Extra sparse dimensionality reduction

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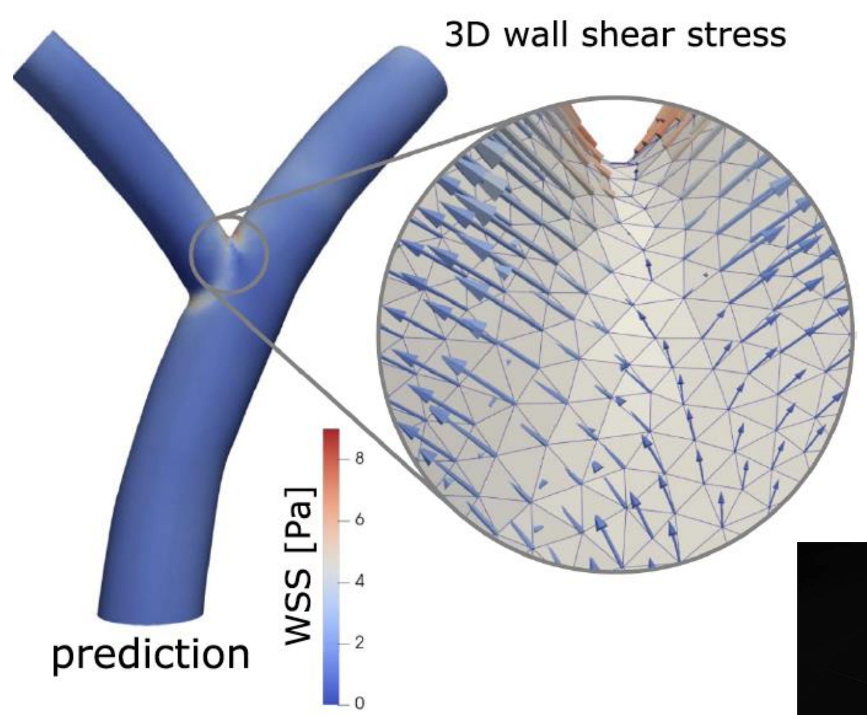
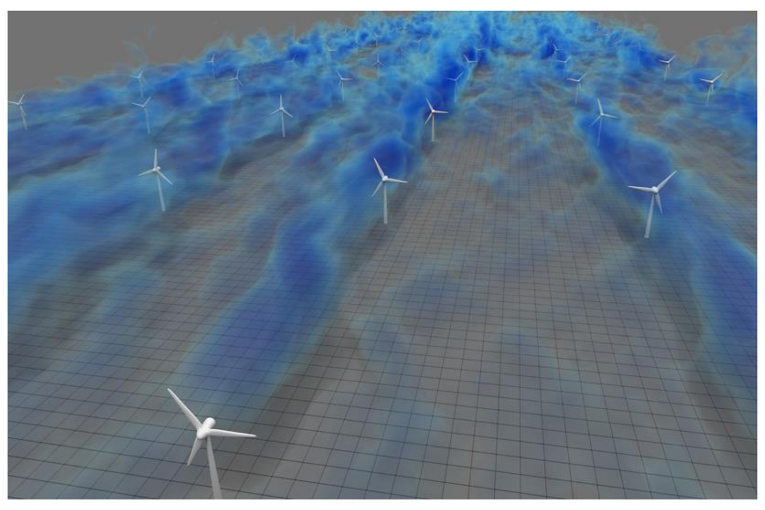
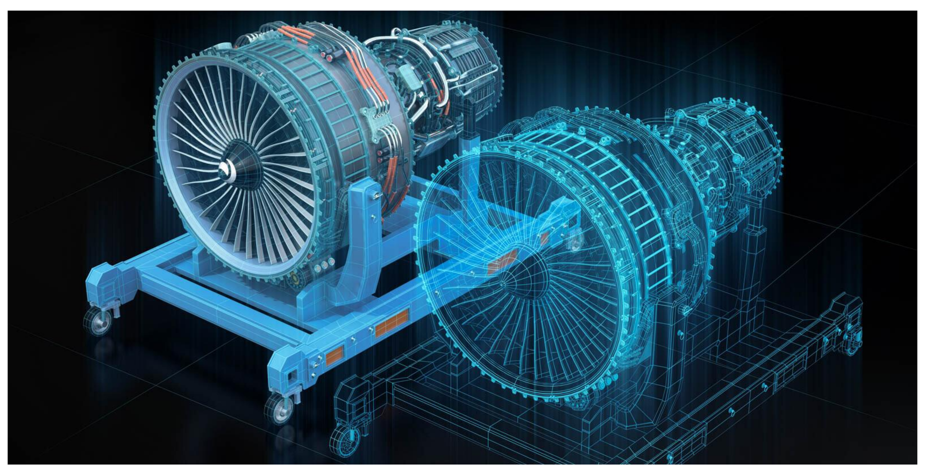
Mathematics of Imaging & AI

