18 Seriation

In dating any class of objects, such as spears, adzes, fibulae, combs, etc., the process is to look out the age or the range of age of each of the graves in which such objects are found.

Petrie (1899: 299)

This chapter is the first of several that deal explicitly with the chronological dimension of archaeological analysis. Seriation involves methods by which archaeologists attempt to put assemblages of artifacts into a serial order, that is, to create an ordinal chronological scale. Seriation orders units—assemblages, artifacts, or other entities—along a single dimension, assumed to be time, so that adjacent units are more similar to one another than to nonadjacent units. Some of the grouping methods already discussed in Chap. 3 can help accomplish this.

These and the methods described below depend on the characteristics of the artifacts found in assemblages and on the assumption that changes in technology, style, and sources of exotic artifacts cause the character of artifact populations to differ over time (Baxter 2003; Laxton 1990; Lyman and O'Brien 2006; Marquardt 1978; McCafferty 2008; O'Brien and Lyman 2002).

Although numismatists had already been using a version of seriation to analyze coin hoards since Bartolomeo Borghesi introduced it about 1820 (Crawford 1990), the classic early archaeological use of seriation was Flinders Petrie's (1899) "sequencing" of predynastic Egyptian graves (Fig. 18.1). Petrie informally used what archaeologists call the **concentration principle** to arrange 900 graves into a serial order on the basis of 804 types and varieties of pottery and other artifacts. The concentration principle is a preference for arrangements that minimize the ranges of varieties over the sequence, in other words, that make the types "clump" in the sequence. Other superficially similar methods, such as those of General Pitt-Rivers (1875), ordered artifacts instead by their assumed evolutionary development.

18.1 Incidence and Frequency Seriation

The simplest seriation, **incidence seriation**, uses a dichotomous model inspired by Petrie's "sequence dating" but formalized much later. This involves recording the presence (1) or absence (0) of each type instead of its relative abundance and following the rule that, once a 0 follows a 1, it can only be followed by more zero values. For example, you could have 0, 0, 1, 1, 0, 0, but not 0, 1, 0, 1, 0, 0. This is the concentration principle: the ones must cluster together with no intervening zeroes (Table 18.1). In honor of Petrie, a series ordered in this way is called a **P-matrix**. In some instances, two or three contexts are interchangeable—their order does not affect the concentration principle—so that we can only treat them as contemporaneous.

It is more common for archaeologists to use **frequency seriation**, based on a model that describes the way the relative abundances of "types," or classes of artifact, not just their presence or absence, are expected to change over time. For typologically-based seriation, the key assumption of this model—the **Kendall model**—is that each artifact type, once introduced, grows in its relative abundance (incipience), eventually reaches a peak (fluorescence), and then declines as other new, competing artifact types begin to displace it (Fig. 18.2; Buck et al. 1996: 328; Dempsey and Baumhoff 1963; Kendall 1971; Phillips et al. 1951; Robinson 1951). If drawn as a bar graph with the bars arranged horizontally and centered, this pattern forms a "battleship curve" (Fig. 18.3; Ford 1962). Because the data are proportions or percentages, the various types are not independent of one another (when one goes up, something else has to go down in relative frequency).

