

SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS

Subject Title: Applied Operation Research

Subject Code: SBAA5202

Course: M.B.A

Semester: II

UNIT – 1 – INTRODUCTION AND LINEAR PROGRAMMING

Concept of Operations Research - Meaning and Models in OR - Utilization of OR Models for Managerial Decision Making. Linear Programming Problems(LPP) - Formulation, Graphical Method and Simplex Method of Solving LPP.

1. INTRODUCTION TO OPERATIONS

1.1 RESEARCH ORIGIN OF OPERATIONS RESEARCH (OR)

The term Operations Research (OR) was first coined by MC Closky and Trefthen in 1940 in a small town, Bowdsey of UK. The main origin of OR was during the second world war – The military commands of UK and USA engaged several inter-disciplinary teams of scientists to undertake scientific research into strategic and tactical military operations. Their mission was to formulate specific proposals and to arrive at the decision on optimal utilization of scarce military resources and also to implement the decisions effectively. In simple words, it was to uncover the methods that can yield greatest results with little efforts. Thus it had gained popularity and was called “ An art of winning the war without actually fighting it.”

The name Operations Research (OR) was invented because the team was dealing with research on military operations. The encouraging results obtained by British OR teams motivated US military management to start with similar activities. The work of OR team was given various names in US: Operational Analysis, Operations Evaluation, Operations Research, System Analysis, System Research, Systems Evaluation and so on. The first method in this direction was simplex method of linear programming developed in 1947 by G.B Dantzig, USA. Since then,

new techniques and applications have been developed to yield high profit from least costs. Now OR activities has become universally applicable to any area such as transportation, hospital management, agriculture, libraries, city planning, financial institutions, construction management and so forth. In India many of the industries like Delhi cloth mills, Indian Airlines, Indian Railway, etc are making use of OR techniques.

1.2 HISTORY OF OR

The term OR coined by Mc.Closkey and Tref in the year 1940 in U.K. OR was first used in military operations for optimum utilization of resources.

YEAR	EVENTS
1940	Term OR was coined by Mc.Closky and Trefthen in U.K
1949	<ul style="list-style-type: none"> <input type="checkbox"/> OR unit was set up in India in Hyderabad. (The Regional Research Lab) <input type="checkbox"/> OR unit was set up at defence science lab.
1951	<ul style="list-style-type: none"> <input type="checkbox"/> The National Research Council (NRC) in US formed a committee on OR. <input type="checkbox"/> The first book was published called “Methods on OR” by Morse and Kimball.
1952	<input type="checkbox"/> OR Society of America was formed.
1953	<input type="checkbox"/> OR unit was set up in Calcutta in the “Indian Statistical Institute”.
1995	<input type="checkbox"/> OR society of India was established.

OR gained its significance first in the defence during the World War II (1939-1945) in order to make the best use of limited military resources and win the war. The effectiveness of OR in defence spread interest in Government departments and industry.

1.3 CONCEPT AND DEFINITION OF OR

Operations research signifies research on operations. It is the organized application of modern science, mathematics and computer techniques to complex military, government, business or industrial problems arising in the direction and management of large systems of men, material, money and machines.

The purpose is to provide the management with explicit quantitative understanding and assessment of complex situations to have sound basics for arriving at best decisions. Operations research seeks the optimum state in all conditions and thus provides optimum solution to organizational problems.

DEFINITION

“OR is defined as the application of Scientific methods, tools and techniques to problems involving the operations of a system so as to provide to those in control of the system, with optimum solutions to the problem”.

“OR is defined as the application of Scientific methods by interdisciplinary team to problems involving control of organized system, so as to provide solutions which serve best to the organization as a whole.”

OR is, otherwise, called as the “Science of use”.

OR is the combination of management principles and mathematical concepts (Quantitative techniques) for managerial decision-making purpose.

1.4 CHARACTERISTICS OF OR

- ☐ Aims to find solutions for problems of organized systems.
- ☐ Aims to provide optimum solution. Optimization means the best minimum or maximum for the criteria under consideration.
- ☐ It is the application of scientific methods, tools and techniques.
- ☐ Interdisciplinary team approach is used to solve the problems.
- ☐ The solutions that serve best to the organization as a whole is taken into consideration.

1.5 APPLICATION OF OR

1. Production:

- ☐ Production scheduling
- ☐ Project scheduling
- ☐ Allocation of resources
- ☐ Equipment replacement
- ☐ Inventory policy
- ☐ Factory size and location

2. Marketing

- ☐ Product introduction with timing
- ☐ Product mix selection
- ☐ Competitive strategies
- ☐ Advertising strategies
- ☐ Pricing strategies.

3. Accounts

- ☐ Cash flow analysis (optimum cash balance)
- ☐ Credit policies (optimum receivables)

4. Finance

- ☐ Optimum dividend policy
- ☐ Portfolio analysis

5. Personnel Management

- ☐ Recruitment and selection
- ☐ Assignment of jobs
- ☐ Scheduling of training programs

6. Purchasing

- ☐ Rules for purchasing
- ☐ EOQ-Economic Order Quantity (how much to order)
- ☐ Timing of purchase (when to purchase)

7. Distribution

- ☐ Deciding number of warehouses.
- ☐ Location of warehouses
- ☐ Size of warehouses
- ☐ Transportation strategies

8. Defence

- ☐ Budget allocation
- ☐ Allocation of resources

9. Government Departments

- ☐ Transportation
- ☐ Budget fixation
- ☐ Fiscal policies.

10. R & D (Research and Development)

- ☐ Project introduction
- ☐ Project control
- ☐ Budget allocation for projects

1.6 THE MAIN PHASES OF OR

- ☐ Formulation of the problem
- ☐ Construction of a model (Mathematical model)
- ☐ Solve the model
- ☐ Control and update the model
- ☐ Test the model and validate it
- ☐ Implement the model

1.6.1. FORMULATION OF THE PROBLEM

- ☐ The first task is to study the relevant system and develop a well-defined statement of the problem. This includes determining appropriate objectives, constraints, interrelationships and alternative course of action.
- ☐ The OR team normally works in an advisory capacity . The team performs a detailed technical analysis of the problem and then presents recommendations to the management.
- ☐ Ascertaining the appropriate objectives is very important aspect of problem definition. Some of the objectives include maintaining stable price, profits, increasing the share in market, improving work morale etc.
- ☐ OR team typically spends huge amount of time in gathering relevant data.
 - ☐ To gain accurate understanding of problem
 - ☐ To provide input for next phase.
- ☐ OR teams uses Data mining methods to search large databases for interesting patternsthat may lead to useful decisions.

1.6.2. CONSTRUCTION OF A MATHEMATICAL MODEL

This phase is to reformulate the problem in terms of mathematical symbols and expressions. The mathematical model of a business problem is described as the system of equations and related mathematical expressions. Thus

1. Decision variables (x_1, x_2, \dots, x_n) – ‘n’ related quantifiable decisions to be made.
 2. Objective function– measure of performance (profit) expressed as mathematical function of decision variables. For example $P = 3x_1 + 5x_2 + \dots + 4x_n$
 3. Constraints– any restriction on values that can be assigned to decision variables in terms of inequalities or equations. For example $x_1 + 2x_2 \geq 20$
 4. Parameters– the constant in the constraints (right hand side values)
- The advantages of using mathematical models are

- ☐ Describe the problem more concisely
- ☐ Makes overall structure of problem comprehensible
- ☐ Helps to reveal important cause-and-effect relationships
- ☐ Indicates clearly what additional data are relevant for analysis
- ☐ Forms a bridge to use mathematical technique in computers to analyze

1.6.3. SOLVE THE MODEL

This phase is to develop a procedure for deriving solutions to the problem. A common theme is to search for an optimal or best solution. The main goal of OR team is to obtain an optimal solution which minimizes the cost and time and maximizes the profit

1.6.4. TESTING THE MODEL

After deriving the solution, it is tested as a whole for errors if any. The process of testing and improving a model to increase its validity is commonly referred as Model validation. The OR group doing this review should preferably include at least one individual who did not participate in the formulation of model to reveal mistakes. A systematic approach to test the model is to use Retrospective test. This test uses historical data to reconstruct the past and then determine the model and the resulting solution. Comparing the effectiveness of this hypothetical performance with what actually happened, indicates whether the model tends to yield a significant improvement over current practice.

