

# Results of the 2016 IEEE WCCI/CEC Competition on Niching Methods for Multimodal Optimization

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# Outline

- 1 Introduction
- 2 Participants
- 3 Results
- 4 Winners
- 5 Summary

# Introduction

- Many real-world problems are “multi-modal” by nature, i.e., multiple satisfactory solutions exist
- **Niching methods:** promote and maintain formation of multiple stable subpopulations within a single population
  - **Aim:** maintain diversity and locate multiple globally optimal solutions.
- **Challenge:** Find an efficient optimization algorithm, which is able to **locate multiple global optimal solutions** for multi-modal problems with various characteristics.

# Competition: CEC 2013/2015/2016

Provide a common platform that encourages fair and easy comparisons across different niching algorithms.

**X. Li, A. Engelbrecht, and M.G. Epitropakis, “Benchmark Functions for CEC’2013 Special Session and Competition on Niching Methods for Multimodal Function Optimization”, Technical Report, Evolutionary Computation and Machine Learning Group, RMIT University, Australia, 2013**

- 20 benchmark multi-modal functions with different characteristics
- 5 accuracy levels:  $\varepsilon \in \{10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}\}$
- The benchmark suite and the performance measures have been implemented in: C/C++, Java, MATLAB, (Python soon)

# Benchmark function set

X. Li, A. Engelbrecht, and M.G. Epitropakis, “**Benchmark Functions for CEC’2013 Special Session and Competition on Niching Methods for Multimodal Function Optimization**”, Technical Report, Evolutionary Computation and Machine Learning Group, RMIT University, Australia, 2013

| Id       | Dim.     | # GO   | Name                     | Characteristics  |
|----------|----------|--------|--------------------------|--|
| $F_1$    | 1        | 2      | Five-Uneven-Peak Trap    | Simple, deceptive  |
| $F_2$    | 1        | 5      | Equal Maxima             | Simple   |
| $F_3$    | 1        | 1      | Uneven Decreasing Maxima | Simple   |
| $F_4$    | 2        | 4      | Himmelblau               | Simple, non-scalable, non-symmetric                                    |
| $F_5$    | 2        | 2      | Six-Hump Camel Back      | Simple, not-scalable, non-symmetric                                    |
| $F_6$    | 2,3      | 18,81  | Shubert                  | Scalable, #optima increase with D, unevenly distributed grouped optima |
| $F_7$    | 2,3      | 36,216 | Vincent                  | Scalable, #optima increase with D, unevenly distributed optima         |
| $F_8$    | 2        | 12     | Modified Rastrigin       | Scalable, #optima independent from D, symmetric                        |
| $F_9$    | 2        | 6      | Composition Function 1   | Scalable, separable, non-symmetric                                     |
| $F_{10}$ | 2        | 8      | Composition Function 2   | Scalable, separable, non-symmetric                                     |
| $F_{11}$ | 2,3,5,10 | 6      | Composition Function 3   | Scalable, non-separable, non-symmetric                                 |
| $F_{12}$ | 2,3,5,10 | 8      | Composition Function 4   | Scalable, non-separable, non-symmetric                                 |

## Measures:

**Peak Ratio** (PR) measures the average percentage of all known global optima found over multiple runs:

$$PR = \frac{\sum_{run=1}^{NR} \# \text{ of Global Optima}_i}{(\# \text{ of known Global Optima}) * (\# \text{ of runs})}$$

## Who is the winner:

- The participant with **the highest average Peak Ratio performance on all benchmarks wins**.
- In all functions the following holds: the higher the PR value, the better

# Participants

## Submissions to the competition:

- **(rlsis)**: Restarted Local Search with Improved Selection of Starting Points, Simon Wessing
- **(rs-cmsa-es)**: Benchmarking Covariance Matrix Self Adaption Evolution Strategy with Repelling Subpopulations, Ali Ahrari, Kalyanmoy Deb and Mike Preuss
- **(ascga)**: Adaptive species conserving genetic algorithm, Jian-Ping Li, Felician Campean
- **(nea2+)**: Niching the CMA-ES via Nearest-Better Clustering: First Steps Towards an Improved Algorithm, Mike Preuss