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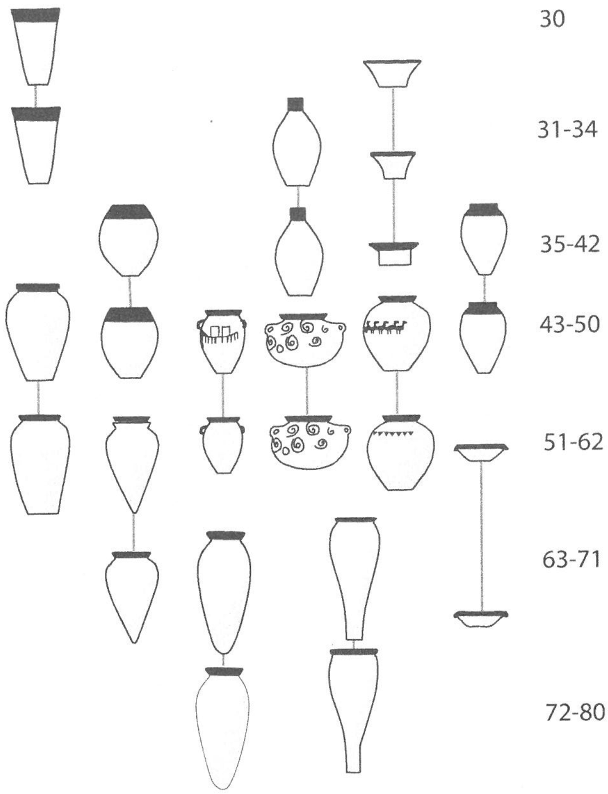

Fig. 18.1 Abbreviated version of one of Petrie's seriations, which Petrie labelled numerically with so-called "sequence numbers" at right, and line segments showing continuity of types. (Y. Salama, modified from Petrie 1899: fig. 1)

Table 18.1 An incidence matrix (left) showing the presence (1) or absence (0) of each of four artifact types A-D among eight different archaeological contexts; and (right) a seriation of those contexts by the concentration principle to create a "P-matrix"

	Artifact type					Artifact type			
Context	A	B	C	D	Context	A	B	C	D
1	1	0	1	0	2	0	0	0	1
2	0	0	0	1	8	0	1	0	1
3	1	1	0	1	6	1	1	0	1
4	1	1	0	0	3	1	1	0	1
5	0	0	1	0	4	1	1	0	0
6	1	1	0	1	7	1	0	1	0
7	1	0	1	0	1	1	0	1	0
8	0	1	0	1	5	0	0	1	0

From Kendall (1971: 220)

Notice how the "1" values are now clustered, with no intervening zeroes

Other assumptions of this model are that each unit corresponds with a brief and comparable period of time, that the content of each unit is a representative sample of a population of artifacts that were in use during that time and place (see Chap. 6), that all units belong to the same cultural tradition, and that all the units come from a reasonably small region (Dunnell 1970: Marquardt 1978: 261–297; O'Brien and Lyman 2002: 117–119). The last two assumptions are to ensure that the dimension along which we order the units is time, rather than space or cultural affiliation, while the assumption of a representative sample is an important one whose implications merit further attention, below. Further implicit assumptions, not always recognized, are that the populations from which the artifacts came are themselves serially ordered, and the units are samples from different populations. In other words, none of the assemblages are exactly contemporary. As you can imagine, many of these assumptions pose difficulties, as our imposition of an "episodic" model—treating each assemblage as a sort of glimpse of time—ignores the likelihood that assemblages accumulated over somewhat extended and somewhat different intervals of time.

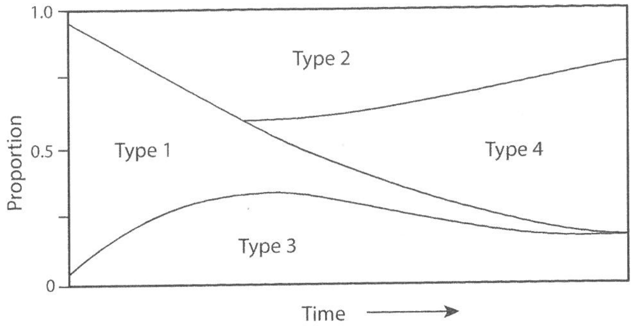

Fig. 18.2 Kendall's model for the way the proportions of four types of artifact would vary over time. Each type has a unimodal distribution such that it gradually increases, reaches a peak, and then decreases in abundance. (After Doran and Hodson 1975: 272)

Mathematically, we can represent the relative abundances of types in a matrix of m rows and n columns, where m is the number of archaeological units (layers, graves, components, etc.) and n is the number of artifact types. Each cell in the matrix contains a value, $a_{i,j}$, meaning the jth entry in the ith row, or the number of artifacts of type j in the ith archaeological unit. We conclude that the matrix is seriated if the numbers going down each column, once they have decreased, never rise again. We refer to this property of never increasing again as decreasing **monotonically**. For example, in the left side of Table 18.2, the archaeological contexts are in arbitrary order but, on the right, reordering of the same contexts results in the relative abundance of each type never increasing once it has started to decrease, a **Q-matrix**.

