CUSUM Trading Strategy

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1 PROBLEM STATEMENT

The goal of the project is to develop a trading strategy based on trading signals generated by the CUSUM techniques. When an upward (downward) shift of a stock's daily return is detected, the trading system generates a long (short) trading signal. This active trading strategy is evaluated against the actual return over the same historical time period to see if it can beat the long-and-hold strategy. The comparison is carried out in two dimensions. We use the S&P 500(SPY) vs Hang Seng Index(HSI) as the proxies of the US and China stock markets to assess the strategy's effectiveness in different regions. We also trade the SPDR® Portfolio S&P 500® Growth ETF (SPYG) vs the SPDR® Portfolio S&P 500® Value ETF (SPYV) as the proxies of growth stocks and value stocks in the US market to assess if the categories of stocks impact the performance of this strategy.

2 LITERATURE REVIEW

E.S. Page in his paper *Continuous Inspection Schemes*[PAGE 1954] addressed the problem of detecting value changes in parameters, θ as CUSUM. Lam and Yam showed CUSUM inspection is a generalized version of filter trading strategy that can be applied in the financial market and gives higher returns[Lam and Yam 1997]. They used the CUSUM model with drift parameter k and threshold parameter h. Let y_t denote the observation on the t^{th} day and $x_t = y_t - k$. They proved that the logic using CUSUM statistics passing thresholds as signs of distribution change is the same as using the asset's cumulative returns passing certain expectations as signs to buy or sell this asset, which means the CUSUM trading strategy is a kind of filter trading strategy. Given the example of the buy signal triggering mechanism and p_t denoting the asset price on the t^{th} day, the proof is as follows:

The process for CUSUM Z_t with $Z_0 = 0$ and $k = 0$ is defined as:

$$y_t = \log(\frac{p_t}{p_{t-1}})$$

which is very similar to the log return of the asset's price:

$$\ln(1 + return) = \ln(\frac{p_t}{p_{t-1}})$$

Since in a CUSUM procedure, a change is detected at the first n satisfying $Z_t \geq h$, it can be described as:

$$Z_t = \log(p_t) - \min_{0 \leq i \leq t} \log(p_i) \geq h$$

$$\therefore \frac{p_t - \min_{0 \leq i \leq t} p_i}{\min_{0 \leq i \leq t} p_i} \geq e^h - 1 \Longleftrightarrow return \geq R$$

where $R(= e^h - 1)$ denotes a threshold for traditional filter trading strategies to buy the asset when the percentage change exceeds that limit.

Generalize the case to $h > 0$ and $k > 0$, k can be viewed as the daily percentage change and h the percentage change over a span of days.

To prove the feasibility of CUSUM in the financial market, they used Heng Seng Index to calculate possible combinations of parameters used in *average run length*, which is denoted as $l(\cdot)$ here. Given the distribution of run length L from Page's paper and $Z_0 = z$ for the initial value in

CUSUM. Assume $y_i \sim N(\mu, \sigma)$ and y_i is i.i.d. then $E(L)$ and $Var(L)$ are functions of process deviate $\theta = \frac{(\mu-k)\sqrt{n}}{\sigma}$ and standardized decision interval $H = \frac{h\sqrt{n}}{\sigma}$. They calculated the average lengths of buy runs and sell runs and hence the theoretical CUSUM strategy payoff with y_t historical data:

$$\begin{cases} E(\text{Payoff}_{\text{buyrun}}) = E(L_{\text{buy}}) * E(y_t) & y_t = \log(\frac{p_t}{p_{t-1}}) \\ E(\text{Payoff}_{\text{sellrun}}) == E(L_{\text{sell}}) * E(y_t) & y_t = \log(\frac{p_{t-1}}{p_t}) \end{cases} \tag{1}$$

By comparing them with the historical index returns, they concluded that theoretically, CUSUM can generate a better payoff. However, this research didn't conduct actual detection using CUSUM and overlooked differences in outcome caused by difficulties implementing CUSUM to real data.

