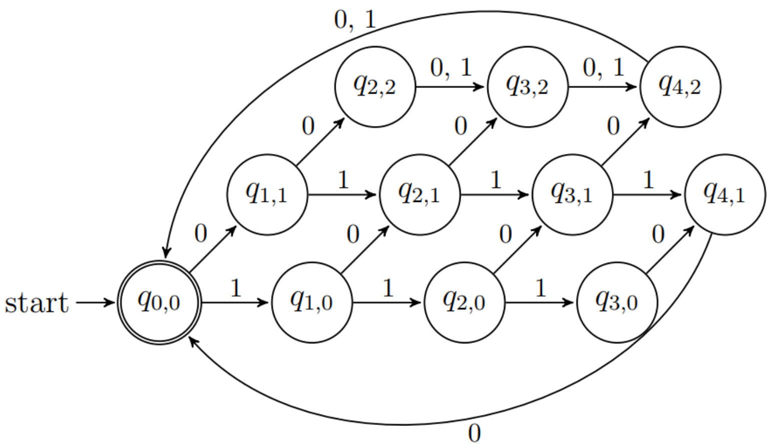
第1次作业：有穷自动机参考答案

2.2.5

(a)

将字符串划分为五个一段，每一段都至少有两个0

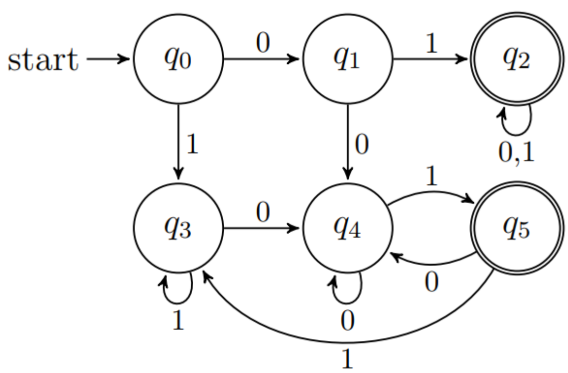


(b)

$A = (Q, \Sigma, \delta, q_0, F)$

- $Q = \{q_i : i \in \{0,1,\cdots,1023\}\}$
- $\Sigma = \{0,1\}$
- $\delta(q_i, a) = q(i \cdot 2+\mathrm{a})\, mod\ 1024$
- $F = \{q_i : i \geq 512\}$

(c)



(d)

$A = (Q, \Sigma, \delta, q_{0,0}, F)$

$\cdot\ Q = \{q_{i,j} : i \in \{0,1,2\}, j\epsilon\{0,1,2,3,4\}\}$

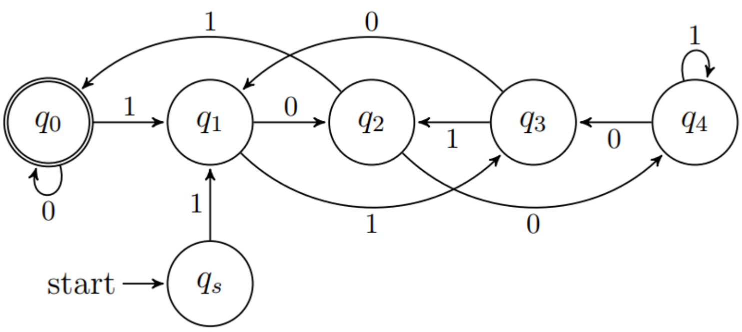
$\cdot\ \Sigma = \{0,1\}$

$\cdot\ \delta(q_{i,j}, 0) = q_{i,(j+1)}\ mod\ 5,\ \delta(q_{i,j}, 1) = q(i+1)\ mod\ 3, j$

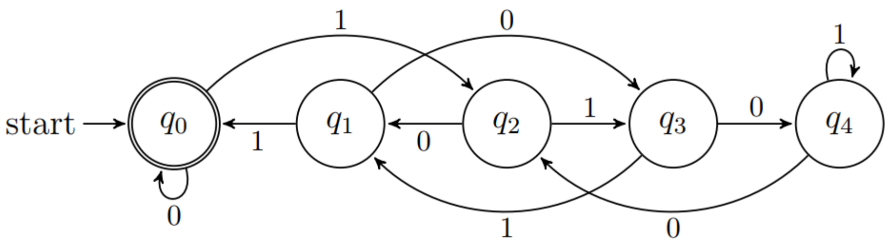
$\cdot\ F = \{q_{0,0}\}$

2.2.6

(a)



(b)



2.2.9

(a)

$w \neq \epsilon \implies |w| \geq 1$

Basis: If $|w| = 1$, then $w = a, a \in \Sigma$.