Although the seriation arranges the contexts in a linear order (4, 1, 7, 5, 3, 8, 2, 6), the direction of the ordering is arbitrary. Except by reference to some other data, we cannot rule out that the order 6, 2, 8, 3, 5, 7, 1, 4 is the correct one.

There are, however, problems with the Kendall method (McNutt 2005). First, it is not always true that in correctly ordered populations of artifacts the relative abundance of a

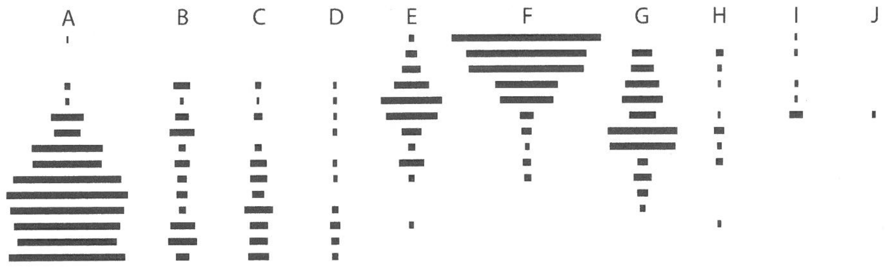

Fig. 18.3 Example of a "battleship curve" for Allegheny Region pottery types labelled A through J (Y. Salama, after Ford 1962: fig. 9). Each horizontal row of bars represents pottery frequencies from a single assemblage. Note that it is not necessary to center the bars in order to seriate the assemblages correctly, as the key feature is that each type increases, reaches a peak, then declines in relative frequency

Table 18.2 On the left, a matrix with the percentages of four different artifact types in eight different contexts, in arbitrary order. On the right, the same eight contexts seriated so that the percentages never increase again after they begin to decrease (a Q-matrix, after Kendall 1971: 219)

	Artifact type					Artifact type			
Context	A	B	C	D	Context	A	B	C	D
1	10	20	1	70	4	0	10	0	90
2	20	0	70	10	1	10	20	1	70
3	40	10	30	20	7	30	30	5	35
4	0	10	0	90	5	40	20	15	25
5	40	20	15	25	3	40	10	30	20
6	10	0	90	0	8	30	5	50	15
7	30	30	5	35	2	20	0	70	10
8	30	5	50	15	6	10	0	90	0

type will never increase after it has decreased. More realistically, there are fluctuations that the Kendall model does not accommodate. Even Ford's (1962) examples of battleship curves, such as those for surface collections in Virginia (Fig. 18.3), show numerous fluctuations that deviate from the Kendall model, and such anomalies persist in more recent applications (e.g., Wesler 1999). Second, since any particular context is only a sample of a population, and probably not a random one at that (Madsen 1988), typical archaeological cases may violate the assumption of representative sampling or just suffer from insufficient sample size; it is easy to imagine cases where a particular sample could omit an artifact type that was, in reality, fairly common at the time of assemblage formation. The Kendall model makes no allowance for sampling error, preservation issues and other factors that create noise in the data (Buck and Sahu 2000). Third, the algorithms for finding the best ordering of units do not account for the possibility that there is more than one possible order, including some that are almost as likely as the "best" one (Buck and Litton 1991; Buck et al. 1996: 329). That is why most researchers have favored a probabilistic approach to seriation—finding the best, yet imperfect, fit to the Kendall model, rather than a deterministic one that assumes there is only a single solution. One reason that such alternative orders are possible is that some of the samples could be from the same population, or from two contemporary populations.

