SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS

Subject Title: Applied Operation Research **Subject Code: SBAA5202**

Course: M.B.A **Semester: II**

UNIT 2 TRANSPORTATION **12 Hrs.**

Transportation Problems - Initial Solution by North West Corner Method, Least Cost Method and VAM Method; MODI Method of Deriving Optimum Solution. Assignment Problems-Hungarian Method of Solving Minimization and Maximization Problems – Restricted Assignment problems – Travelling Salesman Problem.

1. TRANSPORTATION PROBLEMS

1.1. INTRODUCTION

quantities to be shifted from each source to destination, so that the total transportation cost is minimum.

Suppose a factory owns ware houses in 3 different locations in a city and has to dispatch the monthly requirement of the product manufactured by them to 5 different wholesale markets located in the same city. The cost of transporting one unit of the product from the i-th warehouse to the j-th market is known and is cij. It is assumed that the total cost is a linear function so that the total transportation cost of transporting xij, units of the product from the i-th warehouse to the j-th market is given by Σcijxij.

It is clear that the factory management will be interested in obtaining a solution that minimizes the total cost of transportation. During the process of transportation they will also face the constraints that from a warehouse they cannot transport more than what is stored or available in the warehouse (supply) and that they need to transport to a market the total monthly requirement of the market (demand).

1.2. ASSUMPTIONS

- Quantity of supply at each source is known.
- Quantity demanded at each destination is known.

The cost of transportation of a commodity from each source to destination is known.

1.3. PROCEDURE TO SOLVE TRANSPORTATION PROBLEM

Step I : Deriving the initial basic feasible solution.

Step II : Deriving the final optimal solution.

1.4 DERIVING THE INITIAL BASIC FEASIBLE SOLUTION

- North West corner method.
- Matrix minimum method.
- Vogel's approximation method (VAM Method) penalty method.

1.5 DERIVING THE FINAL SOLUTION

Modified distribution method / Modi method / UV method.

- If total demand = total supply, then it is a balanced transportation problem.
- If the total supply not equal to total demand, then the transportationproblem is unbalanced transportation problem.

2. NORTH WEST CORNER METHOD

1. Check if Demand=Supply. If not add dummy row or column.
2. Select the North West (upper left hand) corner cell.
3. Allocate as large as possible in the North West corner cell.
4. If demand is satisfied, strike off the respective column and deduct supply accordingly

If supply is exhausted, strike off the respective row and deduct demand accordingly

5. From the resultant array, locate the North West corner cell and repeat the procedure

Note : The assignment done is not taking cost into consideration.

6. Continue allocation until all demand is satisfied and all supply is exhausted.
7. Multiply the allocated quantity *cost of transportation for each occupied cell and add it to find the total cost.

3. LEAST COST METHOD

1. Check if Demand=Supply. If not add a dummy row /column.
2. The lowest cost cell in the matrix is allocated as much as possible based on demand and supply requirement.
    - If there are more than one least cost cell, select the one where maximum units can be allocated.

- If the tie exist, follow the serial order.

3. If demand is satisfied, strike off the respective column and deduct supply accordingly. If supply is exhausted, strike off the respective row and deduct demand accordingly.
4. From the resultant array, locate the least cost cell and repeat the procedure.
5. Continue allocation until all demand is satisfied and all supply is exhausted.
6. Find total cost.

4. VOGEL'S APPROXIMATION METHOD (VAM)

This method gives better initial solution in terms of less transportation cost through the concept of 'penalty numbers' which indicate the possible cost penalty associated with not assigning an allocation to given cell.

4.1. STEPS IN VOGEL'S APPROXIMATION METHOD (VAM)

1. Check if demand = supply, if not add a dummy row or column.
2. Calculate penalty of each row & column by taking the difference between the lowest unit transportation cost. This difference indicates the penalty or extra cost which has to be paid for not assigning an allocation to the cell with the minimum transportation cost.
3. Select the row or column which has got the largest penalty number.(If there is a tie it can be broken by selecting the cell where the maximum allocation can be made.)
4. In that row or column choose the minimum cost cell and allocate accordingly.
   - If there are more than one minimum cost cell, select the one where maximum units can be allocated.
   - If the tie exists, follow the serial order.
5. If demand is satisfied, strike off the respective column and deduct supply accordingly. If supply is exhausted, strike off the respective row and deduct demand accordingly.
6. From the resultant array, calculate penalty and repeat the procedure.
7. Continue allocation until all demand is satisfied and all supply is exhausted.
8. Find the total cost.

4.2. OPTIMAL SOLUTION

Work out the basic feasible solution using by any one method

a) Northwest corner method

b) Least cost method

c) VAM/Penalty method. (preferably VAM)

STEP 1:

Check if the number of occupied cells is m+n-1 (i.e., number of rows +number of columns-1) Note : Rows & columns include dummy rows & columns.

