) COMPUTATION OF LEADING (L—R)

**Leading (E) = { +, Leading(T) } = { +, \*, (, id }**

**Leading (T) = { \*, Leading(F) } = { \*, (, id }**

**Leading (F) = { (, id }**

### (2) COMPUTATION OF TRAILING (R---L)

**Trailing (E) = { +, Trailing (T) } = { +, \*, ), id }**

**Trailing (T) = { \*, Trailing (F) } = { \*, ), id }**

**Trailing (F) = { ), id }**

### (3) COMPUTATION OF PRECEDENCE TABLE

**Rules :**

**1. \$ <. Leading (Starting NT)**

**\$ <. Leading (E)**

**\$ <. { +, \*, (, id }**

**2. Trailing (Starting NT) .> \$**

**Trailing(E) .> \$**

**{ +, \*, ), id } .> \$**

**3. On the RHS of the production rule,**

**(a) Terminal NT => Terminal <. Leading (NT)**

**+ T => + <. Leading(T) => + <. { \*, (, id }**

**\* F => \* <. Leading(F) => \* <. { (, id }**

**( E => ( <. Leading(E) = ( <. { +, \*, (, id }**

**(b) NT Terminal => Trailing (NT) .> Terminal**

**E + => Trailing(E) .> +**

**=> { +, \*, ), id } .> +**

$T * \Rightarrow \text{Trailing}(T) .> *$

$\Rightarrow \{ *, ), id \} .> *$

$E ) \Rightarrow \text{Trailing}(E) .> )$

$\Rightarrow \{ +, *, ), id \} .> )$

(c)  $\text{Ter1 NT Ter2} \Rightarrow \text{Ter1} = \text{Ter2}$

( = )

	+	*	id	(	)	\$
+	.>	<.	<.	<.	.>	.>
*	.>	.>	<.	<.	.>	.>
id	.>	.>	e	e	.>	.>
(	<.	<.	<.	<.	=	e
)	.>	.>	e	e	.>	.>
\$	<.	<.	<.	<.	e	Accept

(4) Parsing the string  $\rightarrow id+id*id$

Actions : push, pop, accept, error

Rules:

If the tos <. input, push

If the tos .> input, pop  $\rightarrow$  continue popping the symbols until the tos is related by <. to the recently popped symbol

Stack	Input String	Action
\$	id+id*id \$	\$ <.id [Push]
\$id	+id*id \$	id .> + [Pop]
\$	+id*id \$	\$ <. + [Push]
\$+	id*id \$	+ <. id [Push]
\$+id	*id \$	id .>* [Pop]
\$+	*id \$	+ <. * [Push]
\$+*	id \$	* <. id [Push]
\$+*id	\$	id .> \$ [Pop]
\$+*	\$	*.>\$ [Pop]
\$+	\$	+ .> \$ [Pop]
\$	\$	Accept

String : **id\*+id**

Stack	Input String	Action
\$	id*+id\$	\$ <.id [Push]
\$id	*+id\$	id .> * [Pop]
\$	*+id\$	\$ <. * [Push]
\$*	+id\$	*.>+ [Pop]
\$	+id\$	\$ <.+ [Push]
\$+	id\$	+ <.id [Push]
\$+id	\$	id .>\$ [Pop]
\$+	\$	+.>\$ [Pop]
\$	\$	Accept