One way to address the first of these problems is to turn to the overall similarity between units, rather than assuming that each and every type will increase to a peak and then decrease monotonically. To do this, we can make a new matrix that is n x n instead of n x m, that is, with archaeological contexts along both axes (Table 18.3). For each pair of contexts, we then measure the degree of similarity by Robinson's (1951) Index of Agreement (IA):

$$IA_{jk} = 200 - \left(\sum_{i=1}^{m} \left| x_{ji} - x_{ki} \right| \right)$$

or the sum of the absolute values of the difference between the percentages of each type for each pair of archaeological units (j and k), subtracted from 200. In other words, if two archaeological contexts have exactly the same distribution of types, the summed differences between them (a measure of dissimilarity) will be 0 and the IA will be 200. If, on the other hand, the two contexts are maximally different (i.e., when one has 100% of one type and the other has 100% of another type), the summed differences will be 200% and IA will be 0. Subtracting from 200 simply turns a dissimilarity coefficient into a similarity coefficient.

Returning to the data from the matrix in Table 18.2, the absolute values of differences between the four types for contexts 1 and 2 are 20-10, 20-0, 70-0, and 70-10. The sum of these differences is 10 + 20 + 70 + 60 = 160, so that IA = 200 − 160 = 40. For contexts 1 and 3, they are 30 + 10 + 30 + 50 = 120, so IA = 80. We can continue this process for each pair of contexts to produce the matrix in Table 18.3. We only have to fill out half the matrix, as it is perfectly symmetrical about the diagonal, where all values of IA are 200.

The next step is to rearrange the rows and columns so that higher IA values cluster near the diagonal and IA decreases toward the upper right corner. Some of the highest values are 170 for the pair (3,5) and 160 for (5,7). Meanwhile, the pair (4,6) shows no agreement at all (IA = 0), indicating that they should be at opposite ends of the sequence. This kind of reasoning provides the basis for ordering the contexts as in Table 18.4.

Dempsey and Baumhoff's (1963: 499) variation on Robinson's IA reduces the data to a dichotomous scale "so

Table 18.3 Matrix of Robinson's Indices of Agreement (IA) for the data in Table 18.2

Context	1	2	3	4	5	6	7	8
1	200	40	80	140	110	20	130	60
2		200	120	20	110	160	70	160
3			200	60	170	80	130	160
4				200	50	0	90	40
5					200	50	160	130
6						200	30	125
7							200	110
8								200

Table 18.4 Rearranged matrix of the Robinson's Indices of Agreement (IA) for the data from Tables 18.2 and 18.3. In this case, the same order results as in the right half of Table 18.2. For the most part, the IA values fall off with distance from the diagonal

Context	4	1	7	5	3	8	2	6
4	200	140	90	50	60	40	20	0
1		200	130	110	80	60	40	20
7			200	160	130	110	70	30
5				200	170	130	110	50
3					200	160	120	80
8						200	160	125
2							200	160
6								200

that the presence (or absence) of any one type contains no necessary implication concerning the presence (or absence) of any other type." In other words, it counters the problem of inter-type dependence in the Kendall model. Their approach uses similarity coefficients in the same way as in clustering (pp. 30–32). Although it solves the problem of interdependence of proportions, it has the usual problem of presence/absence data: a single artifact of one type has just as much weight as hundreds of examples of another type (Marquardt 1978: 268). Dempsey and Baumhoff (1963: 498) argue that this is an advantage as rare types may be chronologically "diagnostic."

Marquardt (1978) reviews many other variations on and alternatives to the Robinson (1951) method. Some interesting ones are Cowgill's (1972), which measures similarity, not between pairs of contexts, but pairs of types, Wilkinson's (1974), which treats artifacts and contexts equally, and LeBlanc's (1975), which considers distributions of attributes, rather than types.