1.6.5. IMPLEMENTATION

The last phase of an OR study is to implement the system as prescribed by the management. The success of this phase depends on the support of both top management and operating management. The implementation phase involves several steps

1. OR team provides a detailed explanation to the operating management
2. If the solution is satisfied, then operating management will provide the explanation to the personnel, the new course of action.
3. The OR team monitors the functioning of the new system
4. Feedback is obtained
5. Documentation

1.7 SCOPE OF OR

- ☐ Linear programming model
- ☐ Transportation
- ☐ Sequencing and scheduling
- ☐ Assignment of jobs to minimize cost or maximize profit
- ☐ Game theory
- ☐ Inventory model
- ☐ Maintenance and Replacement
- ☐ Waiting line models
- ☐ Network analysis
- ☐ Shortest route problems like traveling sales person problem
- ☐ Resource allocation problems

1.8 LIMITATIONS OF OR

1. **Magnitude of computation:** In order to arrive at an optimum solutions OR takes into account all the variables that affect the system. Hence the magnitude of computation is very large.

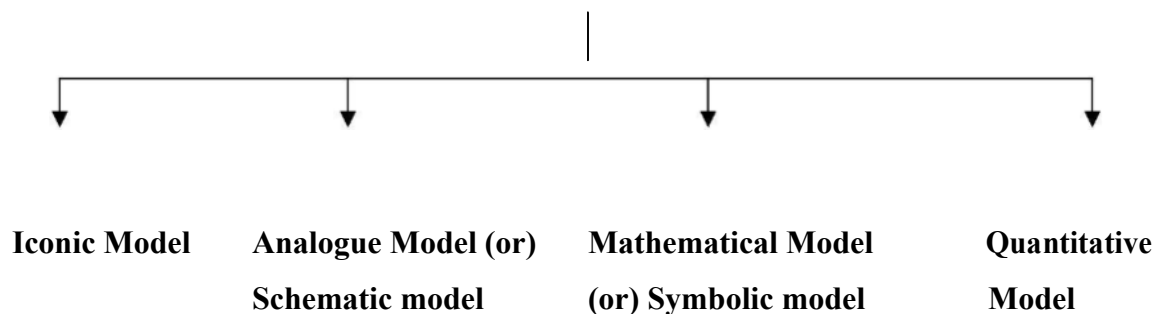
2. **Non-Quantifiable variables:** OR can give an optimum solution to a problem only if all the variables are quantified. Practically all variables in a system cannot be quantified.
3. **Time and Cost:** To implement OR in an organization, it consumes more time and cost. If the basic decision variables change, OR becomes too costly for an organization to handle it.
4. **Implementation of OR:** Implementation of OR may lead to HR problems. The psychology of employees should be considered and the success of OR depends on co-operation of the employees.
5. **Distance between Manager and OR Specialist:** Managers may not be having a complete overview of OR techniques and has to depend upon an OR Specialist. Only if good link is established OR can be a success.

2. MODELS IN OR

Model is a reasonably simplified representation of reality. It is an abstraction of reality. It helps to arrive at a well-structured view of reality.

2.1 TYPES OF MODELS

I -BASED ON NATURE



□ **ICONIC MODELS:**

- It is a pictorial representation or a physical representation of a system. A look alike correspondence is present.

Eg: miniature of a building, toys, globe etc.

- Iconic Models are either scaled up or scaled down. Scaled up - eg: Atom. Scaled down – eg: globe.
- Iconic models are either two-dimensional or three-dimensional.

□ **ANALOGUE MODEL OR SCHEMATIC MODEL**

This model uses one set of properties to describe another set of properties. There is no look alike correspondence. It is more abstract.

Eg: a set of water pipes that are used to describe current flow. Eg: Maps, (different colors are used to depict water, land etc Eg: Organizational chart.

□ **MATHEMATICAL MODEL**

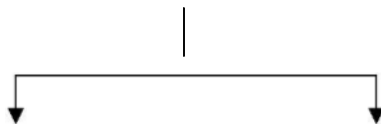
This uses a set of mathematical symbols (letters and numbers) to represent a system.

$$\begin{array}{ccccccc} V & = & I * R & \longrightarrow & \text{(Resistance)} \\ \downarrow & & \downarrow & & \\ \text{(Voltage)} & & \text{(Current)} & & \end{array}$$

□ **QUANTITATIVE MODELS**

Quantitative models are those, which can measure the observation. Eg: Models that measure temperature.

II -BASED ON VARIABILITY



STATIC

DYNAMIC

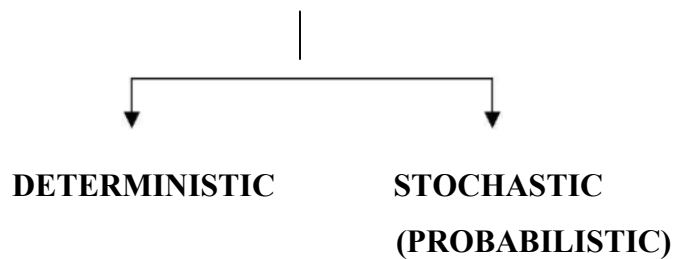
□ **STATIC MODEL**

This model assumes the values of the variables to be constant (do not change with time)
eg: Assignment Model.

□ **DYNAMIC MODEL**

This model assumes that the values of the variable change with time. Eg: Replacement model.

III -BASED ON RISK COSIDERATION



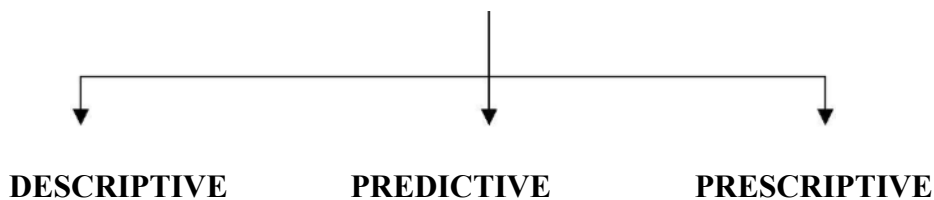
□ **DETERMINISTIC MODEL**

This model does not take uncertainty into consideration.
Eg: Linear programming, Assignment etc

□ **STOCHASTIC (PROBABILISTIC) MODEL**

This model considers uncertainty as an important factor. Eg: Stochastic Inventory models.

IV -BASED ON PRESENTATION



□ **DESCRIPTIVE MODEL**

This model just describes the situation under consideration. Eg: collecting an opinion regarding tendency to vote.

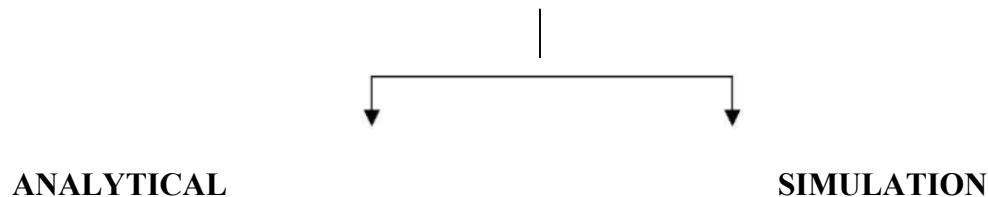
□ **PREDICTIVE MODEL**

This is a model, which predicts the future based on the data collected. Eg: predicting the election results before actual counting.

