lets consider a graph with $A_1 \dots A_m$ components.
we can consider the Adjancy matrix and degree matrix as below:

$$\begin{bmatrix} D_1 & & & \\ & D_2 & & \\ & & \ddots & \\ & & & D_M \end{bmatrix} \qquad \begin{bmatrix} A_1 & & & \\ & A_2 & & \\ & & \ddots & \\ & & & A_m \end{bmatrix}$$

because each component is connected with minimun one edge then we can consider each A_1 to A_m and D_1 to D_m as block of semmetric matrix.

$$\text{hence } L = \begin{bmatrix} L_1 & & & \\ & L_2 & & \\ & & \ddots & \\ & & & L_m \end{bmatrix} . \quad ①$$

Now lets consider an eigendecomposition of L_x for eigenvalue zero and eigen vector V $(V \in R^n)$

$$\lambda = 0 = V^T L_x V = V^T D_x V - V^T A_x V$$
$$= \sum_{i=1}^{n} d_i v_i^2 - \sum_{ij} a_{ij} v_i v_j$$
$$= \frac{1}{2}\left(\sum_{i=1}^{n} d_i v_i^2 - 2\sum_{ij} a_{ij} v_i v_j + \sum_{j=1}^{n} d_j v_j^2\right)$$
$$= \frac{1}{2}\sum_{i,j=1}^{n} a_{ij}(v_i - v_j)^2 \quad ②$$