What we want

Many problems require computationally expensive problem solvers:







To remedy this, we want

1) to find a **small** set of representative variables
2) that **accurately** describes the full solution
3) and allows for **fast** computing of the full solution.

The optimisation problem

We look for the best parameters θ for an autoencoder $\phi_\theta = \phi_{\theta,dec} \circ \phi_{\theta,enc}$ using

$$\theta^\dagger \in \arg\min_\theta \underbrace{\sum_i \|x_i - \phi_\theta(x_i)\|_2^2}_{\text{Accurate}} + \lambda\Big(\underbrace{\sum_{\substack{\ell=1 \\ \ell \neq L_{enc}}}^{L} \|W^\ell\|_{1,2}}_{\text{fast}} + \underbrace{\|W^{L_{enc}}\|_*}_{\text{small}} \Big)$$

We solve this using Linearized Bregman iterations

$$v^{(k+1)} = v^{(k)} - \eta\, \nabla \mathrm{L}(\theta^{(k)})$$
$$\theta^{(k+1)} = prox_R(v^{(k+1)})$$

where

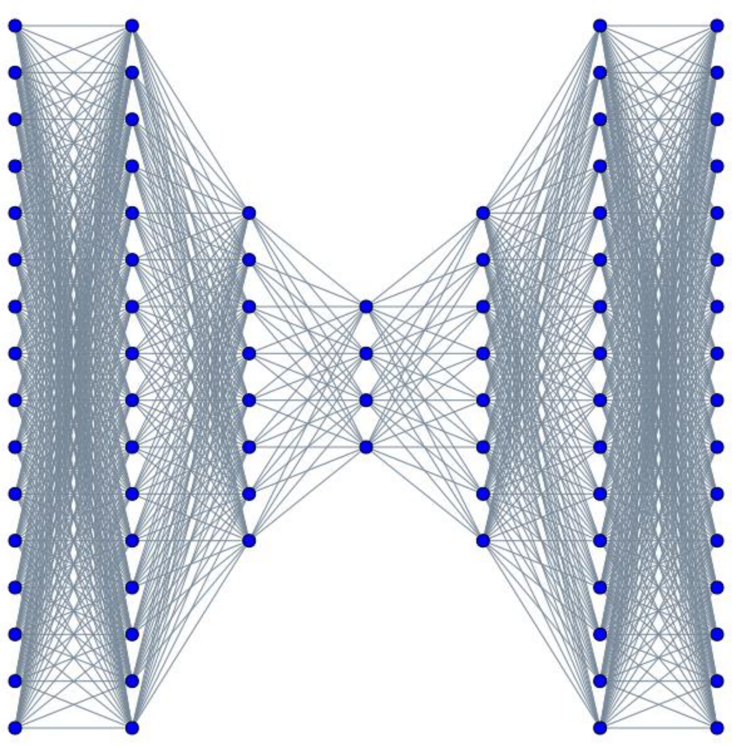
$$prox_R(v) = \arg\min_\theta \frac{1}{2}\|v - \theta\|_2^2 + R(\theta)$$