□ **PRESCRIPTIVE MODEL**

This is a model, which prescribes the course of action to be followed. Eg: Linear programming.

V -BASED ON ACCURACY OR EXACTNESS



□ **ANALYTICAL MODEL**

This is a model that gives an exact solution to the problem.

□ **SIMULATION MODEL**

Simulation model is a representation of reality through the use of some devices, which will react in the same manner under the given set of conditions.

Eg: Simulation of a drive of an Airplane through computer.

3. LINEAR PROGRAMMING

3.1 DEFINITION

Samuelson, Dorfman, and Solow define LP as “the analysis of problems in which linear function of a number of variables is to be maximized (or minimized) when those variables are subject to a number of constraints in the form of linear inequalities”.

3.2 BASIC ASSUMPTIONS OF LINEAR PROGRAMMING:

The following four basic assumptions are necessary for all linear programming models:

1. LINEARITY:

The basic requirements of a LP problem are that both the objectives and constraints must be expressed in terms of linear equations or inequalities. It is well known that if the number of machines in a plant is increased, the production in the plant also proportionately increases. Such a relationship, giving corresponding increment in one variable for every increment in other, is called linear and can be graphically represented in the form of a straight line.

2. DETERMINISTIC (OR CERTAINTY):

In all LP models, it is assumed that all model parameters such as availability of resources, profit (or cost) contribution of a unit of decision variable and consumption of resources by a unit decisions variable must be known and fixed. In other words, this assumptions means that all the **coefficients** in the objectives function and constraints are completely known with certainty and do not change during the period being studied.

3. ADDITIVITY:

The value of the objective function for the given values of decision variables and the total sum of resources used, must be equal to the sum of the contributions (profit or cost) earned from each decision variable and the sum of the resources used by each decision respectively. For example, the total profit earned by the sale of three products A, B and C must be equal to the profits earned separately from A, B and C and similarly, the amount of

resources consumed by A, B, and C individually.

4. DIVISIBILITY:

This implies that solution values of decision variables and resources can take any non-negative values, i.e., **fractional values** of the decision variables are **permitted**. This, however, is not always desirable. For example, it is impossible to produce one-fourth of a bus. When it is necessary to have integer variables, a technique known as integer programming could be used.

3.4 APPLICATIONS OF LINEAR PROGRAMMING:

- (i) **MANUFACTURING PROBLEMS:** to find the number of items of each type that should be manufactured so as to maximize the profit subject to production restrictions imposed by limitations on the use of machinery and labour.
- (ii) **ASSEMBLING PROBLEMS:** To have the best combinations of basic components to produce goods according to certain specifications.
- (iii) **TRANSPORTATION PROBLEMS:** to find the least costly way of transporting shipments from the warehouses to customers.
- (iv) **BLENDING PROBLEM:** To determine the optimal amount of several constitutes to use in producing a set of products which determining the optimal quantity of each product to produce.
- (v) **PRODUCTION PROBLEMS:** To decide the production schedule to satisfy demand and minimize cost in face of fluctuating rates and storage expenses.
- (vii) **DIET PROBLEMS:** To determine the minimum requirement of nutrients subject to availability of foods and their prices.
- (viii) **JOB ASSIGNING PROBLEMS:** To assign job to workers for maximum effectiveness and optimal results subject to restrictions of wages and other costs.

- (ix) **TRIM-LOSS PROBLEMS:** To determine the best way to obtain a variety of smaller rolls of paper from a standard width of roll that it kept its stock and at the same time minimize wastage.
- (x) **STAFFING PROBLEM:** To find optimal staff in hotels, police stations and hospitals to maximize the efficiency.
- (xi) **TELEPHONE EXCHANGE PROBLEMS:** To determine optimal facilities in telephone exchange to have minimum breakdowns.

3.5 APPLICATIONS OF LINEAR PROGRAMMING

- a) Personnel Assignment Problem
- b) Transportation Problem
- c) Efficiency on Operation of system of Dams
- d) Optimum Estimation of Executive Compensation
- e) Agriculture Applications
- f) Military Applications
- g) Production Management
- h) Marketing Management
- i) Manpower Management

3.6 KEY TERMS

Artificial variables: A variable that has no meaning in a physical sense, but acts as tool to help generate an initial LP solution.

Basic variables: The set of variables that are in the solution (i.e., have positive, non-zero values) are listed in the product mix column. The variables that normally take non-zero values to obtain a solution.

Basic solution: A solution to m simultaneous linear equations in n unknowns, $m < n$, with the property that $n-m$ of the variables have the value zero and the values of the remaining m variables are uniquely determined; obtained when a set of non-basic variables are assigned the

value zero.

Basic feasible solution: A basic solution, for which the values of all variables are non-negative, corresponds to a corner of the LP feasible region.

Degeneracy : A condition that arises when there is a tie in the values used to determine which variables indicated will enter the solution next. It can lead to cycling back and forth between two non-optimal solutions.

Degenerate solution: The number of variables in the standard equality form (counting decision variables, surpluses, and slacks) with positive optimal value is less than the number of constraints.

Optimal solution: A solution that is optimal for the given solution.

Pivot column: The column with the largest positive number in the $C_i - Z_j$ row of a maximization problem, or the largest negative $C_j - Z_j$ value in a minimization problem. It indicates which variable will enter the solution next.

Pivot row: The corresponding to the variable that will leave the basis in order to make room for the variable entering (as indicated by the new pivot column). This is the smallest positive ratio found by dividing the quantity column values by the pivot column values for each row.

Slack variable: A variable added to less than or equal to constraints in order to create an equality for a simplex method. It represents a quantity of unused resources.

Surplus variable: A variable inserted in a greater than or equal to constraint to create equality. It represents the amount of resources usage above the minimum required usage.

Unboundedness: A condition describing LP maximization problems having solutions that can become infinitely large without violating any stated constraints.

3.7 ADVANTAGES OF LPP:

- ☐ It provides an insight and perspective into the problem environment. This generally results in clear picture of the true problem.
- ☐ It makes a scientific and mathematical analysis of the problem situations.
- ☐ It gives an opportunity to the decision-maker to formulate his strategies consistent with the constraints and the objectives.
- ☐ It deals with changing situations. Once a plan is arrived through the LP it can also be

revaluated for changing conditions.

- By using LP, the decision maker makes sure that he is considering the best solution.

3.8 LIMITATIONS OF LPP:

- The major limitation of LP is that it treats all relationships as linear but it is not true in many real life situations.
- The decision variables in some LPP would be meaningful only if they have integer values. But sometimes we get fractional values to the optimal solution, where only integer values are meaningful.
- All the parameters in the LP model are assumed to be known constants. But in real life they may not be known completely or they may be probabilistic and they may be liable for changes from time to time.
- The problems are complex if the number of variables and constraints are quite large.
- It deals with only single objective problems, whereas in real life situations, there may be more than one objective.

3.9 FORMULATION OF LPP:

- Identify the objective function
- Identify the decision variables
- Express the objective function in terms of decision variables
- Identify the constraints and express them
- Value of decision variables is ≥ 0 (always non-negativity)

EXAMPLE PROBLEM:

An organization wants to produce Tables and Chairs. Profit of one table is ₹100 and profit of one Chair is ₹50

Particulars	Tables	Chairs	Maximum hours available
Cutting (hours)	4	1	300
Painting (hours)	1	.5	100

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Solution:

- 1) Objective: Maximization of profit
- 2) Decision variables
 No. of Tables to be produced 'x'
 No. of Chairs to be produced 'y'
- 3) Objective function Maxi $Z = 100x + 50y$
- 4) Constraints $4x + 1y \leq 300$
 $1x + 0.5y \leq 100$ $x \geq 0, y \geq 0$
- 5) Formulate
 Maxi $Z = 100x + 50y$
 Subject to
 $4x + 1y \leq 300$
 $1x + 0.5y \leq 100$ $x, y \geq 0$

3.9 STEPS IN GRAPHICAL SOLUTION METHOD:

- ☐ Formulate the objective and constraint functions.
- ☐ Draw a graph with one variable on the horizontal axis and one on the vertical axis.
- ☐ Plot each of the constraints as if they are inequalities.
- ☐ Outline the solution area.
- ☐ Circle the potential solutions points. These are the intersections of the constraints on the perimeter of the solution area. (vertices of the solution space)
- ☐ Substitute each of the potential extreme point values of the two decision variables into the objective function and solve for Z.
- ☐ Select the solution that optimizes Z.