# Participants (2)

## Implemented algorithms for comparisons:

- (**CrowdingDE**) Crowding Differential Evolution [7]
- (**DE/nrand/1**) Niching Differential Evolution algorithms with neighborhood mutation strategies [8]
- (**dADE/nrand/1**) A Dynamic Archive Niching Differential Evolution algorithm for Multimodal Optimization [9]
- (**NEA2**) Niching the CMA-ES via Nearest-Better Clustering [10]
- (**NMMSO**) Niching Migratory Multi-Swarm Optimiser [6]

In the repository: CMA-ES, IPOP-CMA-ES, DE/nrand/1,2, DECG, DELG, DELS-aj, CrowdingDE, dADE/nrand/1,2, NEA1, NEA2, N-VMO, PNA-NSGAI, A-NSGAI, ALNM, MEA, MSSPSO, LSEAGP, LSEAEA, NMMSO, etc



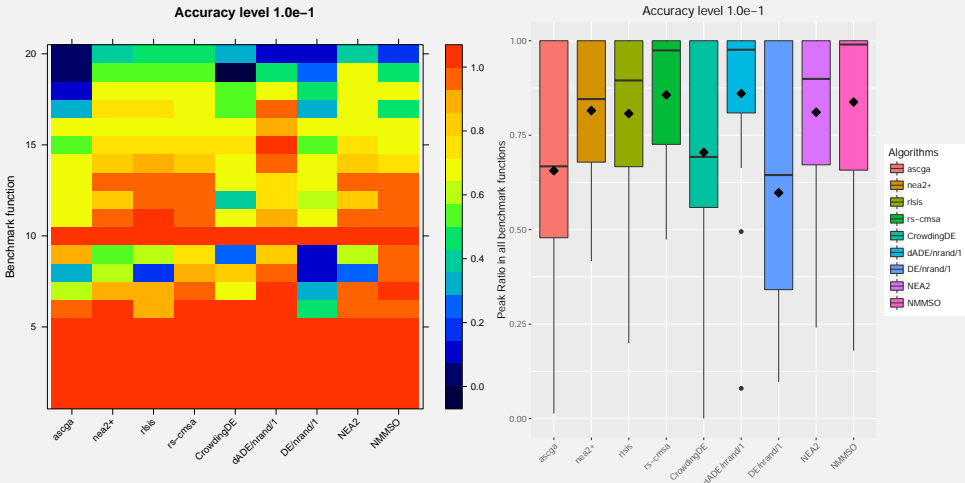
# Results

## Summary:

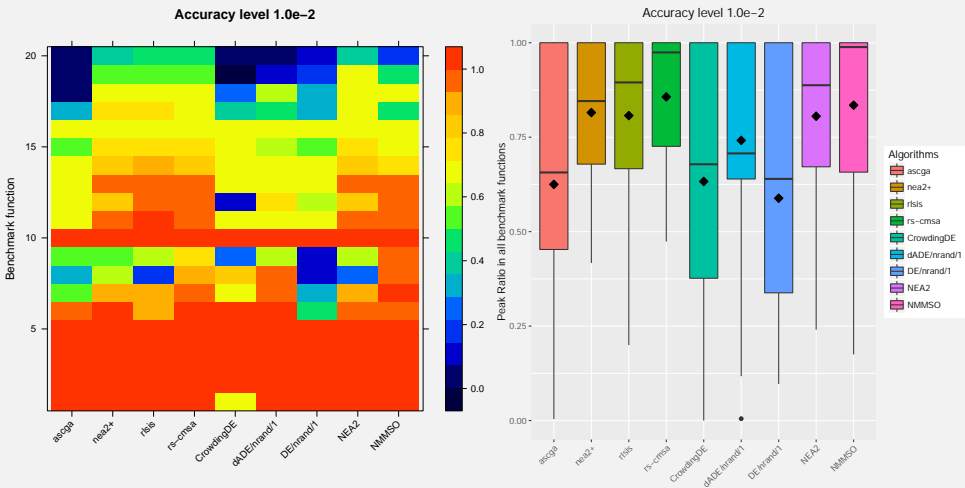
- **4** new search algorithms
- **5** classic comparators (based on CEC 2013, 2015)
- **20** multi-modal benchmark functions
- **5** accuracy levels  $\varepsilon \in \{10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}\}$
- Results: **per accuracy level & over all accuracy levels**
- In total (CEC2013 & CEC2015) more than **21 algorithms** in the repository:

