

* the answer to why from previous page:

We are looking for maximizing the $P(X|y)$ likelihood.

to max $P(X|y) = \frac{P(y|X)}{P(y)}$ we can maximize $P(y|X)$

because $P(y) < 1$. And it will vanish when taking FOC of Lagrange.

Now the optimization problem, by taking log:

$$\begin{aligned}\log P(X, y) &= \log \prod_{i=1}^m (P(y^i) \prod_{k=1}^n \theta_{c,k}^{x_k^i}) = \sum_{i=1}^m \log (P(y^i) \prod_{k=1}^n \theta_{c,k}^{x_k^i}) \\ &= \sum_{i=1}^m \left\{ \log (P(y^i)) + \sum_{k=1}^n \log \theta_{c,k}^{x_k^i} \right\}\end{aligned}$$

$$\text{with constraint } \sum_{k=1}^n \theta_{c,k} = 1 \quad \forall c$$

$\theta_{c,k}^{x_k^i}$ is the likelihood of word x_k from sentence i that belongs to category c .

y^i is the category indicator. Here c is the category hence it can be c

c is variable for category. here is 1 or 0

the Lagrangian optimization problem will be:

$$L(P(X, y)) = \sum_{i=1}^m \left\{ \log (P(y^i)) + \sum_{k=1}^n \log \theta_{c,k}^{x_k^i} \right\} + \lambda \left(\sum_{k=1}^n \theta_{c,k} - 1 \right)$$