3.10 PROCEEDURE FOR SOLVING LPP PROBLEM USING SIMPLEX METHOD

STEP:1

Convert all the inequality functions into equality:

For converting all the inequalities into equalities, we should use slack and surplus variables.

In case of \leq inequalities, we should add Slack variable so as to convert that inequality into equality. For example, $3x + 2y \leq 6$ will become $3x + 2y + S1 = 6$, where S1 is the slack variable.

In case of \geq inequalities, we should deduct Surplus variable

so as to convert that inequality into equation.

For example, $5x + 6y \geq 10$ will become $5x + 6y - S2 = 10$, where S2 is the surplus variable.

In case if the given constraint is an equation category, we should not use either slack variable or surplus variable.

STEP 2:

Find out the basic and non basic variables: Non Basic variable is the variable whose value is zero. Basic variable is the variable which will have either positive or negative value.

After converting all the inequality into equality, we should assume some variables as Non basic variables and find out the values of the other (Basic) variables. This solution is called as initial solution. If all the basic variable values are positive, then that initial solution is called as BASIC FEASIBLE SOLUTION.

STEP:3

Preparation of simplex table: The format of the simplex table is as follows:
Coefficients of Variables in the Objective function

EVALUATION ROW

Coefficients of	Basic				
Basic variables	Variables	Variables		Solution	Ratio

STEP 4:

Calculation of values in Evaluation row: To calculate the values in the evaluation row, we should use the following formula for each variable column:

Evaluation row values = (Variable coefficients x coefficients of basic variables) – Coefficients of the variables in the objective function.

All the values in the Evaluation row should be either positive or zero. Then it indicates that we have reached the optimum stage and thereby we can derive the optimum solution.

If any negative persists, we should proceed further by doing the following steps.

STEP 5:

IDENTIFICATION OF KEY COLUMN: The column that represents least value in the evaluation row is known as KEY COLUMN. The variable in that column is known as ENTERING VARIABLE.

STEP 6:

IDENTIFICATION OF KEY ROW: To find out the Key row, we should calculate the ratio.

Ratio = solution column values / Key column values. The least ratio row is treated as KEY ROW and the value in that row is known as LEAVING VARIABLE. **THE VARIABLE THAT**

PREVAILS IN BOTH KEY ROW AND KEY COLUMN IS KNOWN AS KEY ELEMENT. After finding the key element, we should prepare next simplex table. In that table, should bring the entering variable and should write the new values of the entering

Problems

Problem 1. A person requires 10, 12 and 12 units of chemicals A, B and C respectively for herbal garden. A liquid product contains 5, 2 and 1 units of A, B and C respectively per Jar. A dry product contains 1, 2 and 4 units of A, B and C per cartoon. If the liquid product sells for Rs. 3 per Jar and dry product sells for Rs. 2 per cartoon, how many of each should be purchased to minimise the cost and meet the requirements.

Solution : Requirement	A	B	C	
	10	12	12 units	
Liquid product	5	2	1	units Rs. 3/-per jar
Dry product	1	2	4	Rs. 2/-per Cartons

1. Select decision variable

x_1 – no. of jars of liquid product

x_2 – no. of cartoons of dry product

2. Objective function

Minimize cost (z) = $3x_1 + 2x_2$

3. Constraints :

$$5x_1 + x_2 \geq 10$$

$$2x_1 + 2x_2 \geq 12$$

$$1x_1 + 4x_2 \geq 12$$

4. Add non negativity constraints :

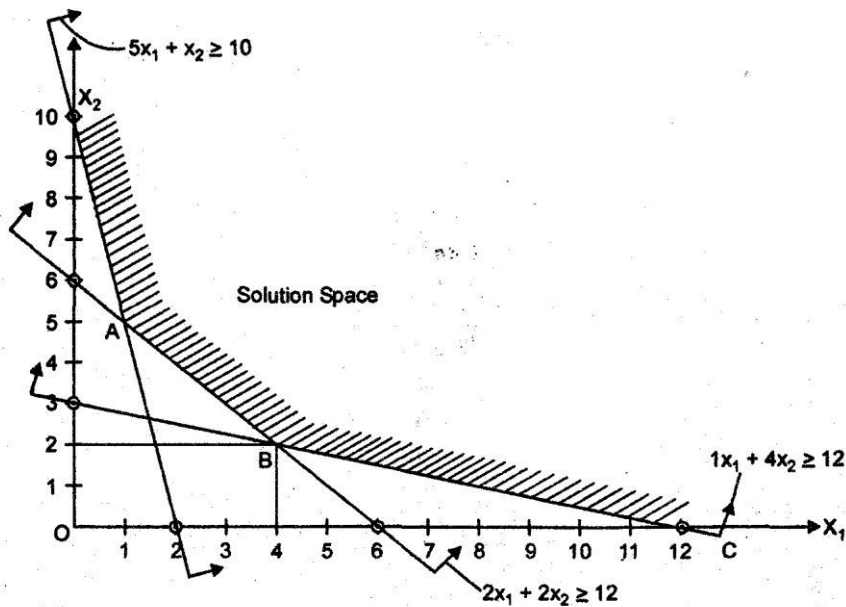
$$x_1 \geq 0 \quad ; \quad x_2 \geq 0$$

Graphical Method :

$$5x_1 + x_2 = 10 \Rightarrow x_1 = 0; x_2 = 10 \text{ and } x_2 = 0; x_1 = 2$$

$$2x_1 + 2x_2 = 12 \Rightarrow x_1 = 0; x_2 = 6 \text{ and } x_2 = 0; x_1 = 6$$

$$1x_1 + 4x_2 = 12 \Rightarrow x_1 = 0; x_2 = 3 \text{ and } x_2 = 0; x_1 = 12$$



Point A (1,5) $Z(A) = 3 \times 1 + 2 \times 5 = 13$
 Point B (4, 2) $Z(B) = 3 \times 4 + 2 \times 2 = 16$
 Point C (12,0) $Z(C) = 3 \times 12 + 2 \times 0 = 36$

Minimum cost at point A i.e. Rs. 13

x_1 (no. of Jar of Liquid product) = 1

x_2 (no. of carton of dry product) = 5

Minimum cost (Z) = Rs. 13.

Problem 2. A firm manufactures pain relieving pills in two sizes A and B, size A contains 4 grains of element a, 7 grains of element b and 2 grains of element c, size B contains 2 grains of element a, 10 grains of element b and 8 grains of c. It is found by users that it requires at least 12 grains of element a, 74 grains of element b and 24 grains of element c to provide immediate relief. It is required to determine that least no. of pills a patient should take to get immediate relief. Formulate the problem as standard LPP.