<https://github.com/mikeagn/CEC2013>

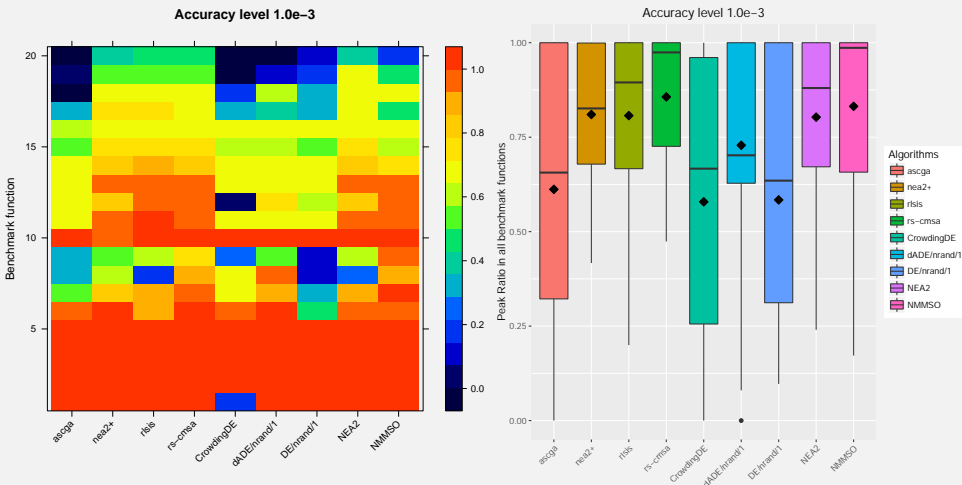
# Accuracy level $\varepsilon = 10^{-1}$



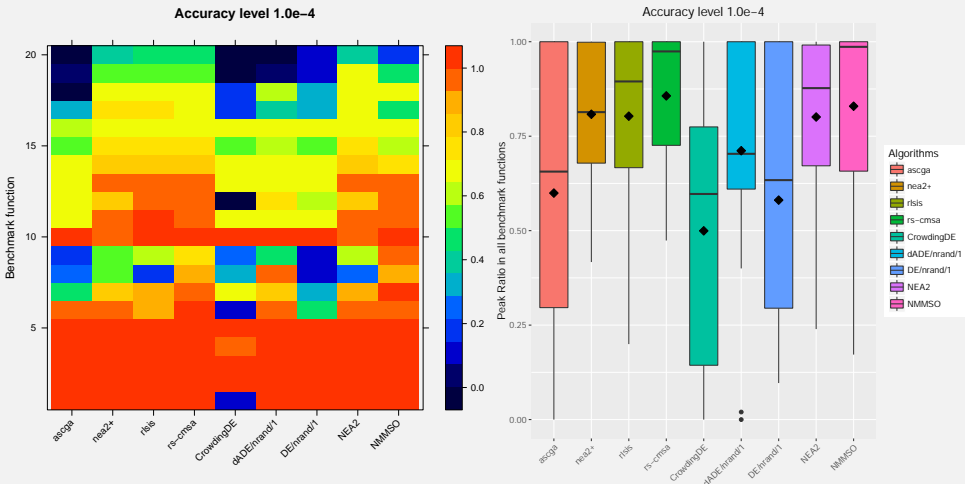
# Accuracy level $\varepsilon = 10^{-2}$