18.2 Multidimensional Scaling (MDS) and Correspondence Analysis (CA)

Just as we could use similarity matrices to group artifacts into types, or to seriate contexts or artifacts as described in the last section, we can also use some of the grouping methods that are based on these matrices. Although the methods are identical, there is a philosophical difference in the application of these methods to either contexts or artifacts. In the former case, we are ordering assemblages, much as with the previous methods and employing many of the same assumptions. In the latter, there is an implicit further assumption of "descent with modification" (Darwin 1859; cf. O'Brien and Lyman 2002), much as with Pitt-Rivers' artifact sequences.

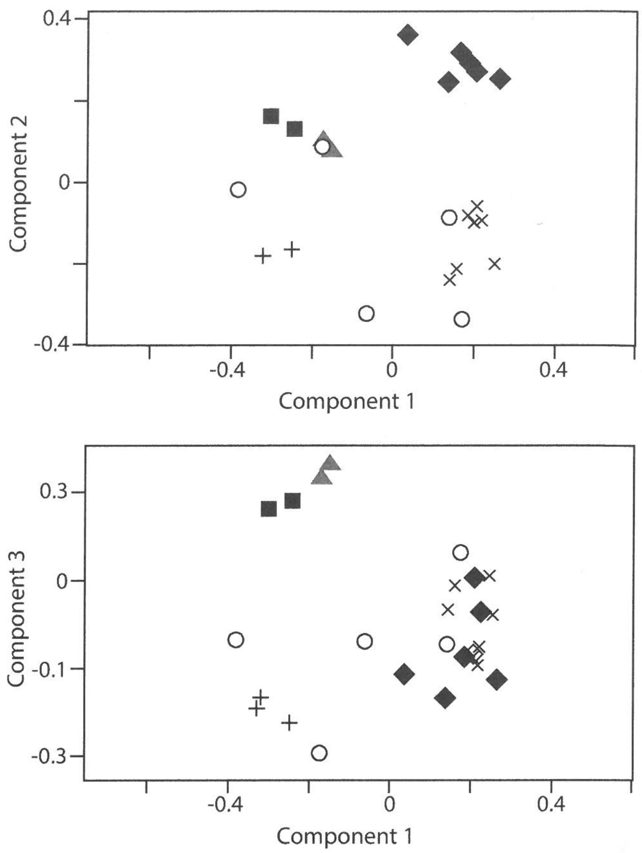

Fig. 18.4 Component plots for a non-metric MDS of a matrix of 19 petrographic characteristics in the fabric of 25 pottery samples from Can Sora, Spain. (Modified from Baxter 2003: 88)

Multidimensional scaling (MDS, p. 33) creates something like a "map" that represents dissimilarities among units (contexts or artifacts) as distances in space. Because the resulting space is multidimensional (one dimension for each attribute or type), representing this "map" on a two-dimensional image requires distortion of distances, and MDS permits these distortions as long as they preserve the rank-order of the distances. Often, the resulting "map" has a near-circular or horseshoe shape (Fig. 18.4) that results from having many units with almost the same dissimilarity with some other unit. If time is the principal dimension along which the units are distributed, then the oldest context or artifact is at one end of the horseshoe and the youngest at the other, so that the units are chronologically ordered.

