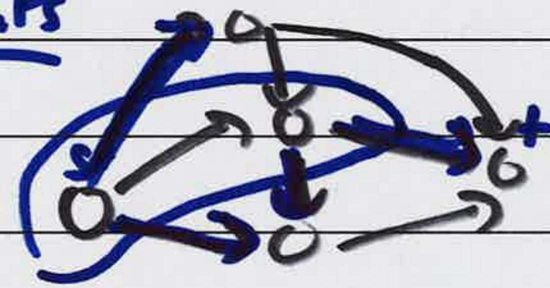
Proof of Max-flow Min-cut th.

(i) $\Rightarrow$ (ii) weak duality lemma : $v(f) \leq cap(A,B)$

True for <u>all cuts</u>



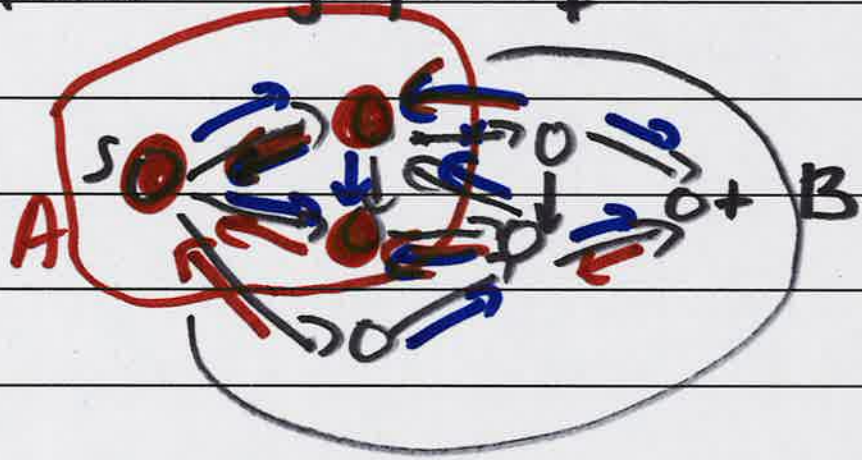
(ii) $\Rightarrow$ (iii) - Let f be a flow(max).

If there exists an augmenting path, then we can improve f (send flow along path).

[redacted]

(iii) $\Rightarrow$ (i) $\rightsquigarrow$ Let f be a flow with <u>no</u> augmenting path. $\rightsquigarrow$ let's build a cut (A,B)

