DP from Midterm 2016

Pb States $\{1..n\}$, state i: p_i (pop)
v_i (votes)

$$V = (\underbrace{\sum_{i=1}^{n} v_i}_{V_{tot}})/2 + 1 \quad \text{required to win}$$

Goal: Find set of states S that minimizes

$$\sum_{i \in S} p_i \quad \text{s.t} \quad \sum_{i \in S} v_i \geq V$$

[redacted]

MinPop(i, v) : Min pop. of a subset of states $\{1, \dots, i\}$ s.t. their votes sum to v (exactly) a/ Prove <u>opt. substructure</u>.

Suppose we have opt. sol. for obtaining v using states $\{1 .. i\}$ → value $MinPop(i, v)$

→ $S(i,v)$ is the set of states included in the vote for this sol.

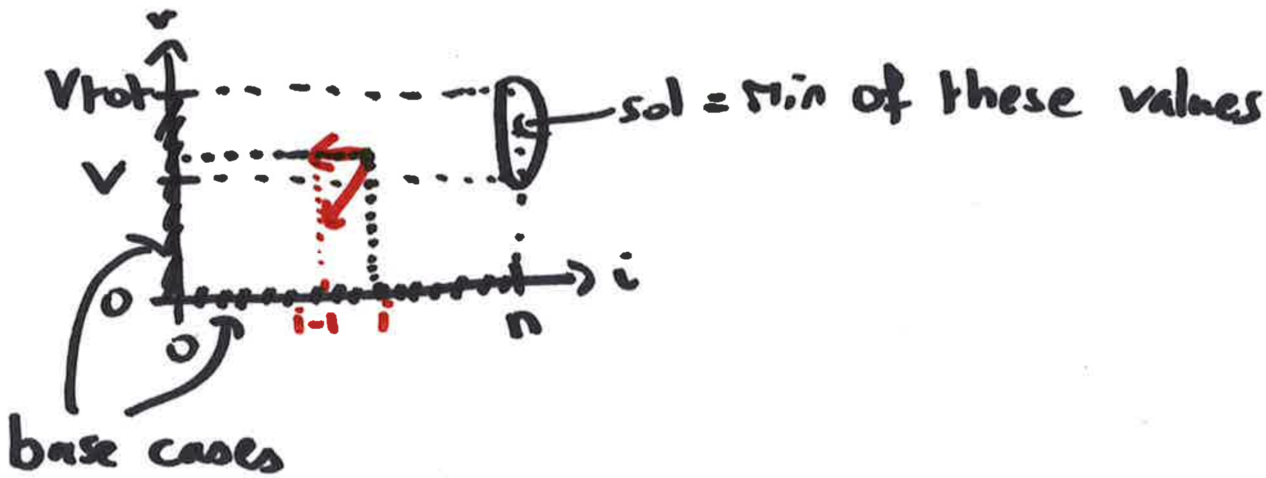
In a solution, either $i \in S(i,v)$ or $i \notin S(i,v)$

- If $i \in S(i,v)$, $S(i,v) \setminus \{i\}$ is a set of states with exactly $v - v_i$ votes. If this set is <u>not</u> the opt. (for states $\{1 ... i-1\}$), then we replace it with the opt. $S(i-1, v-v_i)$, add i to this set → another sol with exactly v votes but less population
- If $i \notin S(i,v)$, then $S(i,v)$ must be opt. way to get v votes from $\{1 .. i-1\}$ [otherwise build better sol]

[redacted]

b/

$$\text{MinPop}(i,v) = \begin{cases} 0 & \text{if } v=0 \\ +\infty & \text{if } v\neq 0 \text{ and } i=0 \\ \min\left(\text{MinPop}(i-1,v),\ \text{MinPop}(i-1,v-v_i)+p_i\right) & \text{otherwise} \end{cases}$$



d/ Time complexity = Space complexity = $O(n \cdot V_{tot})$

e/

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Vtot = 0 ; Ptot = 0;
For j=1 to n, { Vtot += Vj ; Ptot += Pj }
// base cases
For i = 0 to n,  S[i,0] = 0;
For v = 1 to Vtot,  S[0,v] = Ptot + 1;   //(+∞)
// loop
For i=1 to n
    For v = 1 to Vtot
        S[i,v] = Min ( S[i-1,v], S[i-1, v-Vi] + Pi )
                      // (check v ≥ Vi to add)
//sol
Min = Ptot+1
For v = V to Vtot { if S[n,v] < Min
                        Min = S[n,v] }   Return Min
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