Correspondence analysis (CA) is an alternative that, somewhat like multidimensional scaling, often yields a

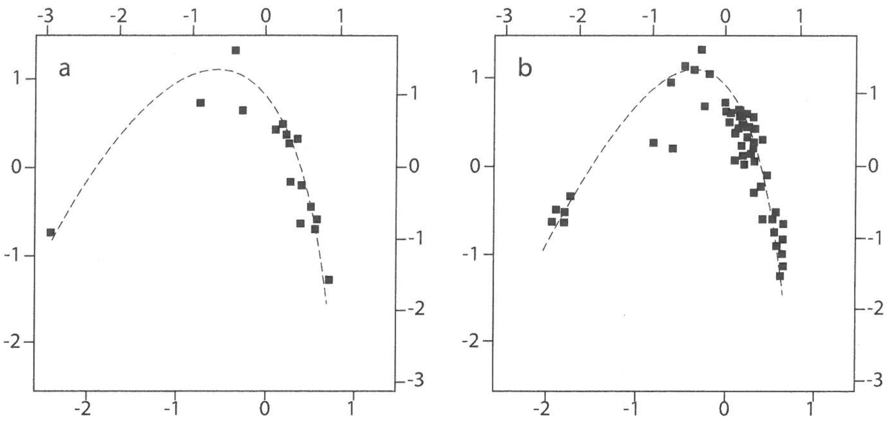

Fig. 18.5 Correspondence Analysis of 52 pottery types in 16 tomb assemblages at the site of Tell al-Farah al-Janubiyyah (after Baxter 2003: 137), with results by tombs (**a**) and pottery types (**b**). Notice how points tend to form horseshoe-shaped curves (dashed), indicating an approximate (chronological) order

U-shaped string of data points in a scatterplot (Fig. 18.5). Unlike the Kendall and Robinson methods, CA does not require the assumption that types first increase and then decrease monotonically in abundance. Consequently, it can, and often does, produce a quite different ordering of the data. However, CA does assume independence between the artifacts and the contexts, something the Kendall-Robinson models do not require, and the presence of many zero values in typical archaeological data tables is a problem for this assumption (Buck and Sahu 2000).

Again, if time is the main contributor to the ordering, and the types or attributes being counted are chronologically sensitive, then the oldest units will be at one end of the curve, and the youngest at the other. Where the distribution of points is not U-shaped, one might take the first axis of the CA as the best seriation order, but most archaeologists consider the "horseshoe" to be a good indicator of a successful seriation. Today, CA is one of the most common ways to "seriate" artifacts and contexts, if only in an exploratory fashion (Baxter 2003: 204–206; Shennan 1997: 342).

18.3 Bayesian Approaches to Seriation

The similarity-matrix and multivariate methods just described provide a single "best" unidimensional order for the units, but real data can be "noisy" and the "best" order ignores the possibility that there could be other orders that are almost as likely (Baxter 2003: 207; Buck et al. 1996: 329; Buck and Sahu 2000; Halekoh and Vach 2004).

Other Bayesian research in this vein has included Buck and Sahu's (2000) introduction of a hierarchical Bayesian model for CA, and Halekoh and Vach's (2004) attempt to seriate graves from the important La Tène cemetery of Münsingen-Rain, near Bern, Switzerland, by using a stochastic model of the unimodal distribution of types (the concentration principle) in an incidence matrix.

Despite their potential, Bayesian approaches to seriation have yet to gain wide acceptance, and Baxter (2003: 207) notes that such acceptance will require more experience with large, realistic data sets.

18.4 A New Approach to Deterministic Seriation

Lipo et al. (2015) have noted that probabilistic methods for seriation, such as MDS or CA, simplify data sets in a way that omits a lot of information and makes it difficult to determine the validity of the resulting order. Meanwhile, deterministic methods become unmanageable, even with modern computers, when there are more than about 14 assemblages to be ordered because there are too many possible permutations of assemblage orders to evaluate in a reasonable time. Their solution is the Iterative Determinitive Seriation Solutions (IDSS), which drastically reduces the number of permutations that need to be examined. IDSS entails an iterative procedure, starting with valid sequences of just a few assemblages (valid in the sense of not violating seriation assumptions), then using these as "building-blocks" for longer sequences. This avoids the step of having to calculate all possible combinations of assemblages.

