→ SMEA 4001

Resource management Techniques.

UNIT - I - Introduction & linear Programing.

UNIT - II - Transportation & Assignment models

UNIT - III - Resource Schedule and Network. Analysis

UNIT - IV - Inventory models

UNIT - V - Queing Theory & Replacement models.

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UNIT - II

Transportation and Assignment models.

Assignment model

→ It is a Part of transportation models.

→ The method we are using is Hungarian model method

Hungarian method :

minimize $Z = \sum_{i=n}^{n} \sum_{j=n}^{n} c_{ij} x_{ij}$

$$\begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & & \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix}$$

→ Taken in matrix form.

Square matrix — Balanced model.

Rectangular matrix — unbalanced model

→ minimizing the cost & maximizing the Profit

Step ①: Find the minimum values from each row, and subtract the first element from ~~remaining~~ each elements of the row

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

Step ②: Same as step ① but column value.

Step ③: Find ~~at~~ whether we have at least one zero from each row & each column.

→ If we find atleast one zero the Problem is over

→ If not go to step ④.

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Assignment models (BT)

→ It is a particular case of Transportation problems in which the object is to assign a number of Jobs/task/source to an equal number of facilities/machine/destination at a minimum cost.

Balanced Problems :

Problem: The assignment cost, of the assigning any one operator to any one machine is given in the following table.

$$\begin{array}{cc} & \text{Operators} \\ & \begin{array}{cccc} I & II & III & IV \end{array} \\ \text{machines} \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 10 & 5 & 13 & 15 \\ 3 & 9 & 18 & 3 \\ 10 & 7 & 3 & 2 \\ 5 & 11 & 9 & 7 \end{bmatrix} \end{array}$$

Find the optimal assignment by Hungarian method.

soln

Step ① - minimum value from each row and subtract from remaining elements.

$$\begin{array}{c} 5\min \\ 3\min \\ 2\min \\ 5\min \end{array} \begin{bmatrix} 5 & 0 & 8 & 10 \\ 0 & 6 & 15 & 0 \\ 8 & 5 & 1 & 0 \\ 0 & 6 & 4 & 2 \end{bmatrix} \rightarrow$$ Reduced matrix from the given matrix.

Step ② : from Reduced matrix find the min value and subtract it from other elements

$$\begin{bmatrix} 5 & 0 & 7 & 10 \\ 0 & 6 & 14 & 0 \\ 8 & 5 & 0 & 0 \\ 0 & 6 & 3 & 2 \end{bmatrix}$$
$$\begin{array}{cccc} 0\min & 0\min & 1\min & 0\min \end{array}$$

Step③: check atleast one zero from each row and each column.

→ If we have atleast one zero from each row & column, The Problem gets over.

→ Here from each row & each column we have one zero

∴ ~~The current assignment is optimal.~~

$$\begin{array}{c|cccc} & I & II & III & IV \\ \hline A & 5 & \boxed{0} & 7 & 10 \\ B & 0 & 6 & 14 & \boxed{0} \\ C & 8 & 5 & \boxed{0} & 0 \\ D & \boxed{0} & 6 & 3 & 2 \end{array}$$

→ find zeros in ~~column~~ row and cross the respective column

From Question

5 : A → II
3 : B → IV
3 : C → III
5 : D → I

each operator is assigned to each machine

~~The optimum value~~ is

~~$= 5 + 3 + 3 + 5 = 16$~~

∴ The current assignment

$$\begin{array}{c|cccc} & I & II & III & IV \\ \hline A & 5 & \boxed{0} & 7 & 10 \\ B & \boxed{0} & 6 & 14 & \boxed{0} \\ C & 8 & 5 & \boxed{0} & \cancel{0} \\ D & \boxed{0} & 6 & 3 & 2 \end{array}$$

already crossed

→ more than one zero leave the rows/columns

If one zero is already striken we should not consider that zero and go to second zero of that row/column

Optimum schedule $\{A \to II,\ B \to IV,\ C \to III,\ D \to I\}$

5 , 3 , 3 , 5

From Question matrix

$= 5 + 3 + 3 + 5$

$= \underline{16}$

Problem ②: consider the Problem of Assigning 5 Jobs to five Persons, the assignment cost ~~is~~ as given as Follows.

$$
\text{Persons}\quad
\begin{array}{c|ccccc}
 & \multicolumn{5}{c}{\text{Jobs}} \\
 & I & II & III & IV & V \\
\hline
A & 8 & 4 & 2 & 6 & 1 \\
B & 0 & 9 & 5 & 5 & 4 \\
C & 3 & 8 & 9 & 2 & 6 \\
D & 4 & 3 & 1 & 0 & 3 \\
E & 9 & 5 & 8 & 9 & 5
\end{array}
$$

Determine the optimum schedule & value.

