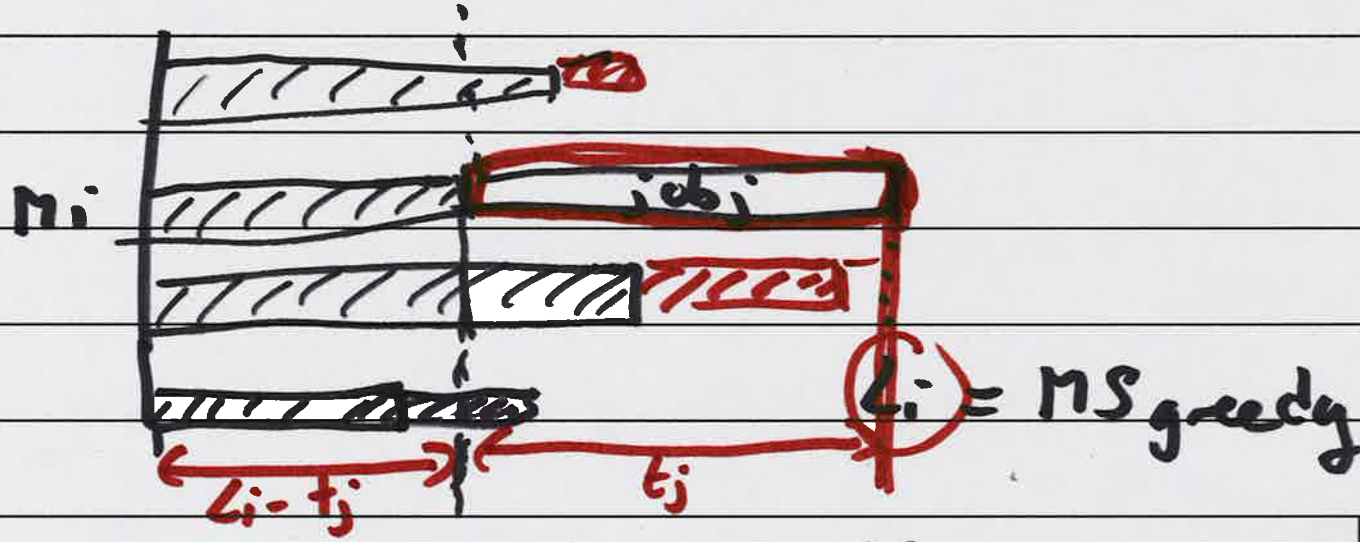
Th. Greedy is a 2-approx.

$$MS_{greedy} = L_i = \max_{1 \le x \le m} L_x \quad \text{"bottleneck machine"}$$

j: last job added to machine i by Greedy



when j was added to M_i,
M_i had the smallest load (greedy rule).
It was $L_i - t_j$
$\Rightarrow L_i - t_j \le L_k \quad \forall 1 \le k \le m$

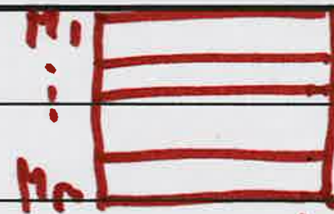
[redacted]

$L_i - t_j \leq L_1$

$L_i - t_j \leq L_2$

$\dots$

$L_i - t_j \leq L_m$



$\frac{1}{m} \sum t_p \leq MS_{opt}$

MS_{opt} is not necessary $=$, but must be $\geq$.

$$m(L_i - t_j) \leq L_1 + L_2 + \dots + L_m = \sum_{k=1}^{m} L_k = \sum_{p=1}^{n} t_p$$

(all jobs are scheduled)

$$MS_{greedy} = L_i \leq \underbrace{\frac{1}{m}\sum_{p=1}^{n} t_p}_{\leq MS_{opt}} + \underbrace{t_j}_{\leq MS_{opt}}$$

($\leq MS_{opt}$ (perfect load balance))

($\leq MS_{opt}$: Job j must go on a machine in any sol!)

$n = m = 2$ $\quad t_1 = 1$, $t_2 = 3$

[redacted]

$\Rightarrow MS_{opt} \leq MS_{greedy} \leq 2 \cdot MS_{opt}$ $\rightarrow$ 2-Approx!

Center selection pb

obs. All centers in greedy sol. C are at least $r(C)$ apart.

