UNIT – IV

Inventory control:

An inventory may be defined as an idle resource that possesses economic value. It is an item stored (or) reserved for meeting future demand. Such items may be materials, machines, money or even human resource.

Types of inventory:

A convenient classification of the types of inventory is as follows:

(i) Raw materials – the materials, components fuels etc. used in the manufacturing of product

(ii) Work-in-progress: partly finished goods and material such assemblies etc. held between manufacturing stages.

(iii) Finished goods: completed products ready for sale or distribution.

Objective of inventory control:

The objective of inventory control is to maintain stock levels so that the combined costs, mentioned earlier are at a minimum

This is done by establishing two sectors:

(i) item to order and (ii) how many units to order

These factors are subject of inventory control and to deal with these we need some basic terminologies.

Inventory control terminologies:

(i) Demand (ii) Economic order Quantity (EOQ)

(iii) Lead time (iv) Buffer stock (v) Maximum order

(vi) Reorder level.

Demand: The amount of quantity required by sales (or) products usually expressed as the rate of demand for week or month or year etc.

EOQ: This is a calculated ordering quantity which minimises the balance between inventory holding costs and reorder costs.

↓

Economic lot size

Lead time: The period of time between ordering and replenishment (full up again)

Buffer stock stock (safety stock)

It is stock allowance to cover errors in forecasting the lead time on the demand during the lead time.

Inventory models:

There are two types of Inventory models,

(i) Deterministic inventory model → Demand known.

(ii) Stochastic inventory model → unknown demand.
(use probabilities).

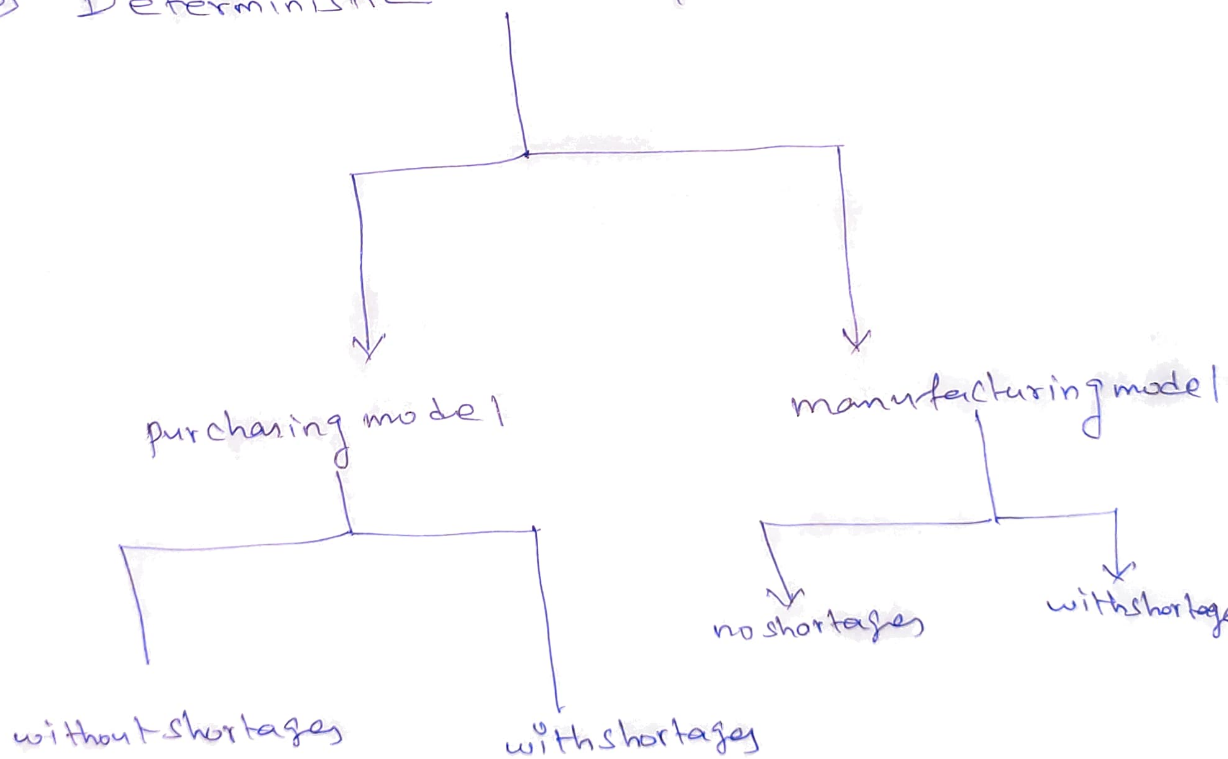
Types of control system:

There are two types of control system

(i) fixed order quantity system

(ii) Periodic Review system.