Soln: Step ①:

Reduced matrix

$$
\begin{bmatrix}
7 & 3 & 1 & 5 & 0 \\
0 & 9 & 5 & 5 & 4 \\
1 & 6 & 7 & 0 & 4 \\
4 & 3 & 1 & 0 & 3 \\
4 & 0 & 3 & 4 & 0
\end{bmatrix}
\begin{array}{l}
1\,\text{min} \\
0\,\text{min} \\
2\,\text{min} \\
0\,\text{min} \\
5\,\text{min}
\end{array}
$$

Step ②:

$$
\begin{array}{c}
\begin{bmatrix}
7 & 3 & 0 & 5 & 0 \\
0 & 9 & 4 & 5 & 4 \\
1 & 6 & 6 & 0 & 4 \\
4 & 3 & 0 & 0 & 3 \\
4 & 0 & 2 & 4 & 0
\end{bmatrix} \\
\begin{array}{ccccc}
0\,\text{min} & 0\,\text{min} & 1\,\text{min} & 0\,\text{min} & 0\,\text{min}
\end{array}
\end{array}
$$

Step ③: We have atleast one zero from each column/row.

$$
\begin{array}{c|ccccc}
 & I & II & III & IV & V \\
\hline
A & 7 & 3 & \cancel{0} & 5 & \boxed{0} \\
B & \boxed{0} & 9 & 4 & 5 & 4 \\
C & 1 & 6 & 6 & \boxed{0} & 4 \\
D & 4 & 3 & \boxed{0} & \cancel{0} & 3 \\
E & 4 & \boxed{0} & 2 & 4 & \cancel{0}
\end{array}
$$

$\cancel{0}$ – already striken zero won't be considered.

$$
\left.
\begin{array}{ccccc}
A \to V, & B \to I, & C \to IV, & D \to III, & E \to II \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
1 & 0 & 2 & 1 & 5
\end{array}
\right\}\ \text{optimal schedule}
$$

$$= 1 + 0 + 2 + 1 + 5 = \underline{9} \;\Big\}\ \text{optimal cost}$$

Problem ③

Jobs machine

~~machine~~ Jobs

$$\begin{bmatrix} 5 & 7 & 11 & 6 \\ 8 & 5 & 9 & 6 \\ 4 & 7 & 10 & 7 \\ 10 & 4 & 8 & 3 \end{bmatrix}$$

From the table find the assignment schedule and minimum cost ↳ 23

Soln:

Step ①:

$$\begin{bmatrix} 0 & 2 & 6 & 1 \\ 3 & 0 & 4 & 1 \\ 0 & 3 & 6 & 3 \\ 7 & 1 & 5 & 0 \end{bmatrix}$$

Step ②:

$$\begin{bmatrix} 0 & 2 & 2 & 1 \\ 3 & 0 & 0 & 1 \\ 0 & 3 & 2 & 3 \\ 7 & 1 & 1 & 0 \end{bmatrix}$$

Step ③:

$$\begin{bmatrix} \boxed{0} & 2 & 2 & 1 \\ 3 & \boxed{0} & \cancel{0} & 1 \\ \cancel{0} & 3 & 2 & 3 \\ 7 & 1 & 1 & \boxed{0} \end{bmatrix}$$

follow unbalanced Problem to complete the Problem.

Step ④:

$$\begin{bmatrix} [0] & 1 & 1 & 1 \\ 4 & 0 & 0 & 2 \\ \cancel{0} & 2 & 1 & 3 \\ 7 & 0 & 0 & [0] \end{bmatrix}$$

Unbalanced Assignment Problem

→ If number of rows & columns in the cost matrix is not equal, Then The assignment Problem is Said to be unbalanced

→ we have to convert The unbalanced Problem by introducing dummy (row) column all are zero.

Problem ①: A company has four machines to do Three Jobs. each Job can be assigned to one and only machine. The cost of each Job on each machine is given in The following table.

machine

$$
\begin{array}{cc}
 & \begin{array}{cccc} I & II & III & IV \end{array} \\
\begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 18 & 24 & 28 & 32 \\ 8 & 13 & 17 & 19 \\ 10 & 15 & 19 & 22 \end{bmatrix}
\end{array}
$$

Jobs

→ It is unbalanced, so add a dummy row.

$$
\begin{bmatrix} 18 & 24 & 28 & 32 \\ 8 & 13 & 17 & 19 \\ 10 & 15 & 19 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

Step ①:

$$
\begin{bmatrix} 0 & 6 & 10 & 14 \\ 0 & 5 & 9 & 11 \\ 0 & 5 & 9 & 12 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

Step ②:

$$
\begin{bmatrix} 0 & 6 & 10 & 14 \\ 0 & 5 & 9 & 11 \\ 0 & 5 & 9 & 12 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

→ This Problem is not optimal because we are not having marked elements from each row & each column.

Step ③:

$$
\begin{bmatrix} \boxed{0} & 6 & 10 & 14 \\ \cancel{0} & 5 & 9 & 11 \\ \cancel{0} & 5 & 9 & 12 \\ \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} \end{bmatrix}
$$

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Select the min
Step (4): from the unstriken elements and subtract from all the unstriken elements and add the min to the element of intersection

$$\begin{bmatrix} \boxed{0} & & 5 & 9 \\ 0 & \boxed{0} & 4 & 6 \\ 0 & 0 & 4 & 7 \\ 5 & 0 & \boxed{0} & \cancel{0} \end{bmatrix}$$

→ Still the Problem is not optimal because ~~do~~ we are not having ~~all~~ the marked elements in all rows and columns, so repeat step (4)

$$\begin{bmatrix} \boxed{0} & 1 & 1 & 5 \\ \cancel{0} & 0 & 0 & 2 \\ \cancel{0} & 0 & 0 & 3 \\ 0 & 4 & \cancel{0} & \boxed{0} \end{bmatrix}$$