- If number of occupied cells = m+n-1, then the solution to the transportation problem is **basic feasible solution**.
- If number of occupied cells < m+n-1, then the solution is **degenerate solution**.

Degeneracy may occur either at the initial stage or at an intermediate stage at some subsequent iteration.

- In case of degeneracy, we allocate an extremely small amount, close to zero [(ξ) epsilon] to one or more empty cells of the transportation table (unoccupied least cost cell). So that total no of occupied cells equals to m+n-1.

STEP 2:

If the basic feasible solution is achieved then MODI method is used to obtain final optimal solution

□Defining the occupied cells.

$c_{ij}= u_i + v_j$ where, c_{ij} □co st.

u_i □row.

V_j □column.

□Assume any one u_i or v_j is to be zero such that max. no of allocations are done in that row(i) or column (j) & find value of all other u_i's & v_j's

STEP 3:

□Evaluate the unoccupied cells.

$d_{ij}= u_i + v_j - c_{ij}$

□If all evaluation val ues are either negative or zero, then the initial solution is optimal solution.

□If any positive value exist, initial solution is not an optimal solution.

STEP 4:

□Identify the **entering variable.**

The highest **positive** evaluated value (dij) cell is treated as entering variable cell.

STEP 5:

□Identify leaving variable.

- To identify the leaving variable, construct a closed loop.
- Loop starts at the entering variable cell.
- Loop can go clockwise or anticlockwise.
- The turning point should be occupied cell.
- Loop can cross each other.

STEP 6:

□Start assigning positive & negative.

□Assign positive (+) for the entering variable & negative (-) alternatively. □Where is the **minimum allocation quantity** among the negative □cells.

□Add □to the allocated value in the positive □cells and deduct □to the allocated value in the negative □cells.

□Cell/cells which have zero allocation (after deducting □) is the leaving variable.

STEP 7:

□Prepare a new transportation table.

□The values in the loop will get changed as per the step 6 and all other allocations not in the loop remains the same.

STEP 8:

□Check for optimality using step 1 to step 3

- If the solution is optimal calculate the minimum transportation cost from the allocations and the unit costs given.
- Repeat the procedure from step 4 to step 8, if the solution is not optimal.

5. ASSIGNMENT

In a printing press there is one machine and one operator is there to operate. How would you employ the worker? Your immediate answer will be, the available operator will operate the machine. Again suppose there are two machines in the press and two operators are engaged at different rates to operate them. Which operator should operate which machine for maximizing profit?

Similarly, if there are n machines available and n persons are engaged at different rates to operate them. Which operator should be assigned to which machine to ensure maximum efficiency? While answering the above questions we have to think about the interest of the press, so we have to find such an assignment by which the press gets maximum profit on minimum investment. Such problems are known as "assignment problems".

Assignment problem is a particular case of the transportation problem in which objective is to assign number of task to equal number of facilities at minimum cost and maximum profit. Suppose there are 'm' facilities and 'n' jobs and the effectiveness of each facility for each job are given, the objective is to assign one facility to one job so that the given measure of effectiveness is optimized.

If the matrix contains the cost involved in assignment the aim is to **minimize the cost**.

If the matrix contains revenue or profit the aim is to **maximize the revenue or profit**.

Ex.:

JOBS

	A	b	c	d
1	C1a	C1b	C1c	C1d
2	C2a	C2b	C2c	C2d

FACILITIES 3 C3a C3b C3c C3d

4 C4a C4b C4c C4d

1,2,3,4 indicates the facilities and a, b, c, d indicates the jobs. The matrix entries are the cost associated with the assignment of facilities with the jobs. The objective is to assign one facility to one job, so that the total cost is minimum.

Ex.:

Facility Job

1 d

2 c

3 a

4 b

Total cost = C1d + C2c +C3a +C4b

5.1. HUNGARIAN METHOD OR ASSIGNMENT ALGORITHM

STEP 1: Balancing the problem

- Check if the No. of Rows is equal to the No. of Columns, if not add a dummy row or a dummy column.

STEP 2: Row wise calculation (row reduced matrix)

- Select the min cost element from each row and subtract it from all the elements in the same row.

STEP 3: Column wise calculation (column reduced matrix)

- From the resultant matrix, select the minimum cost element from each column and subtract it from all the other elements in the same column.

STEP 4: Assigning the zeroes

- Starting with first row of the resultant matrix received in first step, examine the rows one by one until a row containing exactly one zero is found. Then an experimental assignment indicated by ' ' is marked to that zero. Now cross all the zeros in the column in which the assignment is made. This procedure should be adopted for each row assignment.
- When the set of rows has been completely examined, an identical procedure is applied successively to columns. Starting with column 1, examine all columns until a column containing exactly one zero is found. Then make an experimental assignment in that position and cross other zeros in the row in which the assignment was made. Continue these successive operations on rows and columns until all zero's have either been assigned or crossed-out.
- If all the zeros are assigned or crossed out, i.e., we get the maximal assignment.