Deterministic inventory models



Purchasing model		Manufacturing model	
without Shortages	with shortage	without shortage	with shortage
(i) EOQ $Q^* = \sqrt{\frac{2DC_3}{C_1}}$	$Q^* = \sqrt{\frac{2DC_3}{C_1}\left(\frac{C_1+C_2}{C_2}\right)}$	$Q^* = \sqrt{\frac{2DC_3}{C_1} \times \left(\frac{P}{P-D}\right)}$	$Q^* = \sqrt{\frac{2DC_3}{C_1} \times \left(\frac{P}{P-D}\right)\left(\frac{C_1+C_2}{C_2}\right)}$
Optimum inventory cost (minimum cost) (ii) $T.C^* = \sqrt{2DC_1C_3}$	$T.C^* = \sqrt{2DC_1C_3\left(\frac{C_2}{C_1+C_2}\right)}$	$T.C^* = \sqrt{2DC_1C_3\left(\frac{P-D}{P}\right)}$	$TC^* = \sqrt{2DC_1C_3\left(\frac{P-D}{P}\right)\left(\frac{C_2}{C_1+C_2}\right)}$
(iii) Optimum number of order $n = \frac{D}{Q^*}$	$n = \frac{D}{Q^*}$	$n = \frac{D}{Q^*}$	$n = \frac{D}{Q^*}$
Optimum time interval between order $t^* = \frac{Q^*}{D}$	$t^* = \frac{Q^*}{D}$	$t^* = \frac{Q^*}{D}$	$t^* = \frac{Q^*}{D}$
Total cost = Total ordering cost + Total carrying cost $= nC_3 + \frac{1}{2}QC_1$		TC = = cost of material + ordering cost + carrying cost	

where,

C_1 — inventory carrying (holding cost)

C_2 — shortage cost

C_3 — setup (ordering) cost

D — Demand per year

P — Production rate per year

To check:

(i) Purchasing model → No production [No production rate]

(ii) Manufacturing model → with production [production rate given]

(iii) without shortages → Shortage cost not given

(iv) with shortages → Shortage cost given.

Problem 1:

The annual demand for an item is 3200 units. The unit cost is Rs. 6 and inventory carrying charges 25% per annum. If the cost of one procurement (ordering) is Rs. 150. Determine

(i) Economic Order quantity

(ii) Number of orders per year.

(iii) Time between two consecutive order

(iv) the optimal cost.

Note: (i) Production rate not given ⇒ purchasing model

(ii) Shortage cost not given ⇒ purchasing without shortage.

Soln:

Given:

Demand $D = 3200$ units/year.

(ordering) $C_3 = 150$

Carrying cost $C_1 = 6 \times 25\%$

$= 6 \times \frac{25}{100} = Rs. 1.50$

(i) EOQ $Q^* = \sqrt{\frac{2DC_3}{C_1}}$

$= \sqrt{\frac{2 \times 3200 \times 15}{150}} = 800$ units.

(ii) Number of orders $= \frac{D}{Q^*} = \frac{3200}{800} = 4.$

(iii) Optimum time between two consecutive order

$t^* = \frac{Q^*}{D} = \frac{800}{3200} = \frac{1}{4}$ year (or) 3 month

(iv) Optimum cost $TC^* = \sqrt{2DC_1C_3}$

$= \sqrt{2 \times 3200 \times 1.50 \times 150}$

$= 1200$

Problem 2: Company buys in lots 500 boxes which is a 3 months supply. The cost per box is Rs 125 and the ordering cost is Rs. 150. The inventory carrying cost is estimated at 20% of unit value. Find

(i) Total inventory cost

(ii) EOQ (Economic lot size)

(iii) Total minimum cost

(iv) Total saving cost.