→ If the Problem cannot be continued, consider the diagonal zero.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} \mathrm{I} & \mathrm{II} & \mathrm{III} & \mathrm{IV} \end{array} \\ \begin{bmatrix} \boxed{0} & 1 & 1 & 5 \\ \cancel{0} & \boxed{0} & \cancel{0} & 2 \\ \cancel{0} & \cancel{0} & \boxed{0} & 3 \\ 0 & 4 & \cancel{0} & \boxed{0} \end{bmatrix} \end{array}$$

here number of marked elements is equals to number of rows and columns

∴ The optimal schedule

A → I, B → II, C → III, D → IV

optimal cost

$= 18 + 13 + 19 + 0$

$= \underline{50}$

Problem (2):

Note: To maximize the cost, subtract all the elements by largest value of the cost matrix

Problem: A company has a team of 4 salesmen, there are 4 districts where the company wants to start its business, after taking into account, the capabilities of salesmen and the nature of the districts, the company estimates that the profit per day in rupees for each salesmen in each district as follows

$$
\text{Salesmen}\quad
\begin{array}{c|cccc}
 & \multicolumn{4}{c}{\text{districts}} \\
 & I & II & III & IV \\
A & 16 & 10 & 14 & 11 \\
B & 14 & 11 & 15 & 15 \\
C & 15 & 15 & 13 & 12 \\
D & 13 & 12 & 14 & 15
\end{array}
$$

Find the assignment of salesmen to various districts, which will yield a maximum profit.

Soln: **Step ①:** Find the greatest value & subtract from all elements in the matrix

$$
\begin{bmatrix}
0 & 6 & 2 & 5 \\
2 & 5 & 1 & 1 \\
1 & 1 & 3 & 4 \\
3 & 4 & 2 & 1
\end{bmatrix}
$$

[16 is max in the matrix]

Step ②: Row Reduction.

$$
\begin{array}{c}
0\min \\ 1\min \\ 1\min \\ 1\min
\end{array}
\begin{bmatrix}
0 & 6 & 2 & 5 \\
1 & 4 & 0 & 0 \\
0 & 0 & 2 & 3 \\
2 & 3 & 1 & 0
\end{bmatrix}
$$

Step ③: column reduction

$$
\begin{array}{c}
\begin{bmatrix}
0 & 6 & 2 & 5 \\
1 & 4 & 0 & 0 \\
0 & 0 & 2 & 3 \\
2 & 3 & 1 & 0
\end{bmatrix} \\
\begin{array}{cccc}
0\min & 0\min & 0\min & 0\min
\end{array}
\end{array}
$$

Step ③:

$$
\begin{bmatrix}
\boxed{0} & 6 & 2 & 5 \\
1 & 4 & \boxed{0} & \cancel{0} \\
\cancel{0} & \boxed{0} & 2 & 3 \\
2 & 3 & 1 & \boxed{0}
\end{bmatrix}
$$

$A \to I,\ B \to III,\ C \to II,\ D \to IV$

$= 16 + 15 + 15 + 15$

Problem ②: Solve the AP fo maximization of given Profit matrix

$$\begin{bmatrix} 51 & 53 & 54 & 50 \\ 47 & 50 & 48 & 50 \\ 49 & 50 & 60 & 61 \\ 63 & 64 & 60 & 60 \end{bmatrix}$$

Travelling sales man Problem:

→ The salesman should go through every city exactly once except the Home city (starting city).

→ The salesman starts from one city (Home) and comes back to the same city (Home).

Def: If the salesman starts from his home city and Passes through each city exactly once. and. returns to the Home city.

→ The diagonal elements are empty

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & (x) & & & \\ B & & (x) & & \\ C & & & (x) & \\ D & & & & (x) \end{array}$$

Problem ①: Solve the following TSP!

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$$\begin{array}{c|cccc} & A & B & C & D \\ A & - & 46 & 16 & 40 \\ B & 41 & - & 50 & 40 \\ C & 82 & 32 & - & 60 \\ D & 40 & 40 & 36 & - \end{array}$$

Schedule.

soln:

$$\begin{bmatrix} \infty & 46 & 16 & 40 \\ 41 & \infty & 50 & 40 \\ 82 & 32 & \infty & 60 \\ 40 & 40 & 36 & \infty \end{bmatrix}$$

Step ①: Row Reduction

$$\begin{bmatrix} \infty & 30 & 0 & 24 \\ 1 & \infty & 10 & 0 \\ 50 & 0 & \infty & 28 \\ 4 & 4 & 0 & \infty \end{bmatrix}$$

Step ②: column Reduction

$$\begin{bmatrix} \infty & 30 & 0 & 24 \\ 0 & \infty & 10 & 0 \\ 49 & 0 & \infty & 28 \\ 3 & 4 & 0 & \infty \end{bmatrix}$$

Step ③

$$\begin{bmatrix} \infty & 30 & \boxed{0} & 24 \\ \boxed{0} & \infty & 10 & 0 \\ 49 & \boxed{0} & \infty & 28 \\ 3 & 4 & 0 & \infty \end{bmatrix}$$

→ subtract ~~Adding~~ min to all unstriken elements and add it to intersection Point step ①:

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & 30 & [0] & 21 \\ B & 0 & \infty & 13 & [0] \\ C & 46 & [0] & \infty & 25 \\ D & [0] & 4 & 0 & \infty \end{array}$$

Each row & each column we have an encircled cost (value)

∴ cost schedule

$A \to C,\ B \to D,\ C \to B,\ D \to A$

$\Rightarrow A \to C \to B \to D \to A$

→ The Root condition is satisfied.

minimum cost $= 16 + 32 + 40 + 40$

$= 128$

Problem ②: Solve the following travelling salesman problem to minimize the cost price per cycle.