3 DATA

We obtained the daily prices of the SPY, HSI, SPYG, and SPYV from yahoo finance. Our sample covers the period from 2007-01-01 to 2022-11-30, yielding over 4005 and 3922 observations. It covers 2 major financial crises including the financial crisis in 2008 and the pandemic outbreak in 2020. Figure 1 displays the daily log returns of the SPY and HSI in that period. A kernel density plot is given by Figure 3. In Figure 2, we display the cumulative log-return of the S&P 500 during the sample period. Table 1 summarizes the statistics of the log return for the 4 indices. All securities demonstrate a strongly leptokurtic distribution of the log returns. SPY, SPYG, and SPYV are left-skewed whereas HSI is slightly right-skewed.

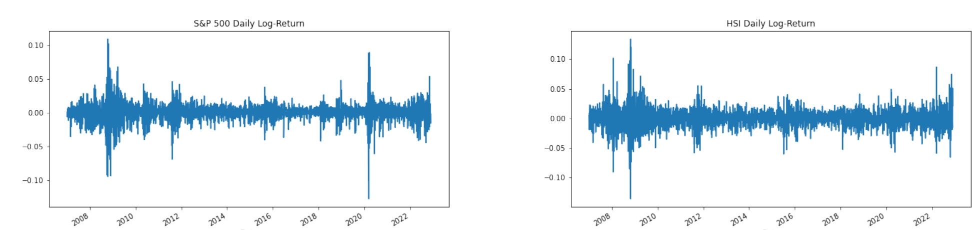

Fig. 1. SPY(left) and HSI(right) Daily Log-Return from 2007-01-01 to 2022-11-30

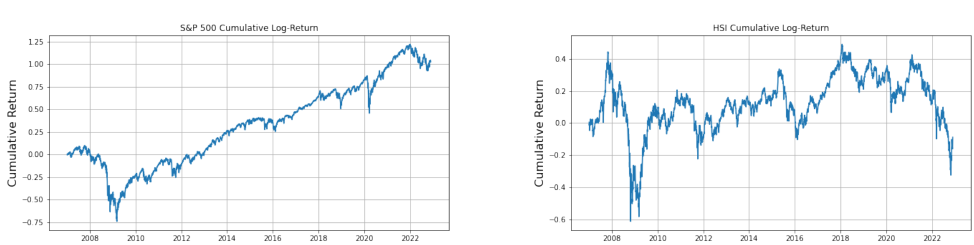

Fig. 2. SPY(left) and HSI(right) Cumulative Log-Return from 2007-01-01 to 2022-11-30

4 METHODOLOGY

4.1 CUSUM

The CUSUM procedure was first introduced in [PAGE 1954] to detect a small deviation of a monitored variable from a target value. Consider independent observations $X_1, X_2, \ldots, X_n, \ldots$ arising from a

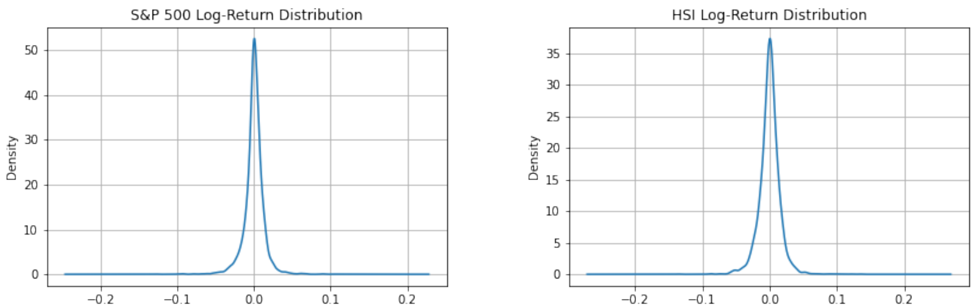

Fig. 3. SPY(left) and HSI(right) Daily Log-Return Kernel Density

Table 1. Descriptive Statistics of Daily Log-Return for SPY, HSI, SPYG, SPYV

	count	min	max	median	mean	std	skew	kurtosis
SPY	4005	-0.12765	0.109572	0.00069	0.000257	0.013031	-0.52036	11.8898
HIS	3922	-0.13582	0.134068	0.000442	-2.2E-05	0.015473	0.065705	7.693831
SPYG	4005	-0.12763	0.112519	0.00096	0.000394	0.013131	-0.51965	9.87039
SPYV	4005	-0.1186	0.099824	0.000736	0.000265	0.012889	-0.64743	12.23025

sequential sampling process. We assume there exists a deterministic time $\tau \in \{0, 1, 2, \ldots\}$ such that the distribution of X_t is $f_1(t)$ before τ and $f_2(t)$ after τ. The up-sided CUSUM Z_t^+ is used to detect the up-shift of the observation X_t:

$$\begin{cases} Z_0^+ = 0 \\ Z_t^+ = \max(Z_{t-1}^+ + \log \frac{f_2(X_t)}{f_1(X_t)}, 0) \end{cases} \tag{2}$$