Th. If. C^* is opt. sol., then

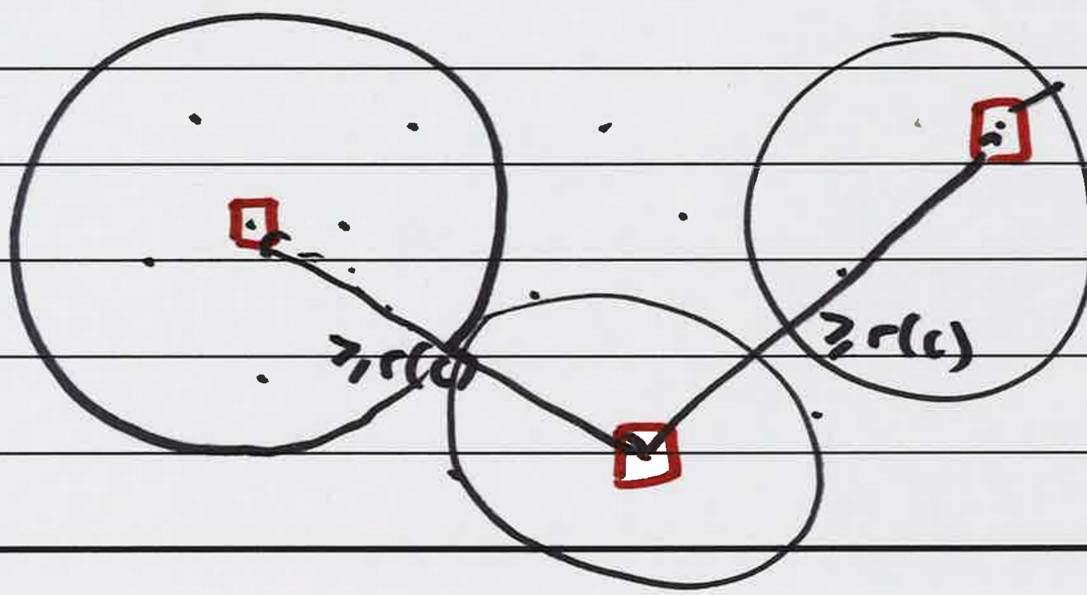
$$\underset{\text{greedy}}{r(C)} \le 2 \cdot \underset{\text{opt.}}{r(C^*)} \Rightarrow \text{Approx. ratio of } 2$$

Pf by contradiction

Assume $r(C) > 2\, r(C^*)$, i.e., $r(C^*) < \frac{1}{2} r(C)$

~~sites~~ sites : •

~~centers~~ centers in C: ▣



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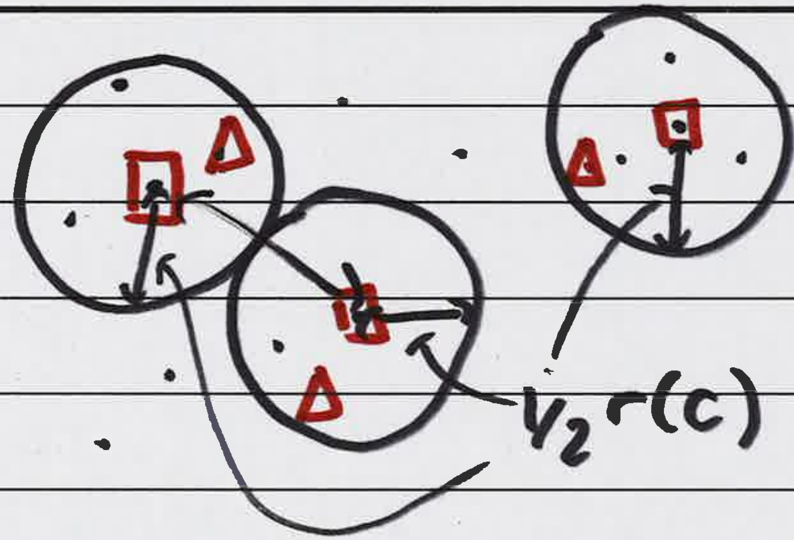


C : □ greedy centers

C^* : Δ opt. centers

For each center c_i in C, consider a ball of radius $\frac{1}{2} r(C)$ $\Rightarrow$ No 2 balls can overlap.

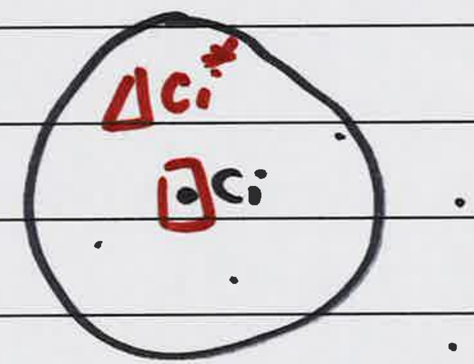
$$\text{dist}(c_i, C^*) \leq r(C^*) < \frac{1}{2} r(C)$$

by assumption

$\Rightarrow$ the closest opt. center in C^* to c_i has to be inside its ball.

[redacted]

$\Rightarrow$ At least 1 opt. center (Δ) c_i^* in
every ball



$\left\{\begin{array}{l}\text{No 2 balls overlap.} \\ k \text{ opt. centers in } C^* \\ \quad \text{and } k \text{ balls}\end{array}\right.$

$\Rightarrow$ every ball has exactly
one opt. center.

Pick any site s
$\rightarrow$ Let $c_i^* \in C^*$ is closest center in C^*
$\rightarrow$ c_i greedy center $c_i \in C$ ~~in this set~~
in whose ball c_i^* falls.

$dist(c_i, c_i^*) \leq \frac{1}{2} r(C)$

$dist(s, C) \leq dist(s, c_i)$

[redacted]

$$\operatorname{dist}(s, C) \le \underbrace{\operatorname{dist}(s, c_i^*)}_{\le r(C^*)} + \underbrace{\operatorname{dist}(c_i^*, c_i)}_{r(C^*)}$$

triangle inequality.

[redacted]