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & - & 3 & 6 & 2 & 3 \\ B & 3 & - & 5 & 2 & 3 \\ C & 6 & 5 & - & 6 & 4 \\ D & 2 & 2 & 6 & - & 6 \\ E & 3 & 3 & 4 & 6 & - \end{array}$$

Transportation model / Problem:

→ It deals with the transportation of 'm' product (comm... from 'm' source (supply) to 'n' destination (sink) or (demand)

→ The following conditions should be assumed (i) level of supply at each source and the amount of demand at each destination

(ii) The unit transportation cost of the commodity from each source to each destination are known.

Mathematical model of Transportation Problem.

→ let a_i be the supply at each source 'i' and b_j be the demand at each sink/destination 'j', c_{ij} be the unit transportation cost in the i^{th} source and j^{th} demand.

→ x_{ij} be the number of units shifted from source i to j^{th} demand.

$$\text{minimize } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to the condition

$$(i) \sum_{j=1}^{n} x_{ij} = a_i \quad i = 1, 2, 3 \dots \infty$$

$$(ii) \sum_{i=1}^{m} x_{ij} = b_j \quad j = 1, 2, 3 \dots \infty$$

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3 methods - North west corner method
least cost method
vogel's method (vogel's approximation)

North west corner method

Problem (1): find the initial feasible solution of the transportation problem by (i) North west corner method (ii) least cost method (iii) vogel's approximation method

destination

Source				supply
	1	2	6	$a_1 = 7$
	0	4	2	$a_2 = 12$
	3	1	5	$a_3 = 11$
demand	$b_1 = 10$	$b_2 = 10$	$b_3 = 10$	30

$\underline{\text{soln}}$ → Given transportation Problem is balance

$$\Sigma a_i = \Sigma b_j$$

$$30 = 30$$

(i) North west corner

1	2	6	7
0	4	2	12
3	1	5	11
10	10	10	

→ In North west corner, b. the min from supply & demand will be allocat and subtract the min ~~with~~ ~~and~~ ~~so~~ from supply & demand.

⇓

1 $\boxed{7}$	2 ×	6 ×	~~7~~ 0
0	4	2	12
3	1	5	11
~~10~~ 3	10	10	

→ If we get zero cancle the respective row or column. and select the North west corner of remaining elements

⇓

1 $\boxed{7}$	2 ×	6 ×	0
0 $\boxed{3}$	4	2	~~12~~ 9
3 ×	1	5	11
~~3~~ 0	10	10	

⇒ Repeat the Process

⇓

1 $\boxed{7}$	2 ×	6 ×	0
0 $\boxed{3}$	4 $\boxed{9}$	2 ×	~~9~~ 0
3 ×	1	5	11
0	~~10~~ 1	10	

⇐

1 $\boxed{7}$	2 ×	6 ×	0
0 $\boxed{3}$	4 $\boxed{9}$	2 ×	0
3 ×	1 $\boxed{1}$	5	~~11~~ 10
0	~~1~~ 0	10	

⇓

1 $\boxed{7}$	2 ×	6 ×	0
0 $\boxed{3}$	4 $\boxed{9}$	2 ×	0
3 ×	1 $\boxed{1}$	5 $\boxed{10}$	~~10~~ 0
0	0	~~10~~ 0	

here = $m+n-1 \Rightarrow 3+3-1 = 5$

= No. of allocated elements

∴ The IFS is non degeneracy

Transportation cost

$= (1\times7)+(0\times3)+(4\times9)+(1\times1)+(5\times10) = 94$ ₹

(ii) least cost method

→ from all elements select the min value and ~~select~~ allocate the min of supply and demand and subtract the allocated element from both supply & demand and cross the zeroth row/column elements

1 ×	2	6	7
0 $\boxed{10}$	4	2	~~12~~ 2
3 ×	1	5	11
~~10~~ 0	10	10	

⇓

1 ×	2 ×	6	7
0 $\boxed{10}$	4 ×	2	2
3 ×	1 $\boxed{10}$	5	~~11~~ 1
0	~~10~~ 0	10	

⇒

1 ×	2 ×	6	7
0 $\boxed{10}$	4 ×	2 $\boxed{2}$	~~2~~ 0
3 ×	1 $\boxed{10}$	5	1
0	0	~~10~~ 8	

⇓

1 ×	2 ×	6 $\boxed{1}$	7
0 $\boxed{10}$	4 ×	2 $\boxed{2}$	0
3 ×	1 $\boxed{10}$	5 $\boxed{1}$	~~1~~ 0
0	0	~~8~~ 7	

⇐

1 ×	2 ×	6 $\boxed{7}$	~~7~~ 0
0 $\boxed{10}$	4 ×	2 $\boxed{2}$	0
3	1 $\boxed{10}$	5 $\boxed{1}$	0
0	0	~~7~~ 0	

Here $m+n-1$

$= 5$

= no. of allocations

⇒ The given IFS is non degenerous

Transport cost

$= (6\times 7) + (0\times 10) + (2\times 2) + (1\times 10) + (5\times 1)$

$= 61$ Rs

(iii) vogel's method.