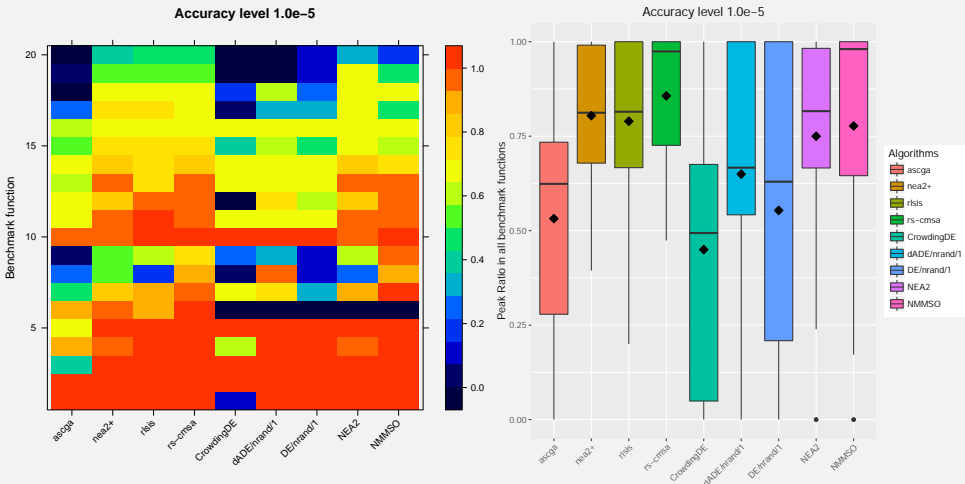
# Accuracy level $\varepsilon = 10^{-3}$



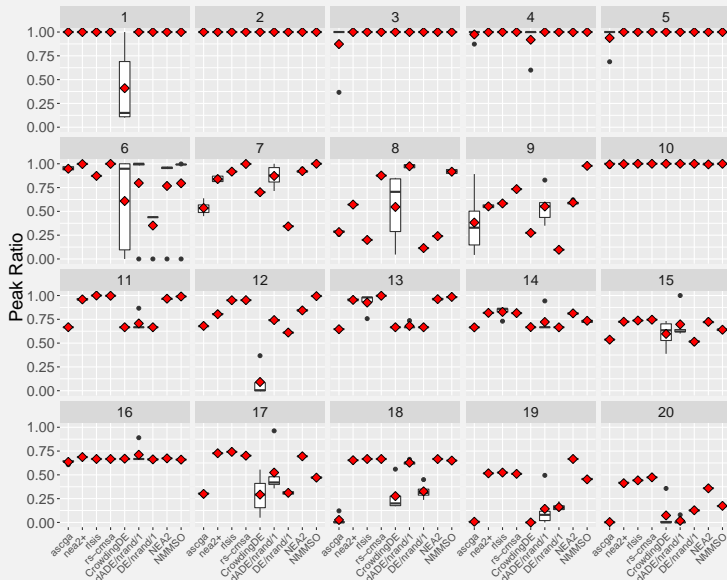
# Accuracy level $\varepsilon = 10^{-4}$



