

$$T(n) \leq \begin{cases} 0 & \text{if } n=1 \\ T(\lceil n/2 \rceil) + T(\lfloor n/2 \rfloor) + n \end{cases}$$

Th. $T(n) \leq n \cdot \lceil \log n \rceil$

Pf by induction

- ① base case $n=1$ $T(1) = 0 \leq 1 \lceil \log 1 \rceil$
- ② ind. hyp: assume true for $1 \dots n-1$
- ③ ind. step: show the result for n .

$$n_1 = \lceil n/2 \rceil \quad n_2 = \lfloor n/2 \rfloor$$

$$T(n) \leq T(n_1) + T(n_2) + n$$

$$\leq n_1 \lceil \log n_1 \rceil + n_2 \lceil \log n_2 \rceil + n \quad [\text{hyp. ind}]$$

$$\leq (n_1 + n_2) \lceil \log n \rceil + n \quad [n_1 \leq n_2]$$

$$= n \lceil \log n \rceil + n$$

$$\lceil \log n \rceil \leq \lceil \log n \rceil - 1 \Rightarrow \text{to prove.}$$

(simple math!)