→ from each row subtract the min 2 values from each other and write as penality for both rows & columns.

				Penality		
1 $\boxed{7}$	2	6 ×	~~7~~ 0	(1)	(1)	(1)
0 $\boxed{2}$	4 ×	2 $\boxed{10}$	~~12~~ ~~10~~ 0	(2)	(4)	
3 $\boxed{1}$	1 $\boxed{10}$	× 5	~~11~~ ~~1~~ 0	(2)	(2)	(2)
~~8~~ ~~1~~ 0	~~10~~ 0	~~10~~ 0				
(1)	(1)	(3)				
(1)	(1)					
(2)	(1)					

M.C = 40

soln → Here the penalty ... and ... the
corresponding ~~row~~/column and subtract
min value of supply & demand
the values

(i

				Penality
1	2	6 ×	7	(1)
0	4	2 [10]	~~12~~ 2	(2)
3	1	5 ×	11	(2)
10	10	~~10~~ 0		
(1)	(1)	(3)		

→ For each step find the Penality and continue the Process

				Penality
1	2	6 ×	7	(1)
0 [2]	4 ×	2 [10]	~~2~~ 0	(4)
3	1	5 ×	11	(2)
~~10~~ 8	10	0		
(1)	(1)	–		

⇓

				Penality
1 [7]	2 ×	6 ×	~~7~~ 0	(1)
0 [2]	4 ×	2 [10]	0	–
3	1	5 ×	11	(2)
8	10	0		
(2)	(1)	–		

⇓

1 [7]	2 ×	6 ×	0
0 [2]	4 ×	2 [10]	0
3 [1]	1	5 ×	~~11~~ 10
~~1~~0	10	0	

⇒

1 [7]	2 ×	6 ×	0
0 [2]	4 ×	2 [10]	0
3 [1]	1 [10]	5 ×	~~10~~ 0
0	~~10~~ 0	0	

Transport cost

$= (1\times 7) + (0\times 2) + (2\times 10) + (3\times 1) + (1\times 10)$

$= 40$ Rs

modi's method :

$2-(-2+1) =$

1 [7]	2 [3]	6 [3]	$u_1 = 1$
0 [2]	4	2 [10]	$u_2 = 0$
3 [1]	1 [6] [10]	5 [0]	$u_3 = 3$
$v_1 = 0$	$v_2 = -2$	$v_3 = 2$	

20 50 30

maximum col | row

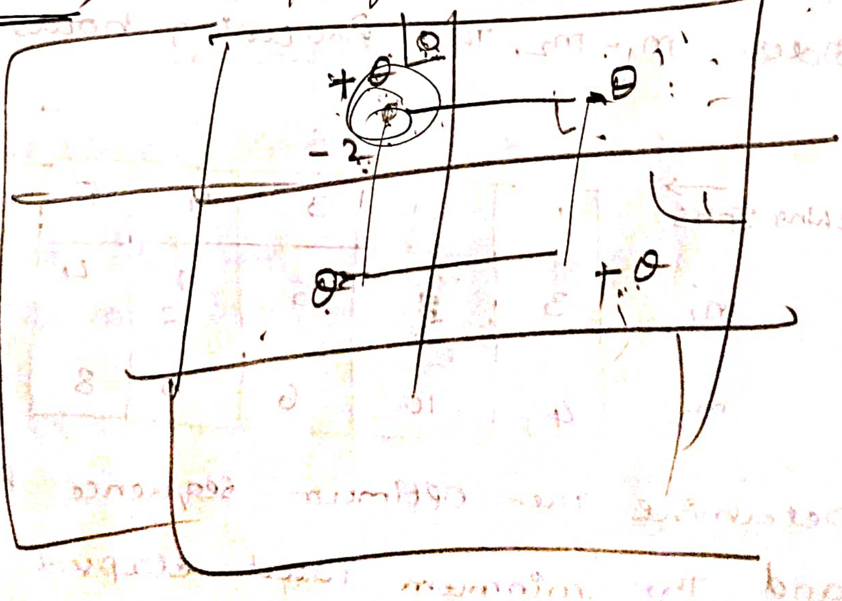
allocated cells $u_i + v_j$

nonallocated cells $d_{ij} = c_{ij} - (u_i + v_j)$

$d_{ij} > 0$ and one $d_{ij} = 0$. $d_{ij} > 0$ mean's the soln is optimal, we have another optimal soln

If $d_{ij} > 0$, there is unique optimal soln

If $d_{ij} < 0$, ~~step~~ go to next step.



UNIT - III

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Resource scheduling & Network Analysis.

* Problems of sequencing
(i) n Jobs with 2 machines Problems.
(ii) n Jobs with 3 machines
(iii) n Jobs with m machines
(iv) 2 Jobs with m machines [Graphical method]

→ Problem of determining the appropriate order (sequence) for a series of Jobs to be done on a finite number of service facilities to minimize the total time taken for finishing the Job.

Total elapsed time (T)

→ The time between starting the first Job and completion the last Job is known as total elapsed time.

Ideal time.

Ideal time
→ on a machine is the time, the machine remains idle during the total elapsed time.