The down-sided CUSUM Z_t^- is used to detect the down-shift of the observation X_t:

$$\begin{cases} Z_0^- = 0 \\ Z_t^- = \min(Z_{t-1}^- + \log \frac{f_2(X_t)}{f_1(X_t)}, 0) \end{cases} \tag{3}$$

Let δ_1 and δ_2 denote the thresholds for Z_t^+ and Z_t^-. The first time $Z_t^+ > \delta_1$ or $Z_t^- > \delta_2$ is τ and we declare that the distribution of X_t has changed before τ.

4.2 CUSUM for Stock Trading

[Lam and Yam 1997] showed that the filter trading rule proposed by [Alexander 1961] is equivalent to the CUSUM test. The independent observations X_t in CUSUM corresponds to the log return series of trading security. When $Z_t^+ > \delta_1$ ($Z_t^- < \delta_2$), it is equivalent to a long (short) signal.

4.2.1 Geometric Brownian Motion.

In this project, we first assume that stock prices follow the Geometric Brownian Motion (GBM) process.

$$\frac{\Delta S}{S} = \mu \Delta t + \sigma \epsilon \sqrt{\Delta t} \tag{4}$$

Where μ is the expected return of a stock that will earn over a short time period of time Δt. σ is the expected volatility of the stock. ϵ is a random draw from a standard normal distribution. According to GBM, the log return of stocks is normally distributed:

$$\ln \frac{S_t}{S_0} \sim \mathcal{N}(\mu - \frac{\sigma^2}{2}, \sigma^2) \tag{5}$$

Where S_0 is the stock price at time t_0 and S_t is the price at time t. In this study, Δ_t is 1 day because we are interested in detecting the change in the daily return of a stock.

4.2.2 *Maximum Likelihood Estimator.*

Then we plug equation 5 into equation 2 and 3:

$$X_t = \ln \frac{S_t}{S_{t-1}}$$

$$f_1(X_t) = \mathcal{N}(\mu_1 - \frac{\sigma_1^2}{2}, \sigma_1^2)$$

$$f_2(X_t) = \mathcal{N}(\mu_2 - \frac{\sigma_2^2}{2}, \sigma_2^2)$$

Where μ_1 and σ_1 are estimated by the Maximum Likelihood Estimator (MLE) based on the 90 observations before t_0:

$$\hat{\mu} = \hat{\mu_1} - \frac{\hat{\sigma_1}^2}{2} = \frac{1}{T}\sum_{t=1}^{T} X_t \tag{6}$$

$$\hat{\sigma} = \hat{\sigma_1} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(X_t - \hat{\mu})^2} \tag{7}$$

Where $T = 90$, equation 6 and 7 are the solutions that maximize the log likelihood:

$$\underset{x}{\operatorname{argmax}} \sum_{t=1}^{T} \log \mathcal{N}(\hat{\mu}, \hat{\sigma})$$

4.2.3 *Rolling CUSUM Tests with Fixed Window Limit.*

We start trading with $Z_t^+ = Z_t^- = 0$. μ_1 and σ_1 are estimated based on the previous 90 samples. Let $\mu_1 \equiv \mu_2$, $\sigma_1 \equiv \sigma_2$, which in the financial market means the filter trading rule monitors whether the drift of the return series is significantly larger than zero (bull market) or significantly less than zero (bear market). The first time $Z_t^+ > \delta_1$ ($Z_t^- < \delta_2$), we take a long (short) position. Once a new position is taken, the up-sided CUSUM Z_t^+ and down-sided CUSUM Z_t^- are reset to 0; Up till an opposite position is taken, μ_1, μ_2, σ_1, σ_2 remain unchanged. Inspired by Xie et al's work [Xie et al. 2022],the two CUSUM statistics are updated daily with a new stock price is available. When $Z_t^+ > \delta_1$ ($Z_t^- < \delta_2$), we will evaluate the current position. If it triggers the opposition position, we close the current cycle and start a new one by resetting Z_t^+ and Z_t^- to 0 and re-calibrating μ_1 and σ_1. We use grid search to tune δ_1 and δ_2 to balance the false alarm rate and the detection delay.

