DP from Midterm 2016

Pb States $\{1..n\}$, state i : p_i (pop)
v_i (votes)

$$V = \underbrace{\left(\sum_{i=1}^{n} v_i\right)}_{V_{tot}} / 2 + 1$$

required to win

Goal : Find set of states S that minimizes

$$\sum_{i \in S} p_i \quad \text{s.t} \quad \sum_{i \in S} v_i \geq V$$

$MinPop(i, v)$: Min pop. of a subset of states $\{1, \ldots, i\}$ s.t. their votes sum to v (exactly)

[redacted]

a/ Prove <u>opt. substructure</u>.

Suppose we have opt. sol. for obtaining v using states $\{1 .. i\}$ $\rightsquigarrow$ value $MinPop(i, v)$

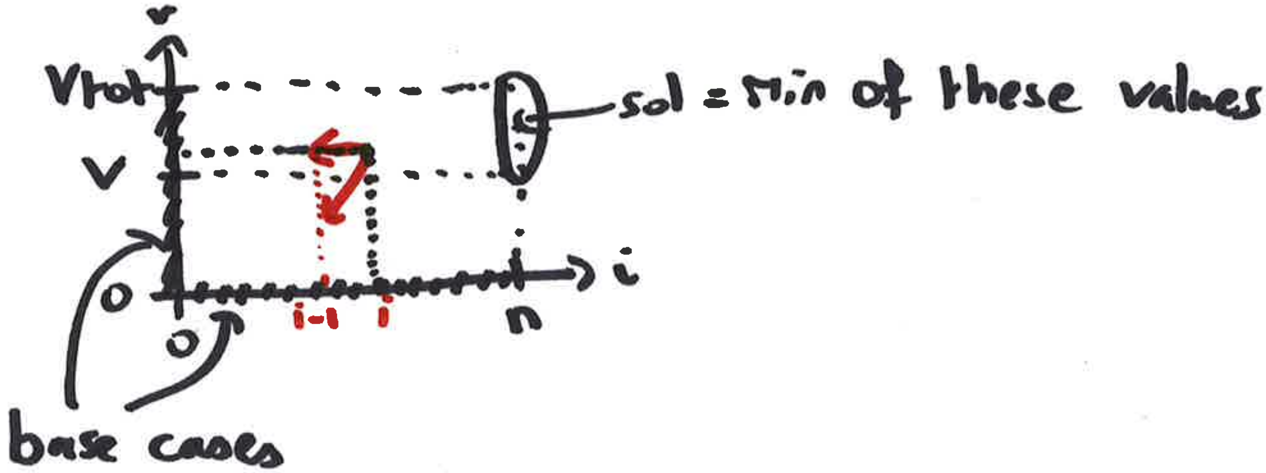
$\rightarrow$ $S(i,v)$ is the set of states included in the vote for this sol.

In a solution, either $i \in S(i,v)$ or $i \notin S(i,v)$

- If $i \in S(i,v)$, $S(i,v) \setminus \{i\}$ is a set of states with exactly $v - v_i$ votes. If this set is <u>not</u> the opt. (for states $\{1 ... i-1\}$), then we replace it with the opt. $S(i-1, v-v_i)$, add i to this set $\rightarrow$ another sol with exactly v votes but less population

[redacted]

- If $i \notin S(i,v)$, then $S(i,v)$ must be opt. way to get v votes from $\{1 .. i-1\}$ [otherwise build better sol] b/ $$MinPop(i,v) = \begin{cases} 0 & \text{if } v=0 \\ +\infty & \text{if } v \neq 0 \text{ and } i=0 \\ \min(MinPop(i-1,v),\ MinPop(i-1, v-v_i)+p_i) & \text{otherwise} \end{cases}$$



d/ Time complexity = Space complexity = $O(n.V_{tot})$

e/ Vtot = 0 ; Ptot = 0;
For j=1 to n, { Vtot += V_j ; Ptot += P_j }
// base cases
For i = 0 to n, S[i,0] = 0;
For v = 1 to Vtot, S[0,v] = Ptot +1; // ($+\infty$)
// loop
For i=1 to n
  For v = 1 to Vtot
    S[i,v] = Min (S[i-1, v], S[i-1, v-V_i] + P_i)
    // (check $v \geq V_i$ to add)

//sol
Min = Ptot +1
For v = V to Vtot { if S[n,v] < Min
  Min = S[n,v] } Return Min