Note: In case, if two zeros are remained by assignment or by crossing out in each row or column. In this situation we try to exclude some of the zeros by trial and error method.

STEP 5: Check for optimality

.

- ✓ If each job is assigned to each facility, then assignment is optimal. If any job or facility is left without assignment move to step 6

STEP 6: Draw of minimum lines to cover zeros

Draw the minimum possible straight lines covering all the zeros in the matrix by the following procedure

- ✓ Mark (√) rows in which the assignment has not been done.
- ✓ Locate zero in marked (√) row and then mark (√) the corresponding column.

- ✓ In the marked (√) column, locate assigned zeros & then mark (√) the corresponding rows.
- ✓ Repeat the procedure, till the completion of marking.
- ✓ Draw the lines through **unmarked rows and marked columns**.

Note: If the above method does not work then make an **arbitrary assignment**. If the number of these lines is equal to the order of the matrix then it will be an optimal solution and then go to step9 Otherwise proceed to step 7.

STEP 7: Modified Matrix

- ➢ Identify covered elements, uncovered elements and junction point
  - **Covered Elements** – where the lines passes through
  - **Uncovered Elements –** where the line does not pass through.
  - **Junction Point-** where the lines intersects
- ➢ Select the smallest element from the uncovered elements.
- ➢ Subtract this smallest element from the uncovered elements.
- ➢ Add this smallest element to the junction point
- ➢ Covered elements remain untouched

Thus we have increased the number of zero's

STEP: 8

- ➢ Repeat the procedure of assigning the zeroes as step 4.
- ➢ Repeat the procedure of checking for optimality as step 5.
- ➢ If optimality is arrived move to step 9 otherwise repeat steps 6 to 8.

STEP : 9

- ➢ Write separately the assignment (ONE TO ONE) and calculate the total cost taking corresponding values from the problem data.

NOTE:

Multiple optimal solutions

- ➢ If the final matrix (for zero assignment) is having more than one zero on rows and columns at independent positions (not possible to assign or cancel row-wise or column-wise) choose arbitrarily one zero for assignment and cancel all zeros in the corresponding rows and columns.

- Repeat the procedure by choosing another zero for assignment till all such zeroes are considered.
- Each assignment by this procedure will provide different set of assignments keeping the total minimum cost as constant. This implies multiple optimal solutions with the same optimal assignment cost.

5.2. SOLVING MAXIMISATION PROBLEMS IN ASSIGNMENT USING HUNGARIAN METHOD

- The maximization problem can be converted in to a minimization problem by subtracting all the elements of the matrix from the highest value.
- Follow the steps 1 to 9 of Hungarian Algorithm.

Note: While calculating the total profits take corresponding values from initial assignment problem (data before conversion of the problem)

5.3. RESTRICTED ASSIGNMENT PROBLEMS

The assignment technique assumes that the problem is free from practical restrictions and any task could be assigned to any facility. But in some cases, it may not be possible to assign a particular task to a particular facility due to space, size of the task, process capability of the facility, technical difficulties or other restrictions. This can be overcome by assigning a very high processing time of cost element ($\propto$ infinity) to the corresponding cell.

- Use Hungarian method for assignment steps 1 to 9.

NOTE:

- For maximization problems in restricted assignments, convert the problem in to a minimization problems given in the procedure above.
- Substitute $\propto$ (infinity) in the matrix for the restricted assignments.

- Use Hungarian method for assignment steps 1 to 9.

5.4. TRAVELLING SALESMAN PROBLEM

A salesman normally visits numbers of cities starting from high head quarters. The distance (or time or cost) between every pair of cities are assumed to be known. If a salesman has to visit ‘n’ cities, then he will have a total of (n-1)! Possible round trips. The problem of finding the shortest distance (or minimum time or minimum cost) if the salesman starts from his headquarters and passes through each city under his jurisdiction exactly once and returns to the headquarters is called the Travelling salesman problem or A Travelling Salesperson problem. A travelling salesman problem is very similar to the assignment problem with the additional constraints.

a) **Route Conditions**:

- The salesman should go through every city exactly once except the starting city (headquarters).
- The salesman starts from one city (headquarters) and comes back to that city (headquarters).

b) Obviously going from any city to the same city directly is not allowed (i.e., no assignments should be made along the diagonal line).

5.2.1. Steps to solve travelling salesman problem:

i. Assigning an infinitely large element ($\propto$) in each of the squares along the diagonal line in the cost matrix.

ii. Solving the problem as a routine assignment problem.

iii. Scrutinizing the solution obtained under (ii) to see if the ‘route’ conditions are satisfied.

iv. If not, making adjustments in assignments to satisfy the condition with minimum increase in total cost (i.e. to satisfy route condition, ‘next best solution’ may require to be considered).

i.

