Sept 5 (1)

Cut property: S : <u>any</u> set of nodes

$e = (u,v)$ is the min cost edge from cutset of S

$\Rightarrow$ every MST T^* contains e.

<u>Pf</u> [exchange argument]

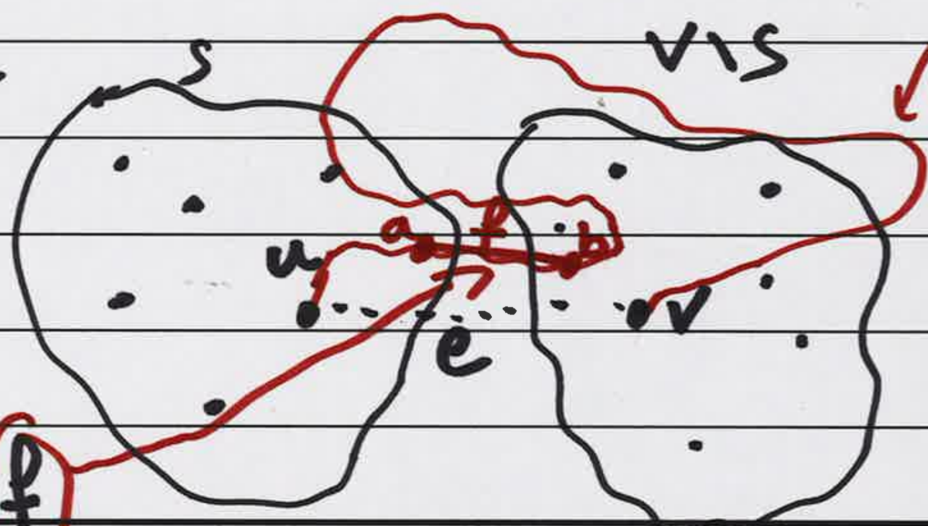
Given e, T^*, S

Suppose e does <u>not</u> belong to T^*

T^* ST $\Rightarrow$ contains path $u \rightsquigarrow v$

u, v on opposite sides of cut S

$\Rightarrow$ the path $u \rightsquigarrow v$ in T^* has at least one edge crossing the cut: f



[redacted]

Exchange f for e

$$T' = T^* - f + e$$

$\Rightarrow T'$ is still a spanning tree. ok!

- still connected? $f = (a,b)$

  using original path $+ e$

  $\Rightarrow$ new path $a \rightsquigarrow b$ in T'

- # edges in T' = # edges in T^*

  $\Rightarrow n-1$ edges

[redacted]

- $T' = T^* - f + e$ is a spanning tree

- T' is no worse than T^*

  $\text{cost}(T') = \text{cost}(T^*) - c(f) + c(e)$

  $c(e) < c(f)$ because e is min cost edge for cut S

  $\Rightarrow \text{cost}(T') < \text{cost}(T^*)$

  Contradiction!

  $\Rightarrow$ [e should have been in T^*]!

[redacted]