SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS

Subject Title: Applied Operations Research **Subject Code: SBAA5202**

Course: M.B.A **Semester: II**

UNIT 3: NETWORK ANALYSIS 12 Hrs.
Network Analysis- CPM-Network Diagram Construction, Identification of Critical Path, Calculation of Floats. PERT- Calculation of Estimated Time, Standard Deviation and Probability. Sequencing- Sequencing of 'N' number of jobs on Two, Three, Four and N Machines.

NETWORK ANALYSIS

3.1 PROJECT

A **project** defines a combination of interrelated **activities** that must be executed in a certain order before the entire task can be completed. The activities are interrelated in a logical sequence in the sense that some activities cannot start until others are completed. An activity in a project is usually viewed as a job requiring time and possibly resources (like manpower, money, material, machinery etc.) for its completion.

3.1.1PHASES OF PROJECT

Any type of project scheduling consists of three basic phases namely:

- **PLANNING**

The planning phase is initiated by breaking down the project into distinct activities with their associated logical sequence. The time estimates for each of the activities are then determined.

- **SCHEDULING**

The scheduling phase constructs a time table giving the start and finish times of each activity as well as its relationship to the other activities in the project.

- **CONTROLLING**

The final phase is project control where periodic progress is reviewed and, depending upon the situation revised time-table for the remaining part of the project is worked out.

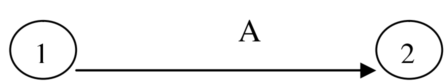
With growing sophistication of technology, the projects at organizational level have tended to become more and more complex, demanding efficient method of planning. Considering the inherent adequacies for planning big and complex projects, some efforts were made in USA and other western countries during 1950s to develop certain more efficient techniques. The outcome was the development of **CPM** (Critical Path Method) and **PERT** (Project Evaluation and Report Technique), which are two important techniques for planning and scheduling of large projects. These techniques are most widely used in industry and services around the globe.

CPM was first developed by E. I. du Pontde Bemours & Company as an application to construction projects and was later extended to a more advanced status by Mauchly Associates. However, PERT was developed for the U.S. Navy by a consulting firm for scheduling the research and Project Scheduling development activities for the Polaris missile program.

Although these two methods were developed independently, they are similar. The most important difference is that the time estimates for the activities are assumed to be deterministic in CPM and probabilistic in PERT. But, the underlying basis of both the techniques is the Network diagram.

3.1.2Network Diagram

A network (or arrow) diagram is a graphic representation of the project, describing the logical sequence and the interdependence of the activities. Moreover, construction of network diagram helps studying all the activities more critically. The basic elements of a network diagram are Arrow and Node



Tail Event Head Event

So, in a network diagram, an arrow is used to represent an activity, with its head indicating the direction of progress of the concerned project.

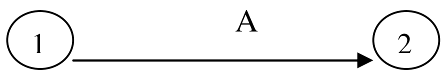
3.1.3 Terminologies related to network diagram

- **Activity**

An activity represents a job or an individual operation for a project. It consumes time, money, or resources in doing the work.

Every activity has a head event and tail event. Event 1(tail event) indicates start and event 2 (head event) indicates completion of activity A. Activity B can start only after completion of activity A.

Activity A is the predecessor activity and Activity B is the successor activity.



Tail Event Head Event

❖ **Dummy activity**

An activity which does not consume time, money and resources but merely depicts the technological dependence. It is an imaginary activity represented by a dotted line. Purpose for having a dummy activity is to create logic and avoid ambiguity.

Ex. Sending invitation cards for a function:

A

Printing of cards

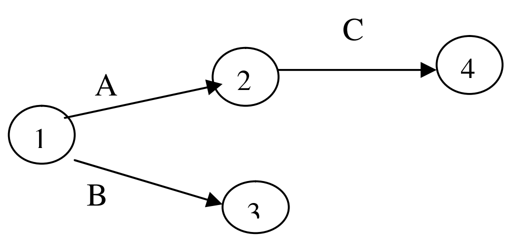
B

Collection of address

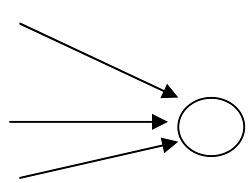
C

Despatching

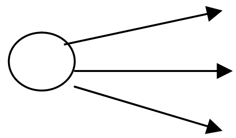
A & B can be done simultaneously but C can be done only after A & B hence to get the network logic we draw dummy activity.



Two or more activities ending in a single node is **merged.**



Two or more activities starting in a single node is **Burst**



3.1.4 HINTS FOR DRAWING A NETWORK DIAGRAM

(a) In network diagram, an arrow represents an activity a d each node signifies either of the two events - start time or completion time. The length of an arrow is arbitrary. It has no relationship with the duration of the activity. The orientation of an arrow indicates the direction of its completion.

(b) The tail-event and head-event of an arrow represent the start and completion of the concerned activity respectively.

(c) Only one activity can span across any given pair of events.

(d) Event numbers should not get repeated.

(e) A dummy activity follows the same logic of precedence relationship as any other normal activity, but consumes no resource (including time).