Solution : Pain relieving pills

	a	b	c
Size A	4	7	2
Size B	2	10	8
Min. requirement	12	74	24

Step 1. Select decision variable

x_1 - no. of pills of size A

x_2 - no. of pills of size B

Step 2. Objective function

Minimum (no. of pills) $z = x_1 + x_2$

Step 3. Constraints

$$4x_1 + 2x_2 \geq 12$$

$$7x_1 + 10x_2 \geq 74$$

$$2x_1 + 8x_2 \geq 24$$

Step. 4. Add non negativity constraints

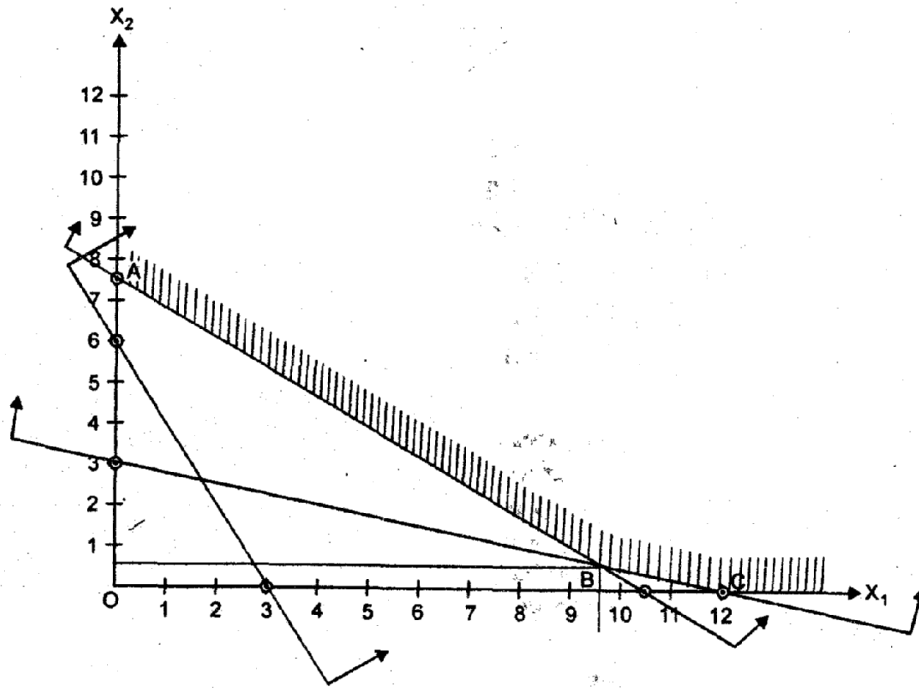
$$x_1 \geq 0 ; x_2 \geq 0$$

Determining the value of x_1 and x_2 by graphical method

$$4x_1 + 2x_2 = 12 \quad x_1 = 0; x_2 = 6 \text{ and } x_2 = 0; x_1 = 3$$

$$7x_1 + 10x_2 = 74 \quad x_1 = 0; x_2 = 7.4 \text{ and } x_2 = 0; x_1 = 10.57$$

$$2x_1 + 8x_2 = 24 \quad x_1 = 0; x_2 = 3 \text{ and } x_2 = 0; x_1 = 12$$



Point A (0, 7.4) $Z(A) = 0 + 7.4 = 7.4$ (Minimum)

Point C (12, 0) $Z(C) = 12 + 0 = 12$

Point B (9.6, 0.6) $Z(B) = 9.6 + 0.6 = 10.2$

No. of pills of size A = 0

No. of pills of size B = $7.4 \approx 8$ pills

Minimum no. of pills = 8 pills.

Problem 3. An automobile manufacturer makes automobiles and trucks in a factory that is divided into two shops. Shop A which perform the basic assy operation must work 5 man days on each truck but only 2 man days on each automobile. Shop B which perform finishing operations must work 3 man days for each automobile or truck that it produces. Because of men and machine limitations shop A has 180 man days per week available while shop B has 135 man days per week. If the manufacturer makes a profit of Rs. 300 on each truck and Rs. 200 on each automobile; how many of each should be produced to maximize his profit?

Solution :

	Shop A	Shop B	Profit
Automobile	2 man days	3 man days	Rs. 200
Trucks	5 man days	3 man days	Rs. 300
Availability	180 man days/week	135 man days/week	

1. Select decision variable

x_1 - no. of automobile to be produced/week

x_2 - no. of trucks to be produced/week

2. Objective function

$$\text{Maximize } Z = 200x_1 + 300x_2$$

3. Constraints

$$2x_1 + 5x_2 \leq 180$$

$$3x_1 + 3x_2 \leq 135$$

4. Add non negativity constraints

$$x_1 \geq 0; x_2 \geq 0$$

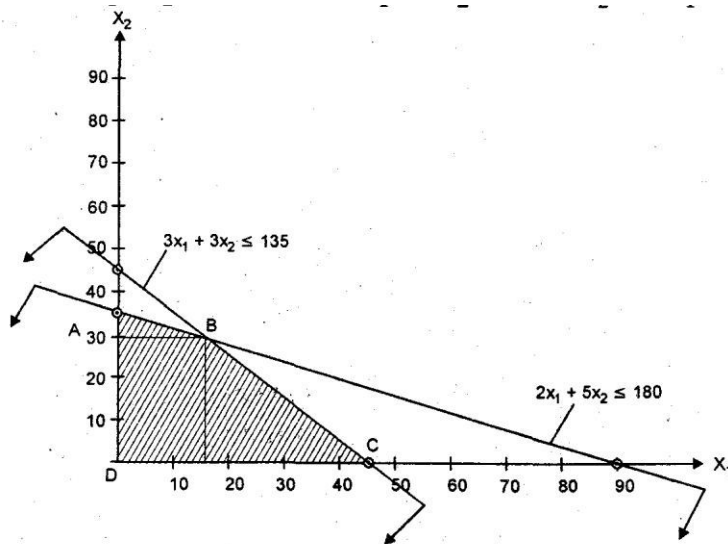
Determine the value of x_1 and x_2 by graphical method

$$2x_1 + 5x_2 = 180$$

$$x_1 = 0; x_2 = 36 \text{ and } x_2 = 0; x_1 = 90$$

$$3x_1 + 3x_2 = 135$$

$$x_1 = 0; x_2 = 45 \text{ and } x_2 = 0; x_1 = 45$$



Point D (0, 0) $Z(D) = 200 \times 0 + 300 \times 0 = 0$

Point A (0, 36) $Z(A) = 200 \times 0 + 300 \times 36 = 10800$

Point C (45, 0) $Z(C) = 200 \times 45 + 300 \times 0 = 9000$

Point B (15, 30) $Z(B) = 200 \times 15 + 300 \times 30 = 3000 + 9000 = 12000$

Maximum Profit at Point B (15, 30) i.e. Rs.12000/-

$$x_1 = \text{no. of automobile/week} = 15$$

$$x_2 = \text{no. of trucks/week} = 30$$

$$\text{Maximum profit} = 12000/-$$

Problem 4. On completing the construction of house a person discovers that J square feet of plywood scrap and 80 square feet of white pine scrap are in use!able m for the construction of tables and book cases. It takes 16 square feet of plywood 8 square feet of white pine to make a table, 12 square feet of plywood and 16 Llare feet of white pine are required to construct a book case. By selling the finishing duct to a local furniture store the person can realize a profit of Rs. 25 on each table d Rs. 290 on each book case. How may the man most profitably use the left over ood ? Use graphical method to solve problem.

