Heat transfer in laser based additive manufacturing

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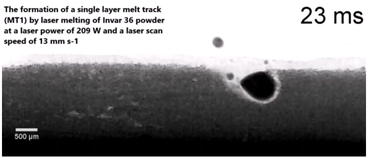
August 16, 2023

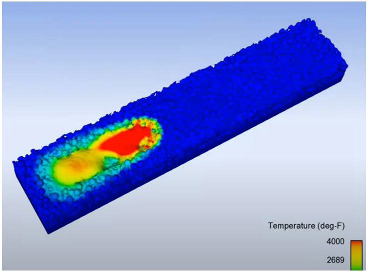
Acknowledgement

Figure: Sponsors and Partners

Videos



molten pool experiments



flow-3d simulation

In engineer's eyes

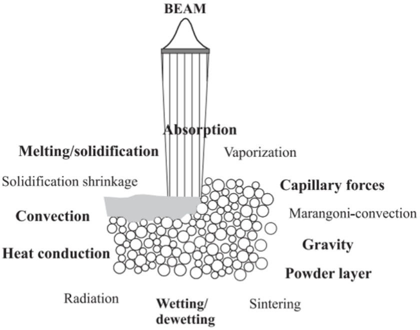
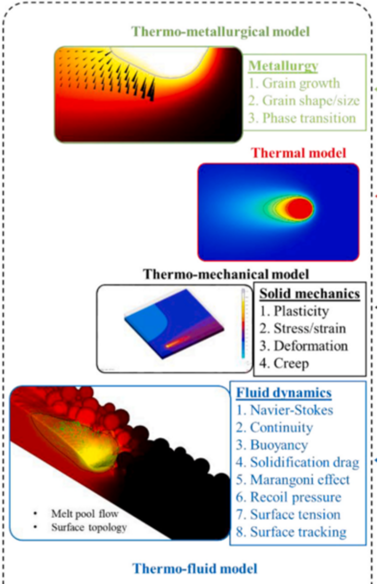


Fig. 1. Physical phenomena during selective beam melting.



Thermal model: Heat Transfer of a moving laser

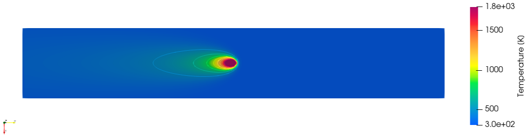


Figure: Temperature distribution

Governing equation

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T \tag{1}$$

$$= \alpha \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] \tag{2}$$

T temperature in Kelvin, α thermal diffusivity in m^2/s, x, y, z are Cartesian coordinates.

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T = \alpha \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] \tag{3}$$

In general, you can **NOT** find a close form solution to the above equation, but We can approximate the solution using numerical methods, see slide 9

Assumptions

2D, infinite and semi-infinite solids, insulated at the infinite boundary

2D analytical solution

$$T - T_0 = \frac{q}{2\pi k R} \exp\left[\frac{-v(x - vt + R)}{2\alpha} \right] \tag{4}$$

T_0 initial temperature, q is the heat flux $(J/(s \cdot m^2))$, k is the heat conductivity of metal $(J/(s \cdot m \cdot Kelvin))$, $R = \sqrt{(x - vt)^2 + y^2}$, v is the heat source moving velocity (m/s), α is the thermal diffusivity

Exercise: calculate and plot temperature for a laser

$$T - T_0 = \frac{q}{2\pi kR} \exp\left[\frac{-v(x - vt + R)}{2\alpha}\right] \tag{5}$$

$$\alpha = \frac{k}{\rho c_p} \tag{6}$$

Parameters	Value	Units
T_0	300	K(Kelvin)
q	840	w/m^2
k	35	$w/m/K$
v	1	m/s
c_p	800	$J/kg/K$
ρ	7600	kg/m^3