Problems

Problem 1. Find the optimum solution to the following problem.

	I	3	4	6	8	8	20
	II	2	10	1	5	30	30
From	III	7	11	20	40	15	15
	IV	2	1	9	14	18	13
		40	6	8	18	6	

Solution:

1. Make a transportation model

I	3	4	6	8	8	20
II	2	10	1	5	30	30
III	7	11	20	40	15	15
IV	2	1	9	14	18	13
	40	6	8	18	6	78

Find basic feasible solution (VAM method)

1.

$\boxed{14}$ 3	4	6	8	$\boxed{6}$ 8	20/14/0	[1] [1] [5]
$\boxed{4}$ 2	10	$\boxed{8}$ 1	$\boxed{18}$ 5	30	30/22/18	[1] [3] [3]
$\boxed{15}$ 7	11	20	40	15	15/0	[4] [4] ←
$\boxed{7}$ 2	$\boxed{6}$ 1	9	14	18	13/7/0	[1] [1] [12] ←
40/25 18/4/0	6/0	8/0	18/0	6/0	78	
[0]	[3]	[5]	[3]	[7]		
[1]	[3]	↑	[3]	↑		

Transportation cost = (14 × 3) + (6 × 8) + (4 × 2) + (8 × 1) +(18 × 5) + (15 × 7) + (7 × 2) + (6 × 1)

= 42 + 48 + 8 + 8 + 90 + 105 + 14 + 6

= Rs. 321.

3. Check for optimality (MODI Test) m (a) Cost matrix of allocated cell.

+ n - 1 = 8 (no. of allocation)

u_j \ V_i	1	2	3	4	5
1	3				8
2	2		1	5	
3	7				
4	2	1			

$$
\begin{array}{ll}
 & V_1 = 0 \\
u_1 + v_1 = 3 & u_1 = 3 \\
u_1 + v_5 = 8 & v_5 = 5 \\
u_2 + v_1 = 2 & u_2 = 2 \\
u_2 + v_3 = 1 & v_3 = -1 \\
u_2 + v_4 = 5 & v_4 = 3 \\
u_3 + v_1 = 7 & v_3 = 7 \\
u_4 + v_1 = 2 & u_4 = 2 \\
u_4 + v_2 = 1 & v_2 = -1
\end{array}
$$

(*b*) Opp. cost matrix

	0	–1	–1	3	5
3	·	2	2	6	·
2	·	1	·	·	7
7	·	6	6	10	12
2	·	·	1	5	7

(c) Cell evaluation matrix

·	2	4	2	·
·	9	·	·	23
·	5	14	30	3
·	·	8	9	11

Since all the elements of cell evaluation matrix are positive so optimality test is passed.

Minimum Transportation Cost = Rs. 321.

Problem 2. Solve the following cost-minimizing transportation problem.

	D_1	D_2	D_3	D_4	D_5	D_6	Available
O_1	2	1	3	3	2	5	50
O_2	3	2	2	4	3	4	40
O_3	3	5	4	2	4	1	60
O_4	4	2	2	1	2	2	30
Required	30	50	20	40	30	10	180

Ans. 1. Make a transportation model.

1. Find basic feasible solution

2	$\boxed{50}$ 1	3	3	2	5	50/0	[1]
3	2	$\boxed{20}$ 2	4	$\boxed{20}$ 3	4	40/20/0	[0][1][1][0]
$\boxed{30}$ 3	5	4	$\boxed{10}$ 2	$\boxed{10}$ 4	$\boxed{10}$ 1	60/50/40/10/0	[1][1][1][1]
4	2	2	$\boxed{30}$ 1	2	2	30/0	[1][1]
30/0	50/0	20/0	40/10/0	30/20/0	10/0		
[1]	[1]	[0]	[1]	[0]	[1]		
[0]		[0]	[1]	[1]	[1]		
[0]		[2]	[2]	[1]	[3]		

Check for optimality test (m + n -1) > no. of allocation (8)

2	$\boxed{50}$ 1	3	3	$\boxed{\varepsilon}$ 2	5
3	2	$\boxed{20}$ 2	4	$\boxed{20}$ 3	4
$\boxed{30}$ 3	5	4	$\boxed{10}$ 2	$\boxed{10}$ 4	$\boxed{10}$ 1
			$\boxed{30}$		

m + n- 1 = no. of allocation 9

9. Cost matrix of allocated cell

	1			2	
		2		3	
3			2	4	1
			1		

$$
\begin{array}{ll}
u_1 + v_2 = 1 & v_1 = 0 \\
u_1 + v_5 = 2 & v_2 = 0 \\
u_2 + v_3 = 2 & u_1 = 1 \\
u_2 + v_5 = 3 & v_3 = 0 \\
u_3 + v_1 = 3 & u_2 = 2 \\
u_3 + v_4 = 2 & u_3 = 3 \\
u_3 + v_5 = 2 & v_4 = -1 \\
u_3 + v_6 = 1 & v_5 = 1 \\
u_3 + v_4 = 1 & v_6 = -2 \\
 & u_4 = 2
\end{array}
$$