(f) No unnecessary dummies.

(g) Crossing of arrows should be avoided.

(h) The start-time as well as the completion time of the project must be represented by unique events.

(i) No dangling of arrows. There should not be more than one terminal node.

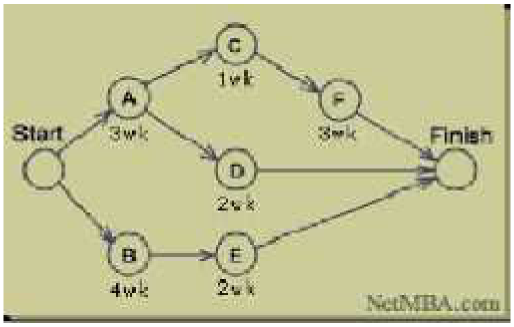
(j) The logic of inter-dependencies between the activities is governed by the following rules.

(i) An event can occur only when all activities leading into it, have been completed.

(ii) No activity can commence until its tail event has occurred.

3.2 Critical Path Method (CPM)

A path in a network diagram is a continuous chain of activities that connects the initial event to the terminal event. The length of a path is the sum of the durations of all the activities those lie on it. Critical Path defines the longest path consisting of critical activities that connects the start and end nodes of the network. To shorten the time for completion of the project, we must reduce the duration of the activities lying on the critical path. In order to complete the project in specified time, no delay is allowed in execution of the critical activities. It may be achieved by diverting allocated resources of non-critical activities to critical activities. However, this calls for information on the slack of each non-critical activity and Critical Path Method finds the same. They are extremely useful to a project-manager.



3.2.1.Steps in CPM Project Planning

1. Specify the individual activities.
2. Determine the sequence of those activities.
3. Draw a network diagram.
4. Estimate the completion time for each activity.
5. Identify the critical path (longest path through the network)
6. Update the CPM diagram as the project progresses.

1. Specify the individual activities

All the activities in the project are listed. This list can be used as the basis for adding sequence and duration information in later steps.

2. Determine the sequence of the activities

Some activities are dependent on the completion of other activities. A list of the immediate predecessors of each activity is useful for constructing the CPM network diagram.

3. Draw the Network Diagram

Once the activities and their sequences have been defined, the CPM diagram can be drawn. CPM originally was developed as an *activity on node* network.

4. Estimate activity completion time

The time required to complete each activity can be estimated using past experience. CPM does not take into account variation in the completion time.

5. Identify the Critical Path

The critical path is the longest-duration path through the network. The significance of the critical path is that the activities that lie on it cannot be delayed without delaying the project. Because of its impact on the entire project, critical path analysis is an important aspect of project planning.

The critical path can be identified by determining the following four parameters for each activity:

- ES - earliest start time: the earliest time at which the activity can start given that its precedent activities must be completed first.

- EF - earliest finish time, equal to the earliest start time for the activity plus the time required completing the activity.
- LF - latest finish time: the latest time at which the activity can be completed without delaying the project.
- LS - Latest start time, equal to the latest finish time minus the time required to complete the activity.

The *slack time* for an activity is the time between its earliest and latest start time, or between its earliest and latest finish time. Slack is the amount of time that an activity can be delayed past its earliest start or earliest finish without delaying the project.

The critical path is the path through the project network in which none of the activities have slack, that is, the path for which ES=LS and EF=LF for all activities in the path. A delay in the critical path delays the project. Similarly, to accelerate the project it is necessary to reduce the total time required for the activities in the critical path.

6. Update CPM diagram

As the project progresses, the actual task completion times will be known and the network diagram can be updated to include this information. A new critical path may emerge, and structural changes may be made in the network if project requirements change.

3.2.2 FLOATS IN CPM CALCULATION

Total Float

1. Total float is the time by which a particular activity can be delayed for non-critical activity.
2. It is a difference between latest finish & earliest finish or latest start & earliest start.
3. If the total float is positive, it indicates resources are more than adequate.
4. If the total float is negative, it indicates resources are inadequate.
5. If the total float is zero, it indicates resources are thus adequate.

Free Float:

It is the portion of Total Float. It is that amount of time where the activity can be rescheduled without affecting succeeding activity.

Free Float = Total Float – Slack of Head event

Where Slack = Latest Occurrence time – Earliest occurrence time

Individual Float

It is that amount of time where activity can be rescheduled without affecting both preceding & succeeding activity. It is a portion of Free Float

3.2.3 CPM Benefits

- Provides a graphical view of the project.
- Predicts the time required to complete the project.
- Shows which activities are critical to maintaining the schedule and which are not.

3.2.4 CPM Limitations

While CPM is easy to understand and use, it does not consider the time variations that can have a great impact on the completion time of a complex project. CPM was developed for complex but fairly routine projects with minimum uncertainty in the project completion times. For less routine projects there is more uncertainty in the completion times, and this uncertainty limits its usefulness.

3.3 Project Evaluation &Review Technique(PERT)

The *Program Evaluation and Review Technique* (PERT) is a network model that allows for randomness in activity completion times.