N Jobs with 2 machines : (Johnson method)

Q! There are five Jobs each of which is to be Processed through machine M_1 and M_2 in the order $M_1 - M_2$, The Processing hours are as follows.

machine ↓ Jobs →	1	2	3	4	5
M_1	3	8	5	7	4
M_2	4	10	6	5	8

Determine the optimum sequence for the five Jobs and the minimum total elapsed time also find the ideal time of machines M_1 & M_2

Optimum Sequence:

	1	2	3	4	5
M_1	$\boxed{3}$	8	$\boxed{5}$	7	$\boxed{4}$
M_2	4	10	6	$\boxed{5}$	8

$M_1 - M_2$

→ find the min from both machines and go on ascending order

→ If there are two min numbers find the difference between with the other machines and go with the minimum.

1	5	3	2	4

$M_1 \rightarrow$ $\leftarrow M_2$

Total elapsed time:

Jobs	M_1 In	M_1 Out	M_2 In	M_2 Out
1	0	0+3=3	3	3+4=7
5	3	3+4=7	7	7+8=15
3	7	7+5=12	15	15+6=21
2	12	12+8=20	21	21+10=31
4	20	20+7 =27	31	31+5=36

→ Out of 1st will be In of 2nd Job

→ Initial time is zero

→ Out time of 1st Job(m_1) is always In time for machine 2(m_2)

→ compare the In of 2nd Job of m_2 with out of m_1's 2nd Job

$m_1 > m_2$ → In of m_2 will be greater

→ continue the above step with all Jobs.

∴ Total elapsed time = 36-0 = 36 hrs

Ideal time of m_2 = 3hrs " 3-0 = 3"

Ideal time of m_1 = 36-27 = 9hrs

N Jobs and 3 machines:

let m_1, m_2, m_3 be 3 machines with n Jobs

→ we need to convert to n Jobs and 2 machines

for that conditions

(i) min. time taken by m_1 $\geq$ max time taken to complete the Job by m_2

(or)

(ii) min time taken by m_3 $\geq$ max time taken to complete the Job by m_2

→ The conditions (i) & (ii) should be satisfied. if not the method fails to find the optimal solution.

→ If the condition satifies then

$$\left.\begin{aligned} m_1 + m_2 &= H \\ m_2 + m_3 &= K \end{aligned}\right\} \text{New machines}$$

finally we've converted to n Jobs & 2 machines

Problem: Find the sequence that minimize the total elapsed time required to complete the following task on the machines in the order 1-2-3. Find also the min total elapsed time and ideal times on all three machines

Task.	A	B	C	D	E	F	G
m_1	3	8	7	4	9	8	7
m_2	4	3	2	5	1	4	3
m_3	6	~~5~~ 7	5	11	5	6	12

soln: (i) min time taken by $m_1 = 3$, $\cancel{\geq}$ max time taken by $m_2 = 5$

(iii) min time taken by $m_3 = 5 \geq$ max time taken by $m_2 = 5$

condition (ii) is satisfying

	A	B	C	D	E	F	G
H	$\boxed{7}$	11x	9x	$\boxed{9}$	10x	x 12	$\boxed{10}$
K	10x	$\boxed{10}$	$\boxed{7}$	16 x	$\boxed{6}$	$\boxed{10}$	15

Now Perform n jobs and 2 machines

optimal sequence:

H →

A	D	G	F	B	C	E

← K

Total time elapsed:

Jobs	m_1 In	m_1 Out	m_2 In	m_2 Out	m_3 In	m_3 Out
A	0	0+3=3	3	3+4=7	7	7+6=13
D	3	3+4=7	7	7+5=12	13	13+11 = 24
G	7	7+7=14	14>12 = 14	14+3 = 17	24	24+12 = 36
F	14	14+8=22	22>17 22	22+4 = 26	36	36+6 = 42
B	22	22+8=30	30>26 30	30+3 = 33	42	42+7 = 49
C	30	30+7=37	37>33 37	37+2=39	49	49+5 = 54
E	37	37+9=46	46>39 46	46+1 = 47	54	54+5 = 59

Total elapsed time (T) = 59 − 0 = 59 hrs

Ideal time of m_3 = 7

Ideal time of

m_1 = 59 − 46 = 13 hrs

m_2 = (59−47)+3+2+5+4+4+7

= 12+25

= 37 hrs

let $A_1, A_2, \ldots A_m$ be the machines

Conditions

$$\min_i A_{i1} \geq \max_i A_{ij} \quad j = 1, 2, 3 \ldots m-1$$

(or)

$$\min_i A_m \geq \max_i A_{ij} \quad j = 1, 2 \ldots m-1$$

New machines $H_i = A_{i1} + A_{i3} + \ldots + A_{im-1}$

$i = 1, 2, 3 \ldots n$

$K_i = A_{i2} + A_{i3} + \ldots + A_m$

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Q: Solve the following Problem of 4 Jobs on 6 machines

Jobs		m_1	m_2	m_3	m_4	m_5	m_6
	A	19	8	8	3	11	24
	B	18	6	9	6	9	18
	C	12	5	8	5	7	15
	D	20	5	3	4	8	11

Soln:

$\min \left(\begin{array}{l} \text{Processing Time} \\ \text{given by } m_1 \end{array} \right) = 12$

$12 \geq 8, 9, 6, 11$

$\max \left(\begin{array}{l} \text{Processing time} \\ \text{given by} \end{array} \right) = 8, 9, 6, 11$

m_2, m_3, m_4, m_5

$\min \left(\begin{array}{l} \text{Processing time} \\ \text{given by } m_6 \end{array} \right) = 11 \not\geq \max \left(\begin{array}{l} \text{Processing time} \\ \text{given by } m_2, m_3, m_4, m_5 \end{array} \right)$

$8, 9, 6, 11$

→ Here two conditions are ~~be~~ satisfying.

