

(C) According to bayes rule  $P(y|x) = \frac{P(x|y) P(y)}{P(x)}$

based on naive bayes rule  $\Rightarrow \underbrace{P(y|x)}_{\text{Posterior}} = \underbrace{P(x|y)}_{\text{Likelihood}} \underbrace{P(y)}_{\text{Prior}}$

if  $P(y=i|x) > P(y=j|x) \Rightarrow y=i$  else  $\Rightarrow y=j$

Because we assume independence for all data incidence  
Then we can write the probability for data  $X$  to  
be  $y=1$  (spam) or  $y=0$  (ham) can be written as:

$$P(y|X) = P(X, y) = P(x^1, y^1) \cdot P(x^2, y^2) \cdot \dots \cdot P(x^m, y^m)$$

$m$  is total number of records of data

$$= \prod_{i=1}^m P(x^i, y^i)$$

Likelihood

$$\stackrel{\text{naive}}{=} \prod_{i=1}^m P(x^i|y^i) P(y^i)$$

$$= \prod_{i=1}^m P(y^i) \prod_{k=1}^n \theta_{c,k}^{x_{c,k}^i}$$

(why in next page)  
we want to maximize this equation.  
this is constrained optimization

for simplification we take the  $\log$  from both sides.  
 $\log$  is monotonic so the maximization result will hold

The constrained is:  $\sum_{k=1}^n \theta_{c,k} = 1 \quad \forall c$