Residual graph G_f



Let A be the set of vertices reachable from s in G_f

$s \in A$, $t \notin A$

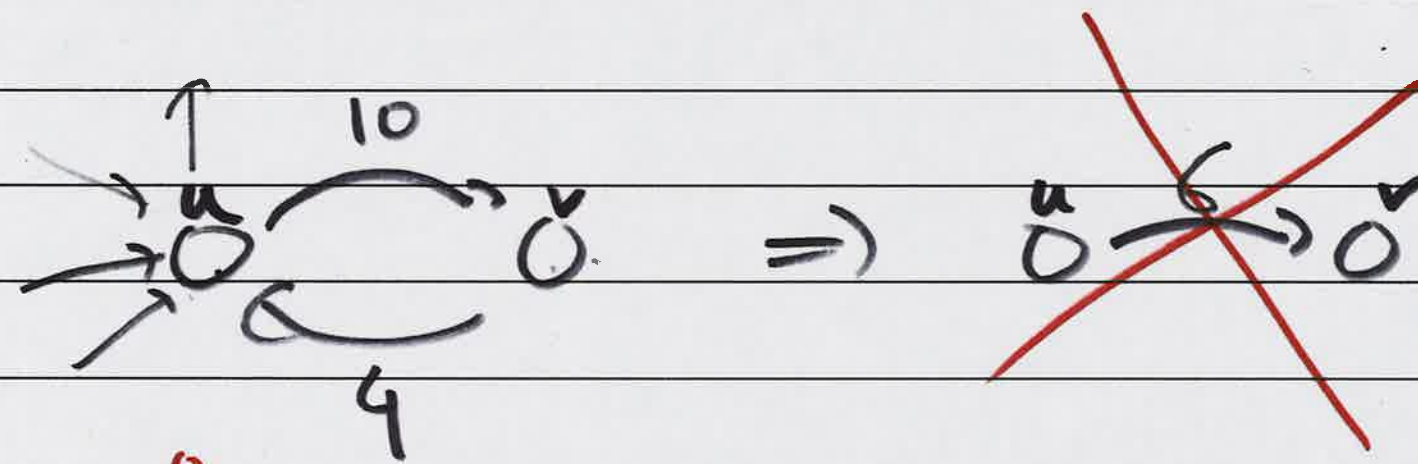
$\Rightarrow$ we have a cut

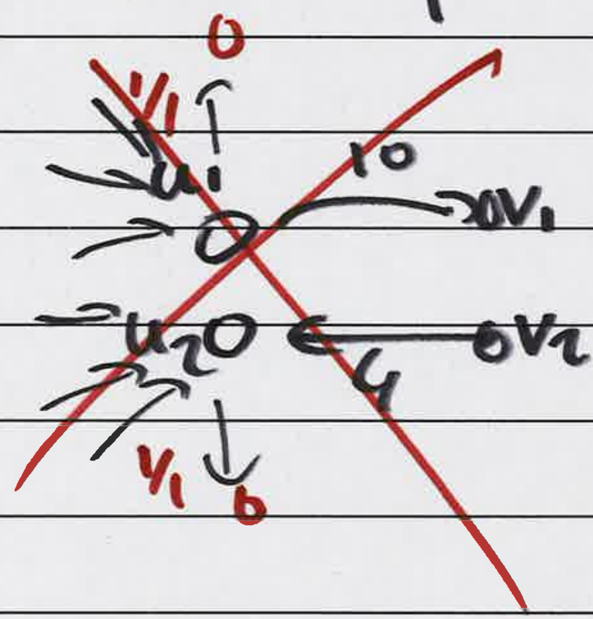
$$v(f) = \sum_{e \text{ out of } A} f(e) - \sum_{e \text{ into } A} f(e)$$

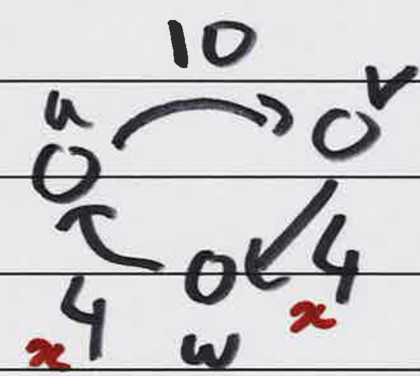
$$\sum_{e \text{ out of } A} f(e) = \sum_{e \text{ out of } A} c(e) = \text{cap}(A,B)$$

$$\sum_{e \text{ into } A} f(e) = 0$$

[redacted]







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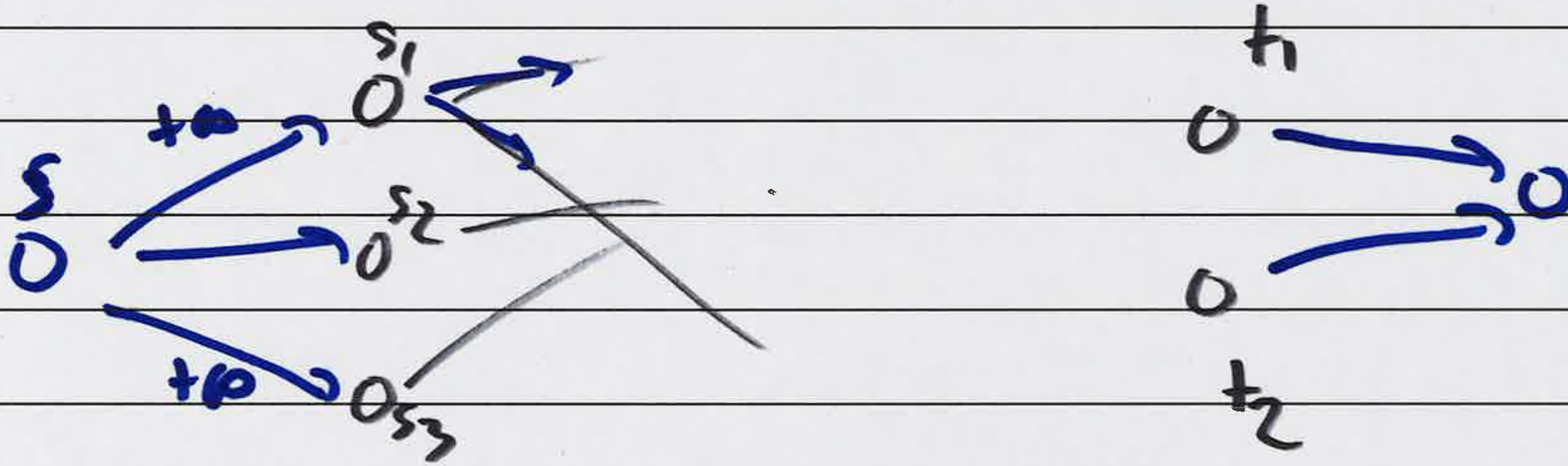

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