Solution :

	Plywood	White pine	Profit	
Table	16	8	Rs. 25	each table
Book case	12	16	Rs. 290	each book case
Availability	100	80		

1. Select decision variable

$$x_1 - \text{no. of table}$$

$$x_2 - \text{no. of book case}$$

2. Objective function

$$\text{Maximize profit } (Z) = 25x_1 + 290x_2$$

3. Constraints

$$16x_1 + 12x_2 \leq 100$$

$$8x_1 + 16x_2 \leq 80$$

4. Add non negativity constraints

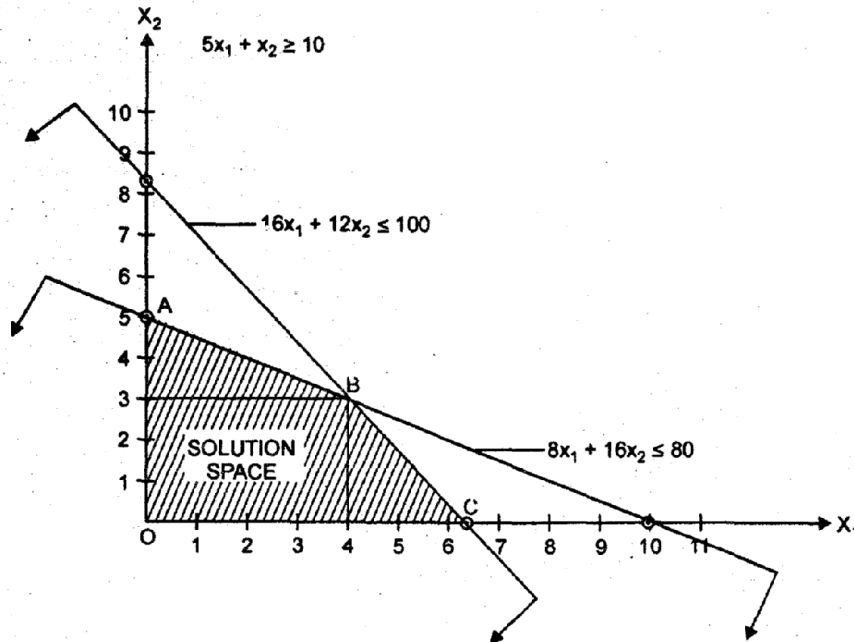
$$x_1 \geq 0$$

$$x_2 \geq 0$$

Determine the value of x_1 and x_2 using graphical method

$$16x_1 + 12x_2 = 100 \quad x_1 = 0; x_2 = 8.3 \text{ and } x_2 = 0; x_1 = 6.25$$

$$8x_1 + 16x_2 = 80 \quad x_1 = 0; x_2 = 5 \text{ and } x_2 = 0; x_1 = 10$$



Point O (0, 0) $Z(O) = 25 \times 0 + 290 \times 0 = 0$

Point A (0, 5) $Z(A) = 25 \times 0 + 290 \times 5 = 1450$

Point C (6.25, 0) $Z(C) = 25 \times 6.25 + 290 \times 0 = 156.25$

Point B (4, 3) $Z(B) = 25 \times 4 + 290 \times 3 = 100 + 870 = 970$

Maximum profit (Z) at point A i.e. Rs. 1450.

x_1 - no. of table = 0 Max. Profit (Z) = Rs. 1450/-

x_2 - no. of bookcase = 5.

Problem 5. A truck can carry a total of 10 tons of product. Three types of products are available for shipment. Their weight and values are tabulated. Assuming that at least one of each type must be shipped. Determine the loading which will maximize the total value. Formulate the problem.

Type	Value (Rs)	Weight (tons)
A	20	1
B	50	2
C	60	2

Solution : 1. Select decision variable

x_1 - no. of type A products

x_2 - no. of type B products

x_3 - no. of type C products

2. Objective function

Maximize (total value) $Z = 20x_1 + 50x_2 + 60x_3$

3. Constraints

$$x_1 + 2x_2 + 2x_3 \leq 10$$

$$x_1 \geq 1 \Rightarrow x_1 = 1+x$$

$$x_2 \geq 1 \Rightarrow x_2 = 1+y$$

$$x_3 \geq 1 \Rightarrow x_3 = 1+z$$

Put these values in constraint 1.

$$(1+x) + 2(1+y) + 2(1+z) \leq 10$$

$$1+x+2+2y+2+2z \leq 10$$

$$x+2y+2z \leq 5$$

4. Add non negativity constraints

$$x \geq 0; y \geq 0; z \geq 0$$

Objective function in terms of x, y, z

$$\begin{aligned} \text{Maximize (V)} &= 20(1+x) + 50(1+y) + 60(1+z) \\ &= 20 + 20x + 50 + 50y + 60 + 60z \end{aligned}$$

$$\text{Max. (U)} = 20x + 50y + 60z + 130$$

Subject to

$$x + 2y + 2z \leq 5$$

$$x \geq 0; y \geq 0; z \geq 0;$$

Simplex method

2. Find initial basic feasible solution

Substituting $x_1 = x_2 = x_3 = 0$ in equation (i), (ii) and (iii)

$$S_1 = 8$$

$$S_2 = 10$$

$$S_3 = 15$$

3. Perform optimality test

FR	C_B	C_j Basis	3 x_1	5 x_2	4 x_3	0 S_1	0 S_2	0 S_3	b	θ
Key row	0	S_1	2	key element 3	0	1	0	0	8	8/3
2/3	0	S_2	0	2	5	0	1	0	10	5
2/3	0	S_3	3	2	4	0	0	1	15	15/2
		Z_j	0	0	0	0	0	0	0	
		$C_j - Z_j$	3	5	4	0	0	0		
				key column						

Key row ←

4. Iterate towards optimal solution

	C_J		3	5	4	0	0	0		
FR	C_B	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b	θ
	5	x_2	2/3	1	0	1/3	0	0	8/3	∞
	0	S_2	-4/3	0	5	-2/3	1	0	14/3	14/15
4/5	0	S_3	5/3	0	4	-2/3	0	1	29/3	29/12
		Z_J	10/3	5	0	5/3	0	0	40/3	
		$C_J - Z_J$	-1/3	0	4	-5/3	0	0		

Key column

Key row

FR	C_B	C_J Basis	3 x_1	5 x_2	4 x_3	0 S_1	0 S_2	0 S_3	b	θ
$\frac{10}{41}$	5	x_2	2/3	1	0	1/3	0	0	8/3	4
$-\frac{4}{41}$	4	x_3	-4/15	0	1	-2/15	1/5	0	14/15	-7/2
	0	S_3	41/15	0	0	-2/15	-4/5	1	89/15	89/41
		Z_J	34/15	5	4	17/15	4/5	0	256/15	
		$C_J - Z_J$	11/15	0	0	-17/15	-4/5	0		

Key row ←

	C_J	3	5	4	0	0	0		
C_B	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b	θ
5	x_2	0	1	0	45/123	8/41	-10/41	50/41	
4	x_3	0	0	1	-6/41	33/205	4/41	62/41	
3	x_1	1	0	0	-2/41	-12/41	15/41	89/41	
	Z_J	3	5	4	135/41	152/41	11/41	765/41	
	$C_J - Z_J$	0	0	0	-135/41	-152/41	-11/41		

Since all element of $C_J - Z_J$ row are negative or zero so optimality test is passed

$$x_1 = \frac{89}{41}$$

$$x_2 = \frac{50}{41}$$

$$x_3 = \frac{62}{41}$$

$$\text{Max } z = 3 \times \frac{89}{41} + 5 \times \frac{50}{41} + 4 \times \frac{62}{41}$$

$$\text{Ans. } z = \frac{765}{41}$$

Problem 6. . Show that there is an unbounded solution to the following L.P. problem.

$$\begin{aligned} \text{Maximize } Z &= 4x_1 + x_2 + 3x_3 + 5x_4 \\ \text{Subject to } 4x_1 - 6x_2 - 5x_3 - 4x_4 &\geq -20 \\ -3x_1 - 2x_2 + 4x_3 + x_4 &\leq 10 \\ -8x_1 - 3x_2 + 3x_3 + 2x_4 &\leq 20 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

Solution : Standard form

Multiplying the first constraint by -1

$$\begin{aligned} -4x_1 + 6x_2 + 5x_3 + 4x_4 &\leq 20 \\ -4x_1 + 6x_2 + 5x_3 + 4x_4 + S_1 &= 20 \\ -3x_1 - 2x_2 + 4x_3 + x_4 + S_2 &= 10 \\ -8x_1 - 3x_2 + 3x_3 + 2x_4 + S_3 &= 20 \\ x_1, x_2, x_3, x_4, S_1, S_2, S_3 &\geq 0 \end{aligned}$$