A distinguishing feature of PERT is its ability to deal with uncertainty in activity completion times. For each activity, the model usually includes three time estimates:

- **Optimistic estimate** (t_o) is **a** minimum time duration of any activity when everything goes on well about the project. It can be also written as '**a**'.
- **Pessimistic estimate** (t_p) is **maximum** time duration of any activity when everything goes against our will and lot of difficulties is faced in the project. It can be also written as '**b**'.
- **Most likely estimate** (t_m,) means the time required in normal course when something goes on very well and something goes on bad during the project. It can be also written as '**m**'.

Then, given any activity, we compute its expected duration and variance induration are given by the following relations.

(a) **Expected duration (t_e) =** $\dfrac{t_o + 4t_m + t_p}{6}$

(b) **Standard deviation** = $\dfrac{t_p - t_o}{6}$

(c) **Variance** = $\left[\dfrac{t_p - t_o}{6}\right]^2$

3.3.1 Benefits of PERT

PERT is useful because it provides the following information:

- Expected project completion time.
- Probability of completion before a specified date.
- The critical path activities that directly impact the completion time.
- The activities that have slack time and that can lend resources to critical path activities.
- Activities start and end dates.

3.3.2 Limitations of PERT

The following are some of PERT's limitations:

- The activity time estimates are somewhat subjective and depend on judgment. In cases where there is little experience in performing an activity, the numbers may be only a guess. In other cases, if the person or group performing the activity estimates the time there may be bias in the estimate.

- The underestimation of the project completion time due to alternate paths becoming critical is perhaps the most serious.

3.3.3 Basic difference between PERT and CPM

Difference Point	PERT	CPM
Stands for	PERT stands for "***Program Evaluation and Review Technique"***.	CPM stands for "**Critical Path Method".**
Model	It is a **probabilistic** model under which the result estimated in a manner of probability.	It is a **deterministic** model under which the result is ascertained in a manner of certainty
Time	It deals with the activities of **uncertain** time.	It deals with the activities of precise well known time.
Jobs	It is used for onetime projects that involve activities of **non-repetitive** nature .	It is used for completing of projects that involve activities of **repetitive** nature.
Orientation	It is **activity oriented** in as much as its result is calculated on the basis of the activities.	It is **even oriented**, in as much as its results are calculated on the basis of the events.
Dummy Activities	It does **not** make use of **dummy activities**.	It makes use of **dummy activities** to represent the proper sequencing of the activities.
Cost	It has nothing to do with cost of a project.	It deals with the cost of a project schedules and their minimization.
Estimation	t finds out expected time of each activity on the basis of three types of estimates.	Its calculation is based on one type of time estimation that is precisely known.

Time	PERT is restricted to time variable.	CPM includes **time-cost trade off.**

3.4 DEFINITION OF SEQUENCING

The selection of an appropriate order for a series of jobs to be done on a finite number of service facilities is called sequencing. The objective is to determine the optimal order of performing the jobs in such a way that the total elapsed time will be minimum. The total cost involved may be minimum if the total elapsed time is made minimum in the business situation.

Consider there are jobs 1,2,3,……………..n to be processed through m machines. (The machines may be A, B, C……………)There are actually (n!)^m combinations. The objective is to find the technologically feasible solution, such that the total elapsed time is minimum.

.: Consider 5 jobs and 2 machines.

Possible sequences = $(5!)^2$ = 14400. From these (14400) sequences the best sequence(having minimum total elapsed time) has to be selected.

Consider a printing press. Each job is processed through two machines MI and M2. Documents arrive there for printing books, articles, magazines etc. Printing is done with desired number of copies on machine MI. Binding of the materials is done on machine M2.The press has at present, five jobs on hand. The time estimates for printing and binding for each job are worked out as follows:

	Time (hours) for	
Job No.	Printing	Binding
1	22	50
2	18	25
3	55	45
4	42	50
5	35	20

How do you sequence the jobs in order to minimize the finish time (the total time devoted by the press) of all the jobs?

3.4.1 IMPORTANT TERMS

- **No of machines** means the number of service facilities through which the jobs must be passed for processing.
- **Processing order** – is the order in which the machines are required for processing the job.
- **Processing time** – is the time taken by each job at each machine.
- **Total elapsed time** – is the time interval between starting the first job and completing the last job.
- **Idle time** – is the time during which the machine remains idle during the total elapsed time.
- **No passing rule** –refers to the rule of maintaining the same order of processing for all the jobs. Each job should be processed in the particular order.

3.4.2 ASSUMPTIONS OF SEQUENCING:

- Only one operation is carried out in a machine at a time.
- Processing times are known and they do not change.
- Each operation as well as the job once started must be completed.
- Only one machine of each type is available.
- The transportation time in moving jobs from one machine to another is negligible.

- No inventory aspect of the problem is considered.
- Only on completion of an operation, the next operation can start.
- Processing times are independent of the order in which the jobs are performed.
- Jobs are completely known and are ready for processing when the period under consideration starts.