String : id id + id

Stack	Input String	Action
\$	id id + id	\$ <.id [Push]
\$id	id + id	Error

## Operator Precedence Graph

### Precedence Table

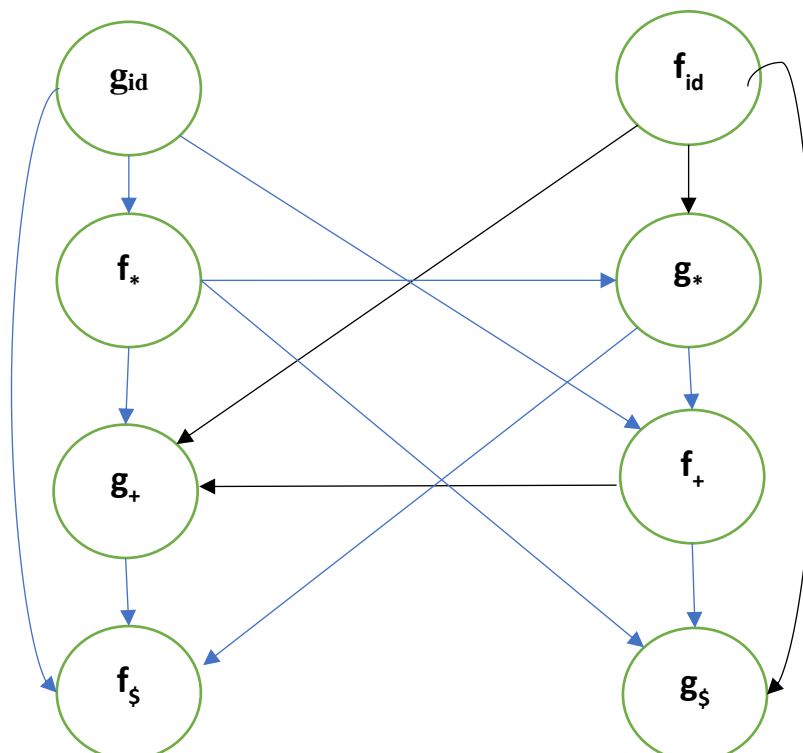
$\downarrow$   
f

$\longrightarrow$  g

	id	+	*	\$
id		>	>	>
+	<	>	<	>
*	<	>	>	>
\$	<	<	<	

fid, f+, f\*, f\$, gid, g+, g\*, g\$

### Operator Precedence Graph



$f_{id} \rightarrow g_{\$} (1)$

$f_{id} \rightarrow g_{*} \rightarrow f_{+} \rightarrow g_{\$} (3)$

$f_{id} \rightarrow g_{+} \rightarrow f_{\$} (2)$

$f_{id} \rightarrow g_{*} \rightarrow f_{\$} (2)$

$f_{id} \rightarrow g_{*} \rightarrow f_{+} \rightarrow g_{+} \rightarrow f_{\$} (4)$

$g_{id} \rightarrow f_{\$} (1)$

$g_{id} \rightarrow f_{*} \rightarrow g_{+} \rightarrow f_{\$} (3)$

$g_{id} \rightarrow f_{+} \rightarrow g_{\$} (2)$

$g_{id} \rightarrow f_{+} \rightarrow g_{+} \rightarrow f_{\$} (3)$

$g_{id} \rightarrow f_{*} \rightarrow g_{*} \rightarrow f_{\$} (3)$

$g_{id} \rightarrow f_{*} \rightarrow g_{*} \rightarrow f_{+} \rightarrow g_{+} \rightarrow f_{\$} (5)$

## Function Table

	id	+	*	\$
f	4	2	4	0
g	5	1	3	0

## Precedence between 2 operators

+      \*

f+      g\*

2   <   3

+   < .   \*

\*      id

f\*      gid

4   <   5

\*   < . id

## Predictive Parsing

1. Given a grammar,

$E \rightarrow E + T \mid T$

$T \rightarrow T * F \mid F$

$F \rightarrow (E) \mid id$

**Input String: id + id \* id**

Verify whether the given input string is valid or not using Predictive parsing method.

Soln:

**Since there is left recursion in the grammar, eliminate the left recursion**

**$E \rightarrow TE'$**

**$E' \rightarrow +TE'$**

**$E' \rightarrow \epsilon$**

**$T \rightarrow FT'$**

**$T' \rightarrow *FT'$**

**$T' \rightarrow \epsilon$**

**$F \rightarrow (E)$**

**$F \rightarrow id$**

**Steps :**

1. Computation of FIRST
2. Computation of FOLLOW
3. Construction of Predictive Parsing Table
4. Parsing the String

### **FIRST**

**$FIRST(E) = \{FIRST(T)\} = \{FIRST(F)\} = \{ (, id \}$**

**$FIRST(E') = \{ +, \epsilon \}$**

**$FIRST(T) = \{FIRST(F)\} = \{ (, id \}$**

**$FIRST(T') = \{ *, \epsilon \}$**

**$FIRST(F) = \{ (, id \}$**

### **FOLLOW**

**Follow has to be computed for all the non-terminals in the grammar.**

- **While finding the FOLLOW (Starting NT), include \$ in the set.**
- **Then you have to inspect the RHS of the production rules..**
  - **If followed by the NT, if there is a terminal, include it in the set**
  - **If followed by the NT, if there is a NT, then find the FIRST(NT) and include it in the set.**
  - **If followed by the NT, if there is nothing ( $\epsilon$ ), find the FOLLOW(LHS NT) and include it in the set.**

