① Sept. 28

I_1 : instance of VC $\Rightarrow$ I_2: instance of SC.

I_1 has a solution ($VC(I_1) = yes$)

$\Leftrightarrow$ I_2 has a sol ($SC(f(I_1) = I_2) = yes$)

$sol(I_1) \Rightarrow sol(I_2)$

$VC(I_1)$ has a sol, set $V' \subseteq V$

$V' = \{i_1, \ldots, i_p\}$, $p \leq k$

Consider $A = \{S_{i_1}, \ldots S_{i_p}\}$

For the sake of contradiction, assume A is <u>not</u> a sol. to $SC(I_2)$

[redacted]

- # of sets in A is $p \leq k$ ok!
- We must have $S_{i_1} \cup \cdots \cup S_{i_p} \neq \mathcal{U}$

$\Rightarrow \exists e \in \mathcal{U}$ that is not in $S_{i_1} \cup \ldots \cup S_{i_p}$

e corresponds to an edge in the VC

$e = (u,v)$,

$\Rightarrow$ S_u and S_v cannot be in A

$\Rightarrow$ $u, v \notin V'$ (by construction of A)

$\Rightarrow$ edge $e = (u,v)$ would not have been covered by V'

$\Rightarrow$ Contradiction! (because V' is $sol(I_1)$ of VC)

$\Rightarrow I_2$ has a sol. (A).

$sol(I_1) \Rightarrow sol(I_2)$

Need to prove also that

$sol(I_2) \Rightarrow sol(I_1)$

[redacted]

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$sol(I_2) \Rightarrow sol(I_1)$

SC is a yes instance

$\rightarrow$ sol $A = \{ S_{i_1}, \dots, S_{ip} \}$ with $p \leq k$

$S_{i_1} \cup \dots \cup S_{ip} = \mathcal{U}$

Consider vertex set $V' = \{ i_1, \dots, ip \}$

we have $p \leq k$

contradiction: V' is not a VC sol.

$\exists e = (u, v) \in E$ s.t. $u \notin V'$, $v \notin V'$

S_u, S_v were not in A

$e \notin S_{i_1} \dots S_{ip} = \mathcal{U}$

$\Rightarrow$ contradiction

V' is a sol. to VC

$\Rightarrow$ we have a sol. to I_1

[redacted]

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$X = \{0, 1\}$ Boolean variable

$\quad\;\; F \quad T$

X	$\neg X, \bar{X}$
0	1
1	0

non-X.

X_1	X_2	$X_1 \wedge X_2$ (AND)	$X_1 \vee X_2$ (OR)
1	1	1	1
1	0	0	1
0	1	0	1
0	0	0	0

[redacted]