3.4.3 SEQUENCING FOR PROCESSING OF 'n' JOBS THROUGH TWO MACHINES [JOHNSON'S ALGORITHM]

- Let the jobs be 1,2,3,…………………….n
- Let the two machines be A & B.
- Let the processing order be A-B.
- Let the processing time in A be A_1, A_2, A_3…………………..A_n
- Let the processing time in B be B_1, B_2, B_3,…………………..B_n

STEP 1:

Examine the available processing time on Machine A & Machine B and find the smallest Value.

STEP 2:

a) If the minimum value falls on A schedule it first. If it occurs in B schedule it last.

b) If there is a tie of equal minimum values, one in A and one in B for different jobs then schedule the job in A first and schedule the job in B last.

c) If there is a tie equal minimum values both in A, choose the job with the minimum value in B and schedule it first and the next job consequently.

d) If there is a tie of equal minimum values both in B, choose the job with the minimum value in A and schedule it last and the next job previously.

e) If there is a tie of equal min values both in A and B for the same job, choose the job and schedule it either first or last. (Preferably first)

STEP 3:

Cancel the scheduled job along with the processing times Repeat the same procedure from step 1 till all the jobs are scheduled, to get the optimum sequence.

3.4.4 SEQUENCING FOR PROCESSING OF 'n' JOBS THROUGH THREE MACHINES

- Let the 3 machines be A, B and C.
- Let the processing order be ABC
- Let the jobs be 1, 2, 3,…………………….n.
- Let the processing time in A be A_1, A_2, A_3………………..…..A_n
- Let the processing time in B be B_1, B_2, B_3,…………….……..B_n
- Let the processing time in C be C_1, C_2, C_3…………………...C_n

The three-machine problem can be converted in to a two-machine problem and Johnson's method can be applied for finding the optimum sequence if either of the following condition is satisfied:

[Min Processing time in A >= Max processing time in B]

OR

[Min Processing time in C >= Max processing time in B]

Convert the 3-machines in to 2 fictitious (imaginary) machines to apply Johnson's method for finding the optimum sequence. Let the two fictitious machines be X and Y.

$\mathbf{X_i = A_i + B_i}$

$\mathbf{Y_i = B_i + C_i}$

Follow the same procedure of Johnson's method as for 2 machines to find out the sequence.

Note : Consider all the three actual machines(A, B & C) to find out the total elapsed time & find idle time.

3.4.5 SEQUENCING FOR PROCESSING OF ‘n’ JOBS THROUGH ‘m’ MACHINES

- Let the machines be A,B ,C………m
- Let the processing order be ABC..m
- Let the jobs be 1,2,3,…………………….n.
- Let the processing time in A be A_1, A_2, A_3……………….…..A_n
- Let the processing time in B be B_1, B_2, B_3,…………….……..B_n
- Let the processing time in C be C_1, C_2, C_3…………………..…..C_n
- Let the processing time in m be m_1, m_2, m_3…………………..…..m_n

The m machine problem can be converted in to a two-machine problem and Johnson’s method can be applied for finding the optimum sequence if either of the following condition is satisfied:

[Min Processing time in A >= Max processing time in B,C,D….m-1]

OR

[Min Processing time in m >= Max processing time in B,C,D….m-1]

Convert the m machines in to 2 fictitious (imaginary) machines to apply Johnson’s method for finding the optimum sequence. Let the two fictitious machines be X and Y.

$\mathbf{X_i = A_i + B_i \ldots\ldots (m-1)_i}$

$\mathbf{Y_i = B_i + C_i \ldots . m_i}$

Follow the same procedure of Johnson’s method as for 2 machines to find out the sequence.

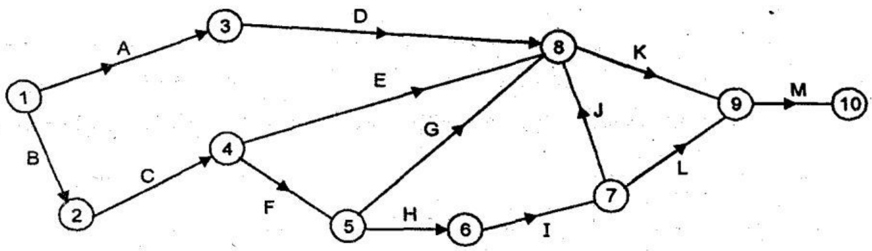
Note : Consider all the actual machines(A, B,C,D,E, …) to find out the total elapsed time & find idle time.