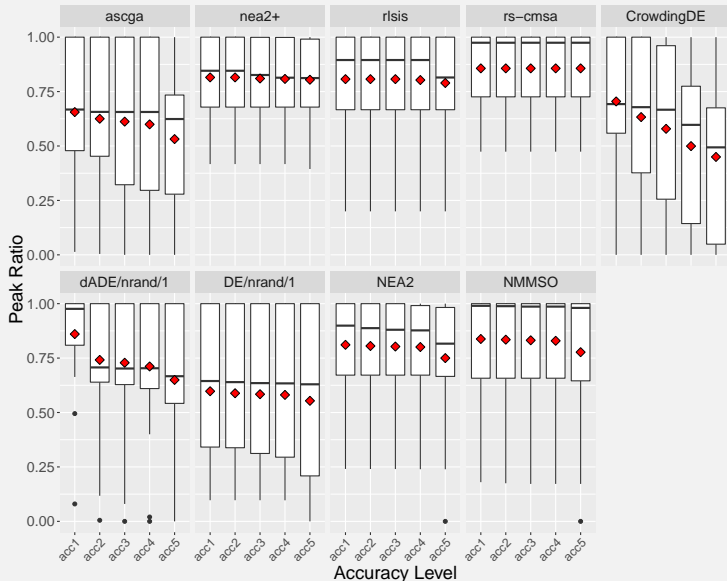
# Accuracy level $\varepsilon = 10^{-5}$



# Performance per benchmark across all accuracy levels

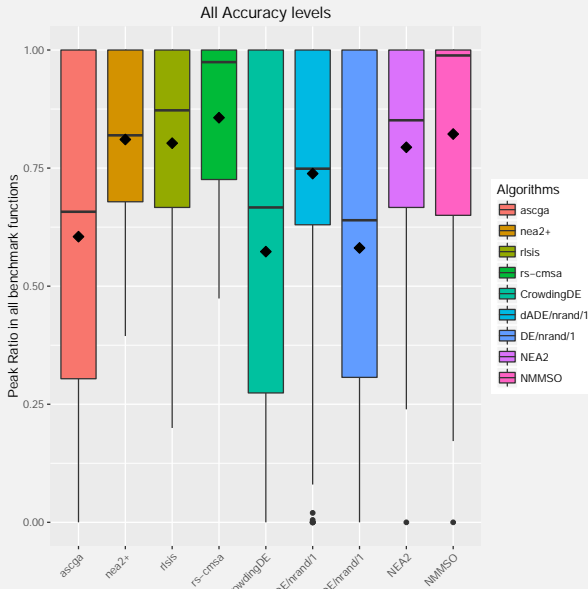


# Performance per algorithm





# Overall performance (1)



# Overall performance (2)

|           | Algorithm    | Statistics   |              |       |          |
|-----------|--------------|--------------|--------------|-------|----------|
|           |              | Mean         | Median       | St.D. | Rank     |
| CEC2016   | ascga        | 0.604        | 0.657        | 0.348 | 7        |
|           | nea2+        | <b>0.810</b> | 0.819        | 0.190 | <b>3</b> |
|           | rlsis        | 0.802        | <b>0.872</b> | 0.225 | 4        |
|           | rs-cmsa      | <b>0.856</b> | <b>0.974</b> | 0.174 | <b>1</b> |
| CEC2013/5 | NMMSO        | <b>0.822</b> | <b>0.988</b> | 0.253 | <b>2</b> |
|           | NEA2         | 0.794        | 0.851        | 0.233 | 5        |
|           | DE/nrand/1   | 0.580        | 0.639        | 0.333 | 8        |
|           | dADE/nrand/1 | 0.738        | 0.748        | 0.301 | 6        |
|           | CrowdingDE   | 0.573        | 0.666        | 0.361 | 9        |