New machines

	H	K
A	$\boxed{49}$	54 ×
B	$\boxed{48}$	48 ×
C	$\boxed{37}$	40 ×
D	40	$\boxed{31}$

Now
The Problem of sequencing is similar to n Jobs and 2 machines.

Optimal sequence

C	B	A	D

H →   K ←

Total elapsed time :

Jobs	M_1 In	M_1 Out	M_2 In	M_2 Out	M_3 In	M_3 Out
C	0	12	12	12+5=17	17	17+8 = 25
B	12	12+18=30	30>17 30	30+6=36	36>25 36	36+9 = 45
A	30	30+19 = 49	49>36 49	49+8 = 57	57>45 57	57+8 = 65
D	49	49+20 = 69	69>57 69	69+5 = 74	74>65 74	74+3 = 77

M_6 In	M_6 Out		M_4 In	M_4 Out	M_5 In	M_5 Out
37	37+15 = 52	C	25	25+5 = 30	30	30+7 = 37
66>52 60	60+18 = 78	B	45>30 45	45+6 = 51	51>37 51	51+9 = 60
79>78 79	79+24 = 103	A	65>51 65	65+3 = 68	68>60 68	68+11 = 79
103>81 103	103+11 = 114	D	79>68 79	79+4 = 81	81>79 81	81+8 = 89

Total elapsed time = 114.

~~Ideal~~ Idle time of $m_1 = 114 - 69 = 45$ hrs

Initial time of

Idle " $m_2 = 114 - 74 + \underbrace{13 + 13 + 12}$ + (12) = 90

Idle " $m_3 = 114 - 77 + \underbrace{11 + 12 + 9} + 17 = 86$

Idle " $m_4 = 114 - 81 + 15 + 14 + 9 + 25 = 96$ hrs

" " $m_5 = 114 - 89 + 14 + 8 + 2 + 30 = 79$ hrs

Initial time.

" " $m_6 =$ (37) $+ 8 + 1$

$= 46$ hrs

2 Jobs with m machines [graphical method]:

→ The conditions to follow the method.

(i) The technological ordering of each of the Jobs through m machines is known in advance

(ii) Each machine can process only one Job at a time

(iii) The exact processing time on all the 'm' machines are known.

Formula: The min Total elapsed time = Processing time of Job 1 + ~~Ideal~~ Idle time of Job 1

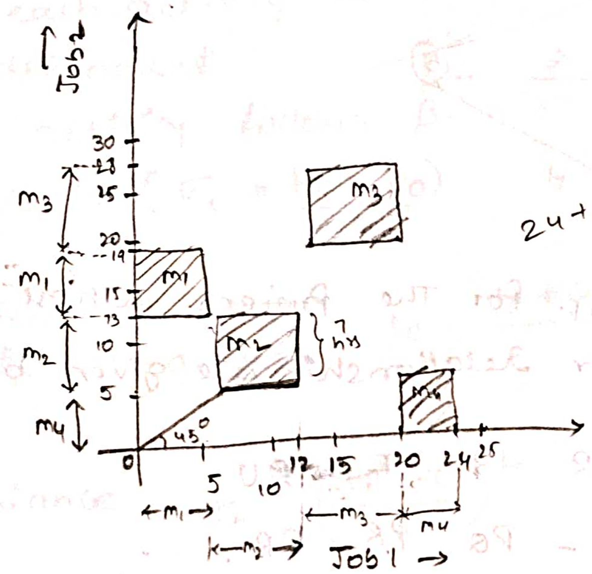
(or)

= Processing time of Job 2 + Ideal time of Job 2

→ The optimal sequence of the processing is found the graph drawn.

Q: Determine the min time need to process the two Jobs on 4 machines m_1, m_2, m_3, m_4, the Technological order is given below

Job 1	m_1 5	m_2 7	m_3 8	m_4 4
Job 2	m_4 5	m_2 8	m_1 6	m_3 9



$24+7 = 31$ hrs

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Network Analysis

Scheduling by PERT & CPM

→ A project is defined as the combination of inter-related activities all of which must be executed in a certain order to achieve a set of Goal.

A systematic scientific approach has been considered to complete the task.

(i) Program Evaluation Review technique (PERT)

(ii) critical Path method (CPM)

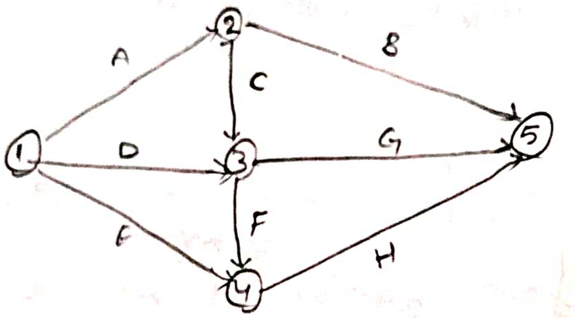
→ The three main ~~con~~ manageble functions are

(i) Planning
(ii) scheduling &
(iii) controlling.