(*b*) Opportunity cost matrix

	0	0	0	–1	1	–2
1	1	·	1	0	·	–1
2	2	2	·	1	·	0
3	·	3	3	·	·	·
2	2	2	2	·	3	0

(c) Cell evaluation matrix

1	·	2	3	·	6
1	0	·	3	·	4
·	2	1	·	·	·
2	0	0	·	–1√	2

identified cell

Iteration for optimal solution.

1.

	50			ε	
		20		20	
30			+10	10–	10
			–30	√+	

Check for optimality test

	50			ε	
		20		20	
30			20		10
			20	10	

2nd feasible solution

(a). Cost matrix of allocated cell.

	1			2	
		2		3	
3			2		1
			1	2	

$u_1 + v_2 = 1$ $\quad v_1 = 0$
$u_1 + v_5 = 2$ $\quad v_2 = -1$
$u_2 + v_3 = 2$ $\quad u_1 = 2$
$u_2 + v_5 = 3$ $\quad v_3 = -1$
$u_3 + v_1 = 3$ $\quad u_2 = 3$
$u_3 + v_6 = 1$ $\quad u_3 = 3$
$u_4 + v_4 = 1$ $\quad v_4 = -1$
$u_4 + v_5 = 2$ $\quad v_6 = -2$
$u_4 = 2$
$v_5 = 0$

(*b*) Opp. cost matrix

	0	−1	−1	−1	0	−2
2	2	·	1	1	·	0
3	3	2	·	2	·	1
3	·	2	2	·	3	·
2	2	1	1	·	·	0

(*c*) Cell evaluation matrix

0	·	2	2	·	5
0	0	·	2	·	3
·	3	2	·	1	·
2	1	1	·	·	2

Since all elements of cell evaluation matrix are non negative so 2hldI feasible solution is the optimum solution.

Transportation cost

$= 50 \times 1 + 20 \times 2 + 20 \times 3 + 30 \times 3 + 20 \times 2 + 10 \times 1 + 20 \times 1 + 10 \times 2$
$= 50 + 40 + 60 + 90 + 40 + 10 + 20 + 20$
$= 330/-$

Problem 3. **Goods have to be transported from factories F_1, F_2, F_3 to ware house W_1, W_2, W_3 and W_4. The transportation cost per unit capacities and requirement of the ware house are given in the following table**

	W_1	W_2	W_3	W_4	Capacity
F_1	95	105	80	15	12
F_2	115	180	40	30	7
F_3	195	180	95	70	5
Requrement	5	4	4	11	

Solution. 1. Make a transportation model

	W_1	W_2	W_3	W_4	
F_1	95	105	80	15	12
F_2	115	180	40	30	7
F_3	195	180	95	70	5
	5	4	4	11	24 Balanced Model

2. Find a basic feasible solution
VAM method

F_1	95	$\boxed{4}$ 105	80	$\boxed{8}$ 15	12/8/0	[65] ←
F_2	$\boxed{5}$ 115	180	$\boxed{2}$ 40	30	7/2/0	[10]
F_3	195	180	$\boxed{2}$ 95	$\boxed{3}$ 70	5/3/0	[25]
	5/0	4/0	4/2/0	11/3/0		
	[20]	[75]	[40]	[15]		
	[80]	↑	[55]	[50]		
	↑		↑			

3. Optimality test

$$m + n - 1 = \text{number of allocations}$$
$$6 = 6$$

(a)Cost matrix of allocated cell

$u_j \backslash v_i$	1	2	3	4
1	·	105		15
2	115		40	
3		·	95	70

	$V_1 = 0$
$u_1 + V_2 = 105$	$V_2 = -10$
$u_1 + V_4 = 15$	$u_1 = 115$
$u_2 + V_1 = 115$	$u_2 = 115$
$u_2 + V_3 = 40$	$V_3 = -75$
$u_3 + V_3 = 95$	$u_3 = 170$
$u_3 + V_4 = 70$	$V_4 = -100$

(*b*) Opp. cost matrix

	0	−10	−75	−100
115	115	·	40	·
115	·	105	·	15
170	170	160	·	·

(*c*) Cell evaluation matrix

−20	·	40	·
·	75	·	15
25	20	·	·

4. Iteration for optimal solution

+	4		8 −
5 −		2 +	
		2 −	3 +

2	4		6
3		4	
			5

2nd feasible solution

5. Check for optimality

(*a*) Cost matrix of allocated cell

$u_j \backslash v_i$	1	2	3	4
1	95	105		15
2	115		40	
3				70

	$V_1 = 0$
$u_1 + V_1 = 95$	$u_1 = 95$
$u_1 + V_2 = 105$	$V_2 = 10$
$u_1 + V_4 = 15$	$V_4 = -80$
$u_2 + V_1 = 115$	$u_2 = 115$
$u_2 + V_3 = 40$	$V_3 = -75$
$u_3 + V_4 = 70$	$u_3 = 150$

1. Opp. cost matrix

	0	10	−75	−80
95	·	·	20	·
115	·	125	·	35
150	150	160	75	·

Cell evaluation matrix

·	·	60	·
·	55	·	√ –5
45	20	20	·

Iteration for optimal solution.