4.3 Implementation

Based on the CUSUM methodology, we designed a trading strategy that can be applied to multiple scenarios in the real market. The Pseudo-Code of our CUSUM-based trading strategy is provided in Algorithm 1. We also provide an example of tuning δ_1 and δ_2 by maximizing the previous 60 days period cumulative return. The Pseudo-Code of the grid search is provided in Algorithm 2.

Algorithm 1 CUSUM-Based Trading Strategy

Require: Historical Return Data R, Trading record T, CUSUM records Z^+ and Z^-, Threshold parameters a_1 and a_2, Cumulative Return cr
Ensure: Updated Trading record T, Cumulative Return cr

```
Calculate today's return r_t
if T is empty then
    Calibrate μ_0 and σ_0 using R, calculate f_0
    CUSUM Z^+, Z^- ← [0], [0]
else
    Retrieve μ_0, σ_0 and f_0 from T
end if
μ_1, μ_2 ← μ_0, μ_0; σ_1, σ_2 ← σ_0, σ_0
h_1 ← μ_1 + a_1 * σ_0; h_2 ← μ_2 - a_2 * σ_0
Today's return r ← R[−1]
Calculate f_1, f_2 and CUSUM Z_t^+, Z_t^-
if Z_t^+ ≥ h_1 then
    Buy or hold a long position, direction ← 1
    Renew CUSUM Z^+, Z^- ← [0], [0]
else if Z_t^- ≤ h_2 then
    Short or hold a short position, direction ← −1
    Renew CUSUM Z^+, Z^- ← [0], [0]
else
    if T is empty then
        Hold cash and cr ← cr + r_f
    else
        Retrieve last direction from T
        Hold current position and cr ← cr + direction ∗ r
        Z^+, Z^- append Z_t^+, Z_t^-
    end if
end if
Update T with direction and distribution
return Trading record T, Cumulative Return cr
```

Algorithm 2 CUSUM Grid-Search

Require: Threshold parameters list $alist$ of length n
Ensure: Cumulative return matrix mcr

```
Initial mcr ← n × n matrix of [0]
for all a1 in alist do
    for all a2 in alist do
        Run CUSUM-Based Trading Strategy, mcr[a_1][a_2] ← cr
    end for
end for
return Cumulative return matrix mcr
```

5 RESULTS AND EVALUATION

In this section, we want to discuss whether our CUSUM trading strategy illustrated above will be profitable or not with real-world data. We first fit our trading strategy to the markets we want to study. We consider four indices: SPY, HSI, SPYG, and SPYV. The period we use for our studies is from 2007-01-01 to 2022-11-30.

5.1 CUSUM Statistics & Trading Signals

In this part, we observe the CUSUM statistics and trading signals for SPY. The left side graph in Figure 4 shows the CUSUM statistics from September 01, 2022, to November 30, 2022, and the right side graph is from January 01, 2022, to November 30, 2022. We set the rolling window to 90 days and tuned the threshold h1 and h2.

Figure 5 shows the long and short trading signals corresponding to the CUSUM statistics of SPY. We can observe from this plot that normally, our strategy will take a long position in a relatively low place and take a short position in a relatively high place, which makes sense since the stock price will likely drop when it strikes too high, and vice versa.