$$L(\theta) = \sum_i \|x_i - \phi_\theta(x_i)\|_2^2$$

Last layer of the encoder

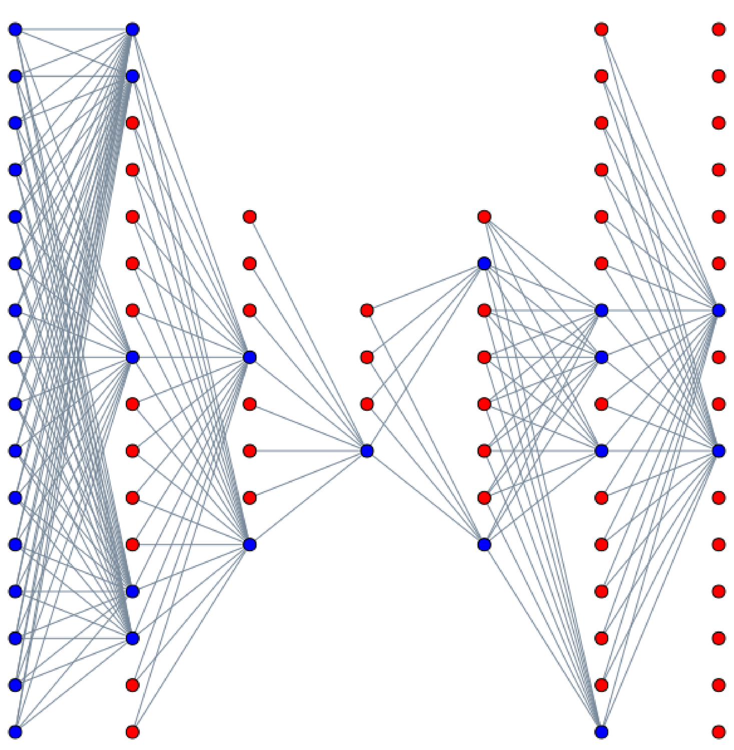
$$\mathrm{R}(\theta) = \lambda\Big(\sum_{\ell=1, \ell \neq L_{enc}}^{L} \|W^\ell\|_{1,2} + \|W^{L_{enc}}\|_* \Big)$$

Similar accuracy
Sparser network and smaller latent dimensionaility



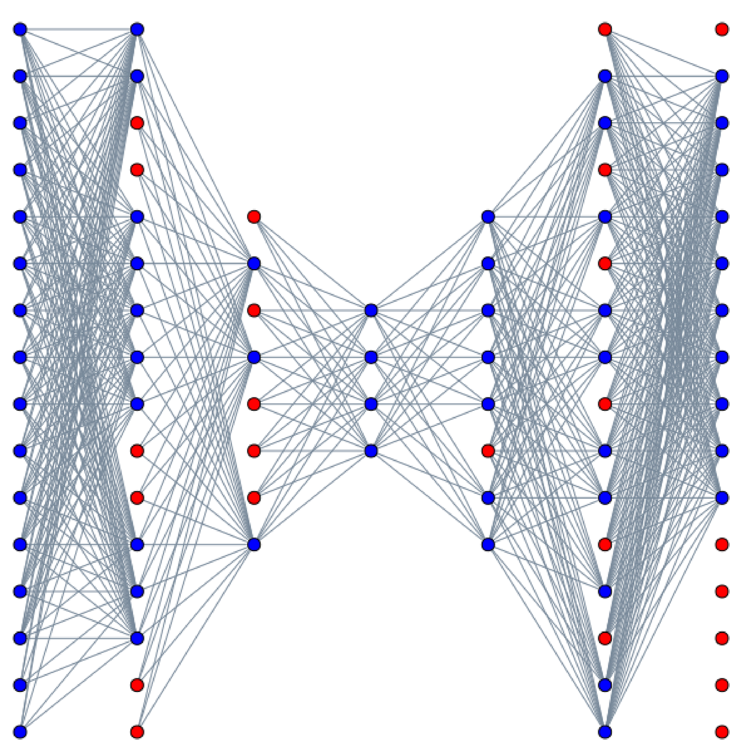
Sparsification

We make the matrices row-sparse.