2. ibfs

$$\begin{aligned} x_1 = x_2 = x_3 = x_4 &= 0 \\ S_1 = 20; S_2 = 10; S_3 = 20; Z &= 0 \end{aligned}$$

3. Iterations for optimal solution

C_B	C_J	4	1	3	5	0	0	0	b	θ
	Basis	x_1	x_2	x_3	x_4	S_1	S_2	S_3		
0	S_1	-4	6	5	4	1	0	0	20	5
0	S_2	-3	-2	4	1	0	1	0	10	10
0	S_3	-8	-3	3	2	0	0	1	20	10
	Z_J	0	0	0	0	0	0	0	0	
	$C_J - Z_J$	4	1	3	5	0	0	0		

← Key row



	C_j	4	1	3	5	0	0	0		
C_B	Basis	x_1	x_2	x_3	x_4	S_1	S_2	S_3	b	θ
5	x_4	-1	3/2	5/4	1	1/4	0	0	5	-5
0	S_2	-2	-7/2	11/4	0	-1/4	1	0	5	-5/2
0	S_3	-6	-6	1/2	0	-1/2	0	1	10	-5/3
	Z_j	-5	15/2	25/4	5	5/4	0	0	25	
	$C_j - Z_j$	9	-13/2	-13/4	0	-5/4	0	0		

Since all replacement ratios (θ) are negative the problem has no bounded solution and further computation stop. (Unbounded Solution)

Problem 7. Maximize $Z = x_1 + 2x_2 + 3x_3 - x_4$

subject to $x_1 + 2x_2 + 3x_3 = 15$

$2x_1 + x_2 + 5x_3 = 20$

$x_1 + 2x_2 + x_3 + x_4 = 10$

$x_1, x_2, x_3, x_4 \geq 0$

Solution : Step 1. Standard form

Maximize $Z = x_1 + 2x_2 + 3x_3 - x_4 - MA_1 - MA_2 - MA_3$

Subject to $x_1 + 2x_2 + 3x_3 + 0x_4 + A_1 + 0A_2 + 0A_3 = 15$

$2x_1 + x_2 + 5x_3 + 0x_4 + 0A_1 + A_2 + 0A_3 = 20$

$x_1 + 2x_2 + x_3 + x_4 + 0A_1 + 0A_2 + A_3 = 10$

$x_1, x_2, x_3, x_4, A_1, A_2, A_3 \geq 0$

Step 2. ibfs

$x_1 = x_2 = x_3 = x_4 = 0$

$A_1 = 15$

$A_2 = 20$

$A_3 = 10$

$Z = -45M$

Step 3. Check for optimality test

	C_j	1	2	3	-1	-M	-M	-M		
C_B	Basis	x_1	x_2	x_3	x_4	A_1	A_2	A_3	b	θ
-M	A_1	1	2	3	0	1	0	0	15	5
-M	A_2	2	1	(5)	0	0	1	0	20	4
-M	A_3	1	2	1	1	0	0	1	10	10
	Z_j	-4M	-5M	-9M	-M	-M	-M	-M	-45M	
	$C_j - Z_j$	1+4M	2+5M	3+9M	-1+M	0	0	0		

Key row

C_B	C_J	1	2	3	-1	-M	-M		
	Basis	x_1	x_2	x_3	x_4	A_1	A_3	b	θ
-M	A_1	-1/5	(7/5)	0	0	1	0	3	15/7
3	x_3	2/5	1/5	1	0	0	0	4	20
-M	A_3	3/5	9/5	0	1	0	1	6	10/3
	Z_J	$\frac{6-2M}{5}$	$\frac{3-16M}{5}$	3	-M	-M	-M	12-9M	
	$C_J - Z_J$	$\frac{-1+2M}{5}$	$\frac{7+16M}{5}$	0	-1+M	0	0		

↑

C_B	C_J	1	2	3	-1	-M		
	Basis	x_1	x_2	x_3	x_4	A_3	b	θ
2	x_2	-1/7	1	0	0	0	15/7	∞
3	x_3	3/7	0	1	0	0	25/7	∞
-M	A_3	6/7	0	0	(1)	1	15/7	15/7
	Z_J	$\frac{7-6M}{7}$	2	3	-M	-M	$\frac{105-15M}{7}$	
	$C_J - Z_J$	$\frac{6M}{7}$	0	0	-1+M	0		

↑

C_B	C_J	1	2	3	-1		
	Basis	x_1	x_2	x_3	x_4	b	θ
2	x_2	-1/7	1	0	0	15/7	-15
3	x_3	3/7	0	1	0	25/7	25/3
-1	x_4	6/7	0	0	1	15/7	5/2
	Z_J	1/7	2	3	-1	90/7	
	$C_J - Z_J$	6/7	0	0	0		

↑

	C_j	1	2	3	-1	
C_B	Basis	x_1	x_2	x_3	x_4	b
2	x_2	0	1	0	1/6	5/2
3	x_3	0	0	1	-1/2	5/2
1	x_1	1	0	0	7/6	5/2
	Z_j	1	2	3	0	15
	$C_j - Z_j$	0	0	0	-1	

$C_j - Z_j$ is either zero or negative under all columns, The optimal feasible solution has been obtained

$$x_1 = \frac{5}{2}, x_2 = \frac{5}{2}, x_3 = \frac{5}{2}, x_4 = 0$$

$$A_1 = A_2 = A_3 = 0$$

$$Z_{\max} = 15$$

Problem 8. Use penalty Method to

$$\text{minimize } Z = x_1 + 2x_2 + x_3$$

$$\text{Subject to } x_1 + \frac{x_2}{2} + \frac{x_3}{2} \leq 1$$

$$\frac{3}{2}x_1 + 2x_2 + x_3 \geq 8$$

$$x_1, x_2, x_3 \geq 0$$

Solution : 1. Standard form

$$\text{minimize } z = x_1 + 2x_2 + x_3 + 0S_1 + 0S_2$$

$$\text{Subject to } x_1 + \frac{x_2}{2} + \frac{x_3}{2} + S_1 = 1$$

$$\frac{3}{2}x_1 + 2x_2 + x_3 - S_2 + A_1 = 8$$

$$x_1, x_2, x_3, S_1, S_2 \geq 0$$

2. ibfs

$$\text{Setting } x_1 = x_2 = x_3 = S_2 = 0$$

$$S_1 = 1, A_1 = 8$$

3. Iterations for optimal Solution

C_B	C_J	1	2	1	0	0	M		
	Basis	x_1	x_2	x_3	S_1	S_2	A_1	b	θ
0	S_1	1	1/2	1/2	1	0	0	1	2
M	A_1	3/2	2	1	0	-1	1	8	4
	Z_J	3/2M	2M	M	0	-M	M	8M	
	$C_J - Z_J$	$1 - \frac{3}{2}M$	$2 - 2M$	$1 - M$	0	M	0		

C_j		1	2	1	0	0	M	
C_B	Basis	x_1	x_2	x_3	S_1	S_2	A_1	b
2	x_2	2	1	1	2	0	0	2
M	A_1	-5/2	0	-1	-4	-1	1	4
	Z_j	$4 - \frac{5}{2}M$	2	$2 - M$	$4 - 4M$	$-M$	M	$4 + 4M$
	$C_j - Z_j$	$-3 + \frac{5}{2}M$	0	$-1 + M$	$-4 + 4M$	M	0	

Since $C_j - Z_j$ is non negative under all columns so optimality test passed since A_1 appears in the basis at a positive value, the given problem has no feasible solution