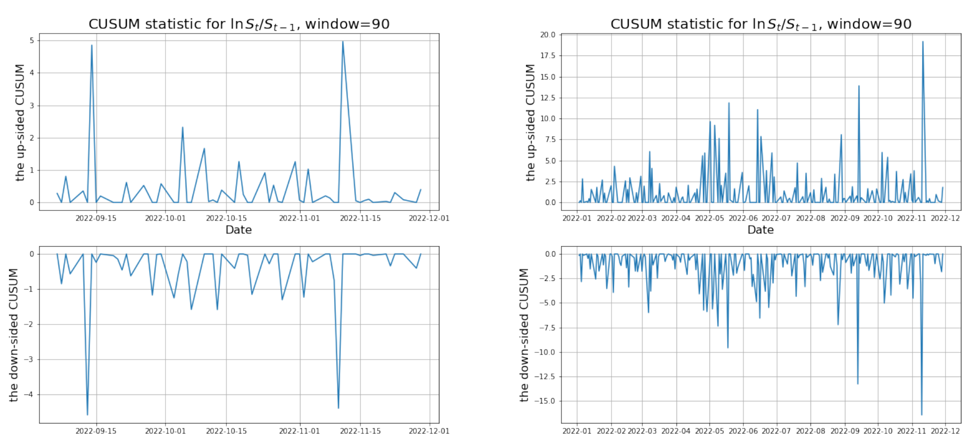

Fig. 4. Last 60 Days and YTD SPY CUSUM Statistics

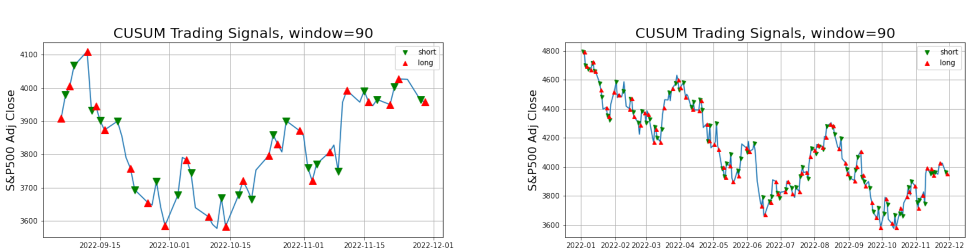

Fig. 5. Last 60 Days and YTD SPY Long and Short Trading Signals

5.2 Cumulative Return Comparison

In this part, we compare our trading strategy's cumulative return with the long-and-hold strategy. We use the SPY vs HSI as the proxies of the US and China stock markets to assess the strategy's effectiveness in different regions. We also trade the SPYG vs the SPYV as the proxies of growth stocks and value stocks in the US market to assess if the categories of stocks impact the performance of this strategy.

5.2.1 US Market vs Chinese Market.

We implement the CUSUM strategy in both the US stock market and the China stock market. The left side graph in Figure 6 shows the YTD cumulative return for SPY representing the US market while the right side graph is for HSI representing the Chinese Hong Kong stock market. The result shows that the CUSUM strategy can beat the long-and-hold strategy with a holding period return of 37.9% compared to -18.6% for the long-and-hold strategy, which means that our strategy can generate value and beat the market during the 2022 time period in the US market. While for the Hong Kong market, the CUSUM strategy has a cumulative return of -63.3%, which is lower than the long-and-hold cumulative return of -22.5%. It illustrates that the trading strategy for HSI underperforms the buy-and-hold strategy in the Hong Kong market.

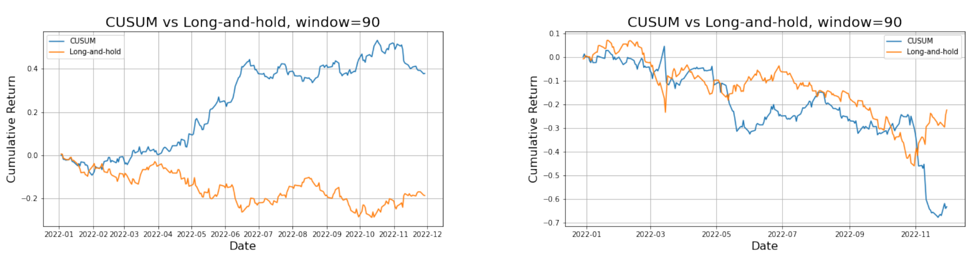

Fig. 6. YTD SPY (left) and HSI (right) Cumulative Return Comparison

We further analyze the strategy within the time period from 2007-01-01 to 2022-11-30 and plot the SPY and HSI cumulative return as shown below in Figure 7. We could conclude that the CUSUM strategy can beat the long-and-hold strategy in the year 2022 in the US market while it can not in the Hong Kong market. However, for a much longer time span from 2007 to 2022, the strategy can not beat the long-and-hold strategy in both US and Chinese Hong Kong markets while the HSI CUSUM strategy did perform better than the long-and-hold from 2020 ahead.