18.5 Die-Linkage of Coins

One very specialized class of artifact—coinage—offers another kind of seriation that would not work for others. Die-linking is based on the fact that most ancient and mediaeval coins were made by striking a flat disk of metal between two dies that wear out after a period of use (see p. 222). Because the upper (reverse) die, which is struck with a hammer, wore or cracked more quickly than the lower (obverse) die, which was set into an anvil, coiners changed the upper and lower dies at different times. In addition, it is likely that coiners kept reverse dies in a secure storage box overnight, and next morning they could be paired with different obverse dies. These factors resulted in the association

Example

Buck and Litton (1991) use an iterative algorithm called a Gibbs sampler, some arbitrary starting values, likelihoods modelled by the multinomial and prior probability by Dirichlet distributions to create seriations that take into account the stochastic nature of real data. For example, site deposits can contain "residual" artifacts (see p. 323) that belong to earlier periods, or some artifact types can be missing from a tomb because of small-sample effects. By carrying out many iterations of this process, each of which may result in a somewhat different ordering, they can discover which order is most probable, and which others fairly probable. Because this takes seriously the possibility that the "best" order might be incorrect, it is an approach that is quite relevant to the theme of quality in data analysis.

After 1000 iterations, using the same data that Laxton and Restorick (1989) used to compare the Kendall method with CA (Table 18.5), they found that the order resulting from the Kendall method (2, 5, 3, 6, 1, 4) has the highest probability but other orders had significant probabilities (Table 18.6). Surprisingly, the other orders found do not include the one that Laxton & Restorick found by CA (3, 6, 5, 2, 1, 4), which would suggest that it is improbable. It is also notable that the three most probable solutions, with a combined probability of 0.893, are very similar, only varying in the order of 1 and 4 or 3 and 6.

In this example, there was no outside information to guide the selection of prior probabilities, but one potentially great advantage of this method is that it can, like any Bayesian method, incorporate information that has a bearing on the correct order. For example, there could be stratigraphic information or radiocarbon dates for some, but not all, of the contexts being ordered, and this would have definite bearing on the "best" outcome by allowing informative priors.

Table 18.5 Fictitious artifact counts (from Laxton and Restorick 1989) that Buck and Litton (1991) use for demonstrating Bayesian seriation

	Artifact type						
Site	1	2	3	4	5	6	7
1	20	3	4	42	18	0	13
2	85	3	12	0	0	0	0
3	26	40	8	0	0	26	0
4	20	1	4	13	58	0	4
5	67	10	23	0	0	0	0
6	26	29	8	3	0	33	1

Table 18.6 Results from 1000 iterations of ordering the six sites in the demonstration of Bayesian seriation by Buck and Litton (1991: 98)

Order	Frequency	Posterior probability
2, 5, 3, 6, 1, 4	679	0.679
2, 5, 3, 6, 4, 1	128	0.128
2, 5, 6, 3, 1, 4	86	0.086
4, 1, 6, 3, 2, 5	46	0.046
2, 5, 6, 3, 4, 1	28	0.028
3, 6, 5, 2, 1, 4	16	0.016
4, 1, 3, 6 2, 5	8	0.008
3, 6 5, 2, 4, 1	4	0.004
5, 2, 3, 6 4, 1	3	0.003
4, 1, 2, 5, 3, 6	1	0.001
5, 2, 6, 3, 4, 1	1	0.001

of each obverse die with two or more reverse dies, while some reverse dies were used with more than one obverse die. And because dies prior to the modern period were engraved by hand, no two dies were exactly alike, even when the engraver intended to reproduce a design very faithfully. Consequently, careful examination of coins to associate them with the various dies, makes it possible to work out a sequence of die use (Fig. 18.6). As with other kinds of seriation, it may not be obvious which end of the sequence is early and which is late, but evidence from increasing die wear or formation of a die crack, either of which would only occur after the die had been used for some time, can help us determine the correct direction of the sequence when there are no other indications, such as a Roman emperor's titles, to help us do so (Laing 1969: 26–28).