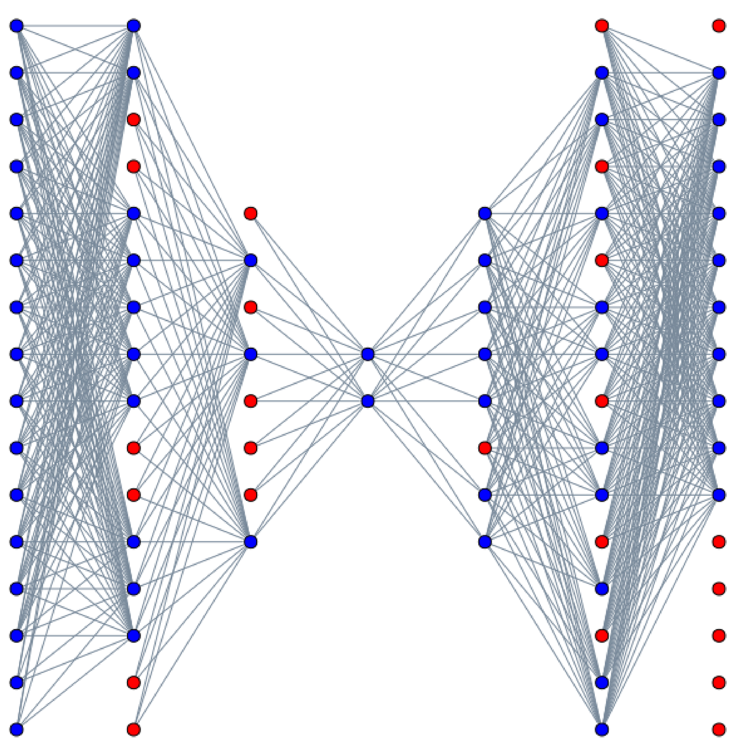
Training

We train the network using Linearized Bregman.



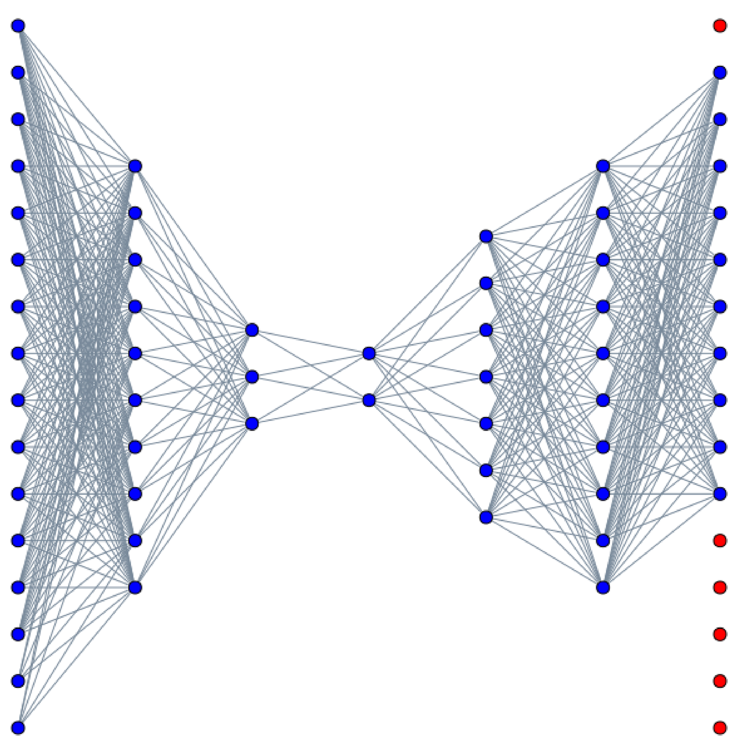
Latent POD

We use POD to truncate the number of latent variables.



Bias propagation[4]

We move all the biases corresponding to matrices with zero-rows to the next layer.



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2. https://www.aip.org/publishing/journal-highlights/wind-energy-grid-checkerboard
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