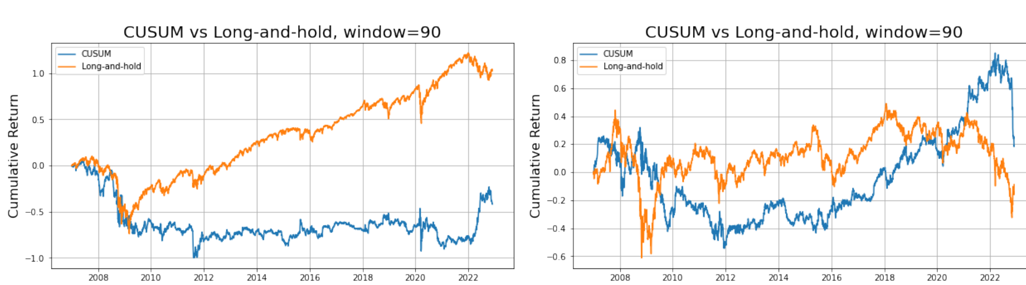

Fig. 7. SPY (left) and HSI (right) Cumulative Return from 2007-01-01 to 2022-11-30

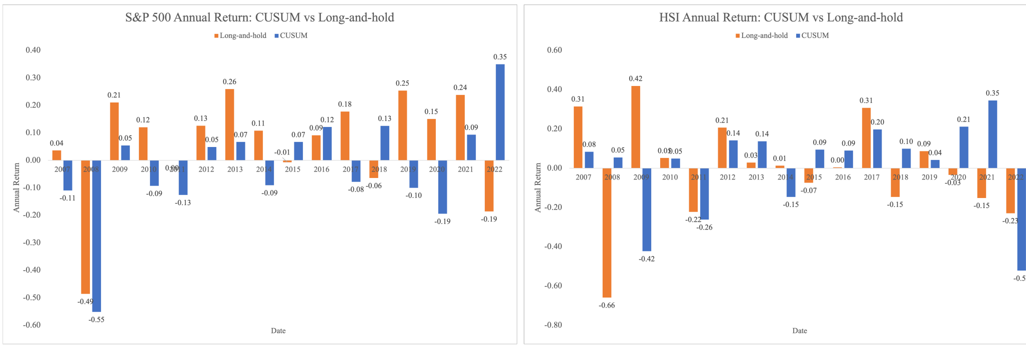

Fig. 8. SPY (left) and HSI (right) Annual Return

5.2.2 *Growth Stock vs Value Stock.*

Given the market is in the US, we now want to compare the performance of the CUSUM strategy for growth stocks and value stocks. Figure 9 shows the cumulative return for SPYG and SPYV from 2007-01-01 to 2022-11-30. We could conclude from the figure that the CUSUM strategy can not beat the long-and-hold strategy for both growth stocks (SPYG) and value stocks (SPYV) for a time span from 2007 to 2022, while the growth stocks did perform better than the value stocks.

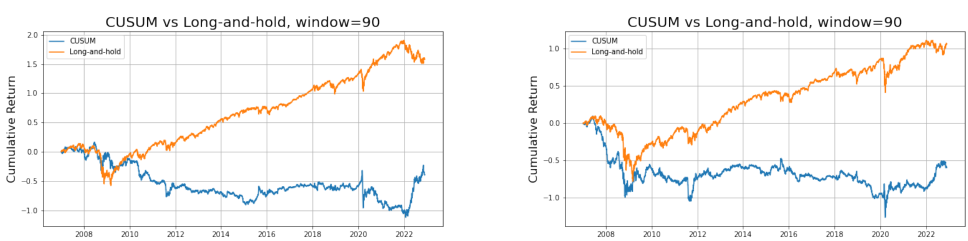

Fig. 9. SPYG (left) and SPYV (right) Cumulative Return from 2007-01-01 to 2022-11-30

5.3 GridSearch

In this section, we conduct grid search to tune threshold parameters a_1 and a_2 respectively for both growth stocks (SPYG) and value stocks (SPYV). In a real implementation, this procedure should only be applied to historical data earlier than the current 60 days. But through the comparison here, we would like to know if CUSUM trading strategies for growth stocks and for value stocks should have generally different thresholds. Given the calibrated $\hat{\sigma}_w$ being very small, we paired up all a values from 1,5,10,15,20,25, and 30.