Because:

- $\hat{\delta}(q_0, w) = \delta(\hat{\delta}(q_0, \epsilon), a) = \delta(q_0, a)$
- $\hat{\delta}(q_f, w) = \delta(\hat{\delta}(q_f, \epsilon), a) = \delta(q_f, a)$
- $\delta(q_0, a) = \delta(q_f, a)$

$\therefore \hat{\delta}(q_0, w) = \hat{\delta}(q_f, w)$

Induction: If the proposition is true for $|w| \leq k (k \geq 1)$, when $|w| = k + 1$, w can be written as $w = xa$, where $x \in \Sigma^k$ and $a \in \Sigma$

$|x| = k \leq k \implies \hat{\delta}(q_0, x) = \hat{\delta}(q_f, x)$

$\therefore \hat{\delta}(q_0, w) = \delta(\hat{\delta}(q_0, x), a) = \delta(\hat{\delta}(q_f, x), a) = \hat{\delta}(q_f, w)$

Conclusion: $\forall w \in \Sigma^*, \hat{\delta}(q_0, w) = \hat{\delta}(q_f, w)$

(b)

$\forall x \in \Sigma^+ \wedge x \in L(A)$

Basis: If $k = 1$, then $x \in L(A) \implies x^1 = x \in L(A)$

Induction: If $x^k \in L(A)(k \geq 1)$, i.e. $\hat{\delta}(q_0, x^k) = q_f$ (because $F = \{q_f\}$), then

$\hat{\delta}(q_0, x^{k+1}) = \hat{\delta}(\hat{\delta}(q_0, x), x^k) = \hat{\delta}(q_f, x^k)$

$\because k > 0 \wedge |x| > 0 \implies x^k \neq \epsilon, \therefore \hat{\delta}(q_f, x^k) = \hat{\delta}(q_0, x^k) = q_f$

$\therefore x^{k+1} \in L(A)$

Conclusion: $\forall k > 0, x^k \in L(A)$

$\therefore \forall x \in \Sigma^+ \wedge x \in L(A) \wedge k > 0, x^k \in L(A)$

2.2.10

Claim:

$L(A) = \{\omega : \omega$包含奇数个$1\}$

Proof:

用归纳法证明以下两个命题:

1. $\hat{\delta}(A,w)=A \iff w$中包含偶数个1
2. $\hat{\delta}(A,w)=B \iff w$中包含奇数个1

Basis: 若 $|w|=0 \implies w=\epsilon$

ϵ 中包含 0 个 1, 0 是偶数, 所以 $\hat{\delta}(A,\epsilon)=A$, 满足命题 1 和命题 2

Induction: 若 $w\in\Sigma^k$ 时命题 1 和命题 2 都满足, 当 $w\in\Sigma^{k+1}$ 时, 把 w 写成 xa, 其中 $x\in\Sigma^k$, $a\in\Sigma$

- 若 x 中包含偶数个 1 且 $a=0$, 根据归纳假设 $\hat{\delta}(A,x)=A$, 所以 $\hat{\delta}(A,w)=\delta(\hat{\delta}(A,x),0)=\delta(A,0)=A$. 同时 $w=x0$ 中包含偶数个 1, 与命题 1 和命题 2 相容
- 若 x 中包含偶数个 1 且 $a=1$, 根据归纳假设 $\hat{\delta}(A,x)=A$, 所以 $\hat{\delta}(A,w)=\delta(\hat{\delta}(A,x),1)=\delta(A,1)=B$. 同时 $w=x1$ 中包含奇数个 1, 与命题 1 和命题 2 相容
- 若 x 中包含奇数个 1 且 $a=0$, 根据归纳假设 $\hat{\delta}(A,x)=B$, 所以 $\hat{\delta}(A,w)=\delta(\hat{\delta}(A,x),0)=\delta(B,0)=B$. 同时 $w=x0$ 中包含奇数个 1, 与命题 1 和命题 2 相容
- 若 x 中包含奇数个 1 且 $a=1$, 根据归纳假设 $\hat{\delta}(A,x)=B$, 所以 $\hat{\delta}(A,w)=\delta(\hat{\delta}(A,x),1)=\delta(B,1)=A$. 同时 $w=x1$ 中包含偶数个 1, 与命题 1 和命题 2 相容