18.6 Quality in the Use and Interpretation of Seriations

Effective use of seriation requires careful attention to assumptions, most notably to have representative samples of artifacts from meaningful contexts that are free of disturbances that could have introduced intrusive or residual artifacts (Marquardt 1978: 292–304; Madsen 1988; McNutt 2005). Any departure from the basic assumptions could result in a seriation along some dimension that is not time, such as space, function, or site-formation process. One way to check to see if the resulting seriation is ordered chronologically is to have at least some chronometric dates, such as radiocarbon assays.

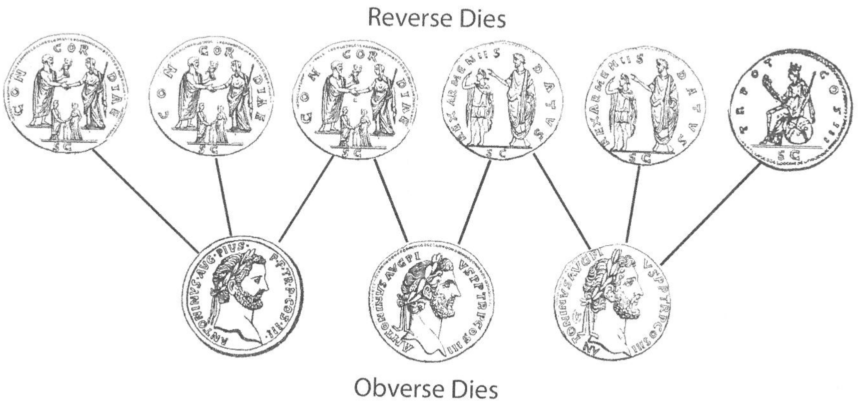

Fig. 18.6 Simplified demonstration of die-linking among coins to establish their relative chronology, using coins of Antoninus Pius (Roman emperor from AD 138 to 161). Because each obverse die was used in combination with several reverse dies, which wore out faster, some obverses share a reverse die, and this provides a basis for ordering the coins from which they were struck. The obverse inscriptions all indicate a date during the emperor's third consulship (AD 140–141)

Scott (1993) additionally notes that archaeologists need to give adequate attention to whether similarity coefficients are based on chronologically sensitive attributes. He worries that giving all contexts equal weight, regardless of the size or nature of their assemblages, could distort results. Scott also notes that, unlike Petrie's old sequence dating, prevalent methods do not help fit new assemblages into previously established sequences. He offers a parametric method that treats the age of each context as a parameter to be estimated. This has the advantages of providing estimates with standard errors, allowing us to judge whether pairs of assemblages are statistically different from one another, and incorporating known dates into the analysis. He models the unimodal variation in the abundance of types over time with a normal distribution, and considers the distribution of artifacts in an assemblage as fitting a Poisson model.

18.7 Summary

- Seriation provides tools to arrange artifacts or archaeological contexts in time on an ordinal scale when evidence from stratigraphy or independent dating is unavailable
- Its key principle, the concentration principle, is that artifact types cluster in time
- Frequency seriation, or the Kendall method, is based on matrices of artifact abundances in different contexts, which may result in an optimal order (Q-matrix with "battleship curves") if all the assumptions are valid
- Incidence seriation, based only on presence or absence of artifact types in each context, results in a P-matrix (after its quasi-originator, Flinders Petrie)
- It is also possible to seriate artifacts whose attributes have "evolved" over time. Although this has often involved an implicit progressivist assumption that was popular in the late nineteenth century but less acceptable today, there are also neo-evolutionary versions of artifact descent through modification
- Multivariate methods such as CA and PCA can order either contexts or artifacts, and it is possible that time is the most influential dimension on that order if most of seriation's assumptions are met
- There are other ordering methods, most notably coinage die-linking, that also seriate specialized classes of artifacts but have principles that do not depend on relative abundance of types or artifacts or the concentration principle

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