Given:

Demand $D = 500$ units (3 month)

$= 500 \times \frac{12}{3}$ per year

$\boxed{D = 2000 \text{ units per year}}$

note
$Q \neq Q^*$

$Q = 500$

$C_3 = \text{Rs. } 150$

$C_1 = 125 \times 20\% = \text{Rs } 25$

$= \text{Rs. } 25$

(i) Total cost $= n C_3 + \frac{1}{2} Q C_1$

$= 4 \times 150 + \frac{1}{2} \times 500 \times 25$

$= 600 + 6250 = 6850$

$= \text{Rs. } 6850$

(ii) $EOQ = Q^* = \sqrt{\frac{2 D C_3}{C_1}} = \sqrt{\frac{2 \times 2000 \times 150}{25}}$

$= 155$ units per year.

(iii) Total minimum cost $TC^* = \sqrt{2 D C_1 C_3}$

$= \sqrt{2 \times 2000 \times 150 \times 25}$

$= \text{Rs. } 3873.$

(iv) Amount of saving

$=$ T.C $-$ total minimum cost

$= 6850 - 3873 = \text{Rs. } 2977$

Problem 3:

The demand for an item in a company is 18000 units per year and the company can produce the item at a rate of 3000 units per month. The cost of one setup is Rs. 500 and the holding cost per month is 15 paise.

Determine

(i) Ecconomic lot size

(ii) Total annual cost

Note: production rate given ⇒ manufacturing model.
Shortage cost not given ⇒ without shortage model.

Soln:

Given:

$D = 18000$ units per year

$P = 3000 \times 12$ per year

$P = 36000$ unit per year

$C_3 = Rs. 500$

$C_1 = Rs. 0.15 \times 12 = Rs. 1.80/\text{year}.$

(i) EOQ $Q^* = \sqrt{\frac{2DC_3}{C_1}\left(\frac{P}{P-D}\right)}$

$= \sqrt{\frac{2 \times 18000 \times 500}{1.80}\left(\frac{36000}{36000-18000}\right)}$

$= 4472$ units.

Total annual cost (including the cost of materials)

$$= \sqrt{2DC_1C_3\left(\frac{P-D}{D}\right)} + \text{unit cost} \times \text{Demand}$$

$$= \sqrt{2 \times 18000 \times 500 \times 180 \times \frac{36000-18000}{18000}} + 2 \times 18000$$

$= Rs. 40025$

Problem 4: Given the following data for an item of units from demand, instaneous delivery time and back order (shortage) facility

Annual demand = 800 units
Cost of an item = Rs. 40
ordering cost = Rs. 800
inventory carrying cost = 40%
Back order cost = Rs. 10

To find EOQ

Note: Production rate not given ⇒ Purchasing Model.
Shortage cost given ⇒ P.M with Shortage

Soln:

Given: $D = 800$, $C_3 = 800$, $C_1 = 40 \times 40\% = 16$

$C_2 = 10$

$$\text{EOQ} \quad Q^* = \sqrt{\frac{2DC_3}{C_1}\left(\frac{C_1+C_2}{C_2}\right)}$$

$$= \sqrt{\frac{2 \times 800 \times 800}{16}\left(\frac{16+10}{10}\right)} = 456 \text{ units}$$

Probabilistic inventory model or stochastic Model

The optimum order quantity is

$$\sum_{r=0}^{z_0-1} p(r) < \frac{c_2}{c_1+c_2} < \sum_{r=0}^{z_0} p(r).$$

where z_0 – the optimum stock level

Problem the probability distribution of monthly sales of an item is

Sales unit :	0	1	2	3	4	5	6
Probability :	0.01	0.06	0.25	0.30	0.22	0.10	0.06

The cost of carrying inventory is Rs. 30 per month and the cost of unit shortage is Rs. 70. Determine optimum stock level

Soln:

Sales:	0	1	2	3	4	5	6
Probability:	0.01	0.06	0.25	0.30	0.22	0.10	0.06
Cummulative probability	0.01	0.07	0.32	0.62	0.84	0.94	1.00