Problems

Problem 11.15. Construct the network for the following activity data:

Activity	Preceded by	Activity	Preceded by
A	–	–	–
B	–	H	F
C	B	I	H
D	A	J	I
E	C	K	D,E,G,J
F	C	L	I
G	F	M	K,L

Solution. Network:



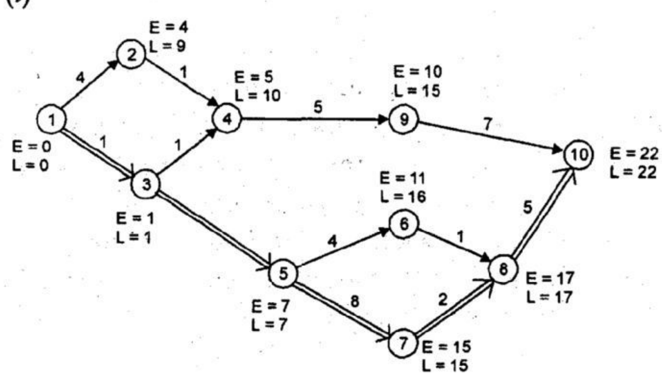
Problem 11.16. A project has the following time schedule :

Activity	1 – 2	1 – 3	2 – 4	3 – 4	3 – 5	4 – 9	5 – 6	5 – 7	6 – 8	7 – 8	8 – 9	8 – 10	9 – 10
Time weeks	4	1	1	1	6	5	4	8	1	2	1	5	7

1. Draw Network diagram and find the critical paths.

2. Calculate float on each activity.

Solution. (*i*)



2.

Activity	Duration (weeks)	Start Time		Finish Time		Total Float
		E	T_{LS}	T_{EF}	L	
1 – 2	4	0	5	4	9	5
1 – 3	1	0	0	1	1	0
2 – 4	1	4	9	5	10	5
3 – 4	1	1	9	2	10	8
3 – 5	6	1	1	7	7	0
4 – 9	5	5	10	10	15	5
5 – 6	4	7	12	11	16	5
5 – 7	8	7	7	15	15	0
6 – 8	1	11	16	12	17	5
7 – 8	2	15	15	17	17	0
8 – 10	5	17	17	22	22	0
9 – 10	7	10	15	17	22	5

Critical path 1—3—5—7—8—10 with project duration of 22 weeks.

Problem. 11.17. The time estimate for the activities of a PERT network are given

below :

Activity	t_0	t_m	t_p
1 – 2	1	1	7
1 – 3	1	4	7
1 – 4	2	2	8
2 – 5	1	1	1
3 – 5	2	5	14
4 – 6	2	5	8
5 – 6	3	6	15

(a) Draw the project network and identify all the path through it.

(b) Determine the expected project length.

(c) Calculate the standard deviation and variance of the project length.

(d) What is the probability that the project will be completed

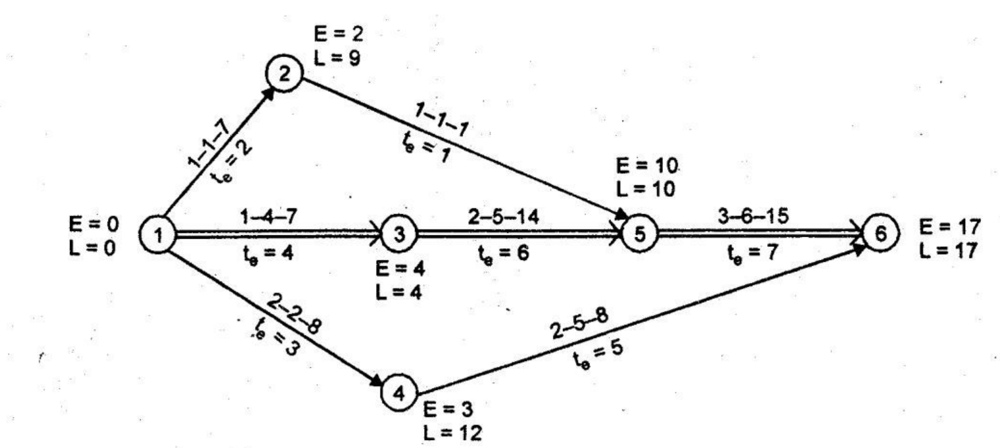
1. At least 4 weeks earlier than expected time.

2. No more than 4 weeks later than expected time.

(e) The probability that the project will be completed on schedule if the schedule completion time is 20 weeks.

(f) What should be the scheduled completion time for the probability of completion to be 90%.

Solution. (a) Network



Activity	t_0	t_m	t_p	$t_e = \frac{t_0 + 4t_m + t_p}{6}$	$\sigma^2 = \frac{(t_p - t_0)^2}{6}$
1 – 2	1	1	7	2	1
1 – 3	1	4	7	4	1
1 – 4	2	2	8	3	1
2 – 5	1	1	1	1	0
3 – 5	2	5	14	6	4
4 – 6	2	5	8	5	1
5 – 6	3	6	15	7	4

Critical path—1 —3—5—6

Project duration = 17 weeks.

