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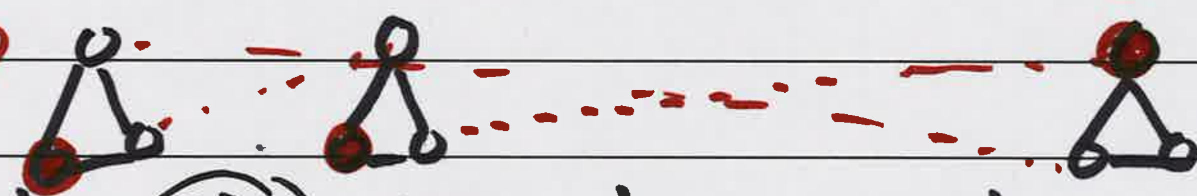
3SAT $\le_p$ IS

I_1 : ϕ : $c_1 \wedge c_2 \ldots\ldots c_m$

$\hookrightarrow I_2$: G, $k = m$



I_1 has a sol. $\iff$ I_2 has a sol.

(ϕ satisfiable) (IS (G, $k=m$))

(3b) $sol(I_1) \Rightarrow sol(I_2)$

If I_1 has a sol, ϕ has a satisfying assignment.

→ Pick exactly one of the true litterals per clause and add it to S. (sol. to I_2, i.e., IS)

* Size of S? $k = m$
* Indep. set?

[redacted]

Is S an I.S. ?

- 1 node per triangle
- If there is an edge between 2 nodes $x_1 - x_2$ with $x_1 \in S$ & $x_2 \in S$ it means that $x_1 = x$, $x_2 = \bar{x}$ (or reverse)

$\Rightarrow$ Contradiction with sol (I1) !

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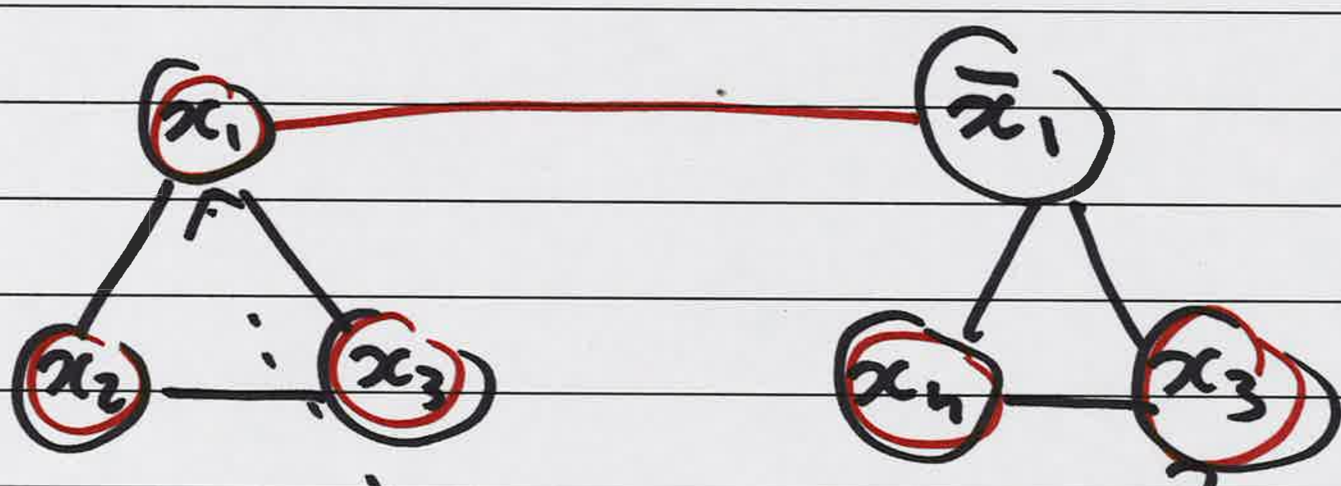
$$(x_1 \vee x_2 \vee x_3) \wedge (\bar{x}_1 \vee x_4 \vee x_3)$$

C_1 C_2



ϕ : sol : x_1, x_2, x_3, x_4, are all True

sol(I_2): - we pick $x_1 \in S$. from C_1

$x_3 \in S$. from C_2

[redacted]

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$sol(I_2) \Rightarrow sol(I_1)$

↳ $IS(G, k=m)$ is yes, let S be a $sol(I_2)$

$|S| \geq k=m$

→ we have m Δ in G, each Δ can have at most 1 vertex in S

⇒ S contains 1 vertex per Δ

→ Set the literal corresponding to each vertex in S to "True".

1 vertex in S per Δ

→ every clause has a true literal

→ ϕ is satisfiable

→ Argue that a variable is not assigned to both True and False

(x_1 is T & $\bar{x}_1$ is T)

[redacted]

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Let there be such a variable x.

→ A vertex labeled x in S

———————— $\bar{x}$ ————

→ By construction, these 2 vertices share an edge x — $\bar{x}$

→ Contradicts the fact that S is an I.S.

[redacted]

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VC $\Leftrightarrow$ IS

Claim: S is an IS in $G \Leftrightarrow V \setminus S$ is a VC of G

$\Rightarrow$ Let S be an IS of G

Consider any edge $(u,v) \in E$

$\Rightarrow$ Either $u \notin S$ or $v \notin S$ or both (def of IS)

$\Rightarrow$ At least one of them in $V \setminus S \Rightarrow$ in the VC!

$V \setminus S$ "covers" (u,v), $\forall (u,v) \in E \Rightarrow V \setminus S$ is a VC.

$\Leftarrow$ Let $V \setminus S$ be a VC of G.

Consider <u>any</u> 2 nodes $u \in S$, $v \in S$

If $(u,v) \in E$, it would <u>NOT</u> be covered by $V \setminus S \Rightarrow (u,v) \notin E$ $\Rightarrow$ S is an IS.

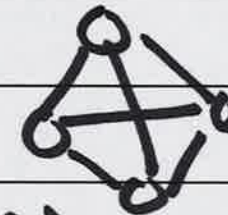
[redacted]

VC $\Leftrightarrow$ Clique

V' is a clique in G $\Leftrightarrow$ $V \setminus V'$ is a VC in G_c

($\Rightarrow$) V' is a clique

Consider any edge $(u,v) \in E_c$



$(u,v) \notin E$ by def of G_c

u and v cannot both be in V' (clique def)

Either $u \notin V'$ or $v \notin V'$

$\rightarrow$ At least one of them is in $V \setminus V'$

$\rightarrow$ $V \setminus V'$ covers edge (u,v), $\forall (u,v) \in E_c$

$\rightarrow$ $V \setminus V'$ is a VC for G_c

[redacted]

($\Leftarrow$) $V \setminus V'$ is VC of G_c

Let $u \in V'$ and $v \in V'$ be any 2 vertices of V'.

If no edge ($(u,v) \notin E$), then $(u,v) \in E_c$

Contrad. with "$V \setminus V'$ is a VC of G_c"

$\Rightarrow (u,v) \in E$, $\forall (u,v) \in V'$

$\Rightarrow V'$ is a clique in G.

[redacted]