$c_1 = 30, \quad c_2 = 70.$

The optimum order quantity

$$\sum_{r=0}^{z_0-1} p(r) < \frac{c_2}{c_1+c_2} < \sum_{r=0}^{z_0} p(r)$$

$$\sum_{r=0}^{z_0-1} p(r) < \frac{70}{70+30} < \sum_{r=0}^{z_0} p(r)$$

$$\sum_{r=0}^{z_0-1} p(r) < 0.70 < \sum_{r=0}^{z_0} p(r)$$

$\Rightarrow$ EOQ $Q^* = \underline{4}$

12.8 Probabilistic Inventory Models (Stocastic)

Model 3 : Instantaneous Demand, No set up cost, Stock in discrete units.

We have the following Assumptions :

(i) t is the time Interval between orders

(ii) Z is the stock (in discrete Units) for time t

(iii) d is the estimated demand with probability $p(d)$

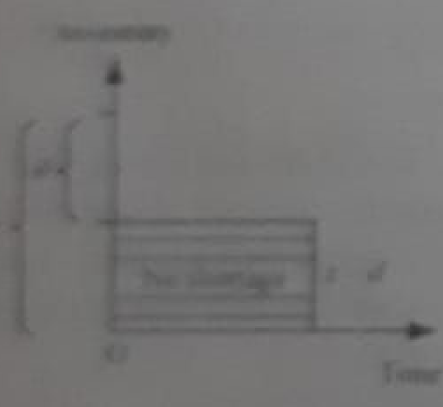
(iv) c_1 is the inventory holding cost per item per t time unit

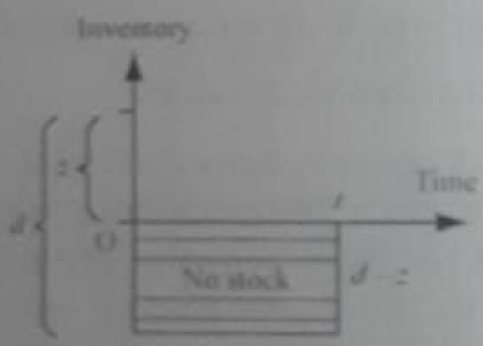
(v) c_2 is the shortage cost per 't' time unit.

(vi) Lead time is zero.

Determine the optimum order level Z. *[MU. BE. Oct '97]*

Solution : In the model with Instantaneous demand, it is assumed that the total demand is filled at the beginning of Each period. Thus depending on the amount d demanded, the inventory position just after the demand occurs may be either surplus or shortage.





Case 1 : When demand d does not exceed the stock $(i.e.)\ d \le z$

$$\text{Now holding cost} = (z-d)\times c_1 \ \text{ for } d \le z$$
$$= C_1 \times 0 \ \text{ for } d > z$$

Case 2 : when $d > z$ then

$$\text{Shortage cost} = C_2 \times 0 \ \text{ for } d \le z$$
$$= (d-z)\,C_2 \ \text{ for } d > z$$

The total expected cost

$$C(z) = \sum_{d=0}^{z} (z-d)\, C_1\, p(d) + \sum_{d=z+1}^{\infty} C_1 \cdot 0\, p(d) + \sum_{d=0}^{z} C_2\, 0\, p(d) + \sum_{d=z+1}^{\infty} (d-z)\, C_2\, p(d)$$

$$\text{or } C(z) = \sum_{d=0}^{z} (z-d)\, C_1\, p(d) + \sum_{d=z+1}^{\infty} (d-z)\, C_2\, p(d)$$