Conclusion: 命题 1 和命题 2 成立

又因为 A 是起始状态, $F=\{B\}$, 所以 $L(A)=\{w:w$中包含奇数个1$\}$

2.2.11

Claim:

$L(A) = \{w : w\text{不包含子串}00\}$

Proof:

用归纳法证明以下三个命题:

1. $\hat{\delta}(A, w) = A \iff w$不包含 00 且 w 不以 0 结尾
2. $\hat{\delta}(A, w) = B \iff w$不包含 00 且 w 以 0 结尾
3. $\hat{\delta}(A, w) = C \iff w$包含00

Basis: 若 $|w| = 0 \implies w = \epsilon$

ϵ 不包含 00 而且不以 0 结尾, 所以 $\hat{\delta}(A, \epsilon) = A$, 满足以上三个命题

Induction: 若 $w \in \Sigma^k$ 时以上三个命题都满足, 当 $w \in \Sigma^{k+1}$ 时, 把 w 写成 xa, 其中 $x \in \Sigma^k$, $a \in \Sigma$

- w 包含 00

  $\iff$ (x 包含 00) $\vee$ (x 不包含 00 且以 0 结尾, 同时 $a = 0$)

  $\iff (\hat{\delta}(A, x) = C) \vee (\hat{\delta}(A, x) = B \wedge a = 0)$

  $\implies (\hat{\delta}(A, w) = \delta(\hat{\delta}(A, x), a) = \delta(C, a) = C) \vee (\hat{\delta}(A, w) = \delta(\hat{\delta}(A, x), 0) = \delta(B, 0) = C)$

  $\implies \hat{\delta}(A, w) = C$

  如果 $\hat{\delta}(A, w) = C$, 即 $\delta(\hat{\delta}(A, x), a) = C$, 因为只有 $\delta(B, 0), \delta(C, 0), \delta(C, 1)$ 等于 C, 所以能推出:

  $(\hat{\delta}(A, x) = C) \vee (\hat{\delta}(A, x) = B \wedge a = 0)$

$\iff w$ 包含 00

因此命题 3 成立

- w 不包含 00 且 $a = 0$

  $\iff x$ 不包含 00 且 x 不以 0 结尾且 $a = 0$

  $\iff \hat{\delta}(A, x) = A \wedge a = 0$

  $\iff \hat{\delta}(A, w) = \delta(\hat{\delta}(x), 0) = \delta(A, 0) = B$ (" $\Longleftarrow$ " 是因为状态 B 只有一条来自状态 A 的字母为 0 的入边)

  因此命题 2 成立

- w 不包含 00 且 $a \neq 0$

  $\iff x$ 不包含 00 且 $a = 1$ (因为 $|w| > 0$)

  $\iff (\hat{\delta}(A, x) = A \vee \hat{\delta}(A, x) = B) \wedge a = 1$

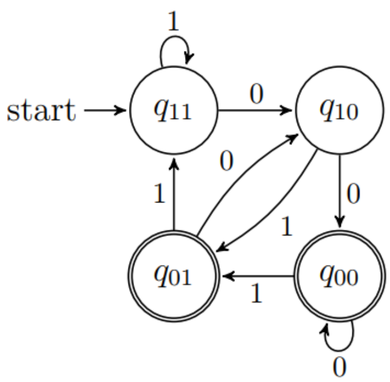
  $\iff \hat{\delta}(A, w) = \delta(\hat{\delta}(x), 1) = \delta(A, 1) = \delta(B, 1) = A$ (" $\Longleftarrow$ " 是因为状态 A 只有来自状态 A, B 的字母为 1 的入边且 $|w| > 1$)