Q: Draw the Network for The Project whose activities and the relationship are given as follows

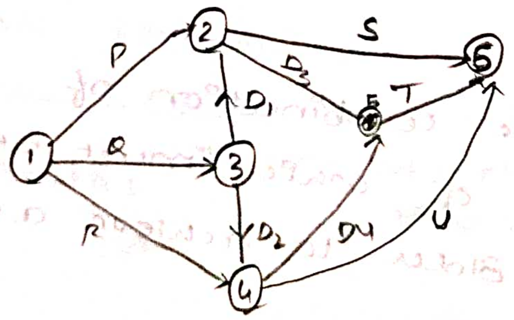
- Activity A, D, E are start simultaneously

B, C > A, G, F > D, C, H > G, F



Q: Draw The Network for The Project whose activities and Their relationship are given below

Activity :	P	Q	R	S	T	U
Predecessor :	–	–	–	PQ	PR	QR



$D_1 D_2 D_3 D_4$ are The dummy activities

PERT method :

→ Pert depends upon 3 time estimates

1. Optimistic time (t_o) - least time
2. Pessimistic time (t_p) - greatest time
3. most likely time. (t_m) - Equal Proportion (or) Normal time

~~optimistic time~~

~~→ In this duration of any activity~~

The two main assumptions to do the Project by the model PERT

→ for each activity the duration of time is independent

→ The activity follows β- distribution with standard deviation ($\sigma_t = \frac{t_p - t_o}{6}$)

→ expected duration of each activity $t_e = \frac{t_o + 4t_m + t_p}{6}$

→ variance $\sigma_t^2 = \left(\frac{t_p - t_o}{6}\right)^2$

→ critical path is the ~~togn~~ longest duration to first complete the Project from the initial node to the last terminal node

PERT

Q! construct the network for the Project whose activities and the three time estimates of these activities are given below.

compute

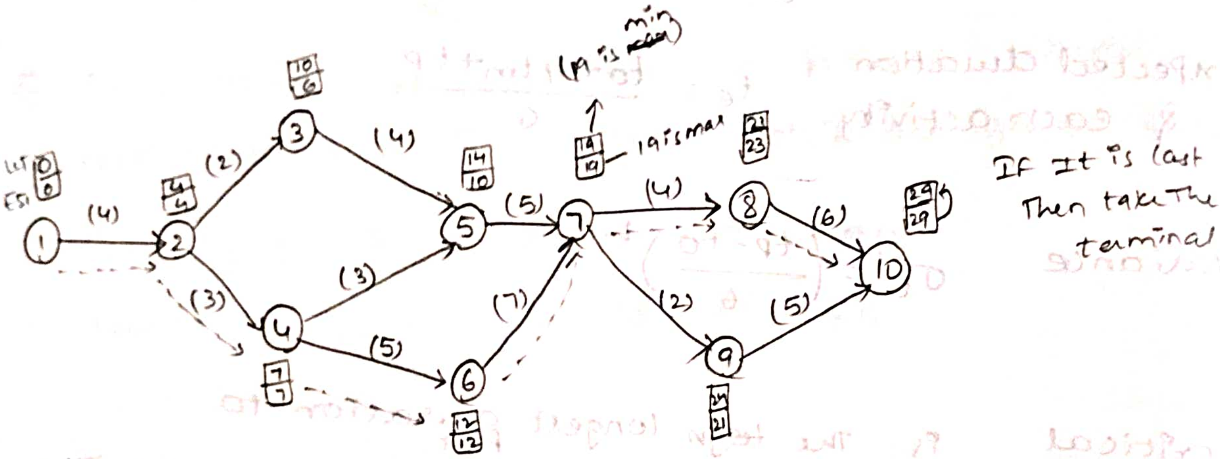
(i) Expected duration of each activity.

(ii) Expected variance of each activity.

(iii) expected variance of the Project Length.

(critical Path)

Activity	to	tm	tp	te	σ_t^2
1 - 2	3	4	5	4	0.11
2 - 3	1	2	3	2	0.11
2 - 4	2	3	4	3	0.11
3 - 5	3	4	5	4	0.11
4 - 5	1	3	5	3	0.44
4 - 6	3	5	7	5	0.44
5 - 7	4	5	6	5	0.11
6 - 7	6	7	8	7	0.11
7 - 8	2	4	6	4	0.44
7 - 9	1	2	3	2	0.11
8 - 10	4	6	8	6	0.44
9 - 10	3	5	7	5	0.44



j = end node
i = start node

ES_j = Earliest start time - forward pass

LS_j = Latest completion time - backward pass

$ES_j = \max_i (ES_i + D_{ij})$

$LC_i = \min_i (LC_j - D_{ij})$

To find the critical path

$ES_i = LC_j$

∴ Critical Path: 1 - 2 - 4 - 6 - 7 - 8 - 10.

Variance of the critical path $= 0.11 + 0.11 + 0.44 + 0.11 + 0.44 + 0.44$

$= 0.33 + 1.32$

$$\begin{array}{r} 0.44 \\ \times 3 \\ \hline 1.32 \end{array}$$

$= \underline{\underline{1.65}}$

Expected Project duration $= 4 + 3 + 5 + 7 + 4 + 6$

$= \underline{\underline{29}}$