**$FOLLOW(E) = \{ \$, ) \}$**

**$FOLLOW(E') = \{ FOLLOW(E) \} = \{ \$, ) \}$**

**$FOLLOW(T) = \{ FIRST(E') \} = \{ +, \epsilon \} = \{ +, FOLLOW(E') \} = \{ +, \$, ) \}$**

$\text{FOLLOW}(T') = \{\text{FOLLOW}(T)\} = \{+, \$, )\}$

$\text{FOLLOW}(F) = \{\text{FIRST}(T')\} = \{*, \epsilon\} = \{*, \text{FOLLOW}(T')\} = \{*, +, \$, )\}$

**Construction of Predictive Parsing Table (Based on the computation of FIRST)**

	id	+	*	(	)	\$
E	<b>E -&gt; TE'</b>			<b>E -&gt; TE'</b>		
E'		<b>E' -&gt; +TE'</b>			<b>E' -&gt; €</b>	<b>E' -&gt; €</b>
T	<b>T -&gt; FT'</b>			<b>T -&gt; FT'</b>		
T'		<b>T' -&gt; €</b>	<b>T' -&gt; *FT'</b>		<b>T' -&gt; €</b>	<b>T' -&gt; €</b>
F	<b>F -&gt; id</b>			<b>F -&gt; (E)</b>		

**Parsing the given input String**

Stack	Input String	Action
<b>\$E</b>	<b>id+id*id\$</b>	<b>E -&gt; TE'</b> [Push]
<b>\$E'T</b>	<b>id+id*id\$</b>	<b>T -&gt; FT'</b> [Push]
<b>\$E'T'F</b>	<b>id+id*id\$</b>	<b>F -&gt; id</b> [Push]
<b>\$E'T'id</b>	<b>id+id*id\$</b>	[Pop]
<b>\$E'T'</b>	<b>+id*id\$</b>	<b>T' -&gt; €</b> [Push]
<b>\$E'</b>	<b>+id*id\$</b>	<b>E' -&gt; +TE'</b> [Push]
<b>\$E'T+</b>	<b>+id*id\$</b>	[Pop]
<b>\$E'T</b>	<b>id*id\$</b>	<b>T -&gt; FT'</b> [Push]
<b>\$E'T'F</b>	<b>id*id\$</b>	<b>F -&gt; id</b> [Push]
<b>\$E'T'id</b>	<b>id*id\$</b>	[Pop]
<b>\$E'T'</b>	<b>*id\$</b>	<b>T' -&gt; *FT'</b> [Push]



$\$E'T'F*$	$*id\$$	[Pop]
$\$E'T'F$	$id\$$	$F \rightarrow id$ [Push]
$\$E'T'id$	$id\$$	[Pop]
$\$E'T'$	$\$$	$T' \rightarrow \epsilon$ [Push]
$\$E'$	$\$$	$E' \rightarrow \epsilon$ [Push]
$\$$	$\$$	ACCEPT

2. Given a grammar,

$S \rightarrow iEtS \mid iEtSeS \mid a$

$E \rightarrow b$

**Input String: ibta**

Verify whether the given input string is valid or not using Predictive parsing method.

Soln:

- There is no left recursion
- There is left factoring. So, eliminate it.

$S \rightarrow iEtSS'$

$S \rightarrow a$

$S' \rightarrow eS$

$S' \rightarrow \epsilon$

$E \rightarrow b$

Computation of FIRST

$FIRST(S) = \{i, a\}$

$FIRST(S') = \{e, \epsilon\}$

$FIRST(E) = \{b\}$

Computation of FOLLOW

$FOLLOW(S) = \{\$, FIRST(S'), FOLLOW(S')\} = \{\$, e, \epsilon\}$

$FOLLOW(S') = \{FOLLOW(S)\} = \{\$, e, \epsilon\}$

$FOLLOW(E) = \{t\}$

### Construction of Predictive Parsing Table (According to FIRST)

	a	b	e	i	t	\$
S	$S \rightarrow a$			$S \rightarrow iEtSS'$		
S'			$S' \rightarrow eS$			$S' \rightarrow \epsilon$
E		$E \rightarrow b$				

### Parsing the given input string (ibta)

Stack	Input String	Action
$\$S$	$ibta\$$	$S \rightarrow iEtSS'$ [Push]
$\$S'StEi$	$ibta\$$	[Pop]
$\$S'StE$	$bta\$$	$E \rightarrow b$ [Push]
$\$S'Stb$	$bta\$$	[Pop]
$\$S'St$	$ta\$$	[Pop]
$\$S'S$	$a\$$	$S \rightarrow a$ [Push]
$\$S'a$	$a\$$	[Pop]
$\$S'$	$\$$	$S' \rightarrow \epsilon$ [Push]
$\$$	$\$$	ACCEPT