  因此命题 1 成立

Conclusion: 命题 1,2,3 都成立

又因为 A 是起始状态, $F = \{A, B\}$, 所以 $L(A) = \{w : w\text{不包含子串}00\}$

2.3.3



$\cdot\ q_{11} = \{q\}$

$\cdot\ \{q_{10} = \{p, q\}\}$

$\cdot\ \{q_{01} = \{p, t\}\}$

$\cdot\{q_{00} = \{p, q, r, s\}\}$

$L(A) = \{\omega\colon \omega$长度至少为2且$\omega$的倒数第二位是$0\}$

2.3.4

(a)

$A = (Q, \Sigma, \delta, q_s, F)$

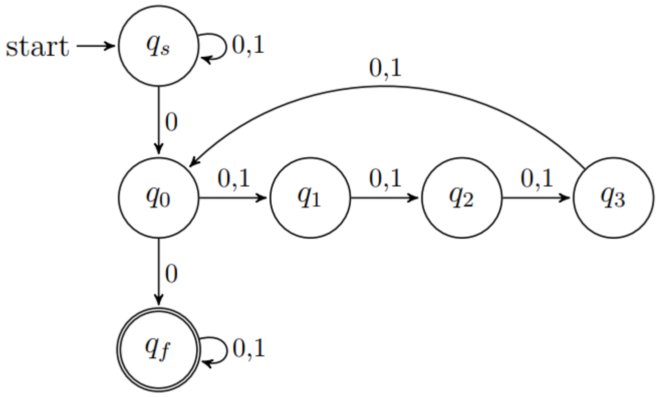
- $Q = \{q_s, q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_f\}$
- $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- $\forall i \in \{0, 1, \ldots, 9\}, \delta(q_s, i) = \{q_s, q_i\}$
  $\forall i \in \{0, 1, \ldots, 9\}, \delta(q_i, i) = \{q_f\}$
  $\forall i, j \in \{0, 1, \ldots, 9\} \wedge i \neq j, \delta(q_i, j) = \{q_i\}$
- $F = \{q_f\}$

(b)

$A = (Q, \Sigma, \delta, q_s, F)$

- $Q = \{q_s, q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_e, q_f\}$
- $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- $\forall i \in \{0, 1, \ldots, 9\}, \delta(q_s, i) = \{q_e\} \cup \{q_j : j \in \{0, 1, \ldots, 9\} \wedge j \neq i\}$
  $\forall i \in \{0, 1, \ldots, 9\}, \delta(q_i, i) = \{q_f\}$
  $\forall i, j \in \{0, 1, \ldots, 9\} \wedge i \neq j, \delta(q_i, j) = \{q_i\}$
- $F = \{q_e, q_f\}$

(c)



2.3.7

Let us define w_{-i} to be the i-th symbol from the end of w when $i > 0$.

我们要证明:

1. $\forall w \in \Sigma^*, q_0 \in \hat{\delta}(q_0, w)$
2. $\forall i > 0, \forall w \in \Sigma^*, q_i \in \hat{\delta}(q_0, w) \iff w_{-i} = 1$

Basis: 对于 $|w| = 0 \implies w = \epsilon$, $\hat{\delta}(q_0, \epsilon) = \{q_0\}$. 因此该命题对 $|w| = 0$ 成立.

Induction: 若对于任意的 $w \in \Sigma^k$ 都成立, 则 $w = xa \in \Sigma^{k+1}$ 时 $(x \in \Sigma^k, a \in \Sigma)$:

此时有 $w_{-1} = a$, $\forall i > 1, w_{-i} = x_{-(i-1)}$

- 如果 $i = 1$, 则 $q_1 \in \hat{\delta}(q_0, w) = \delta(\hat{\delta}(q_0, x), a)$

  $\iff \exists q' \in \hat{\delta}(q_0, x), q_1 \in \delta(q', a)$

  $\iff q_0 \in \hat{\delta}(q_0, x) \wedge a = 1$ (因为 q_0 是唯一有连向 q_1 的边的节点, 且边上字母为 1)

