

Basic probability

Given two random variables X, Y ,

- ▶ The joint probability density function of (X, Y) is given by

$$p(x, y)$$

which satisfies $\int \int p(x, y) dx dy = 1$

- ▶ Marginal probability

$$p(x) = \int p(x, y) dy$$

$$p(y) = \int p(x, y) dx$$

- ▶ When you are dealing with discrete valued x and y , just replace integrals with sums.

► Conditional probability

$$p(x|y) = \frac{p(x, y)}{p(y)}$$

$$p(y|x) = \frac{p(x, y)}{p(x)}$$

- For example, when y is discrete and takes value $\{1, 2, 3\}$ this is particularly simple:

$$p(x|y = i) = \frac{p(x, y = i)}{p(y = i)}, \quad i = 1, 2, 3$$

i.e., for each fixed value $y = i$, evaluate the above.

Alternatively, you can also find a family of probability density functions (pdf), one for each fixed $y = i$ and the pdf is terms of the distribution of x such that the corresponding $y = i$

Bayes formula

Bayes formula relates two conditional probabilities

$$p(x|y) = \frac{p(x, y)}{p(y)} = \frac{p(y|x)p(x)}{p(y)} = \frac{p(y|x)p(x)}{\int p(z, y)dz}$$