# Overall performance: CEC2013 + 2015 + 2016

| Algorithm    | Statistics |        |        |      |
|--------------|------------|--------|--------|------|
|              | Mean       | Median | St.D.  | Rank |
| rs-cmsa      | 0.8566     | 0.9743 | 0.1746 | 1    |
| NMMSO        | 0.8221     | 0.9885 | 0.2538 | 2    |
| nea2+        | 0.8105     | 0.8193 | 0.1902 | 3    |
| rlsis        | 0.8027     | 0.8723 | 0.2250 | 4    |
| NEA2         | 0.7940     | 0.8513 | 0.2332 | 5    |
| LSEAEA       | 0.7477     | 0.9030 | 0.3236 | 6    |
| dADE/nrand/1 | 0.7383     | 0.7488 | 0.3010 | 7    |
| LSEAGP       | 0.7302     | 0.7900 | 0.3268 | 8    |
| CMA-ES       | 0.7137     | 0.7550 | 0.2807 | 9    |
| N-VMO        | 0.6983     | 0.7140 | 0.3307 | 10   |
| dADE/nrand/2 | 0.6931     | 0.7150 | 0.3174 | 11   |
| ALNM         | 0.6594     | 0.7920 | 0.3897 | 12   |
| PNA-NSGAI    | 0.6141     | 0.6660 | 0.3421 | 13   |
| NEA1         | 0.6117     | 0.6496 | 0.3280 | 14   |
| DE/nrand/2   | 0.6082     | 0.6667 | 0.3130 | 15   |
| ascga        | 0.6048     | 0.6575 | 0.3485 | 16   |
| DE/nrand/1   | 0.5809     | 0.6396 | 0.3338 | 17   |
| DELS-aj      | 0.5760     | 0.6667 | 0.3857 | 18   |
| CrowdingDE   | 0.5731     | 0.6667 | 0.3612 | 19   |
| DELG         | 0.5706     | 0.6667 | 0.3925 | 20   |
| DECG         | 0.5516     | 0.6567 | 0.3992 | 21   |
| IPOP-CMA-ES  | 0.3625     | 0.2600 | 0.3117 | 22   |
| MEA          | 0.3585     | 0.2075 | 0.3852 | 23   |
| A-NSGAI      | 0.3275     | 0.0740 | 0.4044 | 24   |
| MSSPSO       | 0.2188     | 0.0000 | 0.3913 | 25   |

# Winners

## Ranking based on average PR values (only CEC2016)

- ➊ **(rs-cmsa-es)**: Benchmarking Covariance Matrix Self Adaption Evolution Strategy with Repelling Subpopulations, Ali Ahrari, Kalyanmoy Deb and Mike Preuss
- ➋ **(nea2+)**: Niching the CMA-ES via Nearest-Better Clustering: First Steps Towards an Improved Algorithm, Mike Preuss
- ➌ **(rlsis)**: Restarted Local Search with Improved Selection of Starting Points, Simon Wessing
- ➍ **(ascga)**: Adaptive species conserving genetic algorithm, Jian-Ping Li, Felician Campean

**Note: The algorithms have not been fine-tuned for the specific benchmark suite!**

# Conclusions

## Summary

- Four new search algorithms (in total 25 algorithms!)
- **Winner: rs-cmsa-es:** Benchmarking Covariance Matrix Self Adaption Evolution Strategy with Repelling Subpopulations, Ali Ahrari, Kalyanmoy Deb and Mike Preuss
  - Competitive on average performance, (CMA-ES, repelling strategy)
  - CMA-ES: Strong local searcher to accurately locate global optima
  - Repelling: To avoid wasting effort in already searched areas

# Conclusions (2)

- The competition gave a boost to the multi-modal optimization community
- New competitive and very promising approaches

## Key characteristics of the algorithms:

- Usage of local models to maintain diversity and exploit locally the neighborhoods
- Methodologies: repelling, restarts, surrogates
- Algorithms: CMA-ES, Evolutionary Algorithms, Multi-swarms, GAs.

# Future Work

## Possible objectives:

- Re-organize the competitions in future
- Enhance the benchmark function set
- Introduce new performance measures
- Further automate the experimental design and results output
- Boost multi-modal optimization community

# Acknowledgment

We really want to thank for their help:

- The participants :-)

Stay tuned!

- IEEE CIS Task Force on Multi-Modal Optimization
- <http://www.epitropakis.co.uk/ieee-mmo/>



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(-: Thank you very much for your attention :-)



Questions ???

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