For $C(z)$ to be minimum

$$\Delta C(z-1) < 0 < \Delta C(z)$$

$$\text{Now } \Delta C(z) = C_1 \sum_{d=0}^{z} [\{(z+1)-d\} - (z-d)]\, p(d) + C_2 \sum_{d=z+1}^{\infty} [\{d-(z+1)\} - (d-z)]\, p(d)$$

$$= C_1 \sum_{d=0}^{z} p(d) - C_2 \sum_{d=z+1}^{\infty} p(d)$$

$$= C_1 \sum_{d=0}^{z} p(d) - C_2 \left[\sum_{d=0}^{\infty} p(d) - \sum_{d=0}^{z} p(d) \right]$$

$$= (C_1 + C_2) \sum_{d=0}^{z} p(d) - C_2 \qquad \therefore \left\{ \sum_{d=0}^{\infty} p(d) = 1 \right\}$$

for minimum $\Delta C(z) > 0$

$$\therefore (C_1 + C_2) \sum_{d=0}^{z} p(d) - C_2 > 0$$

or

$$\sum_{d=0}^{z} p(d) > \frac{C_2}{C_1 + C_2}$$

$\therefore$ The required relationship is,

$$\sum_{d=0}^{z-1} p(d) < \frac{C_2}{C_1 + C_2} < \sum_{d=0}^{z} p(d)$$

Model 12 : If the stock levels are in Continuous units then we replace $p(d)$ by $f(x)\,dx$ where $f(x)$ is p.d.f.

Solution : Let $\int_{x_1}^{x_2} f(x)\,dx$ = the probability of the order in the range x_1 to x_2.

The cost equation is $c(z) = C_1 \int_0^z (z-x) f(x)\,dx + C_2 \int_z^\infty (x-z) f(x)\,dx$

$$\frac{dC(z)}{dz} = C_1 \int_0^z (1-0) f(x)\,dx + C_1\left[(z-x) f(x) \frac{dx}{dz}\right]_{x=0}^{z}$$

$$+ C_2 \int_0^z (0-1) f(x)\,dx + C_2\left[(x-z) f(x)\frac{dx}{dz}\right]_0^\infty$$

$$\left[\because \text{If } C(z) = \int_{a(z)}^{b(z)} f(x,z)\,dx,\right.$$

$$\left.\frac{dC(z)}{dz} = \int_{a(z)}^{b(z)} \frac{\partial f(x,z)}{\partial z}\,dx + \left[f(x,z)\frac{dx}{dz}\right]_{a(z)}^{b(z)}\right]$$

$$= C_1 \int_0^z f(x)\,dx - C_2 \int_z^\infty f(x)\,dx$$

$$= C_1 \int_0^z f(x)\,dx - \left[C_2 \int_0^\infty f(x)\,dx - \int_0^z f(x)\,dx\right]$$

as $\int_0^\infty f(x)\,dx = 1$

$$\frac{dC}{dz} = 0 \Rightarrow (C_1 + C_2) \int_0^z f(x)\,dx - C_2 = 0$$

$$\Rightarrow \int_0^z f(x)\,dx = \frac{C_2}{C_1 + C_2}$$

from which we get the optimum value of z.

Example 18 : **A newspaper boy buys paper for Rs. 1.40 and sells them for Rs. 2.45 He cannot return unsold news papers. Daily demand has the following distribution.**

Customers :	25	26	27	28	29	30	31	32	33	34	35	36
Probability :	.03	.05	.05	.10	.15	.15	.12	.10	.10	.07	.06	.02

If each day's demand is independent of the previous days's, how many papers he should order each day ? *[Meerut M.Sc 92]*

Solution : Given : C_1 = Rs. 1.40, $C_2 = 2.45 - 1.40 = 1.05$

:	25	26	27	28	29	30	31	32	33	34	35	36
$p(d)$:	.03	.05	.05	.10	.15	.15	.12	.10	.10	.07	.06	.02
$\Sigma p(d)$:	.03	.08	.13	.23	.38	.53	.65	.75	.85	.92	.98	1.00

$$\text{Now } \boxed{\frac{C_2}{C_1 + C_2}} = \frac{1.05}{1.40 + 1.05} = \frac{1.05}{2.45} = 0.4285$$

$$\text{Now } 0.38 < 0.4285 < 0.53$$

$\therefore$ No of papers ordered = 30 *[Ans]*

Example 19 : **The probability distribution of the demand for a certain item is as follows :**

Monthly Sales	**:**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
Probability	**:**	**0.01**	**0.06**	**0.25**	**0.35**	**0.20**	**0.03**	**0.10**

The cost of carrying inventory is Rs.30 per unit per month and the cost of unit short is Rs. 70 per month. Determine the optimum stock level which will minimize the total expected cost.