Table: physical parameters for a metal

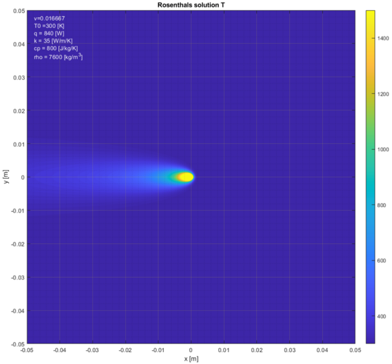


Figure: Exercise example solution

Finite difference unsteady heat transfer: explicit method

Consider a 2D heat conduction

$$\frac{\partial T}{\partial t} = \alpha(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) \quad (7)$$

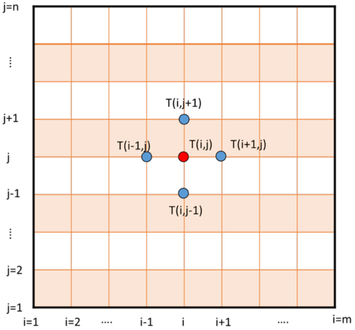


Figure: Illustration of an uniform grid

The righ hand side of Eqn.(7) can be discretized using a central difference method

$$\frac{\partial^2 T}{\partial x^2} = \frac{T^t_{i-1,j} - 2T^t_{i,j} + T^t_{i+1,j}}{\Delta x^2},$$
$$\frac{\partial^2 T}{\partial y^2} = \frac{T^t_{i,j-1} - 2T^t_{i,j} + T^t_{i,j+1}}{\Delta y^2}$$

The left hand side has two options, forward difference (explicit method) or backward difference (implicit method), which will be discussed in the next two slides.

Forward difference in time,

$$\frac{\partial T}{\partial t} = \frac{T_{i,j}^{t+1} - T_{i,j}^{t}}{\Delta t} \tag{8}$$

Explicit Method

$$\underbrace{\frac{T_{i,j}^{t+1} - T_{i,j}^{t}}{\Delta t}}_{\text{forward difference}} = \alpha\left(\frac{T_{i-1,j}^{t} - 2T_{i,j}^{t} + T_{i+1,j}^{t}}{\Delta x^2} + \frac{T_{i,j-1}^{t} - 2T_{i,j}^{t} + T_{i,j+1}^{t}}{\Delta y^2}\right) \tag{9}$$

Assuming $\Delta x = \Delta y = h$

$$T_{i,j}^{t+1} = T_{i,j}^{t} + \frac{\alpha \Delta t}{h^2}\left(T_{i-1,j}^{t} + T_{i+1,j}^{t} + T_{i,j-1}^{t} + T_{i,j+1}^{t} - 4T_{i,j}^{t}\right)$$

Stability requirement: $\Delta t \leq \dfrac{h^2}{2\alpha}$

Implicit method

Backward difference in time,

$$\frac{\partial T}{\partial t} = \frac{T_{i,j}^{t} - T_{i,j}^{t-1}}{\Delta t} \tag{10}$$

Implicit method

$$\underbrace{\frac{T_{i,j}^{t} - T_{i,j}^{t-1}}{\Delta t}}_{\text{backward difference}} = \alpha\left(\frac{T_{i-1,j}^{t} - 2T_{i,j}^{t} + T_{i+1,j}^{t}}{\Delta x^2} + \frac{T_{i,j-1}^{t} - 2T_{i,j}^{t} + T_{i,j+1}^{t}}{\Delta y^2}\right) \tag{11}$$

Assuming $\Delta x = \Delta y = h$, and define $k = \dfrac{\alpha \Delta t}{h^2}$

$$T_{i,j}^{t} = T_{i,j}^{t-1} + \frac{\alpha \Delta t}{h^2}(T_{i-1,j}^{t} + T_{i+1,j}^{t} + T_{i,j-1}^{t} + T_{i,j+1}^{t} - 4T_{i,j}^{t}) \tag{12}$$

$$T_{i,j}^{t} = \frac{1}{1+4k}(T_{i,j}^{t-1} + k(T_{i-1,j}^{t} + T_{i+1,j}^{t} + T_{i,j-1}^{t} + T_{i,j+1}^{t}) \tag{13}$$

Eqn.(13) can be solved using Gauss-Sidel iterative method.
Stability requirement: unconditionally stable, Δt can take any arbitray size