  $\iff a = 1$ (根据归纳假设 1, $q_0 \in \hat{\delta}(q_0, x)$ 必定成立)

  $\iff w_{-1} = 1$
- 如果 $i > 1$, $q_i \in \hat{\delta}(q_0, w) = \delta(\hat{\delta}(q_0, x), a)$

  $\iff \exists q' \in \hat{\delta}(q_0, x), q_i \in \delta(q', a)$

$\iff q_{i-1} \in \hat{\delta}(q_0, x)$ (因为 q_{i-1} 是唯一有连向 q_i 的边的节点, 且边上字母为 $0, 1$)

$\iff x_{-(i-1)} = 1$(根据归纳假设 2)

$\iff w_{-i} = 1$

- $q_0 \in \hat{\delta}(q_0, x)$ (归纳假设 1)

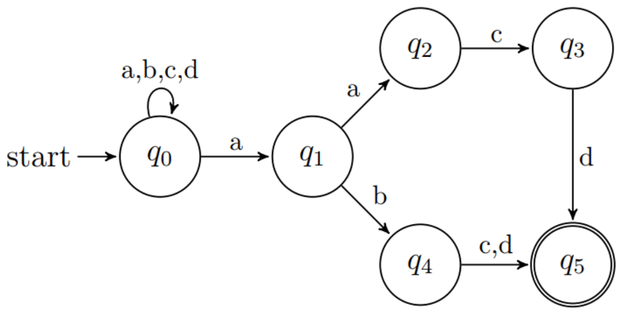
  $\implies \delta(q_0, a) \subset \delta(\hat{\delta}(q_0, x), a) = \hat{\delta}(q_0, w)$