(c) Variance of the project length is the sum of the variance of the activities on the critical.

$$V_{cp} = V_{1-3} + V_{3-5} + V_{5-6} = 1 + 4 + 4 = 9$$
$$\sigma^2 = V \Rightarrow \sigma^2 = 9 \Rightarrow \sigma = 3 \text{ weeks.}$$

(d) (i) Probability that the project will be completed at least 4 week earlier than expected time

Expected time (E_p) = 17 weeks

Scheduled time = 17 – 4 = 13 weeks

$$Z = \frac{13-17}{3} = -1.33$$

$$P(-1.33) = 1 - 0.9082 = 0.0918$$

2. Probability that the project will be completed at least 4 weeks later than expected

Time

Expected time = 17 weeks Scheduled time =17+ 4 =21 weeks

$$Z = \frac{21-17}{3} = 1.33$$

$$P(1.33) = 0.9082 = 90.8\%.$$

(e) Scheduled time = 20 weeks

$$Z = \frac{20-17}{3} = 1$$

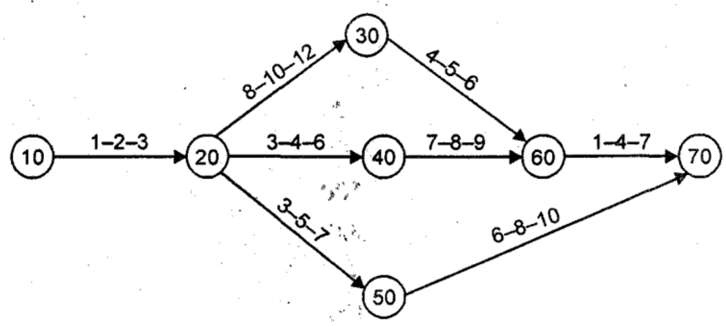
$$P(1) = 84.13\%$$

(f) Value of Z for P = 0.9 is 1.28 (from probability table)

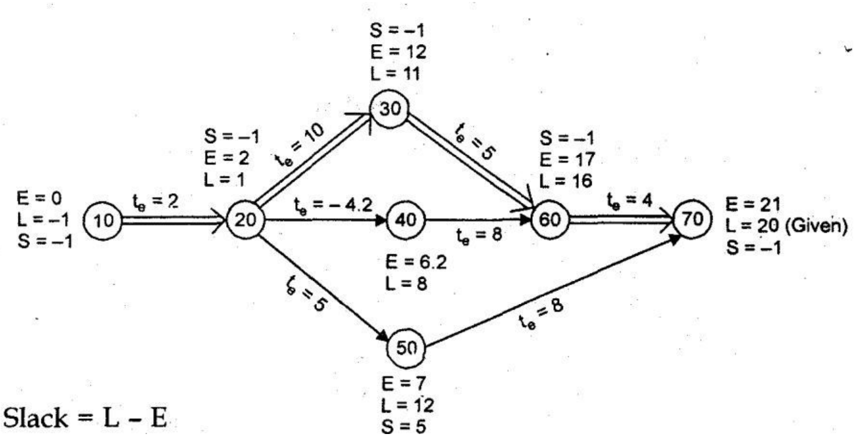
$$1.28 = \frac{T-17}{3}$$

$$T = 17 + 3.84 = 20.84 \text{ weeks.}$$

Problem 11.18. Consider the PERT network given in fig. Determine the float of each activity and identify the critical path if the scheduled completion time for the project is 20 weeks.



Solution.



(s) Slack = L – E

Activity	$t_e = \frac{t_0 + 4tm + t_p}{6}$	Start Time		Finish Time		Total
		E	T_{ES}	T_{EF}	L	Float
10 – 20	2	0	–1	2	1	–1
20 – 30	10	2	1	12	11	–1
20 – 40	4.2	2	3.8	6.2	8	1.8
20 – 50	5	2	7	7	12	5
30 – 60	5	12	11	17	16	–1
40 – 60	8	6.2	8	14.2	16	1.8
50 – 70	8	7	12	15	20	5
60 – 70	4	17	16	21	20	–1

Critical path 10 - 20- 30 - 60 — 70.

Problem. 3.25. There are five jobs each of which just go through two machines A and B in the order of AB.

Processing times are given below. Determine a sequence for five jobs that will

minimize the elapse time and also calculate the total time.

Job	1	2	3	4	5
Time for A	5	1	9	3	10
Time for B	2	6	7	8	4

Determine the sequence for the jobs so as to minimize the process time. Find

total elapsed time.

Solution : Examine the columns of processing time on rn/c A and B and find the

smallest value. If this value falls in column A, schedule the job first on M/c, A, if this

value falls in column B, schedule the jobs last on M/c A. In this way sequence of jobs so as to minimize the process time is

2	4	3	5	1

Job	Machine A		Machine B	
	Time in	Time out	Time in	Time out
2	0	1	1	7
4	1	4	7	15
3	4	13	15	22
5	13	23	23	27
1	23	28	28	30

Ans. Sequence

2	4	3	5	1

Total elapsed time = 30 hours. V

Problem 3.26. Find the sequence that minimize the total elapsed time to complete the following Jobs. Each Job is processed in the order of AB.

Job (Processing time in minutes)

		1	2	3	4	5	6	7
m/c	**A**	**12**	**6**	**5**	**11**	**5**	**7**	**6**
	B	**7**	**8**	**9**	**4**	**7**	**8**	**3**

Determine the sequence for the jobs so as to minimize the process time. Find the total elapsed time and idle time of M/c A and M/c B.