[MU. BE. Nov 96]

Solution : Given : $C_1 = 30,\ C_2 = 70$

$$\therefore \frac{C_2}{C_1 + C_2} = \frac{70}{100} = 0.7$$

Monthly Sales	:	0	1	2	3	4	5	6
$p(d)$	:	0.01	0.06	0.25	0.35	0.20	0.03	0.10
$\Sigma p(d)$	:	0.01	0.07	0.32	0.67	0.87	0.90	1.00

$$\text{Here } 0.67 < \frac{C_2}{C_1 + C_2} < 0.87$$

$\therefore$ Optimum Quantity = 4 *[Ans]*

Example 20 : **The probability distribution of monthly sales of a Certain item is as follows :**

Monthly Sales	:	0	1	2	3	4	5	6
Probability	:	0.02	0.05	0.30	0.27	0.20	0.10	0.06

The cost of carrying inventory is Rs.10 per unit per month. The current policy is to maintain a stock of four items at the beginning of each month. Assuming that the cost of shortage is proportional to both time and quantity short, obtain the imputed cost of a shortage of one item for one time unit. *[MU. 93]*

Solution :

Monthly Sales	:	0	1	2	3	4	5	6
Probability	:	0.02	0.05	0.30	0.27	0.20	0.10	0.6
Cumulative	:	0.02	0.07	0.37	0.64	0.84	0.94	1.00

$C_1 = 10/-$

We know $$\sum_{d=0}^{z-1} p(d) < \frac{C_2}{C_1 + C_2} < \sum_{d=0}^{z} p(d)$$

here $z = 4 \Rightarrow$ $$\sum_{d=0}^{3} p(d) < \frac{C_2}{C_1 + C_2} < \sum_{d=0}^{z} p(d)$$

$$\Rightarrow \quad 0.64 < \frac{C_2}{10 + C_2} < 0.84$$

form $0.64 < \dfrac{C_2}{10 + C_2} \Rightarrow c_2 = 17.7$

from $\dfrac{C_2}{10 + C_2} < 0.84$

$$C_2 = 52.5$$

Shortage Cost $17.7 < C_2 < 52.5$ Rs *[Ans]*

Example 21 : **A company uses to order a new machine after a certain fixed time. It is observed that one of the parts of the machine is very expensive if it is ordered with out the machine . The cost of spare part when ordered with the machine is Rs.500.00. The cost of down time of the machine and the cost of arranging the new part is Rs.10,000.00. From the past records it is observed that spare part is required with the probabilities mentioned below.**

Demand (r)	:	0	1	2	3	4	5	6
Probability $P(r)$	:	0.90	0.05	0.02	0.01	0.01	0.01	0.00

Find the optimal number of spare parts which should be ordered with the order of the machine ?

Solution :

Demand	:	0	1	2	3	4	5
$P(r)$	:	0.90	0.05	0.02	0.01	0.01	0.01
$\Sigma P(r)$	:	0.90	0.95	0.97	0.98	0.99	1.00

$$\text{Now } \frac{C_2}{C_1 + C_2} = \frac{10{,}000}{10{,}500}$$

$$C_2 = \text{Rs.}10{,}000$$

$$C_1 = \text{Rs.}500$$

$$= 0.952$$

Here 0.952 lies between 1 and 2.

Optimum Number of spare parts to be ordered = 2 *[Ans]*

$\boxed{\text{Example 22}}$: **If the demand for a certain product has a rectangular distribution between 4000 and 5000, find the optimal order quantity if storage cost is Re.1 per unit and shortage cost is Rs. 7 per unit.** ***[MKU. B.Sc 80]***