  $\implies q_0 \in \hat{\delta}(q_0, w)$

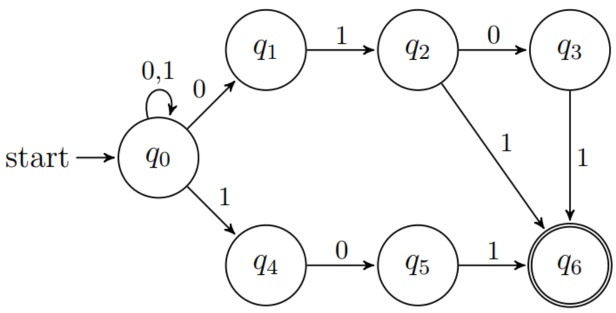
Conclusion: 假设 1,2 都成立

2.4.1

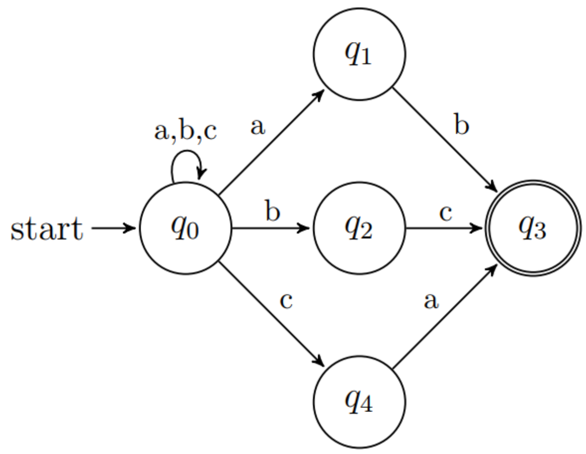
（a）



（b）

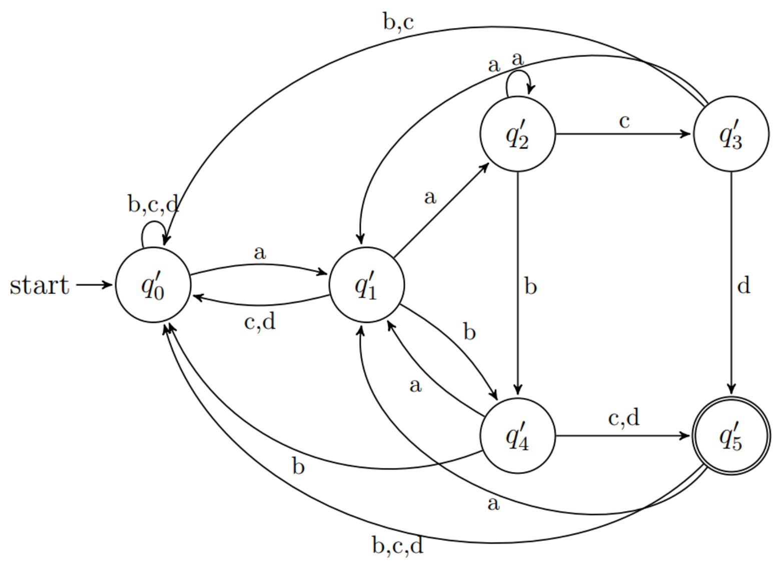


（c）



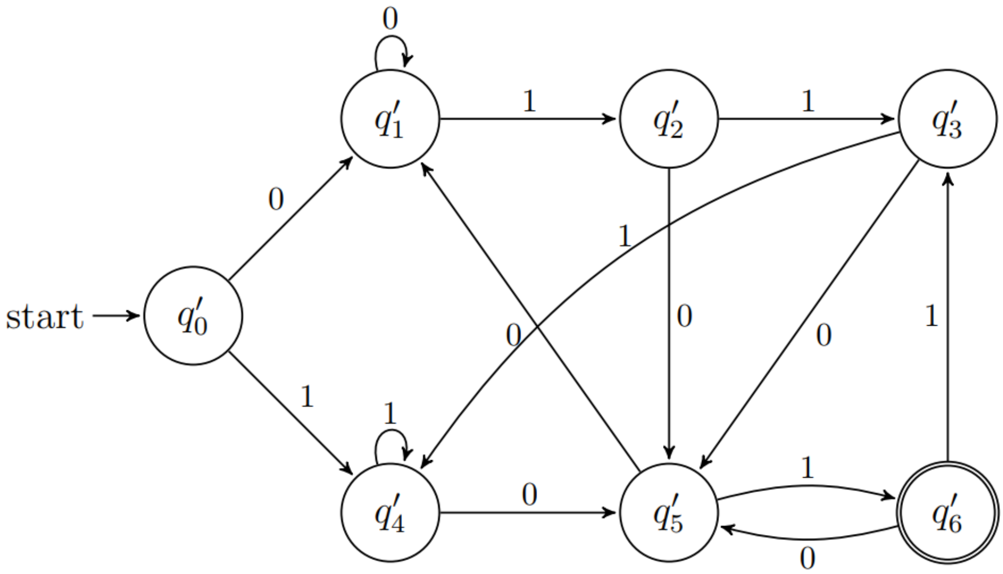
2.4.2

(a)



- $q'_0 = \{q_0\}$
- $q'_1 = \{q_0, q_1\}$
- $q'_2 = \{q_0, q_1, q_2\}$
- $q'_3 = \{q_0, q_3\}$
- $q'_4 = \{q_0, q_4\}$
- $q'_5 = \{q_0, q_5\}$

(b)



- $q'_0 = \{q_0\}$
- $q'_1 = \{q_0, q_1\}$
- $q'_2 = \{q_0, q_2, q_4\}$
- $q'_3 = \{q_0, q_4, q_6\}$
- $q'_4 = \{q_0, q_4\}$
- $q'_5 = \{q_0, q_1, q_5\}$ or $\{q_0, q_1, q_3, q_5\}$
- $q'_6 = \{q_0, q_2, q_4, q_6\}$

(c)

	a	b	c
$\rightarrow q_0'$	q_1'	q_2'	q_3'
q_1'	q_1'	q_4'	q_3'
q_2'	q_1'	q_2'	q_5'
q_3'	q_6'	q_2'	q_3'
$*q_4'$	q_1'	q_2'	q_5'
$*q_5'$	q_6'	q_2'	q_3'
$*q_6'$	q_1'	q_4'	q_3'