Problem 9. Use the two phase simplex method to

$$\text{Maximize } Z = 5x_1 - 4x_2 + 3x_3$$

$$\text{Subject to } 2x_1 + x_2 - 6x_3 = 20$$

$$6x_1 + 5x_2 + 10x_3 \leq 76$$

$$8x_1 - 3x_2 + 6x_3 \leq 50$$

$$x_1, x_2, x_3 \geq 0$$

Solution : Step 1. Standard form

$$2x_1 + x_2 - 6x_3 + A_1 = 20$$

$$6x_1 + 5x_2 + 10x_3 + S_1 = 76$$

$$8x_1 - 3x_2 + 6x_3 + S_2 = 50$$

$$x_1, x_2, x_3, S_1, S_2, A_1 \geq 0$$

The new objective function

$$\text{minimize } w = A_1$$

Subject to

$$2x_1 + x_2 - 6x_3 + 0S_1 + 0S_2 + A_1 = 20$$

$$6x_1 + 5x_2 + 10x_3 + S_1 + 0S_2 + 0A_1 = 76$$

$$8x_1 - 3x_2 + 6x_3 + 0S_1 + S_2 + 0A_1 = 50$$

$$x_1, x_2, x_3, S_1, S_2, A_1 \geq 0$$

Step 2. ibfs

$$x_1 = x_2 = x_3 = 0$$

$$A_1 = 20$$

$$S_2 = 76$$

$$S_3 = 50$$

Step 3. Iterations for optimal Solution :

C_B	C_J		0	0	0	0	0	1		
	Basis	x_1	x_2	x_3	S_1	S_2	A_1	b	θ	
1	A_1	2	1	-6	0	0	1	20	10	
0	S_1	6	5	10	1	0	0	76	$\frac{38}{3}$	
0	S_2	(8)	-3	6	0	1	0	50	$\frac{25}{4}$	← Key row
	Z_J	2	1	-6	0	0	1	20		
	$C_J - Z_J$	-2	-1	6	0	0	0			

↑

C_B	C_J	0	0	0	0	0	1			
	Basis	x_1	x_2	x_3	S_1	S_2	A_1	b	θ	
1	A_1	0	(7/4)	-15/2	0	-1/4	1	15/2	30/7	← Key row
0	S_2	0	29/4	11/2	1	-3/4	0	77/2	154/29	
0	x_1	1	-3/8	3/4	0	1/8	0	25/4	-50/3	
	Z_J	0	7/4	-15/2	0	-1/4	1	15/2		
	$C_J - Z_J$	0	-7/4	15/2	0	1/4	0			

↑

C_B	C_J	0	0	0	0	0	1		
	Basis	x_1	x_2	x_3	S_1	S_2	A_1	b	
0	x_2	0	1	-30/7	0	-1/7	4/7	30/7	
0	S_2	0	0	256/7	1	2/7	-29/7	52/7	
0	x_1	1	0	-6/7	0	1/14	3/14	55/7	
	Z_J	0	0	0	0	0	0	0	
	$C_J - Z_J$	0	0	0	0	0	1		

Since $C-Z$ is non negative under all columns and no artificial variable appears in the basis.

2nd phase (deleting artificial variable column)

	C_j	5	-4	3	0	0	
C_B	Basis	x_1	x_2	x_3	S_1	S_2	b
-4	x_2	0	1	$-30/7$	0	$-1/7$	$30/7$
0	S_2	0	0	$256/7$	1	$2/7$	$52/7$
5	x_1	1	0	$-6/7$	0	$1/14$	$55/7$
	Z_j	5	-4	$90/7$	0	$13/14$	$155/7$
	$C_j - Z_j$	0	0	$-69/7$	0	$-13/14$	

Since $C_j - Z_j$ is either negative or zero under all columns, optimality test has passed.

$$x_1 = \frac{55}{7}, x_2 = \frac{30}{7}, x_3 = 0$$

$$Z_{\max} = \frac{155}{7}$$

variable.

New values of the Entering Variable = Old values of the leaving variable / Key element.

Thereafter, we should write the new values of the other left out rows. The formula is New values of the Left out row = Old values of the left out row – (New values of the entering variable X value in the key column of the Old left out row)

UNIT – 1 – INTRODUCTION AND LINEAR PROGRAMMING

QUESTION BANK

PART – A

1. Define Operations Research.
2. Write the stages in operations research?
3. What are the areas in which operations research is being applied?
4. Name the models being classified based on nature.
5. What do you mean by simulation model?
6. Enumerate the limitations of operations research.
7. What is meant by Linear programming problem?
8. Write the steps involved in formulation of linear programming problem.
9. What are the decision variables in LPP?
10. A person wants to decide the constituents of a diet which will fulfill his daily requirement of protein, fat, and carbohydrates at minimum cost. The choice is to be made from 4 different types of food.

Food type	(Yield/unit)			Cost/unit
	protein	fat	carbohydrates	
1	3	2	6	45
2	4	2	4	40
3	8	7	7	85
4	6	5	4	65
Min	800	200	700	

requirement

12. Consider food stuff A&B. These contain three vitamins V1, V2, V3. Minimum daily requirement of V1 is 1mg, V2 is 50mg and V3 is 10mg. Suppose food A contain 1mg of V1, 100mg of V2 and 10mg of V3. and food B contain 1mg of V1, 10mg of V2. Cost of 1 unit of food A is ₹1 and food B ₹1.5.
13. Formulate the LPP. An organization wants to produce Tables & Chairs Profit of 1 table is ₹100 and profit of 1 Chair is ₹50.

	Tables	Chairs	Maximum hours available
Cutting (hours)	4	1	300
Painting (hours)	1	1/2	100

PART - B

11. Briefly discuss about Models in operations research.

OR

12. Solve the LPP using Graphical method for the given formulation.

$$\text{MAX } Z = 28x + 30y$$

$$\text{Subject to } x + 3y \leq 18, 3x + y \leq 8, 4x + 5y \leq 30 \text{ (x, y} \geq 0).$$

13. Define operations research. Give the scope, characteristics of operations research.

OR

14. Solve the given LPP using Simplex method.

$$\text{MAX } Z = 6X_1 + 12X_2$$

$$\text{Subject to } 3X_1 + 4X_2 \leq 12, 10X_1 + 5X_2 \leq 20, (X_1, X_2 \geq 0).$$

15. Solve the LPP using Graphical method for the given formulation. $\text{MAX } Z = 5x -$

$$2y$$

$$\text{Subject to } 2x + y \leq 9, 2x - 4y \leq 48, 3x + 2y \leq 3 \text{ (x, y} \geq 0).$$

OR

16. Solve the given LPP using Simplex method. MAX

$$Z = 6X_1 + 12X_2$$

Subject to $3X_1 + 4X_2 \leq 12$, $10X_1 + 5X_2 \leq 20$, $(X_1, X_2 \geq 0)$.

17. Solve the LPP using Graphical method for the given formulation. MAX $Z = 4$

$$X_1 + 2X_2$$

Subject to $2X_1 + 3X_2 \leq 18$, $X_1 + X_2 \geq 10$ ($X_1, X_2 \geq 0$).

OR

18. Solve the given LPP using Simplex method. MIN $Z = -$

$$40X_1 - 100X_2$$

Subject to $10X_1 + 5X_2 \leq 250$, $2X_1 + 5X_2 \leq 100$, $2X_1 + 3X_2 \leq 90$ ($X_1, X_2 \geq 0$).

19. Solve the LPP using Graphical method for the given formulation. MAX

$$Z = 2X_1 + 4X_2$$

Subject to $X_1 + X_2 \leq 14$, $3X_1 + 2X_2 \geq 30$, $2X_1 + X_2 \leq 18$ ($X_1, X_2 \geq 0$).

OR

20. Solve the LPP using Graphical method for the given formulation

$$\text{MAX } Z = 2X_1 + 4X_2$$

Subject to $X_1 + X_2 \leq 14$, $3X_1 + 2X_2 \geq 30$,

$$2X_1 + X_2 \leq 18, (X_1, X_2 \geq 0)$$