Solution :

$$\int_0^Q f(x)\,dx = \frac{C_2}{C_1 + C_2} \qquad \text{here } C_2 = \text{Rs. } 7,\ C_1 = \text{Rs.}1$$

$$\Rightarrow \text{here} \int_{4000}^{Q} \frac{1}{5000 - 4000}\,dx = \frac{7}{8}$$

$$\Rightarrow \int_{4000}^{Q} \frac{1}{1000}\,dx = \frac{7}{8}$$

$$Q = \frac{7000}{8} + 4000$$

$\frac{Q}{1000} - \frac{4000}{1000} = 7/8$

$$= 875 + 4000$$

$$= 4875$$ *[Ans]*

Example 23 : An ice cream company sells one of its types of ice-creams by weight. If the product is not sold on the day it is prepared, it can be sold for a loss of 50 paise per pound but there is an unlimited market for one day old ice-creams. On the other hand, the company makes a profit of Rs. 3.20 on every pound of ice-cream sold on the day it is prepared. If daily orders form a distribution with $f(x) = 0.02 - 0.0002x$, $0 \le x \le 100$ how many pounds of ice-cream should the company prepare every-day ? *[Patna, M.Sc 83]*

Solution : Given $C_1 = \text{Rs.}0.50$, $C_2 = \text{Rs.}3.20$

Let Q be the amount of ice-cream prepared every day

$$\text{Now } \int_0^Q f(x)\,dx = \frac{C_2}{C_1 + C_2}$$

$$\Rightarrow \int_0^Q (0.02 - 0.0002x\,dx) = \frac{3.20}{0.50 + 3.20}$$

$$(i.e.,)\ 0.02\,Q - \frac{0.0002}{2}Q^2 = 0.865$$

$$\text{or } 0.0002\,Q^2 - 0.04\,Q + 1.730 = 0$$

$$\text{Solving, } Q = 136.7 \text{ or } 63.5$$

$$Q = 136.7 \text{ not admissible } \{x \le 100\}$$

$$\Rightarrow Q = 63.5 \text{ Pounds (optimum Quantity)}$$ *[Ans]*

Example 24 : Let the probability density of a demand of a certain item during a day be

$$f(x) = \begin{cases} 0.1 & 0 \le x \le 10 \\ 0 & x > 10 \end{cases}.$$

The demand is assumed to occur at uniform pattern during the whole day. Let the unit carrying cost of the item in inventory be Rs. 0.5 per day and unit shortage cost is Rs. 4.5 per day. If Rs. 0.5 bet be purchasing cost per unit, determine the optimum level of inventory. *[MKU. B.Sc 83]*

Solution : Given $C_1 = \text{Rs. } 0.50$, $C_2 = \text{Rs. } 4.5$, $C = \text{Rs. } 0.5$

Let Q be the amount of ice-cream prepared every day

$$\therefore \int_0^Q f(x)\,dx = \frac{C_2 - C}{C_1 + C_2}$$

$$\Rightarrow \int_0^{Q} (0.1)\, dx = \frac{4.5-0.5}{4.5+0.5}$$

$$\text{or } (0.1)\, Q^{\circ} = 0.8$$

$$\Rightarrow Q^{\circ} = 8 \text{ units} \qquad [Ans]$$

Example 25 : Demand for a certain product in a "newsboy" model is normally distributed with mean 100 units and standard deviation 20. Lost profit is Rs.8 per unit and salvage loss is Rs.12. Find the optimal initial Inventory.

Solution :

$$C_1 = \text{Salvage Loss : Rs.12 ;}$$

$$C_2 = \text{Lost profit : Rs.8}$$

$$\mu = 100 \text{ units, } \sigma = 20$$

$$p_c = \frac{C_1}{C_2 + C_1} = 0.60$$

The z value corresponding to 0.60 of the area under the normal curve can be read from the tables as 0.25

$$\text{E o Q} = \mu - z\,\sigma$$

$$= 100 - 0.25 \times 20$$

$$= 95 \text{ units.} \qquad [Ans]$$

EXERCISE 12.3

1. Two products are stocked by a company. The company has limited space and cannot store more than 40 units. The demand distribution of the product are as follows:

Demand(z)	:	0	10	20	30	40
Prob. for I product	:	0.10	0.20	0.35	0.25	0.10
Prob. for II product	:	0.05	0.20	0.40	0.20	0.15

The inventory carrying costs are Rs. 5 and Rs. 10 per unit of the ending inventories for the first and second product respectively. The shortage costs are Rs. 20 and Rs. 50 per unit of the ending shortages for the I and II product respectively. Find the Economic order Quantities for both the products. *[Delhi. MBA 76]*

Ans : EoQ : 20 Units for both products.