Solution : The sequence of jobs so as to minimize the process time is

5	3	2	6	1	4	7

Job	Machine A		Machine B	
	Time in	Time out	Time in	Time out
5	0	5	5	12
3	5	10	12	21
2	10	16	21	29
6	16	23	29	37
1	23	35	37	44
4	35	46	46	50
7	46	52	52	55

Min elapsed time = 55 mins
Idle time of M/c A = 3 mins
Idle time of M/c B = 9 mins.

QUESTIONS

SEQUENCING

1. Find out the optimum sequence for the jobs which are to be processed through two machines. Machines A and B.

	Jobs			
	1	2	3	4
Machine A	1	6	6	5
Machine B	2	8	10	3

2.

	Jobs			
	1	2	3	4
Machine A	1	6	8	5
Machine B	2	10	6	3

3.

	Jobs					
	1	2	3	4	5	6
Machine A	2	4	6	3	3	10
Machine B	4	4	8	4	9	12

4. Find out the appropriate sequence total elapse time and total idle time for jobs to be processed through 2 machines.

	Jobs					
	A	B	C	D	E	F
Machine X	11	7	12	4	6	7
Machine Y	11	11	11	11	11	15

5.

	Jobs				
	1	2	3	4	5
Machine A	4	8	6	8	1
Machine B	3	4	7	8	5

6. Find out the appropriate sequence, idle time, and total elapsed time for processing through 3 machines.

	Jobs				
	1	2	3	4	5
Machine A	4	8	6	4	6
Machine B	2	3	1	1	4
Machine C	6	8	2	4	3

7. Find out the appropriate sequence, idle time, and total elapsed time for processing through 3 machines.

	Jobs					
	A	B	C	D	E	F
Machine 1	8	7	3	2	5	1
Machine 2	3	4	5	2	1	6
Machine 3	8	7	6	9	10	9

8. Find out the optimum sequence, idle time and total elapsed time for the jobs to be processed through 4 machines.

	Jobs			
	A	B	C	D
Machine 1	8	8	4	3
Machine 2	4	2	1	6
Machine 3	6	8	10	12
Machine 4	14	18	20	22

9.

	Machines			
	M1	M2	M3	M4
JOB1	11	8	7	14
JOB 2	10	6	8	19
JOB 3	9	7	5	18
JOB 4	8	5	5	18

NETWORK ANALYSIS
NETWORK CONSTRUCTION AND SCHEDULING

1. Draw the network for the project given :

Activities	**Predecessor**
A	-
B	-
C	-
D	A
E	B
F	B
G	C
H	D
I	E
J	H, I
K	F, G

2. Draw the network for the project given :

Activities	**Predecessor**
P	-
Q	-
R	-
S	P, Q
T	R, Q

3. Draw the network for the project given :

Activities	**Predecessor**
A	-
B	A
C	A
D	-
E	D
F	B, C, E
G	F
H	E
I	G, H

4. Draw the network for the project given :

Activities	**Predecessor**
A	-
B	-
C	A, B
D	B
E	B
F	A, B
G	F, D
H	F, D
I	C, G

5. Draw the network for the project given :

Activities: A, D, and E can start simultaneously. Activities B, C is greater than A; G,F greater than D, C; H> E, F.

Hint:

Activities	**Predecessor**
A	-
B	A
C	A
D	-
E	-
F	D, C
G	D, C
H	E, F

6. A< C, D; B< E; C, E < F, G; D < H; G < I; H, I, < J.

Hint:

Activities	**Predecessor**
A	-
B	-
C	A
D	A
E	B

F	C, E
G	C, E
H	D
I	G
J	H, I

CRITICAL PATH METHOD

7. Draw the network and also find the critical path. Duration of each activity is given below

A < C, D, I; B < G, F; D < G, F; F < H, K; G, H, < J; I, J, K < E.

Activities	**Predecessor**	**Duration**
A	-	5 days
B	-	3
C	A	10
D	A	2
E	I, J, K	8
F	B, D	4
G	B, D	5
H	F	6
I	A	12
J	G & H	8
K	F	9

8. Draw the network and find the critical path. Also find earliest start, earliest finish, latest start and latest finish of each activity.

Activity	Duration
1-2	8 days
1-3	4
2-4	10
2-5	2
3-4	5
4-5	3

9. Draw the network and find the critical path, and also calculate floats

Activity	Duration
1-2	8 days
1-3	7
1-5	12
2-3	4
2-4	10
3-4	3
3-5	5
3-6	10
4-6	7
5-6	4

PROGRAM EVALUATION AND REVIEW TECHNIQUE

10. Draw the network; find the expected duration and the variance of the project. Also find the standard deviation of the project.

Activity	Optimistic time	Moderate time	Pessimistic time
1-2	3	5	8
1-3	3	4	9
1-4	8	10	12
2-4	14	15	16
3-4	3	4	6
2-5	1	3	5
3-5	2	4	6
4-5	3	4	6

11. Draw the network; find the expected duration and the variance of the project. Also find the standard deviation of the project

Activity	a	m	b
1-2	3	6	15
1-3	2	5	14
1-4	6	12	30
2-5	2	5	8
2-6	5	11	17
3-6	3	6	15
4-7	3	9	27
5-7	1	4	7
6-7	2	5	8

What is the probability that project will be completed within 27 days.