## SLR PARSING



### Problem

Construct the SLR parsing table and parse the string abab for the following grammar

$S \rightarrow AA$

$A \rightarrow aA \mid b$

Soln:

#### 1. Augmented Grammar

$$\begin{aligned} S' &\rightarrow S \\ S &\rightarrow AA \\ A &\rightarrow aA \mid b \end{aligned}$$

#### Augmented Grammar

$S' \rightarrow S$

$S \rightarrow AA$

$A \rightarrow aA$

$A \rightarrow b$

#### LR (0) Items

**$I_0$**

$S' \rightarrow .S$

$S \rightarrow .AA$

$A \rightarrow .aA$

$A \rightarrow .b$

**GOTO ( $I_0$ , S) =**

**$I_1$**

$S' \rightarrow S.$

**GOTO (I<sub>0</sub>, A) =**

**I<sub>2</sub>**

S -> A.A

A -> .aA

A -> .b

**GOTO (I<sub>0</sub>, a) =**

**I<sub>3</sub>**

A -> a.A

A -> .aA

A -> .b

**GOTO (I<sub>0</sub>, b) =**

**I<sub>4</sub>**

A -> b.

**GOTO (I<sub>2</sub>, A) =**

**I<sub>5</sub>**

S -> AA.

**GOTO (I<sub>2</sub>, a) =**

**I<sub>3</sub>**

A -> a.A

A -> .aA

A -> .b

**GOTO (I<sub>2</sub>, b) =**

**I<sub>4</sub>**

A -> b.

**GOTO (I<sub>3</sub>, A) =**

**I<sub>6</sub>**

A -> aA.

GOTO (I<sub>3</sub>, a) =

I<sub>3</sub>

A -> a.A

A -> .aA

A -> .b

GOTO (I<sub>3</sub>, b) =

I<sub>4</sub>

A -> b.

## LR (0) Items

I<sub>0</sub>  
S' -> . S  
S -> . AA  
A -> . aA  
A -> . b

Goto (I<sub>0</sub>, S)

I<sub>1</sub>  
S' -> S .

Goto (I<sub>0</sub>, A)

I<sub>2</sub>  
S -> A.A  
A -> . aA  
A -> . b

Goto (I<sub>0</sub>, a)

I<sub>3</sub>  
A -> a.A  
A -> . aA  
A -> . b

Goto (I<sub>0</sub>, b)

I<sub>4</sub>  
A -> b.

Goto (I<sub>2</sub>, A)

I<sub>5</sub>  
S -> AA.

Goto (I<sub>2</sub>, a)

I<sub>3</sub>  
A -> a . A  
A -> . aA  
A -> . b

Goto (I<sub>2</sub>, b)

I<sub>4</sub>  
A -> b.

Goto (I<sub>3</sub>, A)

I<sub>6</sub>  
A -> aA .

Goto (I<sub>3</sub>, a)

I<sub>3</sub>  
A -> a.A  
A -> . aA  
A -> . b

Goto (I<sub>3</sub>, b)

I<sub>4</sub>  
A -> b.



**SLR PARSING TABLE**

State	Action			Goto	
	a	b	\$	S	A
0	S3	S4		1	2
1			ACCEPT		
2	S3	S4			5
3	S3	S4			6
4	R3	R3	R3		
5			R1		
6	R2	R2	R2		

### Reduce Action

**I4 :A -> b.**

Follow(A)=  
FIRST(A)= {a,b,\$}

**I5: S->AA.**

Follow(S)= {\$}

**I6: A -> aA .**

Follow(A)=  
FIRST(A)= {a,b,\$}

1.S -> AA  
2.A -> aA  
3.A -> b

**Parsing the given input string**

Stack	Input String	Action
0	abab\$	Shift(S3)
0a3	bab\$	Shift(S4)
0a3b4	ab\$	Reduce (R3) A -> b
0a3A6	ab\$	Reduce (R2) A -> aA
0A2	ab\$	Shift (S3)
0A2a3	b\$	Shift (S4)
0A2a3b4	\$	Reduce (R3) A -> b
0A2a3A6	\$	Reduce (R2) A -> aA
0A2A5	\$	Reduce (R1) S -> AA
0S1	\$	Accept