- $q_0' = \{q_0\}$
- $q_1' = \{q_0, q_1\}$
- $q_2' = \{q_0, q_2\}$
- $q_3' = \{q_0, q_4\}$
- $q_4' = \{q_0, q_2, q_3\}$
- $q_5' = \{q_0, q_3, q_4\}$
- $q_6' = \{q_0, q_1, q_3\}$

2. 用JFLAP构建接受下列语言的FA，其中，$\Sigma=\{0,1\}$ ：

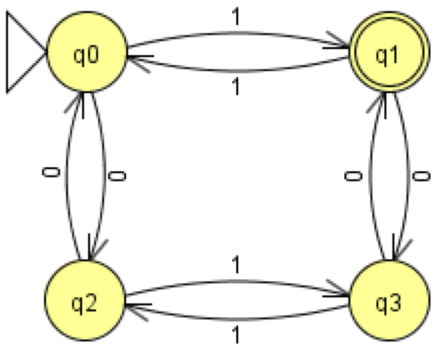
1） 包含偶数个0和奇数个1;

2） 包含偶数个0，且至少2个1;

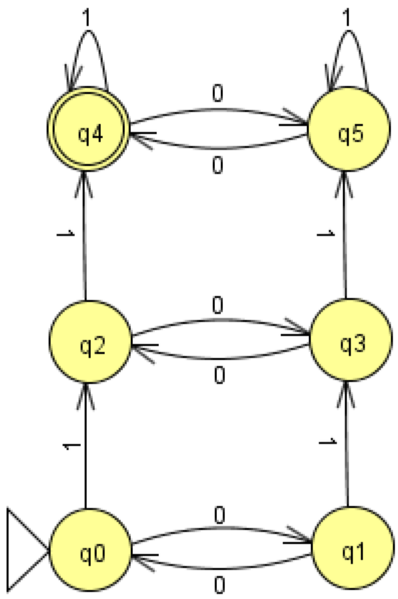
3） 0和1的个数要么都是偶数，要么都是奇数;

4） 任意个0后面跟随偶数个1;