- What is the probability that project will be completed within 33 days.
- What is the probability that project will take above 33 days.
- What is the probability that project will be completed within 25 days or probability that the project is just completed on the expected duration.
- What is the probability that project will be completed between 20-25 days.

12. Draw the network; find the expected duration and the variance of the project. Also find the standard deviation of the project

Activity	to	tp	tm
1-2	0.8	1.2	1
2-3	3.7	9.9	5.6
2-4	6.2	15.4	6.6
3-4	2.1	6.1	2.7
4-5	0.8	3.6	3.4
5-6	0.9	1.1	1

UNIT – III RESOURCE SCHEDULING AND NETWORK ANALYSIS

MODEL QUESTION PAPER

PART - A

1. What do you mean by sequencing? Explain the objectives of sequencing.
2. Explain the procedure of sequencing.
3. What is meant by a) Total elapsed time b) Idle time.
4. Write the conditions to convert (i) 3 machines problem into 2 machines problem (ii) 4 machines problem into 2 machines problem in sequencing.
5. Write short notes on i) Total float ii) Free float iii) Independent float
6. What do you mean by critical path?
7. Explain the procedure for constructing network diagram
8. Explain the steps in CPM project planning.
9. Differentiate between PERT and CPM.
10. What is meant by a) Project b) Earliest Start and Earliest Finish b) Latest start and Latest finish?
11. Find the optimum Sequence for the following tasks:

Tasks

		A	B	C	D	E	F	G	H	I
	M1	2	5	4	9	6	8	7	5	4
Machines	M2	6	8	7	4	3	9	3	8	11

12. Construct the network for the project whose activities and their precedence relationships are as given below.

    A, B, C can start simultaneously

    A<F, E: B<D; C, E, D<G.

PART - B

1. The time in hours to process six known batches J1 – J6 through the washer and cooker is given below:

Batches

	J1	J2	J3	J4	J5	J6
Washer (M1)	4	7	3	12	11	9
Cooker (M2)	11	7	10	8	10	13

Find out the optimum sequence and also find out total elapsed time and idle time.

2. There are six jobs which must go through two machines A and B. Processing time is given below. Find out the optimum sequence, idle time, and total elapsed time.

Job	1	2	3	4	5	6
Machine A	8	9	11	12	16	20
Machine B	7	15	10	14	13	9

3. Find out the optimum sequence, idle time, and total elapsed time for the given 3 machines problem.

	Jobs				
Machines	1	2	3	4	5
Machine A	4	8	6	4	6
Machine B	2	3	1	1	4
Machine C	6	8	2	4	3

4. Find out the optimum sequence, idle time and total elapsed time for the jobs to be processed through 4 machines.

	Jobs			
	A	B	C	D
Machine 1	8	8	4	3
Machine 2	4	2	1	6
Machine 3	6	8	10	12
Machine 4	14	18	20	22

5. Draw the network and find the critical path. Find earliest start, earliest finish, latest start, and latest finish of each activity.

Activity	Duration	Preceding Activity

A	6	-
B	8	A
C	4	A
D	9	B
E	2	C
F	7	D

6. Draw the network and find the critical path, and also calculate floats

Activity	Duration
1-2	8 days
1-3	7
1-5	12
2-3	4
2-4	10
3-4	3
3-5	5
3-6	10
4-6	7
5-6	4

7. Draw the network; find the expected duration and the variance of the project. Also find the standard deviation of the project

Activity	a	m	b
1-2	3	6	15
1-3	2	5	14
1-4	6	12	30
2-5	2	5	8
2-6	5	11	17
3-6	3	6	15
4-7	3	9	27
5-7	1	4	7
6-7	2	5	8

What is the probability that project will be completed within 27 days.

- What is the probability that project will be completed within 33 days.
- What is the probability that project will take above 33 days.
- What is the probability that project will be completed within 25 days or probability that the project is just completed on the expected duration.
- What is the probability that project will be completed between 20-25 days.

8. Draw the network and find the critical path. Find earliest start, earliest finish, latest start, latest finish, total float, free float and independent float for each activity.

Activity	Preceding Activity	Duration
A	-	2
B	A	6
C	A	6
D	B	5
E	C,D	3
F	-	3
G	E,F	1

9. Draw the network and find the critical path. Find earliest start, earliest finish, latest start, latest finish A< C, D; B< E; C, E < F, G; D < H; G < I; H, I, < J.

Hint:

Activities	**Predecessor**
A	-
B	-
C	A
D	A
E	B
F	C, E
G	C, E
H	D
I	G
J	H, I

10. Draw the network; find the expected duration and the variance of the project. Also find the standard deviation of the project

Activity	To	tp	tm
1-2	0.8	1.2	1
2-3	3.7	9.9	5.6
2-4	6.2	15.4	6.6
3-4	2.1	6.1	2.7
4-5	0.8	3.6	3.4
5-6	0.9	1.1	1