

$$\Rightarrow \frac{1}{2} \sum_{i,j=1}^n a_{ij} (v_i - v_j)^2 = 0$$

\downarrow
 positive

The equation above can only equal to zero if $v_i = v_j$ for each i, j .

This means for component x , the eigenvector is scaled on identity vector.

if $v_i = v_j = v_x = v_x^T I_x$ (in the indicator vector)

we can generalize this concept to all components

$$V^T L V = \begin{bmatrix} v_1^T I_1 & v_2^T I_2 & \dots & v_m^T I_m \end{bmatrix}$$

(v_1, \dots, v_m) are values

This shows that the eigenvectors to eigenvalue zero are linear combination of I_1, \dots, I_m .