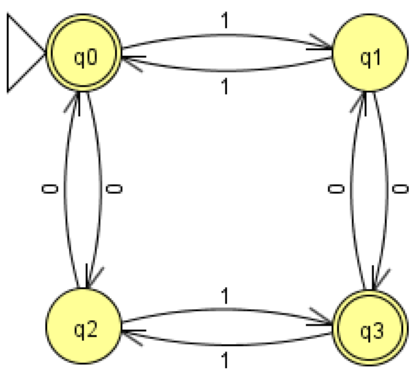
2.1


q0
q1
q2
q3
1
1
0
0
0
0
1
1

2.2

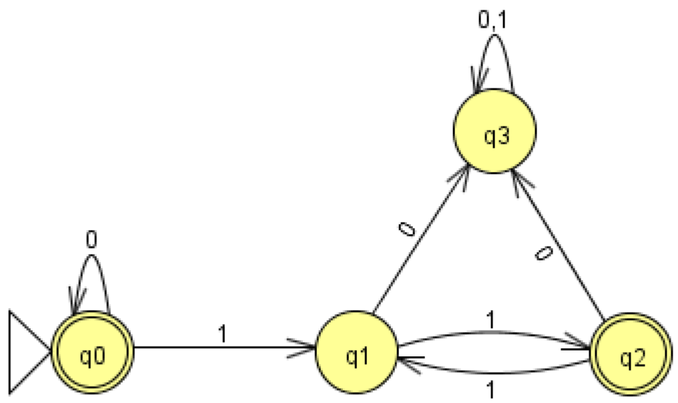

q0
q1
q2
q3
q4
q5
1
1
0
0
1
1
0
0
1
1
0
0

2.3

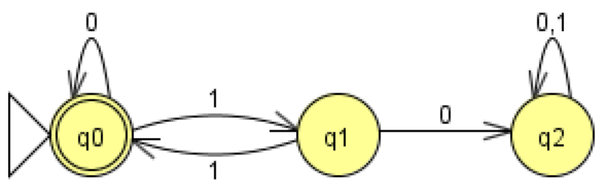

q0
q1
q2
q3
1
1
0
0
0
0
1
1

2.4

理解1：0，1不能交错出现（只接受0011，不接受00110011）

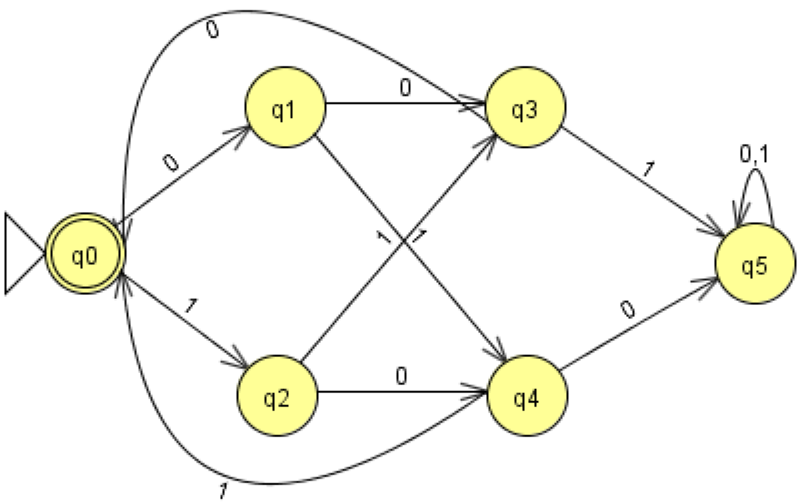


理解2：0，1可以交错出现（既接受0011，也接受00110011）

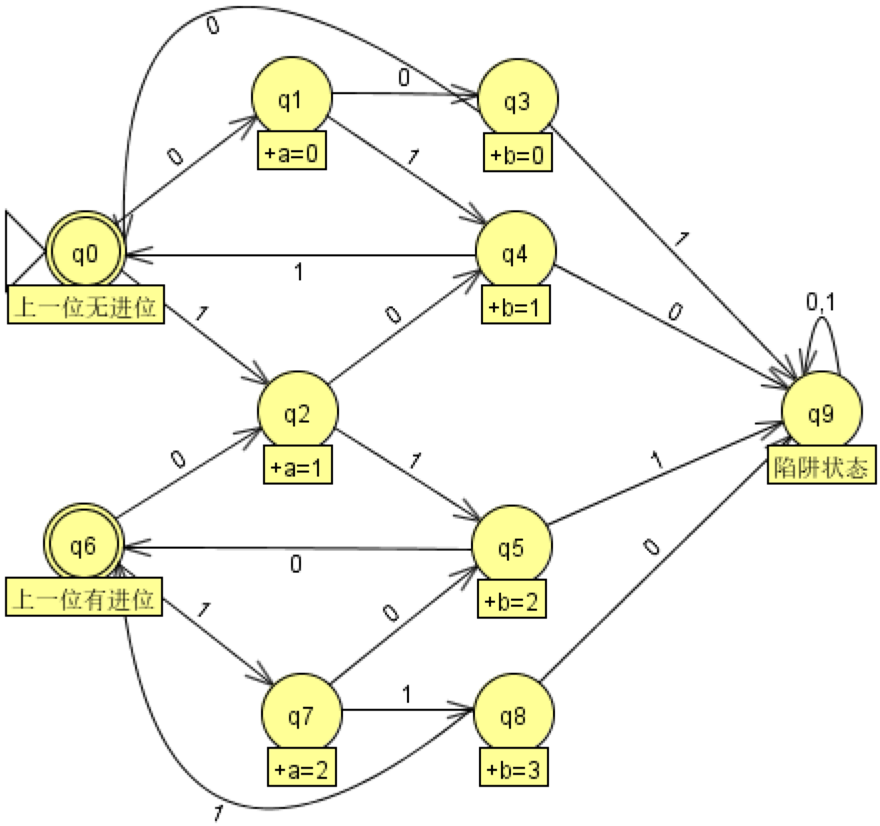


3.

理解1：不考虑进位（相当于异或）



理解2：考虑进位


q0
上一位无进位
q1
+a=0
q2
+a=1
q3
+b=0
q4
+b=1
q5
+b=2
q6
上一位有进位
q7
+a=2
q8
+b=3
q9
陷阱状态
0,1
0
1