+ 2	4		6 –
– 3		4	√ +
			5

5	4		3
		4	3
			5

3rd feasible solution

Check for optimality test (a)

Cost matrix of allocated cell

95	105		15
		40	30
			70

$u_1 + V_1 = 95$ $\quad v_1 = 0$

$u_1 + V_2 = 105$ $\quad u_1 = 95$

$u_1 + V_4 = 15$ $\quad V_2 = 10$

$u_2 + V_3 = 40$ $\quad V_4 = -80$

$u_2 + V_4 = 30$ $\quad V_3 = -70$

$u_3 + V_4 = 70$ $\quad u_2 = 110$

$u_3 = 150$

(*b*) Opp. cost matrix

	0	10	–70	–80
95	·	·	15	·
110	110	120	·	·
150	150	160	80	

(*c*) Cell evaluation matrix

·	·	65	·
5	60	·	·
45	20	15	·

Since all elements of cell evaluation matrix are non negative. Hence 3rd feasible solution is the optimum solution.

Transportation cost = (5 × 95) + (4 × 105) + (3 × 15) + (4 × 40) + (3 × 30) + (5 × 70) = 475 + 420 + 45 + 160 + 90 + 350 = Rs 1540/-

Problem 4. **Solve the following assignment problem.**

work	1	2	3	4
A	12	30	21	15
B	18	33	9	31
C	44	25	24	21
D	23	30	28	14

Solution:

1. Prepare a square matrix.

2. Reduce the matrix

0	5	12	1
6	8	0	17
32	0	15	7
11	15	19	0

→

3. Check if optimal assignment can be made in the current solution or not

	1	2	3	4
A	$\boxed{0}$	5	12	1
B	6	8	$\boxed{0}$	17
C	32	$\boxed{0}$	15	7
D	11	5	19	$\boxed{0}$

Since there is one assignment in each row and each column, the optimal assignment can be made in the current solution.

Minimum total cost = 12 ×1 + 9 ×1+ 25 ×1 + 14 ×1
= 12 +9 + 25+14
= 60

Ans.
A ⟶ 1
B ⟶ 3
C ⟶ 2
D ⟶ 4

Minimum cost = Rs. 60.

Problem 5. **Find the optimal assignment for the assignment problem with the following cost matrix.**

	I	II	III	IV
A	5	3	1	8
B	7	9	2	6
C	6	4	5	7
D	5	7	7	6

Solution: 1. Prepare a square matrix

2. Prepare a reduced matrix.

0	0	0	2
2	6	1	0
1	1	4	1
0	4	6	0

0	0	0	2
2	6	1	0
0	0	3	0
0	4	6	0

3. Check if optimal assignment can be made in the current solution.

	I	II	III	IV
A	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	2
B	2	6	1	$\boxed{0}$
C	$\cancel{0}$	$\boxed{0}$	3	$\cancel{0}$
D	$\boxed{0}$	4	6	$\cancel{0}$

since each row and each column have assignment so optimal assignment can be made.

A – III
B – IV
C – II
D – I

Cost = 1+ 6 + 4 + 5 = 16.

Problem 6. **Four different jobs are to be done on four different machines. Table below indicate the cost of producing job i on machine j in rupees.**

		m/c 1	2	3	4
Job	1	5	7	11	6
	2	8	5	9	6
	3	4	7	10	7
	4	10	4	8	3

Solution : 1. Reduced matrix

0	2	6	1
3	0	4	1
0	3	6	3
7	1	5	0

0	2	2	1
3	0	0	1
0	3	2	3
7	1	1	0

3. Check if optimal assignment can be made in the current solution or not

$\boxed{0}$	2	2	1	✓
3	$\boxed{0}$	$\cancel{0}$	1	
$\cancel{0}$	3	2	3	✓
7	1	1	$\boxed{0}$	

Cross marked column and unmaked row.
Since no. of lines
$\leq$ Rank of matrix
$3 \leq 4$

4. Iterate towards optimality

$\cancel{0}$	1	1	$\cancel{0}$	✓
4	$\boxed{0}$	$\boxed{0}$	1	
$\boxed{0}$	2	1	2	✓
8	1	1	$\boxed{0}$	✓
✓			✓	

number of lines (3) $\leq$ Rank of matrix (4)

		m/c 1	2	3	4
	1	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$
Job	2	5	$\boxed{0}$	$\cancel{0}$	2
	3	$\boxed{0}$	1	$\cancel{0}$	2
	4	8	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$

Since each row and column have assignment so optimality condition is satisfied.