Using the Algorithm 2, we get a cumulative returns matrix of 7×7 as shown in Figure 10. As we can see, when a_1 and a_2 are small, the CUSUM trading strategy performs better in growth stocks (SPYG); when a_1 and a_2 are large, CUSUM trading strategy performs better in value stocks (SPYV). Generally speaking, cases with $a_1 > a_2$ perform better and SPYV cumulative returns outperform SPYG significantly in their best cases. This result possibly implies that for values stocks, the CUSUM strategy should be more conservative, and small turbulation will always reverse to its true value, so holding is very important and the CUSUM strategy can be successful if focuses on large changes; for growth stocks, CUSUM trading strategy should focus on earning from small turbulations for the longer trend of such stocks could be pretty unstable. However, for more solid conclusions, future research should apply to more categories of stocks and longer periods.

SPYV		a2						
		1	5	10	15	20	25	30
	1	-0.0634	0.0150	0.0483	0.0230	0.0271	0.0271	0.0705
	5	-0.0089	0.0150	0.0483	0.0271	0.0271	0.0211	0.0705
	10	-0.0196	0.0043	0.0483	0.0211	0.0271	0.0271	0.0705
a1	15	-0.0361	-0.0062	0.0006	0.0105	0.0105	0.0105	0.0366
	20	-0.0547	0.0349	0.0788	0.0821	0.0821	0.0046	0.1082
	25	0.0109	0.0349	0.0788	0.0476	0.0821	0.1190	0.1451
	30	0.0056	0.0349	0.0476	0.0821	0.0821	0.1190	0.1451

SPYG		a2						
		1	5	10	15	20	25	30
	1	0.0253	-0.0955	-0.1104	-0.1805	-0.1484	-0.1622	-0.0196
	5	0.0119	-0.0955	-0.0883	-0.1395	-0.1484	-0.1647	-0.0196
	10	-0.0224	-0.0364	-0.0241	-0.2463	-0.2792	-0.1875	-0.0448
a1	15	0.0517	0.0165	0.0190	-0.0477	-0.0272	-0.1875	-0.0448
	20	-0.0316	-0.0557	-0.0582	-0.0867	-0.0662	-0.0448	-0.0473
	25	-0.0785	-0.1051	-0.1051	0.0028	-0.0686	-0.1325	-0.1325
	30	-0.1182	-0.1448	-0.1448	-0.0829	-0.0853	-0.1325	-0.1325

Fig. 10. SPYV and SPYG GridSearch for Parameter a1 and a2

6 CONCLUSION

To sum up, our study applies CUSUM techniques to create an active trading strategy and compares its performance with the long-and-hold strategy in different stock markets (US vs China) and for different stock types (growth vs value). We find that the strategy works differently in the US and Chinese Hong Kong markets as every market has its unique personality with distinct economic situations and public policy. The volatility and momentum of different markets can vary significantly.

As the usual CUSUM strategy may not beat the market, we look at ways to improve our trading strategy. First, due to the limited time, we only tuned the thresholds at a fixed time point. It would be interesting to try tuning the thresholds in each rolling iteration. The window length can also be tuned to optimize the performance. In addition, we notice that some long signals are lagging behind the best buying point. When the strategy detected an upward shift and triggered a long signal, the price is already too high to make profits. If the price tanked the following day, the strategy appears to buy at the high point. Even though it immediately triggers a sell signal and short the security the next day, the strategy still loses money. Therefore, it would be interesting to apply the strategy with higher frequency data to see if it can react to signals more quickly. Lastly, we assume no transaction cost in this project. Our strategy involves frequent changes in positions. If transaction cost is accounted into the total profit, the performance may be different.

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