Job 1 – M/c 3
Job 2 – M/c 2
Job 3 – M/c 1
Job 4 – M/c 4

Cost = 11 +5 + 4 +3 = 23

QUESTIONS BANK

TRANSPORTATION
INITIAL SOLUTION
NORTH WEST CORNER METHOD

1.

	Destination				Supply
Sources	D1	D2	D3	D4	
S1	2	4	6	2	100
S2	8	6	5	2	60
S3	9	10	7	5	40
Demand	40	60	80	20	

2.

	Destination				Supply
Sources	D1	D2	D3	D4	
S1	2	4	6	2	70
S2	8	6	5	2	30
S3	9	10	7	5	50
Demand	80	10	20	30	

LEAST COST METHOD

3.

	Destination				Supply
Sources	D1	D2	D3	D4	
S1	2	4	6	2	100
S2	8	6	5	2	60
S3	9	10	7	5	40
Demand	40	60	80	20	

4. .

	Destination				Supply
Sources	D1	D2	D3	D4	
S1	6	1	9	3	70
S2	11	5	2	8	55
S3	10	12	14	7	70
Demand	85	35	50	45	

VOGEL'S APPROXIMATION METHOD

5.

	Destination				Supply
Sources	D1	D2	D3	D4	
S1	7	3	6	8	60
S2	4	2	5	0	100
S3	2	6	5	1	40

Demand	20	50	50	80

6.

	Destination			Supply	
Sources	D1	D2	D3	D4	
S1	6	1	9	3	70
S2	11	5	2	8	55
S3	10	12	14	7	70
Demand	85	35	50	45	

7.

	Destination			Supply
Sources	D1	D2	D3	
S1	2	7	4	5
S2	3	3	1	8
S3	5	4	7	7
S4	1	6	2	14
Demand	7	9	18	

FINAL OPTIMAL SOLUTION (UV METHOD)

8.

	Destination				Supply
Sources	D1	D2	D3	D4	
S1	7	3	8	6	60
S2	4	2	5	10	100
S3	2	6	5	1	40
Demand	20	50	50	80	

9.

		Destination		Supply
Sources	D1	D2	D3	
S1	5	1	7	10
S2	6	4	6	80
S3	3	2	5	15
S4	5	3	2	40
Demand	75	20	50	

10.

	Destination			Supply	
Sources	D1	D2	D3	D4	
S1	6	1	9	3	70
S2	11	5	2	8	55
S3	10	12	4	7	70
Demand	85	35	50	45	

11.

Sources	D1	D2	D3	D4	D5	Supply
	Destination					
S1	3	5	8	9	11	20
S2	5	4	10	7	10	40
S3	2	5	8	7	5	30
Demand	10	15	25	30	40	

DEGENERACY

12.

Sources	D1	D2	D3	Supply
	Destination			
S1	16	20	12	50
S2	14	8	18	50
S3	26	24	16	50
Demand	50	50	50	

13.

Sources	D1	D2	D3	D4	Supply
	Destination				
S1	13	25	12	21	18
S2	18	23	14	9	27
S3	23	15	12	16	21
Demand	14	12	23	27	

14.

Sources	D1	D2	D3	D4	Supply
	Destination				
S1	42	48	38	37	160
S2	40	49	52	51	150
S3	39	38	40	43	190
Demand	80	90	110	160	

ASSIGNMENT

1.

JOBS	Persons 1	2	3	4
A	10	5	13	15
B	3	9	18	3
C	10	7	3	2
D	5	11	9	7

2.

JOBS	Persons 1	2	3	4
A	5	8	4	2
B	1	4	6	3
C	0	4	2	6
D	4	7	5	4

3.

JOBS	Persons 1	2	3	4
A	8	8	4	3
B	4	2	1	6
C	6	8	10	12
D	14	18	20	22

UNBALANCED ASSIGNMENT MODELS

4.

JOBS	Persons 1	2	3	4
A	24	27	18	20
B	26	23	20	31
C	24	22	34	26
D	19	21	21	22
E	30	25	28	27

MAXIMIZATION CASE IN ASSIGNMENT PROBLEMS

5.

	Territories			
	T1	T2	T3	T4
P1	60	50	40	30
P2	40	30	20	15
P3	40	20	35	10
P4	30	30	25	20

6.

	Territories			
	T1	T2	T3	T4
P1	10	22	12	4
P2	16	18	22	10
P3	24	20	12	18
P4	16	14	24	20

RESTRICTED ASSIGNMENT MODEL

7.

	Territories			
	T1	T2	T3	T4
R1	4	-	-	8
R2	9	-	4	3
R3	8	1	2	-

TRAVELLING SALESMAN PROBLEM

8.

		TO			
		A	B	C	D
	A	-	46	16	40
	B	41	-	50	40
FROM	C	82	32	-	60
	D	40	40	36	-

9.

		TO				
		A	B	C	D	E
	A	-	3	6	2	3
	B	3	-	5	2	3
FROM	C	6	5	-	6	4
	D	2	2	6	-	6
	E	3	3	4	6	-

PROBLEMS FOR PRACTICE

10.

	Persons				
JOBS	1	2	3	4	5
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

11.

	Persons			
	1	2	3	4
JOBS				
A	8	3	2	1
B	4	5	6	3
C	2	2	9	4
D	1	3	6	5
E	9	3	6	5

MAXIMIZATION CASE IN ASSIGNMENT PROBLEMS

12.

	Territories		
	T1	T2	T3
S1	80	40	30
S2	20	10	10
S3	40	40	60
S4	90	30	40

13. Territories

	T1	T2	T3
P1	20	26	42
P2	24	32	50
P3	32	34	44

RESTRICTED ASSIGNMENT MODEL

14.

Territories

	R1	R2	R3	R4
C1	4000	5000	-	-
C2	-	4000	-	4000
C3	3000	-	2000	-
C4	-	-	4000	5000

TRAVELLING SALESMAN PROBLEM

15. TO

		A	B	C	D	E
	A	-	4	7	3	4
	B	4	-	6	3	4
FROM	C	7	6	-	7	5
	D	3	3	7	-	7
	E	4	4	5	7	-

UNIT – II – TRANSPORTATION AND ASSIGNMET MODEL
MODEL QUESTION PAPER

PART - A

1. Define and specify the objective of transportation model.
2. What is meant by unbalanced problem in transportation? How will you convert unbalanced problem into balanced problem in transportation?
3. List the methods used to find initial solution in transportation?
4. What is degeneracy in transportation?
5. Define and list out the objectives of assignment?
6. Specify the route conditions in travelling salesman problem.
7. How to convert maximization problem into minimization problem in assignment?
8. Differentiate between assignment problem and transportation problem.
9. Find the initial solution for the given transportation problem using North-West corner method.

		Destination				
		D1	D2	D3	D4	Supply
	O1	6	1	9	3	100
Origins	O2	11	5	2	8	60
	O3	10	12	4	7	40
	Demand	40	60	80	20	

10. A Computer centre has got 4 programmers. The centre needs 4 application programmes to be developed. The centre head, after studying carefully the programmes to be developed, estimates the computer time (in minutes) required by the respective experts to develop the application programmes as follows:

		Programmes			
		A	B	C	D
	1	120	100	80	90
Programmers	2	80	90	110	70
	3	110	140	120	100
	4	90	90	80	90

Assign the programmers to the programmes in such a way that the total computer time gets minimized.

PART - B

1. What is meant by transportation? Specify the objectives of transportation tool. Write the procedure for making unbalanced problem into balanced problem with an example.

OR

2. Solve the transportation problem using MODI method.

	Destination				
Sources	D1	D2	D3	D4	Supply
S1	7	3	8	6	60
S2	4	2	5	10	100
S3	2	6	5	1	40
Demand	20	50	50	80	

3. Write the procedure to solve transportation problem using MODI method.

OR

4. For the given transportation problem, find the initial solution using North-west corner method and final optimal solution using MODI method.

		Destination				
Sources		D1	D2	D3	D4	Supply
S1	6	1	9	3	70	
S2	11	5	2	8	55	
S3	10	12	4	7	70	
Demand	85	35	50	45		

5. Write the procedure for a) North-West corner method b) Least-Cost method c) Vogel's approximation method.

OR

6. Using U-V method, solve the given transportation problem.

	Destination			
Sources	D1	D2	D3	Supply
S1	5	1	7	10
S2	6	4	6	80
S3	3	2	5	15
S4	5	3	2	40
Demand	75	20	50	

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7. Write Hungarian algorithm.

OR

8. Given are the costs for assigning jobs to the persons working in an organization, find the minimum cost using the given information.

		Persons			
		P1	P2	P3	P4
	A	8	8	4	3
Jobs	B	4	2	1	6
	C	6	8	10	12
	D	14	18	20	22

9. Write the procedure for a) Maximization problem in assignment with an example b) Restricted assignment problem with an example c) Conditions to solve Travelling salesman assignment problem.

OR

10. Solve the given assignment problem in which profits are given for various territories.

		Territories			
		T1	T2	T3	T4
	P1	10	22	12	4
Profits	P2	16	18	22	10
	P3	24	20	12	18
	P4	16	14	24	20